# A new Hybrid Algorithm for Simultaneous Size and Semi-rigid Connection Type Optimization of Steel Frames

Ali Hadidi and Amin Rafiee\*

 $B$  Civil Engineering, Tabri $B$ , Tabri $\alpha$ , Tabri $\alpha$ , Tabriz, Tabriz, Tabriz, Tabriz, Tabriz, Tabriz, Iran, Iran

#### Abstract

A hybrid algorithm based on Harmony Search (HS) and Big Bang-Big Crunch (BB-BC) optimization methods is proposed for optimal design of semi-rigid steel frames. The algorithm selects suitable sections for beams and columns and assigns suitable semi-rigid connection types for beam-to-column connections, such that the total member plus connection cost of the frame, is minimized. Stress and displacement constraints of AISC-LRFD code together with the size constraints are imposed on the frame in the design procedure. The nonlinear moment-rotation behavior of connections and P-∆ effects of beam-column members are taken into account in the non-linear structural analysis. Three benchmark steel frames are designed and the results are compared with those of standard BB-BC and of other studies. The comparisons demonstrate that proposed algorithm performs better than standard BB-BC and HS methods in all examples and that the total cost of a frame can be reduced through suitable selection of its beam-to-column connection types.

Keywords: Big Bang-Big Crunch, Harmony Search, hybrid optimization algorithm, size optimization, steel frames, semi-rigid connections

# 1. Introduction

It is obvious that the actual complex behavior of a structure has to be simplified for analysis by feasible modeling. To yield to this aim, in the analysis and design of steel framed building structures, it is convenient to use one of the two extreme behaviors of either fully rigid or perfectly pinned behavior to model the beam-to-column connections. This idealization simplifies the analysis; however, such a modeling cannot provide a realistic prediction of response of the frame. This is because, these connections possess some flexural stiffness between two extremes, i.e. are semi-rigid connections. This semi-rigid behavior is also nonlinear in nature and therefore it is sound that the designers make the use of suitable relationships to take into account the effects of the actual behavior of the beam-to-column connections on the response of a frame, in its analysis.

The moment-rotation behavior of beam-to-column connections in steel frames strongly influences their stability and strength; hence, it is necessary to accurately

Received September 16, 2013; accepted September 23, 2014; published online March 31, 2015 © KSSC and Springer 2015

\*Corresponding author Tel: +00989381723903, Fax: +00984113344287 E-mail: a.rafiee@tabrizu.ac.ir

model the stiffness, strength and ductility of connection. To this aim, numerous studies have been conducted (Frye and Morris, 1975; Abdalla and Chen, 1995; Chisala, 1999; Kim et al., 2010; Wu et al., 2012). In addition, analysis and design of steel frames with semi-rigid connections have been extensively investigated (Bayo et al., 2006; Kaveh and Moez, 2008; Chiorean, 2009; Ihaddoudène et al., 2009; Valipour and Bradford, 2013; Nguyen and Kim, 2013). As a convenient activity of research in the field of structural optimization, optimal design of steel frames with semi-rigid connections has also been investigated by means of mathematical programming techniques (Alsalloum and Almusallam, 1995; Simoes, 1996) and of meta-heuristics (Kameshki and Saka, 2003; Hayalioglu and Degertekin, 2005, 2010; Rafiee et al., 2013).

Current steel specifications such as British Standard, BS5950 (1990), Eurocode3 (1992) and American Institute of steel construction (AISC) have investigated the semirigid behavior of beam-to-column connections. AISC-ASD specification (1989) describes three types of steel constructions: rigid, simple (unrestrained) and semi-rigid (partially restrained) framing, whereas, in AISC-LRFD (2001) two types of steel construction namely FR (fully restrained) and PR (partially restrained) types are described. The behavior of the construction type PR, which is considered to be semi-rigid, is described on the basis of experimental and numerical studies.

A number of efficient meta-heuristic optimization algorithms, have been applied to optimal design of structures. One of the well-known, recently proposed optimization algorithms, which has already received a lot of attention, is the Big Bang-Big Crunch (BB-BC), optimization method (Erol and Eksin, 2006). The BB-BC relies on one of the theories of the evolution of the universe namely, the Big Bang and Big Crunch theory. In the Big Bang phase of this theory, energy dissipation produces disorder and randomness is the main feature of this phase; whereas, in the Big Crunch phase, randomly distributed points (masses) are drawn into an order. The BB-BC optimization method similarly generates random points in the Big Bang phase and shrinks them to a single representative point via a center of mass in the Big Crunch phase. After a number of successive Big Bangs and Big Crunches, where the distribution of randomness within the search space during the Big Bang becomes smaller and smaller about the average point computed during the Big Crunch, the algorithm converges to a socalled optimal solution.

BB-BC algorithm has been used to solve various engineering optimization problems and was proved to be of high computation efficiency, easy implementation and stable convergence. Different types of structural optimization examples, including optimal design of space trusses, domes and planar rigidly-connected steel frames have been solved using this method by Kaveh and Talatahari (2009, 2010a, 2010b). Parameter estimation in structural systems using BB-BC is performed by Tang et al. (2010). Later, Afshar and Motaei (2011) used BB-BC to determine the optimal solution of reservoir operation problems. Camp and Assadollahi (2013) used a hybrid BB-BC algorithm for design of reinforced concrete footings. Recently, Rafiee et al. (2013) developed an algorithm on the basis of BB-BC for optimal design of semi-rigidly connected steel frames.

Another famous optimization algorithm, namely, harmony search (HS) was proposed by Geem et al. in 2001, inspiring the performance process of natural music. In HS a memory of best solutions, called Harmony Memory (HM), is created and updated successively during optimization. The search strategy is such that, a number of design variables are selected randomly among allowable values, whereas, the others are chosen from HM. Moreover, for some of those chosen from HM, a pitch adjusting decision is applied. The rates of HM consideration and pitch adjusting determine that which of design variables should be chosen from HM or be pitching adjusted.

