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Dynamic Games in the Economics of Natural Resources: A Survey

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Abstract This article provides a comprehensive survey of models of dynamic games in the exploitation of renewable and exhaustible resources. It includes dynamic games at the industry level (oligopoly, cartel versus fringe, tragedy of the commons) and at the international level (tariffs on exhaustible resources, fish wars, entry deterrence). Among more recent topics are international strategic issues involving the link between resource uses and transboundary pollution, the design of taxation to ensure efficient outcomes under symmetric and asymmetric information, the rivalry among factions in countries where property rights on natural resources are not well established. Various extensions are considered, such as (i) modeling the effects of the concern for relative performance (relative income, relative consumption, and social status) on the over-exploitation of resources, (ii) applying the tragedy of the commons paradigm to the declining effectiveness of antibiotics and pesticides. Outcomes under Nash equilibria and Stackelberg equilibria are compared. The paper ends with some suggestions for future research.

Keywords Exhaustible resources · Renewable resources · Over-exploitation · Property rights

Introduction

Natural resources play an important role in economic development and economic growth. They are also potential sources of conflicts among nations, and among rival groups within a nation. The fact that many resources are common properties has raised serious concerns about excessive exploitation and the potential collapse of many species. The use of several resources has been linked to pollution problems which are themselves subject of international disputes. Analysis of these national and global issues requires the use of dynamic games.

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At the microeconomic level, to develop a thorough understanding of pricing and supply behavior of resource-extracting firms, economists have also relied on dynamic games as a main tool of analysis. Advances in dynamic games have shaped progress in the economic theory of natural resources. Conversely, in studying natural resource problems, economists have come up with a number of questions that help sharpen the theoretical tools of dynamic games.

The purpose of this article is to provide an overview of the main issues in natural resource economics that have been successfully analyzed with the help of dynamic games, and to point to some issues that are emerging as possible interesting future topics for research in this important field.

Exhaustible Resources

Formal dynamic analysis of exhaustible resources began with Hotelling [79] who used the calculus of variations to study the optimal behavior of mining firms. The famous Hotelling Rule describes the equilibrium condition for the evolution of resource prices. However, Hotelling's work contained no game-theoretic considerations. Dynamic games began to be used by resource economists around 1975–1980. The concepts of open-loop Nash equilibrium (OLNE) and open-loop Stackelberg equilibrium (OLSE) were featured in a number of early articles and two early books on dynamic models of exhaustible resources, namely [35] and [92]. Since then, the emphasis has shifted to feedback Nash equilibrium (also called Markov-perfect Nash equilibrium, MPNE) and feedback Stackelberg equilibrium (FBSE), and issues concerning mechanism designs, such as dynamic corrective taxation for resource markets.[1]

Exhaustible Resources and Industrial Structures

While the theory of the extractive industry under perfect competition and under monopoly was formally developed by Hotelling in 1931, it took a long time before theories of other forms of imperfect competition in the extractive industry came into being. We discuss below the theory of cartel versus fringe and the theory of oligopoly in the context of an exhaustible resource, such as oil.

A Cartel and a Competitive Fringe

One of the first game-theoretic papers on exhaustible resources is [132] who investigates the (open-loop) equilibrium resource-extraction path in an industry consisting of a dominant firm and a fringe of perfectly competitive firms (the price takers).

Salant assumes zero extraction cost and a stationary inverse demand curve $P = f(Q)$ with $f'(Q) < 0$, and $f(0) \equiv \overline{P} > 0$.

The representative fringe firm takes the price path $P(t)$ as given, and chooses its extraction rate $q_i(t)$ to maximize the integral of its stream of discounted profits,

$$\max \int_0^\infty e^{-rt} P(t) q_i(t)\, dt$$

[1] The crucial distinction between OLNE and MPNE was made transparent in [128]. Dockner et al. [45] offered a more detailed treatment, with many examples in resource economics.

(where $r > 0$ is the discount rate) subject to $q_i(t) \geq 0$ and

$$\int_0^\infty q_i(t)\,dt = X_i,$$

where X_i is the initial size of its deposit. Let $Q^c(t)$ denote the fringe's aggregate extraction at t.

By definition of an open-loop Nash–Cournot equilibrium, the dominant firm takes the time path $Q^c(t)$ as given and chooses its own time path of extraction $Q^d(t)$ to maximize the integral of its stream of discounted profits,

$$\max \int_0^\infty e^{-rt} f\big(Q^c(t) + Q^d(t)\big) Q^d(t)\,dt$$

subject to $Q^d(t) \geq 0$ and

$$\int_0^\infty Q^d(t)\,dt = X^d,$$

where X^d is the initial size of its deposit.

Salant shows that the equilibrium extraction path consists of two phases, of lengths θ_1 and θ_2, respectively. In Phase 1, the price rises at the rate of interest, $\dot{P}(t)/P(t) = r$ for all $t \in [0, \theta_1]$. In Phase 2, only the dominant firm supplies, and its discounted marginal revenue at any two points of time must be equalized. Suppose that initially the industry is perfectly competitive. Let a subset of competitive firms form a cartel (a dominant firm). It can be shown that the sum of discounted profit of each extractor rises. However, the profit of each non-member jumps by a larger percentage than the profit of members of the cartel.

Extending Salant's model, Ulph and Folie [146] derive the open-loop Nash equilibrium under constant marginal extraction costs, possibly different between the cartel and the fringe. Other related studies (such as [100, 125, 133]) also use the open-loop Nash equilibrium.

Salant's dynamic game of cartel versus fringe is somewhat different from the standard static model of cartel versus fringe that one encounters in most textbooks, where the cartel does not take the output of the fringe as given, but rather commits in advance to a price (or output) to induce an output response from the fringe, i.e., the cartel is a Stackelberg leader.

Gilbert [66] considers the case where an exhaustible-resource cartel is an open-loop Stackelberg leader while the fringe firms are followers. The cartel determines its extraction path first, and the fringe reacts to that. A problem with the open-loop leader–follower formulation is that, in general, the open-loop Stackelberg equilibrium is not time-consistent: in the absence of binding commitment, the leader will have an incentive to renege on its announced path. This was discussed in [147]. An interesting feature of the cartel–fringe model with an exhaustible resource is that there is a small subset of parameter values such that the open-loop Stackelberg solution is time-consistent [70, 122, 145].

To avoid the problem of time inconsistency, Groot et al. [71] propose a model of cartel and fringe under the feedback Stackelberg assumption. Taking the limit case where the number of fringe firms becomes arbitrarily large, they propose that the value function for each fringe member is linear in its own stock and independent of the stock of any other firm (fringe or cartel): $V_i^f = \lambda^f S_i^f$. Here λ^f is treated by the fringe firm as a constant, though, of course, it would depend on the stock of the cartel. Given this assumption (a form of myopia imposed on the fringe), the authors obtain the value function for the cartel in terms of some transformed variables which themselves are implicit functions of the stocks. It turns out that for those parameter values such that the open-loop Stackelberg solution is time-consistent, the open-loop solution path and the feedback solution path coincide.

Oligopoly in an Extractive Industry

Loury [115] characterizes the open-loop Nash equilibrium in a game among n oligopolists that have the same marginal cost k but different initial deposit sizes $\overline{X}_i$, $i = 1, 2, \dots, n$. Let $q_i(t)$ be the rate of extraction of firm i at time t. The total quantity sold is

$$Q(t) = \sum_{i=1}^{n} q_i(t).$$

The price is $P = P(Q)$, $P'(Q) < 0$. Each firm realizes that it can influence the price $P(t)$ by its choice $q_i(t)$. Let $Q_{-i}(t) = Q(t) - q_i(t)$. Each firm i takes the time path $Q_{-i}(t)$ as given, and chooses $q_i(t)$ to maximize its discounted stream of profit

$$\int_0^{\infty} e^{-rt}\big[P\big(Q_{-i}(t) + q_i(t)\big) - k\big]q_i(t)\,dt$$

subject to $q_i(t) \geq 0$, and

$$\dot{X}_i(t) = -q_i(t), \qquad X_i(0) = \overline{X}_i, \qquad \lim_{t\to\infty} X_i(t) \geq 0.$$

Loury shows that in an open-loop Nash equilibrium (a) the average and marginal return on resource stocks are decreasing in the initial stock sizes, (b) aggregate output falls over time, (c) firms with smaller deposits exhaust their stocks before firms with larger deposits, and (d) industry production maximizes a weighted average of profits and consumer's welfare.

Lewis and Schmalensee [100] consider firms which differ in extraction costs. They show that, contrary to what would be dictated by a social planner, in an open-loop Nash equilibrium the lowest cost deposit may not be exhausted first. An empirical test is performed by Polasky [127]. Further results on open-loop oligopoly in exhaustible resources are obtained by Benchekroun et al. [13, 14], under the assumption that there are two groups each consisting of identical firms, and firms can differ across groups both in deposit size and in marginal cost. Under linear demand, they show that in open-loop Nash equilibrium there almost always exists a phase where both types of firms simultaneously produce. Interestingly, when the high cost deposits are exploited by the group of firms whose number goes to infinity, the equilibrium approaches the cartel-versus-fringe model. Deposits with lower extraction costs may not be exhausted first. They also find that an increase in the aggregate stock of the fringe (which has higher extraction cost) may reduce social welfare. This result is a dynamic counterpart of a result obtained in static oligopoly models [97, 112], where a fall in the marginal cost of higher cost firms may induce them to increase their market shares, leading to productive inefficiency and welfare loss.

Gaudet and Long [60] study the effect of a marginal transfer of resource from one firm to another in an extractive duopoly. Restricting attention to open-loop equilibrium, they show that if the initial deposits are sufficiently different in size, a transfer that renders the stock distribution more unequal will increase the industry's profit. In contrast, when the sizes of deposits are similar, a marginal transfer has no effect on the industry's output and profit. These results have static counterparts in the static Cournot oligopoly model of Bergstrom and Varian [18], who consider an increase in the marginal cost of one firm accompanied by an equal decrease in the marginal cost of its rival. Benchekroun et al. [12] show how the anticipation of a future oligopoly phase may influence the extraction path of an oil cartel.

