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A Criterion Describing the Dynamic Yield Behavior of PMMA

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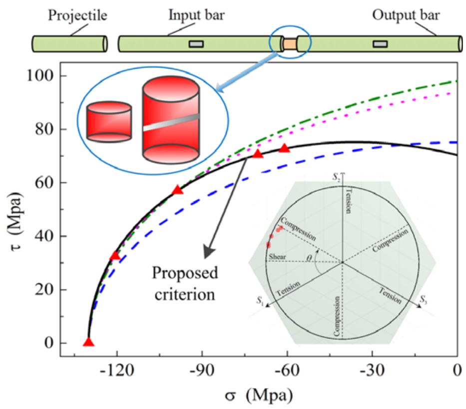
Abstract: Dynamic uniaxial and multiaxial tests were conducted to characterize the mechanical behavior of poly(methyl methacrylate) (PMMA). In this case, the multiaxial yield behavior of materials can be captured by the shear-compression samples (SCS). The experimental results were compared with different yield criteria, and which does not have a good agreement between theoretical prediction and experimental yield loci of PMMA undergoing combined shear-compression. Therefore, a phenomenological yield criterion is developed, and it notably involves hydrostatic stress sensitivity, lode angle, and one parameter. The parameter can be expressed by the asymmetry of yield strength between the tension and compression. The proposed criterion was verified by the utilization of differently combined shear-compression tests. The investigative results reveal that the proposed criterion can be successfully applied to describe the complex yield behavior of PMMA under dynamic loading conditions. The present study shows an efficient method to develop yield criterion of the reviewed materials.



Keywords: PMMA, complex stress state, yield condition, dynamic loading.

1. Introduction

The mechanical properties of poly(methyl methacrylate) (PMMA) have been widely researched and modeled.[1,2] Many mechanical tests, including uniaxial tension,[3] uniaxial compression,[4] and confined behavior,[5] have been conducted to provide an accurate representation for the mechanical response of polymer over a wide range of temperatures and strain rates. As a typically amorphous polymer, PMMA is frequently exposed to dynamic loading situations where complex stresses occur and in many cases are even dominating. These complex stresses may be either local or global. The split Hopkinson pressure-bar is the basic method for obtaining stress-strain curves at very high strain rates. The dynamic polymer properties of compressive and tensile tests under high strain rate loading have been investigated by many authors.[6-8] Forquin *et al*. analyzed the mechanical behavior of PMMA at high strain-rates for unconfined as well as confined conditions and observed that the compression stress is weakly influenced by the level of pressure and is much more sensitive to strain-rate.[9] Nasraoui *et al.*[10] investigated the strain-rate and temperature sensitivity of PMMA polymer through uniaxial compression tests under dynamic loading conditions at room temperature. The above discussion indicated that most of the prior researches on the mechanical behavior of PMMA mainly focus on simple stress state. In addition, there are some literatures also reported the mechanical behavior of polymer when subjected to complex stress state. Such as, Jin *et al.*[11] discussed the deformation modes and the failure locus of PMMA by using cylindrical specimen with beveled ends under quasi-static loading. Zhou *et al.*[12] completed the combined shear-compression stress state by employing the complex loading device and provided a failure criterion. The investigation for the mechanical response of polymer under complex stress state is limited, especially in dynamic loading.

In classical plasticity, the first aim is to find a function of all the components of stress which reaches a critical value for all tests. This is the yield criterion, and in its most general form is $f(I_1, J_2, J_3)$. The effect of hydrostatic pressure on several amorphous and crystalline polymers was first investigated by Holliday *et al.* in 1964.[13] Subsequently, several experiments were conducted in order to testify the result by many other authors in 1970s.[14] Silano[15] proposed a polynomial yield function based on hydrostatic pressure plays such an important role in the initial yield of polymers. This yield criterion was generalized under different strain rate by Farrokh *et al.*[16] and the strain rate sensitivity was considered in the yield criterion to simulate the yield behavior of polymer. Lee *et al.*[17] think that if the polymers are conducted a twist, the deformation involves both pure shear and

