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ELEMENTARY WAVE INTERACTIONS IN MAGNETOGASDYNAMICS[1]

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This paper is mainly concerned with the interactions of the elementary waves for the one-dimensional ideal Magnetogasdynamics with transverse magnetic field. By applying the method of the characteristic analysis, we obtain constructively the solutions of the all possible wave interactions when the initial data are three piecewise constant states. We find that the result is very different from that of the corresponding case of the conventional gas dynamics. However, the result is consistent with that of the corresponding case for Euler equations when the magnetic field vanishes.

Key words : Wave interaction; Riemann problem; magnetogasdynamics; shock wave; rarefaction wave; contact discontinuity.

1. Introduction

It is well known that Magnetogasdynamics plays a very important role in studying engineering physics and many other aspects ([1, 4, 8, 9, 10, 13, 14, 19] and the references cited therein) and it is also an important example of the hyperbolic system's theory.

One-dimensional inviscid and perfectly conducting compressible fluid, subject to a transverse magnetic field, is described by the following conservation laws

$$\begin{cases} \tau_t - u_x = 0, \\ u_t + (p + \frac{B^2}{2\mu})_x = 0, \\ (E + \frac{B^2\tau}{2\mu})_t + (pu + \frac{B^2 u}{2\mu})_x = 0, \end{cases} \tag{1.1}$$

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under the assumption $B = k\rho$, where k is positive constant [6, 15, 16], τ, u, p, E and $B \geq 0$ denote the specific volume, velocity, pressure, the specific total energy and transverse magnetic field, respectively. $E = e + \frac{u^2}{2}$ and e is specific internal energy. Here $\rho = \frac{1}{\tau}$ is the density, μ is the magnetic permeability. For the polytropic gas, $e = \frac{p\tau}{\gamma - 1}$ where γ is the adiabatic gas constant and $1 < \gamma < 3$ for most gases.

Hu and Sheng [6] studied the system (1.1) with the following initial data

$$(\tau, p, u)(x, 0) = (\tau^{\pm}, p^{\pm}, u^{\pm}), \quad \pm x > 0, \tag{1.2}$$

where $\tau^{\pm}, p^{\pm}, u^{\pm}$ are arbitrary constants, and $\tau > 0$ is the specific volume. They obtained constructively the unique solution of the Riemann problem (1.1) and (1.2) with the characteristic method.

Raja Sekhar and Sharma [15] studied the Riemann problem for one-dimensional unsteady simple flow of an isentropic, inviscid and perfectly conducting compressible fluid, subject to a transverse magnetic field

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p + \frac{B^2}{2})_x = 0, \end{cases} \tag{1.3}$$

and they obtained the Riemann solutions constructively. Moreover, they discussed the interactions of the elementary waves.

Shen [16] studied the Riemann problem for (1.3) further and found that the Riemann solutions converge to the corresponding Riemann solutions of the transport equations by letting both the pressure and the magnetic field vanish.

In [11], we removed the above assumption $B = k\rho$ and mainly consider the Riemann problem of the one-dimensional unsteady flow of an inviscid, perfectly conducting compressible fluid, subject to a transverse magnetic field for the magnetogasdynamic system

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p + \frac{B^2}{2})_x = 0, \\ (B)_t + (Bu)_x = 0, \end{cases} \tag{1.4}$$

where the pressure p is given by $p = A\rho^{\gamma}$ for polytropic gas, A is positive constant and γ is the adiabatic constant.

The Riemann problem of the conventional gas dynamics is studied by lots of people ([2, 3, 5, 7, 18], etc.). While the results of magnetogasdynamics flow are less than that of the conventional gas dynamics since the governing equations are highly nonlinear and complicated even for the one-dimensional flow.

It is noticed that although the governing equations of magnetogasdynamics are more complex than that of the conventional gas dynamics system, many results are similar except for the contact discontinuity. Unlike the conventional gas dynamics, where the image of the contact discontinuity in the space (τ, p, u) is a straight line parallel to the τ-axis and the projection on the plane (p, u) is a point, here the contact discontinuity is a plane curve in the space (τ, p, u) and the projection on the plane (p, u) is a straight line parallel to the p-axis. It induce that the Riemann solutions are more complex than that of the conventional gas dynamics.

It is important to study the interactions of the elementary waves not only because of their significance in practical applications in magnetogasdynamics system such as comparison with the numerical and experimental results, but also because of their basic role as building blocks for the theory of magnetogasdynamics.

In this paper we are concerned with the wave interactions of the elementary waves of (1.1) with the following initial data

$$(B, \rho, u)(x, 0) = \begin{cases} (B_l, \rho_l, u_l), & -\infty < x \leq x_1, \\ (B_m, \rho_m, u_m), & x_1 < x \leq x_2, \\ (B_r, \rho_r, u_r), & x_2 < x < \infty, \end{cases} \tag{1.5}$$

for arbitrary $x_1, x_2 \in R$.

There are many results for the wave interactions of the elementary waves of the hyperbolic system and we refer the readers to the references [2, 3, 11, 12, 15, 16, 17].

Based on investigating the important properties of the elementary waves containing the shock wave, rarefaction wave and the contact discontinuity in the phase plane (u, p), we obtain constructively the existence and uniqueness of the solution of the initial value problem (1.1) and (1.5) which embodies the internal mechanism of this model.

The detailed discussions are divided into two cases: one is that the wave interactions containing no rarefaction wave, the other one is that the wave interactions containing rarefaction wave. For the former case, we obtain uniquely the global solution by solving a new Riemann problem, while for the latter case we construct the unique local solution which is still important since it can be used to construct the approximate solution by Glimm's scheme and to describe the asymptotic behavior of the solution as the time tends to infinity.

Note that we should deal with the contact discontinuity carefully since it is much more complicated than that of the conventional gas dynamics. We find that the result is very different from that of the corresponding case of the conventional gas dynamics. However, the result is consistent with that of the corresponding case for Euler equations when the magnetic field B vanishes which indicates that there is a close connection between the two hyperbolic systems.

The rest of this paper is organized as follows. Section 2 restates the Riemann problem (1.1) and (1.2) for our later discussions. In Section 3, when the initial date are three pieces of constant states, the interactions of the elementary waves are considered case by case by investigating the wave curves in the phase plane (u, p) and we construct uniquely the solution of the initial value problem (1.1) and (1.5).

2. Preliminaries

In this section, we firstly sketch the results of the Riemann problem for (1.1) with the initial data (1.2), and we refer the readers to [6] for more details.

The system (1.1) can be rewritten, when we consider a smooth solution, as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (e+\frac{B^2}{2\mu})\tau & u & e_p \end{pmatrix} \begin{pmatrix} \tau \\ u \\ p \end{pmatrix}_t + \begin{pmatrix} 0 & -1 & 0 \\ \frac{BB_\tau}{\mu} & 0 & 1 \\ \frac{uBB_\tau}{\mu} & p+\frac{B^2}{2\mu} & u \end{pmatrix} \begin{pmatrix} \tau \\ u \\ p \end{pmatrix}_x = 0. \tag{2.1}$$

It defines the eigenvalues $\lambda_0 = 0$, $\lambda_\pm = \pm\sqrt{\frac{p - e_p\frac{BB_\tau}{\mu} + e_\tau}{e_p}}$. If $e_p > 0$ and $e_\tau + p > 0$, they are real and distinct, thus (1.1) is a strictly hyperbolic system. It is easily shown that the characteristic fields $\lambda_\pm$ are genuinely nonlinear and the characteristic field λ_0 is linearly degenerate.

2.1 *Rarefaction waves*

There are piecewise smooth solutions of (1.1), which are of the form $U(\frac{x}{t})$, such that

$$U(x,t) = \begin{cases} U_l, & \frac{x}{t} \le \lambda_\pm(U_l), \\ U(\frac{x}{t}), & \lambda_\pm(U_l) \le \frac{x}{t} \le \lambda_{\pm(U_r)}, \\ U_r, & \lambda_\pm(U_r) \le \frac{x}{t}. \end{cases} \tag{2.2}$$

If we set $\xi = \frac{x}{t}$, the system (1.1) becomes

$$\begin{cases} \lambda \mathrm{d}\tau = -\mathrm{d}(u), \\ \lambda \mathrm{d}u = \mathrm{d}(p + \frac{B^2}{2\mu}), \\ \lambda \mathrm{d}(E + \frac{B^2\tau}{2\mu}) = \mathrm{d}(pu + \frac{B^2 u}{2\mu}). \end{cases} \tag{2.3}$$

Besides the constant state solution $(\tau, p, u) = const.$, for the polytropic gas, the forward or backward rarefaction wave in the (τ, p, u) space passing though the point $Q_0(\tau_0, p_0, u_0)$ is given by

$$\overleftrightarrow{R} \ : \begin{cases} p\tau^\gamma = p_0\tau_0^\gamma, \\ u = u_0 \pm \int_{p_0}^{p} \frac{\sqrt{\gamma p\tau + \frac{B^2\tau}{\mu}}}{\gamma p} \mathrm{d}p. \end{cases} \tag{2.4}$$

2.2 *Discontinuity*

For the system (1.1), the Rankine-Hugoniot (RH) jump conditions are

$$\begin{cases} \sigma[\tau] = -[u], \\ \sigma[u] = [p + \frac{B^2}{2\mu}], \\ \sigma[E + \frac{B^2\tau}{2\mu}] = [pu + \frac{B^2 u}{2\mu}], \end{cases} \tag{2.5}$$

where $[u] = u_r - u_l$, etc.

By solving (2.5) we obtain two kinds of discontinuities as follows.

Contact discontinuity:

$$J : \begin{cases} \sigma = 0, \\ [u] = [p + \frac{B^2}{2\mu}], \end{cases} \tag{2.6}$$

and it is easy to see that J is a curve with $u = Const.$ in the (τ, p, u) space and the projection on the (p, u) plane is a straight line parallel to the p-axis.

