

Oil production optimization: a mathematical model

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Abstract In this paper, a mathematical model is developed to study crude oil extraction from oil wells. Due to advancement in technical, procedural and reservoir management the estimated lifetime and subsequently field hardware utilization has increased. Continuous high-quality and efficient production can lead to complacency about the integrity of the wellhead, but can also cause lack of focus on routine maintenance. Lack of critical component maintenance and over exposure can take its toll on mechanical assemblies, increasing the likelihood of production downtime caused by equipment failure. In extreme cases it may also lead to an emergency shutdown. The objective of the paper is to optimize the total cost per unit time in light of critical factors which involve spills or leakage, system failure and maintenance cost for optimum extraction. The model is valid with empirical data and sensitivity analysis is carried out.

Keywords Crude oil production · Leakage · Optimization

List of symbols

V_0	Expected production
β_1	Rate of production (before peak)
β_2	Rate of declination (after peak)
A	Total area of leakage

SC	Set up cost per unit time
g	Maintenance cost
η	Cost associated with failure of system
δ	Rate of loss of production due to system failure
h	Holding cost
C_L	Leakage cost

Introduction

Petroleum system

The petroleum system consists of a mature source rock, migration pathway, reservoir rock, trap and seal. Appropriate relative timing of formation of these elements and the processes of generation, migration and accumulation are necessary for hydrocarbons to accumulate and be preserved.

The formation of hydrocarbon liquids from an organic rich source rock with kerogen and bitumen to accumulate oil or gas takes place in Source Rock. Generation depends on three main factors, i.e., the presence of organic matter rich enough to yield hydrocarbons, adequate temperature and sufficient time to bring the source rock to maturity. Pressure and the presence of bacteria and catalysts also affect generation. Generation is a critical phase in the development of a petroleum system.

Migration is movement of hydrocarbons from their source into reservoir rocks. The movement of newly generated hydrocarbons out of their source rock is primary migration, also called expulsion. The further movement of the hydrocarbons into reservoir rock in a hydrocarbon trap or other area of accumulation is secondary migration.

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Migration typically occurs from a structurally low area to a higher area in the sub-surface because of the relative buoyancy of hydrocarbons in comparison to the surrounding rock. Migration can be local or can occur along distances of hundreds of kilometers in large sedimentary basins, and is critical to the formation of a viable petroleum system. Accumulation is development of a petroleum system during which hydrocarbons migrate into and remain trapped in a reservoir.

Reservoir is subsurface body of rock having sufficient porosity and permeability to store and transmit fluids. Sedimentary rocks are the most common reservoir rocks because they have more porosity than most igneous and metamorphic rocks and they form under temperature conditions at which hydrocarbons can be preserved. A reservoir is a critical component of a complete petroleum system.

An impermeable rock that acts as a barrier to further migration of hydrocarbon liquids is referred as Seal (cap rock). Rocks that form a barrier or cap above and around reservoir rock forming a trap such that fluids cannot migrate beyond the reservoir. The permeability of a seal capable of retaining fluids through geologic time is $\sim 10^{-6}$ to 10^{-8} darcies. Commonly known cap rocks are shale, mudstone, anhydrite, salt.

A configuration of rocks suitable for containing hydrocarbons and sealed by a relatively impermeable formation through which hydrocarbons will not migrate is known as Trap. Traps can be of types (1) structural traps—hydrocarbon traps that form in geologic structures such as folds and faults. (2) Stratigraphic traps—hydrocarbon traps that result from changes in rock type or pinch-outs, unconformities, or other sedimentary features such as reefs or buildups.

The oil exploration and development cycle

It comprise of five phases—exploration, appraisal, development, production and abandonment. In practice, the advancement from one phase to the next is conditional on continued verification of a positive assessment of the commercial potential of a discovery or field.

