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Asymmetric Information and Differentiated Durable Goods Monopoly: Intra-Period Versus Intertemporal Discrimination

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Abstract

A durable good monopolist faces a continuum of heterogeneous customers who make purchase decisions by comparing present and expected price-quality offers. The monopolist designs a sequence of price-quality menus to segment the market. We consider the Markov perfect equilibrium (MPE) of a game where the monopolist is unable to commit to future price-quality menus. We obtain the novel results that: (a) under certain conditions, the monopolist covers the whole market in the first period (even when a static Mussa–Rosen monopolist would not cover the whole market), because this is a strategic means to convince customers that lower prices would not be offered in future periods and that (b) this can happen only under the stage-wise Stackelberg leadership assumption (whereby consumers base their expectations on the value of the state variable at the end of the period). Conditions under which MPE necessarily involves sequentially trading are also derived.

Keywords Product quality · Durable good monopoly · Second-degree price discrimination · Coase conjecture

JEL Classification C73 · D42 · L12

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1 Introduction

In their celebrated paper on the Coase Conjecture, Gul et al. [7] pointed out that the monopolist's inability to commit to a future path of prices is a fundamental feature of a dynamic theory of monopoly. They wrote (p. 155) that "*a dynamic theory of monopoly must take into account the fact that a monopolist cannot normally sign contracts to guarantee that the future prices of his output will be above some minimal level.*" Indeed, *"without repeat purchases monopoly rents must depend substantially on a monopolist's ability to commit to prices or quantities offered in the future."*[1] In the limiting case where the time interval that must elapse between two different offers becomes arbitrarily small, the only possible equilibrium outcome is that the market opens at a price equal either to the marginal cost (MC) when MC is higher than the valuation of the lowest consumer type (the No Gap case) or to the lowest type's valuation when it is higher than the MC (the Gap case). In both cases, the monopolist loses all or part of its monopoly power.[2] This result is known as Coase conjecture.

It is illuminating to consider the mechanism underlying this result. A monopolist who is unable to practice discrimination among potential customers in a given period may have a strong incentive to practice discrimination among them over different periods. The firm is tempted to offer sequentially lower and lower prices to customers with smaller and smaller willingness to pay. Since consumers rationally expect such behavior, they have an incentive to delay their purchase until the price is lower. When the interval between two different offers becomes infinitesimal, they wait until the price equals the (constant) marginal cost (or, in the Gap case, the lowest type's valuation of the durable). This ends up being the only rational outcome, given that the firm cannot credibly commit not to change the price of the good in the future.

The present paper (i) shows generally that this result may not hold when the monopolist is able to discriminate, within any given period, among its heterogeneous customers by offering them, in the Mussa–Rosen [26] way, a set of (durable) goods that differ in quality and (ii) provides some new insights into durable goods monopoly theory. To the best of our knowledge, this question has only been studied in the literature of market dynamics for durable goods by Inderst [9] who considered only two types of consumers with different preferences for quality who can be offered at least two different qualities each period. Nonetheless, it constitutes a particularly relevant question when we consider the recent advances in digital technologies. Indeed, the digitization of many economic activities has drastically changed market design and business practices in a vast range of sectors, opening the road for smart factories and for massive product customization. For example, in the automotive industry, car manufacturers like Tesla now allow their clients to customize their own car model according to their preferences.[3] Luxury brands like Louis Vuitton or Gucci allow customers to create their own personalized luxury items. Audiovisual content distributors like Netflix allow customers to choose their own consumption bundle from a wide range of audiovisual contents.

In this paper, we extend [9] by showing in a model with a continuum of types that firms' ability to design increasingly rich menus of quality-price options to their customers (by tailoring the product-price specification to the tastes of different segments of consumers) may result in market dynamics that depart from standard Coasian dynamics. We obtain the

[1] p.156.

[2] Technically, as noticed by Inderst [9],*"while the real time in which the market is served goes to zero as the time between periods shrinks, the number of periods it takes to clear the market also increases"*.

[3] More precisely, consumers may use the Tesla online platform (https://www.tesla.com/en_GB/models/design#battery) to parametrize a wide range of characteristics of the car (including the personalized dashboard). At the end, different consumers may end up getting differentiated variants of their cars.

novel results that (a) under certain conditions, the monopolist covers the whole market in the first period (even when a static Mussa–Rosen monopolist would not cover the whole market), because this is a strategic means to convince customers that lower prices would not be offered in future periods, and that (b) this can happen only under the stage-wise Stackelberg leadership assumption (whereby consumers, having seen the monopolist's move, decide whether to buy or to wait, based on their future rent expectations conditioned on the value of the anticipated state variable at the end of the period). Conditions under which MPE necessarily involves sequentially trading are also derived.

We assume that the firm does not know the consumers' types.[4] Thus, in each period, the only form of discrimination is second-degree price-quality discrimination. Apart from quality discrimination (which intends to capture firms' recent business practices regarding massive customization and market hyper-segmentation), other features of our model are standard. A firm sells an infinitely durable good to a continuum of infinitely lived customers. Each consumer buys at most one unit of the good in their lifetime. Their preferences for quality are private information. The innovative feature of our model vis-à-vis the mainstream literature on durable goods pricing is that the monopolist may propose within each period a menu of different qualities at different prices to different customers (e.g., in the Tesla example, the firm ends up offering a menu of quality-price differentiated specifications for a given car and consumers end up picking up a given product from the set of all variants available in the Tesla online platform).

We consider that in each period, a period-specific price-quality schedule is made available to all potential customers and the firm cannot commit to future price-quality schedules (equilibrium outcomes are also obtained for the following useful benchmarks: the social planners' welfare maximizing solution and the full commitment monopoly solution).

By combining the features of the standard durable good dynamic model with those of the static model of monopoly and product quality proposed by Mussa and Rosen [26], our model can be conceived as a dynamic version of the Mussa–Rosen model, in which the durable good monopoly offers in each period a new price-quality schedule for customers who have not bought a durable good before. Consumers decide when to make their purchase, knowing the current price-quality schedule, and having rational expectations about the monopolist's future price-quality schedules. In a Markov Perfect Equilibrium (MPE), the monopolist quality/price strategy maximizes his discounted lifetime profit, given consumers' expectations, and given the monopolist's sales strategy, the consumers' expectations are rational. We assume that firms and consumers move sequentially (and we also discuss equilibrium results under the alternative assumption of simultaneous moves).

Since, within any given period, the firm is able to offer a menu of price-quality pairs to its customers, it may seem reasonable to conjecture that the monopolist's temptation to practice intertemporal price discrimination would disappear. In other words, the ability to offer to heterogeneous customers different price-quality pairs might render the commitment problem a non-issue, thus invalidating Coase conjecture. One of the striking results of this paper is to provide general sufficient conditions which ensure that there is indeed a MPE in which the Coasian conjecture regarding profit erosion does not hold, though the market is instantaneously covered. This MPE, which we show to be unique, corresponds to a modified

[4] See Laussel et al. [18] for a model in which a monopolist firm producing a non-durable good gets full information on customers' preferences after their first purchase, using such information to make personalized price-quality offers (first-degree price discrimination). Laussel et al. [19] also look at the case of non-durable goods. Their setup departs from the present model (and also from [18]) since there, the monopolist is unable to adjust the specification of the product (quality) when it gets information on customers' tastes (after the customers' first purchase).

version of the static Mussa–Rosen monopoly equilibrium, the modification being that the firm is constrained to cover the whole market. Thus, lack of commitment affects only the range of consumers covered; of course, the lower bound of that range changes all price-quality pairs.[5]

We obtain that, for distributions of preferences such that welfare maximization leads to full market coverage, the monopolist covers all the market either when the static Mussa–Rosen monopolist always does it or, when the latter does not cover it but the discount factor is close to one (which amounts to saying that the length of the commitment period must be small enough, though it may be strictly bounded away from zero).

Looking then at the linear-quadratic case, we derive two even less demanding sufficient conditions and we show that at least one of them must be satisfied for immediate full market coverage to be an equilibrium strategy. In this MPE, the monopolist actually profits from offering customizable Mussa–Rosen price-quality menus to its customers. Thus, when this MPE prevails, we identify one theoretical rationale (among other factors) explaining why an increasing number of firms announce that they are committed to the new product customization paradigm.[6]

We argue that this paper not only fills a gap between the two literatures, but also provides some new insights. Our immediate full market-covering result, while at first sight might look like a standard result in the standard durable good monopoly literature, displaying a seemingly Coasian flavor, is actually obtained under quite different conditions and for quite different reasons. Without quality differentiation, immediate full market coverage is a limit result when the length of commitment tends toward zero. In our paper, it may obtain even when the length of commitment is strictly positive. In the standard durable good monopoly literature, immediate full market is due to consumers' expectations that the monopolist will infinitely quickly lower prices to attract new customers. In contrast, in our model, it is a strategic choice to serve immediately all customers so as to credibly commit not to lower prices to attract unserved consumers in subsequent periods. Moreover, our analysis unveils that these results depend significantly on consumers' market expectations about their future surplus, when they contemplate delaying their purchases. As mentioned earlier, in our baseline model we assume that the monopolist decisions precede consumers' ones. This means that, in any period, the expectations of surplus which delay their purchase depend on the expected value of the state variable at the end of the period (or equivalently, the size of potential customers—who have not bought the good yet—in the subsequent period).

We also investigate how our results on immediate market coverage are affected when this assumption is replaced by the alternative one that consumers and firms move simultaneously (and therefore consumers need to formulate expectations on the market size at the beginning of the period). Our results unveil that in this case immediate full market coverage occurs for a much smaller set of parameter values, and it becomes necessary, but is no longer sufficient, that the static Mussa–Rosen monopolist would cover the whole market. Moreover, in that case, the discount factor must be small enough, so that the model is close enough to the static Mussa–Rosen one.

The rest of the paper is organized as follows: Section 2 considers related literature. Section 3 presents the main ingredients of the model. Section 4 deals with the Markov perfect equilibria under non-commitment. Section 5 derives sufficient conditions for the existence of full-market covering Markov perfect equilibria under general assumptions. Section 6 obtains

[5] We are grateful to an anonymous referee for drawing our attention to this point.

[6] For example, McKinsey [24] argues that *"Personalization is teetering on the edge of the buzzword precipice. But companies that can figure out what it really means and how to take advantage of it are already outstripping their competition."* For more information, visit https://www.mckinsey.com/business-functions/marketing-and-sales/our-insights/perspectives-on-personalization-at-scale [Access date: 25 February 2020] .

more precise sufficient conditions and necessary conditions for immediate full market coverage in the linear-quadratic case. Section 7 concludes.

2 Related Literature

To the best of our knowledge, there is a very scarce literature investigating how the recent product customization trends affect market dynamics in durable-goods industries. Herein, our contribution is to shed some light on this question by bringing together (i) the literature on Coase conjecture and (ii) the literature on quality second-degree discrimination à la Mussa–Rosen [26].

Hence, our paper is closely related to these two streams of literature. First, it contributes to the literature beginning with [2], who argued that, in the No Gap case, a durable-good monopolist who cannot commit to its future prices and outputs will lose all his monopoly power: in equilibrium, the price is equal to the constant marginal cost, and all potential customers are served instantaneously.[7] A number of exceptions have been put forward. See, for example, Kahn [13] for non-constant marginal cost, and Karp [12] for durable goods subject to depreciation. The Coase conjecture may also fail when the durable good is subject to network externalities (e.g., Mason [23], Laussel et al. [20]).[8] In all these cases, however, even when the Coase conjecture fails, non-commitment always reduces monopoly profits relative to the full commitment solution. In Kahn's continuous time model for instance, the market is (only) asymptotically fully covered and the monopolist retains positive profits even in the No Gap case. Herein, we show that the possibility to engage in intratemporal quality and price discrimination may actually lead to positive profits, therefore identifying one additional circumstance where Coasian predictions related to profit erosion do not hold.

The paper also relates closely to the vast literature on firms' product specification strategies, starting with the seminal work by Mussa and Rosen [26] and attracting the attention of a large number of scholars. To the best of our knowledge, the large majority of works in this field has looked at the problem of firms' optimal product design within a static context.[9] Of particular interest to the present paper is the stream of literature that deals with principal-agent problems, which are often relevant when firms choose their product menus under imperfect information about consumers' true tastes.

In particular, the formulation of our model highly benefits from the literature that deals with principal-agent problems in which agents' types are their own private information. Following the path-breaking work of Mirrlees [25], several authors have formalized the revelation principle (e.g., Holmstrom [8], Myerson [27]), which has proved fruitful in many applications, such as models of regulation and incentive contracts (see, for example, Laffont and Tirole [16,17], Laffont and Martimort [17]). Extensions of this mechanism design approach include the multi-dimensional case (e.g., Martimort [22]) and the multi-period case (e.g., the generalization of Mirrlees' model to a dynamic setting, as in Kocherlakota [14], Golosov et al. [5,6], Stantcheva [29]).

7 In the Gap case the price tends instantaneously toward the lowest type's valuation. The profit is substantially eroded but not entirely dissipated. Immediate full market-coverage still holds.

8 Laussel et al. [20] show that the standard results for Coasian dynamics must be modified when the durable-good monopolist participates in two distinct markets with consumption network effects (the primary market and the aftermarket).

9 In static settings, the issue of optimal product design has been widely studied, following the seminal work of Mussa and Rosen [26]. Some recent contributions include Deneckere and McAfee [3] and Johnson and Myatt [11]. Johnson and Myatt [10] look at the problem of optimal product design in a competitive setup.

To the best of our knowledge, there are very few papers on durable goods that take into account the dynamic interplay between the Coasian conjecture and the existence of asymmetric information on consumers' willingness to pay (*à la* [26]). Nava and Schiraldi [28] analyzed a dynamic game where a monopolist offers two horizontally differentiated varieties of a durable. They found that *"in any perfect Bayesian equilibrium of the game: (1) there is skimming, as the measure of buyers in the market at any point in time is a truncation of the original measure and (2) the market clears instantaneously whenever the seller sets static market-clearing prices."* Instead of looking at horizontally differentiated industries with fixed varieties (as done in Nava and Schiraldi [28]), herein we look at industries with vertically differentiated products and we give the monopolist the ability to choose the price-quality menu that maximizes its profit, given consumers' rational expectations about future quality-price menus.

Board and Pycia [1] also study, like we do, a dynamic monopoly with durable-goods when there is a single buyer who privately knows her value for the good and who has an outside option they may exercise each period.[10] They show that their model displays a unique equilibrium in which the firm sets the monopoly price every period. Differently from our model, the outside option ends up leaving low-type consumers out of the market.