Kemp and Long [90], drawing on [88, 89], study the case where firms are uncertain about the size of their deposits. Even if firms are ex-ante identical, each will take account of the possibility that at some future time it may be the only remaining firm with a positive resource stock, so that it becomes a monopoly. They show that in the absence of a complete set of state-contingent markets (i.e., in the absence of a full set of Arrow–Debreu securities), price-taking behavior, in conjunction with a recognition of possible monopoly power in the future, may result in an equilibrium that is socially more inefficient than monopoly.

What can be said about feedback Nash equilibrium in an extractive oligopoly? It turns out that results for this case are scarce. Under the assumption of a constant elasticity of demand and zero extraction cost, explicit value functions for oligopolists can be found, see [51, 128], and [9]. The latter paper considers the case where the firms have different deposit sizes. It is found that a uniform addition (e.g., by new discoveries of deposits) to all reserves could harm firms that have larger stock sizes.

An alternative model of extractive duopoly is proposed by Salo and Tahvonen [134]. There are n firms: firm i owns a resource deposit $\overline{X}_i$ $(i = 1, 2, \dots, n)$. The surface area of each deposit is unity, so the depth at which the last unit of resource can be found is $\overline{X}_i$. The marginal cost of extraction increases with the depth of the mine. Let $S_i(t)$ denote the depth reached by firm i at time t and $q_i(t)$ the rate of extraction of firm i. Then $\dot{S}_i(t) = q_i(t)$. At any time, the cost of extracting q_i is $c_i S_i q_i$, i.e., the marginal cost of extraction is $c_i S_i$. The inverse demand function of the resource good is

$$p = a - \sum_{i=1}^{n} q_i, \quad a > c_i, \tag{1}$$

where p is the price the consumers have to pay per unit. The parameter a is the 'choke price.' It is the marginal utility of consuming the first unit. Let $\overline{S}_i$ denote the depth at which the marginal extraction cost from deposit i equals the choke price, i.e., $c_i \overline{S}_i = a$. Assume that $\overline{X}_i$ is greater than $\overline{S}_i$. Then, efficiency implies that the resource stock of firm i is abandoned at the depth $S_i = a/c_i$, i.e., before physical exhaustion of the stock. This is called economic abandonment. It can be shown that as t tends to infinity, $S_i(t)$ tends to a/c_i. The assumption of economic abandonment enables Salo and Tahvonen [134] to obtain analytically the Nash equilibrium feedback strategies. The implications of their model are quite different from the model based on stock-independent marginal cost (such as Lewis and Schmalensee [100], and Loury [115], who only consider open-loop strategies). These earlier papers predict that small firms will exhaust their stocks before large firms do, leading possibly to eventual monopolization of the market. In contrast, the Salo–Tahvonen model predicts that as the price rises along the demand curve, more firms will become active, and that eventually firms with the same cost parameter have equal market shares, regardless of the different initial depth of their mines, $S_i(0)$.

Generalizing to non-linear demand, and extraction costs that are initially independent of the stocks when these are large, Salo and Tahvonen [134] obtain numerical solution for feedback Nash equilibrium. Extraction paths may at first be similar to the model of Loury [115], but eventually they resemble the equilibrium of the economic abandonment model under linear demand.

Extraction of Exhaustible Resources under Common Access

It has been recognized that not all extractive firms have exclusive access to their own resource stocks. Firms with adjacent and interconnected gas fields may siphon off the gas that "belongs" to their rivals. Similarly, while oil-well owners have the right to extract the oil located under their own properties, oil may seep from one pool to another. This problem has been studied by Khalatbari [93], Dasgupta and Heal [35, Chap. 12], Kemp and Long [92], and Sinn [136], under the assumption that firms use open-loop strategies.[2] McMillan and Sinn [119] review the various open-loop assumptions, and consider conjectural variation with closed-loop (but not Markov-perfect) decision rules. They find that there are infinitely many equilibria, most of which display excessive exploitation.

Analytical expressions for feedback Nash equilibrium extraction of an exhaustible resource under common access can be found for a class of problems, when the utility of each player depends only on his own extraction rates [29, 108]. Long et al. [114] suppose there are n players with different utility functions, $u_i(q_i) = q_i^{\beta_i}$ where $0 < \beta_i < 1$, and different discount rates, $r_i \neq r_j$. They have access to a common stock of exhaustible resource S. Then

$$\dot{S} = -\sum_{i=1}^{n} q_i.$$

Each player i believes that player j has a feedback strategy $\phi_j(\cdot)$ such that $q_i(t) = \phi_j(S(t))$. The optimization problem of player i is

$$\max \int_0^\infty \exp(-r_i t)\big[q_i(t)\big]^{\beta_i}\, dt, \quad \text{where } r_i \neq r_j$$

subject to $q_i(t) \geq 0$, $\dot{S} = -q_i - \sum_{j\neq i} \phi_j(S)$, and $\lim_{t\to\infty} S(t) \geq 0$, $S(0) = S_0$. Long et al. [114] show that there exists a unique feedback equilibrium in linear strategies, with $\phi_j(S) = \gamma_j S$ where in general $\gamma_i \neq \gamma_j$. There is, in general, a continuum of feedback equilibria in non-linear strategies, a problem that is also discussed in, e.g., [104] and [32] in somewhat different contexts.

The case of exploitation of a common pool where the utility of agent i depends on both his extraction q_i and the stock S is studied in [108]. Suppose there are n identical players, each with the utility function $u_i = U(q_i, S) = (q_i^{1/2} S^{1/2})^\alpha$ where $0 < \alpha < 1$. Provided that $2 > \alpha n$, it can be shown that there is a unique feedback equilibrium in linear strategies, $q_i(t) = \gamma S(t)$ where $\gamma = rS/(2 - \alpha n)$.

The effectiveness of an antibiotic drug or a pesticide has also been modeled as an exhaustible resource. The resistance of bacteria to drugs has been a concern in the medical profession. Cornes et al. [32] see an isomorphism between the decline in effectiveness of antibiotics and pesticides (due to repeated uses), and the exhaustibility of natural resources (due to extraction). They consider farmers who as individuals do not fully take into account the social consequence of their applications of doses which contribute to the decline in effectiveness of the pesticide. They offer two dynamic game models, one in discrete time and one in continuous time. Both models have multiple feedback Nash equilibria that can be ranked in terms of social inefficiency. In the continuous time model, there is a feedback Nash equilibrium in linear strategies and a continuum of feedback Nash equilibria in non-linear strategies that result in exhaustion of the effectiveness of the pesticide in finite time.

While most economic articles on exhaustible resources assume that the extracted resources are final goods, some authors (e.g., [35, 139]) specify that the resources are only an input which must be used with capital to produce a final output. Let $K \geq 0$ denote the capital stock, $S \geq 0$ the resource stock, and $R \geq 0$ the rate of extraction. Extraction is costless, and the production function for the final output is

$$Y = AK^\alpha R^{1-\beta}, \quad 0 < \alpha < 1, 0 < \beta < 1.$$

[2] As the speed of seepage becomes infinite, one obtains the pure common pool problem, see [20].

Let C denote consumption and I denote investment, where $C + I = Y$. The dynamic system of equations is

$$\dot{K} = AK^{\alpha}R^{1-\beta} - C, \qquad K(0) = K_0, \qquad K(t) \geq 0,$$
$$\dot{S} = -R, \qquad S(0) = S_0, \qquad S(t) \geq 0.$$

Solow [139] and Dasgupta and Heal [35] were interested in a central planning problem, and did not consider any game-theoretic situation for this two-asset economy. Long and Katayama [106] modify this model and consider a game among n infinitely-lived individuals. In this game, the stock of resource is a common property. The players accumulate their own capital stocks. Each player has an instantaneous utility function $U(C_i) = (1-\gamma)^{-1}C_i^{1-\gamma}$ where C_i is his own consumption, and $0 < \gamma < 1$. Let $\rho > 0$ denote the discount rate. Each chooses his control variables R_i and C_i to solve the following problem

$$\max \int_0^{\infty} e^{-\rho t}(1-\gamma)^{-1}C_i^{1-\gamma}\,dt$$

subject to

$$\dot{K}_i = A_i K_i^{\alpha} R_i^{1-\beta} - C_i, \qquad K_i(0) = K_{i0},$$
$$\dot{S} = -R_i - \sum_{j\neq i} R_j, \qquad S(0) = S_0.$$

Player i thinks that player j uses some feedback extraction strategy $R_j = \phi_j(S, K_j)$ and consumption strategy $C_j = \theta_j(S, K_j)$. Long and Katayama [106] show that in the special case where $A_i = A_j = A$ and $\gamma = \alpha$, if $1 > n(1-\beta)$, there exists a symmetric Markov-perfect equilibrium where each agent uses linear strategies: $R_i = \eta S$ and $C_i = (\rho/\alpha)K_i$, where

$$\eta \equiv \frac{\rho}{1 - n(1-\beta)}.$$

Long and Katayama [106] demonstrate the existence of a phase of capital accumulation followed by a phase of dissaving. The consumption path displays a hump-shaped profile. Net saving becomes negative even before consumption reaches its peak. When agents are heterogeneous in terms of productive efficiency ($A_i \neq A_j$), the more productive agents will invest more in capital. However, all players use the same feedback consumption strategy and the same feedback resource-extraction strategy.

