Applied Water Science (2024) 14:149
https://doi.org/10.1007/s13201-024-02209-z

ORIGINAL ARTICLE

Explicit expression for water permeation flux in forward osmosis desalination process

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Received: 21 November 2022 / Accepted: 21 May 2024 / Published online: 6 June 2024
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Abstract

Forward osmosis (FO) is a membrane process of water separation and purification. FO uses the osmotic pressure difference across a semipermeable membrane. The effective osmotic pressure at the membrane–solution interface on both the feed and permeate sides of the membrane is the main driving force for the generation of the water flux. The major hindrance to the permeation of the water flux is the prevalence of concentration polarization on both sides of the membrane. Concentration polarization inhibits permeate flux by increasing the osmotic pressure at the membrane active layer interface on the feed side of the membrane. This work focused on the development of a mathematical model for water flux in the FO process. Combined film theory model and diffusion transport through the membrane were utilized. The effect of internal concentration polarization and external concentration polarization on the flux was studied. Both internal and external concentration polarization were taken into consideration in both membrane orientations, i.e., active layer facing the feed solution and active layer facing the draw solution. The obtained explicit expression for water permeation flux in forward osmosis desalination process shows excellent agreement with the literature data.

Keywords Forward osmosis · Permeation flux · Water flux · Modeling · FO desalination

Introduction

Forward osmosis (FO) process is a membrane process that is a relatively considered a new membrane technology as compared to other membrane processes. The growth of interest toward forward osmosis (FO) processes as an alternative water desalination method is due to continuous improvements in the FO process, especially in membrane development and the progress in the selection of suitable draw solution.

Forward osmosis (FO) offers sustainable, innovative, and affordable alternative to conventional desalination processes, because of its minimal energy requirements, low operating cost, and low membrane fouling. FO is an osmotic process where water is transported across a semipermeable membrane from the low concentration (feed solution, FS) by the action of osmotic pressure to the high concentration (draw solution, DS). The ideal FO membrane should have a small thickness support layer with high porosity to control internal concentration polarization and an active membrane layer with high water flux and minimal reverse salt flux (ICP).

Forward osmosis process modeling allows the prediction of water permeation flux without performing laboratory tests. Flux modeling studies were performed first by S. Loeb and K. L. Lee and their co-workers (Mehta and Loeb 1978; Loeb 1979; Lee et al. 1981). Their developed models, concepts and approximate estimations created the framework for the current internal concentration polarization (ICP) estimation.

Several studies on modeling forward osmosis process recently were conducted aiming at the evaluation of water permeation flux. Tang et al. (2011) developed an analytical model for double-skinned forward osmosis membranes. Their test results demonstrated that the model could predict FO performances (water flux and solute reverse diffusion) for both single-and double-skinned membranes. In their design, the separation is caused by a dense rejection skin

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that is exposed to the draw solution while a second skin facing the feed solution is intended to foulants away from the porous support layer which is placed between the two skins. Both designs exhibited a decrease in the draw solution concentration and an increase in the feed solution concentration resulting in reduction in FO water flux.

Jeon et al. (2018) developed a simple model for a commercial spiral wound forward osmosis (FO) module. The model incorporates essential FO process mechanisms like internal and external concentration polarization (ICP and ECP), and the fitting process for the reflection of the module characteristics. Both the real flow patterns of feed solution (FS) and draw solution (DS) in FO module were found not evenly distributed. If a uniform distribution is assumed, contradictory results to the theory may be obtained, such as the increases in water flux at higher FS flow rates and the decrease the water flux with an increase in DS flow rates.

Xiao et al. (2012) developed a mathematical model to evaluate the local performances within the FO hollow fiber module as well as the overall performances. They used a flow model that took into account for both ICP and ECP and employing module characteristic and membrane properties. They mentioned that severe flux decline could result from the interaction between concentration polarization and fouling.

Tiraferri et al. (2013) proposed accurate predictions of the water and salt fluxes based on a quick and simple method to determine the permeability coefficients for both water and solutes and the structural parameter of forward osmosis membranes. They mentioned that the intrinsic parameters; the pure water permeability coefficient and the solute permeability coefficient describe the transport across the membrane active layer, and the structural parameter quantifies the mass transport length scale across the membrane support layer. They used these three parameters with the respective governing equations to anticipate the water and salt flux performance of a membrane sample precisely in any laboratory-scale forward osmosis system.

