

HEAT TRANSFER IN MHD OSCILLATORY FLOW OF DUSTY FLUID IN A ROTATING POROUS VERTICAL CHANNEL

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An analysis of the unsteady flow of a dusty viscous, incompressible, electrically conducting fluid in a vertical porous channel rotating with constant angular velocity under the influence of periodic pressure gradient is presented. The left porous plate of the channel is subjected to a uniform injection and the right porous plate to same uniform suction respectively. A magnetic field of uniform strength is applied perpendicular to the planes of the plates. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is negligible. The whole system rotates in unison about the axis normal to the planes of the plates. Analytical solutions for the velocities and temperatures of the fluid and the dust particles are obtained. The influence of the various parameters appearing in the equations of velocities and temperatures of the fluid and the dust particles have been numerically evaluated and expressed graphically.

Key words : Hydromagnetic; oscillatory; dusty; rotating vertical porous channel.

1. INTRODUCTION

A dusty fluid is a mixture of fluid and dust particles. The study of dusty fluid is important due to its applications in industrial filtration, ceramic engineering, powder metallurgy, environmental pollution, and smoke emission from vehicle, emission of effluents from industries, cooling effects of air conditioners, flying ash produced from thermal reactor, formation of rain drops and the study of lunar ash flows which explain many features of lunar soil. The flow behaviour of the suspended matter may consists of solid particles, liquid droplets, gas bubbles, or some combinations of the these

are important in engineering and applied problem concerned with, nuclear reactor cooling, powder technology, acoustics, sedimentation performance of solid fuel nozzles, batch setting, rain erosion of guided missiles, aerosol and paint spraying aircraft icing and many others.

In the field of dynamics of a dusty fluid, the volume of the dust particles is generally assumed to be negligible. Due to technological advances new material of industrial importance have been developed whose rheological properties cannot be adequately characterized by the classical Newtonian model. Many of these material exhibit both elastic and viscous properties. Cosmic dust is widely present in space where gas and dust clouds are primary precursors for planetary system. The zodiacal light seen in the sky on a dark night is produced by sunlight reflected from particles of dust in orbit around the sun. The tails of comet are produced by emission of dust and ionized gases from the body of the comet. Interstellar dust is found between the stars and its high concentration can produce diffuse nebulae and reflection nebulae. Dust sample returned from outer space can provide information about the condition in the early solar system. Several space crafts have been launched in an attempt to gather sample of dust and other materials.

Hassan *et al.* [1] has discussed the hydromagnetic flow of a dusty fluid in a rectangular channel with hall current and heat transfer. Attia [2] investigated the time varying Couette flow with heat transfer of a dusty viscous incompressible electrically conducting fluid under the influence of constant pressure gradient by taking into account the hall current effects. Attia [3] has also studied the unsteady MHD Couette flow and heat transfer of a dusty fluid with variable physical properties. MHD flow and heat transfer of dusty viscoelastic stratified fluid through an inclined channel in porous medium under variable viscosity was studied by Chakraborty [4]. Datta *et al.* [6] has studied the unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel. Unsteady flow of a dusty viscous fluid through rectangular ducts has been studied by Dixit [7]. Ezzat *et al.* [8] has studied the space approach to the hydro-magnetic flow of a dusty fluid through a porous medium. Ghosh and Mitra [9] investigated the flow of a dusty gas through horizontal pipes. Gireesha *et al.* [10] has investigated the pulsatile flow of an unsteady dusty fluid through rectangular channel. Gupta and Gupta [11] discussed the flow of a dusty gas through a channel with arbitrary time varying pressure gradient. Palani and Ganesan [12] have studied the heat transfer effects on dusty gas flow past a semi-infinite inclined plate. Prasad and Ramacharyulu [13] discussed the unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient. Saffman [14] studied the stability of laminar flow of dusty gas neglecting the volume fraction of the dust particles. Shawky [16] have explored the pulsatile flow with heat transfer of dusty magnetohydrodynamic Re-Eyring fluid through a channel. Singh [17] analyzed the unsteady flow of a conducting dusty fluid through a rectangular channel with time dependent pressure gradient.

Motivated by above studies, we purpose to investigate the flow of a dusty, viscous incompressible and electrically conducting fluid under the influence of hall current and periodic variation in pressure in a vertical porous channel. The governing equations for both the fluid and the dust particles phases are analytical solved for velocity and temperature field.

2. DESCRIPTION OF THE PROBLEM

The dusty fluid is assumed to be flowing between two infinite vertical porous plates located at $z^* = \pm \frac{d}{2}$ planes, as shown in the Fig. 1 the fluid is injected with constant velocity w_0^* through the left porous plate and is simultaneously removed with same suction velocity through the right porous plate. Thus the z^* -component of the velocity of the fluid is constant and denoted by w_0^* . The dust particles are assumed to be electrically non-conducting, spherical and uniformly distributed in the fluid and big enough so that they are not pumped in and out through the porous plates therefore have no z^* -component of velocity. The two-plates are assumed to be electrically non-conducting and kept at two different temperatures; T_1^* for the left plate and T_2^* for the right plate with $T_2^* > T_1^*$. A periodic pressure gradient varying with time is applied in x^* -direction. A uniform magnetic field B_0 is applied in the z^* -direction. This is only magnetic field in the problem as the induced magnetic field is neglected by assuming very small magnetic Reynolds number (Crammer and Pai [5]). The whole system is assumed to rotate with constant angular velocity, Ω_0 about the z^* -axis. It is required to obtain the time varying velocity distribution for both the fluid and the dust particles. Since the plates are infinite in x^* and y^* directions, all physical quantities for this fully developed flow depends only on z^* and t^* except the pressure.

3. GOVERNING EQUATIONS

The governing equations for this study are based on the conservation laws of mass, linear momentum and energy for both fluid and dust particles phases.

Let $\vec{u}(\mu, v, w)$ and $\vec{u}_p(\mu_p, v_p, w_p)$ be the fluid and particle velocities respectively. The magnetic field and angular velocity for the present problem are $\vec{B}(0, 0, B_0)$ and $\vec{\Omega}(0, 0, \Omega_0)$ respectively.

