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Application of Model Reduction Techniques to Jacket Structures

Yongjei Lee[1,*], Bitna Seo[2], and Eun-Taik Lee[3]

[1]*Ph.D, PE, General Manager, Seoyeong Engineering Inc., Korea*
[2]*Bitna Seo, Deputy Engineer, SACS Korea Inc., Korea*
[3]*Ph.D., Professor, School of Architecture and Building Science, Chung-Ang University, Korea*

Abstract

Offshore structures like jackets are exposed to various kinds of external loading conditions and examination of broader range of modes and natural frequencies are required for their dynamic analysis. In this study, four model reduction techniques - two modal truncation methods and two balanced truncation methods - are introduced and applied to a jacket structure. The reduction results are compared with those of the conventional Paz condensation methods. Four new methods show better fidelity, especially in high frequency ranges than the conventional methods.

Keywords: Guyan-Paz model reduction, dynamic condensation, modal truncation, balanced truncation, SACS, jacket structures, offshore wind turbine

1. Introduction

A typical jacket structure consists of jacket leg, bracing, jacket pile, pin pile, and joint can. It is commonly used as an offshore structure, especially when the ground is soft and construction is difficult due to wave and current. This structure rarely changes current direction and settles down little after installation. High quality control and easy installation are possible because it is factory manufactured (Hammar *et al.*, 2010). For that reason, the jacket structures are used as substructures for wind turbine generation, ocean plant, and offshore drilling machine. It is also used as breakwater facility, quay, mooring dolphin and berthing dolphin.

The water depth of jacket install site is usually more than 20 m, usually reaching 80 m (Seidel, 2007; Valk, 2013). The structure is exposed to various kinds of external forces with broad range of frequencies; hydrostatic pressure, wind, wave, current, tide, ice, earthquake, temperature, souring, up-lift, buoyancy, and so on. For structural modeling and analysis, the external dynamic forces were often converted to equivalent static forces to make life easier. This method would produce conservative analysis results. Recently, as commercial software for Finite Element (FE) analysis method develops, assigning more realistic dynamic load conditions has become possible. However, counting all ocean conditions in a FE model would make it heavy and slow containing unnecessary information.

The cost of design and building of a jacket structure forms a major part of projects. For example, in Offshore Wind Turbine (OWT) project, it reaches at least 24% (Hugas, 2013) of total costs. That means an optimized jacket design can reduce the total project cost dramatically. It can be obtained by examining numerous structural candidates; therefore, the model reduction techniques are essential to cut down the analysis time and cost. When using commercial FE analysis programs like SACS® (Bentley Institute, 2012), engineers need to decide which nodes to retain and to remove based on engineer's judgment and a question has arisen if their decisions are always true. Especially in the ocean structures like jacket, dynamic properties shall be kept even after model reduction procedure.

2. Model Reduction Techniques

FE models for jacket structure often have more than thousands of degrees of freedom (DOF) and cause very large computation costs. The purpose of model reduction is to get a high-fidelity and low-order model, and to cut down the computational expense, ultimately. The basic concept of model reduction techniques is to remove unnecessary or less necessary DOF's in time domain or

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*Corresponding author
Tel: +82-2-6915-7567; Fax: +82-2-6915-7532
E-mail: yongjei@gmail.com

states in frequency domain (Qu, 2004). Several model reduction techniques have been developed including: modal truncation method; static condensation (Guyan, 1965); dynamic condensation (Zhang 1995); balanced truncation method (Laub, 1980; Moore, 1981; Laub *et al*., 1987), and Linear Matrix Inequalities (Boyd *et al*., 1994; Lee and Johnson, 2004).

