



# Assessing vulnerability to soil erosion based on fuzzy best worse multi-criteria decision-making method

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## Abstract

Soil wearing away or erosion is a chief agent of land loss in agricultural land and is regarded worldwide as a serious environmental hazard. This study performed watershed prioritization using morphometric parameters based on fuzzy best worse method (F-BWM) and GIS integration for Gusru Watershed, India. This study prioritizes sub-watersheds of the study area from viewpoint of soil erosion using five major parameters i.e., stream frequency ( $F_s$ ), relative relief ( $R_r$ ), length of overland flow ( $L_o$ ), relief ratio ( $R_h$ ) and drainage density ( $D_d$ ). Fuzzy based Best Worse Multi-Criteria Decision-Making (F-BWM) Method was used to assigning weights to used criteria and combining them to achieve erosion susceptibility for each sub-watershed. Results showed that sub-watersheds 9, 14, and 5 were most susceptible to soil erosion and sub-watershed 3 was the least from the viewpoint of soil erosion ranking.

**Keywords** Soil erosion · Prioritization · Best worse method · Fuzzy logic · Multi-criteria decision-making method

## List of symbols

F-BWM	Fuzzy best worse method	$R_c$	Circulatory ratio
GIS	Geographical information system	$D_d$	Drainage density
AHP	Analytic hierarchy process	$l, m, u$	Lower, median and upper numbers of $\tilde{A}$
MCDM	Multi criteria decision making	$\tilde{A}$	Relative importance of criterion
PCA	Principal component analysis	$c_B$	Best criterion
DEM	Digital elevation model	$c_W$	Worst criterion
SRTM	Shutter radar topography mission	$\tilde{w}_1^*, \tilde{w}_2^*, \dots, \tilde{w}_n^*$	Optimal fuzzy weight
TFN	Triangular fuzzy number	$\xi$	Consistency ratio
$F_s$	Stream frequency	$c_1, c_1, \dots, c_j, \dots, c_n$	Criteria
$R_r$	Relative relief	$C_c$	Compactness coefficient
$L_o$	Length of overland flow	$R_e$	Elongation ratio
$R_h$	Relief ratio	$R_f$	Farm factor

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## Introduction

Soil erosion is an environmental, economic and social problem. Watersheds are taken as a unit to estimate the erosion problem. It is therefore important to monitor losses due to erosion in a watershed which is a planning unit for the sustainable development of natural resources (Meshram et al. 2017). Among many factors, water is the most important causative factor in soil erosion.

Soil attrition or erosion, excess water flow or runoff, changes in river geometry, degradation of streams, sediment accumulation in river and stream characters are related with morphometry (Meshram et al. 2018). This suggests that the morphometry of a basin is fundamental to the basin hydrology. In present time geo-morphometric analysis using a new technique i.e., Remote Sensing (RS) and Geographic Information Systems (GIS) being is utilized as this tool gives flexibility to analyze spatial data in a new manner (Gajbhiye et al. 2014; Meshram and Sharma 2017).

In this context, various approaches are available to analyze and prioritize sub-watersheds. These include Multi-Criteria Decision Analysis (MCDA) (Akay and Koçyiğit 2020; Chitsaz and Banihabib 2015; Dahmardeh Ghaleno et al. 2020; Sepehri et al. 2019), Soil and Water Assessment Tool (SWAT) (Mishra et al. 2007), Artificial Neural Network (ANN) (Dehghanian et al. 2020), Storm Water Management Model (SWMM) (Babaei et al. 2018), Support Vector Machine (SVM) (Tehrany et al. 2014; Fan et al. 2018) and The Hydrologic Modeling System (HEC-HMS) (Malekinezhad et al. 2017). Among the aforementioned methods, MCDA account takes priority due to its capability to handle nonlinear and complex problems and its usability to prioritize un-gaged watershed.

MCDA are the most usable methods which can be used to manage large amounts of data and solving decision-making under scale, quantitative, qualitative and conflict factors (Fernández and Lutz 2010; Mahmoud and Gan 2018). The Analytic Hierarchy Process (AHP) which was developed by Saaty (1980), due to some reasons such as cost-effectiveness, ease to use and understand has become one of the most popular methods among MCDA (Zou et al. 2013), which has been successful in various natural hazard studies such as landslides (Kayastha et al. 2013; Myronidis et al. 2016; Bahrami et al. 2020), flood magnitude (Sepehri et al. 2017; Swain et al. 2020; Lin et al. 2020), groundwater vulnerability (Sener and Davraz 2013; Abdullah et al. 2018; Das and Pal 2020).

In this regard, several methods were developed to reduce the number of pair-wise comparisons. In recent years, a new method was introduced by Rezaei (2015). This method is a more optimal version of AHP with the

need of less compared data, resulting in more consistent results. However, the weak point of the BWM is related to the type of import data. This method just as the AHP, uses a limited 9-point table. In here, experts face a dilemma of choosing a point of initial weighting to factors causing inconsistency in the results. Therefore, it is better to use the fuzzy number instead of the limited 9-point table because it is more in line with actual situations and can obtain more convincing ranking results (Guo and Zhao 2017; Ali and Rashid 2019).