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rotation, and come to the conclusion that the third deviatoric invariant has a profound effect on yield behavior of polymer by analyzing Mohr-Coulomb criterion. The previous literatures show that the yield strength is highest in compression, followed by tension, and with the lowest value measured in shear in uniaxial testing.[18] Cazacu *et al.*[19] claimed that the asymmetry of compressive strength and tensile strength can be described by J_3, which due to twinning effect for pressure insensitive metals. However, Stachurski[20] concluded that the difference between tensile and compressive yield behavior (strength) was due to the hydrostatic pressure component for polymer by discussing the effect of pressure activation volume, based on the modified Eyring theory.

The above discussions indicated that although the mechanical behavior of PMMA has been investigated by measuring uniaxial and multiaxial stress state under quasi-static, very few studies about the complex stress state have been conducted to determine yield behaviors under dynamic loading conditions. The aim of this paper is to develop a phenomenological yield criterion which can be applied to describe the yield behavior of PMMA under complex stress state when subjected to dynamic loading. This criterion includes the hydrostatic pressure and lode angle, and involves only one parameter that is used to characterize the different between the tensile and compressive strength. Finally, the yield loci predicted by proposed criterion is compared with experimental results, and the agreement is very good.

2. Experimental

In the present tests, all specimens were machined by ordinary commercial PMMA bar (Φ10 mm×1000 mm). The cylindrical sample and shear-compression samples (SCS) were employed and the specific sizes as shown in Figure 1. The SCS can be obtained by introducing a slot on the surface of cylinder sample, so that generate additional shear. The different stress state can be achieved by altering the angle of slot which was defined as loading angle α. In this study, four different loading angles (*i.e.* 15°, 30°, 45°, 50°) were employed, which can be represented by four kinds of lode angles θ (*i.e.* 56.97°, 50.70°, 44.47°, 42.59°).

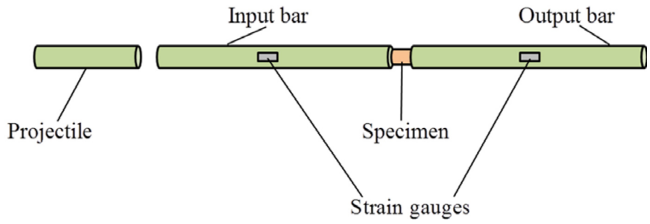

Figure 2. Schematic of SHPB set-up.

The Split Hopkinson pressure bar (SHPB) is a popular tool for the characterization of material behavior under impact loading. In this investigation, a 37 mm diameter SHPB setup is used as basic loading and measuring techniques. The impact velocity is 7.14 m/s. The schematic of SHPB is shown in Figure 2. A compressive longitudinal incident wave $\varepsilon_i(t)$ is generated in the input bar as soon as projectile strikes the input bar. A part of this elastic wave called $\varepsilon_r(t)$ is reflected, when it reaches the bar/sample interface. The other part goes through the sample and develops as the transmitted wave $\varepsilon_t(t)$ in the output bar. Then, the force $F(t)$, deformation $S(t)$ and strain rate $\dot{\varepsilon}(t)$ of sample can be obtained by following expressions:

$$F(t) = S_B E \varepsilon_t(t) \tag{1-1}$$

$$S(t) = 2C_0 \int_0^t \varepsilon_r(t) d\tau \tag{1-2}$$

$$\dot{\varepsilon}(t) = \frac{2C_0}{L_s} \varepsilon_r(t) \tag{1-3}$$

where S_B, E, C_0, and L_s are the cross section area of the bars, Young's modulus of the bars, the wave speed in the bars and the length of sample, respectively.