2.1. Modal truncation method

A dynamic system can be expressed using state equation as Eq. (1) (Chen, 1999).

$$\begin{Bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{Bmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} + \begin{bmatrix} B_{\eta 1} \\ B_{\eta 2} \end{bmatrix} \cdot u \tag{1a}$$

$$y = [C_{\eta 1} \;\; C_{\eta 2}] \begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} + D_\eta \cdot u \tag{1b}$$

where u = input, y = output, η_1=state corresponding to low frequency, η_2=state corresponding to high frequency, Λ=state matrix, B_η=input matrix, C_η=output matrix, D_η=direct transmission matrix.

The modal truncation method can be classified into two methods according to the way to handle the high frequency state. The first method is based on an assumption that the states of high frequencies extinguish rapidly so that η_2 becomes zero. In this case, above state equation is expanded as Eq. (2).

$$\dot{\eta}_1 = \Lambda_1 \eta_1 + B_{\eta 1} u \,, \;\; \dot{\eta}_2 = B_{\eta 2} u \tag{2a, 2b}$$

$$y = C_{\eta 1} \eta_1 + D_\eta \cdot u \tag{2c}$$

The second method is based on an assumption that derivate of the states of high frequencies extinguish rapidly so that $\dot{\eta}_2$ becomes zero. In this case, Eq. (1) is expanded as Eq. (3).

$$\dot{\eta}_1 = \Lambda_1 \eta_1 + B_{\eta 1} u \,, \;\; \dot{\eta}_2 = \Lambda_2^{-1} B_{\eta 2} u \tag{3a,3b}$$

$$y = C_{\eta 1} \eta_1 + (D_\eta + C_{\eta 2} \Lambda_2^{-1} B_{\eta 2}) \cdot u \tag{3c}$$

2.2. Paz condensation methods

Through Paz static condensation method (Guyan, 1965; Paz, 1991), the stiffness matrix is sorted into two parts: partial matrices to be kept (primary part, denoted by *p*) and to be eliminated (subsidiary part, denoted by *s*). A stiffness matrix after sorting can be expressed as Eq. (4).

$$\begin{bmatrix} [K_{ss}] & [K_{sp}] \\ [K_{ps}] & [K_{pp}] \end{bmatrix} \begin{Bmatrix} \{y_s\} \\ \{y_p\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{F_p\} \end{Bmatrix} \tag{4}$$

where $\{y\}$=displacement vector, $\{F_p\}$=external force. The external force corresponding to the parts to be eliminated is assumed zero. After rearrangement of Eq. (4), a stiffness matrix $[\bar{K}]$ is obtained.

$$[\bar{K}]=[T]^T[K][T] \tag{5}$$

where, $[T]=[[\bar{T}] \; [I]]^T$, $[\bar{T}]=-[K_{ss}]^{-1}[K_{sp}]$

With the same token, the condensed mass matrix $[\bar{M}]$ and the condensed damping matrix $[\bar{C}]$ are obtained as

$$[\bar{M}]=[T]^T[M][T] \tag{6a}$$

$$[\bar{C}]=[T]^T[C][T] \tag{6b}$$

Through Gauss-Jordan elimination, Equation 4 can be rearranged as

$$\begin{bmatrix} [I] & [-\bar{T}] \\ [0] & [\bar{K}] \end{bmatrix} \begin{Bmatrix} \{y_s\} \\ \{y_p\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{F_p\} \end{Bmatrix} \tag{7}$$

In Paz dynamic condensation method (Paz 1991, Zhang, 1995), the first eigenvalue of the generalized eigenvalue equation (Eq. (8)) is assumed as zero. After getting transformation matrix, Gauss-Jordan elimination is applied to obtain a condensed eigenvalue equation.

$$\begin{bmatrix} [K_{ss}] - \omega_i^2 [M_{ss}] & [K_{sp}] - \omega_i^2 [M_{sp}] \\ [K_{ps}] - \omega_i^2 [M_{ps}] & [K_{pp}] - \omega_i^2 [M_{pp}] \end{bmatrix} \begin{Bmatrix} \{y_s\} \\ \{y_p\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \tag{8}$$

where ω_i=natural frequency of i th mode.