The use of HS in searching for solutions to various optimization problems has been resulted in effective results (Lee and Geem, 2005; Geem, 2007; Cheng et al., 2008; Mun and Geem, 2009). Together with these studies, HS has also been utilized to optimize the design of structures in a number of researches and the results demonstrated its robustness. Among these work, those in which the main purpose is to minimize the weight of the structure can be summarized as follows: Degertekin, 2008; Saka, 2009; Saka and Erdal, 2009. In addition, Degertekin and Hayalioglu (2010) studied the minimum cost design of steel frames by developing an algorithm on the basis of harmony search. Recently, Hadidi and Rafiee (2014) proposed a HS-based PSO for design of semi-rigid steel frames.

In spite of its simplicity, high speed of convergence and good exploitation, in standard BB-BC algorithm, in each generation only the center of mass of previous iteration is used in computations as previous knowledge, this reduces the exploration ability of BB-BC to find global optimum. On the other hand, although in a standard HS algorithm, a memory of best solutions is used as previous knowledge, this memory is constructed in a random manner with no proper strategy to move the individuals toward the feasible domain. This paper proposes a new optimization algorithm, by hybridizing the BB-BC and HS approaches to give HS-BB-BC algorithm. In HS-BB-BC, the harmony memory (HM) is created and improved by making use of BB-BC. On the other hand, the off-springs generated by BB-BC are modified using HM as previous knowledge.

A computer code is developed based on HS-BB-BC, for optimal sizing design of steel frames, wherein, the optimal arrangement of semi-rigid beam-to-column connection types is taken into account, as well. The developed code selects suitable sections for beams and columns from American Institute of Steel Construction (AISC) wide-flange W-shape sections and assigns suitable semi-rigid connection types for beam-to-column connections, such that the total cost of the frame, is minimized. The total cost includes member and connection cost values. Stress and displacement constraints of AISC-LRFD code together with the size constraints are imposed on the frame in design procedure as problem constraints.

The P-∆ effects of beam-column members are taken into account in the non-linear structural analysis, while, the non-linear moment-rotation behavior of semi-rigid beam-to-column connections is modeled using Frye-Morris odd-power polynomial (Frye and Morris, 1975). Three benchmark design examples are solved and the results are compared with those of standard BB-BC, HS and with those reported in literature. The comparisons demonstrate that HS-BB-BC performs better than standard BB-BC and HS methods in all examples and that the total cost of a frame can be reduced through suitable selection of its beam-to-column connection types.

# 2. Optimization Algorithm: Hybrid HS and BB-BC

In this Section the proposed algorithm is explained. More details about HS and BB-BC algorithms can best be found in Lee and Geem (2005) and Erol and Eksin (2006), respectively. The hybrid algorithm, proposed herein, is

called HS-BB-BC algorithm. In HS-BB-BC the Big Bang-Big Crunch method is used to create and improve harmony memory (HM). On the other hand, the new offsprings generated by BB-BC, which can be considered as new improvised harmonies, are improved through the concepts used in harmony search. In better words, HS-BB-BC is a HS algorithm in which the random generation rule is removed and instead the BB-BC is applied, and at the same time, is a BB-BC approach in which the new positions of individuals are modified by making use of HM consideration rate and pitch adjusting rules.

Indeed, in a standard BB-BC algorithm, in each

generation only the center of mass of previous iteration is used in computations as previous knowledge. To overcome this drawback by increasing the cooperation component of the search algorithm and to use more previous knowledge in optimization process, in HS-BB-BC algorithm, a memory of best individuals (harmonies) is considered. On the other hand, in a standard HS algorithm, harmony memory is constructed in a random manner with no proper strategy to move the individuals toward the feasible domain. It seems that, the big bang and big crunch concepts may be proper strategies, which can result in a better memory of harmonies within a relatively small



Figure 1. Flow chart of the HS-BB-BC algorithm.

number of iterations.

In a brief statement, that is to say, in HS-BB-BC we have a population of harmonies moving toward better solutions by adopting the strategy of finding optimal solutions from BB-BC, together with the scheme of considering a harmony memory and pitch adjusting from HS. The proposed algorithm possesses the merits of both the HS and BB-BC methods, excluding their drawbacks.

The HS-BB-BC algorithm can be explained by the flowchart of Fig. 1. This figure is self-explanatory; however, some points should be noted; (a) m is population size, "Mer<sup>"</sup> is the "Merit function" and  $x_j^{(k,j)}$  is the j-th component of the i-th individual generated in the kth iteration; (b) HMS, HMCR, and PAR are harmony memory size, harmony memory consideration rate and pitch adjusting rate, respectively; (c) In hybrid HS-BB-BC algorithm  $HMS\leq m$  must be satisfied; (d) The random number  $r_i$  used for constructing new solutions is generated with a normal distribution and is independent from those needed for HS-based modification for off-springs; (e)  $\alpha_1$ is a parameter for tuning the standard deviation used in generation of  $r_i$ ; and (f) The random numbers  $r_1$  and  $r_2$ used for HS-based modification for off-springs are generated for each component of each particle independently.

# 3. Problem Formulation of Optimal Sizing Design of Semi-rigid Steel Frames by Taking the Optimal Arrangement of Connections into Account

# 3.1. Design variables

The goal of the optimization problem of this study is to minimize the total cost of steel frame design, subject to the constraints imposed on the frame. Here, the problem is to select suitable sections for beams and columns. At the same time, the suitable selection of semi-rigid beamto-column connection types should be done. Consequently, two groups of design variables are member sections and beam-to-column connection types. The first group is identified by their cross-sectional areas, whereas, the latter one is characterised by rotational stiffness values  $S_i$ .