Takeyama [30] analyzed a two-period model with two types of consumers, allowing for quality upgrading in the second period. She showed that the monopolist may benefit from offering to the low-type consumers a higher quality than in a static setting, in order to ensure that such consumers will not upgrade in period two: if low-type consumers would upgrade in the second period, the monopolist would have to give high-type consumers additional rents to dissuade them from mimicking the low type ones. Kumar [15] considered time-consistent intertemporal price-quality discrimination by a durable goods monopolist in the so-called Gap Case when there is a continuum of types. Assuming away quality upgrading, he showed that the Coase conjecture holds because, as the time between two different offers shrinks to zero, the monopolist's profit tends to the pooling one that would arise if everybody had the lowest type's valuation for quality. This departs from our framework, where, in each commitment period, the possibility of offering a menu of price and qualities allows the firm to make positive profits by extracting some information rents from lower types buying in each period.

The most closely related paper is Inderst [9]. He assumes, however, that there are only two types of consumers. In each period, the monopolist may offer a range of product qualities. He shows that if the fraction of high-valuation consumers is small enough, or if the real time between two consecutive periods becomes sufficiently small, then there exists an equilibrium in which the monopolist serves the whole market in the first period. He points out that the restriction to two types is a limitation of the model, and that the method of proofs in his paper "does not allow for an immediate generalization" (p. 174). Our paper considers a model with a continuum of customer types, with a general distribution of types. We identify sufficient conditions for a non-commitment monopolist to cover the market immediately in the initial period. As mentioned earlier, this occurs if the static Mussa–Rosen monopolist would cover the whole market and/or if the discount factor is close enough to one. In the linear-quadratic case, we obtain more precise and less restrictive sufficient conditions for immediate full market coverage: either the discount factor is smaller than a critical value which is strictly

[10] The outside option may be, for instance, the possibility of buying another product.

smaller than 1 if the customer-base is "*super-strong*" [11] or, if not, the range of consumers' types is small enough. Moreover, one of these conditions should be satisfied for immediate full market coverage.

3 The Model

A monopolist produces an infinitely durable good at different quality levels. Let q be the quality index, where q can take any nonnegative value: $q \in [0, \infty)$. The lowest possible quality is zero. We assume that a unit of durable at quality 0 yields no benefit to any consumer. The unit cost of a durable good at quality level q is $c(q)$. With respect to quantity, we assume constant returns to scale: the cost of producing x units of the durable at quality q is simply $xc(q)$. With respect to quality, $c(q)$ is assumed to be strictly convex and twice differentiable. We refer to $c'(q)$ as the marginal cost of providing quality q. Specifically we suppose that the function $c(q)$ has the following properties:

Assumption A1 $c(0) \geq 0$, $c'(0) \geq 0$, with $c'(q) > 0$ and $c''(q) > 0$ for all $q > 0$.

Remark 1 The possibility that $c(0) > 0$ is admitted. In that case, the cost of producing a unit of durable, even at the lowest quality, is strictly positive (i.e., it implies a fixed cost). Regardless of whether $c(0) = 0$ or $c(0) > 0$, it follows from Assumption A1 that there exists a unique $\widehat{q} \in [0, \infty)$ such that

$$\widehat{q}c'\left(\widehat{q}\right) = c\left(\widehat{q}\right), \tag{1}$$

where $\widehat{q}$ represents the quality level for which the marginal cost of providing quality $\widehat{q}$ equals the corresponding average cost. Obviously, $\widehat{q} > 0$ iff $c(0) > 0$. If $c(0) = 0$, then $\widehat{q} = 0$. As we shall see, $\widehat{q}$ plays an important role in the characterization of (i) the optimum allocation under a benchmark scenario (the social planner's solution), and also of (ii) the equilibrium strategy of the firm (in the case of a durable-good monopoly). In particular, the last consumer served by the welfare-maximizing social planner will get a quality level such that his/her valuation of the provided quality level coincides with the production cost of a durable with that quality level, getting a zero surplus (with the zero marginal surplus condition being equivalent to the one presented in (1)).

Time is a continuous variable, $t \in [0, \infty)$. Consumers are infinitely-lived. There is a continuum of consumer types, indexed by θ, where $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, $\overline{\theta} > \underline{\theta} \geq 0$. We refer to θ as the individual's marginal valuation of quality, which is private information. In addition, the type θ of any given consumer remains the same over her whole lifetime.

We assume that a consumer of type θ who makes use of a unit of durable at quality level q from time t to time infinity derives a utility flow of θq at each instant of time $\tau \in [t, \infty)$. Since we look at an infinitely lived durable good, we assume that once consumers have bought a unit at a given quality q, they will not need to buy the good again. Instead, they exit the market and enjoy the service flow yielded by that unit for their whole lifetime. Thus, by assumption, immediate consumption of the durable dissipates the consumers' need forever.[12] In light of this, consumers who have bought a unit of durable in a given period n do

[11] The customer base is "super strong" if the lowest type customers' maximum net surplus (gross surplus minus production cost) is strictly positive, i.e., if a benevolent social planer would always strictly prefer to supply them.

[12] That does not mean that this consumption does not yield permanent per period benefits.

not consider upgrading it to a higher quality in later periods. This is the so-called permanent exit assumption, which is quite common in the literature, whether it is made implicitly or explicitly. [13] As noticed by Nava and Schiraldi[14] (2019, p. 22), such an assumption is compelling for goods whose immediate consumption dissipates the need forever as it is, for instance, the case for many services.[15] The lifetime utility of a customer of type θ, discounted back to time t at the instantaneous discount rate $r > 0$, is accordingly

$$U_t(\theta, q) \equiv \int_t^{\infty} e^{-r(\tau-t)}\theta q \mathrm{d}\tau = \frac{1}{r}\theta q.$$

Individuals who do not get allocated a unit of the good get zero lifetime utility. In order to streamline our model and analysis, we make the following additional assumption on the distribution of consumer types.

Assumption A2 The cumulative distribution of consumer types, denoted by $F(\theta)$, is continuously differentiable, with $F(\underline{\theta}) = 0$, $F\left(\overline{\theta}\right) = 1$, and the density function $f(\theta) \equiv F'(\theta)$, which is strictly positive for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Assumption A2 allows us to define the "inverse hazard rate" function $h(\theta)$ over the interval $\left[\underline{\theta}, \overline{\theta}\right]$:

$$h(\theta) = \frac{1 - F(\theta)}{f(\theta)} \geq 0, \text{ with } h(\underline{\theta}) = \frac{1}{f(\underline{\theta})} > 0 \text{ and } h\left(\overline{\theta}\right) = 0.$$

As we shall see later, this function will play an important role in the analysis of the strategies of the monopolist.

3.1 A Benchmark Scenario: The Social Planner Solution

In this section, we consider the following benchmark scenario: a social planner wishes to maximize social welfare (SW), given by the integral (over $\left[\underline{\theta}, \overline{\theta}\right]$) of lifetime utilities of all individuals of all types $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, net of the cost $c(q(\theta))$ of supplying individuals of type θ with a unit of durable of quality $q(\theta)$. (We assume that all consumers with the same type θ get the same treatment.) The function SW is specified as follows:

$$SW = \int_{\underline{\theta}}^{\overline{\theta}} \delta(\theta) \left[\frac{1}{r}\theta q(\theta) - c(q(\theta))\right] f(\theta)\mathrm{d}\theta,$$

where $\delta(.)$ is a function, defined over $\left[\underline{\theta}, \overline{\theta}\right]$, that can only take one of two values, 1 or 0. For given $\theta', \theta'' \in \left[\underline{\theta}, \overline{\theta}\right]$, $\delta(\theta') = 1$ means that all consumers of type θ' get allocated one unit of the durable good at time $t = 0$ for use during their lifetime, while $\delta(\theta'') = 0$ means all consumers of type θ'' are not allocated a unit of the durable good and thus have zero lifetime

13 For instance, Kumar ([15], p. 900), wrote that "Each consumer is in the market for only one unit of the good and exits after making the purchase" . Inderst ([9], p. 174) stated that "*Consumers want to buy at most a single good*". ([28], p. 6) wrote that "*In the baseline setting, buyers have unit-demand for the product and exit the market upon purchasing either of the two varieties*." On the contrary, for a model where upgrading is possible, see Takeyama [30].

14 They remark that *"after all, in these models, durability simply amounts to sales permanently depleting the demand for the good.*"

15 For instance, one may think of a cataract operation, or a removal of an internal organ such as "Appendix" or the tonsil.

utility. Conditional on $\delta(\theta')=1$, the quality of the durable assigned to consumers of type θ' is denoted by $q(\theta')$. An allocation is a pair of functions $(\delta(.), q(.))$.

Result 1 *Let $\widehat{\theta}$ be given by*

$$\widehat{\theta} \equiv \begin{cases} \frac{rc(\widehat{q})}{\widehat{q}} & \text{if } c(0) > 0 \\ rc'(0) & \text{if } c(0) = 0 \end{cases}$$

Then, the socially efficient allocation has the following properties:

(i) Consumers of type $\theta < \widehat{\theta}$ are not served, because their valuation of any quality level is strictly lower than the production cost of a durable with that quality level.
(ii) Consumers of type $\theta \geq \widehat{\theta}$ are offered durables of quality $q^{se}(\theta)$, defined by

$$q^{se}(\theta) = c'^{-1}(\theta/r) \text{ for } \theta \geq \widehat{\theta}. \tag{2}$$

so that higher type consumers are offered higher quality and achieve higher surplus.

Proof See "Appendix". □

Example 1 Assume that $c(q) = B + \frac{1}{2}q^2$. It is easy to show that $\widehat{q} = \sqrt{2B}$ and $\widehat{\theta} = rc'(\widehat{q}) = r\sqrt{2B}$. Then, if $\widehat{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, the socially efficient quality level for type θ is

$$q^{se}(\theta) = \frac{1}{r}\theta, \text{ for all } \theta \in \left[r\sqrt{2B}, \overline{\theta}\right] \tag{3}$$

and for consumers of type $\theta < r\sqrt{2B}$, it is socially efficient that they do not get any durable good at all.

Notice that, at the social planner's full information solution, the distribution of rents between consumers and the firm is undermined within bounds. A type $\theta \geq \widehat{\theta}$ consumer cannot be enticed to buy unless $\frac{\theta}{r}q^{se}(\theta) \geq p^{se}(\theta)$, where $p^{se}(\theta)$ is the price of the durable supplied to a type θ -customer. The firm cannot be enticed to supply customers unless $p^{se}(\theta) \geq c(q^{se}(\theta))$. It obviously follows that $p^{se}(\theta) \in [c(q^{se}(\theta)), \frac{\theta}{r}q^{se}(\theta)]$.

In order to restrict the number of cases to be considered in subsequent sections where we will be comparing the outcome under monopoly with the social optimum, let us define the concepts of a customer base, a strong customer base, and a super-strong customer base.

Definition A customer base is a cumulative distribution of types, denoted by $F(\theta)$, defined over $\left[\underline{\theta}, \overline{\theta}\right]$, where $\overline{\theta} > \underline{\theta} \geq 0$. A customer base is said to be strong if

$$\underline{\theta} \geq \widehat{\theta}, \tag{4}$$

and super-strong if

$$\underline{\theta} > \widehat{\theta}. \tag{5}$$

Clearly, when condition (4) is satisfied, then, for the customer base under study, given Assumptions A1 and A2, it is *socially efficient* to serve all consumer types $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. In contrast, under monopoly, the assumption that the customer base is strong is not sufficient to ensure that the monopolist will serve the whole market, as we shall see below.

The concepts of a strong and a super-strong market base are related in a simple way to the Gap / No Gap cases in the standard durable goods model (see, for instance, Fudenberg and Tirole [4], p.400). The Gap case in this literature arises when for all consumers' types (including the lowest type) the utility of one unit of the durable is strictly greater than its marginal cost. Our concept of a super-strong consumer base corresponds to an extension of the Gap case to a model with an endogenous quality level: the consumer base is super-strong iff the maximum social surplus[16] from serving any consumer's type is strictly positive. The No Gap case in the standard durable good literature is when the utility of unit of the durable is not greater than its production cost. In our model with endogenous quality level, this arises when $\widehat{\theta} \geq \underline{\theta}$, i.e., the market is just strong (but not super-strong) or even weak (when $\widehat{\theta} > \underline{\theta}$).

3.2 Monopoly Under Asymmetric Information

In what follows, we study several types of alternative equilibrium outcomes that may arise in a monopoly market, under asymmetric information and consumers' rational expectations. We assume that the monopolist announces at discrete points of time, $(t_0, t_1, t_2, \ldots)$, $t_0 < t_1 < t_2 < \cdots < t_j < \cdots$, a schedule of prices $p_{t_i}(.)$ which applies over the time interval $\left[t_i, t_{i+1}\right)$, where $p_{t_i}(q)$ is the price assigned to a durable of quality q, which is maintained constant over that time interval. Moreover, we assume that $t_{i+1} - t_i = \Delta$, where Δ is the same for all $i = 0, 1, 2, \ldots$. The parameter Δ defines the length of commitment period. We shall investigate how monopoly's profit under each equilibrium type depends on Δ. (It will be convenient to refer to the period commencing at time t_n as period n, where $n = 0, 1, 2, 3, \ldots$).

To streamline our analysis of the monopoly's price-quality offers under asymmetric information, we make some further assumptions:

Assumption A3 (Monotone decreasing inverse hazard rate) The function $h(\theta)$ is monotone decreasing over the interval $\left[\underline{\theta}, \overline{\theta}\right]$.

Assumption A3 is the standard assumption ensuring that, when a monopolist practices second-degree price discrimination, there is "no bunching" in equilibrium (i.e., in equilibrium, each positive quality level is produced in order to supply a unique consumer type). Assumption A3 is satisfied by many distributions, such as the uniform, normal, Poisson, exponential, and binomial distributions.

Assumption A4 The customer base is strong: $\overline{\theta} > \underline{\theta} \geq \widehat{\theta}$.

Note that A4 implies that there exists a range of strictly positive quality levels such that

$$\frac{1}{r}\overline{\theta} q - c(q) > 0.$$

In what follows, we assume that A1, A2, A3, and A4 hold.

Notice that Assumption A4 only means that the whole market is supposed to be economically viable. It is made only for the sake of convenience and is not really restrictive. Suppose indeed on the contrary that the market is weak, i.e., that $\widehat{\theta} > \underline{\theta}$. We can then restrict our attention only to the economically viable parts of the market, those for which there exist potential benefits for trade in durables, i.e., the segment $\left[\widehat{\theta}, \overline{\theta}\right]$, simply with a density function $f(\theta)/1 - F(\widehat{\theta})$ on this interval. The results of Sects. 5 and 6 about the existence and uniqueness of an immediate full market-covering Markov perfect equilibrium remain unchanged provided full market-covering is intended to mean, in line with the durable goods monopoly literature, immediate full covering of the economically viable parts of the market, i.e., the immediate realization of all potential gains from trade.

16 The quality level being selected according to equation (2) so as to maximize the social surplus of each consumer's type.