The condensed mass matrix and the condensed damping matrix are obtained same as Eq. (6), and the condensed stiffness matrix is obtained from Eq. (9).

$$[\bar{K}_i] = [\bar{D}_i] + \omega_i^2 [\bar{M}_i] \tag{9}$$

From Eqs. (8) and (9), following relationship can be obtained.

$$[[\bar{K}_i] - \omega_i^2 [\bar{M}_i]]\{y_p\} = \{0\} \tag{10}$$

After three or four times of repetition of above process, it produces virtually exact eigensolution.

2.3. Balanced truncation

The state-space model shown in Eq. (1) can be rephrased as Eq. (11).

$$\dot{x} = Ax + Bu \tag{11a}$$

$$y = Cx + Du \tag{11b}$$

The basic idea of this method is to find a principle that which states are representing the properties of system "better" than other states. In modern control engineering, the observability gramians and the controllability gramians are used as measuring indexes of certain states. The lower gramians do not necessarily mean the corresponding states are not important in system input and output. However if proper concept which includes both observability and controllability simultaneously exists, the concept of gramian can be used to find more important states (or less important states) in state-space model.

To change basis of gramians, a proper transformation matrix T is used as

$$\tilde{A} = TAT^{-1},\ \tilde{B} = TB,\ \tilde{C} = CT^{-1},\ \tilde{D} = D \tag{12}$$

By the definition of controllability gramians,

$$\bar{X}_c = \int_0^\infty e^{\tilde{A}\tau}\tilde{B}\tilde{B}^* e^{\tilde{A}^*\tau} d\tau = TX_cT^* \tag{13}$$

and by the definition of observability gramians,

$$\tilde{Y}_o = (T^*)^{-1} Y_o T^{-1} \tag{14}$$

For given matrix X and Y, nonsingular matrix T exists satisfying Equation 15 (Chen, 1999).

$$TXT^* = (T^*)^{-1} YT^{-1} = \Sigma \tag{15}$$

where Σ=diagonal positive definite matrix.

When controllability equals to observability, i.e., X_c= X_o=diag (σ_1, σ_2, ..., σ_n), it is called balanced realization. The eigenvalues (σ_1, σ_2, ..., σ_n) are called balanced gramians. If a gramian σ_i is lower than σ_j, its corresponding state has lower controllability and observability than σ_j's corresponding state (Dullerud and Paganini, 2000); therefore, proper balanced truncation procedure can be accomplished by removing i th state with σ_i. Like modal truncation method, two applying methods - truncation of high frequencies and of derivate of high frequencies - are possible in this method.

3. Application of Model Reduction Techniques

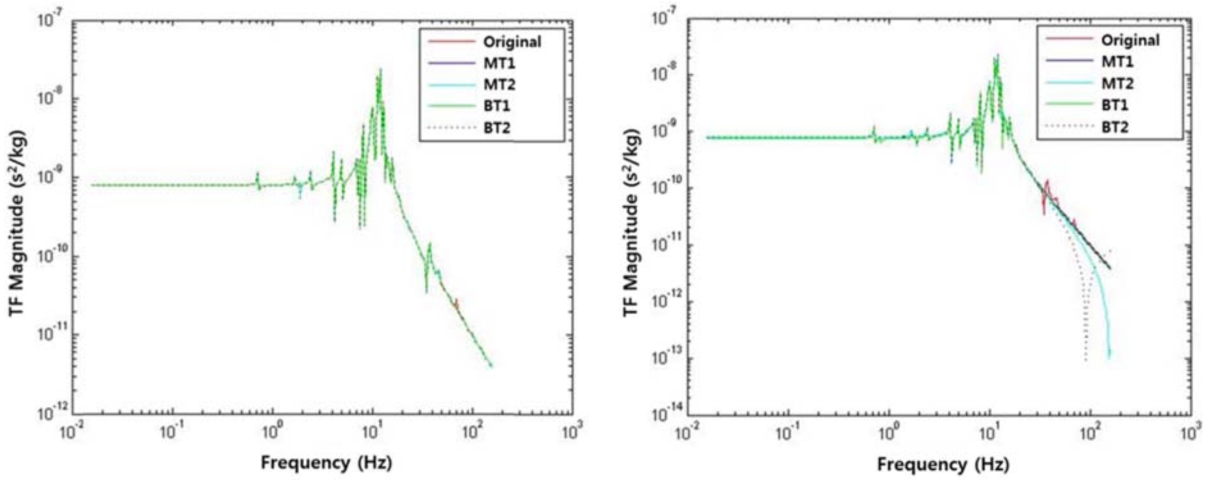
Six model reduction techniques were used to get condensed models of a jacket structure. The jacket structure with 132 DOF's which was designed for U-