For the polytropic gas, the forward or backward shock wave in the (τ, p, u) space passing though the point $Q_0(\tau_0, p_0, u_0)$ is given by

$$\overleftrightarrow{S} \ : \begin{cases} (p + \theta^2 p_0 + \theta^2(\frac{3B^2}{2\mu} + \frac{B_0^2}{2\mu}))\tau = (p_0 + \theta^2 p + \theta^2(\frac{3B_0^2}{2\mu} + \frac{B^2}{2\mu}))\tau_0, \\ u = u_0 \pm (p + \frac{B^2}{2\mu} - p_0 - \frac{B_0^2}{2\mu})(-\frac{\tau - \tau_0}{p + \frac{B^2}{2\mu} - p_0 - \frac{B_0^2}{2\mu}})^{\frac{1}{2}}, \end{cases} \tag{2.7}$$

where $\theta^2 = \frac{\gamma - 1}{\gamma + 1}$ and $B_0 = \frac{k}{\tau_0}$.

For convenience and conciseness, denote the projection of $\overleftrightarrow{R}(\overleftrightarrow{S})$ on the (τ, p) plane and (p, u) plane by $R_u(S_u)$ and $\overleftrightarrow{R}_\tau(\overleftrightarrow{S}_\tau)$, respectively. Denote the contact discontinuity J by $\overset{<}{J}$ when $p_l < p_r,\ \tau_l < \tau_r$, and $\overset{>}{J}$ when $p_l > p_r,\ \tau_l > \tau_r$.

For our later discussions, we restate the following properties of the shock wave curves (see Lemma 3.5. in [6]).

Lemma 2.1 — The wave curve $\overrightarrow{S}_\tau(Q_{0\tau})$ is concave and monotonically increasing, while $\overleftarrow{S}_\tau(Q_{0\tau})$ is convex and monotonically decreasing.

In a similar way with Lemma 3.3.7. in [2], we have the following result and the proof is omitted for simplicity.

Lemma 2.2 — Suppose the point $Q_2 \in \overleftrightarrow{R}_\tau(Q_1) \cup \overleftrightarrow{S}_\tau(Q_1)$, then the curve $\overleftrightarrow{S}_\tau(Q_1)$ does not intersects with $\overleftrightarrow{S}_\tau(Q_2)$ on the side where p increases while $\overleftrightarrow{R}_\tau(Q_1)$ does not intersects with $\overleftrightarrow{R}_\tau(Q_2)$ on the side where p decreases.

In order to construct the Riemann problem of (1.1) and (1.2), we denote $\overleftarrow{W}_{-\tau}(Q_{-\tau}) = \overleftarrow{R}_{-\tau}(Q_{-\tau}) \cup \overleftarrow{S}_{-\tau}(Q_{-\tau})$ and $\overrightarrow{W}_{+\tau}(Q_{+\tau}) = \overrightarrow{R}_{+\tau}(Q_{+\tau}) \cup \overrightarrow{S}_{+\tau}(Q_{+\tau})$, where $Q_{-\tau}$ and $Q_{+\tau}$ are respectively the projections of Q_- and Q_+ on the plane (p, u).

Draw $\overleftarrow{W}_{-\tau}(Q_{-\tau})$ from $Q_{-\tau}$ and $\overrightarrow{W}_{+\tau}(Q_{+\tau})$ from $Q_{+\tau}$ in the plane (p, u) respectively. According to the properties of $\overleftarrow{W}_{-\tau}(Q_{-\tau})$ and $\overrightarrow{W}_{+\tau}(Q_{+\tau})$, they intersect with each other at most once. Therefore, there are five cases: $\overleftarrow{W}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{W}_{+\tau}(Q_{+\tau}) = (\overleftarrow{R}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{R}_{+\tau}(Q_{+\tau}))$ or $(\overleftarrow{S}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{R}_{+\tau}(Q_{+\tau}))$ or $(\overleftarrow{R}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{S}_{+\tau}(Q_{+\tau}))$ or $(\overleftarrow{S}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{S}_{+\tau}(Q_{+\tau}))$ or $\varnothing$.

For the last case, we easily know there is a vacuum solution. In what follows, we just need to consider the first case since the other cases can be studied similarly.

Suppose $\overleftarrow{W}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{W}_{+\tau}(Q_{+\tau}) = \overleftarrow{R}_{-\tau}(Q_{-\tau}) \cap \overrightarrow{R}_{+\tau}(Q_{+\tau}) = \{Q_{*\tau}\}$ (Fig. 2.1.), we know there exists (p_*, u_*) satisfying

$$u_* = u_- + \int_{p_-}^{p_*} \frac{\sqrt{\gamma p\tau + \frac{kB}{\mu}}}{\gamma p} \mathrm{d}p, \quad p\tau^\gamma = p_-\tau_-^\gamma, \tag{2.8}$$

$$u_* = u_+ - \int_{p_+}^{p_*} \frac{\sqrt{\gamma p\tau + \frac{kB}{\mu}}}{\gamma p} \mathrm{d}p, \quad p\tau^\gamma = p_+\tau_+^\gamma. \tag{2.9}$$

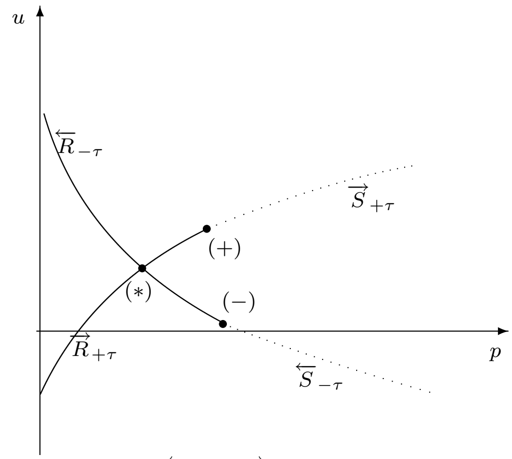

Fig. 2.1. $\overleftarrow{R}_{-\tau} \cap \overrightarrow{R}_{+\tau} = \{Q_{*\tau}\}$.

Denote

$$f_1(p_1) := u_- + \int_{p_1}^{p_-} \frac{\sqrt{\gamma p\tau + \frac{kB(\tau)}{\mu}}}{\gamma p} \mathrm{d}p, \;\; p\tau^\gamma = p_-\tau_-^\gamma, \;\; p_1 \in [0, p_-],$$

$$f_2(p_2) := u_+ - \int_{p_2}^{p_+} \frac{\sqrt{\gamma p\tau + \frac{kB(\tau)}{\mu}}}{\gamma p} \mathrm{d}p, \;\; p\tau^\gamma = p_-\tau_-^\gamma, \;\; p_2 \in [0, p_+],$$

$$g_1(p_1) := p_1 + \frac{B_1^2(\tau_1)}{2\mu},$$

$$g_2(p_2) := p_2 + \frac{B_2^2(\tau_2)}{2\mu},$$

where τ_1 satisfies

$$\overleftrightarrow{S} \; : \begin{cases} (p_1 + \theta^2 p_- + \theta^2(\frac{3B_1^2(\tau_1)}{2\mu} + \frac{B_-^2}{2\mu}))\tau_1 = (p_- + \theta^2 p_1 + \theta^2(\frac{3B_-^2(\tau_1)}{2\mu} + \frac{B_1^2}{2\mu}))\tau_-, \;\; p_1 > p_-, \\ p_1\tau_1^\gamma = p_-\tau_-^\gamma, \;\; p_1 \le p_-, \end{cases} \tag{2.10}$$

and τ_2 satisfies

$$\overleftrightarrow{S} \; : \begin{cases} (p_2 + \theta^2 p_+ + \theta^2(\frac{3B_2^2(\tau_2)}{2\mu} + \frac{B_+^2}{2\mu}))\tau_2 = (p_+ + \theta^2 p_2 + \theta^2(\frac{3B_+^2(\tau_2)}{2\mu} + \frac{B_2^2}{2\mu}))\tau_+, \;\; p_2 > p_+, \\ p_2\tau_2^\gamma = p_+\tau_+^\gamma, \;\; p_2 \le p_+. \end{cases} \tag{2.11}$$

Denote

$$h_1(p_1) := u_- - \sqrt{(p_1 + \frac{B_1^2(\tau_1)}{2\mu} - p_- - \frac{B_-^2}{2\mu})(\tau_- - \tau_1)},$$

$$h_2(p_2) := u_+ + \sqrt{(p_2 + \frac{B_2^2(\tau_2)}{2\mu} - p_+ - \frac{B_+^2}{2\mu})(\tau_+ - \tau_2)},$$

where τ_1 satisfies the first equation of (2.10) and τ_2 satisfies the first equation of (2.11).

Let

$$\begin{cases} f_1(p_1) = f_2(p_2), \\ g_1(p_1) = g_2(p_2), \end{cases} \tag{2.12}$$

$$\begin{cases} f_1(p_1) = h_2(p_2), \\ g_1(p_1) = g_2(p_2), \end{cases} \tag{2.13}$$

$$\begin{cases} h_1(p_1) = f_2(p_2), \\ g_1(p_1) = g_2(p_2). \end{cases} \tag{2.14}$$

In [6], the authors proved that only one of the above three equations (2.12), (2.13) and (2.14) is solvable and the solution is unique, which implies that there exists a unique contact discontinuity J joining the two states which are located on $\overset{\rightleftarrows}{R}$ and $\overset{\leftrightarrows}{S}$ respectively.

Case 1 : $p_-\tau_-^\gamma = p_+\tau_+^\gamma$.

In this case, we have $g_1(p_*) = g_2(p_*)$, and the Riemann solution is $\overleftarrow{R} + \overrightarrow{R}$ where the symbol " $+$ " means "followed by". We notice that for this case there is no contact discontinuity.

Case 2 : $p_-\tau_-^\gamma < p_+\tau_+^\gamma$. In this case, we know that $g_1(p_*) > g_2(p_*)$ and should look for the solution in $\{(p_1, p_2)|0 \le p_1 < p_*,\ p_2 > p_*\}$. There are two possibilities as follows.

Subcase 2.1 : $f_1(0) \le u_+$. (Fig. 2.2.)

It is obvious that there exists a point $\hat{p}_1 \in (p_*, p_+)$ such that $f_1(0) = f_2(\hat{p}_1)$ and $g_1(0) < g_2(\hat{p}_1)$. It follows that there exists a point $(p_1, p_2) :\ \ 0 < p_1 < p_*,\ p_* < p_2 < \hat{p}_1$ and the Riemann solution is $\overleftarrow{R} + \overset{<}{J} + \overrightarrow{R}$.

Subcase 2.2 : $f_1(0) > u_+$. (Fig. 2.3.)