# 3.2. Total cost of a steel frame with semi-rigid connections

The total cost of a steel frame with semi-rigid beam-tocolumn connections, considering member and connection costs, is defined by Xu and Grierson (1993) as follows

$$
Z(\mathbf{x}) = \sum_{i=1}^{NM} W_i A_i + \sum_{i=1}^{NB} \sum_{i=1}^{2} (\beta_{ij} R_{ij} + \beta_{ij}^0)
$$
(1)

where  $A_i$  and  $W_i$  are the *i*-th member cross-sectional area and weight coefficient, respectively  $(W_i=$ material density  $\times$ member length),  $R_{ii}$  and  $\beta_{ii}$  are the connection rotational stiffness and cost coefficient, and  $\beta_{ij}^0$  is the cost of a pinned connection having zero rotational stiffness. The jsubscripts in Eq. (1) correspond to two ends of the semirigid beam member and NM and NB denote the total number of members and beams in a frame, respectively.

The values of  $\beta_{ij}$  for two ends of a semi-rigid beam member are assumed to be equal and calculated as

$$
\beta_i = \frac{0.225 W_i A_i}{S_i} \tag{2}
$$

where  $S_i$  is rotational stiffness of a connection which is a estimated value depending on the stiffness of the connection, equal for the both ends of a beam and lies in the range  $2.26 \times 10^5$  kN·mm/rad to  $5.65 \times 10^8$  kN·mm/rad as it is suggested by Xu and Grierson (1993) and the equal value for  $\beta_{i1}^0$  and  $\beta_{i2}^0$  are accepted to be equal to

$$
\beta_i^0 = 0.125 W_i A_i \tag{3}
$$

## 3.3. Constraints

As it is usually involved in an optimization problem, some constraints should be imposed on the problem during the optimization procedure, which divide the search space into feasible and infeasible domains. The optimal design problem of this study has the following constraints:

(a) The strength constraints of AISC-LRFD (2001) considering the interaction of bending moment and axial force can be formulated in the normalized form, for i-th member of the frame, as follows

$$
V_i^{IER} = \begin{cases} \left(\frac{P_u}{\phi P_n}\right) + \frac{8}{9} \left(\frac{M_{ux}}{\phi_b M_{nx}}\right) - 1.0 & \frac{P_u}{\phi P_n} \ge 0.2\\ \frac{1}{2} \left(\frac{P_u}{\phi P_n}\right) + \left(\frac{M_{ux}}{\phi_b M_{nx}}\right) - 1.0 & \frac{P_u}{\phi P_n} < 0.2 \end{cases}
$$
(4)

where  $P_u$  and  $P_n$  are required and nominal strength of a member (tensile or compressive), respectively and  $\phi$  is resistance reduction factor, which is equal to 0.9 for the member in tension and 0.85 for compressive ones. Moreover,  $M_{ux}$  and  $M_{nx}$  are notations for required and nominal flexural strength of the member about its major axis, respectively and reduction factor that corresponds to bending is denoted by  $\phi_b$  (equal to 0.9). The nominal strength of a compressive member is calculated based on AISC-LRFD (2001) as follows

$$
P_n = AF_{cr} \tag{5}
$$

$$
F_{cr} = \begin{cases} 0.658^{\lambda_c^2} F_y & \lambda_c \le 1.5\\ \frac{0.877}{\lambda_c^2} F_y & \lambda_c > 1.5 \end{cases}
$$
 (6)

$$
\lambda_c = \frac{KL}{r\pi} \sqrt{\frac{F_y}{E}}\tag{7}
$$

where A is cross-sectional area;  $F_v$  is yield stress; and E is modulus of elasticity of steel member.  $L$  and are  $r$  the member length and radius of gyration, respectively. The effective length factor, which is denoted by  $K$  in Eq. (7), is needed in stability evaluation of the columns in the frame. K-factor of columns in an unbraced semi-rigid frame is calculated following the relations proposed by Kishi et al. (1997).

(b) The displacement normalized constraints including the constraints of inter-storey drift and top storey sway can be formulated in general form of

$$
V_j^d = \frac{|\delta_j|}{\delta_j^u} - 1.0 \,, \quad j = 1, \ 2, \ \dots \ g \tag{8}
$$

where  $\delta_i$  is the displacement of the *j*-th restricted

displacements among the total number of g and  $\delta_j^u$  is its allowable upper bound limit determined by the code of practice. In this study, the inter-storey drift and top storey sway values are restricted to storey height/300 and 0.0052 ×total height of frame, respectively.

(c) The other group of constraints imposed on the optimization problem in this study arises from the size adaptations of beams and columns relative to each other. This group consists of two constructional considerations: one consideration implies that flange width of a beam must be smaller than the same value for column in all joints, whereas, the other one considers the fact that the column of each storey cannot be smaller in depth compared to its above storey column. These two constraints can be formulated, respectively, as



Figure 2. Semi-rigid beam-to-column connection types.

$$
V_p = \frac{b_f^{bp}}{b_f^{op}} - 1.0 \,, \quad p=1, \ 2, \ \dots \ nj \tag{9}
$$

$$
V_q = \frac{d_c^{uq}}{d_c^{lq}} - 1.0 \,, \quad q=1, \ 2, \ \dots \ nc \tag{10}
$$

where  $b_f^{bp}$  and  $b_f^{cp}$  are the value of flange width for beam and column in node number  $p$  among the total number of  $nj$  nodes, respectively  $(nj)$  is the total number of nodes of frame except the supports). The  $d_c^{uq}$  and  $d_c^{lq}$  are notations for depths of column sections of upper and lower floor in a node, respectively. nc is the total number of columns in the frame excluding ones for first storey.