3. Results and discussion

The tests of five lode angles which include uniaxial compression (*i.e.* $\theta = 60°$) and combined shear–compression are conducted in this study. The SCSs after testing are shown in Figure 1. It can be seen that the SCSs are broken along gage section, and the fracture surface has an angle with the loading axis, which is

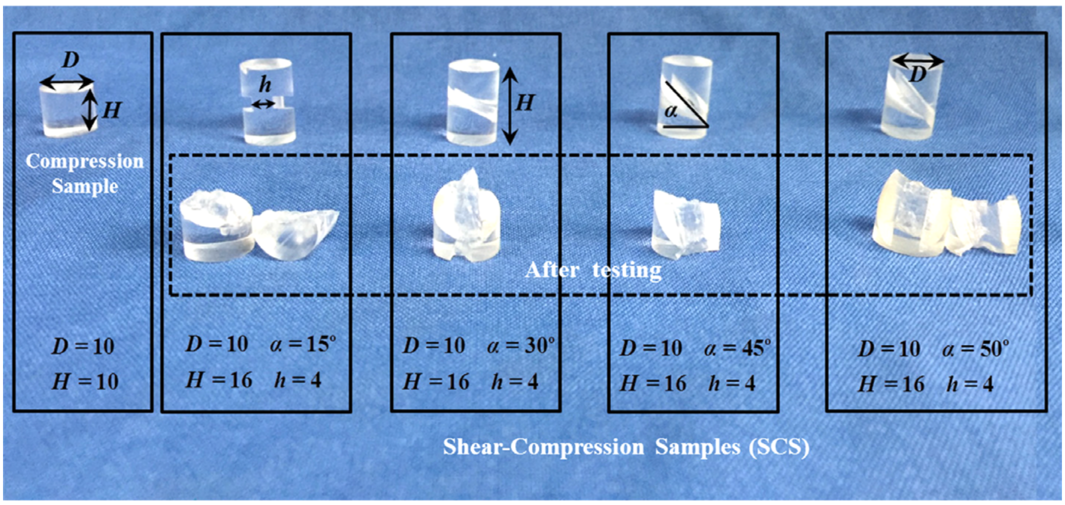

Figure 1. Specific sizes of specimens.

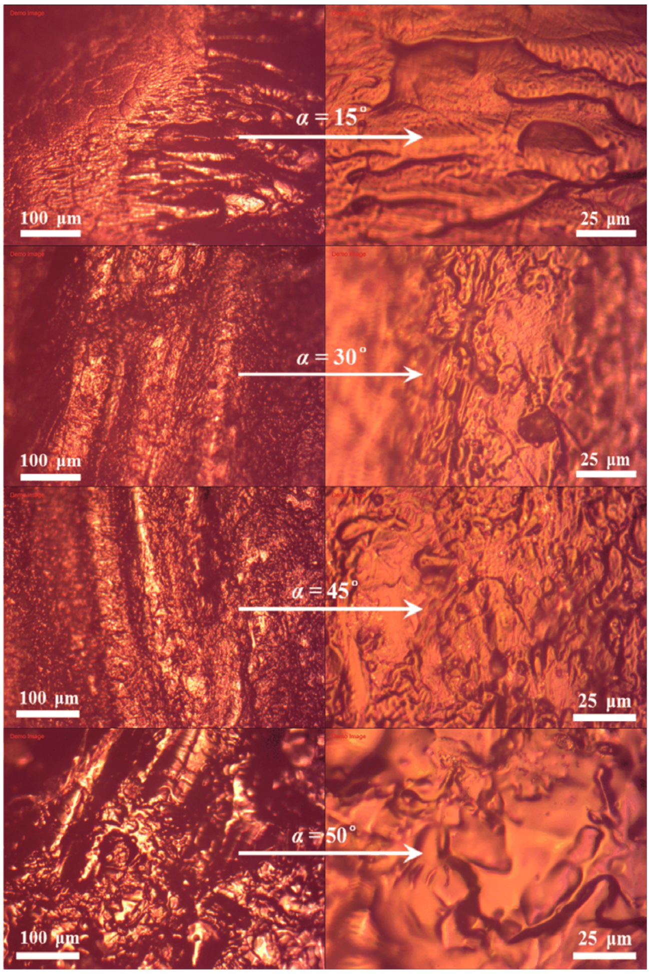

Figure 3. Fracture surfaces of microstructure dynamic SCSs (15°, 30°, 45°, 50°).