Gruber et al. (2016) presented a computational fluid dynamics (CFD) model for an FO module using an open source CFD code. They emphasized that one of the main issues with regard to separation efficiency is concentration polarization at membrane surfaces since this phenomenon significantly reduces the forces that drive the separation process. Hence, it reduced the water flux significantly. They found that as the number of spacers increases, an increase in the average water flux is expected. Going from no spacers to a channel that is heavily filled (≈18 spacers in each channel), 14% in the average water flux is observed.

D'Haese et al. (2017) used the solution-diffusion model to prescribe water and draw solute transport utilizing an FO-only approach. In order to account for the concentration dependency of draw solute diffusivity during transport over the membrane active layer, they used expressions for the solute permeability coefficient. The structural parameter and membrane permeability coefficients were evaluated by fitting the model for each draw solute and each membrane orientation, enabling a comprehensive comparison of the properties of membrane and draw solute.

Francis et al. (2020) found that water flux was much lower than theoretical or predicted values due to several reasons such as internal and external concentration polarization effects (ICP and ECP). Forward osmosis process performance in their work was characterized by the intrinsic parameters of the membrane which are essentially dependent on the membrane structure and solute type. They estimated that the impact of ICP on the FO water flux can be extremely severe, in some cases even resulting in a drop in water flux of over 80%. Although internal concentration polarization (ICP) plays a major role in flux decline in the forward osmosis process, external concentration polarization (ECP) effects cannot be disregarded when using dealing with high salinity solutions, or when operating the forward osmosis at a high water flux (Cath et al. 2006). Therefore, it is important to take the impacts of both internal and external concentration polarization into account while designing and operating the forward osmosis process as well as when evaluating the water flux.

Concentration polarization can be either dilutive when the membrane is operated in AL-FS mode (active layer facing the feed solution), or concentrative when the membrane is operated in AL-DS mode (active layer facing the draw solution). Optimizing the membrane thickness, tortuosity, porosity of the support layer and solute diffusion coefficient by membrane fabrication and modification will reduce the effect of dilutive and concentrative ICP since ICP usually occurs within the support layer (Yadav et al. 2020). Since ECP occurs on the active layer surface, enhancing flow turbulence or velocity will increase the effective concentration on the membrane surface and will reduce the dilute and concentrative ECP (Lee et al. 2020).

Most of the forward osmosis models comprise reverse solute flux (RSF), internal concentration polarization (ICP) or external concentration polarization (ECP) on either active layer faces the feed solution (AL-FS) membrane orientation or active layer faces the draw solution (AL-DS) membrane orientation. In this work, all RSF, ICP, and ECP were considered in both membrane orientations.

This work is in line with the current research for mastering better understanding the forward osmosis process. Formulating the water flux in forward osmosis is among the important issues in the most membrane-based process. Explicit expressions for water permeation flux in forward osmosis desalination process were formulated in this work. Concentrative and dilutive internal and external concentration polarization was considered in this investigation. The mathematical model was derived using the modified film theory and diffusion transport coupled with balance equations and solved simultaneously. The obtained equations for estimating the water flux at different membrane modes were verified against published data.

Modeling of water flux in forward osmosis

Water flux through the membrane can be expressed as a proportionality of the difference in osmotic and hydraulic pressure as is in the following equation:

$$J_w = A(\Delta P - \sigma \Delta \pi) \tag{1}$$

where A is the pure water permeability, ΔP is hydraulic pressure, σ is the reflection coefficient, and $\Delta \pi$ is the osmotic pressure between two solutions.

In forward osmosis (FO), no pressure is applied, $\Delta P = 0$. Osmotic reflection coefficient σ is the relative measure of tightness or leaking of the membrane to the solute. For an ideal semipermeable, no solute passage, $\sigma = 1$.

Therefore, the water flux can be given as:

$$J_w = A(\Delta \pi) \tag{2}$$

Which can be re-written as follows

$$J_w = A\left(\pi_{\mathrm{Db}} - \pi_{\mathrm{Fb}}\right) \tag{3}$$

where π_{Db} is the osmotic pressure of the draw solution and π_{Fb} is the osmotic pressure of the feed solution.