Since there exists a point $\hat{p}_2 \in (0, p_*)$ such that $f_1(\hat{p}_2) = u_+$, we divide it into two subcases.

Subcase 2.2.1 : If $g_1(\hat{p}_2) \leq g_2(p_+)$, we know that there exists a point (p_1, p_2) : $\hat{p}_2 \leq p_1 < p_*,\ p_* < p_2 < p_+$ and it follows that the Riemann solution is $\overleftarrow{R} + \overset{<}{J} + \overrightarrow{R}$.

Subcase 2.2.2 : If $g_1(\hat{p}_2) > g_2(p_+)$, similarly we obtain that there exists a point (p_1, p_2) : $0 < p_1 < \hat{p}_2,\ p_+ < p_2 < \hat{p}_3$ and the Riemann solution is $\overleftarrow{R} + \overset{<}{J} + \overrightarrow{S}$, where $\hat{p}_3 \in (p_+, +\infty)$ satisfying $f_1(0) = h_2(\hat{p}_3)$.

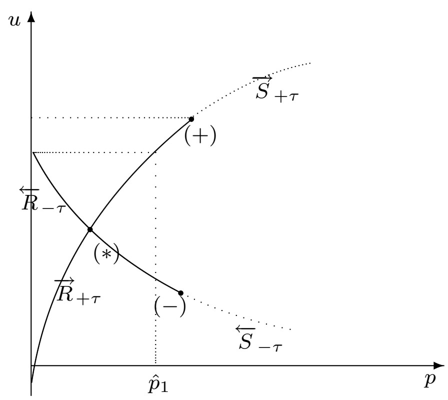

Fig. 2.2. $f_1(0) \leq u_+ = f_2(p_+)$.

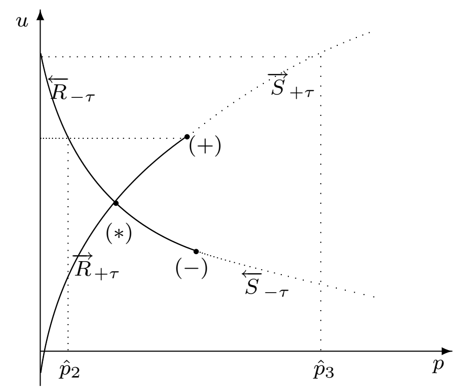

Fig. 2.3. $f_1(0) > u_+ = f_2(p_+)$.

Case 3 : $p_-\tau_-^\gamma > p_+\tau_+^\gamma$. In this case, we know that $g_1(p_*) < g_2(p_*)$ and should look for the solution in $\{(p_1, p_2) | p_1 > p_*,\ 0 \leq p_2 < p_*\}$. We divide it into two subcases as follows.

Subcase 3.1 : $u_- \leq f_2(0)$ (Fig. 2.4.)

It is obvious that there exists a point $\hat{p}_4 \in (p_*, p_-)$ such that $f_1(\hat{p}_4) = f_2(0)$ and $g_1(\hat{p}_4) > g_2(0)$. And we get the Riemann solution $\overleftarrow{R} + \overset{>}{J} + \overrightarrow{R}$.

Subcase 3.2 : $u_- > f_2(0)$ (Fig. 2.5.)

Since there exists a point $\hat{p}_5 \in (0, p_*)$ such that $f_2(\hat{p}_5) = u_-$, we divide it into two subcases.

Subcase 3.2.1 : If $g_1(p_-) \geq g_2(\hat{p}_5)$, similarly as the above discussions there exists a point (p_1, p_2) : $p_* < p_1 < p_-,\ \hat{p}_5 \leq p_2 < p_*$ and the Riemann solution is $\overleftarrow{R} + \overset{>}{J} + \overrightarrow{R}$.

Subcase 3.2.2 : If $g_1(p_-) < g_2(\hat{p}_5)$, since there exists a point (p_1, p_2) : $p_- < p_1 < \hat{p}_6,\ 0 < p_2 < \hat{p}_5$, the Riemann solution is $\overleftarrow{S} + \overset{>}{J} + \overrightarrow{R}$, where $\hat{p}_6$ satisfies $h_1(\hat{p}_6) = f_2(0)$.

From the above discussions, we obtain the following result [6].

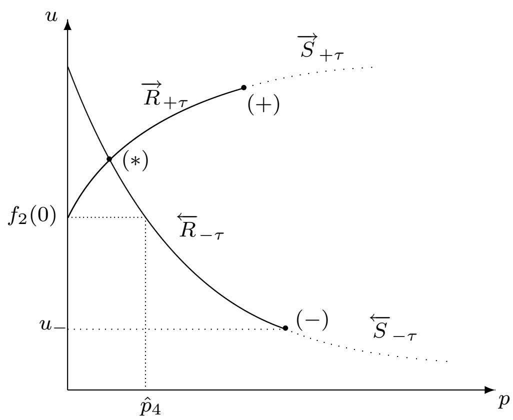

Fig. 2.4. $f_1(p_-) = u_- \leq f_2(0)$.

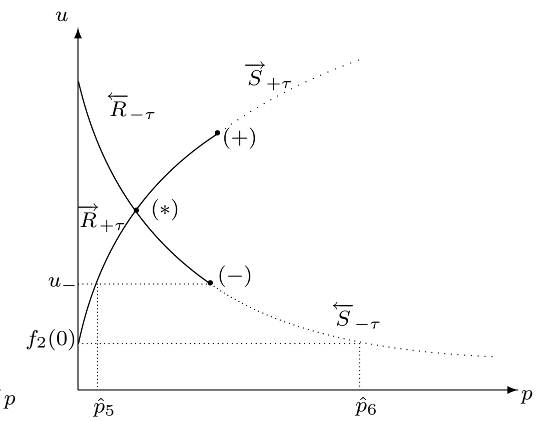

Fig. 2.5. $f_1(p_-) = u_- > f_2(0)$.

***Theorem* 2.1** — *For any initial constant states U_- and U_+, there exists uniquely the entropy solution of the Riemann problem* (1.1) *and* (1.2).

3. Interactions of the Elementary Waves

Now we consider the kinds of interactions of the elementary waves obtained from the Riemann problem (1.1) and (1.2). We divide the discussions into two cases: the interactions of the elementary waves containing no R and the interactions of the elementary waves containing R.

3.1 *Interactions of the elementary waves containing no R*

In this case, we discuss the wave interactions case by case and can obtain the global solution by solving a new Riemann problem.

Case (i) : $\overrightarrow{S}\overset{>}{J}$.

Since

$$\overrightarrow{S}_{r\tau}(Q_r) : u = u_r + \sqrt{(p + \tfrac{B^2}{2\mu} - p_r - \tfrac{B_r^2}{2\mu})(\tau_r - \tau)},$$
$$\overrightarrow{S}_{m\tau}(Q_m) : u = u_m + \sqrt{(p + \tfrac{B^2}{2\mu} - p_m - \tfrac{B_m^2}{2\mu})(\tau_m - \tau)},$$

where $B_r = \frac{k}{\tau_r}$, $B_m = \frac{k}{\tau_m}$ and $u_m = u_r$, $p_m + \frac{B_m^2}{2\mu} = p_r + \frac{B_r^2}{2\mu}$. From the properties of the contact discontinuity, we have $\tau_m > \tau_r \Leftrightarrow B_m < B_r$, it follows that the curve $\overrightarrow{S}_{r\tau}(Q_r)$ lies always above the curve $\overrightarrow{S}_{m\tau}(Q_m)$. Thus, $\overrightarrow{S}_{r\tau}(Q_r)$ intersects with $\overleftarrow{R}_{l\tau}(Q_l)$ at $Q_{*\tau}$ where a new Riemann problem is formed. In order to construct the solution of this new Riemann problem, we discuss as follows.

Case 1 : $\tau_{*l} < \tau_{*r}$. In this case, $g_1(p_*) > g_2(p_*)$ and we should seek a solution in $\{(\bar{p}_1, \bar{p}_2)|0 < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < +\infty\}$.

It is obvious that there exists a point $\hat{p}_1 \in (p_*, +\infty)$ which satisfies $f_1(0) = h_2(\hat{p}_1)$ and $0 = g_1(0) < g_2(\hat{p}_1)$. It follows that there exists a point $(\bar{p}_1, \bar{p}_2):\ 0 < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < \hat{p}_1$ and the solution is given by $\overrightarrow{S}\overset{>}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Case 2 : $\tau_{*l} = \tau_{*r}$. Since there is no contact discontinuity of the new Riemann solution in this case, the state Q_l is connected to the state Q_r by the state Q_* directly and we obtain that the solution is $\overrightarrow{S}\overset{>}{J} \to \overleftarrow{R}\overrightarrow{S}$.

Case 3 : $\tau_{*l} > \tau_{*r}$. This means that $g_1(p_*) < g_2(p_*)$, i.e., $p_{*l} + \frac{B_{*l}^2}{2\mu} < p_{*r} + \frac{B_{*r}^2}{2\mu}$, where $B_{*l} = \frac{k}{\tau_{*l}}$, $B_{*r} = \frac{k}{\tau_{*r}}$.

In view of $u_l > u_r$, if there is a contact discontinuity $(\bar{p}_1, \bar{p}_2)$ of the solution for the new Riemann problem, we know that the following equality

$$\bar{p}_1 + \frac{\bar{B}_1^2}{2\mu} = \bar{p}_2 + \frac{\bar{B}_2^2}{2\mu}, \tag{3.1}$$

holds, where $\bar{p}_1 > p_*$, $0 < \bar{p}_2 < p_*$ (Fig. 3.1.).

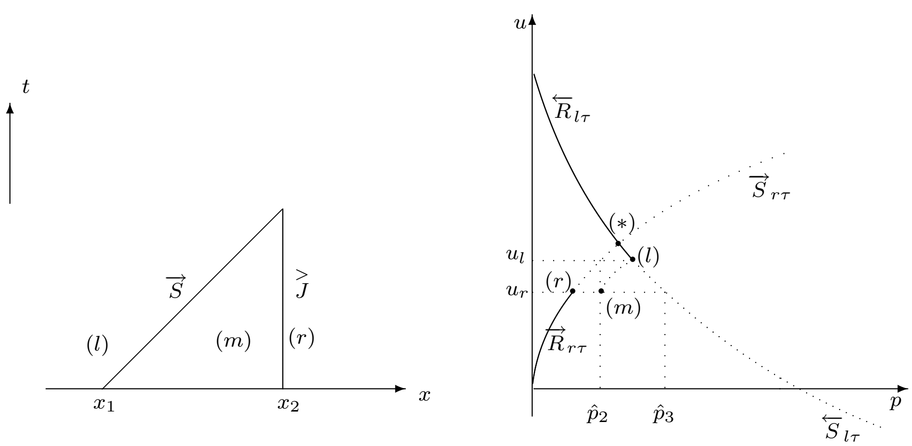

Fig. 3.1. The interaction of $\overrightarrow{S}$ and $\overset{>}{J}$.