Following, Attia [2] the unsteady MHD flow in the vertical channel is governed by the following equations:

Conservation law of mass for the fluid and dust particles is expressed by the following equations

$$\nabla \cdot \vec{u} = 0, \quad \nabla \cdot \vec{u}_p = 0 \quad (1)$$

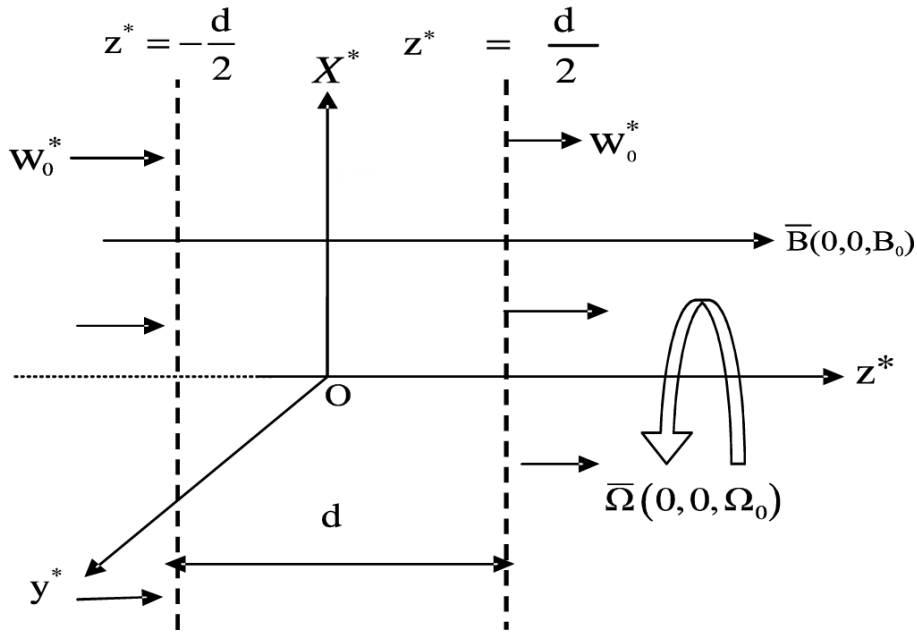


Figure 1: The physical configuration of the problem

Equation of motion of the fluid. The flow of the fluid is governed by the following equation

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} + 2\vec{\Omega} \times \vec{u} = \frac{-1}{\rho} \nabla p + \mu \nabla^2 \vec{u} + \frac{KN}{\rho} (\vec{u}_p - \vec{u}) + \frac{1}{\rho} \vec{J} \times \vec{B} + g\beta (T_p^* - T^*) \tag{2}$$

where ρ is the density of the clean fluid, μ is the viscosity of the clean fluid, \vec{u} is the fluid velocity, \vec{u}_p is the velocity of the dust particles, \vec{J} is the current density, \vec{B} is the magnetic flux density vector, p is the pressure distribution, $\vec{\Omega}$ is the constant angular velocity of the channel, N is the number of dust particle per unit volume, $K = 6\pi\mu a$ is the Stokes constant and a is the average radius of the dust particles.

Equation of motion of the dust particles. The motion of the dust particles is governed by Newton's second law of motion and is given by

$$m_p \left[\frac{\partial \vec{u}_p}{\partial t} + (\vec{u}_p \cdot \nabla) \vec{u}_p + 2\vec{\Omega} \times \vec{u}_p \right] = K(\vec{u} - \vec{u}_p). \tag{3}$$

Here, m_p is the average mass of the dust particles.

The momentum equation (2) for the fluid velocity in its component form is given by

$$\frac{\partial u^*}{\partial t^*} + w_0 \frac{\partial u^*}{\partial z^*} - 2\Omega_0 v^* = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \mu \frac{\partial^2 u}{\partial z^{*2}} + \frac{KN}{\rho} (u_p^* - u^*) - \frac{\sigma B_0^2}{\rho} u^* + g\beta (T_p^* - T^*) \quad (4)$$

$$\frac{\partial v^*}{\partial t^*} + w_0 \frac{\partial v^*}{\partial z^*} + 2\Omega_0 u^* = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \mu \frac{\partial^2 v}{\partial z^{*2}} + \frac{KN}{\rho} (v_p^* - v^*) - \frac{\sigma B_0^2}{\rho} v^* \quad (5)$$

$$0 = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*} \quad (6)$$

The components of the momentum equation (3) for the dust particles phase is given by

$$m_p \left[\frac{\partial u_p^*}{\partial t^*} - 2\Omega_0 v_p^* \right] = KN(u^* - u_p^*) \quad (7)$$

$$m_p \left[\frac{\partial v_p^*}{\partial t^*} + 2\Omega_0 u_p^* \right] = KN(v^* - v_p^*). \quad (8)$$

Boundary conditions relevant to the problem are given by

$$\left. \begin{aligned} z^* = -\frac{d}{2}, u^* = v^* = u_p^* = v_p^* = 0 \\ \text{and} \\ z^* = \frac{d}{2}, u^* = v^* = u_p^* = v_p^* = 0 \end{aligned} \right\} \quad (9)$$

ENERGY EQUATION

The problem deals with two-phase flow, therefore two-energy equations are required (Schlichting [15]; Crammer and Pai [5]). The energy equations describing the temperature distributions for the fluid and dust particles in the absence of viscous dissipation and Joules heating are given by.

$$\rho c_v \frac{\partial T^*}{\partial t} + \rho c_v w_0 \frac{\partial T^*}{\partial z^*} = k \frac{\partial^2 T^*}{\partial z^{*2}} + \frac{\rho_p c_p}{\gamma_T} (T_p^* - T^*) \quad (10)$$

$$\frac{\partial T_p^*}{\partial t} = -\frac{1}{\gamma_T} (T_p^* - T^*) \quad (11)$$

where T is the temperature of the fluid, T_p is the temperature of the dust particles, c_v/c_p is the specific heat capacity of the fluid at constant volume/constant pressure, w_0 is the constant suction/injection velocity, κ is the thermal conductivity of the fluid, ρ_p is the mass of the dust particles per unit volume of the fluid, γ_T the temperature relaxation time.

Boundary conditions relevant to the problem are given by;

$$\left. \begin{aligned} z^* &= -\frac{d}{2}, T^* = 0, T_p^* = 0 \\ \text{and} \\ z^* &= \frac{d}{2}, T^* = T_p^* = T_0 \cos \omega^* t^* \end{aligned} \right\} \quad (12)$$

Introducing the following non-dimensional quantities

$$\begin{aligned} (x, y, z) &= \frac{(x^*, y^*, z^*)}{d}, (u, v, w) = \frac{(u^*, v^*, w_0^*)}{w_0}, (u_p, v_p) = \frac{(u_p^*, v_p^*)}{w_0} \\ G &= \frac{m_p \nu}{KNd^2}, T = \frac{T^*}{T_0}, T_p = \frac{T_p^*}{T_0} \end{aligned} \quad (13)$$

$$\begin{aligned} \omega &= \frac{\omega^* d}{w_0}, t = \frac{t^* w_0}{d}, \Omega = \frac{\Omega_0 d^2}{\nu}, \lambda = \frac{w_0 d}{\nu}, P = \frac{p^*}{\rho w_0^2}, R = \frac{KNd^2}{\mu} \\ M &= B_0 d \sqrt{\frac{\sigma}{\mu}}, L_0 = \frac{\rho d^2}{\mu \gamma T}, G_r = \frac{g \beta T_0 d^2}{w_0 \nu} \end{aligned}$$

in the equations governing the motion and energy of fluid and dust particles phases reduce to:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial z} - \frac{2\Omega}{\lambda} V = -\frac{\partial p}{\partial x} + \frac{1}{\lambda} \frac{\partial^2 u}{\partial z^2} + \frac{R}{\lambda} (u_p - u) - \frac{M^2 u}{\lambda} + \frac{G_r}{\lambda} (T_p - T) \quad (14)$$