## 3.4. Penalization

The optimum design problem, considered in the present work, is a constrained problem; we can transform it into an unconstrained one using a penalty function. Here, we use the penalty function suggested by Rajeev and Krishnamoorthy (1992), so the objective function of the problem can be computed as

$$
\varphi(\mathbf{x}) = Z(\mathbf{x}) \left[ 1 + C \left( \sum_{i=1}^{NM} v_i^{IER} + \sum_{j=1}^{g} v_j^d + \sum_{p=1}^{nj} v_p + \sum_{q=1}^{nc} v_q \right) \right] (11)
$$

where  $Z(x)$  is calculated by Eq. (1); C is a penalty constant, which is equal to 10 in this work;  $v_i^{IER}$ ,  $v_j^d$ ,  $v_p$ , and  $v<sub>a</sub>$  are the violations of normalized interaction equation ratio, displacement, and size considerations for beams and columns, respectively and are computed using Eq. (12).

$$
\{v_i^{IER}, v_j^d, v_p, v_q\} = \max(0, \{V_i^{IER}, V_j^d, V_p, V_q\})
$$
 (12)

#### 3.5. Termination criteria

In this work, two termination criteria are used to stop

the optimal design process. The first criterion stops the algorithm when a predetermined number of iterations (generations) are performed, whereas, the second one terminates the process before reaching the maximum iteration number, if lighter frame is not found during a specified number of successive generations. If one of these criteria is satisfied, the algorithm is terminated and the so-called optimal solution is printed.

# 4. Nonlinear Analysis of Steel Frames with Semi-rigid Beam-to-column Connections

In a structural optimization problem, each structural design (individual) is evaluated through its analysis, which leads to structural response and makes it possible to evaluate the penalty function. On the other hand, it is obvious that the actual complex behavior of a structure must be simplified for analysis by feasible modeling of it. Among the numerous experimental and numerical studies on the modeling of semi-rigid beam-to-column connections, the model proposed by Frye and Morris (1975) is adopted for use in this work, due to its easy-to-implement characteristic. This odd-power polynomial model is reasonably good for simulation of the nonlinear  $M-\theta$ behavior of connections and has been presented as

$$
\theta = c_1(\kappa M) + c_2(\kappa M)^3 + c_3(\kappa M)^5 \tag{13}
$$

where  $\theta$  is the connection rotation and M denotes the moment acting on the connection. The parameter  $\kappa$  is the standardization factor determined by the connection type and geometry, and  $c_1$ ,  $c_2$ , and  $c_3$  are curve-fitting constants obtained by using the method of least squares. For several types of beam-to-column connections, which are shown in Fig. 2, the values of the constants  $c_1$ ,  $c_2$ , and  $c_3$  and the parameter  $\kappa$  for each type, are illustrated in Table 1 (Faella *et al.*, 2000). The schematic  $M-\theta$  curves for these

Table 1. The Curve fitting constants and standardization parameters for Frye-Morris polynomial model

Connection		Curve fitting constants	Standardization parameter, $(\kappa)$	
type	$c_1$	c <sub>2</sub>	$c_3$	
1	$4.28 \times 10^{-3}$	$1.45\times10^{-9}$	$1.51\times10^{-16}$	$\kappa = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
$\overline{2}$	$3.66 \times 10^{-4}$	$1.15\times10^{-6}$	$4.57\times10^{-8}$	$\kappa = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
3	$2.23\times10^{-5}$	$1.85 \times 10^{-8}$	$3.19\times10^{-12}$	$\kappa = d^{-1.287} t^{-1.128} t_c^{-0.415} l_a^{-0.694} g^{1.35}$
4	$8.46\times10^{-4}$	$1.01\times10^{-4}$	$1.24\times10^{-8}$	$\kappa = d^{-1.5} t^{-0.5} \bar{l}_a^{-0.7} d_h^{-1.5}$
5	$1.83\times10^{-3}$	$1.04\times10^{-4}$	$6.38\times10^{-6}$	$\kappa = d_g^{-2.4} t_b^{-0.4} d_h^{-1.5}$
6	$1.79\times10^{-3}$	$1.76\times10^{-4}$	$2.04\times10^{-4}$	$\kappa = d_{\varphi}^{-2.4} t_n^{-0.6}$
7	$2.10\times10^{-4}$	$6.20\times10^{-6}$	$-7.60\times10^{-9}$	$\kappa = d^{-1.5} t^{-0.5} l_t^{-0.7} d_h^{-1.1}$
8	$5.10\times10^{-5}$	$6.20\times10^{-10}$	$2.40\times10^{-13}$	$\kappa = d_n^{-2.3} t_n^{-1.6} t_w^{-0.5} g^{1.6}$



Figure 3. Moment-Rotation curves of semi-rigid connection types.



Figure 4. Secant stiffness values of load increments.

eight types of connections are drawn in Fig. 3 according to Chen et al. (1996).

In this study, the displacements method is used to analyze the structure, wherein, the stiffness matrix of the structure is constructed through assembling of the stiffness matrices of members in the global coordinates. In order to consider the P-∆ effects into account in the analyses of frames, an incremental approach is applied, such that, in each increment the stiffness matrices are updated using most recently computed axial force values for beamcolumn elements, in an iterative procedure until the convergence is achieved. Moreover, the secant stiffness approach is applied to consider the semi-rigid connection stiffness nonlinearity of beam members. The connection secant stiffness values corresponding to all load increments are shown in Fig. 4. In each set of iterations convergence criterion is controlled by comparing of the difference between end forces of members with applied incremental loads so that to be smaller than a determined tolerance. A convergent solution of a load increment forms an initial estimate for the next iteration, and the iterative process continues until all load increments are considered. The solutions for all load increments are accumulated to obtain a total nonlinear response.

Table 2. The fixed connection size parameters and adopted rotational stiffness values

Connection type	Fixed connection size parameters (cm)	values in Eq. $(2)$ (kN·mm/rad)
1	$t_a = 2.54$ , $g = 11.43$	$85\times10^{6}$
2	$t_a = 2.858$ , $g = 25.4$	$113\times10^{6}$
3	$t=2.54$ , $t_c=2.54$ , $g=11.43$	$282\times10^{6}$
4	$t = 2.54$ , $d_h = 2.858$	$226\times10^{6}$
5	$t_p = 2.54$ , $d_b = 2.858$	$339\times10^{6}$
6	$t_n = 2.54$	$395\times10^{6}$
7	$t = 3.81, d_h = 2.858$	$452\times10^{6}$
8	$t_p = 2.54$ , $g = 25.4$	$141\times10^{6}$

# 5. Numerical Examples

To demonstrate the efficiency of the proposed algorithm and to find the optimal arrangement of semi-rigid beamto-column connection types, three steel frames including a 9-storey (as a small size frame), a 10-storey (as a median one) and a 24-storey frame (as a large scale frame), are solved. These examples have been solved by Rafiee and coworkers (2013, 2014) using BB-BC, PSO and HS-PSO algorithms, wherein, the connection types of the frame are the same in all joints. In these examples the A36 steel grade is used for all of the members and the sections for these members are selected among a total number of 273 standard sections of American Institute of Steel Construction wide flange W shapes. In addition, the connection types are chosen among the eight types shown in Fig. 2.