Remark 2 If $\underline{\theta} - h\left(\underline{\theta}\right) < 0$, this inequality together with Assumption A3 and the fact that $h\left(\overline{\theta}\right) = 0$ implies that there exists a unique value $\theta^c \in (\underline{\theta}, \overline{\theta}]$, at which the downward sloping curve $h(\theta)$ intersects the 45 degree line, i.e., at which

$$\theta^c - h(\theta^c) = 0. \tag{6}$$

This implies that $\theta - h(\theta) < 0$ iff $\theta \in \left[\underline{\theta}, \theta^c\right)$ and $\theta - h(\theta) > 0$ iff $\theta \in \left(\theta^c, \overline{\theta}\right]$. As is well known, in that case, a (full-committed) monopolist would never supply $q(\theta) > 0$ for $\theta \in \left[\underline{\theta}, \theta^c\right)$, given that $c(q) \geq 0$ for all $q \geq 0$. The full-commitment monopoly is analyzed in the following section.

3.2.1 Another Benchmark: The Full-Commitment Monopoly

The static (or full-commitment) second-degree discrimination model à la Mussa–Rosen provides a useful benchmark. In this full-commitment scenario, the monopolist offers at time $t = 0$ a menu, i.e., a price-quality schedule $(q, p(q))$, and commits to maintain the same schedule for ever. Consumers then either choose an item on the menu to purchase immediately at $t = 0$, or refuse to make a purchase, and then, they exit the market (as there is no point for them to wait for a better deal next period, given that the monopolist credibly commits to make the same offer in the next period). As is well known, making use of the revelation principle, the monopolist's menu can be represented by a pair of functions, $(q(\theta), p(\theta))$ for all $\theta \in \left[\theta^*, \overline{\theta}\right]$, such that $(q(\theta), p(\theta))$ satisfy the usual incentive compatibility constraint,[17] where $\theta^* \geq \underline{\theta}$ is the monopolist's cutoff type, such that consumers of types $\theta \in \left[\underline{\theta}, \theta^*\right)$ will not be served. Following the Mirrlees' trick, we define the informational rent of a type θ customer (where $\theta \in \left[\theta^*, \overline{\theta}\right]$) by

$$U(\theta) = \frac{1}{r}\theta q(\theta) - p(\theta) = \max_{\widetilde{\theta}} \frac{1}{r}\theta q\left(\widetilde{\theta}\right) - p\left(\widetilde{\theta}\right). \tag{7}$$

Then, applying the envelope theorem, we obtain $\frac{\mathrm{d}U}{\mathrm{d}\theta} = \frac{1}{r}q(\theta)$ for $\theta \in \left[\theta^*, \overline{\theta}\right]$. Since θ^* is the cutoff type of customers (the lowest type of customers that are served), it is clear that the monopolist will extract all the surplus from this type, i.e., $U(\theta^*) = 0$, and it follows that, for all $\theta \in \left[\theta^*, \overline{\theta}\right]$,

$$U(\theta) = U(\theta) - U(\theta^*) = \int_{\theta^*}^{\theta} \frac{\mathrm{d}U(\theta')}{\mathrm{d}\theta'}\mathrm{d}\theta' = \int_{\theta^*}^{\theta} \frac{1}{r}q(\theta')\mathrm{d}\theta'. \tag{8}$$

Using equations (7) and (8), we can express the price $p(\theta)$ as

$$p(\theta) = \frac{1}{r}\theta q(\theta) - \int_{\theta^*}^{\theta} \frac{1}{r}q(\theta')\mathrm{d}\theta'. \tag{9}$$

Let us start by characterizing, under full commitment, the monopolist's optimal cutoff type, θ^*, and the quality offered to consumer types above the cutoff type. Conditional on a given cutoff type $\theta^* \geq \underline{\theta}$, the profit of the monopolist is

$$\pi = \int_{\theta^*}^{\overline{\theta}} \left[p(\theta) - c(q(\theta))\right] f(\theta)\mathrm{d}\theta. \tag{10}$$

[17] See below for more details.

Lemma 1 *The monopolist's "virtual surplus" in the full-commitment benchmark, denoted by* $\widetilde{v}(\theta, q(\theta))$ *is given by:*

$$\widetilde{v}(\theta, q(\theta)) = \frac{1}{r}\left[\theta - h(\theta)\right] q(\theta) - c(q(\theta)). \tag{11}$$

Lemma 1 shows that $\widetilde{v}(\theta, q(\theta))$ *is equal to consumers' gross utility,* $\frac{1}{r}\theta q(\theta)$*, minus production cost, minus the term* $h(\theta)\frac{1}{r}q(\theta)$*, which measures the effect of selling* $q(\theta)$ *on the aggregate informational rent* (IR)*, which is equal to*

$$IR = \int_{\theta^*}^{\overline{\theta}} \left[h(\theta)\right] \frac{1}{r} q(\theta) f(\theta) \mathrm{d}\theta$$

as obtained in equation (A.7) in "Appendix" (see also (8)[18]*).*

Remark 3 The "virtual surplus" function $\widetilde{v}(\theta, q(\theta))$ is defined only for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ because $h(\theta)$ is not defined neither for $\theta < \underline{\theta}$, nor for $\theta > \overline{\theta}$.

Given that $\theta^*$is the cutoff type, pointwise differentiation of profit (as given in integral (A.8) in "Appendix") with respect to $q(\theta)$ implies that the monopolist must choose, for customers of type $\theta \in \left[\theta^*, \overline{\theta}\right]$, a value $q(\theta) \geq 0$ that maximizes $\widetilde{v}(\theta, q(\theta))$. Define

$$q^m(\theta) = \arg\max_{q \geq 0} \frac{1}{r}\left[\theta - h(\theta)\right] q - c(q).$$

The maximization yields the FOC that characterizes the optimal quality $q^m(\theta)$ to be offered to consumers of type $\theta \in \left[\theta^*, \overline{\theta}\right]$,

$$q^m(\theta) = \begin{cases} (c')^{-1}\left(\theta/r - h(\theta)/r\right) & \text{if } \theta - h(\theta) \geq 0 \\ 0 & \text{if } \theta - h(\theta) < 0 \end{cases} \tag{12}$$

Comparing with the social optimal solution, it becomes evident the important role that information rents optimization plays on the optimal decision of the fully committed monopolist (whose effect is conveyed through the hazard function $h(\theta)$, which does not affect the planner's solution, while shaping the monopolist's quality provision decision).

Result 2 *(i) If*

$$\frac{1}{r}\left[\underline{\theta} - h(\underline{\theta})\right] q^m(\underline{\theta}) - c(q^m(\underline{\theta})) > 0 \tag{13}$$

then $\theta^{*opt} = \underline{\theta}$.

(ii) If $\frac{1}{r}\left[\underline{\theta} - h(\underline{\theta})\right] q^m(\underline{\theta}) - c(q^m(\underline{\theta})) < 0$ *, then* $\overline{\theta} > \theta^{*opt} > \underline{\theta}$.

(iii) For the monopolist to offer strictly positive quality to consumers of type θ*, it is necessary that* $\theta - h(\theta) > 0$ *(though not sufficient if* $c(0) > 0$ *or* $c'(0) > 0$*).*

Proof See "Appendix". □

Example 1 (continued): Assume the uniform distribution of types, with the support $\left[\underline{\theta}, \overline{\theta}\right]$.[19] We consider the quadratic cost $c(q) = B + \frac{1}{2}q^2$, so that $\widehat{q} = \sqrt{2B}$ and $\widehat{\theta} = rc'\left(\widehat{q}\right) = r\sqrt{2B}$. Moreover, assume that the market is super-strong:

$$\overline{\theta} > \underline{\theta} > \widehat{\theta}.$$

18 Note that $h(\theta)\frac{1}{r}q(\theta)$ is not the information rent $U(\theta)$ of type θ.

19 With the uniform distribution, $h(\theta) = \overline{\theta} - \theta$, and $\theta - h(\theta) = 2\theta - \overline{\theta}$, which is positive iff $\theta \geq \overline{\theta}/2$.

If a type θ is served, the quality offered to that type is:

$$q^m(\theta) = \frac{1}{r}\left[\theta - h(\theta)\right] = \frac{1}{r}(2\theta - \overline{\theta})$$

and the cost of producing the quality level offered to this type θ is

$$c\left(q^m(\theta)\right) = B + \frac{(2\theta - \overline{\theta})^2}{2r^2}.$$

Suppose for now that $\underline{\theta} = \overline{\theta}/2$. Then, if $B = 0$, the market will be fully covered, with $q^m\left(\underline{\theta}\right) = 0$ and $q^m(\theta) > 0$ for all $\theta \in \left(\overline{\theta}/2, \overline{\theta}\right]$, with $v(\underline{\theta}) = \frac{1}{r}(\overline{\theta} - \overline{\theta})0 - B - 0 = 0$. If, on the contrary, $B > 0$, then clearly $\underline{\theta}$ cannot be the optimal cutoff type, because, with $B > 0$, we would have $v(\underline{\theta}) = -B < 0$, which means that the necessary condition is not satisfied. It follows that when $B > 0$, the optimal cutoff type θ^{*opt} must be*greater* than $\overline{\theta}/2$.[20] More generally, when $\underline{\theta} \leq \overline{\theta}/2$, if $B > 0$, the (full-commitment) monopolist's optimal cutoff type θ^{*opt} is an interior one

$$\theta^{*opt} = \frac{\overline{\theta} + r\sqrt{2B}}{2} > \frac{2\underline{\theta} + \widehat{\theta}}{2} > \underline{\theta}.$$

What happens if $\underline{\theta} > \overline{\theta}/2$? In that case, the optimal cutoff point is interior (meaning $\theta^{*opt} > \underline{\theta}$) iff

$$\underline{\theta} < \frac{\overline{\theta} + \widehat{\theta}}{2}.$$

Under our linear-quadratic specification, for the Mussa–Rosen static monopolist to cover the whole market, it is necessary and sufficient that $\underline{\theta} \geq \frac{\overline{\theta}+\widehat{\theta}}{2}$. Notice that the assumption that the customer base is super-strong does not ensure that $\underline{\theta} \geq \frac{\overline{\theta}+\widehat{\theta}}{2}$.

For example, consider the numerical example with $\overline{\theta} = 1$, $\underline{\theta} = 0.55$, $B = 2$ and $r = 0.1$. Then $\widehat{\theta} = 0.2$, and

$$\theta^{*opt} = \frac{\overline{\theta} + r\sqrt{2B}}{2} = \frac{1 + 0.2}{2} = 0.6 > \underline{\theta},$$

which means that the market is not fully covered. (We would need $\underline{\theta} \geq 0.6$ to get full market coverage). We are now ready to state a useful result:

Claim 1 *For the static Mussa–Rosen monopolist to cover the whole market, i.e.,* $\theta^{*opt} = \underline{\theta}$*, it is necessary (though not sufficient) that the market is super-strong, i.e. ,* $\underline{\theta} > \widehat{\theta}$*.*

Proof See "Appendix". □

Remark 4 A special case which will prove to be of interest is what we call the constrained full commitment monopolist or, equivalently, the constrained static MR monopolist. It only differs from the standard Mussa–Rosen monopolist by the fact that the firm is constrained to serve all the consumers' types $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. The quality offered to type θ customers is still defined by Eq. (12), except that it is offered to all types $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, not only to types $\left[\theta^*, \overline{\theta}\right]$. Even customers whose virtual surplus $\widetilde{v}(\theta, q^m(\theta))$ is negative are served.[21]

[20] To compute the optimal cutoff, we use the condition $\frac{1}{r}(2\theta^{*opt} - \overline{\theta})\left[q^m(\theta^*)\right] - B - \frac{1}{2}\left[q^m(\theta^{*opt})\right]^2 = 0 \Rightarrow \frac{1}{r^2}(2\theta^{*opt} - \overline{\theta})^2 - B - \frac{1}{2r^2}(2\theta^{*opt} - \overline{\theta})^2 = 0$,
implying: $\theta^{*opt} = \frac{\overline{\theta}+r\sqrt{2B}}{2} = \frac{2\underline{\theta}+\widehat{\theta}}{2} > \underline{\theta}$.

[21] Function $\widetilde{v}(.)$ is defined in Lemma 1 by Eq. (11).

Prices of course differ since even the lowest type consumers should prefer to buy the good rather than to exit the market. The equilibrium price is simply:

$$p^C(\theta) = \frac{1}{r}\theta q^m(\theta) - \int_{\underline{\theta}}^{\theta} \frac{1}{r} q^m(\theta')\mathrm{d}\theta'.$$

Comparing $p^C(\theta)$ to the unconstrained price $p(\theta)$ from Eq. (9), we see that their difference is simply $p^C(\theta) - p(\theta) = -\int_{\underline{\theta}}^{\theta^*} \frac{1}{r} q^m(\theta')\mathrm{d}\theta'$ which is strictly negative (and independent of θ) for all θ as long as $\theta^* > \underline{\theta}$, i.e., as long as the full market-coverage constraint is binding. Very intuitively, the monopolist must lower its price by a fixed amount in order to induce low type customers to buy.

We will show in Sects. 5 and 6 that, under certain conditions, there exists an immediate full market-covering Markov perfect equilibrium of our dynamic game which exactly mimics the constrained static Mussa–Rosen equilibrium.

3.2.2 Lack of Commitment by the Monopolist: Participation Constraints in a Multi-period Setting

We have shown that if the monopolist is able to commit to future price-quality schedules, he will commit to the infinite repetition of the initial (i.e., period zero) static Mussa–Rosen schedule. If this schedule implies that there exists a cutoff type $\theta^* > \underline{\theta}$, then, under the commitment policy, the unserved customer types (those θ in the interval $\left[\underline{\theta}, \theta^*\right)$) will remain unserved for ever. Knowing this, all customers of type $\theta \in \left[\theta^*, \overline{\theta}\right]$ will choose to buy the durable at time $t = 0$, as there is no point to wait for future offers that will be the same as the offer at time $t = 0$. In this full-commitment case, the surplus of the cutoff type θ^* is zero.

Now, let us turn to the case of a monopolist that cannot commit to future price-quality schedules. In this case, it is possible that potential consumers in period n (where $n = 0, 1, 2, 3, \ldots$) anticipate that the future price-quality schedules differ from the present one, and consequently, some of them might have the incentive to delay the purchase of the durable to take advantage of a better deal in the future. For a consumer to buy in period n rather than in period $n + 1$, it must be the case that her lifetime surplus if she purchases the durable in period n is at least as great as what she can get if she delays the purchase till the next period. Let us formulate the incentive-compatibility constraints and the participation constraints that must be satisfied for customers who buy in period n.