$$\frac{\partial v}{\partial t} + \frac{\partial v}{\partial z} + \frac{2\Omega u}{\lambda} = -\frac{\partial p}{\partial y} + \frac{1}{\lambda} \frac{\partial^2 v}{\partial z^2} + \frac{R}{\lambda} (v_p - v) - \frac{M^2 v}{\lambda} \quad (15)$$

$$\frac{\partial u_p}{\partial t} - 2\frac{\Omega}{\lambda} v_p = \frac{1}{G\lambda} (u - u_p) \quad (16)$$

$$\frac{\partial v_p}{\partial t} + 2\frac{\Omega}{\lambda} u_p = \frac{1}{G\lambda} (v - v_p) \quad (17)$$

$$\frac{\partial T}{\partial t} + \frac{\partial T}{\partial z} = \frac{1}{\lambda P_r} \frac{\partial^2 T}{\partial z^2} + \frac{2R}{3\lambda P_r} (T_p - T) \quad (18)$$

$$\frac{\partial T_p}{\partial t} = -L_0 (T_p - T) \quad (19)$$

The corresponding boundary conditions transformed to

$$\left. \begin{aligned} z &= -\frac{1}{2}, u = v = u_p = v_p = 0, T = 0, T_p = 0, \\ \text{and} \\ z &= \frac{1}{2}, u = v = u_p = v_p = 0, T = \cos \omega t, T_p = \cos \omega t \end{aligned} \right\} \quad (20)$$

where G = particle mass parameter, R = particle concentration parameter, ω = frequency of oscillation, M = magnetic field parameter, λ = suction parameter, L_0 = temperature relaxation time parameter, G_r = Grashoff number, P_r = Prandtl number.

Continuity equation is identically satisfied and $\frac{-1}{\rho} \frac{\partial p^*}{\partial z^*} = 0$, show the constancy of fluid pressure along the axis of rotation. We assume that the fluid flows under the pressure gradient varying with time along the x^* -axis is of the form

$$-\frac{\partial p^*}{\partial x^*} = Ae^{i\omega t} \quad \text{and} \quad \frac{\partial p^*}{\partial y^*} = 0 \quad (21)$$

where, A = the amplitude of the pressure gradient.

Introducing the complex velocities for fluid and the dust particles is of the form

$$F = u + iv \quad \text{and} \quad F_p = u_p + iv_p \quad (22)$$

respectively, where $i = \sqrt{-1}$

Equations (14) & (15) and (16) & (17) can be combined into following equations

$$\frac{\partial F}{\partial t} + \frac{\partial F}{\partial z} + \frac{2i\Omega}{\lambda} F = -\frac{\partial p}{\partial x} + \frac{1}{\lambda} \frac{\partial^2 F}{\partial z^2} + \frac{R}{\lambda} (F_p - F) - \frac{M^2}{\lambda} F + \frac{G_r}{\lambda} (T_p - T) \quad (23)$$

$$\frac{\partial F_p}{\partial t} + \frac{2i\Omega}{\lambda} F_p = \frac{1}{G\lambda} (F - F_p) \quad (24)$$

The boundary conditions transformed to:

$$\left. \begin{aligned} z = -\frac{1}{2}; F = F_p = 0; \\ \text{and} \\ z = \frac{1}{2}; F = F_p = 0 \end{aligned} \right\} \quad (25)$$

4. SOLUTION OF THE PROBLEM

To solve the equations (18), (19), (23) and (24) we assume the solution of the following form:

$$\left. \begin{aligned} -\frac{\partial p}{\partial x} = Ae^{i\omega t}, \quad F(z, t) = \phi(z)e^{i\omega t}, \quad F_p(z, t) = \psi(z)e^{i\omega t}, \\ \text{and} \\ T(z, t) = \theta(z)e^{i\omega t}, \quad T_p(z, t) = \xi(z)e^{i\omega t} \end{aligned} \right\} \quad (26)$$

Using equation (26) in equations (18) & (19) and (23) & (24) they transformed to

$$\frac{\partial^2 \theta}{\partial z^2} - \lambda P_r \frac{\partial \theta}{\partial z} - n^2 \theta = 0 \quad (27)$$

$$\xi(z) = \left(\frac{L_0}{L_0 + i\omega} \right) \theta(z) \quad (28)$$

$$\begin{aligned} \frac{\partial^2 \phi}{\partial z^2} - \lambda \frac{\partial \phi}{\partial z} - \phi \left\{ R + M^2 + (2\Omega + \lambda\omega)i - \frac{R}{1 + iG(2\Omega + \lambda\omega)} \right\} \\ = -A\lambda - G_r \{ \xi(z) - \theta(z) \} \end{aligned} \quad (29)$$

$$\psi = \frac{\phi}{[1 + iG(\lambda\omega + 2\Omega)]} \quad (30)$$

The corresponding boundary condition transformed to

$$\left. \begin{aligned} z = -\frac{1}{2}, \quad \theta = \xi = \phi = \psi = 0, \\ \text{and} \\ z = \frac{1}{2}, \quad \phi = \psi = 0, \theta = \xi = 1, \end{aligned} \right\} \quad (31)$$

Solution of the equations (27) to (30) under the boundary conditions (31) are given by

$$\theta(z) = \frac{1}{e^{\frac{n_1-n_2}{2}} - e^{-\frac{n_1-n_2}{2}}} \left(e^{-\frac{n_2}{2}} e^{n_1 z} - e^{-\frac{n_1}{2}} e^{n_2 z} \right) \quad (32)$$

$$\xi(z) = \left(\frac{L_0}{L_0 + i\omega} \right) \frac{1}{e^{\frac{n_1-n_2}{2}} - e^{-\frac{n_1-n_2}{2}}} \left(e^{-\frac{n_2}{2}} e^{n_1 z} - e^{-\frac{n_1}{2}} e^{n_2 z} \right) \quad (33)$$

$$\begin{aligned} \phi = & \left\{ \left[\frac{e^{p_1 z}}{\sinh\left(\frac{p_1 - p_2}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_2}{2} - R \sin\left(\frac{n_1 - p_2}{2}\right) + S \sinh\left(\frac{n_2 - p_2}{2}\right) \right) \right] \right\} \\ & + \left\{ \left[\frac{e^{p_2 z}}{\sinh\left(\frac{p_2 - p_1}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_1}{2} - R \sin\left(\frac{n_1 - p_1}{2}\right) + S \sinh\left(\frac{n_2 - p_1}{2}\right) \right) \right] \right\} \\ & + \left[\frac{A\lambda}{p^2} + R e^{n_1 z} - S e^{n_2 z} \right] \end{aligned} \quad (34)$$

$$\begin{aligned}
\psi = & \frac{1}{[1 + iG(\lambda\omega + 2\Omega)]} \\
& \left[\left\{ \frac{e^{p_1 z}}{\sinh\left(\frac{p_1 - p_2}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_2}{2} - R\sin\left(\frac{n_1 - p_2}{2}\right) + S\sinh\left(\frac{n_2 - p_2}{2}\right) \right) \right\} \right. \\
& + \left\{ \frac{e^{p_2 z}}{\sinh\left(\frac{p_2 - p_1}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_1}{2} - R\sin\left(\frac{n_1 - p_1}{2}\right) + S\sinh\left(\frac{n_2 - p_1}{2}\right) \right) \right\} \\
& \left. + \frac{A\lambda}{p^2} + Re^{n_1 z} - Se^{n_2 z} \right] \tag{35}
\end{aligned}$$