Maximum number of iterations for first two examples is 600, however, if after 300 iterations the best design is not improved during ten successive generations the algorithm is terminated. These numbers for third example are 300 and 200, respectively. Selection of these values is judgmental and depends on experience, population size and size of search space. In spite of these termination criteria, in Fig. 8 the convergence histories are plotted up

Table 3. The optimal results of nine-storey frame for  $HMS=m/7$ 

Total cost (kg)		<b>PAR</b>						
		0.3	0.4	0.5				
	0.3	15,045	15,642	15,494				
<b>HMCR</b>	0.5	14,789*	14,973	15,470				
	0.7	14,908	14,987	15,001				

\*In the case of  $HMS=(2/7)m$  this value is equal to 14.610 kg.

to the maximum number of iterations. By doing so, the robustness of proposed algorithm is demonstrated when the number of frame analyses remains unchanged for HS, BB-BC and HS- BB-BC.

In this study, to simplify the problem, some of the connection size parameter values required in Frye-Morris polynomial model of  $M-\theta$  curve is considered to be fixed during the optimum design procedure. Following Rafiee et al. (2013), these fixed values are selected according to Table 2, whereas, the values of angle length, beam height, the vertical distance between bolt groups, web thickness of beam are calculated based on dimensions of W-shape section assigned to the beam member throughout the optimal design procedure. During design process, the bolt numbers and diameters will be computed according to AISC-LRFD (2001) for bending moment and shear considering grade 8.8 of ordinary bolts, except for connection type 4, 5 and 7 where the diameter is given in Table 2. Moreover, the size of end plate and welding detail in connection type 5, 6 and 8 are calculated based on the W-shape section assigned to the corresponding beam throughout the optimization and shear and bending moment values. The last column of Table 2, gives the estimated rotational stiffness values,  $S_i$  for each type of semi-rigid connections. These are the case for all of the design examples.

On the other hand, as it is evident from Fig. 1, in the HS-BB-BC algorithm for a HMCR value of zero the algorithm is simplified to BB-BC, hence one can see the HMCR as a HSCR parameter, which determines the contribution of harmony search (HS) scheme to the proposed hybrid algorithm. Here, to determine the optimum PAR, HMS, and HMCR values, the first example is solved for several values of these parameters and the results are listed in Table 3. These results depicts that the optimum PAR, HMS, and HMCR values are equal to 0.3,  $(2/7)m$ and 0.5, respectively, where m is population size. These values are used for the remainder of the examples.

## 5.1. Nine-storey, single-bay frame

The geometry, member grouping and the service loading conditions for the nine-storey, one-bay frame are illustrated in Fig. 5. The applied loads W,  $W_1$ , and  $W_2$  are equal to 17.8 kN, 27.14 kN/m, and 24.51 kN/m, respectively. In order to impose the fabrication conditions on the construction



Figure 5. Nine-storey, single-bay frame.

of the frame, the 27 members of this frame are separated to seven groups of members. The global sway corresponding to the roof level is limited to a maximum value of 154 mm. In this frame the beam-to-column connections are grouped, as well. This grouping is such that the connections of each storey level to be of one type, i.e. nine connection groups are defined.

Table 4 presents the optimal designs developed by the HS-BB-BC algorithm for this frame. Figure 8 shows the convergence history for the optimum design of this frame. It is clear from this figure that the proposed algorithm performs better than standard BB-BC method. Moreover, the numerical results presented in Table 5 show that for this frame 15 and 11% reduction in cost is obtained if one uses HS-BB-BC instead of BB-BC and HS, respectively. Furthermore, the use of various connection types in different stories of the frame is accompanied by the reduction of cost. The results implies that taking the optimal arrangement of beam-to-column connections into account in the optimization process makes the search space bigger and the standard BB-BC fails to find the optimal solution and the use of a robust algorithm is needed.

Member	9-storey frame				10-storey frame		24-storey frame		
group no.	W-shape sections	Story no.	connection types	W-shape sections	Story no.	connection types	W-shape sections	Story no.	connection types
1	$33\times118$	1	3	$27\times102$	1	4	$24\times55$	1,2	7
2	$24\times55$	2	7	$30\times124$	2	3	$12\times30$	3,4	7
3	$14\times34$	3	7	$24\times94$	3	3	$30\times90$	5,6	7
4	$21\times 44$	4	7	$21\times93$	$\overline{4}$	6	$21\times55$	7,8	7
5	$24\times55$	5	7	$21\times57$	5	5	14×233	9,10	
6	$18\times 46$	6	3	$21\times 48$	6	3	$14\times211$	11,12	7
7	$18\times 40$	7	3	$12\times72$	$\overline{7}$	3	$14\times159$	13,14	7
8		8	8	$16\times77$	8	3	$14\times145$	15,16	6
9		9	4	$21\times 44$	9	8	$14\times120$	17,18	3
10				$16\times 45$	10	3	$14\times61$	19,20	3
11				$21\times 44$			$14\times61$	21,22	4
12				$18\times 40$			$14\times 48$	23,24	$\overline{4}$
13							14×426		
14							$14\times370$		
15							14×342		
16							$14\times211$		
17							$14\times145$		
18							$14\times109$		
19							$14\times99$		
$20\,$							$14\times90$		