At the beginning of each period n, the monopolist offers to all potential new customers (those who have not bought the durable in some earlier period) a monotone increasing price-quality schedule $p_n = \phi_n(q)$,where ϕ_n is a mapping from the domain of feasible qualities $[0, \infty)$ to the space of nonnegative prices, $R_+^2 = [0, \infty)$. As is well known, according to the revelation principle, without loss of generality we can restrict attention to mechanisms that induce the consumers to reveal their true type. This means that in any period n, all consumers who have not bought the durable in a previous period are facing a set of (quality, price) pairs $\left(q_n(\widetilde{\theta}), p_n(\widetilde{\theta})\right)$, which depend on their reported type $\widetilde{\theta}$ when buying in period n. A type θ-customer, where $\theta \in \left[\theta_{n+1}, \theta_n\right]$, who purchases the durable in period n would be willing to report her true type if and only if it does not pay to pretend to be a different type, i.e., iff

$$\theta = \arg\max_{\widetilde{\theta}} \frac{1}{r}\theta q_n(\widetilde{\theta}) - p_n(\widetilde{\theta}). \tag{14}$$

Accordingly, facing a truth-inducing menu of (quality, price) pairs, the net utility (over the entire lifetime) of a type $\theta \in [\theta_{n+1}, \theta_n]$ customer who buys the durable good in period n

is $\frac{1}{r}\theta q_n(\theta) - p_n(\theta)$. We denote this by $U_n(\theta)$:

$$U_n(\theta) \equiv \max_{\widetilde{\theta}} \frac{1}{r}\theta q_n(\widetilde{\theta}) - p_n(\widetilde{\theta}) = \frac{1}{r}\theta q_n(\theta) - p_n(\theta). \tag{15}$$

The envelope theorem implies that the quality-price schedule $(q_n(\theta), p_n(\theta))$ is incentive compatible iff

$$U_n'(\theta) = \frac{1}{r} q_n(\theta). \tag{16}$$

Note that by a standard revealed preference argument, $U_n(\theta)$ is convex, which implies $U_n'(\theta)$ is monotone increasing: for any two values θ' and θ'' in $[\theta_{n+1}, \theta_n]$, with $\theta'' > \theta'$, it holds that $q_n(\theta'') \geq q_n(\theta')$, i.e., $q_n(\theta)$ is non-decreasing.

By integrating (16), we find that for all $\theta \in \left[\theta_{n+1}, \theta_n\right]$, it holds that

$$U_n(\theta) = U_n(\theta_{n+1}) + \int_{\theta_{n+1}}^{\theta} \left(\frac{1}{r} q_n(s)\right) \mathrm{d}s. \tag{17}$$

The integral on the RHS of Eq. (17) is the difference between the informational rent (i.e., lifetime surplus) of a type-θ consumer over the informational rent of the marginal type θ_{n+1}. By definition, the latter is indifferent between *(a)* buying in period n, at the bottom of the rung, and *(b)* buying in period $n+1$, at the top of the rung. Notice that, whatever the quality-price schedule $(q_n(\theta), p_n(\theta))$, given (16), (17) and the very definition of θ_{n+1}, any consumer of type $\theta > \theta_{n+1}$ is better off to buy the good in period n rather than waiting to buy later. For later use, let us denote by $R(\theta_n, \theta_{n+1})$ the difference between the informational rent of a customer at the top rung in period n and that of a customer at the bottom rung in period n:

$$R(\theta_n, \theta_{n+1}) \equiv \int_{\theta_{n+1}}^{\theta_n} \frac{1}{r} q_n(s) \mathrm{d}s. \tag{18}$$

A consumer who chooses to delay the purchase in period n by waiting for the next price-quality offer (available in period $n+1$) must forgo the utility flow arising from consuming the service of the durable good in period n. Recall that the length of the monopolist's commitment period is Δ. Let β denote the discount factor across periods, i.e.,

$$\beta \equiv e^{-r\Delta} < 1,$$

where $r > 0$ is the instantaneous rate of discount. Then, a quality-price schedule $(q_n(\theta), p_n(\theta))$ is *incentive-feasible* if, in addition to the incentive compatibility condition (16) (which is equivalent to (17)), it also satisfies the following participation constraint (PC) for first-time buyers in period n:

$$U_n(\theta) \geq \beta \left[\frac{1}{r}\theta q_{n+1}^b(\theta) - p_{n+1}^b(\theta)\right], \forall \theta \in \left[\theta_{n+1}, \theta_n\right], \tag{19}$$

where the RHS of Eq. (19) is the (discounted) lifetime net utility obtained by a consumer of type $\theta \in [\theta_{n+1}, \theta_n]$ who delays her purchase to period $n+1$ and $\left(q_{n+1}^b(\theta), p_{n+1}^b(\theta)\right)$ denote the best price-quality pair which a type-θ consumer may choose in period $n+1$ when she delays her purchase of the durable good to period $n+1$. It can be shown[22] that, for $\theta \in [\theta_{n+1}, \theta_n]$, a type-$\theta$ customer, when assessing the benefits of delaying the purchase of the

[22] See Laussel et al. [18], Claim 1, for a proof of this result (in "Appendix") in a different context (a model with *non-durable* goods, where the monopolist gets full information on consumers' type θ after their first purchase).

durable good to period $n+1$ instead of buying it in period n, will find it optimal (conditional on deviating) to identify herself as the highest type among the set of consumers getting the good in period $n+1$, or equivalently, $\widetilde{\theta} = \theta_{n+1}$. Thus, it follows that the participation constraint (19) for a type $\theta \in [\theta_{n+1}, \theta_n]$ may be written as

$$\Delta_n(\theta) \equiv U_n(\theta) - \beta \left(\frac{1}{r}\theta q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1}) \right) \geq 0, \tag{20}$$

$\forall \theta \in \left[\theta_{n+1}, \theta_n\right]$.

Since θ_{n+1} is defined as the type who is indifferent between buying the durable good at n or at $n+1$, it holds that, for the (marginal) type θ_{n+1}, condition (20) is satisfied with equality. Thus we have $\Delta_n(\theta_{n+1}) = 0$. Then, to ensure that the participation constraint (20) is satisfied for all the infra-marginal types, $\theta \in (\theta_{n+1}, \theta_n]$, we require that $\Delta_n'(\theta) \geq 0$, which using Eq. (16), is equivalent to $q_n(\theta) \geq \beta q_{n+1}(\theta_{n+1})$. Since $q_n(\theta)$ is non-decreasing, a necessary and sufficient condition for the latter inequality to hold is[23]

$$q_n(\theta_{n+1}) - \beta q_{n+1}(\theta_{n+1}) \geq 0. \tag{21}$$

Inequality (21) requires that the*lowest* quality offered to new customers in period n is greater than the (discounted) *highest* quality offered to new customers in period $n+1$.

Recall that $q_{n+1}(\theta_{n+1})$ is the quality level intended for the highest consumer type among those who purchase the durable in period $n+1$. One may expect that the"*no distortion at the top* " property (which often arises in static settings) also applies in our dynamic model. Consequently, we would expect $q_{n+1}(\theta_{n+1}) = q^{se}(\theta_{n+1})$,where $q^{se}(\theta)$ is the first-best quality for type θ, as defined by Eq. (2). As shown later in this paper, this actually turns out to be the case in equilibrium. When this is indeed the case, constraint (21) can be rewritten as:

$$q_n(\theta_{n+1}) \geq \beta q^{se}(\theta_{n+1}). \tag{22}$$

This inequality requires that the quality level offered to the bottom-rung customer in period n must be at least as large as the (discounted) first-best quality for that type.

Finally, the condition $\Delta_n(\theta_{n+1}) = 0$ implies that

$$U_n(\theta_{n+1}) = \beta U_{n+1}(\theta_{n+1}), \tag{23}$$

in line with the result obtained in Laussel et al. [18] for the case of durable goods where the monopolist is able to uncover consumers' exact quality valuation θ, after her first purchase. Equation (23) indicates that in equilibrium the marginal consumers in period n, those of type θ_{n+1}, are indifferent between being at the top rung of all new consumers in period $n+1$ and being at the bottom rung of all new consumers in period n. Evaluating Eq. (17) at $n+1$, the indifference condition (23) implies:

$$U_n(\theta_{n+1}) = \beta \left(U_{n+1}(\theta_{n+2}) + \int_{\theta_{n+2}}^{\theta_{n+1}} \left[\frac{1}{r} q_{n+1}(\theta) \right] \mathrm{d}\theta \right). \tag{24}$$

Using (24) and (18), we deduce that

$$U_n(\theta_{n+1}) = \sum_{j=1}^{\infty} \beta^j \left(\int_{\theta_{n+j+1}}^{\theta_{n+j}} \left[\frac{1}{r} q_{n+j}(\theta) \right] \mathrm{d}\theta \right) \tag{25}$$

23 A similar condition has been obtained in Laussel et al. [18] for the case of a monopolist that sells non-durable goods (and gets full information on consumers' preferences after their first purchase).

$$= \sum_{j=1}^{\infty} \beta^j R(\theta_{n+j}, \theta_{n+j+1}). \tag{26}$$

Recall that $U_n(\theta_{n+1})$ is the rent of the marginal new customer in period n. The above equation shows that, in equilibrium, this rent must be equal to the discounted sum (over j) of the differences between the informational rent of the customer who has purchased the good at the top rung and the one at the bottom rung in period $n + j$, similarly to what was obtained in Laussel et al. [18] and companion papers (although the authors studied repeated consumption of a non-durable good).

The profit (evaluated at the beginning of period n) which the monopolist makes in period n from selling the durable good to customers in period n is

$$\pi_n = \int_{\theta_{n+1}}^{\theta_n} \left[(p_n(\theta) - c(q_n(\theta))\right] f(\theta) \mathrm{d}\theta.$$

Using Eq. (15), π_n can be rewritten as

$$\pi_n = \int_{\theta_{n+1}}^{\theta_n} \left[\frac{1}{r}\theta q_n(\theta) - c(q_n(\theta)) - U_n(\theta)\right] f(\theta) \mathrm{d}\theta. \tag{27}$$

After integration by parts, we obtain

$$\pi_n = \int_{\theta_{n+1}}^{\theta_n} \left\{ \begin{array}{c} \frac{1}{r}\left[\theta - h(\theta, \theta_n)\right] q_n(\theta) \\ -c(q_n(\theta)) - U_n(\theta_{n+1}) \end{array} \right\} f(\theta) \mathrm{d}\theta, \tag{28}$$

where $U_n(\theta_{n+1})$ is given by Eq. (25), and where the function $h(\theta; \theta_n)$ is defined by

$$h(\theta; \theta_n) \equiv \frac{F(\theta_n) - F(\theta)}{f(\theta)} \text{ for } \theta \in \left[\theta_n, \overline{\theta}\right]. \tag{29}$$

In the next section, we study the Markov perfect equilibria arising in a monopoly under asymmetric information. Throughout the subsequent analysis, we assume that the monopolist is unable to commit to future prices and qualities.

4 Markov Perfect Equilibria Under Non-commitment

Having spelled out the participation constraints, let us now explore possible Markov perfect equilibria when the monopolist cannot make commitment beyond the current period of length Δ, and consumers have rational expectations.

4.1 The Monopolist's Markovian Cutoff Rule and Consumers' Expectation Rule

Now, in any period n, the firm cannot commit to offer in subsequent periods $j > n$ to potentially new consumers (those who have not purchased the durable) the same price quality schedule which it offers to consumers in period n. Thus, in any period, the firm will only offer (quality, price) pairs that are optimal for the intertemporal profit maximization problem starting from that period onwards. We ask the following questions: Can the repetition of the static Mussa–Rosen equilibrium constitute an equilibrium under non-commitment? If so, under which conditions?

In order to investigate this matter, we focus on equilibria in which the monopolist's strategy is a Markovian function of a state variable, $\Theta(n)$, which denotes the lowest type among customers who have bought the durable in the periods prior to n, allowing us to follow the market expansion at each point in time (or equivalently, the fraction of customers who remain out of the market). We assume that in each period, the monopolist is the *first mover*, announcing the additional number of customers that he will serve (namely, $\Theta(n) - \Theta(n+1)$), and the menu of (quality, price) to be offered to these new customers (he is able to commit to this for the duration Δ, which establishes the length of the commitment period). Consumers are *second movers* within the same period: having seen the monopolist's move, they decide whether to buy or to wait, based on their forecast future rent (their forecasting being a function of $\Theta(n+1)$ which the monopolist is able to commit to, as long as he remains within his the commitment period of length Δ). This means that our game has the *stage-wise Stackelberg leadership property*.[24] We shall see later that equilibrium properties under this assumption of sequential moves actually lead to different results from the ones obtained under simultaneous moves.

In the sequential equilibrium, the consumers' expectations function must be a best reply to the monopolist's strategy, such that the consumers expectations are correct given the monopolist's strategy, i.e., they are rational. Moreover, starting from any (date, state) pair $(n, \Theta(n))$, the monopolist's strategy maximizes its profit, given the anticipated consumers' expectations function. Let us now formally define below the state variable, the monopolist's strategy, and the consumers expectation function.

Letting, in any period n, $X(n) \in [0, 1]$ denote the fraction of the customer base that has purchased the product in previous periods $0, 1, 2, .., n-1$, with $X(0) = 0$, the state variable $\Theta(n)$ is defined in the following simple way:

$$\Theta(n) \equiv F^{-1}(1 - X(n)),$$

where $F(.)$ is the cumulative distribution of θ. Then $\Theta(0) = \overline{\theta} \equiv \theta_0$, and $\Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$.

We specify that the firm's Markovian strategy is a pair (ψ, η), that consists of two components: (a) a *Markovian cutoff rule* $\psi(.)$, which, at the beginning of each period n, given $\Theta(n)$, specifies the next $\Theta(n+1)$, thus determining the fraction of the currently unserved customer base that will be served in period n and (b) a *Markovian quality-schedule rule* $\eta(.)$, defining the monopolist's type-dependent quality offers to consumers who buy the durable good in period n.

In other words, the firm's Markovian cutoff rule $\psi(.)$ is a function, non-increasing and bounded below by $\underline{\theta}$, which maps any currently observed value $\Theta(n)$, into a value $\Theta(n+1) = \psi(\Theta(n)) \leq \Theta(n)$. The value $\Theta(n+1) = \psi(\Theta(n))$ is to be interpreted as the lowest-type of customers that the firm intends to serve in period n.

A quality-schedule mapping $\eta(.)$ determines for any given currently observed value $\Theta(n)$ an associated quality schedule $q_n(.|\Theta(n))$, which is itself a function that assigns to each $\theta \in [\Theta(n+1), \Theta(n)]$ a value $q \geq 0$, where $\Theta(n+1) = \psi(\Theta(n))$. The value $q_n(\theta|\Theta(n))$ is to be interpreted as the quality level offered to a customer of type $\theta \in [\Theta(n+1), \Theta(n)]$ in period n, given the value $\Theta(n)$ of the state variable.