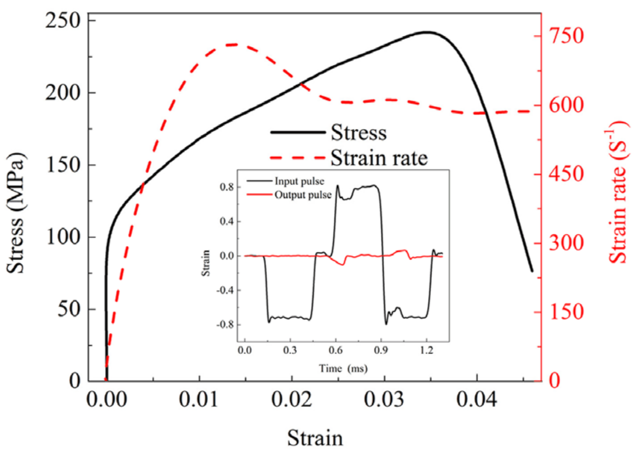

Figure 4. Input and output pulses, and stress–strain curves under uniaxial compression.

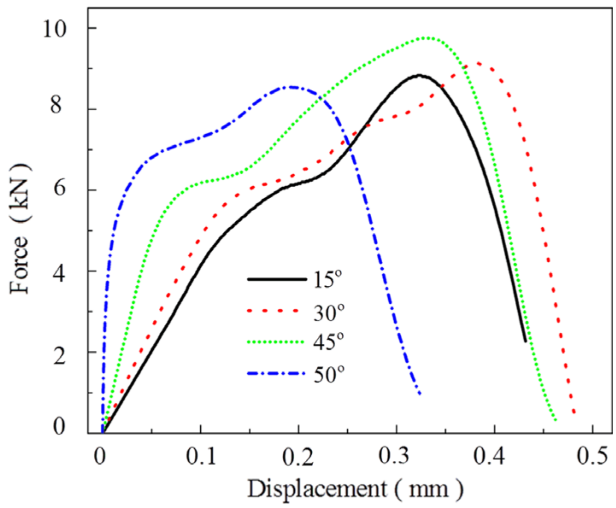

Figure 5. Dynamic pressure curves of combined shear–compression of different loading angles.

accordance with loading angle α. The micrographs of fracture surface are presented in Figure 3, which indicates a ductile fracture mode by a lot of dimples. It is interesting to mention that the primary failure is caused by shear stress because the dimples are anisometric, and slip bands are formed.

The typical dynamic compression result is shown in Figure 4. The hardening behavior is observed after the yield point. All the combined shear-compression tests are shown in Figure 5. The slope of the ascending segment to reach the initial yield increases with loading angle α. The peak force increases firstly, and then decreases with the increment of loading angle, and reaches the maximum value at 45°, which indicates much more difficult during the initial collapse of PMMA when the loading angle reaches 45°.

The state of stress at a point within the gage section for the SCS was determined by Vural *et al.*[21] The normal stress σ and shear stress τ can be expressed as:

$$\sigma = \frac{E}{Dh}\cos^2\alpha \quad \tau = \frac{E}{Dh}\cos\alpha\,\sin\alpha \tag{2}$$

where D and h are the diameter and the thickness of the gage section of SCS, respectively. Based on above analyzing, the components of compression and shear stresses are listed in Table 1.