Subcase 3.1 : $g_1(p_l) \geq g_2(\hat{p}_2)$, where $\hat{p}_2$ is determined by $u_l = u_{\overrightarrow{S}_{r\tau}}(\hat{p}_2)$, i.e., we choose $\hat{p}_2$ such that the value of u along the curve $\overrightarrow{S}_{r\tau}$ as $p = \hat{p}_2$ equals to the value of u_l. Therefore there exists a point $(\bar{p}_1, \bar{p}_2):\ p_* < \bar{p}_1 < p_l,\ \hat{p}_2 < \bar{p}_2 < p_*$ and the solution lies between u_l and u_* which is given by $\overrightarrow{S}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{S}$.

Subcase 3.2 : $g_1(p_l) < g_2(\hat{p}_2)$. On the other hand, we have

$$\hat{p}_3 + \frac{\hat{B}_3^2}{2\mu} > p_r + \frac{B_r^2}{2\mu}, \tag{3.2}$$

where $\hat{p}_3$ is determined by $u_{\overleftarrow{S}_{l\tau}}(\hat{p}_3) = u_r$. In fact, due to $p_m + \frac{B_m^2}{2\mu} = p_r + \frac{B_r^2}{2\mu}$,

$$u_m = u_r = u(\hat{p}_3) = u_l - \sqrt{(\hat{p}_3 + \frac{\hat{B}_3^2}{2\mu} - p_l - \frac{B_l^2}{2\mu})(\tau_l - \hat{\tau}_3)},$$

and

$$u_m = u_l - \sqrt{(p_l + \frac{B_l^2}{2\mu} - p_m - \frac{B_m^2}{2\mu})(\tau_m - \tau_l)},$$

where $\hat{p}_3 > p_l > p_m$. From Lemma 2.1., $\frac{\mathrm{d}\tau}{\mathrm{d}p} < 0$ holds for $\overleftarrow{S}$ which yields that $\hat{\tau}_3 < \tau_l < \tau_m$. It follows that

$$\hat{p}_3 + \frac{\hat{B}_3^2}{2\mu} > p_l + \frac{B_l^2}{2\mu} > p_m + \frac{B_m^2}{2\mu} = p_r + \frac{B_r^2}{2\mu},$$

that is to say, (3.2) holds.

Hence there exists a point $(\bar{p}_1, \bar{p}_2)$: $p_l < \bar{p}_1 < \hat{p}_3,\ p_r < \bar{p}_2 < \hat{p}_2$ and the solution is described by $\overrightarrow{S}\overset{>}{J} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{S}$.

Similarly, the interaction between $\overset{<}{J}$ and $\overleftarrow{S}$ can also be obtained and here omitted.

***Theorem* 3.1** — *When a shock collides with a contact discontinuity which is of a jump increase in density in the propagating direction of the shock, the shock will cross the contact discontinuity at once and a new rarefaction wave or a new shock wave propagating in the opposite direction will appear. Furthermore, after the interaction the contact discontinuity may appear or disappear.*

Case (ii) : $\overrightarrow{S}\overset{<}{J}$. Similar discussions as the above case, it follows that the curve $\overrightarrow{S}_{m\tau}(Q_m)$ lies always above the curve $\overrightarrow{S}_{r\tau}(Q_r)$. Thus, there are two possibilities: $\overleftarrow{S}_{l\tau}(Q_l)$ intersects with $\overrightarrow{S}_{r\tau}(Q_r)$ at $Q_{*\tau}$ where a new Riemann problem is formed, or $\overleftarrow{S}_{l\tau}(Q_l)$ intersects with $\overrightarrow{R}_{r\tau}(Q_r)$ at $Q_{*\tau}$ where a new Riemann problem is formed. In what follows, we construct the solution of the new Riemann problem as follows.

Case 1 : $\hat{p}_1 \geq p_r$, where $\hat{p}_1$ satisfies $u_r = u_{\overleftarrow{S}_{l\tau}}(\hat{p}_1)$. In this case, we know that $Q_{*\tau} \in \overleftarrow{S}_{l\tau}(Q_l) \cup \overrightarrow{S}_{r\tau}(Q_r)$ (Fig. 3.2.).

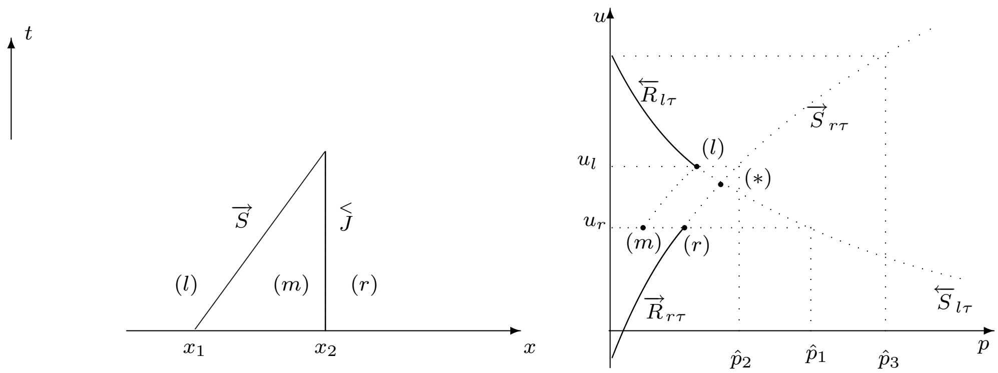

Fig. 3.2. The interaction of $\overrightarrow{S}$ and $\overset{<}{J}$, $\hat{p}_1 \geq p_r$.

Subcase 1.1 : $\tau_{*l} < \tau_{*r}$. This means that $g_1(p_*) > g_2(p_*)$. If there exists a contact discontinuity $(\bar{p}_1, \bar{p}_2)$ of the new Riemann problem, (3.1) must hold. There are two possibilities.

Subcase 1.1.1 : $g_1(p_l) \leq g_2(\hat{p}_2)$, where $\hat{p}_2$ is determined by

$$u_l = u_{\overrightarrow{S}_{r\tau}}(\hat{p}_2). \tag{3.3}$$

Therefore the solution lies between u_l and u_* and the result is $\overrightarrow{S}\overset{<}{J} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{S}$.

Subcase 1.1.2 : $g_1(p_l) > g_2(\hat{p}_3)$. On the other hand, $0 = g_1(0) < g_2(\hat{p}_3)$ holds obviously, where $\hat{p}_3$ is determined by $u_{\overleftarrow{R}_{l\tau}(Q_l)}(0) = u_{\overrightarrow{S}_{r\tau}}(\hat{p}_3)$. It follows that there exists $(\bar{p}_1, \bar{p}_2)$ which satisfies $0 < \bar{p}_1 < p_l$, $\hat{p}_2 < \bar{p}_2 < \hat{p}_3$ and $\overrightarrow{S}\overset{<}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Subcase 1.2 : $\tau_{*l} = \tau_{*r}$. There is no contact discontinuity of the new Riemann solution and the result is given by $\overrightarrow{S}\overset{<}{J} \to \overleftarrow{S}\,\overrightarrow{S}$.

Subcase 1.3 : $\tau_{*l} > \tau_{*r}$.

This means that $g_1(p_*) < g_2(p_*)$. On the other hand, it is evident that

$$p_r + \frac{B_r^2}{2\mu} = p_m + \frac{B_m^2}{2\mu} < \hat{p}_1 + \frac{\hat{B}_1^2}{2\mu}.$$

It yields that there exists $(\bar{p}_1, \bar{p}_2) : p_* < \bar{p}_1 < \hat{p}_1, p_r < \bar{p}_2 < p_*$ such that (3.1) holds. Thus, we have $\overrightarrow{S}\overset{<}{J} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{S}$.

Case 2 : $\hat{p}_1 < p_r$. In this case, we can see that $Q_{*\tau} \in \overleftarrow{S}_{l\tau}(Q_l) \cup \overrightarrow{R}_{r\tau}(Q_r)$ (Fig. 3.3.).

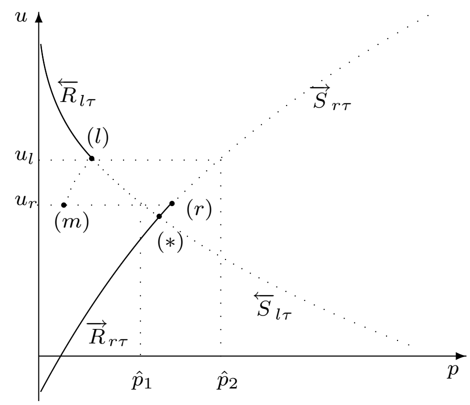

Fig. 3.3. The interaction of $\overrightarrow{S}$ and $\overset{<}{J}$, $\hat{p}_1 < p_r$.

Subcase 2.1 : $\tau_{*l} < \tau_{*r}$. This means that $p_{*l} + \frac{B_{*l}^2}{2} > p_{*r} + \frac{B_{*r}^2}{2}$. Therefore, we have obviously that

$$\hat{p}_1 + \frac{\hat{B}_1^2}{2\mu} > p_m + \frac{B_m^2}{2\mu} = p_r + \frac{B_r^2}{2\mu}.$$

and there is no solution between u_* and u_r.

Subcase 2.1.1 : $g_1(p_l) \leq g_2(\hat{p}_2)$, where $\hat{p}_2$ satisfies (3.3). Thus, there exists a point $(\bar{p}_1, \bar{p}_2)$ which satisfies $p_l < \bar{p}_1 < p_*,\ p_r < \bar{p}_2 < \hat{p}_3$ and $\overrightarrow{S}\overset{<}{J} \rightarrow \overleftarrow{S}\overset{<}{J}\overrightarrow{S}$.

