



Gene Expression Programming (GEP) to predict coefficient of discharge for oblique side weir

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Abstract

In irrigation and drainage structures, side weir is widely used for flow diversion from main to branch channels. Side weir is also used as a measuring device for discharge measurements, so discharge coefficient was mainly studied in many previous studies. Skew side weir was not taking a good highlight in previous studies and literature, so the present work discharge coefficient calculation for the skew side weir was adopted and studied. Multiple Linear Regression (MLR) and Gene Expression Programming (GEP) tools were used in the present study and compared with observed values of C_d . The mean absolute error for C_d observed and calculated in MLR and GEP was not exceeded 5%. The C_d values for skew side weir ranged from (0.65) to (0.85), while its values for straight vertical side from previous literature weir ranged from (0.45) to (0.65); this mean skew side weir can be used for increase in discharge diversion to the branch channel at the same water levels by 27%. The Akaike information criteria (AIC) with (AICs), root-mean-square error (RMSE), mean absolute relative error (MARE) and scatter index (SI) are used in this study for measuring the GEP model performance. From results, the GEP model has $AIC = -216.51$, $AICs = -918.51$, $RMSE = 0.004653$, $MARE = 0.005234$, $R^2 = 0.994$ and $SI = 0.006231$ performed the best. According to previous results, the new equation presented through GEP can be adopted for discharge coefficient calculation in skew side weir.

Keywords Skew side weir · Coefficient of discharge · Gene Expression Programming (GEP) · MLR · Open channel hydraulic · RMSE · AIC · MARE · SI

List of symbols

MLR	Multiple Linear Regression	y	Head of water (L)
GEP	Gene Expression Programming	v	Flow velocity (L/T)
Q_{act}	Actual discharge (L^3/T)	g	Gravity acceleration (L/T^2)
H	Head over standard weir (L)	P	Weir height (L)
Q_1	Actual discharge in main channel when side channel is closed (L^3/T)	q	Discharge per unit length ($L^3/T L$)
Q_2	Actual discharge in main channel when side channel is opened (L^3/T)	S	Longitudinal slope
Q_3	Actual side channel discharge after subtracting Q_1 and Q_2 (L^3/T)	C_d	Coefficient of discharge
E	Specific energy (L)	y_1	Flow depth (L)
		L	Weir length (L)
		b	Channel width (L)
		θ	Side weir angle
		F_r	Froude number $\left(\frac{v}{\sqrt{gy}}\right)$
		C_1-C_5	Constants
		R^2	Coefficient of determination
		RMSE	Root-mean-square error
		AIC	Akaike information criteria
		AICs	AIC with a correction for small sample sizes
		MARE	Mean absolute relative error
		SI	Scatter index
		BIAS	Errors of fitting

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- x_i and y_i Are the actual and modeled C_d values, respectively
- \bar{x} and \bar{y} Are the mean actual and modeled C_d values, respectively
- k The number of estimated parameters

Introduction

Side weirs are an overflow weir installed on the side of the main channel, which allows flow when water rises above the crest. This type of flow is considered as a spatially varied flow. Side weirs are usually used as a control structure and as head regulators in irrigation structure. Many studies deal with side weir hydraulics; some of these studies deal with sharp crested side weirs such as El-Khashab and Smith (1976), Uyumaz and Smith (1991) Swamee et al. (1994), Hager (1987), Masoud (2003), Singh et al. (1994), Rao and Pillai (2008), Delkash and Babak (2014) and other investigated deals with inclined and oblique side weir such as Mwafaq and Ahmed (2011). Honar and Javan (2007) and Amir et al. (2016). The numerical analysis on inclined side weir was investigated by Ahmed (2011), Ahmed et al. (2013) and Ahmed (2015). A powerful tool is recently used to solve complex nonlinear and multi-linear regression equations in hydraulic engineering such as artificial neural network (ANN), genetic programming (GP) and statistic's analysis using Monte Carlo method, Kisi et al. (2012), Ahmed (2018), Hayawi et al. (2019) and Ahmed and Anna (2020). In the recent years GEP and learning machine were used to model of nonlinear problems of predicting discharge coefficient in side weir such as Isa et al. (2015) and Reza et al. (2020). The aim of this study is to estimate MLR equation and compare equation modeled from GEP for coefficient of discharge calculation from skew side weir and then compare these values with values of C_d estimated from the rectangular side weir.