There are two modes of membrane orientation based on the position of the active layer (AL) as shown in Fig. 1. In the first mode, the active layer faces the feed solution and abbreviated as AL-FS. The active layer faces the draw solution (AL-DS) in the second mode. In either mode, there are two types of concentration polarization (CP), namely, external concentration polarization (ECP) on both side of membrane and internal concentration polarization (ICP) in the support layer. ICP takes place within the porous support layer of the membrane. ICP could decrease the water flux since it is considered one of the most significant phenomena in osmotically driven membrane processes.

Forward osmosis water flux model AL-FS mode

Two draw solute fluxes (convictive flux and diffusive flux) transport in opposing directions in the support layer. At steady state, the salt flux J_{s}, so called reverse solute flux (RSF), can be given as:

$$J_{\mathrm{s}} = -J_w\, C + D_{\mathrm{s}} \frac{dc}{dx} \tag{4}$$

where the first term $\left(J_w\, C\right)$ is the convictive flux and the second term $\left(D_{\mathrm{s}} \frac{dc}{dx}\right)$ is the diffusive flux, C is the solute concentration and D_{s} is the solute diffusion coefficient in the support layer which can be expressed as follows

$$D_{\mathrm{s}} = \frac{D\varepsilon}{\tau} \tag{5}$$

where D is the bulk diffusion coefficient, ε is the porosity of the support layer and τ is the tortuosity of the support layer.

Equation (4) can be re-written as

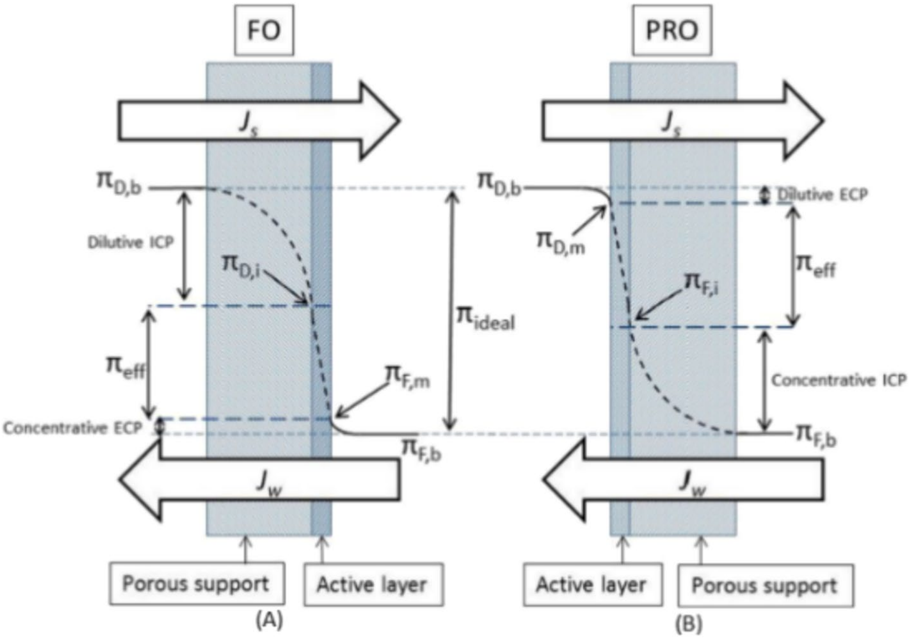

Fig. 1 Modes of membrane orientation. **a** AL-FS mode. **b** AL-DS mode (Johnson et al. 2018)

$$J_{\mathrm{s}} + J_w\ C = D_{\mathrm{s}}\frac{dc}{dx} \tag{6}$$

Divided both side by $\left(J_{\mathrm{s}} + J_w\ C\right)$.

And let $\frac{J_{\mathrm{s}}}{J_w}$ equals to C_p.

Then,

$$\frac{D_{\mathrm{s}}}{J_w\left(C + C_p\right)}\frac{dc}{dx} = 1 \tag{7}$$

According to Fig. 1, Eq. (7) can be solved using the following boundary condition.