Table 1. DOF's to be removed for each reduction case

Reduction Methods	Modal Truncation of high freq.	Modal Truncation of derivate of high freq.	Paz Static Condensation	Paz Dynamic Condensation	Balanced Truncation of high freq.	Balanced Truncation of derivate of high freq.
Abbreviation	MT1	MT2	Guy-S	Guy-D	BT1	BT2
Test 1	33	33	-	-	33	33
Test 2	66	66	-	-	66	66
Case 1	24	24	24	24	24	24
Case 2	48	48	48	48	48	48
Case 3	66	66	66	66	66	66
Case 4	78	78	78	78	78	78

(a) Case 1 (24 DOF) (b) Case 2 (48 DOF) (c) Case 3 (66 DOF) (d) Case 4 (78 DOF)

Figure 1. Nodes to reduce for Paz methods (marked as circle).



(a) Case 1 (33 DOF's reduced) (b) Case 2 (66 DOF's reduced)

Figure 2. Transfer function of Case 1 and Case 2.

project (2013) is used here. The model was modified and simplified from the original version to save efforts. The structure is made mostly by steel frames. The main design codes used in this project are BS 6349-2 (BSI, 2010) and API RP 2A-WSD (API, 2010). The model reduction schemes are summarized in Table 1.

First, two reduction tests are conducted to verify the feasibility of two modal truncation methods and two balanced truncation methods. Through test 1, total 33 DOF's are removed among 132 DOF's. In test 2, half of the DOF's (66 DOF's) of the system are removed.

Unlike other methods, Paz static and dynamic condensation methods need to select DOF's to reduce based on engineer's judgment. Frequently, the nodes connected with fewer member directions are selected to be removed because they are thought to be less important than those connected with more directional members. For example, the spots connecting the horizontal bracings, marked with circles in Fig. 1(a), do not have connection with vertical members so that they are the candidates to be reduced. 24 DOF's in this category from first 2 stories are selected for Case 1 (Fig. 1(a)). Additional 24 DOF's from upper stories are selected for Case 2 (Fig. 1(b)). Those are engineer's first judgment in choosing DOF's to reduce. When a floor is braced and thought to be relatively rigid, displacement of retained nodes could be very similar or even identical to the others on the same level. For this reason, the subsidiary nodes of each level are the candidates to be reduced. 18 DOF's in this category from first 2 stories are added for Case 3 (Fig. 1(c)). Another 18 DOF's from upper stories are selected for Case 4 (Fig. 1(d)). Those are engineer's second judgment in choosing DOF's to reduce.