Exploration Oil exploration typically depends on highly sophisticated geophysical technology to detect and determine the extent of potential structures. Areas thought to contain hydrocarbons are initially subjected to a gravity survey, magnetic survey and regional seismic reflection surveys to detect large scale features of the sub-surface geology. Features of interest (known as “leads”) are subjected to more detailed seismic surveys to refine the understanding of the sub-surface structure. Finally, if a prospect is identified and positively evaluated, an exploration well is drilled in an attempt to conclusively determine the presence or absence of oil or gas. Oil and gas exploration

is an expensive, risky operation with a high likelihood that nothing will be found, or that hydrocarbons will be found in such small quantities that it is not worthwhile producing them. In the North Sea, only about one in eight exploration wells find quantities of oil and gas that are economic to develop. It often takes several years from being awarded an exploration license to the drilling of the first well.

Appraisal of a discovery involves drilling further wells to reduce the degree of uncertainty in the size and quality of the potential field. If an exploratory well shows that hydrocarbons are present, more seismic data may be gathered and one or more appraisal wells may be drilled. Based on the data from this process it is possible to estimate the quantities and production ability of oil and gas in the field.

Development If commercially profitable accumulations of oil and gas are found during appraisal drilling, the development phase begins. This phase involves planning and deciding on how to develop the discovery. Crucial factors for value creation in this phase include choosing the most cost-effective type of development and production activity and ensuring that the project can be completed on schedule. This phase involves considerable investment, especially when the production facilities are located offshore.

Production It involves production of oil and gas and also water, in different proportions. Value creating factors in this phase are production well planning, maintaining the rate of production and maximizing the life of the accumulation by injecting gas or water into specifically designed injector wells to maintain the pressure.

Abandonment It is the last phase of a hydrocarbon development project and involves the decommissioning of any installations and subsea structures associated with the field.

Oil in place and URR

All oil and gas fields represent a limited geological structure, and consequently, they have an upper limit of how much hydrocarbons they contain. The size of the trap and reservoir, which can be defined by geological and geophysical methods, gives an estimate of the potential volume of oil in the field, before the drilling has begun. As borehole data and production data become available, the reserve estimate will tend toward increasing accuracy (Dake 2004). The total volume of oil in a field is commonly referred to as either oil initially in place (OIIP) or oil originally in place (OOIP) or sometimes just oil in place (OIP). This is equivalent to the total amount of oil residing in the pores of one or more reservoirs making up a field (Robelius 2007). It is relatively straightforward to calculate OIIP if the areal extent and thickness of the reservoir is known together with the average porosity and saturation

levels (Robelius 2007). In practice, OIIP estimates get more complicated since both porosity and saturation varies throughout the reservoir.

Far from all the oil in place can be recovered from a given reservoir. The recoverable amount of the oil in place is classified as the reserve. The recovery factor (RF) is a dynamic value, representing the estimated percentage of the total oil in place volume that can be recovered. RF depends on numerous parameters, such as rock and fluid properties, reservoir drive mechanism and production technology, variations in the formation and the development process (Robelius 2007). In some modern reservoir simulators it is not necessary to use OIIP or RF at all to estimate reserves.

Production modeling

Production profiles of giant fields generally have a long plateau phase, rather than the sharp “peak” often seen in smaller fields. The end of the plateau phase is the point where production enters the decline phase. We adopted the end-of-plateau as the point where production lastingly leaves a 4 % fluctuation band, as Hirsch and Robert (2008) postulated in a prior study.

In this analysis the exponential decline model, originally developed by Arps (1945), was used to model field behaviors and to forecast future production. One advantage of the decline curve analysis is that it generally applies independent of the size and shape of the reservoir or the actual drive-mechanism (Doublet et al. 1994), avoiding the need for more detailed reservoir data. This approach is the same used by CERA (2007). Accordingly, each field is assumed to have a constant decline rate, and the production for an individual oil field fluctuates around some average value over time.

Some fields can also show complex behavior with several exponential decline phases or even production collapses, where the decline can be doubled in the end stage of the field life. In other cases introduction of new technology can revive the field and significantly dampen the decline temporarily. This is the case in some Russian fields, which were reworked after the fall of the Soviet Union. However, Höök et al. (2009) found that such events are likely to result in higher decline rates later on, compensating the temporal decrease in decline rate.