$C = C_{Di}$ at $x = 0$, at the support layer surface.

$C = C_{Dm}$ at $x = t$, at the end of the support layer surface.

where C_{Di} is the concentration of solution inside the porous support near the active layer and C_{Dm} is the concentration of solution on the support layer adjacent to the bulk solution.

Appling the boundary conditions, then upon integration

$$\int\limits_{C_{Di}}^{C_{Dm}} \frac{1}{C + C_p}dc = \int\limits_{0}^{t} \frac{J_w}{D_{\mathrm{s}}}dx \tag{8}$$

$$\ln\left[C_{Dm} + C_p\right] - \ln\left[C_{Di} + C_p\right] = \frac{J_w}{D_{\mathrm{s}}}t \tag{9}$$

$$\ln\left[\frac{C_{Dm} + C_p}{C_{Di} + C_p}\right] = \frac{J_w}{D_{\mathrm{s}}}t \tag{10}$$

Since,

$$K = \frac{t}{D_{\mathrm{s}}} \tag{11}$$

K is the solute resistance inside the porous support layer and t is the thickness of the support layer.

$$\frac{C_{Dm} + C_p}{C_{Di} + C_p} = e^{\left(J_w K\right)} \tag{12}$$

For high solute rejecting membrane, J_{s} almost equals zero,.i.e., $J_{\mathrm{s}} = 0$.

where $C_p = \frac{J_{\mathrm{s}}}{J_w} = 0$

$$C_{Di} = \frac{C_{Dm}}{e^{\left(J_w\ K\right)}} \tag{13}$$

The salt concentration ratio is approximately proportional to the osmotic pressure ratio, since is given by $\pi = RTC$ as in van`t Hoff equation, where R is the universal gas constant, T is the absolute temperature and C is the concentration.

$$\pi_{Di} = \frac{\pi_{Dm}}{e^{\left(J_w\ K\right)}} \tag{14}$$

In this case, AL-FS mode for the ICP in the support layer, water flux can be given as

$$J_w = A\left(\pi_{Di} - \pi_{\mathrm{Fm}}\right) \tag{15}$$

Substitute Eq. (14) into Eq. (15), the actual flux through the membrane will be

$$J_w = A\left(\pi_{Dm}e^{\left(-J_w\ K\right)} - \pi_{Fm}\right) \tag{16}$$

External concentration polarization (ECP)

To estimate ECP using the same differential equation Eq. (6), the following appropriate boundary conditions between the membrane surface and bulk solution on the feed side will be applied:

$C = C_{\mathrm{Fb}}$ at $x = 0$, at the bulk feed solution.

$C = C_{Fm}$ at $x = \delta_F$, at the active layer surface.

where the concentration of solute on the membrane surface is C_{Fm}, the concentration of solute in the bulk feed solution is C_{Fb} and δ_F is the thickness of concentration boundary layer on the active layer of the membrane.

Appling the boundary condition, then upon integration

$$\int\limits_{C_{\mathrm{Fb}}}^{C_{Fm}} \frac{1}{C + C_p}dc = \int\limits_{0}^{\delta_F} \frac{J_w}{D}dx \tag{17}$$

$$\ln\left[C_{Fm} + C_p\right] - \ln\left[C_{\mathrm{Fb}} + C_p\right] = \frac{J_w}{D}\delta_F \tag{18}$$

$$\ln\left[\frac{C_{Fm} + C_p}{C_{\mathrm{Fb}} + C_p}\right] = \frac{J_w}{D}\delta_F \tag{19}$$

The mass transfer coefficient on the feed side of the membrane, k_F, may be expressed as

$$k_F = \frac{D}{\delta_F} \tag{20}$$

$$\frac{C_{Fm} + C_p}{C_{\mathrm{Fb}} + C_p} = e^{\left(\frac{J_w}{k_F}\right)} \tag{21}$$

where is $C_p = \frac{J_{\mathrm{s}}}{J_w}$

$$C_{Fm} = \left(C_{\mathrm{Fb}+}\frac{J_{\mathrm{s}}}{J_w}\right)e^{\left(\frac{J_w}{k_F}\right)} - \frac{J_{\mathrm{s}}}{J_w} \tag{22}$$