Then the first invariant of stress, the second invariant of the deviatoric stress and the third invariant of the deviatoric stress are denoted by I_1, J_2, and J_3, which can be respectively written as:

Table 1. Values of yield stresses under different loading conditions

Sample	Compression	SCS			
α (°)	0	15	30	45	50
θ (°)	60	56.97	50.70	44.47	42.59
σ (MPa)	-130.00	-120.68	-98.68	-70.37	-60.98
τ (MPa)	0	32.36	57.03	70.37	72.58

Table 2. Values of invariants under different loading conditions

Sample	Compression	SCS			
θ (°)	60	56.97	50.70	44.47	42.59
I_1 (MPa)	-130.00	-120.68	-98.68	-70.37	-60.98
$\sqrt{J_2}$ (MPa)	75.06	76.83	80.61	81.33	80.67
$\sqrt[3]{J_3}$ (MPa)	-55.49	-54.20	-51.35	-48.06	-45.78

$$I_1 = \sigma$$

$$J_2 = \frac{1}{3}\cdot\sigma^2 + \tau^2$$

$$J_3 = \frac{2}{27}\cdot\sigma^3 + \frac{1}{3}\cdot\tau^2\sigma \tag{3}$$

Based on above discussion, the values of experimental yield stresses are tabulated in Table 2.

3.1. Yield behavior

The mathematical modeling of the yield surface is known to be related to the molecular mobility and architecture of polymer.[22] The two most widely yield criteria are the von-Mises and the Drucker-Prager yield criterion for polymer. The comparisons between the experimental yield points of PMMA and the von-Mises, D-P criteria in the shear-compression stress space are shown in Figure 6. Obviously, the experimental yield locus can be better described by D-P criterion than that of Mises, but the predicted values of D-P criterion are higher than the experimental results for the loading angles of 45° and 50°. Therefore, neither of the two criteria predicts the yield behavior of PMMA when subjected to combined shear-compression stress under dynamic loading.

The effect of hydrostatic pressure on the yield behavior have been highlighted by previous studies. D-P criterion take into consideration the impact of hydrostatic pressure on experimental yielding stress; however, it cannot give an appropriate characterization of the yield behavior when the main deformation mechanism caused by shear stress. Figure 7 shows the projection in the π plane (the plane normal to the hydrostatic axis $\sigma_1 = \sigma_2 = \sigma_3$) of the experimental yield loci, which are not on a same circle (*i.e.* the distance between the yield loci and hydrostatic axis varies as lode angle changes). Hence, a conclusion that the yield behavior of PMMA is dependent on lode angle θ can be obtained.

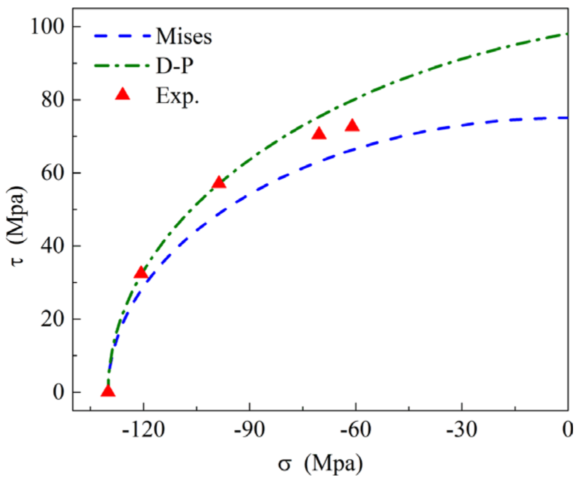

Figure 6. Comparison between the stress yield loci predicted by Mises and D-P with experiments in σ-τ space.

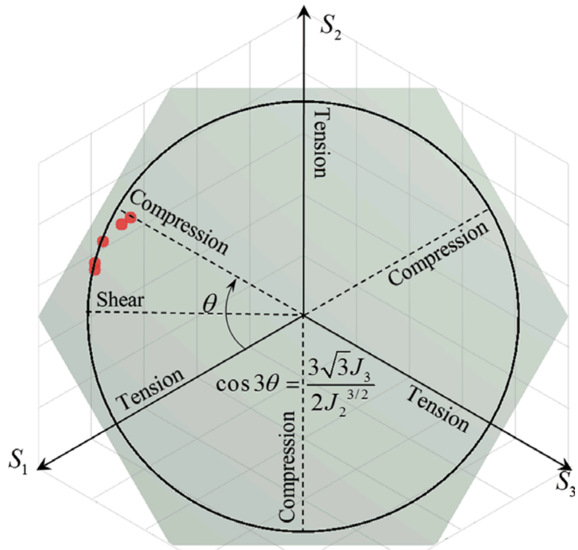

Figure 7. Yield loci of PMMA on π-plane (unit: MPa).