Experimental methodology

According to Al-Talib (2012), the experimental works were achieved in rectangular laboratory channel 10 m long, 0.3 m wide and 0.45 m depth, while the side channel dimensions were 0.15 m wide, 0.3 m depth and 2 m long. The discharge was measured using standard sharp crested weir (0.15 * 0.3 * 0.01) m dimensions at main channel; the side weirs were fixed at the entrance of the side channel by different angles starting from (90°) (perpendicular to the side channel) decreasing to (30°). Five different angles were taken (90°, 75°, 60°, 45° and 30°) inclined to the left of flow direction. Figure 1.

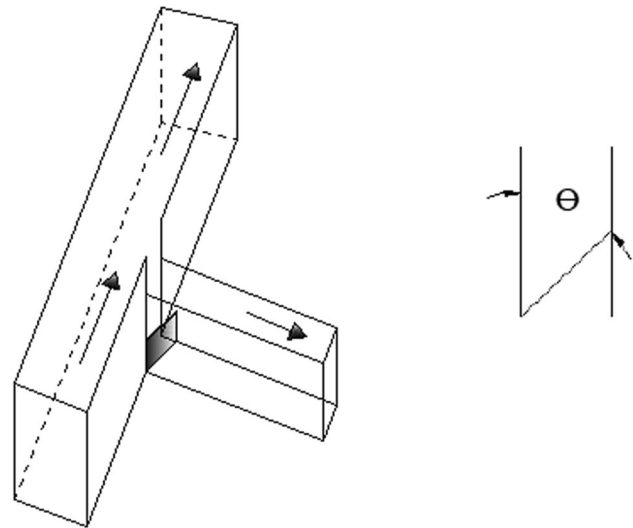


Fig. 1 Sketch of side channel with skew side weir installed

Five discharges were taken ranging from (7.3 to 16.5 L/s); the actual discharge for the main channel was calculated from the equation.

$$Q_{\text{act.}} = 0.58 \times H_{1.5} \quad (1)$$

where $Q_{\text{act.}}$ = actual discharge and H = head over standard weir.

Equation (1) can be calculated by trial and error from volumetric calculations. The actual discharge was measured by closed side channel and measured depth of water over the standard weir at the end of main channel, then from Eq. 1 found Q_1 , then open side channel and measured water depth over the standard weir at the end of the main channel once again and from Eq. 1 found the discharge again, but discharge measured in this case (when side channel open) was Q_2 . Actual side channel discharge Q_3 used Eq. 2.

$$Q_3 = Q_1 - Q_2 \quad (2)$$

where Q_1 = actual discharge in main channel when side channel is closed, Q_2 = actual discharge in main channel when side channel is opened, and Q_3 = actual side channel discharge after subtracting Q_1 and Q_2 .

Theoretical methodology

The general flow through the side weir derived depends on head of water over the side weir as well as the velocity of flow through it, according to specific energy assumption (De Marchi 1934).

$$E = y + \frac{V^2}{2g} \tag{3}$$

where E = specific energy, y = head of water, v = flow velocity and g = gravity acceleration.

Depending on Q , Eq. 3 can be written as discharge,

$$Q = b \times y \sqrt{2g(E - P)} \tag{4}$$

where $(b \times y)$ = cross-sectional area, P = weir height.

Depending on De Marchi, Eq. 4 can be written as,

$$q = -\frac{dQ}{dS} = \frac{2}{3} C_d \sqrt{2g} (E - P)^{3/2} \tag{5}$$

where q = discharge per unit length, S = longitudinal slope and C_d = coefficient of discharge.

Equation 5 satisfies rectangular channel and side weir perpendicular to channel bed, so in skew side weir it is not perpendicular to channel bed; the angle for inclined side weir must have taken, and then, Eq. 5 must change depending on these angles Fig. 1.