Therefore, by using equation (26) the following expressions for the fluid velocity, dust particles velocity, fluid temperature and dust particles temperature are obtained

$$\begin{aligned}
F(z, t) = & e^{i\omega t} \left[\left\{ \frac{e^{p_1 z}}{\sinh\left(\frac{p_1 - p_2}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_2}{2} - R\sin\left(\frac{n_1 - p_2}{2}\right) + S\sinh\left(\frac{n_2 - p_2}{2}\right) \right) \right\} \right. \\
& + \left\{ \frac{e^{p_2 z}}{\sinh\left(\frac{p_2 - p_1}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_1}{2} - R\sin\left(\frac{n_1 - p_1}{2}\right) + S\sinh\left(\frac{n_2 - p_1}{2}\right) \right) \right\} \\
& \left. + \frac{A\lambda}{p^2} + Re^{n_1 z} - Se^{n_2 z} \right] \tag{36}
\end{aligned}$$

$$\begin{aligned}
F_p(z, t) = & \frac{e^{i\omega t}}{[1 + iG(\lambda\omega + 2\Omega)]} \\
& \left[\left\{ \frac{e^{p_1 z}}{\sinh\left(\frac{p_1 - p_2}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_2}{2} - R\sin\left(\frac{n_1 - p_2}{2}\right) + S\sinh\left(\frac{n_2 - p_2}{2}\right) \right) \right\} \right. \\
& + \left\{ \frac{e^{p_2 z}}{\sinh\left(\frac{p_2 - p_1}{2}\right)} \left(\frac{A\lambda}{p^2} \sinh\frac{p_1}{2} - R\sin\left(\frac{n_1 - p_1}{2}\right) + S\sinh\left(\frac{n_2 - p_1}{2}\right) \right) \right\} \\
& \left. + \frac{A\lambda}{p^2} + Re^{n_1 z} - Se^{n_2 z} \right] \tag{37}
\end{aligned}$$

$$T(z, t) = \frac{e^{i\omega t}}{e^{\frac{n_1-n_2}{2}} - e^{-\frac{n_1-n_2}{2}}} \left(e^{-\frac{n_2}{2}} e^{n_1 z} - e^{-\frac{n_1}{2}} e^{n_2 z} \right) \quad (38)$$

$$T_p(z, t) = \left(\frac{L_0}{L_0 + i\omega} \right) \frac{e^{i\omega t}}{e^{\frac{n_1-n_2}{2}} - e^{-\frac{n_1-n_2}{2}}} \left(e^{-\frac{n_2}{2}} e^{n_1 z} - e^{-\frac{n_1}{2}} e^{n_2 z} \right) \quad (39)$$

5. RESULTS AND DISCUSSION

The expressions obtained above have been numerical evaluated and expressed graphically. The following discussions bring out the effects of some important parameters such as the particle concentration parameter (R), the magnetic field parameter (M), the suction parameter (λ), the frequency of oscillation (ω), Grashof number (G_r), the amplitude of the pressure gradient (A), the temperature relaxation time (L_0), the particle mass parameter (G) and Prandtl number (P_r) on the fluid and dust particles velocities.

We observed from Fig. (2-7) that the velocity of fluid increases with the increasing value of the suction parameter (λ) Prandtl number (P_r), amplitude of the pressure gradient (A), the temperature relaxation time (L_0). On the other hand velocity of the fluid decreases with the increasing value of the particle concentration parameter (R), the magnetic field parameter (M), the frequency of oscillation (ω), Grashof number (G_r) and rotation parameter (Ω).

The dust particles velocity variations with these parameters have been shown in Fig. (8-12). The study of these figures shows that the dust particles velocity increases with the increasing value of the amplitude of the pressure gradient (A), the suction parameter (λ), Prandtl number (P_r) and the temperature relaxation time (L_0). It is also observed that the velocity of dust particles decreases with the increasing value of the particle concentration parameter (R), the magnetic field parameter (M), the frequency of oscillation (ω), Grashof number (G_r), the particle mass parameter (G) and rotation parameter (Ω).

The fluid and dust particles velocities decrease with the increasing value of (M), the Hartmann number physically it means that the flow is dragged backward due to the increasing strength of magnetic field (Lorentz force). Also it decreases with the increasing value of particle concentration parameter (R), physically it means that as the number of particles increases the interaction between the fluid and dust particles increases which reduce the velocities of the fluid and the dust particles.

From Fig. 13, we observe that the temperature of the fluid decreases with the increasing value of the suction parameter (λ), Prandtl number (P_r), frequency of the oscillation (ω) and the particle concentration parameter (R).

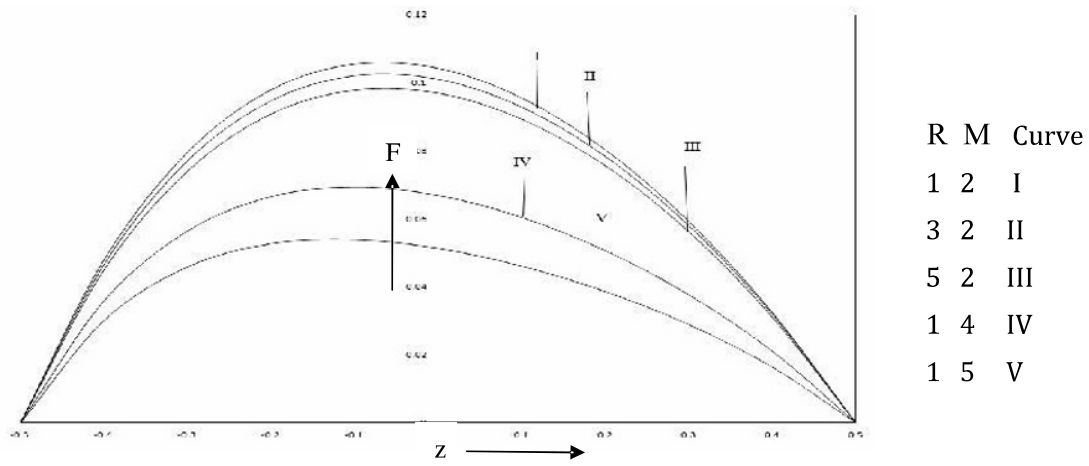


Fig. 2. Variation of fluid velocity with R and M for fixed value of $\lambda = 0.5, G = 1, A = 5, P_r = 0.71, G_r = 2, L_0 = 0.1, \Omega = 5$