The consumers' Markovian expectations rule, denoted by $\Phi(.)$, predicts, given the anticipated value $\Theta(n+1)$ of the state variable, the lifetime rent of the marginal first-time customer in period n, i.e., $U_n(\theta_{n+1})$. The function $\Phi(.)$ maps $\Theta(n+1)$ into the set of positive real numbers. We interpret $\Phi(\Theta(n+1))$ as the predicted value of the lifetime rent of the marginal customer in period n who is, in equilibrium, indifferent between (a) buying the durable good

[24] See, e.g., Long [21] for an exposition of the concept of stage-wise Stackelberg leadership and for a review of that literature.

in period n, located at the bottom rung of the set of shoppers in period n, and (b) being a top-rung shopper in period $n+1$.

In light of these definitions, we are now ready to formally introduce the stage-wise Stackelberg leadership assumption:

Assumption A5 (Stage-wise Stackelberg Leadership Assumption): At each stage n, given $\Theta(n)$, the monopolist moves first and announces a value $\Theta(n+1) \leq \Theta(n)$, and then, consumers' expectations of the period n marginal customer's lifetime net surplus are given by $U_n(\theta_{n+1}) = \Phi(\Theta(n+1))$.

Assumption A5 is crucial for the analysis that follows. An alternative to the stage-wise Stackelberg leadership assumption would be the stage-wise simultaneous moves assumption, that is, replacing A5 by the assumption that consumers in period n form their expectations of $U_n(\theta_{n+1})$ by using a rule $\widetilde{\Phi}(.)$ based on the currently observed state variable, $\Theta(n)$, i.e., $U_n(\theta_{n+1}) = \widetilde{\Phi}(\Theta(n))$ instead of $U_n(\theta_{n+1}) = \Phi(\Theta(n+1))$. Later, we will comment briefly on some implications of such an alternative formulation. Returning to the investigation of the properties of the MPE, when Assumption A5 holds, we have that when consumers have the ability to perfectly anticipate future market outcomes, condition (25) implies that the expectations function $\Phi(.)$ must reflect rational expectations, i.e.,

$$\Phi(\Theta(n+1)) = \sum_{j=1}^{\infty} \beta^j \left(\int_{\Theta^*(n+j+1)}^{\Theta^*(n+j)} \left[\frac{1}{r} q_{n+j}(s) \right] \mathrm{d}s \right) = U_n(\theta_{n+1}), \tag{30}$$

where $\{\Theta^*(.)\}_{n+1}^{\infty}$ is the path of the state variable Θ induced by the strategic behavior of the monopolist from period n, when the state variable takes the value $\Theta(n)$, and where

$$q_{n+j}(s) = q_{n+j}\left(s|\Theta^*(n+j)\right),$$

i.e., the quality schedule that the consumers expect to be offered in period $n+j$ is the same as the schedule that the monopolist's equilibrium strategy would select. Note that, due to (24), $\Phi(\Theta(n+1))$ satisfies the following equation:

$$\Phi(\Theta(n+1)) = \beta \left(\Phi(\Theta(n+2)) + \int_{\theta_{n+2}}^{\theta_{n+1}} \left[\frac{1}{r} q_{n+1}(s) \right] \mathrm{d}s \right). \tag{31}$$

In what follows, we analyze in more detail the firm's optimal Markovian sales and quality strategies. On the firm's side, a Markovian strategy $(\eta(.), \psi(.))$ maximizes the monopolist's profit, given the anticipated consumer expectations function $\Phi(.)$ if (a) it yields a sequence of cutoff values θ_{n+1} and schedules $q_n(.)$ that maximize the monopolist's expected profits from any starting (date, state) pair $(n, \Theta(n))$, and (b) the rational expectations condition (30) is satisfied by such a sequence.

Using our definition of the modified inverse hazard rate, i.e., Eq. (29), the Bellman equation for the monopolist is then

$$V(\Theta(n) = \max_{q_n(.),\theta_{n+1}} \left\{ \begin{array}{c} \int_{\theta_{n+1}}^{\theta_n} \left[\begin{array}{c} \frac{\theta - h(\theta;\Theta(n))}{r} q_n(\theta|\Theta(n)) \\ -c(q_n(\theta|\Theta(n))) - \Phi(\Theta(n)) \end{array} \right] f(\theta)\mathrm{d}\theta \\ +\beta V(\Theta(n+1)) \end{array} \right\}, \tag{32}$$

where the RHS is to be maximized with respect to $q_n(\theta|\Theta(n))$ and $\Theta(n+1)$, subject to the constraint

$$q_n(\Theta(n+1)|\Theta(n)) \geq \beta q_{n+1}(\Theta(n+1)|\Theta(n+1)). \tag{33}$$

Given $\Theta(n)$, pointwise maximization of the RHS of the Bellman equation with respect to $q_n(\theta|\Theta(n))$ subject to the constraint (33) yields the necessary condition that determines the quality offered to customers of type θ in period n . Clearly, if constraint (33) is not binding for a type $\theta \in [\Theta(n+1), \Theta(n)]$, then the monopolist's optimal quality for that type is given by:

$$q^{**}(\theta|\Theta(n)) \equiv c'^{-1}\left[\frac{\theta - h(\theta;\Theta(n))}{r}\right]. \tag{34}$$

It follows that, if

$$q^{**}(\Theta(n+1)|\Theta(n)) > \beta q^{**}(\Theta(n+1)|\Theta(n+1)) \equiv \beta c'^{-1}\left[\frac{\Theta(n+1)}{r}\right],$$

then the monopolist's offers, $q^{**}(\theta|\Theta(n))$ for all types $\theta \in [\Theta(n+1), \Theta(n))$, satisfy constraint (33) with strict inequality.

If, on the contrary, there exists a type $\theta^{**}(\theta_n, \theta_{n+1}) \in [\Theta(n+1), \Theta(n))$ such that

$$q^{**}(\theta^{**}(\theta_n, \theta_{n+1})|\Theta(n)) = \beta q^{**}(\Theta(n+1)|\Theta(n+1)),$$

then the monopolist's optimal offers are equal to $q^{**}(\theta|\Theta(n))$ only for the types $\theta \in \left[\theta^{**}(\theta_n, \theta_{n+1}), \Theta(n)\right]$, while offers for consumers whose type belong to $\Big[\Theta(n+1), \theta^{**}(\theta_n, \theta_{n+1})\Big]$ are bunched. It follows that the quality offered to type θ-new customers in period n is given by:

$$q^m(\theta|\Theta(n)) = \max\{q^{**}(\theta|\Theta(n), \beta q^{se}(\Theta(n+1))\}. \tag{35}$$

It should also be noticed that consumers at the top of the rung are always offered the first-best quality, i.e.,

$$q^m(\Theta(n)|\Theta(n)) = q^{se}(\Theta(n)).$$

5 Immediate Full Market-Covering Markov Perfect Equilibria

In this section, keeping Assumptions A1 to A5, we investigate which conditions may ensure that, along the equilibrium play, the market is fully covered in the initial period, $n = 0$, as in the conventional Mussa–Rosen setting. For this to hold, the monopolist's Markov perfect equilibrium strategy must be such that, at the beginning of any period $n \geq 0$, given any observed value of the concurrent state variable $\Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$, the monopolist's equilibrium cutoff rule is $\psi(\Theta(n)) = \underline{\theta}$, i.e., all the customers that have not made a purchase will be served in period n . The consumers, having received the message sent by the monopolist at the beginning of period n, then rationally expect, in period n, that if for some reason (e.g., because of "trembling hands") the lowest type of consumers that actually make a purchase in period n (denoted by $\Theta(n+1)$) turns out to be higher than $\underline{\theta}$, then in any case the market will be covered in the following period, so that the lifetime surplus of type θ_{n+1} consumers is equal to the (discounted) surplus she will get if she delays her purchase until period $n+1$. This means that Eq. (31) reduces to:

$$\Phi(\Theta(n+1)) = \beta \int_{\underline{\theta}}^{\theta_{n+1}} \left[\frac{1}{r} q_{n+1}(s)\right] \mathrm{d}s. \tag{36}$$

According to the stage-wise Stackelberg leadership assumption, the monopolist, when choosing in period n the value of $\Theta(n+1)$, i.e., the number of new customers it will serve in that period, is fully taking into account the effect of this choice on the lifetime rent of the marginal customer in period n, i.e., it accounts for (36). This assumption is crucial. Without it, immediate full market-covering equilibria may not happen even in the case where a static Mussa–Rosen monopolist would cover the whole market, as we shall show later.

Given θ_n which is observed at the beginning of period n, if the firm chooses to cover immediately the market in period n (i.e., even the lowest type, $\underline{\theta}$, will be served in period n), its profit for period n will be[25]

$$Z(\theta_n, \underline{\theta}) \equiv \int_{\underline{\theta}}^{\theta_n} \left[\frac{\theta - h(\theta; \Theta(n))}{r} q^m(\theta|\Theta(n)) - c(q^m(\theta|\Theta(n))) \right] f(\theta) \mathrm{d}\theta, \qquad (37)$$

where $q^m(\theta|\Theta(n))$ is defined by (35) and $h(\theta; \Theta(n))$ is the modified inverse hazard rate such as defined by Eq. (29).

It will be convenient to refer to the bracketed term in Eq. (37) as $v(\theta; \Theta(n))$ and call it the virtual surplus (starting from the beginning of period n, given that all types $\theta \in [\Theta(n), \overline{\theta}]$ have bought their durable goods):

$$v(\theta; \Theta(n)) \equiv \frac{[\theta - h(\theta; \Theta(n))]}{r} q^m(\theta|\Theta(n)) - c(q^m(\theta|\Theta(n))).$$

Notice that the quality schedule $q^m(\theta|\Theta(n))$ is identical to the static Mussa–Rosen's schedule if $\Theta(n) = \overline{\theta}$. We are here studying the Markov perfect equilibrium of a monopolist who cannot commit to future offers. An interesting feature of MPE when the firm cannot commit to future offers is that, as we will demonstrate, it may be optimal for the firm to fully cover the market immediately, even if the virtual surplus function $v(\theta; \Theta(n))$ turns out to be negative for a subset of types $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon]$ for some strictly positive ε.

This is in sharp contrast to the case of a static (or full-commitment) Mussa–Rosen monopolist, who would optimally choose not to make any offers to these low-type consumers but this exactly corresponds to the constrained static MR monopolist, defined in Remark 4. The reason why in a MPE a monopolist may be willing to make a loss on its sales to the low types, $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon]$ in order to fully cover the market is as follows. If the monopolist leaves these customers unserved in period 0, then at the beginning of period 1, his future self may be tempted to serve them, offering them higher quality at attractive prices. Anticipating this, higher-type consumers in a subset of $(\underline{\theta} + \varepsilon, \overline{\theta}]$ may deviate, by refraining from buying in period 0. To prevent this deviation, the monopolist may find it advantageous to strategically sell to all types of consumers in period 0.

To study the conditions under which full market coverage is optimal, it is useful to note some properties of the function $v(\theta; \Theta(n))$.

Lemma 2 *(i) The function $v(\theta; \Theta(n))$ is decreasing in $\Theta(n)$ and increasing in θ;*

(ii) Customers of type θ_n, if they have not bought the good prior to period n, will be offered the socially efficient quality $q^{se}(\theta_n)$, implying no distortion at the top;

(iii) If Assumption A4 holds (i.e., the customer base is strong), starting from any given $\theta_n \in (\underline{\theta}, \overline{\theta}]$, we obtain that the profit from full market-coverage is positive.

Example 1 (continued): In the linear-quadratic case, straightforward computations show that profit (37) from full market coverage equals

$$\frac{1}{r^2}(\Theta(n) - \underline{\theta}) \left[(\Theta(n) - \underline{\theta})^2 + 3 \left(\underline{\theta}^2 - (\widehat{\theta})^2 \right) \right],$$

[25] This follows from Eq. (28), where θ_{n+1} is replaced with $\underline{\theta}$, and where $U(\underline{\theta}) = 0$.

which is always positive if $\underline{\theta} \geq \widehat{\theta}$, i.e., if the customer base is strong.

Having proved that the monopolist never loses money by fully covering the market does not necessarily mean that an immediate full market-coverage is the profit-maximizing strategy. In order to prove that the monopolist's immediate full market-coverage strategy, $\psi(\Theta(n)) = \underline{\theta}$, $\forall(\Theta(n)) = \underline{\theta} \in (\underline{\theta}, \overline{\theta}]$, is the equilibrium cutoff strategy, it is necessary and sufficient to rule out a *one-shot* deviation to some value $\Theta(n+1) \in (\underline{\theta}, \Theta(n)]$, i.e., to show that it is better for the monopolist to fully cover the market in period n rather than in period $n+1$. Notice that it is not enough to show that this holds when $\Theta(n) = \overline{\theta}$, since one must prove that the candidate equilibrium strategy of full market coverage satisfies the subgame perfection requirement, i.e., a deviation from it would not be profitable.

What is then the monopolist's profit for such a deviation? In light of constraint (22), two cases should be considered and therefore we obtain two different sufficient conditions which are given in Proposition 1.[26] The two cases are analyzed in detail in "Appendix" (see the proof of Proposition 1).

Proposition 1 *Under Assumptions A4 and A5, immediate full market-coverage is the unique Markov perfect equilibrium strategy if*

(i) either the static Mussa–Rosen monopolist would cover all the market, i.e., $\underline{\theta} = \theta^*$*;*
(ii) or, despite the fact that the static Mussa–Rosen monopolist would not cover all the market, i.e., $\underline{\theta} < \theta^*$*, the discount factor* β *is close enough to 1.*

Proof See "Appendix". □

Proposition 1 establishes that when there exists an immediate full market-coverage Markov perfect equilibrium of the game it is the unique one and that this MPE corresponds to the constrained Mussa–Rosen static monopoly equilibrium defined in Remark 4. The game exhibits basically non-Coasian features. On the one hand, when the length Δ of the commitment period is short enough (so that the discount factor is close enough to 1), instantaneous full market-coverage is a Markov perfect equilibrium of the game, which at first sight might seem to look like a Coasian feature, but a more careful reflection reveals that the underlying mechanism is completely different: it is a strategic choice of the monopolist which yields an implicit credible commitment not to lower future prices (since all customers are here served at the beginning of the game when β is sufficiently close to 1, or equivalently, Δ is short enough).[27] On the other hand, the monopolist's strategy of covering immediately the whole market avoids the erosion of profits, which remain always strictly positive, whereas in the No Gap case of the standard model they tend to zero as the time between two period becomes infinitesimal. This is clearly a non-Coasian feature of our MPE.

The last part of the sufficient condition (ii) in the above Proposition turns out to be somewhat too restrictive. Indeed, as will be shown in the following section, which considers the linear quadratic specification, in the case when the static Mussa–Rosen monopolist would not cover all the market but the customer base is strong (A4 holds), the discount factor needs not be very high for immediate full market-coverage to be a Markov perfect equilibrium of the game.

[26] The first case (Case A) arises when $\Theta(n+1)$ is such that $q^m(\Theta(n+1)|\Theta(n)) > \beta q^{se}(\Theta(n+1))$, whereas the second case (Case B) arises when $\Theta(n+1)$ is such that $q^m(\Theta(n+1)|\Theta(n)) < \beta q^{se}(\Theta(n+1))$.