Rezaei (2015) introduced BWM as one of the most recent MCDM approaches. The premise of this strategy is to weight criteria using paired comparisons, such as AHP, with two obvious benefits: fewer pair wise comparisons and a greater consistency ratio. Traditional BWM compares clean values but fails to identify weights in an ambiguous context. As a result, fuzzy BWM was created (Guo and Zhao 2017; Hafezalkotob and Hafezalkotob 2017). Zhang et al. (2015) reported an enhanced fuzzy MCDM methodology for evaluating renewable electricity sources In Jiangsu Province, China. Photovoltaic energy was the top option in their study, followed by wind, biomass, and nuclear power facilities. Because fuzzy BWM inherits some distinguishing characteristics from BWM, it can produce weights of criteria using fuzzy numbers rather than crisp values. As a result, the uniqueness of weight data can be carefully preserved (Guo and Zhao 2017). Shojaei et al. (2017) evaluated Iranian airports using an integrated Taguchi loss function, VIKOR, and BWM method. Ahmed et al. (2017) used BWM to identify the most critical elements affecting gas supply sustainability.

The Gusru watershed in view of soil erosion and its related financial and ecological losses can be regarded as one of the most critical areas in central of India. However, no comprehensive and efficient works have been done to reduce the soil erosion. Thus the main objective of this study is to assess soil erosion based on the fuzzy best worst multi-criteria decision-making method of efficient prioritization of sub-watersheds. The outcomes of this study will be important for water resources management.

## Materials and methods

### Case study

Gusru River watershed is situated in the Madhya Pradesh state lying Satna Panna districts, in India, and it lies between  $80^{\circ} 32' 50.23''$  E and  $80^{\circ} 37' 31.14''$  E longitude,  $24^{\circ} 6' 32.75''$  and  $24^{\circ} 16' 24.07''$  N latitude (Fig. 1). It occupies an area of  $155 \text{ km}^2$  having an elevation range between 339 and 628 m above mean sea level. The Gusru River runs from east to west and confluences with Tons river at Sagwania village.