Dimensional analysis

Dimensional analysis is important to study the effects of the angle skew side weir to calculate coefficient of discharge (C_d) from standard side weir in rectangular channel. The parameters involved is calculated C_d in skew side weir as:

$$C_d = f_1(v_1, y_1, P, L, b, g, \theta) \tag{6}$$

where C_d = coefficient of discharge, v = flow velocity, y_1 = flow depth, P = weir height, L = weir length, b = channel width and g = acceleration due to gravity, θ = side weir angle.

By using Buckingham Pi theorem, parameters on Eq. 6 can be used to develop a non-dimensional equation below:

$$C_d = \phi\left(F_r, \frac{P}{y_1}, \frac{L}{b}, \theta\right) \tag{7}$$

where F_r = Froude number $\left(\frac{v}{\sqrt{gy}}\right)$.

Modeling of skew side weir using MLR

There are many applications for involved regression analysis. These applications deal with linear and nonlinear analysis, depending on variables that involve in the problem. In order to obtain a general equation for skew side weir, several trials with several equation models

examined using (Statistical Package Social Sciences SPSS user guide).

According to Ahmed (2015), from Eq. 7 using several models of SPSS, Eq. 8 can be developed as MLR with a coefficient of determination R^2 (0.958)

$$C_d = C_1 + C_2 Fr + C_3 P/y + C_4 L/b + C_5 \theta \tag{8}$$

where $C_1 - C_5$ = constants, and θ in radian.

GEP modeling for side weir

Gene Expression Programming was an artificial procedure to solve genotype system. This way was invented by Ferreira (2001, 2006), and GEP was similar to (GA) genetic algorithms and (GP) genetic programming; GA deals with individuals as a linear string of length fixed (chromosomes), while GP deals with individuals as nonlinear entities for different parse tree structure. In GEP, the individuals deal with encoded linear strings (chromosomes) which are expressed as nonlinear entities. In GEP, there are two important players: the tree structure (ETS) and chromosomes. The decoding of the process information is called translation that implies obviously a type of code and rules. The genetic code of GEP was simple; a relation between the symbol of the chromosomes and the node is represented in the tree. The rules of GEP determine nodes in the trees and then the type of the interaction in sub-ETS. GEP programming depends on two principal languages: the genetic language and expression trees language. This bilingual notation in GEP is named as Karva. Figure 2 shows the expression tree (ET) for an example of mathematical expressions $(xb + \sqrt{c + d})$ Mohd et al.(2015) and Khalid and Negm (2008), This ET is encoded in Karva language, and then, the expression is called K-expression. Each gene starts at the first left position, then scans all symbols in all directions every time when a symbol is finally added to the K-expression, and then, the K-expression mentioned above can be written as $(+x\sqrt{ab + cd})$.

Figure 3 shown the steps of GEP, include some steps at the begin with the randomly generate of the chromosome from initial population. Then, these chromosomes were expressed and excluded the tree expression to evaluate fitness. The individual is then selected with respect to their fitness to reproduce with the modification; these individuals are subject to the same development. This process was repeated several times until a good solution is found. (Ferreira 2004) The basis of GEP is established on the structure of GEP gene. The simple structure of genes allows the encoding of thinkable program and allows their dynamic evolution due to these multilateral structural arrangements; a powerful set of genetics worker can be implemented to search efficiently solution Ferreira 2002.