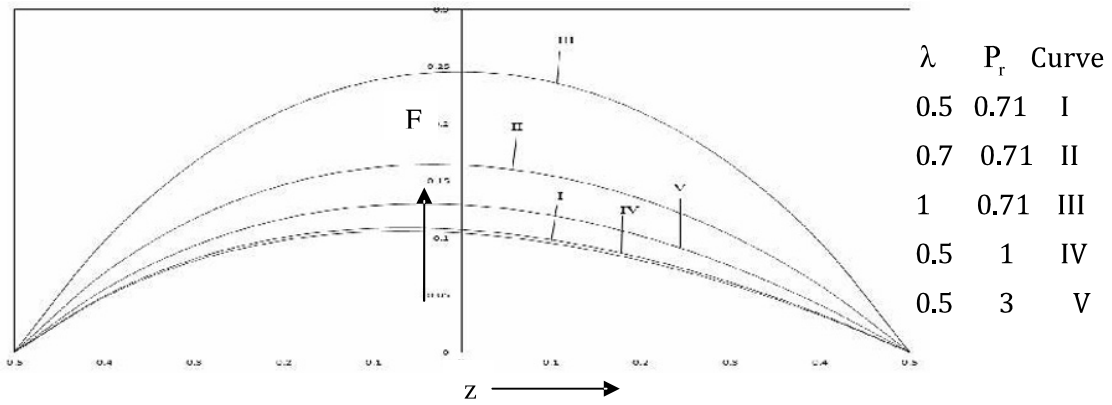


Fig. 3. Variation of fluid velocity with P_r and λ for fixed value of $R = 1, G = 1, A = 5, M = 2, G_r = 2, L_0 = 0.1, \Omega = 5$

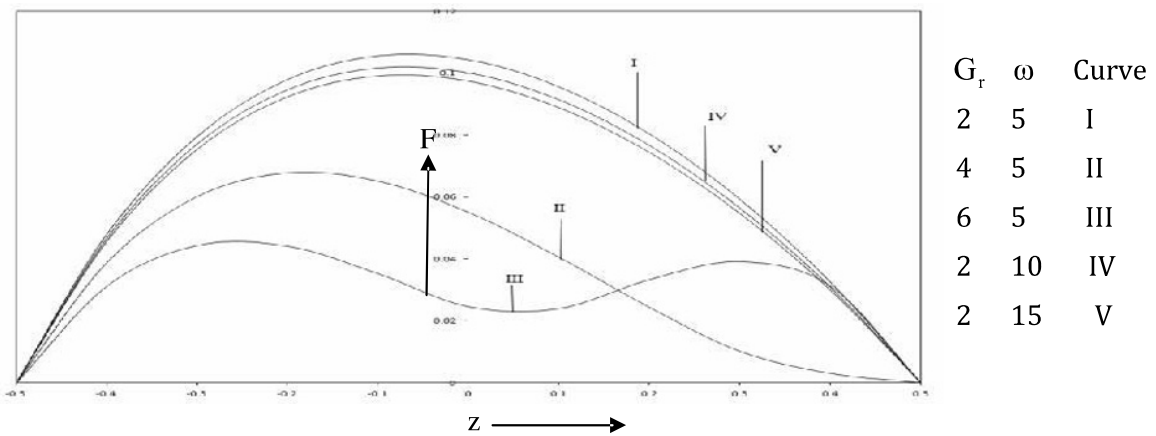


Fig. 4. Variation of fluid velocity with G_r and ω for fixed value of $R = 1, G = 1, A = 5, M = 2, P_r = 0.71, L_0 = 0.1, \Omega = 5$

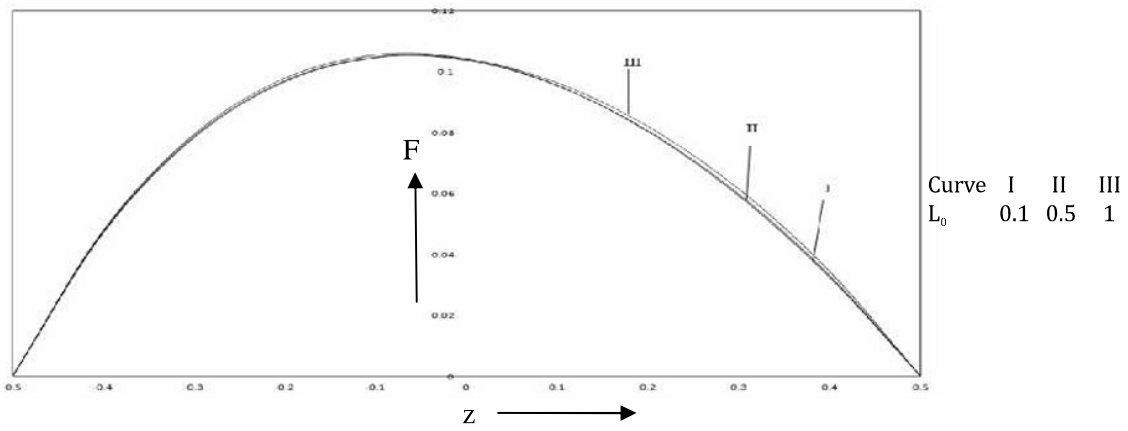


Fig. 5. Variation of fluid velocity with L_0 for fixed value of $R = 1, G = 1, A = 5, M = 2, P_r = 0.71, G = 1$

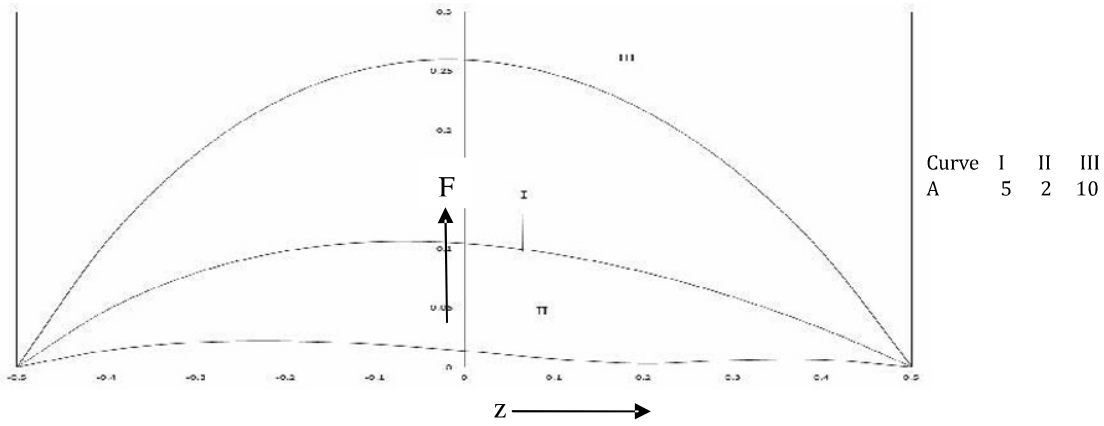


Fig. 6. Variation of fluid velocity with A for fixed value of $R = 1, G = 1, L_0 = 0.1, M = 2, P_r = 0.71, G = 1, \Omega = 5$