Table 4. The optimal designs of steel frames obtained using HS-BB-BC

Table 5. The optimal results for nine-storey, single-bay frame

			Semi-rigid connection types								
			2	3	4	5	6	7	8		
	T. c. $(kg)^*$	40,520	36,235	16,881	25,786	33,488	35,799	53,601	46,146		
<b>BB-BC</b> (Rafiee <i>et al.</i> , 2013)	W. $(kg)**$	38,718	32,617	14,809	23,956	30,804	33,481	43,450	44,527		
	T. s. (mm)	56	55 54 44 66 76 65 71 16,499 14,970 14,787 17,886 15,464 15,773 21,757 14,288 12,901 19,722 13,182 13,468 12,136 11,590 71 73 70 69 69 73 70 17,201 14,512								
HS-PSO	T. c. $(kg)$	21,486									
(Hadidi and Rafiee,	$W.$ (kg)	18,693									
2014)	T. s. (mm)	79									
	T. c. (kg)										
<b>BB-BC</b> (present work)	$W_{\cdot}$ (kg)										
	T. s. (mm)	71									
	T. c. $(kg)$		16,495								
<b>HS</b>	W. (kg)		13,960								
(present work)	T. s. (mm)		75								
	T. c. $(kg)$					14,610					
HS-BB-BC (present work)	$W_{\cdot}$ (kg)					12,218					
	T. s. (mm)					77					

\*T. c.=Total cost; W.=Weight; T. s.=Top-storey sway.

\*\*Steel weight per floor area=Weight/(9×(9.525×1))=W./85.725 (kg/m<sup>2</sup>).

# 5.2. Ten-storey, four-bay frame

The second design example is a 10-storey, 4-bay frame with 90 members. Figure 6 shows the twelve groups of members, acting loads and dimensions for this frame. The values of loads are: W=44.49 kN, W<sub>1</sub>=47.46 kN/m, W<sub>2</sub>=

42.91 kN/m. The values of top storey sway for this frame is restricted to 158 mm based on AISC-LRFD specifications. Like the previous example, in this frame the beam-tocolumn connections of each storey level are grouped to be of one type, *i.e.* ten connection groups are defined.



Figure 6. Ten-storey, four-bay frame.

Table 6. The optimal results for ten-storey, four-bay frame

			Semi-rigid connection types							
			2	3	4	5	6	7	8	
	T. c. $(kg)^*$	140,744	237,050	106,868	93,255	123,743	113,055	204,773	136,881	
<b>BB-BC</b> (Rafiee <i>et al.</i> , 2013)	W. $(kg)**$	128,418	195,578	100,254	87,432	111,865	103,357	150,274	126,120	
	T. s. (mm)	67	25	35	58	37	40	26	56	
HS-PSO	T. c. (kg)	58,939	55,118	46,328	47,788	46,407	46,469	47,328	53,489	
(Hadidi and Rafiee,	W. (kg)	52,196	43,746	40,040	41,853	38,532	37,950	38,737	47,018	
2014)	T. s. (mm)	76	62	58	68	63	48	49	75	
	T. c. (kg)	120,891								
<b>BB-BC</b>	W. (kg)	114,133								
(present work)	T. s. (mm)	41								
	T. c. $(kg)$	60,691								
<b>HS</b>	W. (kg)	50,772								
(present work)	T. s. (mm)					55				
	T. c. (kg)					44,343				
HS-BB-BC	W. (kg)					38,115				
(present work)	T. s. (mm)		68							

\*T. c.=Total cost; W.=Weight; T. s.=Top-storey sway. \*\*Steel weight per floor area=Weight (10×(4×7.112×1))=W./284.48 (kg/m<sup>2</sup> ).



Figure 7. Twenty four-storey, three-bay frame.

The optimum design procedure for ten-storey frame results in the results listed in Table 4. The convergence history for the optimum design of this frame is also shown in Fig. 8. The minimum cost results obtained for this frame are presented in Table 6. These results demonstrate that HS-BB-BC leads to 63 and 27% low cost frame compared to BB-BC and HS, respectively. Moreover, the use of various connection types in different stories of the frame reduces its cost, as it is the case for the first example.

#### 5.3. Twenty four-storey, three-bay frame

The topology, service loading conditions, four beam groups and sixteen column groups of 24-storey, 3-bay frame consisting of a total number of 168 members are shown in Fig. 9. Applied loads including point (W) and uniformly distributed  $(W_1$  through  $W_4$ ) loads have the values of W=25.628 kN, W<sub>1</sub>=4.378 kN/m, W<sub>2</sub>=6.362 kN/ m,  $W_3=6.917$  kN/m and  $W_4=5.954$  kN/m. Starting from the foundation, the beam-to-column connections of every three consecutive stories are combined into a group (a total number of 12 connection groups are defined).

In this frame, each of the four beam element groups may choose from all 273 W-shapes, while the 16 column element groups are limited to W14 sections. AISC-LRFD limits the top storey sway of this frame to a maximum value of 456 mm. Tables 4 and 7 show the optimal designs obtained using HS-BB-BC algorithm and minimum cost values for this frame, respectively. The comparison of results demonstrates that the proposed algorithm performs better than BB-BC (42% reduction in cost for 24-storey frame). The bolded values in Table 7 correspond to infeasible designs. The convergence history of optimal design procedure of this frame is also shown in Fig. 8.

To investigate the optimal stiffness distribution of the connections over the height of the frame, Fig. 9 provides an illustration. In this figure the rotational stiffness values of connections of different stories of frames are plotted versus the normalized frame height. The figure implies that the use of various connection types in different stories of the frame reduces its cost. As it is clear from this figure and Table 4, for first and third examples connection type 7 is the best connection for most of the stories, however, if we set all the connection types to be of type 7 meanwhile the member sections remain unchanged the costs will increase. Analogously, for most of stories of second example best choice is type 3, but if one sets all the connection types to be of this type, unchanging the member sections, not only the cost will increase but also the design will not be feasible.