The three most common forms of decline curves are exponential, hyperbolic, and harmonic. It is assumed that the production will decline on a reasonably smooth curve, and so allowances must be made for wells shut in and production restrictions. The curve can be expressed mathematically or plotted on a graph to estimate future production. It has the advantage of (implicitly) including all reservoir characteristics. It requires a sufficient history to establish a statistically significant trend, ideally when

production is not curtailed by regulatory or other artificial conditions.

Maintenance

Standard maintenance of pipeline is done by surveying through air, by foot and road patrols to check leaks or pipe settling or shifting.

Second way of maintenance of pipeline is by pigs—mechanical devices sent through the pipeline to perform a variety of functions. The most common pig is the scraper pig, which removes wax that precipitates out of the oil and collects on the walls of the pipeline. The colder the oil, the more wax buildup. This buildup can cause a variety of problems, so regular “piggings” are needed to keep the pipe clear. A second type of pig travels through the pipe and looks for corrosion. Corrosion-detecting pigs use either magnetic or ultrasonic sensors. Magnetic sensors detect corrosion by analyzing variations in the magnetic field of the pipeline’s metal. Ultrasonic testing pigs detect corrosion by examining vibrations in the walls of the pipeline. Other types of pigs look for irregularities in the shape of the pipeline, such as if it is bending or buckling. “Smart” pigs, which contain a variety of sensors, can perform multiple tasks. Typically, these pigs are inserted at Prudhoe Bay and travel the length of the pipeline.

A third type of common maintenance is the installation and replacement of sacrificial anodes along the subterranean portions of pipeline. These anodes reduce the corrosion caused by electrochemical actions that affect these interred sections of pipeline. Excavation and replacement of the anodes is required as they corrode. The pipeline gets damaged due to sabotage, human error, maintenance failures, and natural disasters which cause leakage.

Methods for forecasting crude oil production along with measures to increase production rate and managing reduction in costs associated with maintenance, system failure and leakage are crucial.

Aim

This study is to optimize total cost of production when oil spill occurs during oil production. In this paper, cost associated with system failure has also been considered. Peak time and decline analysis has been conducted mathematically. Sensitivity analysis is carried out to analyze the effect of different parameters on time period, total production and total cost. Findings are in accordance with the results established by Licentiate thesis Mikael Höök (2009) Global Energy Systems, Department for Physics and Astronomy, Uppsala University, May 2009.

The enormous growth and development of society in the last 200 years has been driven by rapid increase in the

extraction of fossil fuels. Consequently, reliable methods for forecasting their production, especially crude oil, will be of great significance.

Mathematical model

Let us divide production into two phases' t_1 and t_2 . t_1 represents time period from first oil to peak oil whereas t_2 is time period from peak to abandonment.

Where, $T = t_1 + t_2$ total production time.

Differential equations governing production in both the phases are follows:

$$\frac{dQ_1}{dt} = v_0(1 + \beta_1 t) - A Q_1(t) \quad 0 \leq t \leq t_1 \quad (1)$$

Boundary conditions will be, $Q_1(0) = 0, \quad Q_1(t_1) = P_m$.

$$\frac{dQ_2}{dt} = v_0(1 - \beta_2 t) - A Q_2(t) \quad 0 \leq t \leq t_2 \quad (2)$$

Boundary conditions will be, $Q_2(0) = P_m, \quad Q_2(t_2) = 0$.

Solving differential Eqs. (1) and (2), putting on boundary conditions gives $t_1 = \beta_2 t_2^2 - t_2$.

Total production cost per time unit (PC): $\frac{1}{T} [\frac{g}{P} + \eta P^\delta]$ where, $P = P_1 + P_2$

Total holding cost per time unit (HC): $\frac{h}{T} [\int_0^{t_1} Q_1(t) dt + \int_0^{t_2} Q_2(t) dt]$

Total leakage cost per time unit (LC):

$$\frac{AC_L}{T} \left[\left(V_0 - \int_0^{t_1} v_0(1 + \beta_1 t) dt \right) + \left(V_m - \int_0^{t_2} v_m(1 - \beta_2 t) dt \right) \right]$$

Total cost = set up cost per unit time (S) + total production cost per time unit (PC) + total holding cost per time unit (HC) + total leakage cost per time unit (LC)