[27] It should be noticed in addition that this occurs here in one period (the initial one) while in the standard durable model it takes (in the limit, as the duration of each period becomes infinitesimal) an infinite number of periods.

Remark 5 If Assumption A5 (the stage-wise Stackelberg leadership assumption) were dropped and, instead, $U_n(\theta_{n+1}) = \Phi(\Theta(n))$ (i.e., the monopolist and the consumers move simultaneously within each period), then condition $\theta^* = \underline{\theta}$ in part (i) would become necessary but it would cease to be sufficient for immediate full market-coverage to be a Markov perfect equilibrium strategy.

Necessity is rather obvious. Assume indeed to the contrary that $\theta^* > \underline{\theta}$. Then, in the initial period, the monopolist could deviate to $\Theta(1) = \theta^*$, not serving in that period consumers for which the virtual surplus is negative while keeping, due to expectations slackness, the rent to marginal customers equal to zero and then cover in period 1 the rest of the market with a positive benefit (provided that the market base is strong).[28] Such a deviation ensures greater profits than full market coverage in the initial period. To show that, under the simultaneous move specification, the condition $\theta^* = \underline{\theta}$ is not sufficient, it is enough to provide an example. To this end, let us return to Example 1 and consider $\overline{\theta} = 4$, $\underline{\theta} = 3$, $B = 0.5$ and $r = 1$. It is easy to check that $\frac{\overline{\theta}+r\sqrt{2B}}{2} = 2.5$ so that a static (or full-commitment) Mussa–Rosen monopolist would serve all consumers in the initial period. Without full commitment, monopolist's aggregate profit from serving in the initial period only consumers with $\theta \in [\Theta(1), 4]$ for some $\Theta(1) \in [3, 4]$, and serving the remaining customers in the next period equals

$$\frac{1}{6}\left[(52 - 99\beta) + \Theta(1)^2(24 - 9\beta) + \Theta(1)^3(-4 + \beta) + \Theta(1)(-45 + 51\beta)\right].$$

This expression takes its maximum at $\Theta(1) = 3$ iff $\beta \leq 3/8$. This shows that immediate full market coverage is not a Markov perfect equilibrium strategy for all $\beta > 3/8$.

There is a simple intuition for $\theta^* = \underline{\theta}$ not being a sufficient condition for immediate full market coverage equilibrium when consumers and the monopolist make simultaneous moves. Deviating allows the firm to reoptimize and select in period 1 a quality-schedule $q^m(\theta|\Theta(1))$ for types $\theta \in \left[\underline{\theta}, \Theta(1)\right]$ which gives it greater profits per period than the quality schedule $q^m(\theta|\overline{\theta})$. The cost of this deviation is that subsequent profits are worth less than profits in period 1 as long as the discount factor $\beta < 1$. Accordingly, the deviation is profitable when the discount factor is high enough (in the numerical example above, we should have $\beta > \frac{3}{8}$). Not surprisingly, under consumers' expectations slackness, immediate full market coverage requires both that the Mussa–Rosen static monopolist would cover the whole market and that the agents discount the future heavily enough, so that the model is close enough to a static one.

6 The Linear-Quadratic Case

Let us now focus on the more tractable linear-quadratic case which has already been presented in Example 1. Proposition 2 provides *necessary* and sufficient conditions for immediate full-market coverage to be a Markov perfect equilibrium strategy under the stage-wise Stackelberg leadership property (Assumption A5). In Remark 6, we will comment again on the alternative scenario where consumers' expectations depend on the state variable at the beginning of the period ($\Theta(n)$) rather than at the end ($\Theta(n + 1)$).

Proposition 2 *Under Assumption A5 and the assumption that the customer base is super-strong (i.e., $\widehat{\theta} < \underline{\theta} < \overline{\theta}$), in the linear-quadratic case,*

[28] The proof is analogous to the proof of Lemma 1.

(i) *if* $\beta \in (\overline{\beta}, 1]$, *where we define* $\overline{\beta} \equiv \left(\frac{\widehat{\theta}}{\underline{\theta}}\right)^2 < 1$, *immediate full market coverage is a Markov perfect equilibrium strategy, regardless of how big the ratio* $\overline{\theta}/\underline{\theta}$ *is.*

(ii) *if* $\beta \in [0, \overline{\beta}]$, *immediate full market coverage is a Markov perfect equilibrium strategy if* $\overline{\theta}/\underline{\theta} \leq \left[(2-\beta) - \sqrt{(1-\beta)(\overline{\beta}-\beta)}\right] \equiv \mu(\beta)$.

(iii) *for full market coverage to be a Markov perfect equilibrium strategy, it is necessary that either* $\beta > \overline{\beta}$ *or* $\overline{\theta}/\underline{\theta} \leq \mu(\beta)$.

Proof See "Appendix". □

Proposition 2 is not a simple corollary of Proposition 1. It provides conditions which are not only sufficient but necessary as well. These conditions are less restrictive than the ones in Proposition 1. Consider, for instance, the condition $\overline{\theta}/\underline{\theta} \leq \mu(\beta)$. When $c(0) = 0$, the inequality $\overline{\theta}/\underline{\theta} \leq \mu(\beta)$ is equivalent to the condition that the static Mussa–Rosen monopolist would cover the whole market if (but not only if) $\beta = 0$. With $\beta = 0$, the sufficient condition in (ii) above reduces to $\overline{\theta}/\underline{\theta} \leq (2 - \sqrt{\overline{\beta}}) \Leftrightarrow \underline{\theta} \geq \frac{\overline{\theta}+\widehat{\theta}}{2}$. Since $\mu(\beta)$ is increasing in β for all $\beta \in [0, \overline{\beta}]$, the condition in Proposition 2 in less stringent than the one in Proposition 1 (which requires that β is close to 1). On the other hand, it is clear from Proposition 2 that in the linear-quadratic case the condition that β is close enough to 1 has only some bite when $\overline{\beta} < 1$, i.e., when the customer base is super-strong.

In order to highlight even more the role of the stage-wise Stackelberg assumption versus the simultaneous moves assumption (slackness of consumers' expectations), we show in Remark 6 that in the latter case the conditions for the existence of an immediate full market coverage equilibrium are much more restrictive that in the former one.

Remark 6 Consider again Example 1. Then, under slack consumers' expectations, immediate full market coverage is a Markov perfect equilibrium strategy iff (i) $\frac{\overline{\theta}+r\sqrt{2B}}{2} \leq \underline{\theta}$ (the Mussa–Rosen static monopolist would cover the whole market) and (ii) $\beta \leq \frac{(\overline{\theta}-2\underline{\theta})^2-2r^2B}{\underline{\theta}^2-2r^2B} < 1$ (the discount factor is small enough).[29]

Proof See "Appendix". □

Remark 6 simply shows that, under slack consumers' expectations (i.e., simultaneous moves within each period), immediate full market coverage occurs only if the model is close enough to a static model where the monopolist would cover the whole market. The condition that the discount factor should be low enough goes opposite to the sufficient condition in the stage-wise Stackelberg leadership case that immediate full coverage occurs if the discount factor is close enough to 1. The condition in that case is much more restrictive than the ones in Proposition 2.

Finally it may be interesting to compare the equilibrium price-quality under full commitment (the unconstrained MR static monopoly equilibrium) and no commitment (the immediate full market-covering equilibrium). This is done below in the linear-quadratic case when $\overline{\theta} = 10$, $\underline{\theta} = 5$, $c(q) = 2 + \frac{1}{2}q^2$, $r = 1$. Under full commitment (blue curve), a range of qualities $q \in [2, 10]$ are offered at price $p(q) = \frac{1}{4}(10+q)^2 - 24$. Under no commitment (red curve), a wider range of qualities $q \in [0, 10]$ are offered at a price $p(q) = \frac{1}{4}(10+q)^2 - 25$. In the common range of qualities, the price under no commitment is smaller than the price under full commitment by a fixed amount.

[29] Notice that the numerator is positive since $\underline{\theta} \geq \frac{\overline{\theta}+r\sqrt{2B}}{2}$ (full market coverage by the static MR monopolist) and the denominator is positive as well since the market is strong.

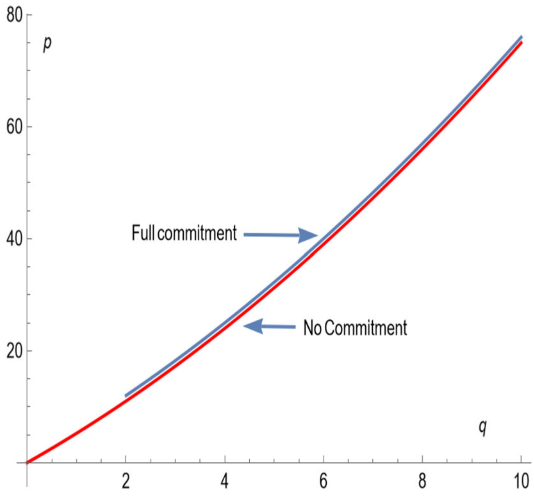

Fig. 1 The price-quality schedules under commitment and non-commitment

Figure 1 illustrates that our immediate full market-covering MPE corresponds exactly to the equilibrium of a Mussa–Rosen static monopolist who is constrained to cover the whole market and that the quality schedule $q(\theta)$ is the same in our model as in the unconstrained MR model (except for the ranges of value of θ), while the price $p(\theta)$ is smaller by a fixed amount.

7 Concluding Remarks

This paper enriches the literature on durable goods monopoly by considering a model in which a durable good monopolist, facing a continuum of customers with private information about their preferences, has the technological ability to offer a continuum of quality levels for the durable good. One of our findings is that, thanks to this additional ability to practice intra-period price discrimination with a price-quality menu, the monopolist's inability to commit to future price offers does not lead to the erosion of monopoly profit (unlike the standard one-quality level durable good monopoly analyzed by Coase, where the lack of commitment leads to lower aggregate profit, which, in the No Gap case, shrinks to zero as the length of the commitment period tends to zero).

Moreover, we found two alternative sufficient conditions for immediate full market coverage to be a Markov perfect equilibrium of the game. Interestingly, such a Markov perfect equilibrium exhibits both Coasian and non-Coasian features. When the length Δ of the commitment period is short enough (so that the discount factor is close enough to 1), instantaneous full market-coverage is a Markov perfect equilibrium of the game, which is a seemingly Coasian feature, but which occurs here under different conditions and for completely different reasons. Moreover, the monopolist's profits under immediate full market coverage remains always strictly positive, which is definitely a non-Coasian feature. The standard Coasian result of profit erosion (under the traditional one-quality assumption) has been interpreted as an outcome of competition between the present monopolist and his future selves. It turns out that this classic time-inconsistency problem in intertemporal price discrimination can be mitigated by replacing intertemporal price discrimination with intra-period price-and-quality discrimination within each cohort of customers, as our model has shown. By covering immediately the whole market, the monopolist is indeed able to commit not to lower its price in subsequent periods: by serving low-type consumers at a loss, he is able to make greater profits on high type ones.

Our result that having the ability to practice intra-period price discrimination mitigates against the curse of time-inconsistency suggests that the lack of commitment on future prices may encourage a durable good monopolist to invest in technology that enables the supply of many versions of the durable good (each with a different quality level), shedding light on an additional theoretically plausible explanation to justify firms' recent investments in mass customization technologies within a wide variety of industries.

Appendix

Proof of Result 1. Clearly, the solution of the social welfare maximization is as follows. For each type $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, the planner must choose $\delta(\theta) \in \{0, 1\}$ and $q(\theta) \in [0, \infty)$ to maximize

$$\delta(\theta)\left[\frac{1}{r}\theta q(\theta) - c(q(\theta))\right]; \tag{A.1}$$

Consider first the case where $c(0) > 0$, i.e., $c(q) = B + g(q)$ where $B > 0$ and $g(0) = 0$, $g'(q) > 0$ and $g''(q) > 0$. Then, there exists a unique $\widehat{q} > 0$ such that $c'(\widehat{q}) = c(\widehat{q})/\widehat{q}$. Let us define a critical consumer type, $\widehat{\theta}$, by the condition that

$$\frac{1}{r}\widehat{\theta} = c'(\widehat{q}). \tag{A.2}$$

Graphically, the straight line $\frac{1}{r}\widehat{\theta}q$ (with slope $\widehat{\theta}/r$) is tangent to the strictly convex cost curve $c(q)$ at $q = \widehat{q}$. It is clear from this observation that the social planner, in solving the maximization problem (A.1) for type $\widehat{\theta}$, finds that the optimal surplus for this type is zero. This means that the planner is indifferent between (i) allocating to type $\widehat{\theta}$ consumers a unit of durable at quality $\widehat{q}$, i.e., $\left\{\delta\left(\widehat{\theta}\right) = 1, \text{ with } q\left(\widehat{\theta}\right) = \widehat{q}\right\}$, requiring them to meet the cost $c(\widehat{q})$, or (ii) not allocating to them any unit of durable good, i.e., $\delta\left(\widehat{\theta}\right) = 0$. Either action yields a net lifetime utility of zero for these consumers. It follows from the convexity of $c(q)$ that, for consumers of type $\theta > \widehat{\theta}$, the planner's optimal solution is $\delta(\theta) = 1$, and they are allocated a unit of durable of quality level $q^{se}(\theta)$, which is defined by the condition

$$\frac{1}{r}\theta = c'\left(q^{se}(\theta)\right), \theta \in \left(\widehat{\theta}, \overline{\theta}\right], \tag{A.3}$$

where the superscript in $q^{se}(\theta)$ indicates that it is the socially efficient quality level. These consumers gain strictly positive lifetime net utility $(\theta/r)q^{se}(\theta) - c\left(q^{se}(\theta)\right)$. By the same token, for consumers of type $\theta < \widehat{\theta}$, the planner's optimal solution is $\delta(\theta) = 0$. Such consumers are not served, and their lifetime net utility is zero.

In the case where $c(0) = 0$, we have $\widehat{q} = 0$. Again, $\widehat{\theta}$ is defined by $\widehat{\theta}/r = c'(\widehat{q}) \geq 0$. The planner's optimal allocation of durable goods is the same as in the case where $B > 0$.

In summary, we define $\widehat{\theta}$ by

$$\widehat{\theta} \equiv \begin{cases} \frac{rc(\widehat{q})}{\widehat{q}} & \text{if } c(0) > 0 \\ rc'(0) & \text{if } c(0) = 0 \end{cases} \tag{A.4}$$

and we denote by $q^{se}(\theta)$ the socially efficient quality level for any given type $\theta \geq \widehat{\theta}$, i.e.,

$$q^{se}(\theta) = c'^{-1}(\theta/r) \text{ for } \theta \geq \widehat{\theta}, \tag{A.5}$$

which corresponds to the quality level identified in Result 1. □

Proof of Lemma 1 Substituting for $p(\theta)$, using Eq. (9), we get

$$\pi = \int_{\theta^*}^{\overline{\theta}} \left[\frac{1}{r}\theta q(\theta) - c(q(\theta))\right] f(\theta)\mathrm{d}\theta - \int_{\theta^*}^{\overline{\theta}} \left[\int_{\theta^*}^{\theta} \frac{1}{r} q(\theta')\mathrm{d}\theta'\right] f(\theta)\mathrm{d}\theta. \tag{A.6}$$

The second integral in (A.6) is called "the aggregate informational rents", or IR, that consumers obtain thanks to their private information about their types.