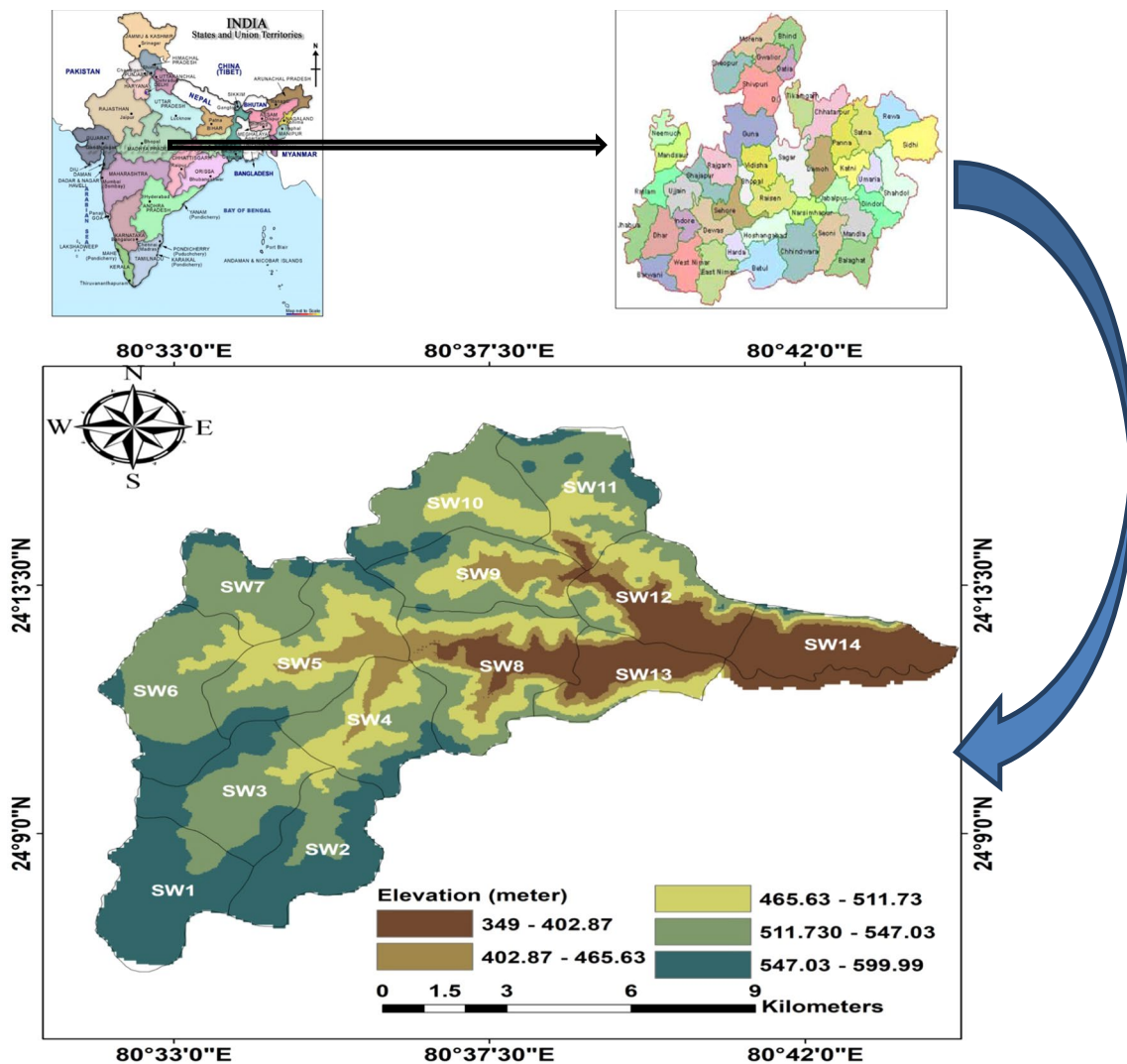


Fig. 1 Location map of the study area

In the eastern part of the watershed, there is a small check dam, which primarily serves as an irrigation outlet. There is no other source of water for irrigation; as a result, rainfed agriculture is primarily practiced. The soil structure in the watershed is primarily sandy loam. The soils under rainfed and irrigated conditions respond to a variety of crops and watershed management. Shale, sandstone and calcareous rocks are the dominant lithological units in the watershed. The study area descends from the plateau of Bhandar and passes through the area between the escarpment of Bhandar and the highlands of Kaimore.

**Methodology**

The used procedure in this study can be summarized in the following stages:

1. Establishing morphometric parameters
2. Applying the Principal Component Analysis (PCA) for redundancy of parameter
3. Applying ensembles of the Fuzzy method and BWM to assigning weights to used indices based on importance of them on soil erosion.

**Morphometric parameters**

Stream network is a basic requirement of any morphometric study and the prioritization of watersheds (Meshram et al. 2022a). Digital Elevation Model (DEM) generated by Shuttle Radar Topography Mission (SRTM) data is a common tool to define a stream network and sub-watershed map (Meshram et al. 2022b). Different drainage network parameters i.e., numbers and lengths and watershed area, perimeter, width and length were determined in GIS environment

(Benzougagh et al. 2022; Meshram et al. 2022c). Then using standard formulae stream frequency, drainage density, circulatory ratio, form factor and elongation ratio were estimated. In order to do fuzzy-BWM analysis, we have adopted the morphometric parameters for the 14 sub-watershed of Gusru watershed from the previous studies of Sharma et al. (2011).

**Principal component analysis**

Most of the time there is relationship between the morphometric parameters such that some of the parameters share the same information. In performing component analysis, the co-ordinates axis is transformed to a new reference frame within the total variable space. This involves assigning new principal components to each variable either through an uncorrelated or an orthogonal transformation. These components are unique in that they consider the maximum variance between the variables (Gajbhiye et al. 2015a, b). The correlation matrix and principal components are thus obtained from the principal component analysis performed on the geomorphic variables. The analysis employs the first factor and rotated the factor loading matrices. The product of the square of a parameter’s loading and the percent of the rotated factor covariance give the order of importance of a parameter. Thus computation is derived from the most commonly used transformation technique involving rotated factor loading matrices based on the varimax criteria (Singh 2006; Ghoderao et al. 2022).

**The proposed F-BWM model**

**Fuzzy sets and triangular fuzzy numbers** The subjective MCDA is sensitive to experts’ judgments, causing difficulty evaluating the weights when the experts uses natural language such as “very better,” “somewhat worse,” or “so much better” to express a kind of general preferences (Hafezalkotob and Hafezalkotob 2017). In mathematics, these natural languages are categorized as crisp sets. The

concept of crisp sets only implied on full membership and non-membership, whereas in fuzzy set each elements can be partially membership (Sepehri et al. 2019; Chen et al. 2020). For the first time, the concept of fuzzy system was introduced and characterized using membership functions by Zadeh (1965) which grading membership between 0 and 1. In decision-making problems, the triangular fuzzy number (TFN) is one of the most used membership functions, which can be donated to triplet (l, m, u), where l < m < u (Dong et al. 2021; Guo and Zhao 2017; Omrani et al. 2018). The triangular fuzzy number is as follow:

$$\mu_{\tilde{A}} = \begin{cases} 0, & x < l \\ \frac{x-l}{m-l}, & l \leq x \leq m \\ \frac{m-x}{u-m}, & m \leq x \leq u \\ 0, & x \geq u \end{cases} \tag{1}$$

where l, m, u are the lower, median and upper numbers of  $\tilde{A}$  (for the basic mathematical calculations of two TFNs, can be referred to (Carlsson and Fullér 2001).