Subcase 2.1.2 : $g_1(p_l) > g_2(\hat{p}_2)$. The solution lies between 0 and u_l and $\overrightarrow{S}\overset{<}{J} \rightarrow \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Subcase 2.2 : $\tau_{*l} = \tau_{*r}$. There is no contact discontinuity of the new Riemann solution and the result is $\overrightarrow{S}\overset{<}{J} \rightarrow \overleftarrow{S}\overrightarrow{R}$.

Subcase 2.3 : $\tau_{*l} > \tau_{*r}$. This means that $p_{*l} + \frac{B_{*l}^2}{2\mu} < p_{*r} + \frac{B_{*r}^2}{2\mu}$. It is obvious that $0 < \hat{p}_4 + \frac{\hat{B}_4^2}{2\mu}$, where $\hat{p}_4$ satisfies $u_{\overrightarrow{R}_{r\tau}(Q_r)}(0) = u_{\overleftarrow{S}_{l\tau}(Q_l)}(\hat{p}_4)$. Therefore there exists $(\bar{p}_1, \bar{p}_2) : \hat{p}_1 < \bar{p}_1 < \hat{p}_4, 0 < \bar{p}_2 < p_r$ such that (3.1) holds which indicates that $\overrightarrow{S}\overset{<}{J} \rightarrow \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

Similarly, the interaction between $\overset{>}{J}$ and $\overleftarrow{S}$ can be investigated and omitted for simplicity.

***Theorem* 3.2** — *When a shock collides with a contact discontinuity which is of a jump decrease in density in the propagating direction of the shock, the shock will cross the contact discontinuity at once or a new rarefaction wave will appear, and after the interaction the contact discontinuity may appear or disappear. Furthermore, a new shock wave or a new rarefaction wave propagating in the opposite direction will appear.*

Case (iii) : $\overrightarrow{S}\overleftarrow{S}$. In this case, it is obvious that $\overrightarrow{S}$ will intersect with $\overleftarrow{S}$ in a finite time and a new Riemann problem is formed. From Lemma 2.2., we know that $\overleftarrow{S}_\tau(Q_l)$ does not intersect with $\overleftarrow{S}_\tau(Q_m)$ and $\overrightarrow{S}_\tau(Q_r)$ does not intersect with $\overrightarrow{S}_\tau(Q_m)$, respectively. It follows that $Q_* \in \overleftarrow{S}_\tau(Q_l) \cup \overrightarrow{S}_\tau(Q_r)$ (Fig. 3.4.) and we construct the solution as follows.

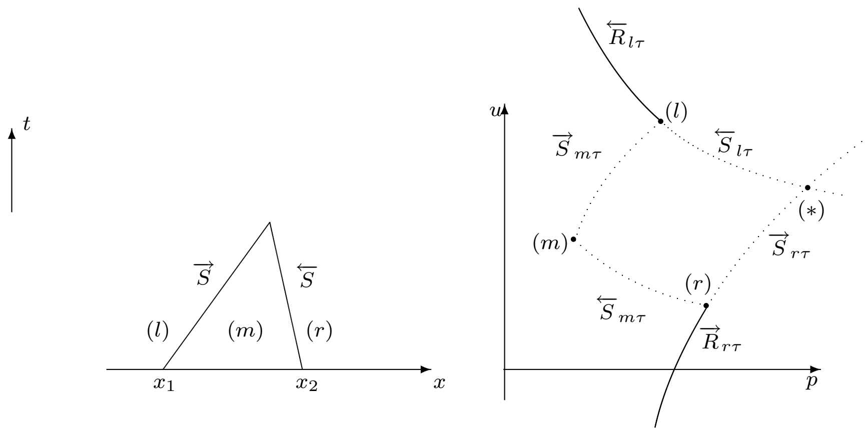

Fig. 3.4. The interaction of $\overrightarrow{S}$ and $\overleftarrow{S}$.

Case 1 : $\tau_{*l} > \tau_{*r}$. In this case, $g_1(p_*) < g_2(p_*)$ and we should seek a solution in $\{(p_1, p_2)|p_1 > p_*,\ 0 < p_2 < p_*\}$. Obviously, there exist $\hat{p}_1$ and $\hat{p}_2$ which satisfies respectively that $u_r = u_{\overleftarrow{S}_{l\tau}}(\hat{p}_1),\ \hat{p}_1 \in (p_*, \hat{p}_2)$ and $u_{\overrightarrow{R}_{r\tau}}(0) = u_{\overleftarrow{S}_{l\tau}}(\hat{p}_2),\ \hat{p}_2 > \hat{p}_1 > p_*$.

Subcase 1.1 : $g_1(\hat{p}_1) \geq g_2(p_r)$. From the continuity of the wave curves, we know there exists a point $(\bar{p}_1, \bar{p}_2)$ such that $p_* < \bar{p}_1 < \hat{p}_1,\ p_r < \bar{p}_2 < p_*$, and the solution is $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{S}$.

Subcase 1.2 : $g_1(\hat{p}_1) < g_2(p_r)$. Similarly, we know there exists a point $(\bar{p}_1, \bar{p}_2)$ satisfying $\hat{p}_1 < \bar{p}_1 < \hat{p}_2$ and $0 < \bar{p}_2 < p_r$ and the result is described by $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

Case 2 : $\tau_{*l} = \tau_{*r}$. In this case, $g_1(p_*) = g_2(p_*)$ and there is no contact discontinuity of the new Riemann solution, the state Q_l is connected to the state Q_r by the state Q_* directly and we obtain that the solution is $\overrightarrow{S}\overset{>}{J} \to \overleftarrow{S}\overrightarrow{S}$.

Case 3 : $\tau_{*l} < \tau_{*r}$. This means that $g_1(p_*) > g_2(p_*)$ and we should seek solution in $\{(\bar{p}_1, \bar{p}_2)|0 < \bar{p}_1 < p_*,\ \bar{p}_2 > p_*\}$. It is easily shown that there exist $\hat{p}_3$ and $\hat{p}_4$ which satisfies respectively that $u_l = u_{\overrightarrow{S}_{r\tau}}(\hat{p}_3)$ and $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{S}_{r\tau}}(\hat{p}_4),\ p_* < \hat{p}_3 < \hat{p}_4$.

Subcase 3.1 : $g_1(\hat{p}_3) \geq g_2(p_l)$. From the continuity of the wave curves, we know there exists a point $(\bar{p}_1, \bar{p}_2)$ satisfying $p_l < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < \hat{p}_3$, and the solution is given by $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{S}$.

Subcase 3.2 : $g_1(\hat{p}_3) < g_2(p_l)$. Similarly, there exists a point $(\bar{p}_1, \bar{p}_2)$ satisfying $0 < \bar{p}_1 < p_l$ and

$\hat{p}_3 < \bar{p}_2 < \hat{p}_4$ which indicates that the solution is $\overrightarrow{S}\overleftarrow{S} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

***Theorem* 3.3** — *When a forward shock collides with a backward shock, the forward (backward) shock will cross the backward (forward) shock at once or a new forward (backward) rarefaction wave will appear. Moreover, the contact discontinuity may appear or not after the interaction.*

3.2 *Interactions of the elementary waves containing R*

In this case, since there is a process of penetration in the interaction, we can not obtain the global solution by solving the new Riemann problem as in the above section. However, the solution of the corresponding Riemann problem is still important for investigating the global solution because this can be used to construct the approximate solution by Glimm's scheme and for describing the asymptotic behavior of the solution as the time tends to infinity [2]. Now we discuss the wave interactions case by case.

Case (i) : $\overrightarrow{R}\overset{>}{J}$. In this case, we know $u_l < u_m = u_r$ and $p_m > p_r$.

Since

$$\overrightarrow{R}_{r\tau}(Q_r) : u = u_r + \int_{p_r}^{p} \frac{\sqrt{\gamma p\tau + \frac{kB(\tau)}{\mu}}}{\gamma p} \mathrm{d}p,$$
$$\overrightarrow{R}_{m\tau}(Q_m) : u = u_m + \int_{p_m}^{p} \frac{\sqrt{\gamma p\tau + \frac{kB(\tau)}{\mu}}}{\gamma p} \mathrm{d}p,$$

$p_m + \frac{B_m^2}{2\mu} = p_r + \frac{B_r^2}{2\mu}$, $u_m = u_r$ and $p_m > p_r$, it follows that the curve $\overrightarrow{R}_{r\tau}(Q_r)$ lies always above the curve $\overrightarrow{R}_{m\tau}(Q_m)$. Thus, there are two possibilities: $\overleftarrow{R}_{l\tau}(Q_l)$ intersects with $\overrightarrow{R}_{r\tau}(Q_r)$ at $Q_{*\tau}$ where a new Riemann problem is formed, or $\overleftarrow{R}_{l\tau}(Q_l)$ intersects with $\overrightarrow{S}_{r\tau}(Q_r)$ at $Q_{*\tau}$ where a new Riemann problem is formed. In order to construct the solution of this new Riemann problem, we discuss as follows.

Case 1 : $\tilde{p}_1 \geq p_r$, where $\tilde{p}_1$ satisfies $u_r = u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_1)$ which means that $\overleftarrow{R}_{l\tau}(Q_l)$ intersects with $\overrightarrow{S}_{r\tau}(Q_r)$ at $Q_{*\tau}$ (Fig. 3.5.).

Subcase 1.1 : $\tau_{*l} = \tau_{*r}$, which indicates that $g_1(p_*) = g_2(p_*)$. There is no contact discontinuity of the new Riemann solution and the result is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overrightarrow{S}$.

Subcase 1.2 : $\tau_{*l} < \tau_{*r}$, in this case we have $g_1(p_*) > g_2(p_*)$. It follows that there exists a point $(\bar{p}_1, \bar{p}_2)$ which satisfies $0 < \bar{p}_1 < p_* < p_l,\ \bar{p}_2 > p_* > p_r$ and the solution is given by $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Subcase 1.3 : $\tau_{*l} > \tau_{*r}$, similarly it holds that $g_1(p_*) < g_2(p_*)$. In this case, we should seek the solution in $\{(\bar{p}_1, \bar{p}_2)|\bar{p}_1 > p_*,\ 0 < \bar{p}_2 < p_*\}$. In view of $u_l < u_r$, we divide our discussions into the following two subcases.