Applying the formula of integration by parts, the second integral in (A.6), which is the aggregate informational rent (over all types), can be written as:

$$IR = \int_{\theta^*}^{\overline{\theta}} [1 - F(\theta)] \frac{1}{r} q(\theta)\mathrm{d}\theta = \int_{\theta^*}^{\overline{\theta}} [h(\theta)] \frac{1}{r} q(\theta) f(\theta)\mathrm{d}\theta. \tag{A.7}$$

Therefore,

$$\pi = \int_{\theta^*}^{\overline{\theta}} \left[\frac{1}{r}\theta q(\theta) - c(q(\theta)) - h(\theta)\frac{1}{r} q(\theta)\right] f(\theta)\mathrm{d}\theta,$$

where, by definition,

$$h(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)}.$$

Rearranging terms, we get

$$\pi = \int_{\theta^*}^{\overline{\theta}} \left[\frac{1}{r} [\theta - h(\theta)] q(\theta) - c(q(\theta))\right] f(\theta)\mathrm{d}\theta. \tag{A.8}$$

We refer to the term inside the brackets [...] as the "virtual surplus" and denote it by $\widetilde{v}(\theta, q(\theta))$:

$$\widetilde{v}(\theta, q(\theta)) = \frac{1}{r} [\theta - h(\theta)] q(\theta) - c(q(\theta)),$$

as pointed out in Lemma 1. □

Proof of Result 2 Let us now define the "optimized virtual surplus" function $v(\theta)$:

$$v(\theta) \equiv \widetilde{v}(\theta, q^m(\theta)).$$

By the envelope theorem, we obtain the result that $v(\theta)$ is increasing in θ[30]:

$$\frac{\mathrm{d}v(\theta)}{\mathrm{d}\theta} = \frac{\partial \widetilde{v}(\theta, q^m(\theta))}{\partial \theta} = \frac{1}{r} \left[1 - h'(\theta)\right] q^m(\theta) \geq 0 \text{ because of A3.}$$

The monopolist's profit, given the cutoff type $\theta^* \in \left[\underline{\theta}, \overline{\theta}\right]$, is then

$$\begin{aligned}\pi(\theta^*) &= \int_{\theta^*}^{\overline{\theta}} \left[\frac{1}{r} [\theta - h(\theta)] q^m(\theta) - c(q^m(\theta))\right] f(\theta)\mathrm{d}\theta \\ &\equiv \int_{\theta^*}^{\overline{\theta}} v(\theta) f(\theta)\mathrm{d}\theta. \end{aligned} \tag{A.9}$$

[30] Note that by Assumption A4 and $h(\overline{\theta}) = 0$, we have $v\left(\overline{\theta}\right) > 0$.

So far, we have considered an arbitrary cutoff type θ^*. Now, let us turn to the monopolist's choice of the optimal cutoff type, denoted by θ^{*opt}.

$$\theta^{*opt} \equiv \arg\max_{\theta^* \geq \underline{\theta}} \pi(\theta^*). \tag{A.10}$$

When θ^* is evaluated at θ^{*opt}, the following FOC conditions must be met:

$$\frac{\mathrm{d}\pi(\theta^*)}{\mathrm{d}\theta^*} \leq 0, \qquad \theta^* - \underline{\theta} \geq 0, \qquad \left[\theta^* - \underline{\theta}\right] \frac{\mathrm{d}\pi(\theta^*)}{\mathrm{d}\theta^*} = 0,$$

since $\frac{\mathrm{d}\pi(\theta^*)}{\mathrm{d}\theta^*} = -v(\theta^*) f(\theta^*)$, and since $f(\theta) > 0$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$ (by Assumption A2), the FOC conditions may be written as

$$v(\theta^*) f(\theta^*) \geq 0, \qquad \theta^* - \underline{\theta} \geq 0, \qquad \left[\theta^* - \underline{\theta}\right] v(\theta^*) = 0. \tag{A.11}$$

That is, the optimal cutoff is implicitly defined by the condition that

$$\frac{1}{r}\left[\theta^{*opt} - h(\theta^{*opt})\right] q^m(\theta^{*opt}) - c(q^m(\theta^{*opt})) = 0 \text{ if } \theta^{*opt} > \underline{\theta} \tag{A.12}$$

$$\geq 0 \text{ if } \theta^{*opt} = \underline{\theta}. \tag{A.13}$$

This allows us to prove each point in Result 2 as follows:

(i) Since $v(\theta)$ is non-decreasing, if $v\left(\underline{\theta}\right) > 0$, then $v(\theta) > 0$ for all $\theta > \underline{\theta}$, which implies that $\theta^{*opt} - \underline{\theta} = 0$;
(ii) Since $v\left(\overline{\theta}\right) > 0$ by Assumption A4, if $v(\underline{\theta}) < 0$, then there exists θ^{*opt} such that $v\left(\theta^{*opt}\right) = 0$;
(iii) This follows from Eq. (11). □

Proof of Claim 1 Since $\frac{\partial v}{\partial \theta} > 0$, the condition $\theta^* = \underline{\theta}$ is equivalent to the condition that $v(\underline{\theta}, q^m(\underline{\theta})) = \frac{1}{r}\left(\underline{\theta} - h(\underline{\theta})\right) q^m(\underline{\theta}) - c(q^m(\underline{\theta})) \geq 0$. Since $h(\theta)$ is monotone decreasing and $h\left(\overline{\theta}\right) = 0$, we have $h(\theta) > 0$ for $\theta < \overline{\theta}$. Therefore, for $v(\underline{\theta}, q^m(\underline{\theta})) \geq 0$ to hold, it is necessary that $\frac{1}{r}\underline{\theta} q^m(\underline{\theta}) - c(q^m(\underline{\theta})) > 0$. This inequality in turn implies that $\frac{1}{r}\underline{\theta} q^{se}(\underline{\theta}) - c(q^{se}(\underline{\theta})) > 0$, because the very definition of q^{se} implies that $\frac{1}{r}\underline{\theta} q^{se}(\underline{\theta}) - c(q^{se}(\underline{\theta})) \geq \frac{1}{r}\underline{\theta} q - c(q)$ for all $q \geq 0$. Finally, the inequality $\frac{1}{r}\underline{\theta} q^{se}(\underline{\theta}) - c(q^{se}(\underline{\theta})) > 0$ is possible if only if $\underline{\theta} > \widehat{\theta}$, by definition of $\widehat{\theta}$. □

Proof of Lemma 2. First, using the Envelope theorem, it is clear that $v(\theta; \Theta(n))$ is decreasing in $\Theta(n)$ and increasing in θ, leading to point (i) in Lemma 2. As far as concerns point (ii) in Lemma 2, note that since $h(\theta_n; \Theta(n)) = 1$, there is no distortion of the top: customers of type θ_n, if they have not bought the good prior to period n, will be offered the socially efficient quality $q^{se}(\theta_n)$; therefore, $v(\theta_n; \Theta(n)) > 0$ because $\theta_n > \underline{\theta} \geq \widehat{\theta}$.

Now, as far as concerns point (iii) either $v(\underline{\theta}; \Theta(n)) > 0$, or $v(\underline{\theta}; \Theta(n)) \leq 0$. If $v(\underline{\theta}; \Theta(n)) > 0$, then all types $\theta \in \left[\underline{\theta}, \Theta(n)\right]$ will be served, and we are done. If $v(\underline{\theta}; \Theta(n)) \leq 0$, then there exists a unique $\theta \in \left[\underline{\theta}; \Theta(n)\right]$ such that $v(\theta, \Theta(n)) = 0$. Then let us denote that value by $\Gamma(\Theta(n))$. Notice that $\Gamma(\overline{\theta}) = \theta^{*opt}$, the optimal cutoff value for the static Mussa Rosen case. Obviously (i) $v(\theta', \Theta(n)) \geq 0$ for all types $\theta' \geq \Gamma(\Theta(n))$ and (ii) $\Gamma(\Theta(n))$ is increasing in $\Theta(n)$ since when applying the implicit function to equation

$$v(\theta; \Theta(n)) = 0,$$

one obtains

$$\Gamma'(\Theta(n)) = \frac{\mathrm{d}\theta}{\mathrm{d}\Theta(n)} = -\frac{v_{\Theta(n)}}{v_\theta} > 0.$$

Then, a necessary condition for an immediate full market-covering strategy $\psi(\Theta(n)) = \underline{\theta}$, $\forall\Theta(n) \in [\underline{\theta}, \overline{\theta}]$ to be an equilibrium strategy is that the monopolist's profit for covering the market immediately, denoted by $Z(\theta_n, \underline{\theta})$, as defined by Eq. (37), be nonnegative. For, if $Z(\theta_n, \underline{\theta})$ is strictly negative, then, for any value of the discount factor $\beta < 1$, the firm would be better off delaying market coverage to the next period. We now show that $Z(\theta_n, \underline{\theta}) \geq 0$ if Assumption A4 holds, i.e., the customer base is "strong.", leading to point (iii) in Lemma 2.

Taking the derivative of (37) with respect to θ_n, and making use of the Envelope theorem and noting that there is no distortion at the top, i.e., for type θ_n, one obtains

$$\frac{\partial Z(\theta_n, \underline{\theta})}{\partial \theta_n} = f(\theta_n)\left[\left(\frac{\theta_n}{r}q^{se}(\theta_n) - c(q^{se}(\theta_n))\right) - \int_{\underline{\theta}}^{\theta_n} \frac{1}{r}q^m(\theta|\Theta(n))\mathrm{d}\theta\right].$$

Under Assumption A4, this derivative is positive if θ_n is evaluated at $\underline{\theta}$. To show that it is positive for any $\theta_n > \underline{\theta}$, it suffices to show that the bracketed term is increasing in θ_n. Differentiating it wrt θ_n and using again the Envelope theorem,[31] one obtains $-\int_{\underline{\theta}}^{\theta_n} \frac{1}{r}\frac{\partial q^m(\theta|\Theta(n))}{\partial\Theta(n)}\mathrm{d}\theta$ which is > 0 since $\frac{\partial q^m(\theta|\Theta(n))}{\partial\Theta(n)} < 0$ □

Proof of Proposition 1 **(a) Existence:**

According to constraint (22), we need to consider two cases when investigating if the monopolist profits from deviating from full market coverage:

Case A: When $\Theta(n+1)$ is such that $q^m(\Theta(n+1)|\Theta(n)) > \beta q^{se}(\Theta(n+1))$, the deviation profit, denoted by D, equals

$$D = \int_{\theta_{n+1}}^{\theta_n} \begin{pmatrix} \frac{[\theta - h(\theta;\Theta(n))]}{r} q^m(\theta|\Theta(n)) \\ -c(q^m(\theta|\Theta(n))) - \Phi(\Theta(n+1)) \end{pmatrix} f(\theta)\mathrm{d}\theta + \beta\int_{\underline{\theta}}^{\theta_{n+1}} \begin{pmatrix} \frac{[\theta - h(\theta;\Theta(n+1))]}{r} q^m(\theta|\Theta(n+1)) \\ -c\left[q^m(\theta|\Theta(n+1))\right] \end{pmatrix} f(\theta)\mathrm{d}\theta.$$

Given Eq. (36), D may be conveniently rewritten as

$$D = \int_{\theta_{n+1}}^{\theta_n} \begin{pmatrix} \frac{[\theta - h(\theta;\Theta(n))]}{r} q^m(\theta|\Theta(n)) \\ -c\left[q^m(\theta|\Theta(n))\right] \end{pmatrix} f(\theta)\mathrm{d}\theta + \beta\int_{\underline{\theta}}^{\theta_{n+1}} \begin{pmatrix} \frac{[\theta - h(\theta;\Theta(n))]}{r} q^m(\theta|\Theta(n+1)) \\ -c\left[q^m(\theta|\Theta(n+1))\right] \end{pmatrix} f(\theta)\mathrm{d}\theta. \tag{A.14}$$

The difference, $d \equiv Z(\theta_n, \underline{\theta}) - D$, between profits as given by (37) and the deviation profits (A.14), equals

$$d = \int_{\underline{\theta}}^{\theta_{n+1}} \left(\frac{[\theta - h(\theta;\,\Theta(n))]}{r} q^m(\theta|\Theta(n)) - c\left[q^m(\theta|\Theta(n))\right]\right) f(\theta)\mathrm{d}\theta - \beta\int_{\underline{\theta}}^{\theta_{n+1}} \left(\frac{[\theta - h(\theta;\,\Theta(n))]}{r} q^m(\theta|\Theta(n+1)) - c\left[q^m(\theta|\Theta(n+1))\right]\right) f(\theta)\mathrm{d}\theta. \tag{A.15}$$

We can re-write d as follows:

$$(1-\beta)\int_{\underline{\theta}}^{\theta_{n+1}} \begin{pmatrix} \frac{[\theta - h(\theta;\Theta(n))]}{r} q^m(\theta|\Theta(n)) \\ -c\left[q^m(\theta|\Theta(n))\right] \end{pmatrix} f(\theta)\mathrm{d}\theta +$$

31 Notice that $q^{se}(\theta) = \arg\max_q \frac{\theta}{r}q - c(q)$.

$$\beta\left[\begin{array}{c}\int_{\underline{\theta}}^{\theta_{n+1}}\left(\frac{[\theta-h(\theta;\Theta(n))]}{r}q^m(\theta|\Theta(n))-c\left[q^m(\theta|\Theta(n))\right]\right)f(\theta)\mathrm{d}\theta-\\ \int_{\underline{\theta}}^{\theta_{n+1}}\left(\frac{[\theta-h(\theta;\Theta(n))]}{r}q^m(\theta|\Theta(n+1))-c\left[q^m(\theta|\Theta(n+1))\right]\right)f(\theta)\mathrm{d}\theta\end{array}\right]. \quad \text{(A.16)}$$

The second, bracketed, term is clearly positive since $q^m(\theta|\Theta(n))$ has been defined as the period n virtual surplus maximizer for types- θ customers. The first term is positive or negative according as the integral of virtual surpluses is positive or negative over the interval $\left[\underline{\theta}, \theta_{n+1}\right]$. Using the Envelope theorem, the virtual surplus for type θ customers is a decreasing function of $\Theta(n)$. Therefore, a sufficient condition for the first term to be positive is that it is positive for $\Theta(n) = \overline{\theta}$ and all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. This is equivalent to condition that the static Mussa–Rosen monopolist finds it optimal to serve the whole market (i.e., $\underline{\theta} = \theta^*$). An alternative sufficient condition is that β be close enough to 1.