**Fuzzy best–worst method (F-BWM)** Best–worst method (BWM) proposed by Rezaei (2015) is a new subjectively MCDA which can be used to derive optimal weights of criteria set {c<sub>1</sub>, c<sub>1</sub>, ..., c<sub>j</sub>, ..., c<sub>n</sub>}. In this content, it is necessity to determine the best (e.g., the most favorable) and the worst (e.g., the least favorable) of criteria by experts. Afterward, these criteria are compared relative to each other based on natural language (Mohtashami 2021). In F-BWM, it is necessity to transfer the natural language to fuzzy rating based on rules of transformation in Table 1 (Dong et al. 2021; Guo and Zhao 2017; Khanmohammadi et al. 2018). The fuzzy comparison can be showed as follows:

$$\tilde{A} = \begin{bmatrix} \tilde{a}_{11} & \cdots & \tilde{a}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \cdots & \tilde{a}_{nn} \end{bmatrix} \tag{2}$$

**Table 1** Saaty’s 9-level linguistic scale

Preference factor	Degree of preference	Explanation
1	Equally	Two factors contribute equally to the objective
3	Moderately	Experience and judgment slightly to moderately favor one factor over another
5	Strongly	Experience and judgment strongly or essentially favor one factor over another
7	Very strongly	A factor is strongly favored over another and its dominance is showed in practice
9	Extremely	The evidence of favoring one factor over another is of the highest degree possible of an affirmation
2,4,6,8	Intermediate	Used to represent compromises between the preferences in weights 1, 3, 5,7and 9
Reciprocals	Opposite	Used for inverse comparison

where each element of the matrix  $\tilde{A}$  represents the relative importance of criterion  $i$  to criterion  $j$ ,  $a_{ij} = (1, 1, 1)$  when  $i = j$ . It must be noted that in BWM method, there is no need to  $n$  fuzzy performance comparison to obtain a completed matrix  $\tilde{A}$ .

In the current study, the details of F-BWM algorithm to calculate the fuzzy weights can be briefly described as follows (Dong et al. 2021; Guo and Zhao 2017; Ecer and Pamucar 2020):

1. Provide a set of desired criteria  $\{c_1, c_1, \dots, c_j, \dots, c_n\}$ , ( $c_1, c_1, \dots, c_j, \dots, c_n =$  morphometric parameter)
2. Determine the best ( $c_B$ ) and worst ( $c_W$ ) criterion
3. Provide  $\tilde{A}_B$  which shows fuzzy reference comparisons of  $c_B$  over all the criteria.

$$\tilde{A}_B = [\tilde{a}_{B1}, \tilde{a}_{B2}, \dots, \tilde{a}_{Bn}] \tag{3}$$

where  $\tilde{a}_{Bj}$  is the fuzzy preference of  $c_B$  over  $c_j$

$$\tilde{a}_{Bj} = (a_{Bj}^l, a_{Bj}^m, a_{Bj}^u), j = 1, 2, \dots, n \text{ and } \tilde{a}_{BB} = (1, 1, 1)$$

4. Provide  $\tilde{A}_W$  which shows fuzzy reference comparisons of all the criteria over  $c_W$ .

$$\tilde{A}_W = [\tilde{a}_{1W}, \tilde{a}_{2W}, \dots, \tilde{a}_{nW}] \tag{4}$$

where  $\tilde{a}_{jW}$  is the fuzzy preference of  $c_j$  over  $c_B$

$$\tilde{a}_{jW} = (a_{jW}^l, a_{jW}^m, a_{jW}^u), j = 1, 2, \dots, n \text{ and } \tilde{a}_{WW} = (1, 1, 1).$$

5. Determine the optimal fuzzy weight  $\tilde{w}^* = [\tilde{w}_1^*, \tilde{w}_2^*, \dots, \tilde{w}_n^*]$ , where  $\tilde{w}_j^* = (w_j^{*l}, w_j^{*m}, w_j^{*u})$  shows the optimal fuzzy weight of  $c_j$  which is calculated using below model:

$$\min \max_j \left\{ \left| \frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{a}_{Bj} \right|, \left| \frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{a}_{jW} \right| \right\} \tag{5}$$

$$s.t. \begin{cases} \sum_{j=1}^n R(\tilde{w}_i) = 1 \\ l_j^w \leq m_j^w \leq u_j^w \\ l_j^w \geq 0 \\ j = 1, 2, \dots, n \end{cases}$$

where  $\tilde{w}_B = (l_B^w, m_B^w, u_B^w), \tilde{w}_j = (l_j^w, m_j^w, u_j^w), \tilde{w}_W = (l_W^w, m_W^w, u_W^w), \tilde{a}_{Bj} = (l_{Bj}^w, m_{Bj}^w, u_{Bj}^w), \tilde{a}_{jW} = (l_{jW}^w, m_{jW}^w, u_{jW}^w)$  a n d  $R(\tilde{w}_j) = 1/6(w_j^l + 4w_j^m + w_j^u)$ .