For high solute rejecting membrane, J_{s} almost equals zero,.i.e., $J_{\mathrm{s}} = 0$

$$C_{Fm} = C_{\mathrm{Fb}}e^{\left(\frac{J_w}{k_F}\right)} \tag{23}$$

Then,

$$\pi_{Fm} = \pi_{\mathrm{Fb}}e^{\left(\frac{J_w}{k_F}\right)} \tag{24}$$

By substituting Eq. (24) into Eq. (16)

$$J_w = A\left(\pi_{Dm}e^{(-J_wK)} - \pi_{\mathrm{Fb}}e^{\left(\frac{J_w}{k_F}\right)}\right)$$

As shown on Fig. 1:

$$J_w = A\left(\pi_{\mathrm{Db}}e^{(-J_wK)} - \pi_{\mathrm{Fb}}e^{\left(\frac{J_w}{k_F}\right)}\right) \tag{25}$$

This is an explicit equation for estimating water flux under the effect of ICP in the support layer and ECP on the feed side.

The dilutive ECP on the draw side can be found by using the following boundary condition:

$C = C_{Dm}$ at $x = 0$, at the membrane surface of the support layer side.

$C = C_{\mathrm{Db}}$ at $x = \delta_D$, at the end of bulk draw solution.

where the concentration of solution on the support layer is C_{Dm}, the concentration of the bulk draw solution is C_{Db} and δ_D is the thickness of concentration boundary layer on the porous support.

Appling the boundary condition, then upon integration

$$\int_{C_{Dm}}^{C_{\mathrm{Db}}} \frac{1}{C + C_p}dc = \int_0^{\delta_D} \frac{J_w}{D}dx \tag{26}$$

$$\ln[C_{\mathrm{Db}} + C_p] - \ln[C_{Dm} + C_p] = \frac{J_w}{D}\delta_D \tag{27}$$

$$\ln\left[\frac{C_{\mathrm{Db}} + C_p}{C_{Dm} + C_p}\right] = \frac{J_w}{D}\delta_D \tag{28}$$

When,

$$k_{\mathrm{D}} = \frac{D}{\delta_D} \tag{29}$$

where k_{D} mass transfer coefficient on the draw side of the membrane δ_D is the thickness of concentration boundary layer on the porous support.

$$\frac{C_{\mathrm{Db}} + C_p}{C_{Dm} + C_p} = e^{\left(\frac{J_w}{k_D}\right)} \tag{30}$$

where $C_p = \frac{J_s}{J_w}$

$$C_{Dm} = \left(C_{\mathrm{Db}+}\frac{J_s}{J_w}\right)e^{\left(-\frac{J_w}{k_D}\right)} - \frac{J_s}{J_w} \tag{31}$$

For high solute rejecting membrane, J_s almost equals zero,.i.e., $J_s = 0$

$$C_{Dm} = C_{\mathrm{Db}}\, e^{\left(-\frac{J_w}{k_D}\right)} \tag{32}$$

Then,

$$\pi_{Dm} = \pi_{\mathrm{Db}}e^{\left(-\frac{J_w}{k_D}\right)} \tag{33}$$

The final equation for water flux with ICP in the support layer and ECP on both sides can be found by substitute Eq. (33) into Eq. (25).

$$J_w = A\left(\pi_{\mathrm{Db}}e^{(-J_w\,K)}e^{\left(-\frac{J_w}{k_D}\right)} - \pi_{\mathrm{Fb}}\,e^{\left(\frac{J_w}{k_F}\right)}\right) \tag{34}$$

This equation is only used for a dense asymmetric membrane, where external concentration polarization occurs in both side of the membrane. Such membrane is not used commercially.

Forward osmosis water flux model AL-DS mode

A similar procedure can be applied when the process is operated in the AL-DS mode. The equation for the external concentration polarization ECP can be written as

$$\frac{\pi_{Dm}}{\pi_{\mathrm{Db}}} = e^{\left(-\frac{J_w}{k_D}\right)} \tag{35}$$

and the equation for the internal concentration polarization ICP can be written as

$$\frac{\pi_{Fi}}{\pi_{\mathrm{Fb}}} = e^{(J_w\,K)} \tag{36}$$

Then, the final water flux equation for AL-DS mode is

$$J_w = A\left(\pi_{\mathrm{Db}}e^{\left(-\frac{J_w}{k_D}\right)} - \pi_{\mathrm{Fb}}e^{(J_wK)}\right) \tag{37}$$

This is an explicit equation for estimating water flux under the effect of ICP in the support layer and ECP on the draw side.