Therefore, Ghorbel[23] proposed a yield criterion based on three stresses invariant, I_1, J_2, and J_3 which accounted for shear banding, as follows:

$$f = \frac{3J_2}{\sigma_t}\left(1 - \frac{27J_3^2}{32J_2^3}\right) + \frac{7(m-1)}{8}I_1 - \frac{7}{8}m\sigma_t \qquad m = \frac{\sigma_c}{\sigma_t} \tag{4}$$

where σ_c, σ_t, and m are compression strength, tension strength and the ratio of the compression and tension, respectively. If choosing normal stress and shear stress replace the three invariant of the stress and deviatoric stress in Eq. (4), then the yield criterion can be rewritten as:

$$f = \frac{(\sigma^2+\tau^2)}{\sigma_t}\cdot\left(1 - \frac{27}{32}\cdot\frac{(2/27\cdot\sigma^3 + 1/3\cdot\tau^2\sigma)^2}{[1/3\cdot(\sigma^2+\tau^2)]^3}\right) + \frac{7(m-1)}{8}\cdot\sigma - \frac{7}{8}\cdot m\cdot\sigma_t \tag{5}$$

Figure 8 compares the D-P yield criterion and the yield criterion proposed by Ghorbel in the σ-τ space. It can be found that the Ghorbel yield criterion has higher agreement than the D-P yield criterion; that is because the Ghorbel criterion considers

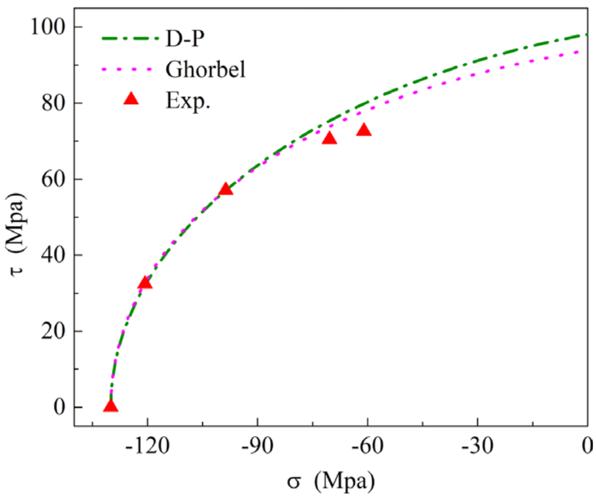

Figure 8. Comparison between the stress yield loci predicted by D-P and Ghorbel with experiments in σ-τ space.

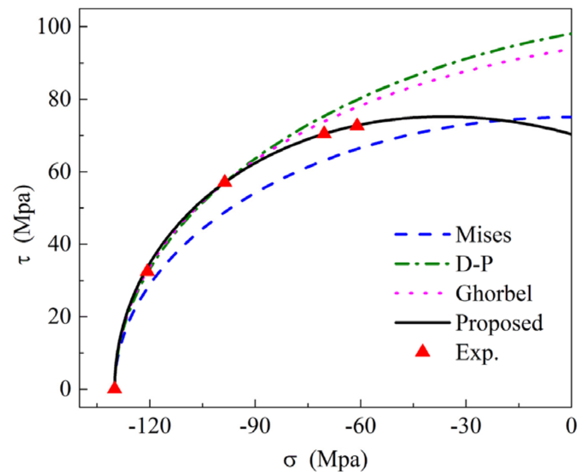

Figure 9. Comparison between the stress yield loci predicted by theory with experiments in σ-τ space.

the effect of the third invariant of the deviatoric stress J_3. However, it does not give an enough characterization of the yield behavior when subject to combined shear–compression stress under dynamic loading.