In a companion paper, Katayama and Long [87] modify the model in [106] by assuming that (i) extraction cost is not zero (i.e., extraction requires labor input L_i), and (ii) an individual's utility is decreasing in effort and depends not only on his consumption but also his status, defined as the ratio of his consumption over some index of average consumption in the community. Specifically, they assume the utility function

$$u_i = \frac{1}{1-\gamma}C_i^{1-\gamma}\left[\frac{C_i}{\delta C_i + (1-\delta)C_{-i}}\right]^{\lambda} - \chi L_i^{\sigma},$$

where

$$C_{-i} \equiv \frac{1}{n-1}\sum_{j\neq i} C_j$$

and $0 < \delta < 1$, $0 < \gamma < 1$, $\lambda > 0$, $\chi > 0$, $\sigma > 0$. To extract the amount R_i, individual i must uses $(\varepsilon R_i)^{\mu}$ units of effort, i.e., $L_i = (\varepsilon R_i)^{\mu}$ where $\mu > 0$ and $\varepsilon > 0$. Another feature of the model is that the capital stock is also a common property. Player i chooses C_i and R_i (which determines L_i) to maximize

$$\int_0^{\infty} e^{-\rho t} u_i \, dt$$

subject to

$$\dot{K} = \left[K^{\alpha} R_i^{1-\beta} - C_i\right] + \sum_{j \neq i} \left[K^{\alpha} R_j^{1-\beta} - C_j\right], \qquad K(0) = K_0,$$

$$\dot{S} = -R_i - \sum_{j \neq i} R_j, \qquad S(0) = S_0.$$

In the special case where $\alpha = \gamma$ and $\mu\sigma = 1 - \beta$, it is found that there exists a Markov-perfect Nash equilibrium where all players use linear feedback strategies. The authors show that the degree of status-consciousness has an important impact on the Markov-perfect Nash equilibrium. A higher degree of status-consciousness (i.e., a higher λ) results in more excessive consumption, and a slower rate of capital accumulation. Under costless extraction (i.e., $\varepsilon = 0$), status-consciousness has no impact on the extraction/resource-stock ratio. In contrast, under costly extraction, this ratio is decreasing in the level of status-consciousness. This result is plausible though at first it might seem puzzling. Since individuals want to surpass their rivals in terms of relative consumption, they find it more advantageous to over-exploit the common man-made capital stock.

van der Ploeg [150] considers a game of exploitation of an exhaustible common pool among N agents (or factions) who can build up their private capital stocks, as in [106]. He assumes that all factions have the same objective: to maximize its minimum level of consumption. Consider the utility function

$$U(C_i) = \frac{1}{1 - (1/\theta)} C_i^{1-(1/\theta)},$$

where θ is the intertemporal rate of substitution. The extreme case where $\theta \to 0$ corresponds to the maximin criterion. Faction i owns a stock of man-made capital K_i. The final output produced by faction i is

$$Y_i = K_i^{\alpha} R_i^{\gamma} L_i^{1-\alpha-\gamma},$$

where $0 < \alpha < 1$, $0 < \gamma < 1$, $\alpha + \gamma < 1$ and $L_i = 1/N$. Since there is no depreciation, $\dot{K}_i = Y_i - C_i$.

van der Ploeg characterizes a feedback Nash equilibrium under the maximin criterion in which each faction i adopts the following extraction strategy

$$R_i = \frac{\sigma S}{K_i}$$

and saves a constant fraction of output Y_i. He shows that this equilibrium differs from the outcome under the social planner (who also has the maximin objective): the Nash equilibrium constant gross output is too high and constant consumption is too low, as factions try to invest excessively in their private capital stocks. van der Ploeg shows that in the Nash equilibrium, genuine saving (using correct shadow prices) is zero. If savings are measured using market prices, they are positive, but such measures are misleading because the market prices fail to reflect social opportunity costs.

A game among N agents that exploit N interconnected pools is modeled by van der Ploeg [151], which is an extension of [150]. Agent i extracts R_i from pool i. The rate of depletion of the stock S_i is

$$\dot{S}_i = -R_i - \sum_{j \neq i} \pi(S_i - S_j),$$

where $\pi \geq 0$ is the seepage rate.[3] Let $K \equiv \sum_i K_i$ denote the aggregate capital stock. van der Ploeg [151] assumes that π is a decreasing function of K, suggesting that property rights improve as the economy moves along its development path. The following specific function form is assumed: $\pi = \varepsilon/K$. Focussing on this extreme case, van der Ploeg shows that there exists a symmetric *open-loop* Nash equilibrium where output, consumption, and investment are constant over time, while the capital stocks increase without bound, and the resource stocks fall to zero asymptotically. Comparing this Nash equilibrium outcome with the standard maximin solution under a social planner, van der Ploeg concludes that under rivalry society ends up with a higher level of output, but a lower level of consumption. Individuals accumulate too much capital, and the aggregate resource stock is being extracted at too fast a rate.

Exhaustible Resources and Pollution

There are links between the exploitation of exhaustible resources and pollution. The most prominent link is the excessive accumulation of carbon dioxide in the atmosphere and the extraction and use of fossil fuels such as oil, gas, and coal. The proposed introduction of carbon taxes by many countries has raised some interesting issues: How do fossil fuel exporting countries (say OPEC) react to carbon taxes, and how do carbon taxes serve to reduce OPEC rents? Some authors have developed models to gain insights into the dynamic interactions between tax policies and the pricing or extraction strategies of resource cartels. In what follows, we survey some theoretical attempts in that direction.

The simplest models contain only one state variable, namely accumulated extraction, and consider it as the stock of pollution. This may be justified on the grounds that the rate of decay of atmospheric CO_2 concentration is very low.

Nash Equilibrium under Constant Extraction Cost and Non-decaying Pollution

Wirl [154] develops a model involving two players: the government of a fossil-fuel importing country and a monopolist seller of fossil fuels from a stock of resource $\overline{S}$. To simplify, assume that the consumption of fossil fuels occurs only in the importing country. The amount consumed at time t is $y(t)$. It generates a flow pollution, with flow damages $\frac{1}{2}\omega y(t)^2$ and contributes to a stock pollution, with damages $\frac{1}{2}DZ(t)^2$ where $Z(t)$ is the stock of pollution, assumed to be identical to accumulated consumption. Simultaneously, the monopolist sets the producer price, $p(t)$, and the importing country sets a tax rate $\tau(t)$ per unit. The consumer price is $p(t) + \tau(t)$. The demand function of the representative consumer is $y = a - (p + \tau)$. The consumer's surplus is $\frac{1}{2}(a - p - \tau)^2$ for $p + \tau \leq a$. The importing

[3] A similar problem of interconnected pools was analyzed by Dasgupta [34, p. 287] in a partial equilibrium setting.

country's instantaneous payoff, U_I, is the sum of consumer surplus and the tax revenue, net of pollution damages,

$$U_I = \frac{1}{2}(a - p - \tau)^2 + \tau(a - p - \tau) - \frac{1}{2}\omega y^2 - \frac{1}{2}DZ^2.$$

Assume that the exporting country does not care about pollution. Its instantaneous payoff is $U_X = py = p(a - (p + \tau))$. The instantaneous global welfare is defined as $U_G = U_I + U_X$. Denote the consumer's price by $\theta \equiv p + \tau$. Let Z^*_∞ denote the steady state stock of pollution under cooperation. Let $\rho > 0$ be the common discount rate. It can be shown that at the cooperative steady state, the present value of the infinite stream of marginal damage cost is equated to the choke price:

$$\frac{DZ^*_\infty}{\rho} = a \tag{2}$$

provided that $\overline{S} \geq \frac{a\rho}{D}$.

Now consider a dynamic game between the two countries. The focus is on feedback Nash equilibrium. The importing country takes as given the feedback pricing strategy of the monopolist seller, denoted by $p(t) = \phi(Z(t))$. Its Hamilton–Jacobi–Bellman (HJB) equation is

$$rJ_I(Z) = \max_\tau \bigl\{U_I\bigl(\tau, Z\bigr) + J_I'(Z)\bigl(a - \phi(Z) - \tau\bigr)\bigr\}. \tag{3}$$

The solution of the importer's HJB equation yields a decision rule $\tau = g(Z)$.

The exporting country believes that the importing country uses a feedback strategy $\tau = g(Z)$. Its HJB equation is

$$\rho J_X(Z) = \max_p \bigl\{p\bigl(a - p - g(Z)\bigr) + J_X'(Z)\bigl(a - p - g(Z)\bigr)\bigr\}.$$

The solution of the exporter's HJB equation yields a decision rule $p = \phi(Z)$. In a Nash equilibrium, expectations are correct. Then

$$p = \frac{1}{2+\omega}\bigl[a + J_I'(Z) - (1+\omega)J_X'(Z)\bigr],$$

$$\tau = \frac{1}{2+\omega}\bigl[a\omega + \omega J_X'(Z) - 2J_I'(Z)\bigr],$$

$$\theta \equiv p + \tau = \frac{1}{2+\omega}\bigl[a(1+\omega) - J_I' - J_X'\bigr].$$

Define the function $J(Z) \equiv J_I(Z) + J_X(Z)$. Then our problem reduces to solving a single differential equation

$$\rho J(Z) = -\frac{1}{2}DZ^2 + \Omega\bigl(a + J'(Z)\bigr), \tag{4}$$

where

$$\Omega \equiv \frac{(3+\omega)}{2(2+\omega)^2}.$$

Now impose the boundary condition that when Z is at the socially optimal steady state, extraction is zero and the flow of welfare is equal to the value of damages caused by the stock:

$$\rho J\left(\frac{a\rho}{D}\right) = -\frac{1}{2}D\left(\frac{a\rho}{D}\right)^2. \tag{5}$$

Equation (4) and the above boundary condition give rise to the following quadratic value function

$$J(Z) = \frac{A}{2}Z^2 + BZ + C,$$

where

$$\begin{aligned}
A &= \frac{1}{4\Omega}\left[\rho - \sqrt{\rho^2 + 2D\Omega}\right] < 0,\\
B &= \frac{2a\Omega A}{\rho - 2\Omega A} < 0,\\
C &= \Omega(a+B)^2 > 0.
\end{aligned}$$

As the stock of pollution accumulates, the consumer price rises gradually toward a. The equilibrium feedback strategies are, for all $Z \leq a\rho/D$,

$$\begin{aligned}
\tau = g(Z) &= a + \frac{2A^2 - 2D(2+\omega)^2}{\rho(2+\omega)^3}\left(\frac{a\rho}{D} - Z\right),\\
p = \phi(Z) &= \frac{D(2+\omega)^2 + 2bA^2}{\rho(2+\omega)^3}\left(\frac{a\rho}{D} - Z\right).
\end{aligned}$$

Along the equilibrium path, the producer price declines gradually to zero, and the tax τ grows toward a.

According to [154], there exist other Nash equilibria where both countries use non-linear strategies, and these equilibria lead to some steady state $Z_\infty^{**} < a\rho/D$. However, one could argue that, in the context of this model, such equilibria are not subgame perfect, in the sense that if both players find themselves at $Z_\infty^{**} < a\rho/D$, they would want to have more output and hence more pollution, and that both would gain by departing from Z_∞^{**}. Wirl and Dockner [156] consider a variation of [154] by allowing a political economy approach to government policy formulation.