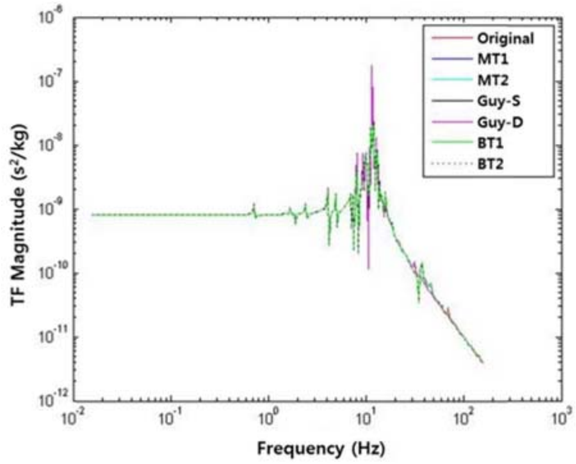
4. Results

4.1. Transfer function

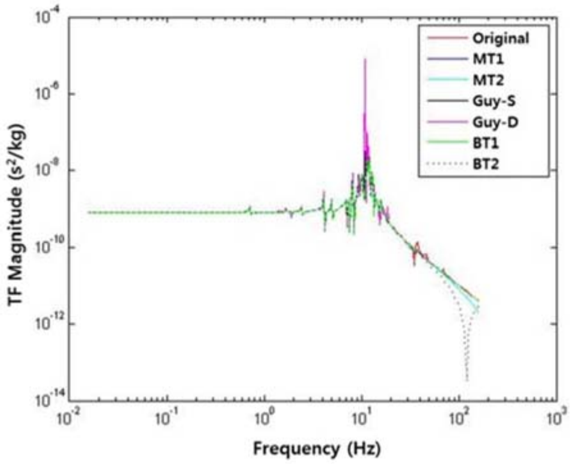
Figure 2(a) shows the transfer function of Test 1. Even after removing 33% of total DOF's, the magnitudes of transfer function of each reduction method are well-matching with original one. Figure 2(b) shows the transfer function of Test 2. The magnitudes of transfer function of all reduction methods show some discrepancy from 15.8 Hz. Overall, the results show good agreement with those of the original model.

Figure 3(a) shows the magnitude of the transfer function of Case 1. After removing 18% of total DOF's, the magnitudes of transfer function of each reduction method are well-matching with original one although two Paz methods produce different results from 10 Hz. Figure 3(b) shows the magnitude of the transfer function of Case 2. Around 10 Hz, Paz methods show discrepancy from the original again. The second balanced truncation method also shows difference in very high frequency range but it is negligible because it doesn't mean to a structure. Overall, they still produce well-matching results with the original. That means the 1st engineer's judgment works well until 36% of total DOF's are removed.

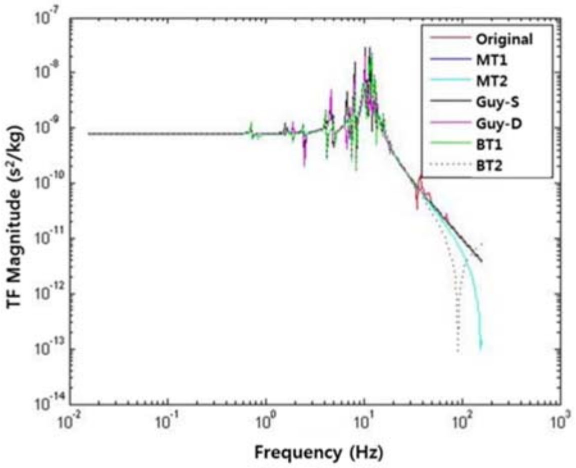
Figure 3(c) shows the magnitude of the transfer function of Case 3. Paz methods show more scattered magnitudes than Case 2. In the meanwhile, the magnitudes of other methods are still matching well with the original. Figure 3(d) shows the magnitude of the transfer function of Case 4. After 64% of total DOF's, no reduction method seems to produce reasonable result.