Subcase 1.3.1 : $u_{\overrightarrow{R}_{r\tau}}(0) < u_l < u_r$.

Subcase 1.3.1.1 : $g_2(p_r) \leq g_1(\tilde{p}_1)$. From the continuity of the wave curves, we know that there exists a point $(\bar{p}_1, \bar{p}_2)$ satisfying $p_* < \bar{p}_1 < \tilde{p}_1$ and $p_r < \bar{p}_2 < p_*$. Thus, the solution is described by $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{S}$.

Subcase 1.3.1.2 : $g_2(p_r) > g_1(\tilde{p}_1)$ and $g_1(p_l) \geq g_2(\tilde{p}_2)$, where $\tilde{p}_2$ is determined by $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_2)$. Thus, there exists a point $(\bar{p}_1, \bar{p}_2)$ which satisfies $\tilde{p}_1 < \bar{p}_1 < p_l$ and $\tilde{p}_2 < \bar{p}_2 < p_r$, and we know that the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{R}$.

Subcase 1.3.1.3 : $g_2(p_r) > g_1(\tilde{p}_1)$ and $g_1(p_l) < g_2(\tilde{p}_2)$. Obviously, it holds that $g_1(\tilde{p}_3) > g_2(0)$, where $\tilde{p}_3$ satisfies $u_{\overrightarrow{R}_{r\tau}}(0) = u_{\overleftarrow{S}_{l\tau}}(\tilde{p}_3)$. Therefore there exists a point $(\bar{p}_1, \bar{p}_2)$ satisfying $p_l < \bar{p}_1 < \tilde{p}_3$ and $0 < \bar{p}_2 < \tilde{p}_2$ and the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

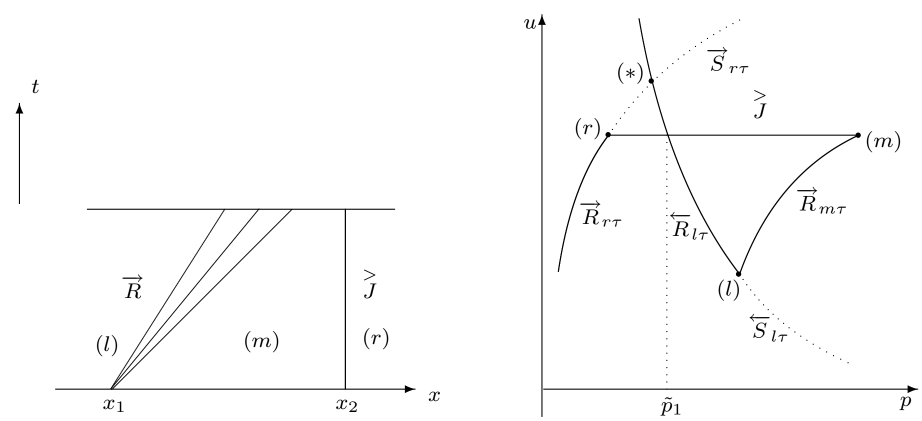

Fig. 3.5. The interaction of $\overrightarrow{R}$ and $\overset{>}{J}$, $\tilde{p}_1 \geq p_r$.

Subcase 1.3.2 : $u_l < u_{\overrightarrow{R}_{r\tau}}(0) < u_r$.

It is obvious that there exists a point $\tilde{p}_4 < p_l$ such that $u_{\overrightarrow{R}_{r\tau}}(0) = u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_4)$.

Subcase 1.3.2.1 : $g_1(\tilde{p}_1) \geq g_2(p_r)$. Since there exists a point $(\bar{p}_1, \bar{p}_2)$ satisfying $p_* < \bar{p}_1 < \tilde{p}_1$ and $p_r < \bar{p}_2 < p_*$, we know that the solution is described by $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{S}$.

Subcase 1.3.2.2 : $g_1(\tilde{p}_1) < g_2(p_r)$. It is obvious that $g_1(\tilde{p}_4) > g_2(0)$. From the continuity, there exists a point $(\bar{p}_1, \bar{p}_2)$ which satisfies $\tilde{p}_1 < \bar{p}_1 < \tilde{p}_4$ and $0 < \bar{p}_2 < p_r$ which indicates that the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{R}$.

Case 2 : $\tilde{p}_1 < p_r$, in this case we know that $\overleftarrow{R}_{l\tau}(Q_l)$ intersects with $\overrightarrow{R}_{r\tau}(Q_r)$ at $Q_{*\tau}$ (Fig. 3.6.).

Subcase 2.1 : $p_l\tau_l^\gamma = p_r\tau_r^\gamma$.

In this case, we have $g_1(p_*) = g_2(p_*)$, and the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overrightarrow{R}$. Note that for this case there is no contact discontinuity.

Subcase 2.2 : $p_l\tau_l^\gamma < p_r\tau_r^\gamma$. In this case, $g_1(p_*) > g_2(p_*)$ and we should look for the solution in $\{(\bar{p}_1, \bar{p}_2) | 0 \le \bar{p}_1 < p_*,\ \bar{p}_2 > p_*\}$. There are two possibilities as follows.

Subcase 2.2.1. $f_1(0) \le u_r$.

It is easily shown that there exists $\hat{p}_1 \in (p_*, p_+)$ such that $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{R}_{r\tau}}(\hat{p}_1)$ and $g_1(0) < g_2(\hat{p}_1)$. It follows that there exists a point $(\bar{p}_1, \bar{p}_2):\ \ 0 < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < \hat{p}_1$ and the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Subcase 2.2.2 : $f_1(0) > u_r$.

Since there exists a point $0 < \hat{p}_2 < p_*$ such that $u_{\overleftarrow{R}_{l\tau}}(\hat{p}_2) = u_r$, we divide it into two subcases.

Subcase 2.2.2.1 : $g_1(\hat{p}_2) \le g_2(p_r)$, we know that there exists a point $(\bar{p}_1, \bar{p}_2):\ \ \tilde{p}_2 \le \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < p_r$ and it follows that the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Subcase 2.2.2.2 : $g_1(\hat{p}_2) > g_2(p_r)$, similarly we obtain that there exists a point $(\bar{p}_1, \bar{p}_2):\ \ 0 < \bar{p}_1 < \hat{p}_2,\ p_r < \bar{p}_2 < \hat{p}_3$, where $\hat{p}_3$ is determined by $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{S}_{r\tau}}(\hat{p}_3)$, and the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

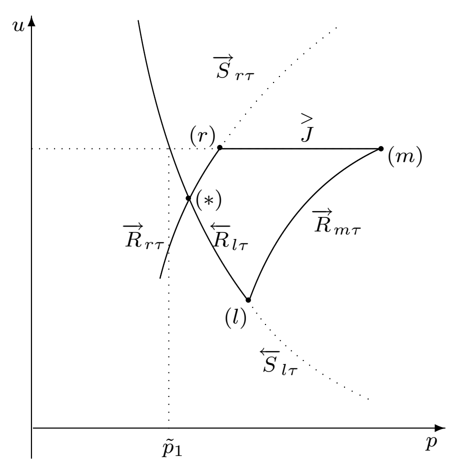

Fig. 3.6. The interaction of $\overrightarrow{R}$ and $\overset{>}{J}$, $\tilde{p}_1 < p_r$.

Subcase 2.3 : $p_l\tau_l^\gamma > p_r\tau_r^\gamma$. In this case, we know $g_1(p_*) < g_2(p_*)$ and should look for the solution in $\{(\bar{p}_1, \bar{p}_2) | \bar{p}_1 > p_*,\ 0 \le \bar{p}_2 < p_*\}$. We divide it into two subcases as follows.

Subcase 2.3.1 : $u_l \le f_2(0)$.

It is obvious that there exists a point $\hat{p}_4 \in (p_*, p_l)$ such that $u_{\overleftarrow{R}_{l\tau}}(\hat{p}_4) = u_{\overrightarrow{R}_{r\tau}}(0)$ and $g_1(\hat{p}_4) > g_2(0)$. And we get the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{R}$.

Subcase 2.3.2 : $u_l > f_2(0)$.

Since there exists a point $\hat{p}_5 \in (0, p_*)$ such that $u_l = u_{\overrightarrow{R}_{r\tau}}(\hat{p}_5)$, we divide it into two subcases.

Subcase 2.3.2.1 : $g_1(p_l) \geq g_2(\hat{p}_5)$, similarly as the above discussions there exists a point $(\bar{p}_1, \bar{p}_2):\ p_* < \bar{p}_1 < p_l,\ \hat{p}_5 \leq \bar{p}_2 < p_*$ and the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{R}$.

Subcase 2.3.2.2 : $g_1(p_l) < g_2(\hat{p}_5)$, since there exists a point $(\bar{p}_1, \bar{p}_2):\ \ p_l < \bar{p}_1 < \hat{p}_6,\ 0 < \bar{p}_2 < \hat{p}_5$, where $\hat{p}_6$ satisfies $u_{\overleftarrow{S}_{l\tau}}(\hat{p}_6) = u_{\overrightarrow{R}_{r\tau}}(0)$, and we get the solution is $\overrightarrow{R}\overset{>}{J} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

Similarly, the interaction between $\overset{<}{J}$ and $\overleftarrow{R}$ can be investigated and omitted for simplicity.

***Theorem* 3.4** — *When a rarefaction wave collides with a contact discontinuity which is of a jump increase in density in the propagating direction of the rarefaction wave, the local solution of the interaction is that the rarefaction wave continues to move forward in its propagating direction or a new shock wave will appear. Meanwhile, a new rarefaction wave or a new shock wave propagating in the opposite direction will appear. Furthermore, the contact discontinuity may appear or not after the interaction.*

Case (ii) : $\overrightarrow{R}\overset{<}{J}$.

In this case, it holds that $u_l < u_m = u_r$ and $p_m < p_r$. Similarly with the discussions in Case (i) of this subsection, we know that the curve $\overrightarrow{R}_{m\tau}(Q_m)$ lies always above the curve $\overrightarrow{R}_{r\tau}(Q_r)$ and the curve $\overleftarrow{S}_{l\tau}(Q_l)$ intersects with $\overrightarrow{R}_{r\tau}(Q_r)$ at the point $Q_{*\tau}$ (Fig. 3.7.).