Stagewise Stackelberg Leadership by the Resource-Exporting Country

Tahvonen [142] uses a model similar to [154] to analyze a leader–follower game. He abstracts from the flow pollution and assumes that the exporter is the leader. The model is limited to stagewise leadership, which can be explained as follows. If the time horizon is finite, and time is discrete, stagewise leadership by the exporter means that in each period, the resource-exporting country moves first by announcing the wellhead price for that period. The importing country reacts to that price, and chooses the per unit tax for that period. Each party's equilibrium payoff for period $T-1$ can be computed as function of the opening stock Z_{T-1}. Working backwards, in period $T-2$, the leader announces his p_{T-2}, and the follower reacts by choosing τ_{T-2}, etc. Extending the stagewise formulation to the case of a continuous-time and infinite-horizon game involves the feeding of one HJB equation into another, as the analysis by Tahvonen shows. As demonstrated below, the stagewise Stackelberg equilibrium for this model is identical to the Nash equilibrium found by Wirl [154].[4]

Define the stock that remains at time t as $x(t)$. Assume $x(0)=\overline{S}$, and $Z(0)=0$. Then, since $Z(t)$ is the cumulative extraction, $\overline{S}-x(t)=Z(t)$. Tahvonen [142] assumes that $\overline{S}>a\rho/D$. For simplicity, assume that extraction is costless.

Unlike the simultaneous-move formulation in [154], in the stagewise Stackelberg formulation by Tahvonen, the importing country observes the actual price $p(t)$ before it chooses its tariff rate $\tau(t)$, and it also knows that $p(t)=h(x(t))$, which is a stationary rule. Tahvonen's approach begins with the following (time-independent) HJB equation of the importing country:

$$\rho V_I(x)=\max_{\tau}\left\{\frac{1}{2}(a-p)^2-\frac{1}{2}\tau^2-\frac{D}{2}(\overline{S}-x)^2-V_I'(x)(a-p-\tau)\right\}, \tag{6}$$

where $p=h(x)$. Now, assume that $h(x)$ is linear, i.e., $h(x)=\alpha x+\beta$ where a and β are to be determined. Then it is reasonable to conjecture that $V_I(x)$ is quadratic. Let us write

$$V_I(x)=\frac{1}{2}A_I x^2+B_I x+C_I.$$

Clearly, A_I, B_I, and C_I should be dependent on α and β. Equation (6) gives the necessary condition $-\tau+A_I x+B_I=0$. Note that the quantity demanded is $y=a-p-A_I x-B_I$ where $p=h(x)=\alpha x+\beta$. Tahvonen obtains the HJB equation for the seller

$$\rho V_X(x)=\max_{p}\left\{p(a-p-A_I x-B_I)-V_X'(x)(a-p-A_I x-B_I)\right\}.$$

This yields a solution that is identical Wirl's Nash equilibrium solution. It would seem that if one maximizes the leader's welfare integral with respect to the parameters α and β, one would obtain higher welfare for the leader; see [57] for the use of this alternative type of Stackelberg leadership, in a different context.

Bilateral Monopoly with Non-decaying Pollution and Stock-Dependent Extraction Cost

Rubio and Escriche [130] modified the model of Wirl [154] by assuming that extraction cost increases as the remaining stock falls. Let $Z(t)$ be accumulated extraction, so that $\dot{Z}(t)=y(t)$. Let $cZ(t)y(t)$ be the cost of extracting $y(t)$. Consider first the Nash equilibrium. The HJB equations yield the following pair of nonlinear differential equations:

$$\rho J_X(Z)=\frac{1}{4}(a-cZ+J_I'+J_X')^2,$$

$$\rho J_I(Z)=-\frac{1}{2}DZ^2+\frac{1}{8}(a-cZ+J_I'+J_X')^2.$$

The consumer price, denoted by θ, satisfies

$$\theta=\frac{1}{2}\left[a+cZ-(J_I'+J_X')\right].$$

[4] For a diagramatic interpretation of this result, see [57].

Rubio and Escriche [130] thought that in the Nash equilibrium, the tax τ is just a Pigouvian Tax without a rent-shifting component. The authors wrote a Bellman equation for the consumers that they interpreted as representing their perfectly competitive behavior. However, that equation, which contains the term $J_I'(Z)y$, means that consumers are not price takers: they realize that more consumption today will increase future values of Z which will, in turn, affect the future wellhead price, according to the seller's feedback pricing rule.

Rubio and Escriche [130] also considered the stagewise Stackelberg leadership by the exporting country. They showed that it is identical to the Nash equilibrium. This is similar to the identity between the Nash equilibrium in [154] and the stagewise Stackelberg equilibrium found by Tahvonen [142] in the case of zero extraction cost.

Turning to the case where the importing country is the stagewise Stackelberg leader, the authors showed that while the long-run pollution stock in this equilibrium is identical to that of the Nash equilibrium, the time paths of pollution in the two cases are substantially different. The initial consumer price and tax rate are lower in the Nash equilibrium than under the importing country's stagewise leadership. The life-time payoff of the importing country is higher under its stagewise leadership than under the Nash equilibrium, and the opposite ranking applies to the exporter's life-time profit.

In an elegant paper, Liski and Tahvonen [102] revisit the Nash equilibrium of the model of Rubio and Escriche [130].They decompose the Nash equilibrium carbon tax into a pure Pigouvian component and a rent-shifting component. Letting $x = \overline{S} - Z$, Liski and Tahvonen find that the Nash equilibrium strategy of the seller can be an increasing function or a decreasing function. If there are no pollution damages ($D = 0$), then it is a decreasing function of x, i.e., the producer price will be rising over time (as x falls over time). However, if D is sufficiently large, then it is an increasing function, i.e., the producer price will be falling over time. The carbon tax rises (respectively, falls) over time if the damage parameter D is large (respectively, small).

Models of Carbon Taxes with Pollution Decay

The one-to-one relationship between the cumulative extraction and the pollution stock no longer exists if pollution is subject to decay. In this case, the dynamic game becomes more complicated, as it involves two state variables. This case is taken up in [155], which is an extension of [154] in two directions: the pollution stock has a constant rate of decay, and the extraction cost increases as the remaining stock dwindles. An analytical solution does not seem possible for this game with two state variables. The author relies on numerical methods to indicate possible paths of tax, price, and pollution. Tahvonen [142] considers a similar two-state variable game, but focuses on the case where the exporting country exercises stagewise Stackelberg leadership.

Exhaustible Resources and International Trade

Optimal Export Tax by a Resource-Exporting Country when the Resource-Importing Country is Passive

Kemp and Long [91] consider trade between a resource-rich and a resource-poor economy. The former exports its resource which is used in the production of a final good produced by the latter. Extraction is costless. The resource-exporting country seeks to influence its terms of trade by choosing a time path of export tax. Let $\tau(t)$ be the ad-valorem export tax rate, defined by

$$p^f(t) = \frac{p^*(t)}{1+\tau(t)},$$

where $p^*(t)$ is the world price, $p^f(t)$ is what the producer gets, and the tariff revenue per unit of export is $\tau(t)p^f(t)$. Acting as an open-loop leader, this country commits to a time path of ad-valorem export tax rate. Kemp and Long show that at the optimum, the leader's export volume $m^*(t)$ (i.e., import volume by the resource-poor economy) and its optimal ad-valorem export tax rate $\tau^*(t)$ satisfy the leader's optimality condition

$$1+\tau^*(t) = \frac{1}{\beta[1+\delta(m^*(t))]},$$

where $\delta(m)$ is the elasticity of marginal productivity of the resource input,

$$\delta(m) \equiv \frac{m}{p^*}\frac{dp^*}{dm},$$

and β is an arbitrary positive constant. This shows that in equilibrium the relationship between the producer's price $p^f(t)$ and the price that importers pay, $p^*(t)$, is given by

$$p^f(t) = p^*(t)\beta\big[1+\delta\big(m^*(t)\big)\big].$$

Since β is an arbitrary positive constant, the above equation indicates that only the rate of change in the tax rate matters, not its level.[5] This is because if the time path $p^f(t)$ is multiplied by a constant, extracting firms will not change their supply behavior, and it is a matter of indifference to citizens of the resource-exporting country whether they receive their income as dividends or tax hand-outs.

In the special case where the production function of the resource-rich country is Cobb–Douglas, so that the input demand is iso-elastic, the resource-rich country has no market power. This is consistent with the result of Weinstein and Zeckhauser [153] and Stiglitz [141] that under zero extraction cost and constant elasticity of demand, the exhaustible resource monopoly has no market power.

Optimal Tariff by Resource-Importing Countries when the Resource-Exporting Country Is Passive

Bergstrom [17] models a game involving n resource-importing countries facing competitive extractive firms operating from a resource-exporting country. Bergstrom assumes that these countries are constrained to choose a *time-invariant* tariff rate.[6] He shows that the importing countries can extract substantial rents from resource-owners.

Kemp and Long [92] allow a resource-poor economy (called Home) to choose a time-varying *ad valorem* tariff rate $\tau(t)$ on the resource good imported from resource-rich country (called Foreign) in which extractive firms are price-takers. Home acts as an open-loop

[5] This result and further results on supply response to resource taxation are explored in more details in [109]. Supply response to tax changes is a key building block in the literature on the "Green Paradox" [65, 69, 78, 137, 138, 152]. Another mechanism behind the Green Paradox is the uncertainty about future carbon taxes, which is akin to uncertainty about future expropriation [103].

[6] Brander and Djajic [21] also assume a constant ad-valorem tariff.

Stackelberg leader. This problem can be analyzed by asking what is an optimal precommitted time path of *per unit* tariff $\theta(t)$. Kemp and Long [92] show that such an optimal tariff path is time-inconsistent, in the sense that at a later date the importing country would want to renege from the precommitted path.

To see this result on time inconsistency most clearly, let us assume that the extraction cost is zero and that there is a third country (called ROW, for "the rest of the world") that imports the resource good without trying to influence the terms of trade. The inverse demand function is $p^c(t) = a - bq(t)$, where $p^c(t)$ is the price facing the consumer (inclusive of the per unit tariff rate θ) and $q(t)$ is the quantity demanded. The price that foreign firms receive is $p^f(t)$. Then, in Home, $p^c_H(t) = p^f(t) + \theta(t) \equiv p^f(t)[1 + \tau(t)]$ while in ROW, which does not impose a tariff, $p^c_R(t) = p^f(t)$, where the subscripts H and R refer to Home and ROW.