## 6. Conclusions

Harmony search (HS) and Big Bang-Big Crunch (BB-BC) are the well-known meta-heuristic optimization algorithms. In this paper a hybrid HS and BB-BC algorithm, called HS-BB-BC is developed and a discrete algorithm based on HS-BB-BC presents for optimal size and connection arrangement design of steel frames. The algorithm finds the member cross-sections and semi-rigid connection types so that the total member plus connection cost is minimized. American Institute of Steel Construction (AISC) wide-flange (W) shape standard steel sections are

		Semi-rigid connection types								
			$\overline{2}$	3	4	5	6	$\tau$	8	
	T. c. $(kg)^*$	502,197	202,737	267,414	249,806	171,868	176,864	385,074	383,738	
<b>BB-BC</b> (Rafiee <i>et al.</i> , 2013)	$W.$ (kg)	381,754	139,161	236,249	211,149	140,536	150,362	359,372	297,834	
	T. s. (mm)	204	245	170	184	237	231	240	190	
HS-PSO	T. c. $(kg)$	505,366	189,791	205,473	210,296	162,582	165,828	156,161	341,798	
(Hadidi and Rafiee,	$W_{\cdot}$ (kg)	384,890	135,368	172,004	175,521	133,930	137,054	125,589	261,722	
2014)	T. s. (mm)	200	245	194	208	238	217	221	203	
	T. c. (kg)	260,152								
<b>BB-BC</b>	$W.$ (kg)	238,721								
(present work)	T. s. (mm)	212								
	T. c. (kg)	289,580								
<b>HS</b> (present work)	W. (kg)	209,040								
	T. s. (mm)					174				
	T. c. $(kg)$					151,481				
HS-BB-BC	W. (kg)					132,313				
(present work)	T. s. (mm)	255								

Table 7. The optimal results for twenty four-storey, three-bay frame

\*T. c.=Total cost; W.=Weight; T. s.=Top-storey sway.

\*\*Steel weight per floor area=Weight  $(24\times((6.096+3.658+8.534)\times1))=W/438.912$  (kg/m<sup>2</sup>).



Figure 8. The convergence histories.

used. Stress and displacement constraints of AISC-Load and Resistance Factor Design (LRFD) specification are considered as the design constraints. Also, in order to find more practical design, size constraints for beams and columns adaptation are imposed on the frame in the optimal design procedure. The P-∆ effects of beam-column members and nonlinear moment-rotation behavior of semi-rigid connections are considered in the analyses.

Three benchmark design examples are investigated and the results of BB-BC, HS and HS-BB-BC are compared. The results are compared with those reported in literature, as well. The comparisons show that in all the examples the proposed algorithm gives the frames with lower cost in comparison with the standard BB-BC and HS methods. The convergence of the HS-BB-BC is better than that of BB-BC and HS, such that in the same number of frame analyses the HS-BB-BC reaches better solutions than the others, while the premature convergence is prevented. A HMCR value of 0.5 and PAR of 0.3 are found to contribute properly the harmony memory to the proposed hybrid algorithm. In addition the optimum ratio of HMS to population size is found to be 2 to 7, respectively.

The proposed algorithm has the advantages of both the BB-BC and HS eliminating their disadvantages. This is because, by moving the individuals toward better solutions the BB-BC can provide with a better memory of harmonies



Figure 9. Optimal connection stiffness distributions.

for HS, within a relatively small number of iterations, on the other hand, harmony memory of HS will provide a proper previous knowledge for BB-BC, improving its exploration. Furthermore, investigating the optimal stiffness distribution of the connections over the height of the frame implies that, blinking at the cost of fabrication difficulty on the diversity of connection type, the use of various connection types in different stories of the frame reduces its total cost. This may be on contrary to the intuition that the use of stiffer connections in an unbraced frame will necessarily reduce its total cost. After all, as it can be inferred from the results, T-stub can be chosen as an economical connection type for an unbraced steel frame if we do not have access to optimization tools.

## References

- Abdalla, K. M. and Chen, W. F. (1995). "Expanded database of semi-rigid steel connections." Computers & Structures, 56(4), pp. 553-564.
- Afshar, M. H. and Motaei, I. (2011). "Constrained big bangbig crunch algorithm for optimal solution of large scale reservoir operation problem." International Journal of Optimization in Civil Engineering, 1(2), pp. 357-375.
- AISC-ASD (1989). Manual of steel construction-Allowable Stress Design. American Institute of Steel Construction, Chicago.
- AISC-LRFD (2001). Manual of steel construction-Load and Resistance Factor Design. American Institute of Steel Construction, Chicago.
- Alsalloum, Y. A. and Almusallam, T. H. (1995). "Optimality and safety of rigidly-jointed and flexibly-jointed steel frames." Journal of Constructional Steel Research, 35(2), pp. 189-215.
- Bayo, E., Cabrero, J. M., and Gil, B. (2006). "An effective component-based method to model semi-rigid connections for the global analysis of steel and composite

structures." Engineering Structures, 28(1), pp. 97-108. BS5950 (1990). Structural use of steelworks in buildings. British Standards Institution, London.