According to the previous analysis, we obtain two different sufficient conditions obtained as follows.

(i) If $\underline{\theta} = \theta^*$, then $\underline{\theta} \geq \Gamma(\Theta(n))$, meaning that the virtual surplus for serving type θ-consumers in period n is positive, whatever $\Theta(n)$. Accordingly, the first term in (A.16) is always positive. Since the second one is positive, whatever the value of β, deviation is never a better strategy than immediate full market-coverage.

(ii) If $\underline{\theta} < \theta^*$, the first term in (A.16) may be negative for some values of θ_{n+1} since the virtual surplus for serving low types of consumers is negative. However, since the second term is positive, a high enough value of β ensures that the sum is positive. Once more, under this condition, deviation is never a better strategy than full market-coverage.

Case B: When $\Theta(n+1)$ is such that $q^m(\Theta(n+1)|\Theta(n)) < \beta q^{se}(\Theta(n+1))$, the monopolist who deviates is constrained to select the quality $\beta q^{se}(\Theta(n+1))$ over the interval $(\theta_{n+1}, \theta^{**}(\theta_n, \theta_{n+1})]$ where $\theta^{**}(\theta_n, \theta_{n+1})$ is the value of θ which satisfies $q^m(\theta|\Theta(n)) = \beta q^{se}(\Theta(n+1))$.[32] She clearly obtains profits which are smaller than in the unconstrained case, namely smaller than (A.14). A sufficient condition for a deviation to be unprofitable is then that (A.16) be positive for all $\theta_{n+1} \in \left[\underline{\theta}, \overline{\theta}\right]$. Since the second, bracketed, term in (A.16) is always positive and the virtual surplus $\frac{1}{r}(\theta - h((\theta; \Theta(n))q^m(\theta|\Theta(n)) - c(q^m(\theta|\Theta(n)))$ is decreasing in $\Theta(n)$, being maximum when $\Theta(n) = \overline{\theta}$.

(b) uniqueness

(i) Suppose first that on the contrary there exists an MPE such that the market is fully covered in a finite number of steps ≥ 2 so that at the last step N, $\theta(N) = \underline{\theta}$ Consider then the two previous steps $N-2$ and $N-1$. At $N-2$, the firm may instead choose to cover instantaneously the market so that $\psi(\theta(N-2)) = \theta(N-1) = \underline{\theta}$. Due to the Stackelberg leadership Assumption A.5., this implies that $\Phi(\theta(N-1)) = 0$. Then, this deviation is profitable iff D defined in the part (a) of Proof of Proposition 1 is negative, i.e., if immediate full-market coverage is an MPE.

(ii) Suppose then that there exists an MPE in which the market is fully covered only asymptotically, in an infinite number of steps. At step n, the static MR monopolist would cover all the market if $\theta(n) < \widetilde{\theta}$ where $\frac{1}{r}\left[\underline{\theta} - h(\underline{\theta}; \widetilde{\theta})\right]q^m(\underline{\theta}) - c(q^m(\underline{\theta})) = 0$. As the market is superstrong $\widetilde{\theta} > \underline{\theta}$ so that $\frac{1}{r}\left[\underline{\theta} - h(\underline{\theta}; \theta)\right]q^m(\underline{\theta}) - c(q^m(\underline{\theta})) > 0$ for all $\theta \in [\underline{\theta}, \widetilde{\theta})$. Since the market is supposed to be asymptotically covered, there always exists some step n at which $\theta(n) \in [\underline{\theta}, \widetilde{\theta})$. It is then more profitable at this period to cover fully the market immediately in one step.

[32] Notice that the strict monotonicity of $q^m(\theta|\Theta(n))$ with respect to θ (which follows from our assumptions) ensures that the solution is unique.

(iii) Suppose finally that there exists an MPE where the market is not fully covered, i.e., such that $\theta(n)- > \theta_{\text{inf}} \geq \widetilde{\theta} > \underline{\theta}$ when n tends toward infinity. There is then always some n great enough so that $\theta(n)$ is arbitrary close to θ_{inf} and then the firm's discounted profits from n to $+\infty$ are arbitrary close to 0. If the market is super-strong we have from Lemma 2 that covering fully and immediately the remaining market yields strictly positive profits and is accordingly a profitable deviation. □

Proof of Remark 5 As far as concerns sufficiency, note that if the static Mussa–Rosen monopolist covers the whole market $\left[\underline{\theta}, \overline{\theta}\right]$, it also covers any smaller market $\left[\underline{\theta}, \Theta(n)\right]$ where $\Theta(n) < \overline{\theta}$ since $\Gamma(\Theta(n)) < \Gamma(\overline{\theta}) = \theta^*$.

The proof of Remark 5 is straightforward. Indeed, under the simultaneous moves assumption, an immediate full market coverage equilibrium must be such that $\Phi(\Theta(n)) = 0$, $\forall \Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$. The profit along the candidate equilibrium path is still given by (37). Suppose that $\theta^* > \underline{\theta}$. Consider $n = 0$ and a deviation to $\Theta(1) = \theta^*$. The firm is then expected to fully cover the market in period 1 and $\Phi(\Theta(1)) = 0$. This deviation yields profits equal to

$$\int_{\theta^*}^{\overline{\theta}} \left[\frac{\theta - h(\theta)}{r} q^m(\theta|1) - c(q^m(\theta|1))\right] f(\theta)\mathrm{d}\theta$$
$$+\beta \int_{\underline{\theta}}^{\theta^*} \left[\frac{\theta - h(\theta;\theta^*)}{r} q^m(\theta|\theta^*) - c(q^m(\theta|\theta^*))\right] f(\theta).$$

□

Proof of Proposition 2 **(a) Proof of parts (i) and (ii) of Proposition 2**

Parts (i) and (ii) provide sufficient conditions for immediate market coverage. As already indicated, we only need to consider conditions that ensure that the value of d, i.e., (A.16), is positive.

Straightforward computations show that (A.16) can be expressed as:

$$(\Theta(n+1) - \underline{\theta}) \times \left\{2(2\underline{\theta}^2 - 3Br^2)(1-\beta) - 6\underline{\theta}\Theta(n)(1-\beta) + 3\Theta(n)^2\right.$$
$$\left.+(4\underline{\theta}(1-\beta) - 6\Theta(n))\Theta(n+1) + (4-\beta)\Theta(n+1)^2\right\}$$

For all $\Theta(n+1) \in (\underline{\theta}, \Theta(n)]$, the sign of this expression is the sign of the bracketed term $\{\ldots\}$. This bracketed term is a U-shaped second-order polynomial in $\Theta(n+1)$ which we shall denote as $P(\Theta(n+1); \Theta(n))$. A deviation is not profitable if whatever $\Theta(n) \in (\underline{\theta}, \overline{\theta}]$, this polynomial either has no real root or, if there exist real roots, the smallest root is greater than $\Theta(n)$.

- The polynomial has no real root if the discriminant of the polynomial is negative. The discriminant is equal to:

$$-(1-\beta)\left[4\underline{\theta}^2 + 2Br^2(-4+\beta) + 2\underline{\theta}(-2+\beta)\Theta(n) + \Theta(n)^2\right].$$

  It is indeed negative if either condition (i) or condition (ii) below holds:

  (i) $\beta > \overline{\beta} \equiv \left(\frac{\widehat{\theta}}{\underline{\theta}}\right)^2$. Notice that $\overline{\beta} < 1$ iff the customer base is super-strong, i.e., $\underline{\theta} > \widehat{\theta}$;

  (ii) $\beta \leq \overline{\beta}$ and $\Theta(n) \leq \underline{\theta}\left[(2-\beta) - \sqrt{(4-\beta)(\overline{\beta}-\beta)}\right]$ for all $\Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$. Notice that the interval of values of β such that this condition is satisfied is non-void iff $\beta > \frac{4\overline{\beta}-1}{2\overline{\beta}+2}$.

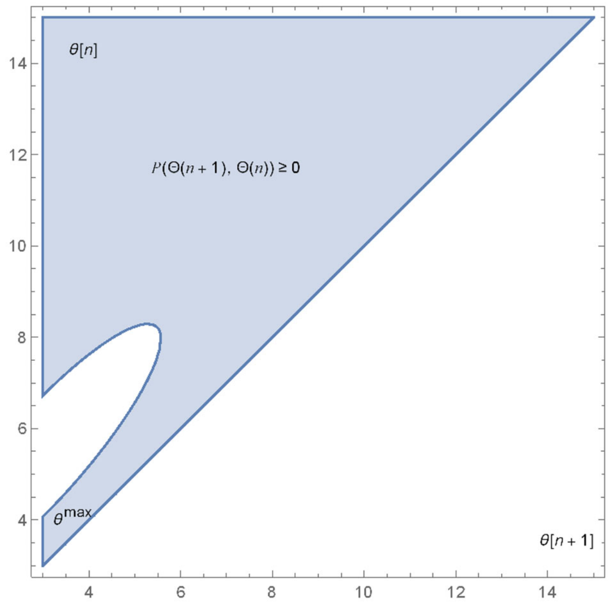

Fig. 2 The area where $P(\Theta(n+1), \Theta(n)) \geq 0$

- Consider the case $\beta \leq \overline{\beta}$. The polynomial $P(\underline{\theta}, \Theta(n))$ is straightforwardly positive if $\Theta(n)/\underline{\theta} \leq \left[(2-\beta) - \sqrt{(1-\beta)(\overline{\beta}-\beta)}\right] \equiv \mu(\beta)$, i.e., if $\Theta(n) \leq \underline{\theta}\mu(\beta)$. Now,

$$P(\Theta(n+1), \underline{\theta}\mu(\beta)) = (4-\beta)(\Theta(n+1) - \underline{\theta}) + 6\underline{\theta}\sqrt{(1-\beta)(\overline{\beta}-\beta)}$$

  is increasing in $\Theta(n+1)$. Accordingly, $\overline{\theta} \leq \underline{\theta}\mu(\beta)$ is sufficient to ensure that immediate full market-coverage is an equilibrium strategy for all $\Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$.[33]
  To help the reader to visualize the argument, we have pictured below, in the $(\Theta(n+1), \Theta(n))$ space, the area (in blue) where $P(\Theta(n+1), \Theta(n)) \geq 0$ and $\Theta(n+1) \leq \Theta(n)$. Figure 2 is drawn for $\underline{\theta} = 3, \overline{\theta} = 8, r = 1, B = 2$, (implying that $\overline{\beta} = 4/9$), and $\beta = 0.2$. Notice that $\underline{\theta}\mu(\beta) = 4.07335$.

When β increases, the white "finger" in the blue area shrinks, disappearing completely when β becomes greater than 4/9.

(b) Proof of part (iii) of Proposition 2

The conditions in parts (i) and (ii) are sufficient conditions, obtained for Case A, where $\Theta(n+1) \geq \frac{\Theta(n)}{2-\beta}$. They are sufficient because the profit differential in Case B is smaller than in Case A. We now show that for immediate market coverage to be the equilibrium outcome it is necessary that either $\beta > \overline{\beta}$ or $\overline{\theta} \leq \underline{\theta}\mu(\beta)$.

Suppose that $\beta \leq \overline{\beta}$ and $\overline{\theta} > \underline{\theta}\mu(\beta)$. Then there exists some $\Theta(n) \leq \overline{\theta}$ belonging to the non-void interval $\underline{\theta}\left[(2-\beta) - \sqrt{(1-\beta)(\overline{\beta}-\beta)}\right], \underline{\theta}(2-\beta)]$, and such that we are in Case A when $\Theta(n+1) = \underline{\theta}$ and $P(\underline{\theta}, \Theta(n)) < 0$. By continuity there is a $\Theta(n+1)$ close enough to $\underline{\theta}$ and a $\Theta(n) \in [\Theta(n+1)\mu(\beta), \Theta(n+1)(2-\beta)]$ such that $P(\Theta(n+1), \Theta(n)) < 0$ so

[33] Notice that this condition is weaker than the condition $\overline{\theta} \leq \underline{\theta}\left[(2-\beta) - \sqrt{(4-\beta)(\overline{\beta}-\beta)}\right]$;.

that, starting from $\Theta(n)$, full market-coverage is not an equilibrium strategy. We conclude that, for full market-coverage to be an equilibrium strategy from any $\Theta(n) \in (\underline{\theta}, \overline{\theta}]$, it is necessary that either $\beta > \overline{\beta}$ or $\overline{\theta} \leq \underline{\theta}\mu(\beta)$. □

Proof of Remark 6 Consider that initially the firm picks a cutoff $\Theta(1) \in \left[\underline{\theta}, \overline{\theta}\right]$, anticipating to cover the whole market in period 1 if not in period 0, and notice that immediate full market coverage corresponds to the special case when $\Theta(1) = \underline{\theta}$. The corresponding profits equal

$$\pi(\Theta(1)) = \int_{\Theta(1)}^{\overline{\theta}} (\frac{1}{2r^2}(2\theta - \overline{\theta})^2 - B) f(\theta) \mathrm{d}\theta \tag{A.17}$$

$$+\beta \int_{\underline{\theta}}^{\Theta(1)} (\frac{1}{2r^2}(2\theta - \Theta(1))^2 - B) f(\theta) \mathrm{d}\theta. \tag{A.18}$$

Remember that, given slack consumers' expectations, the consumers expect the firm to cover the market instantaneously so that $\Phi(\Theta(n)) = 0$, $\forall \Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$, $\forall n \geq 0$. Differentiating (A.17) twice with respect to $\Theta(1)$, one obtains

$$\pi''(\Theta(1)) = 2\overline{\theta} - 4\underline{\theta} + \beta(\Theta(1) - \underline{\theta}),$$

which is negative for all $\beta \in [0, 1]$ and all $\Theta(1) \in \left[\underline{\theta}, \overline{\theta}\right]$ since $\underline{\theta} \geq \frac{\overline{\theta}+r\sqrt{2B}}{2}$. Concavity of (A.17) with respect to $\Theta(1)$ then implies that choosing $\Theta(1) = \underline{\theta}$ maximizes (A.17), i.e., no deviation is profitable iff $\pi'(\Theta(1)) \leq 0$. Straightforward computations show that this is equivalent to $\beta \leq \frac{(\overline{\theta}-2\underline{\theta})^2-2r^2B}{\underline{\theta}^2-2r^2B}$.

Notice then that the same argument holds in any period n, simply replacing $\overline{\theta}$ by $\Theta(n)$ and $\Theta(1)$ by $\Theta(n+1)$. One then obtains the constraint $\beta \leq \frac{(\Theta(n)-2\underline{\theta})^2-2r^2B}{\underline{\theta}^2-2r^2B}$ which is automatically satisfied if $\beta \leq \frac{(\overline{\theta}-2\underline{\theta})^2-2r^2B}{\underline{\theta}^2-2r^2B}$ is satisfied. □

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