3.2. Proposed yield criterion

In general, the yield response of polymers is strongly dependent on the hydrostatic pressure applied on the specimen, and the dependence of all three yield stresses on hydrostatic pressure is linear or non-linear for polymer under quasi-static.[15] Hence, it is reasonable to think that the yield of PMMA also can be effected the hydrostatic pressure under dynamic loading. In addition, the yield stress measured in tension is lower than in compression due to the effect of hydrostatic pressure.[18,20] In other words, the parameter which characterizes the effects on the tension and compression asymmetry should be coupled with I_1. Therefore appropriate material modeling has to take into account the pressure sensitivity of the material.

The conclusion that the yield behavior of PMMA can be affected by lode angle has been obtained through the analysis of Figure 6. However, it is necessary to determine the relationship between J_3 and J_2, I_1 in the yield function. It can be observed that J_3 effects would be decoupled with I_1 from the existing yield criteria. The reason is that, J_3 must be zero because the deformation involves no rotations in purely hydrostatic loading. If a stress-free torsional specimen is subjected to a twist, the deformation involves both pure shear and rotation, and hence J_3 effects would be coupled with J_2. According to above analysis, the following isotropic yield criterion is proposed:

$$f=\left(1-\frac{1}{10}\cdot\cos^2 3\theta\right)\cdot\sqrt{3J_2}+a\cdot\left(I_1+\frac{I_1^2}{|\sigma_c|}\right)-\frac{9}{10}\cdot|\sigma_c| \tag{6}$$

where σ_c is the yield stress in uniaxial compression and a is a material parameter. The physical significance of the parameter a can be revealed from uniaxial tension test. If yielding in a uniaxial tension test occurs when $\sigma_1 = \sigma_t$, $\sigma_2 = \sigma_3 = 0$. Substituting these values in Eq. (6), the material constants a can be expressed as functions of the yield stresses σ_c and σ_t.

$$a=\frac{9(\sigma_c^2-|\sigma_c|\sigma_t)}{10(\sigma_t^2+|\sigma_c|\sigma_t)} \tag{7}$$

Notice that if there is no difference between the yield behavior in tension and compression, then a equals zero. For the yield criterion to be convex, the value of material parameter is limited to not less than zero, and which is independent on the temperature, strain rate and critical molecular mass M_c.[24,25] Therefore, the constant a can be determined by the tensile and compressive stresses under quasi-static conditions. In addition, the parameter a also can be obtained by solving Eq. (6) where an experimental data under combined shear-compressive loading is substituted. Then, the tensile stress can be determined by the value of a, and it is 70 MPa, which is consistent with the report from previous literature.[26]

For combined shear-compression tests, σ_1 is set equal to σ, σ_{12} is set equal to τ and all other stress components are set equal to zero, then the yield criterion Eq. (6) can be rewritten as:

$$f=\left(1-\frac{1}{10}\cdot\cos^2 3\theta\right)\cdot\sqrt{\sigma^2+3\tau^2}+a\cdot\left(\sigma+\frac{\sigma^2}{|\sigma_c|}\right)-\frac{9}{10}\cdot|\sigma_c| \tag{8}$$

The experimental yield data and different theory yield surfaces are summarized in σ-τ stress space, as shown in Figure 9. It is clearly that the proposed yield surface accords better with the experimental yield loci than other yield surface. Therefore, the proposed yield criterion can be used to describe the yield behavior of PMMA under dynamic loading.