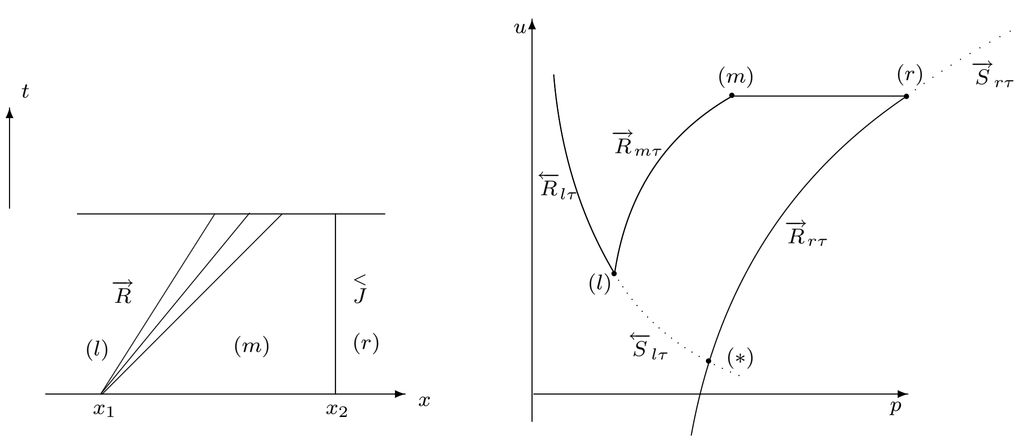

Fig. 3.7. The interaction of $\overrightarrow{R}$ and $\overset{<}{J}$.

Subcase 1 : $\tau_{*l} = \tau_{*r}$.

Similarly, it holds that $g_1(p_*) = g_2(p_*)$ which means that there is no contact discontinuity, and we get the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{S}\overrightarrow{R}$.

Subcase 2 : $\tau_{*l} > \tau_{*r}$. In this case, we know $g_1(p_*) < g_2(p_*)$ and should look for the solution in $\{(\bar{p}_1, \bar{p}_2)|\bar{p}_1 > p_*,\ 0 < \bar{p}_2 < p_*\}$. Obviously, we can define $\tilde{p}_1$ which satisfies $u_{\overrightarrow{R}_{r\tau}}(0) = u_{\overleftarrow{S}_{l\tau}}(\tilde{p}_1)$. Therefore, there exists a point $(\bar{p}_1, \bar{p}_2):\quad p_* < \bar{p}_1 < \tilde{p}_1,\ 0 < \bar{p}_2 < p_*$ and the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

Subcase 3 : $\tau_{*l} < \tau_{*r}$. In this case, we know that $g_1(p_*) > g_2(p_*)$ and should look for the solution in $\{(\bar{p}_1, \bar{p}_2)|0 < \bar{p}_1 < p_*,\ \bar{p}_2 > p_*\}$. There are two possibilities as follows.

Subcase 3.1 : $u_l < u_r < u_{\overleftarrow{R}_{l\tau}}(0)$. Since there exists $0 < \tilde{p}_2 < p_l$ and $p_* < \tilde{p}_3 < p_r$ such that $u_r = u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_2)$ and $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_3)$, respectively.

Subcase 3.1.1 : $g_1(p_l) \leq g_2(\tilde{p}_3)$. Similarly with the above discussions, there exists a point $(\bar{p}_1, \bar{p}_2):\quad p_l \leq \bar{p}_1 < p_*,\ p_* \leq \bar{p}_2 \leq \tilde{p}_3$ and the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.1.2 : $g_1(p_l) > g_2(\tilde{p}_3)$ and $g_2(p_r) \geq g_1(\tilde{p}_2)$. Since there exists a point $(\bar{p}_1, \bar{p}_2):\quad \tilde{p}_2 < \bar{p}_1 \leq p_l,\ \tilde{p}_3 < \bar{p}_2 < p_r$, we get the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.1.3 : $g_1(p_l) > g_2(\tilde{p}_3)$ and $g_2(p_r) < g_1(\tilde{p}_2)$. Since there exists a point $(\bar{p}_1, \bar{p}_2):\quad 0 < \bar{p}_1 < \tilde{p}_2,\ p_r < \bar{p}_2 < \tilde{p}_4$, where $\tilde{p}_4$ satisfies $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{S}_{r\tau}}(\tilde{p}_4)$, we obtain that the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Subcase 3.2 : $u_l < u_{\overleftarrow{R}_{l\tau}}(0) < u_r$. Define $\tilde{p}_5 > p_*$ and $\tilde{p}_5 < \tilde{p}_6 < p_r$ such that $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_5)$ and $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_6)$, respectively.

Subcase 3.2.1 : $g_1(p_l) \leq g_2(\tilde{p}_5)$. Similarly with the above discussions, there exists $(\bar{p}_1, \bar{p}_2):\quad p_l \leq \bar{p}_1 < p_*,\ p_* \leq \bar{p}_2 \leq \tilde{p}_5$ and the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.2.2 : $g_1(p_l) > g_2(\tilde{p}_5)$. In view of $g_2(\tilde{p}_6) > g_1(0)$, it follows that there exists $(\bar{p}_1, \bar{p}_2):\quad 0 < \bar{p}_1 < p_l,\ \tilde{p}_5 < \bar{p}_2 \leq \tilde{p}_6$ and the solution is $\overrightarrow{R}\overset{<}{J} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Similarly, the interaction between $\overset{>}{J}$ and $\overleftarrow{R}$ can be investigated and omitted for simplicity.

***Theorem* 3.5** — *When a rarefaction wave collides with a contact discontinuity which is of a jump decrease in density in the propagating direction of the rarefaction wave, the local solution of the interaction is that the rarefaction wave continues to move forward in its propagating direction or a new shock wave will appear, meanwhile, a new rarefaction wave or a new shock wave propagating in* *the opposite direction will appear. Furthermore, after the interaction the contact discontinuity may appear or not.*

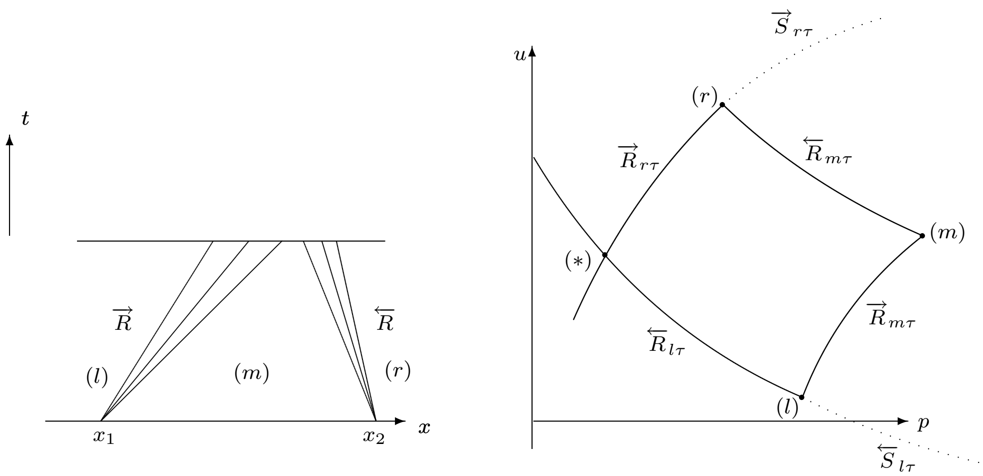

Fig. 3.8. The interaction of $\overrightarrow{R}$ and $\overleftarrow{R}$.

Case (iii) : $\overrightarrow{R}\overleftarrow{R}$. In this case, we know $u_l < u_m$ and $u_m < u_r$. Similar with Case (i) of this subsection, we know that the curve $\overrightarrow{R}_{r\tau}(Q_r)$ lies always above the curve $\overrightarrow{R}_{m\tau}(Q_m)$ and the curve $\overleftarrow{R}_{m\tau}(Q_m)$ lies always above the curve $\overleftarrow{R}_{l\tau}(Q_l)$. It yields that $\overleftarrow{R}_{l\tau}(Q_l)$ intersects with $\overrightarrow{R}_{r\tau}(Q_r)$ at $Q_{*\tau}$ where a new Riemann problem is formed (Fig. 3.8.). In order to construct the solution of this new Riemann problem, we discuss as follows.

Subcase 1 : $p_l\tau_l^\gamma = p_r\tau_r^\gamma$.

It holds that $g_1(p_*) = g_2(p_*)$ and there is no contact discontinuity. Similarly, we get the solution is $\overrightarrow{R}\overleftarrow{R} \rightarrow \overleftarrow{R}\overrightarrow{R}$.

Subcase 2 : $p_l\tau_l^\gamma < p_r\tau_r^\gamma$. In this case, we know that $g_1(p_*) > g_2(p_*)$ and should look for the solution in $\{(\bar{p}_1, \bar{p}_2)|0 < \bar{p}_1 < p_*,\ \bar{p}_2 > p_*\}$. There are two possibilities as follows.

Subcase 2.1 : $u_r \geq f_1(0)$. Obviously, we may define $\tilde{p}_1$ which satisfies $u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_1) = f_1(0)$. Therefore, there exists a point $(\bar{p}_1, \bar{p}_2):\quad 0 < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < \tilde{p}_1$ and the solution is $\overrightarrow{R}\overleftarrow{R} \rightarrow \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Subcase 2.2 : $u_r < f_1(0)$. There exists $0 < \tilde{p}_2 < p_*$ such that $u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_2) = u_r$. We divide into two subcases.

Subcase 2.2.1 : $g_1(\tilde{p}_2) \leq g_2(p_r)$. There exists a point $(\bar{p}_1, \bar{p}_2):\quad \tilde{p}_2 < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < p_r$ and the solution is $\overrightarrow{R}\overleftarrow{R} \rightarrow \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Subcase 2.2.2 : $g_1(\tilde{p}_2) > g_2(p_r)$. There exists a point $(\bar{p}_1, \bar{p}_2)$: $0 < \bar{p}_1 < \tilde{p}_2,\ p_r < \bar{p}_2 < \tilde{p}_3$ and the solution is $\overrightarrow{R}\overleftarrow{R} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$, where $\tilde{p}_3$ satisfies $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{S}_{r\tau}}(\tilde{p}_3)$.