The above model can be transferred as below optimization model which are based on consistency ratio ( $\xi$ ) (next step).

$$s.t. \begin{cases} \min \xi \\ \left| \frac{\tilde{w}_B}{\tilde{w}_j} - \tilde{a}_{Bj} \right| \leq \xi \\ \left| \frac{\tilde{w}_j}{\tilde{w}_W} - \tilde{a}_{jW} \right| \leq \xi \\ \sum_{j=1}^n R(\tilde{w}_j) = 1 \\ l_j^w \leq m_j^w \leq u_j^w \\ l_j^w \geq 0 \\ j = 1, 2, \dots, n \end{cases} \tag{6}$$

where  $\xi = (l^\xi, m^\xi, u^\xi)$  and it can be assumed that  $\xi^* = (k^*, k^*, k^*) \leq l^\xi$ , then Eq. 6 can be transferred as:

**Table 2** Sub-watershed wise morphometric parameters (Sharma et al. 2011)

Sub-watershed	R <sub>h</sub>	R <sub>r</sub>	R <sub>N</sub>	R <sub>b</sub>	D <sub>d</sub>	F <sub>s</sub>	R <sub>c</sub>	R <sub>f</sub>	R <sub>e</sub>	T	L <sub>o</sub>	C <sub>c</sub>	S <sub>a</sub>	HI
1	0.019	0.006	0.304	3.889	3.372	6.264	0.651	0.530	0.822	4.902	0.148	1.239	7.089	0.410
2	0.023	0.008	0.425	4.115	3.293	6.165	0.564	0.340	0.658	4.275	0.152	1.331	9.275	0.700
3	0.025	0.008	0.409	3.521	3.199	5.299	0.573	0.433	0.743	3.776	0.156	1.321	8.121	0.560
4	0.032	0.011	0.499	3.833	3.328	6.663	0.654	0.490	0.790	4.928	0.15	1.236	13.524	0.670
5	0.032	0.010	0.670	3.646	3.488	8.016	0.531	0.414	0.726	6.270	0.143	1.372	12.89	0.520
6	0.022	0.008	0.312	3.643	2.454	3.817	0.56	0.370	0.686	2.859	0.204	1.335	7.467	0.540
7	0.032	0.010	0.420	3.417	3.180	5.700	0.561	0.472	0.775	3.385	0.157	1.335	8.680	0.510
8	0.042	0.012	0.763	3.681	3.670	7.335	0.582	0.591	0.868	6.058	0.136	1.311	20.115	0.560
9	0.046	0.017	0.827	3.705	3.334	6.284	0.631	0.356	0.673	4.495	0.15	1.259	17.845	0.610
10	0.024	0.008	0.462	4.005	3.421	7.426	0.494	0.363	0.680	5.013	0.146	1.422	9.998	0.420
11	0.047	0.015	0.742	3.208	3.285	5.598	0.606	0.473	0.776	4.092	0.152	1.284	14.566	0.750
12	0.044	0.015	0.684	3.113	3.319	6.268	0.758	0.513	0.809	5.217	0.151	1.149	22.295	0.450
13	0.038	0.014	0.737	3.495	3.899	7.322	0.513	0.315	0.634	4.130	0.128	1.395	20.416	0.360
14	0.046	0.015	1.134	3.759	4.994	7.785	0.489	0.381	0.696	4.671	0.1	1.430	11.553	0.230

**Table 3** Inter-correlation Matrix of the Geomorphic Parameters

	R <sub>h</sub>	R <sub>r</sub>	R <sub>N</sub>	R <sub>b</sub>	D <sub>d</sub>	F <sub>s</sub>	R <sub>c</sub>	R <sub>f</sub>	R <sub>e</sub>	T	L <sub>o</sub>	C <sub>c</sub>	Sa	HI
R <sub>h</sub>	1	0.96	0.88	-0.55	0.49	0.32	0.17	0.13	0.13	0.25	-0.49	-0.14	0.76	-0.04
R <sub>r</sub>	0.96	1	0.85	-0.52	0.44	0.26	0.19	-0.06	-0.06	0.13	-0.43	-0.16	0.77	-0.05
R <sub>N</sub>	0.88	0.85	1	-0.24	0.79	0.58	-0.15	-0.09	-0.09	0.36	-0.74	0.19	0.61	-0.32
R <sub>b</sub>	-0.55	-0.52	-0.24	1	0.11	0.25	-0.39	-0.30	-0.31	0.16	-0.09	0.38	-0.47	-0.04
D <sub>d</sub>	0.49	0.44	0.79	0.11	1	0.75	-0.37	-0.09	-0.10	0.38	-0.95	0.42	0.26	-0.62
F <sub>s</sub>	0.32	0.26	0.57	0.25	0.75	1	-0.29	-0.01	-0.02	0.79	-0.84	0.35	0.39	-0.43
R <sub>c</sub>	0.17	0.19	-0.15	-0.39	-0.36	-0.29	1	0.57	0.58	0.13	0.26	-0.99	0.36	0.33
R <sub>f</sub>	0.13	-0.06	-0.09	-0.30	-0.09	-0.01	0.57	1	0.99	0.39	0	-0.58	0.17	0.16
R <sub>e</sub>	0.13	-0.06	-0.09	-0.31	-0.1	-0.02	0.58	0.99	1	0.38	0.01	-0.59	0.15	0.16
T	0.25	0.13	0.36	0.16	0.38	0.79	0.13	0.39	0.39	1	-0.51	-0.09	0.45	-0.12
L <sub>o</sub>	-0.49	-0.43	-0.74	-0.09	-0.96	-0.85	0.27	0	0.01	-0.51	1	-0.32	-0.35	0.51
C <sub>c</sub>	-0.14	-0.16	0.19	0.37	0.42	0.35	-0.99	-0.59	-0.59	-0.09	-0.32	1	-0.32	-0.41
Sa	0.76	0.77	0.60	-0.47	0.26	0.39	0.36	0.17	0.15	0.45	-0.35	-0.32	1	-0.03
HI	-0.04	-0.04	-0.32	-0.04	-0.62	-0.43	0.33	0.16	0.16	-0.12	0.51	-0.41	-0.03	1