Results and discussions

In this work, an explicit equation for estimating water flux for AL-FS mode under the effect of ICP in the support layer and ECP on the feed side was found as follows

$$J_w = A\left(\pi_{\mathrm{Db}} e^{(-J_w\, K)} - \pi_{\mathrm{Fb}} e^{\left(\frac{J_w}{k_F}\right)}\right) \tag{38}$$

Also, an explicit equation for estimating water flux for AL-DS mode under the effect of ICP in the support layer and ECP on the draw side was found as follows

$$J_w = A\left(\pi_{\mathrm{Db}} e^{\left(-\frac{J_w}{k_D}\right)} - \pi_{\mathrm{Fb}} e^{(J_w K)}\right) \tag{39}$$

A comparison of well-known forward osmosis models is presented on Table 1. According to Haupt et al. all models I, II, and III include internal concentration polarization but differ in the extent of external concentration polarization considered (Haupt et al. 2019). Model I and Model II only consider external concentration polarization on the active layer side of the membrane. Model III combines internal concentration polarization and external concentration polarization on the active as well as the support layer of the membrane.

The obtained explicit equations for estimating water flux in this work include reverse solute flux (RSF), internal concentration polarization (ICP) or external concentration polarization (ECP) on either active layer faces the feed solution (AL-FS) membrane orientation or active layer faces the draw solution (AL-DS) membrane orientation were considered.

These obtained explicit equations for estimating water flux were verified against published data of Gordon T. Grey and et al. (Gray et al. 2006) and J. R. McCutcheon and M. Elimelech (McCutcheon and Elimelech 2006) work.

Gordon T. Grey and et al. (Gray et al. 2006) found values of the water flux experimentally by using NaCl with constant concentration of 0.5 M NaCl as draw solution and NaCl with different concentration as feed solution, ranging between from 0.0625 to 0.375 M NaCl. The parameters used in this published work $\left(\pi_{\mathrm{Fb}},\ \pi_{\mathrm{Db}},\ K\right)$ will be utilized in Eq. (25) to find the predicted water flux.

Also, the estimation of the predicted water flux by the obtained Eq. (25) will be compared with another experimental published values. J. R. McCutcheon and M. Elimelech (McCutcheon and Elimelech 2006) found the water flux experimentally by using NaCl in both draw solution of 1.50 M and feed solution with different concentrations (0.0, 0.05, 0.10, 0.50, 1.00 M) for water permeability coefficient.

Another experimental published values of water flux will be used to compare with the predicted water flux values obtained by Eq. (25)C. H. Tan and H. Y. Ng (Tan and Ng 2008) found the water flux experimentally and mathematically by using NaCl as draw solution with different concentration and DI water as feed solution.

The predicted values of water flux matched very well with the experimental values of the published work as shown on Table 2. So, Eq. (25) can be used with confidence.

Table 1 Model equations for water flux in forward osmosis according to the membrane orientation