Efficient allocation in Home requires that $p^c_H(t)$ rises exponentially at a rate equal to the rate of interest r until some time T where the $p^c_H(T) = a$. On the other hand, since extraction cost is zero, Hotelling's Rule requires $\dot{p}^f/p^f = r$. It follows that $\dot{\theta}/\theta = r$, which implies a constant ad-valorem tariff rate, $\tau(t) = \tau$. Note that at time T, consumers in Home stop importing because the consumer price has reached a, but the exporting country is still selling to the Rest of the World, as consumers in ROW are still paying for the resource at a price below the choke price a. Clearly, at T it will be in the interest of Home to cut the tariff rate so that their consumers can benefit from trade. This shows that its originally announced tariff path is time-inconsistent. If producers are smart, they will recognize from the start this time-inconsistency, and they will not believe in the announced tariff path. The time-inconsistency issue is considered in more detail by Karp [83] and Maskin and Newbery [117].

Karp and Newbery [86] consider time-consistent tariff policies. They assume a passive resource-exporting country with a large number of perfectly competitive sellers, and n resource-importing countries that behave strategically. The supply side is represented by a Markovian decision rule: aggregate extraction $y(t)$ is a function of the aggregate stock $S(t)$. In importing country i, the demand function is $q_i = b_i(a - p_i)$ where $p_i = p + \tau_i$ (with p being the world price and τ_i being the per-unit tariff imposed by country i). The instantaneous welfare W_i of importing country i is the sum of consumers surplus and tariff revenue:

$$W_i = \frac{b_i}{2}(a - p - \tau_i)(a - p + \tau_i). \tag{7}$$

Assume $n = 2$. The market-clearing price is $p(t) = a - y(t) - b_1\tau_1(t) - b_2\tau_2(t)$. All players move simultaneously. Suppose that importing country i believes that importing country j uses a Markovian tariff rule $\tau_j = g_j(S)$. Then its instantaneous welfare is

$$W_i(\tau_\iota, S) = \frac{b_i}{2}\bigl[Y(S) + b_j g_j(S) - (1 - b_i)\tau_i\bigr]\bigl[Y(S) + b_j g_j(S) + (1 + b_i)\tau_i\bigr].$$

Let $V_i(S)$ denote the value function for country i. Its HJB equation is

$$rV_i(S) = \max_{\tau_i}\bigl[W_i(\tau_i, S) - V_i'(S)Y(S)\bigr].$$

This gives the first order condition $\tau_i = b_i(a - p)$. Thus, in equilibrium, $\tau_1/\tau_2 = b_1/b_2$, and the equilibrium strategies satisfy

$$g_i(S) = b_i\bigl(1 - b_1^2 - b_2^2\bigr)Y(S).$$

The equilibrium price is a function of S:

$$p(S) = a - \left[1 - \left(b_1^2 + b_2^2\right) + \left(b_1^2 + b_2^2\right)^2\right] Y(S) \equiv a - \mu Y(S).$$

The equilibrium supply function $Y(S)$ can be determined as follows. From the Hotelling Rule, the price must rise at the rate of interest r

$$r = \frac{\dot{p}}{p} = \frac{p'(S)}{p(S)} \dot{S} = \frac{\mu Y'(S) Y(S)}{a - \mu Y(S)}.$$

This first order differential equation and the boundary condition that $Y(0) = 0$ allow Karp and Newbery [86] to compute the equilibrium supply rule $Y(S)$ numerically. Comparison with the free trade case indicates that the importing countries can be worse off compared with free trade. However, free trade is not an equilibrium of the game in which suppliers believe that buyers have market power.

Karp and Newbery [85] propose two different models of market power by resource-importing countries facing perfectly competitive exporters. In their EMF (or exporters move first) model, the sellers move first in each period. In the limit, as the length of each period shrinks to zero, the equilibrium of this model is the same as that of Karp and Newbery [86]. In their IMF (or importers move first) model, the authors assume that in each period, the importers make the first move by choosing quantities. For a wide range of numerical values, the ratio of an importing country's welfare under EMF to that under IMF depends on the initial stock S_0. For small values of S_0, it is disadvantageous for importing countries to be the first mover. In contrast, for very large values of S_0, their welfare under IMF is greater than under EMF.

If extraction costs rise with cumulative extraction, it is not always the case that the entire resource stock eventually will be exhausted. When the extraction cost becomes as high as the choke price the resource will be abandoned. Karp [83] explores the issues of time-consistent tariff under economic abandonment using a two-country model: a resource-importing country and a resource-exporting country that consists of competitive extractive firms. His proposed solution supposes that the importing country behaves as if it would want to maximize world welfare. If the seller is a monopolist, then for this method to work, it would be necessary to assume that the monopolist does not take into account that fact that the importing country uses a feedback decision rule.

Optimal Tariff on an Exhaustible Resource in a Market Characterized by Bilateral Monopoly

We have considered the case of bilateral monopoly in a resource market where its consumption generates pollution; see, e.g., [130]. By dropping the pollution aspect, Rubio [129] obtained similar results in a two-country model of trade in an exhaustible resource where both parties have market power.

Chou and Long [26] derive Markov-perfect tariff strategy when the resource-exporting country exercises its monopoly power, while two importing countries either form a coalition or they choose their tariff strategies non-cooperatively. They restrict attention to price-setting behavior. Several interesting results are obtained in the case where the two importing countries are not identical. As the two importing countries become more asymmetric in terms of their relative size, their aggregate gain from trade is more likely to be higher under non-cooperative tariff war than under global free trade. They also consider the case where the resource-exporting country can commit to earmark the resource stocks to serve the two importing countries separately. The optimal division is that which divides up the resources in proportion to relative size of the importing countries. However, such commitment makes the exporting country worse off compared with the case of undivided reserves.

Fujiwara and Long [57] extend the analysis of Chou and Long [26] to the case where the importing country is a global (as opposed to stagewise) feedback Stackelberg leader, and to the case where it is a global feedback Stackelberg follower. They show that being a global feedback Stackelberg leader makes the importing country better off than being a stagewise Stackelberg leader. On the other hand, world welfare under Nash equilibrium is strictly higher than under global Stackelberg equilibrium. Regardless of which country is the leader, world welfare under stagewise Stackelberg leadership is higher than under global Stackelberg leadership.[7]

R&D on a Substitute for an Exhaustible Resource

Research and development on a substitute for an exhaustible resource is one of the policy priorities of major resource-importing countries. Davidson [39] analyzes optimal control problems involving R&D for a substitute. Hoel [75–77] study the optimal extraction path of a monopolist facing the possibility of substitute production by competitive firms.

A number of authors study a game-theoretic situation where a resource-importing country, M, seeks to reduce reliance from a resource-exporting country, X, by investing in a substitute. Dasgupta et al. [36], Gallini et al. [58], and Olson [123] determine the optimal timing of innovation, T. They assume that M can commit to its choice of T before country X chooses its output path.[8] Dasgupta et al. [36] raise the possibility that by delaying T, M may be able to induce X to hasten extraction. However, Olson [123] points out that the range of parameter values consistent with this possibility is very small.

There are two main weaknesses in the above-mentioned approach. First, it is implausible that the innovation date is deterministic. Second, one cannot expect that a country can commit to a time path of R&D independently of its resource stock level.

Lewis et al. [101] consider a three-period model in which country M can invest in capacity that becomes productive in the following period. They show that M may over-invest in order to induce a more advantageous extraction path.

Harris and Vickers [73] develop a model of R&D where the date of discovery is uncertain. They seek a Markov-perfect equilibrium for the game between M and X. They rely on an approximate reformulation of the concept of Markov strategy, by allowing X to choose a time path for the resource stock (rather than a decision rule on extraction rate) subject to a set of consistency conditions. They find that M increases its R&D intensity as the resource stock dwindles. This induces X to reduce the rate of decline in the stock. Thus a non-monotone extraction path may emerge.

Optimal Taxation of Resource-Extracting Firms

In designing a tax-subsidy scheme to induce efficient extraction by a monopolist, the government is acting as a leader. Bergstrom et al. [19] consider an open-loop formulation and

[7] For further discussion of (non-stagewise) feedback Stackelberg equilibrium, see [45, 135], and [111].

[8] In contrast, Hung and Quyen [80] argue that country X is a natural Stackelberg leader. In their model, country M can at any time invent the substitute by paying a fixed lump-sum cost.

show that by choosing an appropriate time path of subsidy, a government can induce a monopolist to extract at the socially optimal rate. Interestingly, there is a family of such time paths. As pointed out by Karp and Livernois [84], if the government is unable to commit to its chosen time path of subsidy, the monopolist will have an incentive to deviate from the extraction path the government wants because by doing so he can force the government to change the subsidy path. In other words, the subsidy policies advanced by Bergstrom et al. are not subgame perfect. Karp and Livernois demonstrate the existence of a family of linear Markovian subsidy rule that would induce the monopolist to extract at the socially efficient rate; for similar results in a more general setting, see [6, 8, 11].[9]

The problem of extracting rent from a mining firm under asymmetric information is addressed by Gaudet et al. [62]. The firm's cost is its private information. The government designs an incentive scheme to maximize social welfare subject to the extractive firm's rationality constraint. The government is a Stackelberg leader and the firm is the follower. The optimal non-linear resource royalty schedule is characterized. For related analyses of principal agent problems in a dynamic context, see [63, 64].

Renewable Resources

Renewable resources are natural resources that, under careful management, could be maintained at positive steady-state levels. Examples of renewable resources include forests, aquifers, fish species and other animal species. Many renewable resources are threatened by excessive exploitation, partly because of lack of cooperation among agents who have common access to them. This problem is known as the tragedy of the commons [27, 67, 72].

The Tragedy of the Commons

When agents have common access to a resource stock, over-exploitation tends to occur. While institutions and social norms can be developed to mitigate the tragedy of the commons [74, 124], obvious instances of over-exploitation abound. In 2007, 80% of fish stocks are exploited at or beyond their maximum sustainable yield [54]. Grafton et al. [68] provide evidence of serious over-exploitation of several fish species. McWhinnie [120] finds that shared fish stocks are prone to excessive exploitation.

The fishery model has been used as a metaphor for almost any kind of renewable resource, especially when property rights are not well defined [22, 25, 31].