**Table 4** Total Variance Explained

Total variance explained										
Component	Initial eigenvalues			Extraction sums of squared loadings			Rotation sums of squared loadings			
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	
1	5.236	37.402	37.402	5.236	37.402	37.402	4.195	29.964	29.964	
2			4.054	4.054	28.954	66.356	3.81	27.212	57.176	
3	2.158	15.413	81.769	2.158	15.413	81.769	3.443	24.592	81.769	
4	0.985	7.037	88.806							
5			0.649							
6	0.509	3.639	97.081							
7	0.195	1.393	98.474							
8	0.153	1.093	99.567							
9	0.049	0.351	99.919							
10	0.008	0.055	99.974							
11	0.003	0.021	99.994							
12	0.001	0.005	99.999							
13	0	0.001	100							
14	1.001E-16	7.15E-16	100							

Extraction Method: Principal Component Analysis

$$\begin{aligned}
 & \min \xi^* \\
 & \left. \begin{aligned}
 & \left| \frac{(l_B^w, m_B^w, u_B^w)}{(l_j^w, m_j^w, u_j^w)} - (l_{Bj}, m_{Bj}, u_{Bj}) \right| \leq (k^*, k^*, k^*) \\
 & \left| \frac{(l_j^w, m_j^w, u_j^w)}{(l_W^w, m_W^w, u_W^w)} - (l_{jW}, m_{jW}, u_{jW}) \right| \leq (k^*, k^*, k^*) \\
 & \sum_{j=1}^n R(\tilde{w}_i) = 1 \\
 & l_j^w \leq m_j^w \leq u_j^w \\
 & l_j^w \geq 0 \\
 & j = 1, 2, \dots, n
 \end{aligned} \right\} \text{s.t.} \tag{7}
 \end{aligned}$$

By solving above model, the optimal fuzzy weight  $(\tilde{w}_1^*, \tilde{w}_2^*, \dots, \tilde{w}_n^*)$  can be calculated.

### Results and discussion

Morphometric parameters of Gusru watershed adapted from Sharma et al. (2011) are presented in Table 2. For redundancy of morphometric parameter, PCA has been applied. A hierarchical tree from the most effective morphometric results is used to prioritize the sub-watersheds.



Fuzzy best worst method (F-BWM) was applied to establish the relative weights of parameters or criteria and for watershed prioritization.

The SPSS 22.0 software is employed to assess the inter-relationships of morphometric variables through a correlation matrix (Table 3). Very high correlations ( $R > 0.9$ ) exist between the different morphometric parameters that is between; relief ratio ( $R_h$ ) and relative relief ( $R_r$ ); elongation ratio ( $R_e$ ) and farm factor ( $R_f$ ), drainage density ( $D_d$ ) and length of overland flow ( $L_o$ ), and between the circulatory ratio ( $R_c$ ) and compactness coefficient ( $C_c$ ). In addition, moderately high correlations ( $R > 0.70$ ) are observed between;  $R_N$  and  $R_h/R_r/D_d/L_o$  and between  $F_s$  and  $D_d/T/L_o$ ,  $S_a$  and  $R_h/R_r$ . Because there are no significant correlations between HI or  $R_b$  with any of the parameters under consideration, it is practically impossible to put the parameters into component groups. Therefore, the subsequent step makes use of the principal component analysis technique.

The correlation matrix obtained from the previous step is used to generate the first unrotated factor loading matrix (Table 4). The results show that about 81.76% of the total explained variance is attributed to the combination of the first three components with eigen values above one. It is observed that a strong correlation ( $R > 0.9$ ) between  $R_N$  and the first component (Table 5A). Relatively high correlations are also found between the first component and each of the variables;  $D_d$ ,  $L_o$ ,  $R_h$ ,  $F_s$ , and  $R_r$ . On the other hand,  $R_c$ ,  $R_e$ ,  $R_f$  and  $C_c$  have high correlations with second component. No significant correlations exist between the third component and any one of the parameters.

Redistribution of the observed variance is performed so that better factor loadings can be obtained. This is done by carrying out analytical rotations those components whose eigen value exceeds one. The outcome of varimax rotation is shown in Table 5B.