M	ALFS	ALDS	References
I	$J_w = \frac{D_{Ds}}{S} \ln\left[\frac{B+A\pi_{DS}}{B+A\left[\pi_{Fs}\exp\left(\frac{J_w}{k_{fs}}\right)\right]+J_w}\right]$	$J_w = \frac{D_{Fs}}{S} \ln\left[\frac{B+A\pi_{DS}\exp\left(-\frac{J_w}{k}\right)-J_w}{B+A\pi_{Fs}}\right]$	McCutcheon and Elimelech (2006)
II	$J_w = A\frac{\pi_{Ds}\exp\left(-\frac{J_wS}{D_{DS}}\right)-\pi_{FS}\exp\left(\frac{J_w}{k_{FS}}\right)}{1+\frac{B}{J_w}\left[\exp\left(\frac{J_w}{k_{FS}}\right)-\exp\left(\frac{J_wS}{D_{DS}}\right)\right]}$	$J_w = A\frac{\pi_{Ds}\exp\left(-\frac{J_w}{D_{DS}}\right)-\pi_{FS}\exp\left(\frac{J_wS}{D_{FS}}\right)}{1+\frac{B}{J_w}\left[\exp\left(\frac{J_wS}{D}\right)-\exp\left(\frac{J_w}{k_{DS}}\right)\right]}$	Tiraferri et al. (2013); Yip et al. (2011)
III	$J_w = A\frac{\pi_{Ds}\exp\left[-J_w\left(\frac{1}{k_{DS}}+\frac{S}{kD_{DS}}\right)\right]-\pi_{FS}\exp\left(\frac{J_w}{k_{FS}}\right)}{1+\frac{B}{J_w}\left[\exp\left(\frac{J_w}{k_{FS}}\right)-\exp\left[-J_w\left(\frac{1}{k_{DS}}+\frac{S}{kD_{DS}}\right)\right]\right]}$	$J_w = A\frac{\pi_{Ds}\exp\left(-\frac{J_w}{k_{DS}}\right)-\pi_{FS}\left[J_w\left(\frac{1}{k_{FS}}+\frac{S}{D_{FS}}\right)\right]}{1+\frac{B}{J_w}\left[\exp\left[J_w\left(\frac{1}{k_{FS}}+\frac{S}{D_{FS}}\right)\right]-\exp\left(-\frac{J_w}{k_{DS}}\right)\right]}$	Bui et al. (2015)

Table 2 Comparison between published (experimental) and predicted model values of water flux using AL-FS mode

Parameters	Gray et al. (2006)	McCutcheon and Elimelech (2006)
Bulk feed solution osmotic pressure, π_{Fb} (atm)	5.58	3.06
Membrane surface osmotic pressure on the draw side, π_{Dm} (atm)	22.43	70.04
Solute resistance inside the porous support, K (d/m)	3.96	3.240
Experimental Flux J_w m/d (gfd)	0.165 (4.05)	0.406 (9.97)
Predicted Flux J_w m/d (gfd), this work	0.161 (3.94)	0.40 (9.89)
Error (%)	2.45	0.80

Table 3 Comparison between published (experimental) and predicted model values of water flux using AL-DS mode

Parameters	Gray et al. (2006)	McCutcheon and Elimelech (2006)
Bulk feed solution osmotic pressure, π_{Fb} (atm)	2.88	23.35
Membrane surface osmotic pressure on the draw side, π_{Dm} (atm)	22.43	58.65
Solute resistance inside the porous support, K (d/m)	4.52	2.731
Experimental Flux J_w m/d (gfd)	0.302 (7.41)	0.269 (6.60)
Predicted Flux J_w m/d (gfd), this work	0.301 (7.73)	0.268 (6.57)
Error (%)	0.33	0.37

In the same manner, the derived Eq. (37) for the water flux with AL-DS mode including ECP and ICP was compared with Gordon T. Grey and et al. (Gray et al. 2006) work.

Also, the estimation of the predicted water flux by the obtained Eq. (37) will be compared with another experimental published values. J. R. McCutcheon and M. Elimelech (McCutcheon and Elimelech 2006) found the water flux experimentally by using NaCl in both draw solution of 1.50 M and feed solution with different concentrations (0.0, 0.05, 0.10, 0.50, 1.00 M).

The predicted values of water flux matched very well with the experimental values of the published work as shown on Table 3. So, this Eq. (37) can be used with confidence.

Conclusion

Explicit mathematical models for the prediction of water flux in forward osmosis were developed. Internal and external concentration polarization was taken into consideration in both cases, i.e., active layer facing the feed solution (AL-FS) and active layer facing the draw solution (AL-DS). The model was developed analytically to predict the water flux in forward osmosis (FO) desalination process.

The forward osmosis flux model was verified against experimental published data from the literature. The model was in excellent agreement with the literature data. In future work, the obtained effect of different variables on the water flux in forward osmosis process will be investigated to illustrate their influence of the water flux.

Author contributions All authors had equal contribution in writing, review, and final approval of the paper.

Funding Funding information is not applicable. No funding was received. “The authors received no specific funding for this work.”

Declarations

Conflict of interest The authors declare that there is no conflict of interest.

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