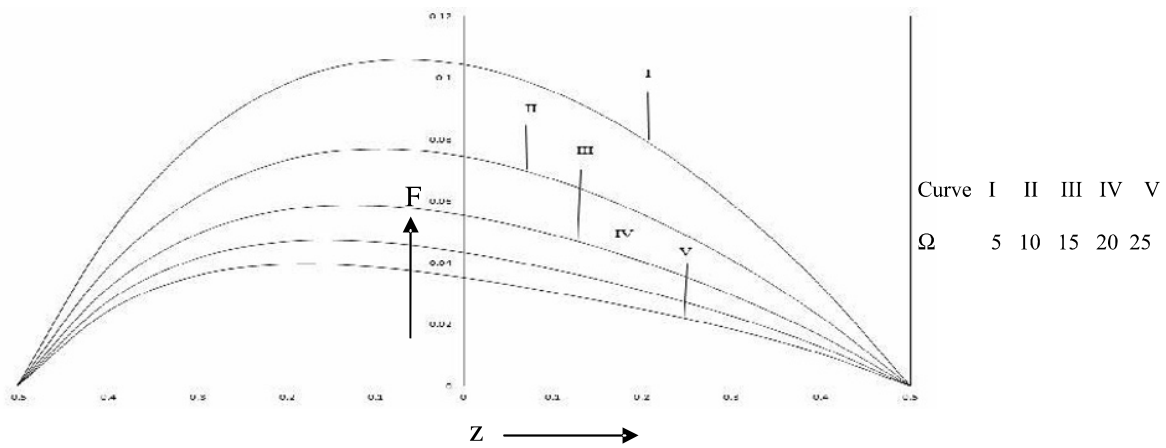


Fig.7. Variation of fluid velocity with Ω for fixed value of $G = 1, L_0 = 0.1, A = 5, P_r = 0.71, \omega = 5, R = 1, M = 2$

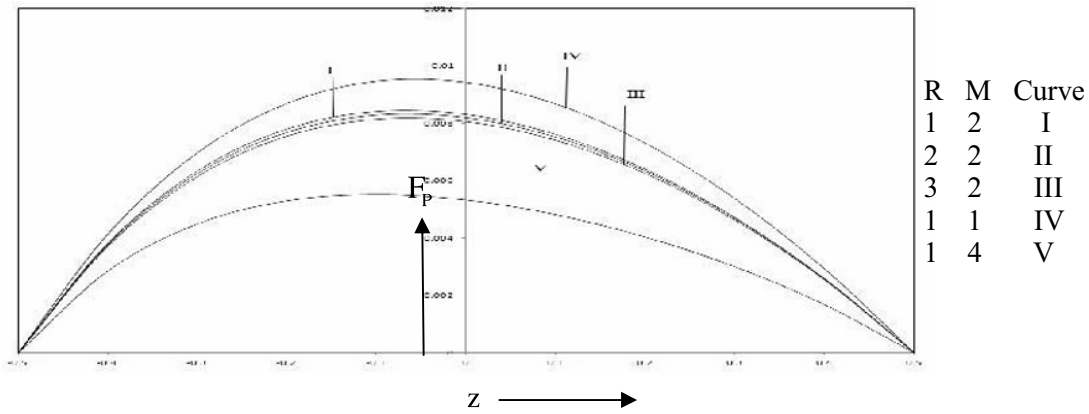


Fig. 8. Variation of dust particles velocity with R and M for fixed value of $G = 1$, $L_0 = 0.1$, $A = 5$, $P_r = 0.71$, $\omega = 5$, $P_r = 0.71$, $\Omega = 5$

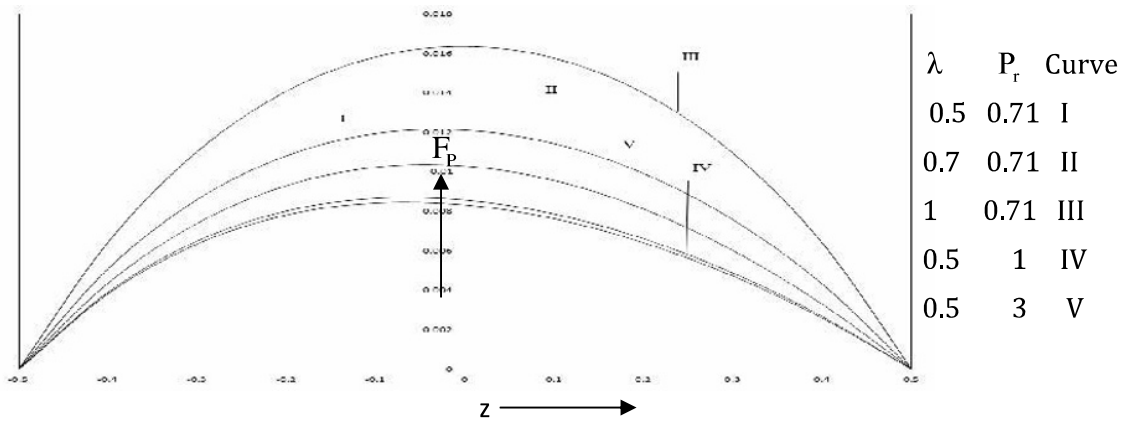


Fig. 9. Variation of dust particles velocity with P_r and λ for fixed value of $G = 1$, $L_0 = 0.1$, $A = 5$, $P_r = 0.71$, $\omega = 5$, $R = 1$, $M = 2$, $\Omega = 5$

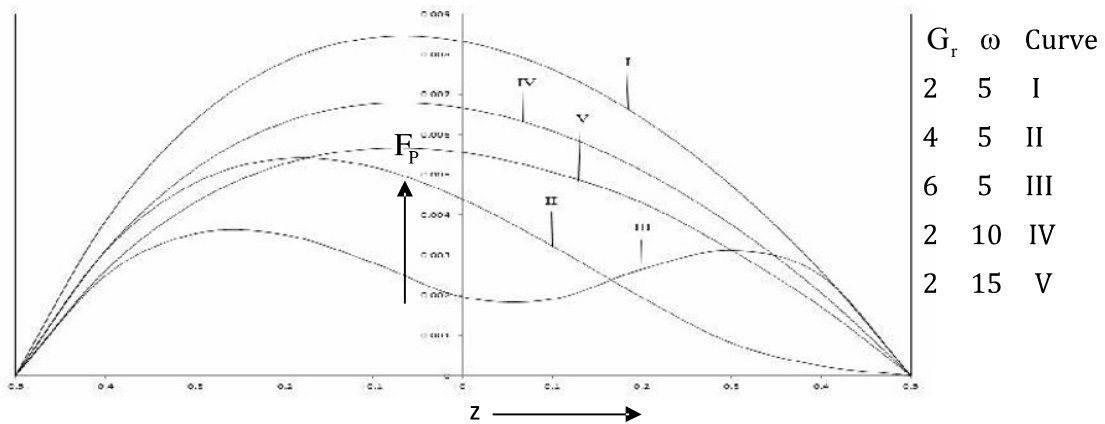


Fig. 10. Variation of dust particles velocity with G_r and ω for fixed value of $G = 1$, $L_0 = 0.1$, $A = 5$, $P_r = 0.71$, $\omega = 5$, $R = 1$, $M = 2$, $\Omega = 5$