Over-exploitation of a Renewable Resource: An Open-Loop Approach

Clark and Munro [28] propose an open-loop differential game involving N fishermen who share a fishing ground. The price of fish is P per unit. Each fisherman's effort denoted by $E_i \in [0, \overline{E}]$. His landing is $Q_i = \omega E_i X^{\eta}$ where $\eta > 0$ and $\omega > 0$. The fish stock X evolves according to the dynamic equation

$$\dot{X} = g(X) - \sum_{i=1}^{N} \omega E_i X^{\eta},$$

[9] An alternative mechanism for achieving efficiency (at least approximately) is to build up cooperation as an outcome of a differential game [10].

where $g(X)$ is the natural growth function. Let $c > 0$ denote the effort cost per unit. Player i's profit is

$$\pi_i = p\omega E_i X^\eta - cE_i.$$

The utility of profit is

$$U(\pi_i) = \frac{\pi_i^{1-\theta}}{1-\theta},$$

where $\theta \geq 0$.[10]

All players have the same discount rate $\rho > 0$. Player i chooses his time path of effort $E_i(t)$ to maximize his life-time payoff,

$$J_i = \int_0^\infty e^{-\rho t} U(\pi_i)\, dt.$$

If the players cooperate and coordinate their effort to maximize the sum of their payoffs, the resulting steady-state stock, denoted by X_∞^{so}, will satisfy the following "modified golden rule"

$$\rho = g'(X_\infty) + \eta\left(\frac{g(X_\infty)}{X}\right)\left[\frac{c}{p\omega(X_\infty)^\eta - c}\right]. \tag{8}$$

The left-hand side of (8) is the rate of impatience. The right-hand side of (8) is the rate of return of leaving a fish in the pool instead of catching it. It is the sum of two terms. The first term is the marginal natural growth rate of the stock (called the biological rate of interest) and the second term is the marginal benefit (in terms of cost reduction) of keeping an extra fish in the pool: it is equal to the *group* steady-state harvest per unit of stock, multiplied by the cost/profit ratio (per unit of effort), and the elasticity of harvest with respect to stock, η.

Without cooperation, each player does not take into account the fact that his effort today will raise other fishermen' costs tomorrow via its effect on tomorrow's stock. The symmetric open-loop Nash equilibrium results in a steady state X_∞^{OL} that satisfies the following "externality-distorted modified golden rule":

$$\rho = g'\big(X_\infty^{\text{OL}}\big) + \frac{\eta}{N}\left(\frac{g(X_\infty^{\text{OL}})}{X}\right)\left[\frac{c}{p\omega(X_\infty^{\text{OL}})^\eta - c} - (N-1)\right]. \tag{9}$$

In the second term on the right-hand side of (9), only the *individual* steady-state harvest is counted in the marginal benefit term, and the steady-state harvest of the other $N-1$ agents is considered as a reduction in the individual's rate of return in leaving an additional fish in the pool. Notice that if $N = 1$ then the two equations (9) and (8) would be identical.

This model has been generalized by Long and McWhinnie [107], who assume that fishermen care about relative income, i.e.,

$$U = \frac{1}{1-\theta}(\pi_i - \gamma\overline{\pi})^{1-\theta}$$

where $\overline{\pi}$ is the average profit in the industry and $0 \leq \gamma \leq 1$. They show that to achieve efficiency, two taxes are required: a tax on relative profit, and a tax on output.

[10] Clark and Munro [28] consider the case where $\theta = 0$.

Note that the steady-state distortion in the model of Clark and Munroe arises because the stock X is an argument the harvesting function, i.e., $\eta > 0$. If the harvest is independent of X (i.e., $\eta = 0$) then there is no externality in the steady state. In fact, in a multi-species model, Chiarella et al. [24] show that when the stock level has no effects on harvesting, there exist open-loop Nash equilibria that are Pareto-efficient (and other OLNEs that are not Pareto-efficient). These results are confirmed by Dockner and Kaitala [42]. On the other hand, as demonstrated by Martín-Herrán and Rincón-Zapareto [116], if fishermen use feedback strategies that are continuous in the state variable, the resulting feedback Nash equilibrium is inefficient.

Feedback Nash Equilibrium Exploitation of a Common-Property Renewable Resource

One of the most well-known papers on feedback Nash equilibrium is the fish-war model of Levhari and Mirman [99]. Let s_t be the stock of fish at time t. It is exploited by two countries, country 1 and country 2. Their harvest rates are denoted by q_t^1 and q_t^2. The evolution of the fish stock obeys the difference equation

$$s_{t+1} = \left(s_t - q_t^1 - q_t^2\right)^\kappa, \quad 0 < \kappa < 1.$$

The utility function of country i is $\ln(q_t^i)$. Let $\beta \in (0, 1)$ be the discount factor.

If the two countries cooperate, s_t will converge to a unique steady state $\widehat{s} = (\beta\kappa)^{\kappa/(1-\kappa)}$. In the non-cooperative case, country i thinks that country j uses the harvesting strategy

$$q_t^j = \phi^j(s_t),$$

where $\phi^j(0) = 0$ and $\phi_s^j(s) > 0$. Country i takes $\phi^j(\cdot)$ as given, and finds the path q_t^i to maximize

$$\sum_{t=0}^{\infty} \beta^t \ln q_t^i$$

subject to $s_{t+1} = (s_t - \phi^j(s_t) - q_t^i)^\kappa$. The Bellman equation for country i is

$$V^i(s_{t+1}) = \max_{q_t^i}\left\{\ln q_t^i + \beta V^i\left[\left(s_t - \phi^j(s_t) - q_t^i\right)^\alpha\right]\right\}.$$

Suppose country i thinks that country j uses the linear feedback strategy $\phi^j(s) = \omega^j s$ where $0 < \omega^j < 1$.

Player i's response is to choose the feedback strategy $q^i = (1 - \beta\kappa)(1 - \omega^j)s \equiv \omega^i s$. In a symmetric Nash equilibrium,

$$\omega_1 = \omega_2 = \omega = \frac{1 - \beta\kappa}{2 - \beta\kappa}. \tag{10}$$

Under this feedback equilibrium, the evolution of the fish stock is

$$s_{t+1} = [s_t - 2\omega s_t]^\alpha = \left(\frac{\beta\kappa}{2 - \beta\kappa}\right)^\kappa s_t^\kappa.$$

This results in the non-cooperative steady-state stock

$$\widetilde{s} = \left(\frac{\beta\kappa}{2 - \beta\kappa}\right)^{\kappa/(1-\kappa)} < \widehat{s}.$$

Thus, under feedback strategies, the tragedy of the commons can occur even if the harvesting function depends only on the effort and is independent of the stock. This result contrasts sharply with the open-loop result found by Chiarella et al. [24].

The model of Levhari and Mirman [99] has been generalized by Koulovatianos et al. [95] to the case where the evolution of the fish stock takes the form

$$s_{t+1} = \theta\left[\kappa(y_t)^{\frac{\lambda-1}{\lambda}} + (1-\kappa)k^{\frac{\lambda-1}{\lambda}}\right]^{\frac{\lambda}{\lambda-1}},$$

where $y_t = s_t - \sum q_t^i$ and k is a positive constant.[11] They postulate the utility function

$$u(q) = \frac{q^{\frac{\lambda-1}{\lambda}} - 1}{\frac{\lambda-1}{\lambda}},$$

where $\lambda > 0$ is the intertemporal rate of substitution. The limiting case where $\lambda \to 1$ corresponds to the model of Levhari and Mirman [99]. Note the assumption that the utility function and the transition function have the same parameter λ. This coincidence of parameter values permits the existence of a Markov-perfect equilibrium in linear feedback strategies. Treating θ is a random variable, Koulovatianos et al. [95] show that changes in riskiness will have an effect of the equilibrium feedback strategy (except in the case where $\lambda = 1$). Under the assumption that θ has a log-normal distribution they demonstrate that an increase in the variance will magnify the tragedy of the commons if $\lambda < 1$.

Additional contributions of differential games of common access fisheries include [7, 15, 33, 38, 43, 53, 118, 121, 126], and [4, 5, 55], among others. Dutta and Sundaram [47, 48] and Duffie et al. [46] examine conditions for the existence of a MPNE (with or without random disturbance). Amir [2] proves some existence results for fishery games with convex transitions under uncertainty. For empirical models of dynamic fishery games, see, for example, [96] and [120].

It is well-known that in static repeated games if players use trigger strategies to punish deviation from an allocation, non-cooperative players can achieve Pareto-efficient outcomes, provided that their discount factor is close enough to 1. In dynamic games, trigger strategies can also serve to achieve a cooperative outcome (see, e.g., [15, 43]). Trigger strategies require memory of the history of the play, and therefore are not Markov-perfect.[12]

Martin-Herrán and Rincón-Zapareto [116] show that under certain conditions a Markov-perfect Nash equilibrium may achieve Pareto efficiency, and apply these conditions to a fishery game proposed by Clemhout and Wan [29]. In this game, despite common access, under certain parameter values, a Markov-perfect Nash equilibrium can be efficient if each player derives pleasure from other players' consumption. This is because the positive externalities of altruism and the negative externalities of common access cancel each other out under suitable parameter values.

Linear Feedback Strategies in Fishery Problems

In general, it is difficult to obtain closed-form solution for feedback Nash equilibria in fishery games. An exception is the class of games where each player's problem can be transformed into an optimization problem that is linear in the (transformed) state variable in such

[11] When the number of players is 1, this model reduces to the case studied by Benhabib and Rustichini [16].

[12] Another route to achieve efficiency is via the concept of "incentive equilibrium"; see, e.g., [49, 50] and [82].

a way that the first order condition with respect to the control variable is independent of the state variable. For example, consider the Levhari–Mirman model [99]. Assume $q_t^i = \omega_t^i s_t$ where $\omega_t^i \in (0,1)$. Then the transition equation is $\ln s_{t+1} = \kappa \ln s_t + \kappa \ln(1-\omega_t^i - \omega_t^j)$. Consider the transformation of variable $y_t = \ln s_t$. Then both the objective function and the state dynamic equation become linear in z_t

$$\max \sum_{t=0}^{\infty} \beta^t \big[y_t + \ln \omega_t^i \big]$$

subject to

$$y_{t+1} = \kappa y_t + \kappa \ln\big(1 - \omega_t^i - \omega_t^j\big).$$

This gives rise to a Bellman equation that is linear in the new state variable z:

$$V_i(y_t) = \max_{\gamma_t^i} \big\{ z_t + \ln \omega_t^i + \beta V_i \big(\kappa z_t + \kappa \ln\big(1 - \omega_t^i - \omega_t^j\big)\big)\big\}. \tag{11}$$

We can conjecture that $V_i(z)$ is linear. Then, assuming symmetry, $V'(y) = A$, a constant. Then we can show

$$\omega = \frac{1-\kappa\beta}{2-\kappa\beta}$$

which is identical to (10).