- Camp, C. V. and Assadollahi, A. (2013). "CO<sub>2</sub> and cost optimization of reinforced concrete footings using a hybrid big bang-big crunch algorithm." Structural and Multidisciplinary Optimization, 48(2), pp. 411-426.
- Chen, W. F., Goto, Y., and Liew, J. Y. R. (1996). Stability design of semi-rigid frames. John Wiley & Sons Inc, New York, USA.
- Cheng, Y. M., Li, L., Lansivaara, T., Chi, S. C., and Sun, Y. J. (2008). "An improved harmony search minimization algorithm using different slip surface generation methods for slope stability analysis." Engineering Optimization, 40(2), pp. 95-115.
- Chiorean, C. G. (2009). "A computer method for nonlinear inelastic analysis of 3D semi-rigid steel frameworks." Engineering Structures, 31(12), pp. 3016-3033.
- Chisala, M. L. (1999). "Modeling M–è Curves for standard beam-to-column connections." Engineering Structures, 21(12), pp. 1066-1075.
- Degertekin, S. O. (2008). "Harmony search algorithm for optimum design of steel frame structures: a comparative study with other optimization methods." Structural Engineering and Mechanics, 29(4), pp. 391-410.
- Degertekin, S. O. and Hayalioglu, M. S. (2010). "Harmony search algorithm for minimum cost design of steel frames with semi-rigid connections and column bases." Structural and Multidisciplinary Optimization, 42(5), pp. 755-768.
- Erol, O. K. and Eksin, I. (2006). "A new optimization method: big bang-big crunch." Advances in Engineering Software, 37(2), pp. 106-111.
- Eurocode 3 (1992). Design of Steel Structures Part I: General rules and rules for buildings. Committee European de Normalisation (CEN), Brussels.
- Faella, C., Piluso, V., and Rizzano, G. (2000). Structural steel semi-rigid connections. CRC Press, Boca Raton.
- Frye, M. J. and Morris, G. A. (1975). "Analysis of flexibly connected steel frames." Canadian Journal of Civil Engineering, 2(3), pp. 280-291.
- Geem, Z. W. (2007). "Optimal scheduling of multiple dam system using harmony search algorithm." Lecture Notes in Computer Science, 4507, pp. 316-323.
- Geem, Z. W., Kim, J. H., and Loganathan, G. V. (2001). "A new heuristic optimization algorithm: harmony search." Simulation, 76(2), pp. 60-68.
- Hadidi, A. and Rafiee, A. (2014). "Harmony search based, improved particle swarm optimizer for minimum cost design of semi-rigid steel frames." Structural Engineering and Mechanics, 50(3), pp. 323-347.
- Hayalioglu, M. S. and Degertekin, S. O. (2005). "Minimum cost design of steel frames with semi-rigid connections and column bases via genetic optimization." Computers & Structures, 83(21-22), pp. 1849-1863.
- Ihaddoudène, A. N. T., Saidani, M., and Chemrouk, M. (2009). "Mechanical model for the analysis of steel frames with semi rigid joints." Journal of Constructional Steel Research, 65(3), pp. 631-640.
- Kameshki, E. S. and Saka, M. P. (2003). "Genetic algorithm based optimum design of nonlinear planar steel frames with various semi-rigid connections." Journal of Constructional Steel Research, 59(1), pp. 109-134.
- Kaveh, A. and Moez, H. (2008). "Minimal cycle bases for analysis of frames with semi-rigid joints." Computers & Structures, 86(6), pp. 503-510.
- Kaveh, A. and Talatahari, S. (2009). "Size optimization of space trusses using big bang-big crunch algorithm." Computers & Structures, 87(17-18), pp. 1129-1140.
- Kaveh, A. and Talatahari, S. (2010a). "A discrete big bangbig crunch algorithm for optimal design of skeletal structures." Asian Journal of Civil Engineering, 11(1), pp. 103-122.
- Kaveh, A. and Talatahari, S. (2010b). "Optimal design of schwedler and ribbed domes via hybrid big bang–big crunch alghoritm." Journal of Constructional Steel Research, 66(3), pp. 412-419.
- Kim, J. H., Ghaboussi, J., and Elnashai, A. S. (2010). "Mechanical and informational modeling of steel beamto-column connections." Engineering Structures, 32(2), pp. 449-458.
- Kishi, N., Chen, W. F., and Goto, Y. (1997). "Effective length factor of columns in semi-rigid and unbraced frames." Journal of Structural Engineering, ASCE,

123(3), pp. 313-320.

- Lee, K. S. and Geem, Z. W. (2005). "A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice." Computer Methods in Applied Mechanics and Engineering, 194(36-38), pp. 3902-3933.
- Mun, S. and Geem, Z. W. (2009). "Determination of viscoelastic and damage properties of hot mix asphalt concrete using a harmony search algorithm." Mechanics of Materials, 41(3), pp. 339-353.
- Nguyen, P. C. and Kim, S. E. (2013). "Nonlinear elastic dynamic analysis of space steel frames with semi-rigid connections." Journal of Constructional Steel Research, 84(5), pp. 72-81.
- Rafiee, A., Talatahari, S., and Hadidi, A. (2013). "Optimum design of steel frames with semi-rigid connections using big bang-big crunch method." Steel and Composite Structures, 14(5), pp. 431-451.
- Rajeev, S. and Krishnamoorthy, C. S. (1992). "Discrete optimization of structures using genetic algorithms." Journal of Structural Engineering, ASCE, 118(5), pp. 1233-1250.
- Saka, M. P. (2009). "Optimum design of steel sway frames to BS5950 using harmony search algorithm." Journal of Constructional Steel Research, 65(1), pp. 36-43.
- Saka, M. P. and Erdal, F. (2009). "Harmony search based algorithm for the optimum design of grillage systems to LRFD-AISC." Structural and Multidisciplinary Optimization, 38(1), pp. 25-41.
- Simoes, L. M. C. (1996). "Optimization of frames with semi-rigid connections." Computers & Structures, 60(4), pp. 531-539.
- Tang, H., Zhou, J., Xue, S., and Xie, L. (2010). "Big bangbig crunch optimization for parameter estimation in structural systems." Mechanical Systems and Signal Processing, 24(8), pp. 2888-2897.
- Valipour, H. R. and Bradford, M. A. (2013). "Nonlinear P- $\ddot{A}$  analysis of steel frames with semi-rigid connections." Steel and Composite Structures, 14(1), pp. 1-20.
- Wu, Z., Zhang, S., and Jiang, S. F. (2012). "Simulation of tensile bolts in finite element modeling of semi-rigid beam-to-column connections." International Journal of Steel Structures, 12(3), pp. 339-350.
- Xu, L. and Grierson, D. E. (1993). "Computer automated design of semi-rigid steel frameworks." Journal of Structural Engineering, ASCE, 119(6), pp. 1740-1760.