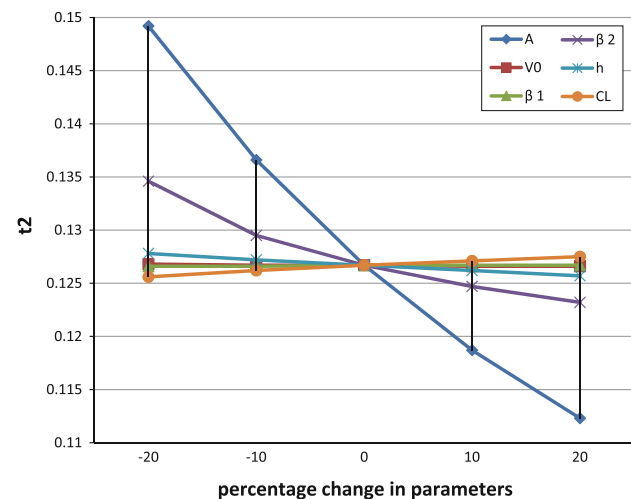


Fig. 1 Variation of time from peak to abandonment

Total cost function optimized as a function of t_2 .

Data testing and analysis

Standard values

$S = 10^7$ dollar, $V_0 = 3,000$ gallon, $\beta_1 = 20, \beta_2 = 10, A = 150 \text{ cm}^2, g = 2 \times 10^5$ dollar, $\eta = 1/5,000$ dollar, $\delta = 3, h = 2$ dollar/unit, $CL = 200$ dollar

$t_2 = 0.1267$ years, $TC = 6.632 \times 10^8$ dollar, $P = 2.5184$ gallons.

Changes in A

A	120	135	150	165	180
t_2	0.1492	0.1366	0.1267	0.1187	0.1123
TC	3.7884×10^8	5.1356×10^8	6.6323×10^8	8.2462×10^8	9.9097×10^8
P	15.9568	6.2948	2.5184	0.9875	0.3686

Changes in V_0

V_0	2,400	2,700	3,000	3,300	3,600
t_2	0.1268	0.1267	0.1267	0.1266	0.1266
TC	5.4326×10^8	6.0324×10^8	6.6323×10^8	7.2322×10^8	7.8321×10^8
P	2.0421	2.2802	2.5184	2.7566	2.9949

Changes in β_1

β_1	16	18	20	22	24
t_2	0.1266	0.1266	0.1267	0.1267	0.1267
TC	6.6415×10^8	6.6369×10^8	6.6323×10^8	6.6276×10^8	6.6230×10^8
P	2.3529	2.4355	2.5184	2.6017	2.6855

Changes in β_2

β_2	8	9	10	11	12
t_2	0.1346	0.1295	0.1267	0.1247	0.1232
TC	7.1771×10^8	7.0407×10^8	6.6323×10^8	6.1701×10^8	5.7127×10^8
P	0.2292	0.9379	2.5184	5.2410	9.3568

Changes in h

h	1.6	1.8	2.0	2.2	2.4
t_2	0.1278	0.1272	0.1267	0.1262	0.1257
TC	6.5007×10^8	6.5700×10^8	6.6323×10^8	6.6888×10^8	6.7407×10^8
P	2.8348	2.6640	2.5184	2.3924	2.2820

Changes in C_L

C_L	160	180	200	220	240
t_2	0.1256	0.1262	0.1267	0.1271	0.1275
TC	5.5400×10^8	6.0887×10^8	6.6323×10^8	7.1712×10^8	7.7508×10^8
P	2.2609	2.3931	2.5184	2.6377	2.7514

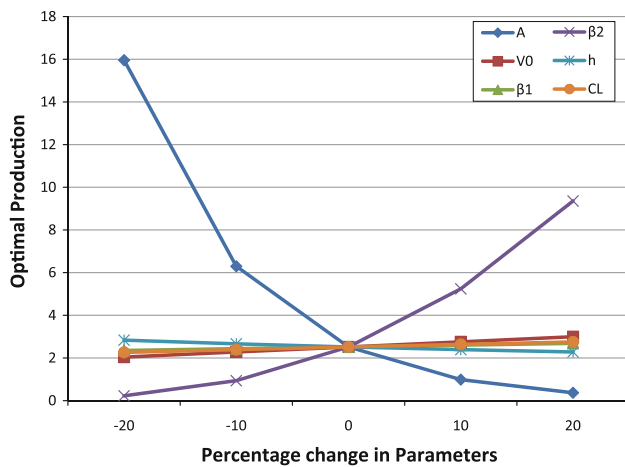


Fig. 2 Variations in optimal production

Observations

Optimal time period (t_2) after achieving peak

As in this paper we are discussing production of crude oil, it is always suggested that after achieving peak it is good to achieve abandonment as early as possible.