For continuous time fishery models, a pair of linear harvest rules, $q^i = \omega^i s$, $i = 1, 2$, constitutes a Markov-perfect Nash equilibrium if by a suitable transformation of variables, the value function is linear in s. To illustrate, suppose the stock of fish is $s(t)$ and its net growth rate is

$$\dot{s}(t) = Bs(t)^\eta - \delta s(t) - q_1(t) - q_2(t), \qquad B > 0,$$

where q_i is the harvest by country i. Assume $0 < \eta < 1$, $\delta > 0$ and $B > 0$.

Assume the utility function is

$$U(q_i) = \frac{q_i^{1-\beta}}{1-\beta}.$$

Now consider the very special case where $\beta = \eta$. Transform the state variable

$$Y \equiv s^{1-\eta}.$$

Define the variable $\omega_i(t)$ as the catch rate per unit of stock. Then

$$\dot{Y} = (1-\eta)B - (1-\eta)(\delta + \omega_1 + \omega_2)Y. \tag{12}$$

Country i's objective function is

$$\int_0^\infty e^{-\rho t} \left[\frac{(\omega_i(t)s(t))^{1-\beta}}{1-\beta} \right] dt.$$

Then, with $\beta = \eta$, the optimization problem is linear in the state variable

$$\max_{\omega_i} \int_0^\infty e^{-\rho t} \left[\frac{(\omega_i(t))^{1-\eta}}{1-\eta} X(t) \right] dt$$

subject to (12).

This implies that $\omega_i(t)$ is independent of ω_j. Conjecture the value function $V_i(Y) = E_i + K_i Y$, where $K_i > 0$. The maximized HJB equation is

$$\rho E + \rho K Y = \frac{Y}{1-\eta}\big[(1-\eta)K\big]^{(\eta-1)/\eta} + (1-\eta)AK + (1-\eta)KY\big[\delta + 2\big[(1-\eta)K\big]^{-1/\eta}\big],$$

and it is simple to determine K and E.

Consider next the case of a natural growth function that has a finite derivative at $s = 0$. Suppose

$$\dot{s}(t) = rs(t) - Bs(t)^{\eta} - q_1(t) - q_2(t),$$

where $r > 0$, $B > 0$, and $\eta > 1$. Use the transformation $Z(t) \equiv s(t)^{1-\eta}$. Notice that since $1-\eta < 0$, higher value of Z means lower value of the fish stock s. As $Z \to \infty$, $s \to 0$. So we expect the shadow price of X to be negative. Again, writing $q_i(t) = \omega_i(t)s(t)$, we get

$$\dot{Z} = (\eta-1)B + (\eta-1)(\omega_1 + \omega_2 - r)Z.$$

Assume the utility function

$$U(q_i) = \frac{q_i^{1-\beta}}{1-\beta}.$$

Take the case where $\beta = \eta$. Since $\eta > 1$, the utility function is bounded above by zero. Try the value function $V_i(Z) = D_i - G_i Z$ where $G_i > 0$. This means $V_i'(Z) = -G_i < 0$ as we would expect since high Z means low s. The HJB equation for player i is

$$\rho V_i(Z) = \max_{\omega_i}\left\{\frac{Z\omega_i^{1-\eta}}{1-\eta} + V_i'(Z)\big[(\eta-1)B + (\eta-1)(\omega_i + \omega_j - r)Z\big]\right\}.$$

This yields

$$\omega_i^{-\eta} = (\eta-1)G_i > 0.$$

In conclusion, to have Markov-perfect equilibrium harvesting strategies of the form $q_i = \omega_i s$, the natural growth function and the utility function must have a common parameter [29, 59, 148, 149].

Koulovatianos [94] proposes a more general model, with two fish species. Their stocks are $u(t)$ and $v(t)$. The aggregate harvest rates from these stocks are denoted by $q_u(t)$ and $q_v(t)$.

The dynamic equations are

$$\dot{u}(t) = A_u u(t)^{\eta} - \left\{\delta_u + D_u\left[\frac{v(t)}{u(t)}\right]^{1-\eta}\right\}u(t) - q_u(t),$$

$$\dot{v}(t) = A_v v(t)^{\eta} - \left\{\delta_v + D_v\left[\frac{u(t)}{v(t)}\right]^{1-\eta}\right\}v(t) - q_v(t).$$

Assume $0 < \eta < 1$, $\delta_u > 0$, $\delta_v > 0$.

The case $D_u > 0$ and $D_v > 0$ implies competing species. In contrast, if $D_u < 0$ and $D_v > 0$, then the two species bear a predator–prey relationship. Consider the linear strategies

$q_u(t) = b_u u(t)$ and $q_v(t) = b_v v(t)$. Transform the state variables, so that $U \equiv u^{1-\eta}$ and $V \equiv v^{1-\eta}$. Then we can write

$$\dot{Z} = A + JZ, \tag{13}$$

where

$$Z \equiv \begin{bmatrix} \mathrm{U} \\ \mathrm{V} \end{bmatrix}, \qquad A \equiv (1-\eta)\begin{bmatrix} A_u \\ A_v \end{bmatrix},$$
$$J \equiv (1-\eta)\begin{bmatrix} -\delta_u - b_u & -D_u \\ -D_v & -\delta_v - b_v \end{bmatrix}.$$

Assume that $\det(J) > 0$. Then both eigenvalues are negative. The system has a unique and stable steady state $Z^{\mathrm{ss}} = -J^{-1}A$. Thus,

$$U^{\mathrm{ss}} = \frac{(1-\eta)[A_u(\delta_v + b_v) - D_u A_v]}{(\delta_u + b_u)(\delta_v + b_v) - D_u D_v}.$$

Parameter values must be restricted such that U^{ss} and V^{ss} are positive. Koulovatianos [94] shows that the case where the growth rates are affected by Brownian motions can be analyzed without much complication.

Oligopoly in Renewable Resource Markets

When players are large enough to influence the market price, we have an oligopolistic fishery. Models of this type include Dockner et al. [44], Jørgensen and Yeung [81], Benchekroun [4, 5], Fujiwara [55, 56]. Benchekroun [4] shows that an exogenous unilateral restriction in one firm's harvest can lead to a decrease in the steady-state stock. Benchekroun [5] shows that an increase in the number of firms results in a lower steady-state industry output. Another kind of oligopolistic dynamic games involves the rivalry between a producer of industrial material derived from a virgin source (e.g., paper from cutting down trees) and a producer of industrial material derived from recycling [61].

Jørgensen and Yeung [81] specify the following stochastic differential equation for a fish stock s exploited by N oligopolists:

$$ds = \left[as^{1/2} - bs - \sum_{i=1}^{N} q_i \right] dt + \sigma s\, dW, \tag{14}$$

where W is a Wiener process, i.e., dW is normally distributed with mean zero and variance σ^2. Here $q_i(t)$ is agent i's harvest at time t. The parameter b is the death rate.

The total amount of fish caught at time t is

$$Q(t) = \sum_{i=1}^{N} q_i(t).$$

It is assumed that the inverse demand function is $P = Q^{-1/2}$. Player i's total cost of catching q_i fish is

$$C(q_i, s) = \frac{c}{\sqrt{s}} q_i.$$

Player i seeks to maximize

$$E_0\left\{\int_0^\infty e^{-\rho t}\left[Pq_i - \frac{c}{\sqrt{s}}q_i\right]dt\right\}.$$

Jørgensen and Yeung [81] show that the value function of i is $V_i(s) = A\sqrt{s} + B$, where A is the unique positive root of the cubic equation

$$\frac{1}{4}\phi A^3 + \phi c A^2 + \left[\phi c^2 + \frac{4N^2 - 8N + 3}{8N^2}\right]A = \frac{(2n-1)c}{4n^3}$$

and $B = \frac{(2N-1)c}{4N^3}$, where $\phi \equiv \rho + \frac{\sigma^2}{8} + \frac{b}{c}$. Assuming symmetry, the Nash equilibrium feedback strategies are

$$q_1(s) = q_2(s) = \left(\frac{(2N-1)c}{4N^3}\right)\left(c + \frac{A}{2}\right)^{-2} s.$$

This equilibrium implies that at any given fish stock, the harvest rate is increasing in the death rate b and in the variance σ^2, but it is decreasing in the cost parameter c. The model can be generalized as follows. Consider a more general transition equation

$$ds = \left[as^\theta - bs - \sum_{i=1}^{N} q_i\right]dt + \sigma s\, dW, \quad \text{where } \theta \in (0,1). \tag{15}$$

Assume $P = Q^{-(1-\theta)}$ and

$$C(q_i, s) = \frac{cq_i}{s^{1-\theta}}.$$

Define the harvesting intensity of firm i as

$$\omega_i(t) = \frac{q_i(t)}{s(t)}.$$

The profit of firm i is

$$\begin{aligned}
\pi_i &= Pq_i - \frac{cq_i}{s^{1-\theta}} = \frac{\omega_i s}{\left[\omega_i + \sum_{j\neq i}\omega_j\right]^{1-\theta} s^{1-\theta}} - \frac{c\omega_i s}{s^{1-\theta}} \\
&= s^\theta\left[\frac{\omega_i}{\left(\omega_i + \sum_{j\neq i}\omega_j\right)^{1-\theta}} - c\omega_i\right].
\end{aligned} \tag{16}$$

Perform a transformation of variable by defining $Y = s^\theta \equiv F(s)$. Then

$$dY = \left[\theta a - \theta Y\left(b + \frac{1}{2}(1-\theta)\sigma^2 + \sum_{j=1}^{N}\omega_j\right)\right]dt + \theta\sigma Y\, dW. \tag{17}$$

Thus we have transformed the generalized model into a differential game that is linear in the state variable. The solution is now straightforward.