The first component is very highly correlated with  $F_s$  and highly correlated with  $D_d$ ,  $L_o$  and  $T$ . A strong correlation also exists between the second component with  $R_h$  and  $R_r$ , while moderately high correlations are obtained with  $R_N$  and  $S_a$ . A very strong correlation is also apparent for the third component with  $R_e$  and with  $R_f$  and at the same time moderately correlated with  $C_c$  and  $R_c$ .

Table 6 depicts the ordering of each parameter with respect to importance. The order of priority in descending order is given as  $F_s > R_r > L_o > R_h > D_d$ .

At the watershed scale, the sub-watersheds, based on their morphometric and hydrologic properties have different hydrological behavior regarding flood degree, erosion and sedimentation. Therefore, prioritization of sub-watersheds is a crucial step for watershed management

strategies. Subjective MCDA is one of the mostly used methods for flood prioritization. These methods based on Smithson (2012) are categorized as knowledge-based methods, so that the results of a desired study are a function of experts' decision, leading to high uncertainty of results. In this regard, BWM can be used as an efficiency method to reduce the number of subjective experts' decisions (Rezaei 2015). However, the existence of qualitative judgments on BWM (i.e., 9-point table) can be considered as one of the main sources of uncertainty in this method, therefore, in this study we used TFN to nearly

**Table 5** Varimax method of the first factor loading matrix (unrotated) and Rotated Factor Loading Matrix of fourteen geomorphic parameters

	Component		
	1	2	3
<i>(A) Component Matrix</i>			
$R_N$	0.936		
$L_o$	-0.880		
$D_d$	0.867	-0.317	
$R_h$	0.793	0.414	-0.388
$F_s$	0.788		0.459
$R_r$	0.745	0.355	-0.538
$S_a$	0.662	0.512	
HI	-0.443	0.410	
$C_c$		-0.896	
$R_c$		0.890	
$R_e$		0.741	0.559
$R_f$		0.739	0.564
$R_b$		-0.624	0.464
T	0.547		0.657
<i>(B) Rotated component matrix</i>			
$F_s$	0.939		
$L_o$	-0.894		
$D_d$	0.864	0.301	
T	0.713		0.510
HI	-0.568		
$R_r$		0.970	
$R_h$		0.936	
$S_a$		0.788	
$R_N$	0.587	0.749	
$R_b$	0.325	-0.687	
$R_f$			0.928
$R_e$			0.927
$C_c$	0.431		-0.772
$R_c$	-0.383		0.765

resolve the drawback of qualitative judgments (Bellman and Zadeh 1970a, b; Guo and Zhao 2017; Zhao and Guo 2014, 2015).

The statistical analysis of F-BWM has been used to prioritize sub-watersheds based on the degree of soil erosion. In this regard, five morphometric parameters i.e.,  $L_o$  (C1),  $F_s$  (C2),  $D_d$  (C3),  $R_r$  (C4) and  $R_h$  (C5) were

used. Based on experts' knowledge and field survey, the  $L_o$  (C1) and  $R_h$  (C5) are considered as the best and worst criteria. Next, the fuzzy preferences to best criterion over other criteria (vector  $\tilde{A}_B$ ) and all criteria over worst criteria (vector  $\tilde{A}_W$ ) were determined. Then, based on step 5, the optimal fuzzy weight was done to obtain the weights (Table 7, Fig. 2).

**Table 6** Order of importance of parameters

Parameters	Rotated Loadings	Square of Rotated loadings	% Rotated factor covariance	Importance percentage	Order of importance
$F_s$	0.939	0.8817	29.964	26.41989	1
$R_r$	0.970	0.9409	27.212	25.60377	2
$L_o$	-0.894	0.7992	29.964	23.94831	3
$R_h$	0.936	0.8761	27.212	23.84032	4
$D_d$	0.864	0.7465	29.964	22.36801	5
$R_r$	0.928	0.8612	24.592	21.17824	6
$R_e$	0.927	0.8593	24.592	21.13262	7
$S_a$	0.788	0.6209	27.212	16.89713	8
$R_N$	0.749	0.5610	27.212	15.26596	9
T	0.713	0.5084	29.964	15.23277	10
$C_c$	-0.772	0.5960	24.592	14.65644	11
$R_c$	0.765	0.5852	24.592	14.39185	12
$R_b$	-0.687	0.4720	27.212	12.84322	13
HI	-0.568	0.3226	29.964	9.66711	14

**Table 7** Prioritization of Micro-watersheds

Sub	$F_s$	$R_r$	$L_o$	$R_h$	$D_d$	Final weight	Final Priority
1	0.583	0.000	0.462	0.000	0.361	0.391	13
2	0.559	0.182	0.500	0.143	0.330	0.426	12
3	0.353	0.182	0.538	0.214	0.293	0.387	14
4	0.678	0.455	0.481	0.464	0.344	0.503	8
5	1.000	0.364	0.413	0.464	0.407	0.556	3
6	0.000	0.182	1.000	0.107	0.000	0.427	11
7	0.448	0.364	0.548	0.464	0.286	0.451	10
8	0.838	0.545	0.346	0.821	0.479	0.544	5
9	0.588	1.000	0.481	0.964	0.346	0.578	1
10	0.859	0.182	0.442	0.179	0.381	0.489	9
11	0.424	0.818	0.500	1.000	0.327	0.520	7
12	0.584	0.818	0.490	0.893	0.341	0.553	4
13	0.835	0.727	0.269	0.679	0.569	0.542	6
14	0.945	0.818	0.000	0.964	1.000	0.563	2