A and β_1 are showing significant change in time period (t_2) (Fig. 1).

Optimal production

It shows highest change by parameter area of leakage or spill (A). It also shows significant changes with respect to parameters (β_2) which represent rate of production during time t_2 after achieving peak or after finishing plateau period.

But, total production shows very less significance toward holding cost, and it is showing almost negligible significance toward leakage cost (C_L) and initial expected production (V_0) (Fig. 2).

Optimal total cost

Our main aim in this paper is to minimize total cost of production. It is very vital to understand parameters those are showing significance affect on total cost in light of sensitivity analysis.

Again for total production cost parameter which shows highest change is area of leakage or spill in pipeline (A). It also shows almost equivalent significance toward rate of production during time t_2 (β_2). It shows an equivalent significance toward leakage cost (C_L) and initial expected production (V_0).

But, total production cost shows very less significance toward holding cost (h) and rate of production during time t_2 (β_1) (Fig. 3).

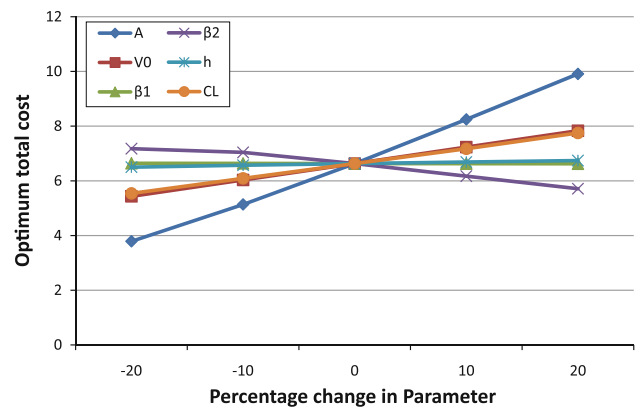


Fig. 3 Variations in optimum total cost

Conclusion

In this paper, a mathematical model is proposed to analyze crude oil extraction from gas wells. It is assumed that the pipeline used suffers from damages of various sizes thus resulting in leakage. Cost associated with the leakage is substantial and renders crude oil extraction cost. The total cost per unit time and sensitivity analysis is carried out to search for critical factors for optimum extraction. The empirical data suggest that leakage area and holding cost contribute maximum to the cost and render production.

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References

Arps JJ (1945) Analysis of decline curves, transactions of the American Institute of Mining. Metall Petro Eng 160:228–247
 CERA (2007) Finding the critical numbers: what are the real decline rates for global oil production? Private report written by Peter M. Jackson and Keith M. Eastwood
 Dake LP (2004) The practice of reservoir engineering, revised edition, developments in petroleum science. Elsevier, London, p 572
 Doublet LE, Pande PK, McCollom TJ, Blasingame TA (1994) Decline curve analysis using type curves—analysis of oil well production data using material balance time: application to field cases. In: Society of Petroleum Engineers paper presented at the International Petroleum Conference and Exhibition of Mexico, 10–13 October 1994, Veracruz, Mexico p 24 (SPE paper 28688-MS)
 Hirsch R (2008) Mitigation of maximum world oil production: shortage scenarios. Energy Policy 36(2):881–889

- Höök M (2009) Depletion and decline curve analysis in crude oil production doctoral thesis, Uppsala University
- Höök M, Söderbergh B, Jakobsson K, Aleklett K (2009) The evolution of giant oil field production behavior. *Nat Resour Res* 18(1):39–56
- Robelius F (2007) Giant oil fields—the highway to oil: giant oil fields and their importance for future oil production. Doctoral thesis, Uppsala University