Entry Deterrence

There are situations where the number of fishermen are endogenous, for example, when incumbents must choose whether to accommodate or to deter entrants. Crabbé and Long [33] consider a country facing foreign poachers. If poachers take the average catch per vessel as given, the country, acting as the Stackelberg leader, can deter entry by overfishing, since a lower stock level raises the harvesting costs of poachers. In contrast, if poachers adopt Cournot behavior, the Stackelberg leader will find it optimal to accommodate entry. Mason and Polasky [118] consider a two-period model with an incumbent facing the potential entry of a rival firm. The incumbent deters entry by increasing its fishing effort, thus driving down the resource stock to raise the rival's cost. Social welfare falls as a result of entry deterrence. There is a parallel between this result and the result on "welfare-reducing enclosure" by Long [104], who shows that when the welfare of poachers is part of the social welfare, property owners' enclosure decision can reduce welfare, even though the final outcome is a competitive equilibrium.

Why Does Capital Fly from Poor Countries to Rich Countries? Resource-Extraction as a Metaphor for Corruption

Tornell and Velasco [144] consider a variant of the fish-war model of Levhari and Mirman [99], re-interpreting it as a model of corruption by two or more rivalrous powerful fractions that dominate the local economy. By allowing these fractions to invest their ill-gotten funds in assets created by advanced economy, they explain capital flows from poor countries to rich countries. Tornell and Lane [143] explain the idea further. They coin the term "voracity effect": an apparently favorable shock, such as increase in the price of the country's exported goods, can perversely generate a disproportionate increase in corruption and have harmful effects on the country's growth rate. A dilution of the concentration of power leads to faster growth and lower voracity.

The Basic Model of Voracity

Let us begin with the simplest model. Let S denote a stock of renewable resource. There are n rival groups (or factions) that exploit this resource. Let $R_i(t)$ be the rate of extraction by group i at time t. Faction i seeks to maximize its infinite-horizon payoff function

$$\int_0^\infty \left(\frac{\sigma}{\sigma-1}\right)\left[R_i(t)\right]^{(\sigma-1)/\sigma} \exp(-\delta t)\, dt$$

subject to

$$\dot{S}(t) = Ak(t) - R_i(t) - \sum_{j\neq i} R_j(t), \qquad A \geq 0,$$

where $R_i = 0$ if $S = 0$. Here $\sigma > 0$ is the intertemporal elasticity of substitution.

Faction i believes that all other factions $j \neq i$ use a linear feedback strategy $R_j = \alpha_j S$. Define

$$\widetilde{\beta} \equiv \sum_{j\neq i} \alpha_j .$$

Let $R_i(t) = \alpha_i(t)S(t)$. Faction i's HJB equation is

$$\delta V_i(k) = \max_{\alpha_i}\left\{\left(\frac{\sigma}{\sigma-1}\right)[k\alpha_i]^{(\sigma-1)/\sigma} + V_i'(k)(A - \widetilde{\beta} - \alpha_i)k\right\}.$$

Conjecture the value function

$$V_i(S) = \frac{B\sigma}{\sigma-1}k^{(\sigma-1)/\sigma},$$

where B is to be determined. The Nash equilibrium intensity of exploitation is

$$\alpha^N = \frac{\sigma\delta + (1-\sigma)A}{n - \sigma(n-1)},$$

where we assume $n - \sigma(n-1) > 0$.

In a more general version of the model, Tornell and Velasco assume that these rivalrous factions can invest their funds in a foreign economy that yields a rate of return $r > 0$. Assume $A > r$. Each faction now extracts $R_i(t)$ from the renewable resource and invest it abroad. Consumption $C_i(t)$ is financed by withdrawing from this privately-owned asset, denoted by X_i. Thus each faction faces two differential equations

$$\dot{S}(t) = AS(t) - R_i(t) - \sum_{j\neq i} R_j(t),$$

$$\dot{X}_i(t) = rX_i(t) + R_i(t) - C_i(t).$$

Assume that there are exogenous bounds on extractions, $b_L S \le R_i \le b_H S$. There are three symmetric Nash equilibria: (i) an interior equilibrium where all factions use the extraction strategy $R_i = \beta^o S$ where $b_L < \beta^o < b_H$, (ii) a pessimistic equilibrium, where they extract at the maximum rate, $R_i = b_H S$, and (iii) an optimistic equilibrium, $R_i = b_L S$.

For a symmetric interior equilibrium, we conjecture that the value function is of the form

$$V_i(X_i, S) = \frac{B\sigma}{\sigma-1}(X_i + S)^{(\sigma-1)/\sigma}.$$

Then

$$\beta^0 = \frac{A-r}{n-1} > 0$$

provided that $b_L < (A-r)/(n-1) < b_H$. The equilibrium consumption strategy is

$$C_i = \left[\delta\sigma + r(1-\sigma)\right](X_i + S) \equiv B^{-\sigma}(X_i + S).$$

This model shows that capital can flow from a poor country dominated by rivalrous factions to a rich country where the rate of return is lower. Each faction knows that while the social of return of holding asset in their home country is A, its own private rate of return is only $A - (n-1)\beta^o$, for its $(n-1)$ rivals appropriate part of the common return. Tornell and Lane [143] point out that the interior equilibrium exhibits "the voracity effect": an increase in A will lead to an increase in the equilibrium β^o. In other words, if the poor country experiences a technical progress or an improvement in its terms of trade, the rivalrous factions will appropriate more than proportionately, resulting in a greater rate of depletion of the country's productive asset.

In the pessimistic Nash equilibrium, extraction is at the upper bound because each player has the self-fulfilling belief that all factions extract as much as they can. In this pessimistic scenario, the value function is

$$V_i(X_i, S) = \frac{B\sigma}{\sigma - 1}(X_i + qS)^{(\sigma-1)/\sigma},$$

where q and B are to be determined. One can show that

$$q = \frac{b_H}{A - r + nb_H} < 1.$$

At the pessimistic Nash equilibrium, the extraction strategy is $R_i = b_H S$ and the consumption strategy is

$$C_i = \left[\delta\sigma + r(1-\sigma)\right](X_i + qS) \equiv B^{-\sigma}(X_i + qS).$$

Extensions of the Model of Voracity

The original authors of the capital flight model thought that including extraction costs or adjustment costs "would add nothing to the insights provided by the model" [143]. That belief turns out to be wrong. Later contributors Sorger [140], and Long and Sorger [110] show that modeling of extraction costs yield important additional insights. Long and Sorger [110] prove that an increase in appropriation costs reduces the growth rate of the common asset. This is a striking result: at a corruption equilibrium, higher costs of money laundering correspond to lower economic growth.

Long and Sorger [110] add the assumption that the agents derive utility not only from consumption but also from wealth.[13] They do not rely on iso-elastic utility functions. They find that an increase in the degree of heterogeneity of cost leads to poorer growth performance, and under certain conditions, a higher elasticity of substitution between wealth and consumption can lead to more voracious extraction, and thus slower the growth rate of the productive asset.

Both Tornell and his co-authors and Long and Sorger share the assumption that the utility of the players depends on their absolute consumption levels and/or absolute wealth levels. That assumption has recently been re-examined because of mounting empirical evidence that individual utility is affected by relative consumption (or relative income): a person's level of satisfaction depends on the comparison of his consumption level with that of other members of his reference group. Thus it becomes important to ask: If agents exploiting a common property resource care about their *relative* consumption, would social welfare and the growth rate of the public asset be more adversely affected? This question is taken up by Long and Wang [113]. Consider a lake shared by a number of municipalities, or provinces. Assume the reward to the administrator of each province is proportional to a relative performance criterion (e.g., relative employment levels or relative GDP growth rates). Would these government officials have stronger incentives to allow local businesses to pollute the lake? Long and Wang [113] explore the effect of the concern for relative performance on the tragedy of the commons, both in the sense of common access natural resources, and in the sense of rent-seeking fiscal appropriations. They also extend the model to the case where

[13] Wealth is a vehicle for achieving social status, see [30].

agents differ with respect to some characteristics. Two sources of heterogeneity are considered: the degree of status-consciousness and the level of appropriation costs. The authors find that social welfare decreases in the degree of heterogeneity in terms of status-seeking, but increases in the degree of heterogeneity in terms of appropriation costs.

New Directions

An interesting extension of dynamic games in natural resource economics is the study of coalition formation in the management of common property resources. For example, in a multi-country version of the fish war model of Levhari and Mirman, one can investigate the possible gainfulness of forming a coalition. A coalition is said to be stable in the sense of d'Aspremont et al. [37] if it is both internally stable (i.e., a member cannot gain from defection, given that all other players maintain their member/non-member status) and externally stable (i.e., no outsider wants to become member). This type of stability has been termed "myopic stability" [40]. An alternative assumption is the farsightedness assumption (e.g., [41]). It is important to explore dynamic games under various possible coalition structures under this assumption.[14]

Another area of research is a dynamic game analysis of the endogenous evolution of property right regimes. Recall that Ploeg [151] assumes that as the aggregate stock of private capital increases, the property rights of natural resources will be strengthened. It would be desirable to replace that assumption by an analysis of a process that leads to stronger enforcement of property rights. Such a process may be of a political-economy nature (e.g., by voting and lobbying for legislative changes).[15] The development of social norms to overcome the tragedy of the commons may also be studied as a differential game, along the line proposed by Benchekroun and Long [10].

Dynamic analysis of the tragedy of the commons has typically assumed that all players begin the game at the same time, and have a common time horizon. An interesting area of research is dynamic games of resource extraction involving forward-looking overlapping generations where later generations may or may not care about the welfare of earlier ones.[16]

Finally, games of common property resources between countries having different philosophies have not been explored. In the real world, countries are heterogeneous not only in terms of technology and endowments, but also in terms of philosophical outlook. What happens if one country has the utilitarian objective while the other country has the maximin objective, or a linear combination of the two objectives?[17]

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[14] For some preliminary steps in this direction, see [23] and [131].

[15] Leonard and Long [98] explore these issues in an overlapping-generation model.

[16] The second case involves an "ancestor-insensitive welfare function," a term coined by Asheim [3]. A sketch of games involving overlapping generations can be found in [105].

[17] For infinite-horizon optimization models using a linear combination of the utilitarian objective and the maximin objective, see [1] and [52].

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