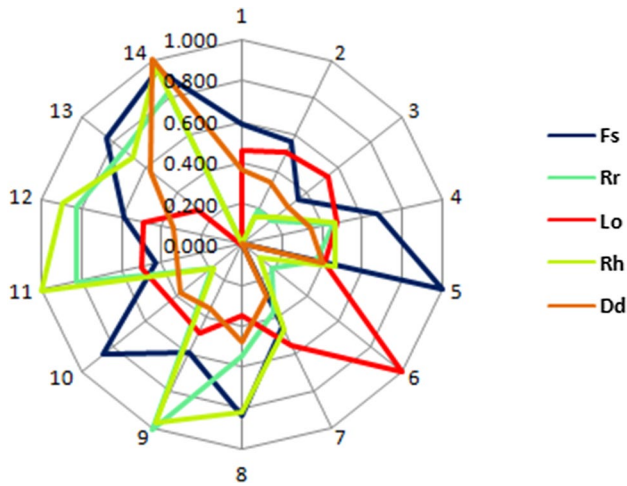


Fig. 2 F-BWM weight to the most effective morphomeric parameter

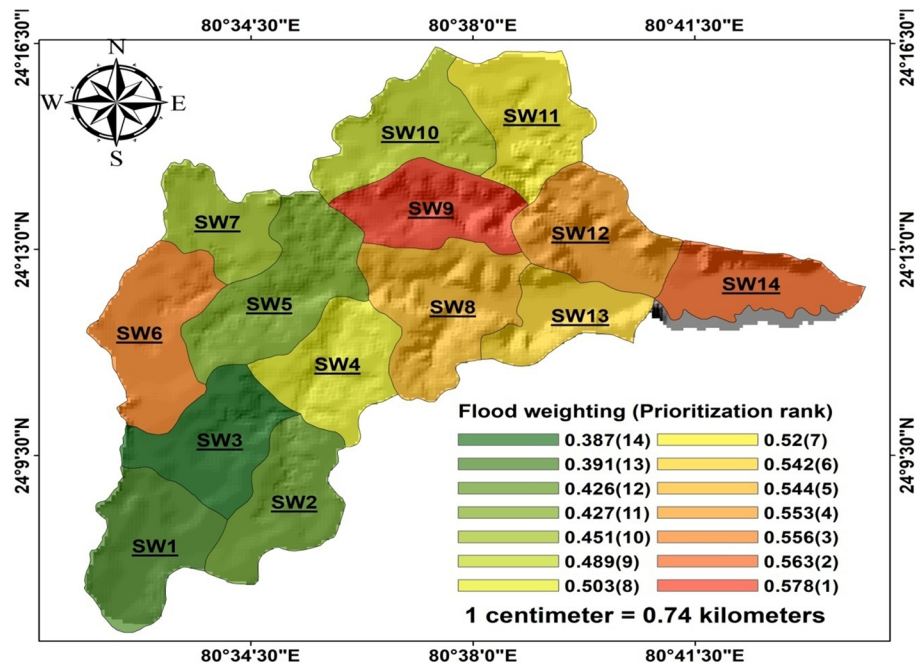
Figure 3 shows the results of F-BWM in watershed prioritization. Based on Table 7, the F-BWM weight of the sub-watershed 9 has the maximum value, so it is located as the first priority. On the contrary, sub-watershed 3 has been located in last rank (14) of the prioritization.

### Conclusion

In the current study, five morphometric parameter i.e.,  $F_s$ ,  $R_r$ ,  $L_o$ ,  $R_h$  and  $D_d$  were used to watershed prioritization in the case study. In this regard, F-BWM as knowledge-based method was used to assigning initial weights to criteria. The conclusion can be drawn that the parameter  $F_s$  is the most important soil erosion related criterion, so that the sub-watersheds 9 and 3 which have first and last rank of prioritization, have the maximum and minimum value of F-BWM weight. In this state, the critical sub-watersheds can be better recognized for doing watershed management strategies.

In this study, there are various elements of improvement for the proposed method, as well as future research objectives. For enhanced input-based consistency ratio and constrained optimization equations, one of the defuzzification approaches is applied first. However, there are a variety of additional defuzzification strategies that can be used with the model, which could be a future study topic. Second, the primary goal of combining the views of several experts is to provide appropriate findings from pair wise comparison matrices. Each

Fig. 3 Soil erosion susceptibility map



methodology has its own set of advantages and disadvantages, and future research might concentrate on the advantages and disadvantages of various aggregation methods.

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**Data availability** The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

## Declarations

**Conflict of interest** The authors declare that they have no competing interests.

**Ethics approval** Not applicable.

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