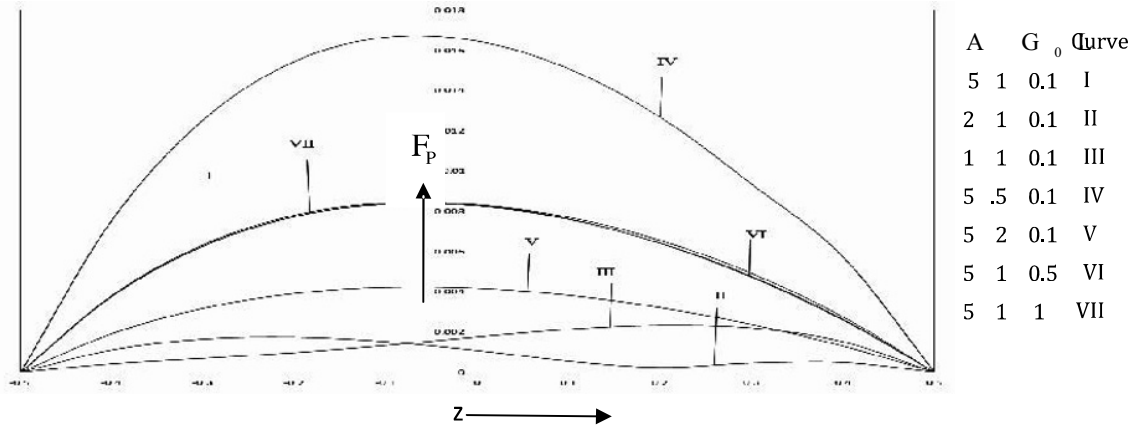


Fig. 11. Variation of dust particles velocity with A, G, L_0 for fixed value of $P_r = 0.71, \omega = 5, R = 1, M = 2, \Omega = 5$

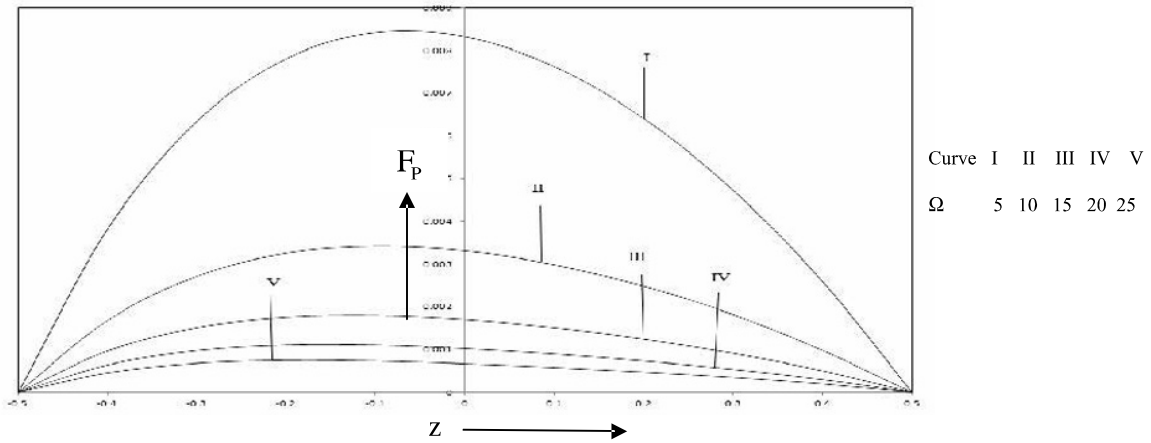


Fig. 12. Variation of dust particles velocity with Ω for fixed value of $G = 1, L_0 = 0.1, A = 5, P_r = 0.71, \omega = 5, R = 1, M = 2$

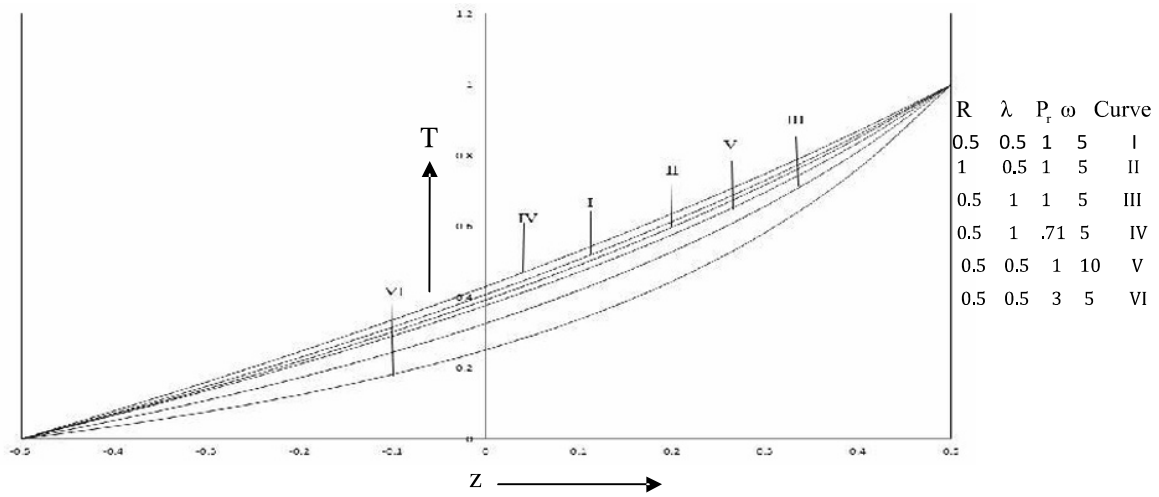


Fig. 13. Variation of fluid temperature with R, λ, P_r and ω

Fig. 14 shows that the temperature of the dust particles increases with the increasing value of temperature relaxation time (L_0). It is also observed that the temperature of the dust particles decreases with the increasing value of the suction parameter (λ), the particle concentration parameter (R), Prandtl number (Pr) and frequency of the oscillation (ω).