4. Conclusions

In this paper, the mechanical responses of PMMA under dynamic loading are investigated by employing cylinder compressive sample and combined shear-compression samples with four different loading angles, respectively. Comparisons between experimental results and Mises, D-P, Ghorbel yield criteria predictions are conducted and discussed. The results show that the yield criteria can not accurately predict the yield loci of combined shear-compression under dynamic condition. Therefore, an isotropic criterion is proposed, which includes I_1, θ, J_2, and a material parameter a. The expression of parameter a concludes two terms of the tensile yield stress and the compressive yield stress. The asymmetry of yield stresses in tension and compression can be described by parameter a for hydrostatic stress sensitive polymers. The accuracy and effectiveness of the proposed yield criterion are verified employing the yield data of differently combined shear-compression stress states. It is believed that the proposed yield criterion permit to predict the yield behavior of PMMA. In addition, an idea that can be used to construct yield surface is exhibited.

References

(1) A. S. M. G. A. Katana, *Int. J. Polym. Mater. Polym. Biomater.*, **60**, 115 (2010).

(2) E. S. Park, E. B. Cho, and D. Kim, *Macromol. Res.*, **15**, 617 (2007).

(3) W. Chen, F. Lu, and M. Cheng, *Polym. Test.*, **21**, 113 (2002).

(4) E. M. Arruda, M. C. Boyce, and R. Jayachandran, *Mech. Mater.*, **19**, 193 (1995).

(5) D. Rittel, and A. Dorogoy, *J. Mech. Phys. Solids*, **56**, 3191 (2008).

(6) M. Jerabek, Z. Major, and R. W. Lang, *Polym. Test.*, **29**, 302 (2010).

(7) P. Yu, X. Yao, S. Tan, and Q. Han, *Macromol. Mater. Eng.*, **301**, 469 (2016).

(8) L. H. Zhang, X. H. Yao, S. G. Zang, and Q. Han, *Mater. Design*, **65**, 1181 (2015).

(9) P. Forquin, M. Nasraoui, A. Rusinek, and L. Siad, *Int. J. Impact Eng.*, **40**, 46 (2012).

(10) M. Nasraoui, P. Forquin, L. Siad, and A. Rusinek, *Mater. Design*, **37**, 500 (2012).

(11) T. Jin, Z. Zhou, Z. Wang, G. Wu, Z. Liu, and X. Shu, *Polym. Testing*, **42**, 181 (2015).

(12) Z. Zhou, B. Su, Z. Wang, Z. Li, X. Shu, and L. Zhao, *Mater. Lett.*, **109**, 151 (2013).

(13) L. Holliday, J. Mann, G. A. Pogany, H. L. D. Pugh, and D. A. Gunn, *Nature*, **202**, 381 (1964).

(14) L. W. Hu, and K. D. Pae, *J. Franklin Institute*, **275**, 491 (1963).

(15) A. A. Silano, S. K. Bhateja, and K. D. Pae, *Int. J. Polym. Mater. Polym. Biomater.*, **3**, 117 (1974).

(16) B. Farrokh and A. S. Khan, *Europ. J. Mech. A Solids*, **29**, 274 (2010).

(17) Y. K. Lee and J. Ghosh, *Int. J. Plasticity*, **12**, 1179 (1996).

(18) K. D. Pae, *J. Mater. Sci.*, **12**, 1209 (1977).

(19) O. Cazacu and F. Barlat, *Int. J. Plasticity*, **20**, 2027 (2004).

(20) Z. H. Stachurski, *Prog. Polym. Sci.*, **22**, 407 (1997).

(21) M. Vural, G. Ravichandran, and D. Rittel, *Metall. Mater. Trans. A*, **34**, 2873 (2003).

(22) A. Dasari and R. D. K. Misra, *Mater. Sci. Eng. A*, **358**, 356 (2003).

(23) E. Ghorbel, *Int. J. Plasticity*, **24**, 2032 (2008).

(24) R. Quinson, J. Perez, M. Rink, and A. Pavan, *J. Mater. Sci.*, **32**, 1371 (1997).

(25) A. J. Lesser and R. S. Kody, *J. Polym. Sci. Part B: Polym. Phys.*, **35**, 1611 (2015).

(26) X. Xiao, *Polym. Test.*, **27**, 164 (2008).