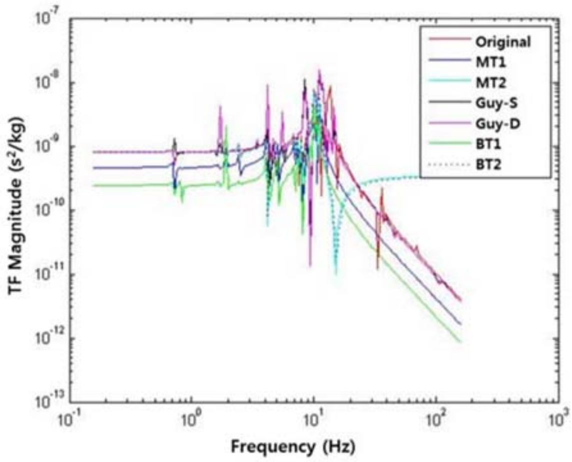
(a) Case 1 (24 DOF's reduced)



(b) Case 2 (48 DOF's reduced)

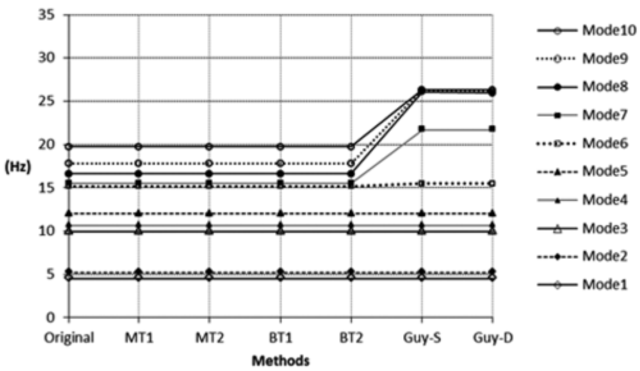


(c) Case 3 (66 DOF's reduced)

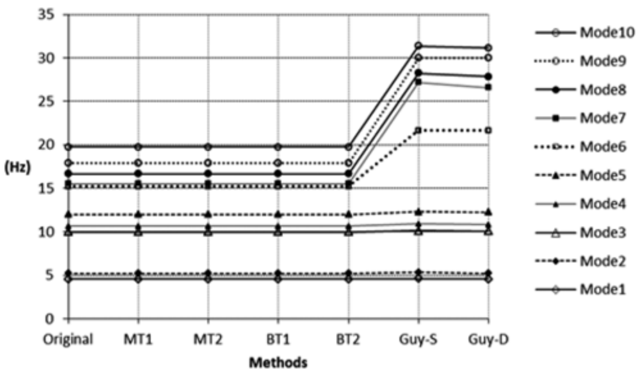


(d) Case 4 (84 DOF's reduced)

Figure 3. Transfer function of Case 1, 2, 3 and 4.



(a) Case 2: 48 DOF's reduced



(b) Case 4: 84 DOF's reduced

Figure 4. Natural frequencies of mode 1 to 10.

4.2. Natural frequencies

The natural frequencies of each reduced model from 1st to 5th modes are almost identical with those of the original model in all Cases. However, as shown in Fig. 4, two Paz condensation methods show discrepancy at mode 6 and above. Paz static condensation method makes 57% bigger frequency than the original at 8th mode in Case 2, and 75% at 7th mode in Case 4. The results from Paz dynamic condensation method are slightly better than the static method but still unacceptable. In Paz methods, those kinds of errors may be inevitable because the selection of DOF's to reduce are subjective and depending on the engineer's judgments.

5. Conclusions

Jacket structures are exposed to various kinds of external loading conditions; therefore broader range of modes and natural frequencies are to be examined for their dynamic analysis. In conventional model reduction like Paz static and dynamic condensations, engineers select degrees of freedom to remove based on their decision. Two modal truncation methods and two balanced truncation methods are introduced and the reduced results are compared with the conventional methods. All reduction methods used here produced satisfying results when the number of condensing DOF's is small. However, as that number increases, the dynamic properties by the conventional methods become different from and do not represent the original model anymore. To exam 6^{th} mode or higher of a structure, modal truncation methods and balanced truncation methods are recommended rather than conventional Paz condensation methods. The selection of model reduction methods may produce significant differences especially in the dynamic analysis of offshore structures.

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