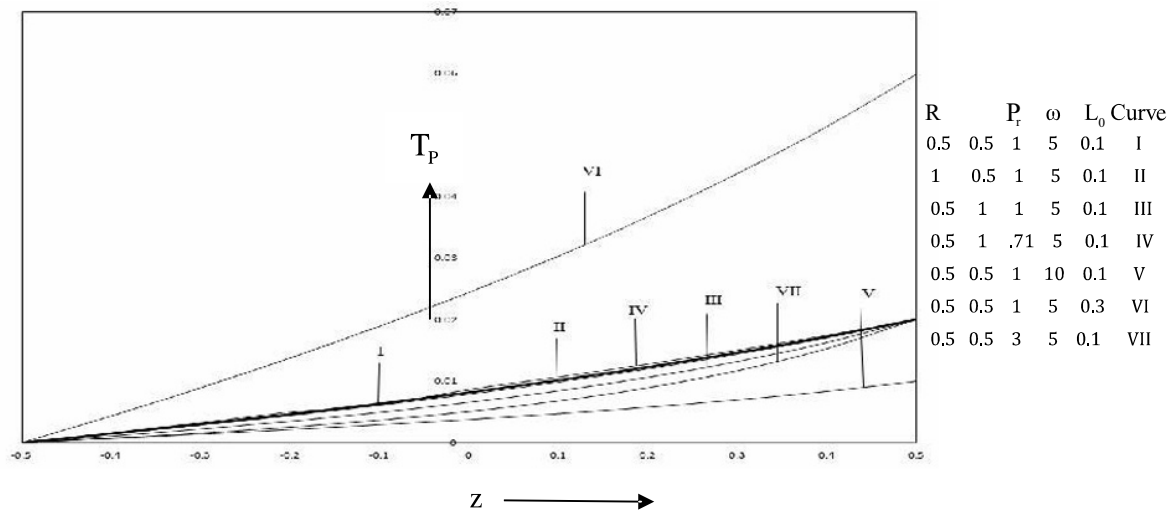


Fig. 14: Variation of dust particles temperature with R , λ , Pr , ω and L_0

6. CONCLUSION

We conclude our research paper with the following observations.

- The fluid and dust particles velocities are enhanced by the amplitude of the pressure gradient, temperature relaxation time, the suction parameter and Prandtl number.
- The fluid and the dust particles velocities decrease with increasing value of Grashof number.
- Fluid and the dust particle velocity are maximum along the centre of the channel.
- Temperature of the dust particles are enhanced with the temperature relaxation time.

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REFERENCES

1. A. L. Aboul-Hasan and H. A. Attia, Hydromagnetic flow of a dusty fluid in a rectangular channel with hall current and heat transfer, *Can. J. Phys.*, **80** (2002), 579-589.
2. H. A. Attia, Time varying hydro magnetic Couette flow with heat transfer of a dusty fluid in the presence of uniform suction and injection considering the Hall Effect, *Turkish Journal of Eng.Env. Sci.*, **30** (2006), 285-297.

3. H. A. Attia, Unsteady MHD couette flow and heat transfer of a dusty fluid with variable physical properties, *Applied Mathematics and Computation*, **177** (2006), 308-318.
4. S. Chakraborty, MHD Flow and heat transfer of a dusty visco-elastic stratified fluid down an inclined channel in porous medium under variable viscosity, *Theoretical and Applied Mechanics*, **26** (2001), 1-14.
5. K. R. Cramer and S. Pai, *Magneto fluid dynamics for engineers and Applied Physicist*, McGraw-Hill New York (1973).
6. N. Datta, D. C. Dalal and S. K. Mishra, Unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel, *International Journal of Heat and Mass Transfer*, **36**(7) (1993), 1783-1788.
7. L. A. Dixit, Unsteady flow of a dusty viscous fluid through rectangular ducts, *Indian Journal of Theoretical Physics*, **28** (1980), 129-136.
8. M. A. Ezzat, A. A. El-Bary and M. M. Morsey, Space approach to the hydro-magnetic flow of a dusty fluid through a porous medium, *Computers and Mathematics with Application*, **59**(8) (2010), 2868-2879.
9. A. K. Ghosh and D. K. Mitra, Flow of a dusty fluid through horizontal pipes, *Rev. Roum. Phys.*, **29** (1984), 631-645.
10. B. J. Gireesha, C. S. Bagewadi and B. C. Prasannakumar, Pulsatile flow of an unsteady dusty fluid through rectangular channel, *Communications in Nonlinear Science and Numerical Simulation*, **14**(5) (2009), 2103-2110.
11. R. K. Gupta and S. C. Gupta, Flow of a dusty gas through a channel with arbitrary time varying pressure gradient, *Journal of Applied Mathematics and Physics*, **27** (1976), 119-128.
12. G. Palani and P. Ganesan, Heat transfer effects on dusty gas flow past a semi-infinite inclined plate, *Forschung im Ingenieurwesen*, **71**(3-4) (2007), 223-230.
13. V. R. Prasad and N.C.P. Ramacharyulu, Unsteady flow of a dusty incompressible fluid between two parallel plates under an impulsive pressure gradient, *Def. Sci. Journal*, **30** (1979), 125-137.
14. P. G. Saffman, On the stability of laminar flow of dusty gas, *Journal of Fluid Mech.*, **13** (1962), 120-128.
15. H. Schlichting, *Boundary layer theory*, McGraw-Hill, New York (1960).
16. H. M. Shawky, Pulsatile flow with heat transfer of dusty magnetohydrodynamic Ree-Eyring fluid through a channel, *Heat and Mass Transfer*, **45**(10) (2009), 1261-1269.

17. K. K. Singh, Unsteady flow of conducting dusty fluid through a rectangular channel with time dependent pressure gradient, *Indian Journal of Pure and Applied Mathematics*, **8** (1976), 1124-1133.

APPENDIX

$$\frac{KNd}{m_p w_0} = \frac{1}{G\lambda}, \quad G = \frac{m_p \nu}{KNd^2}, \quad p_1 = \frac{\lambda + \sqrt{\lambda^2 + 4p^2}}{2}, \quad p_2 = \frac{\lambda - \sqrt{\lambda^2 + 4p^2}}{2}$$

$$n = \sqrt{i\omega \left(\frac{2}{3(L_0 + i\omega)} R + \lambda P_r \right)}, \quad n_1 = \frac{\lambda P_r + \sqrt{(\lambda P_r)^2 + 4n^2}}{2}$$

$$n_2 = \frac{\lambda P_r - \sqrt{(\lambda P_r)^2 + 4n^2}}{2}, \quad p^2 = R + M^2 + (2\Omega + \omega\lambda)i$$

$$\frac{R}{1 + iG(2\Omega + \omega\lambda)}$$

$$E = \frac{G_r i\omega}{L_0 + i\omega} \left(\frac{1}{e^{\frac{n_1 - n_2}{2}} - e^{-\frac{n_1 - n_2}{2}}} \right), \quad H = \frac{e^{-\frac{n_2}{2}}}{n_1^2 - n_1\lambda - p^2}, \quad K = \frac{e^{-\frac{n_1}{2}}}{n_2^2 - n_2\lambda - p^2}$$

$$EH = R, \quad EK = S$$