Subcase 3 : $p_l\tau_l^\gamma > p_r\tau_r^\gamma$. In this case, we know $g_1(p_*) < g_2(p_*)$ and should look for the solution in $\{(\bar{p}_1, \bar{p}_2)|\bar{p}_1 > p_*,\ 0 < \bar{p}_2 < p_*\}$. There are two possibilities as follows.

Subcase 3.1 : $u_l \leq f_2(0)$.

It is obvious that there exists a point $\tilde{p}_4 \in (p_*, p_l)$ such that $u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_4) = u_{\overrightarrow{R}_{r\tau}}(0)$ and $g_1(\tilde{p}_4) > g_2(0)$. And we get the solution is $\overrightarrow{R}\overleftarrow{R} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{R}$.

Subcase 3.2 : $u_l > f_2(0)$. Since there exists a point $\tilde{p}_5 \in (0, p_*)$ such that $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_5)$, we divide it into two subcases.

Subcase 3.2.1 : $g_1(p_l) \geq g_2(\tilde{p}_5)$, similarly wih the above discussions there exists a point $(\bar{p}_1, \bar{p}_2)$: $p_* < \bar{p}_1 < p_l,\ \tilde{p}_5 \leq \bar{p}_2 < p_*$ and the solution is $\overrightarrow{R}\overleftarrow{R} \to \overleftarrow{R}\overset{>}{J}\overrightarrow{R}$.

Subcase 3.2.2 : $g_1(p_l) < g_2(\tilde{p}_5)$, since there exists a point $(\bar{p}_1, \bar{p}_2)$: $p_l < \bar{p}_1 < \tilde{p}_6,\ 0 < \bar{p}_2 < \tilde{p}_5$, where $\tilde{p}_6$ satisfies $u_{\overleftarrow{S}_{l\tau}}(\tilde{p}_6) = u_{\overrightarrow{R}_{r\tau}}(0)$, and we get the solution is $\overrightarrow{R}\overleftarrow{R} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

***Theorem* 3.6** — *When a forward rarefaction wave collides with a backward rarefaction wave, the local solution of the interaction is that the forward (backward) rarefaction wave continues to move forward in its propagating direction or a new forward (backward) shock wave will appear. Furthermore, the contact discontinuity may appear or not after the interaction.*

Case (iv) : $\overrightarrow{R}\overleftarrow{S}$. Similar with the above case, we know that the curve $\overrightarrow{R}_{m\tau}(Q_m)$ lies always above the curve $\overrightarrow{R}_{r\tau}(Q_r)$ and the curve $\overleftarrow{S}_{m\tau}(Q_m)$ lies always above the curve $\overleftarrow{S}_{l\tau}(Q_l)$. It yields that $\overleftarrow{S}_{l\tau}(Q_l)$ intersects with $\overrightarrow{R}_{r\tau}(Q_r)$ at $Q_{*\tau}$ where a new Riemann problem is formed (Fig. 3.9.). In order to construct the solution of this new Riemann problem, we discuss as follows.

Subcase 1 : $\tau_{*l} = \tau_{*r}$. We know $g_1(p_*) = g_2(p_*)$ and there is no contact discontinuity. Thus, the result is given by $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{S}\overrightarrow{R}$.

Subcase 2 : $\tau_{*l} > \tau_{*r}$. This means that $g_1(p_*) < g_2(p_*)$. Since there exists $(\bar{p}_1, \bar{p}_2)$: $p_* < \bar{p}_1 < \tilde{p}_1,\ 0 < \bar{p}_2 < p_*$ where $\tilde{p}_1 > p_*$ satisfies $u_{\overrightarrow{R}_{r\tau}}(0) = u_{\overleftarrow{S}_{l\tau}}(\tilde{p}_1)$. It follows that the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{S}\overset{>}{J}\overrightarrow{R}$.

Subcase 3 : $\tau_{*l} < \tau_{*r}$. This means that $g_1(p_*) > g_2(p_*)$. There are three possibilities as follows.

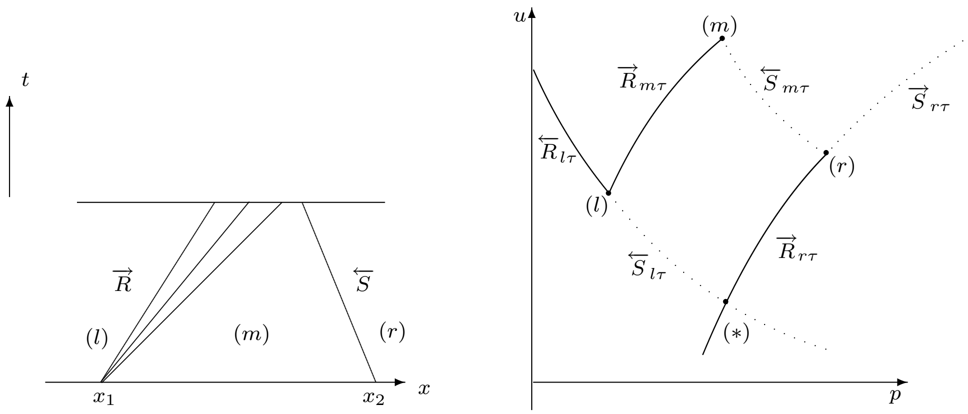

Fig. 3.9. The interaction of $\overrightarrow{R}$ and $\overleftarrow{S}$.

Subcase 3.1 : $u_l \geq u_r$. It is easy to show that there exists $\tilde{p}_2$ which satisfies $u_r = u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_2)$.

Subcase 3.1.1 : $g_1(\tilde{p}_2) \leq g_2(p_r)$, since there exists $(\bar{p}_1, \bar{p}_2) : \tilde{p}_2 < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < p_r$, the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.1.2 : $g_1(\tilde{p}_2) > g_2(p_r)$ and $g_1(p_l) \leq g_2(\tilde{p}_3)$, where $\tilde{p}_3 > p_r$ satisfies $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_3)$. Since there exists $(\bar{p}_1, \bar{p}_2) : p_l < \bar{p}_1 < \tilde{p}_2,\ p_r < \bar{p}_2 < \tilde{p}_3$, the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{S}$.

Subcase 3.1.3 : $g_1(\tilde{p}_2) > g_2(p_r)$ and $g_1(p_l) > g_2(\tilde{p}_3)$. Since there exists $(\bar{p}_1, \bar{p}_2) : 0 < \bar{p}_1 < p_l,\ \tilde{p}_3 < \bar{p}_2 < \tilde{p}_4$, where $\tilde{p}_4$ is determined by $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{S}_{r\tau}}(\tilde{p}_4)$. Thus, the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Subcase 3.2 : $u_l < u_r < u_{\overleftarrow{R}_{l\tau}}(0)$. There exist $0 < \tilde{p}_5 < p_l$ and $p_* < \tilde{p}_6 < p_r$ satisfies respectively that $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_5)$ and $u_r = u_{\overleftarrow{R}_{l\tau}}(\tilde{p}_6)$.

Subcase 3.2.1 : $g_2(\tilde{p}_5) \geq g_1(p_l)$, since there exists $(\bar{p}_1, \bar{p}_2) : p_l < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < \tilde{p}_5$, the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.2.2 : $g_2(\tilde{p}_5) < g_1(p_l)$ and $g_2(p_r) \geq g_1(\tilde{p}_6)$. Since there exists $(\bar{p}_1, \bar{p}_2) : \tilde{p}_6 < \bar{p}_1 < p_l,\ \tilde{p}_5 < \bar{p}_2 < p_r$, the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.2.3 : $g_2(\tilde{p}_5) < g_1(p_l)$ and $g_2(p_r) < g_1(\tilde{p}_6)$. Since there exists $(\bar{p}_1, \bar{p}_2) : 0 < \bar{p}_1 < \tilde{p}_6,\ p_r < \bar{p}_2 < \tilde{p}_7$, where $\tilde{p}_7$ is determined by $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{S}_{r\tau}}(\tilde{p}_7)$. Thus, the result is $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{S}$.

Subcase 3.3 : $u_l < u_{\overleftarrow{R}_{l\tau}(Q_l)}(0) < u_r$. There exist $\tilde{p}_8 > p_*$ and $\tilde{p}_5 < \tilde{p}_9 < p_r$ satisfies respectively that $u_l = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_8)$ and $u_{\overleftarrow{R}_{l\tau}}(0) = u_{\overrightarrow{R}_{r\tau}}(\tilde{p}_9)$.

Subcase 3.3.1 : $g_1(p_l) \leq g_2(\tilde{p}_8)$, there exists a point $(\bar{p}_1, \bar{p}_2)$ which satisfies $p_l < \bar{p}_1 < p_*,\ p_* < \bar{p}_2 < \tilde{p}_8$ and $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{S}\overset{<}{J}\overrightarrow{R}$.

Subcase 3.3.2 : $g_1(p_l) > g_2(\tilde{p}_8)$. Since there exists a point $(\bar{p}_1, \bar{p}_2)$ which satisfies $0 < \bar{p}_1 < p_l,\ \tilde{p}_8 < \bar{p}_2 < \tilde{p}_9$, the solution is given by $\overrightarrow{R}\overleftarrow{S} \to \overleftarrow{R}\overset{<}{J}\overrightarrow{R}$.

Similarly, the interaction between $\overrightarrow{S}$ and $\overleftarrow{R}$ can be investigated and omitted for simplicity.

***Theorem* 3.7** — *When a rarefaction wave collides with a shock wave, the local solution of the interaction is that the rarefaction wave continues to move forward in its propagating direction or a new shock wave will appear, meanwhile, the shock wave continues to move forward in its propagating direction or a new rarefaction wave will appear. Furthermore, the contact discontinuity may appear or not after the interaction.*

So far, we have finished the all possible interactions of the elementary waves and we can see that wave interactions have a more complicated structure for Magnetogasdynamics system (1.1) than the conventional gas dynamics. This is due to the projection of the contact discontinuity on the phase plane (p, u) which is a straight line parallel to the p-axis, however, the projection of the contact discontinuity for the conventional gas dynamics on the phase plane (p, u) which is just a point. Moreover, when the magnetic field B vanishes, our results are consistent with that of the corresponding cases of the conventional gas dynamics.

Based on the above analysis, we obtain the following result.

***Theorem* 3.8** — *There exists uniquely the solution of the initial value problem* (1.1) *and* (1.5).

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