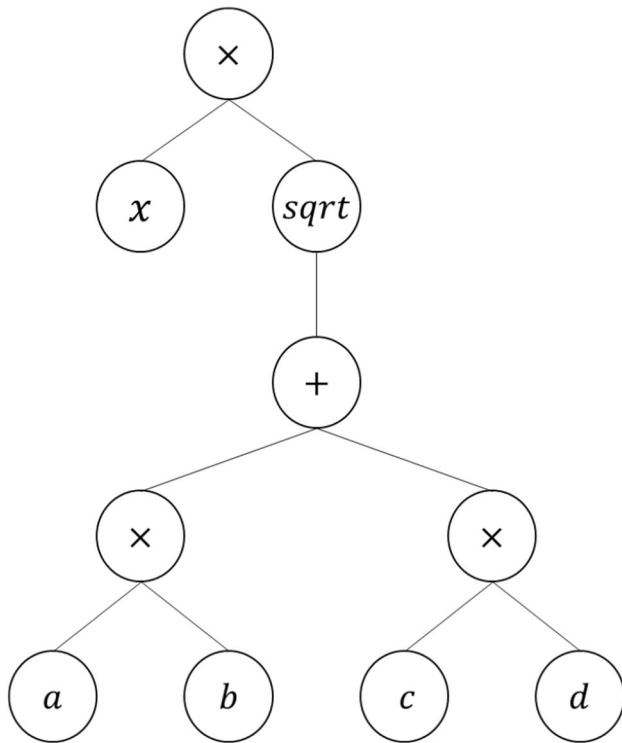


Fig. 2 Expression tree for expression $x\sqrt{ab + cd}$

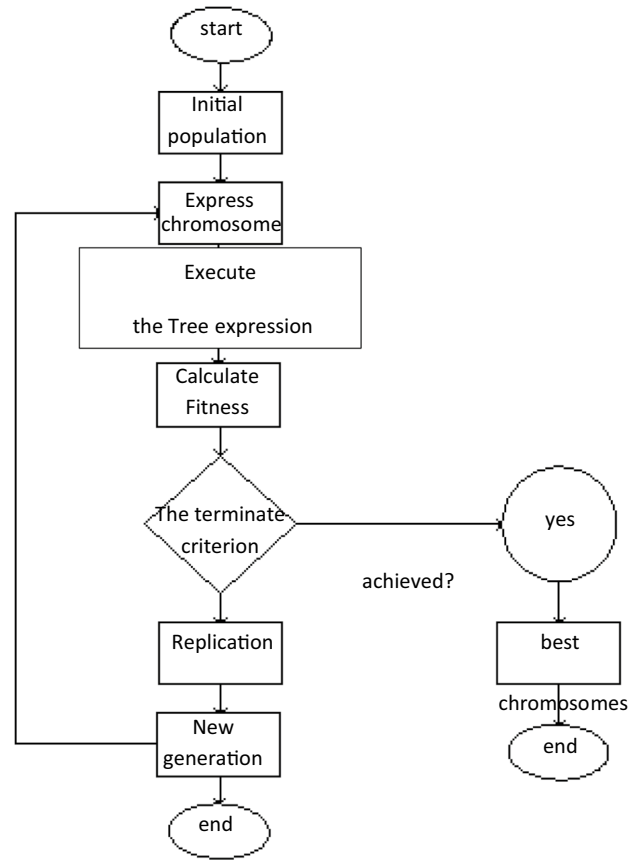


Fig. 3 GEP flow chart

The equation obtained from GEP is given as:

$$\begin{aligned}
 C_d = & \left(A \tan \left(A \tan \left(A \tan \left(\tan \left(1101236493.83311 + \frac{P}{y} \right) + \cos \left(\exp \left\{ \frac{L}{b} \right\} \right) \right) \right) \right) \right)^4 \\
 & + \frac{A \operatorname{coth} \left(\left(\theta \times 8173.64699339967 \right) - 5974.47259811262 \right) \times \left(F_r - \frac{P}{y} \right)}{\frac{\log(\theta)}{3.22324468279425} + \frac{P}{y} - \theta + \sqrt[3]{F_r}} \\
 & + A \operatorname{coth} \left(\left\{ \left\{ \operatorname{Csc} \left(\frac{L/b}{P/y} \right) \times \left(\frac{P}{y} + \frac{L}{b} \right) \right\} + \left\{ \left(6049.86489822289 - \frac{L}{b} \right) - \frac{8224.16588065491}{L/b} \right\} \times A \tanh \left(F_r \right) \right\} \right) \quad (9) \\
 & + \sqrt[4]{\operatorname{Acsch} \left(\exp \{ \theta \} - A \tanh \left(\frac{\log \left(\exp \left\{ \frac{L/b}{560.990764280841} \right\} \right)}{\log \left(\operatorname{Sech} \left\{ \frac{F_r}{P/y} \right\} \right)} \right) \right)} \\
 & + \left(F_r^4 \times \left(\tan \left(\sin \left(6378.3895352031 \times \frac{P}{y} + \theta \right) \times \left(\sqrt{\frac{P}{y}} - \frac{6377.3895352031}{F_r} \right) \right) \right) \right)
 \end{aligned}$$

The corresponding expression tree for the above equation is given in Fig. 4.

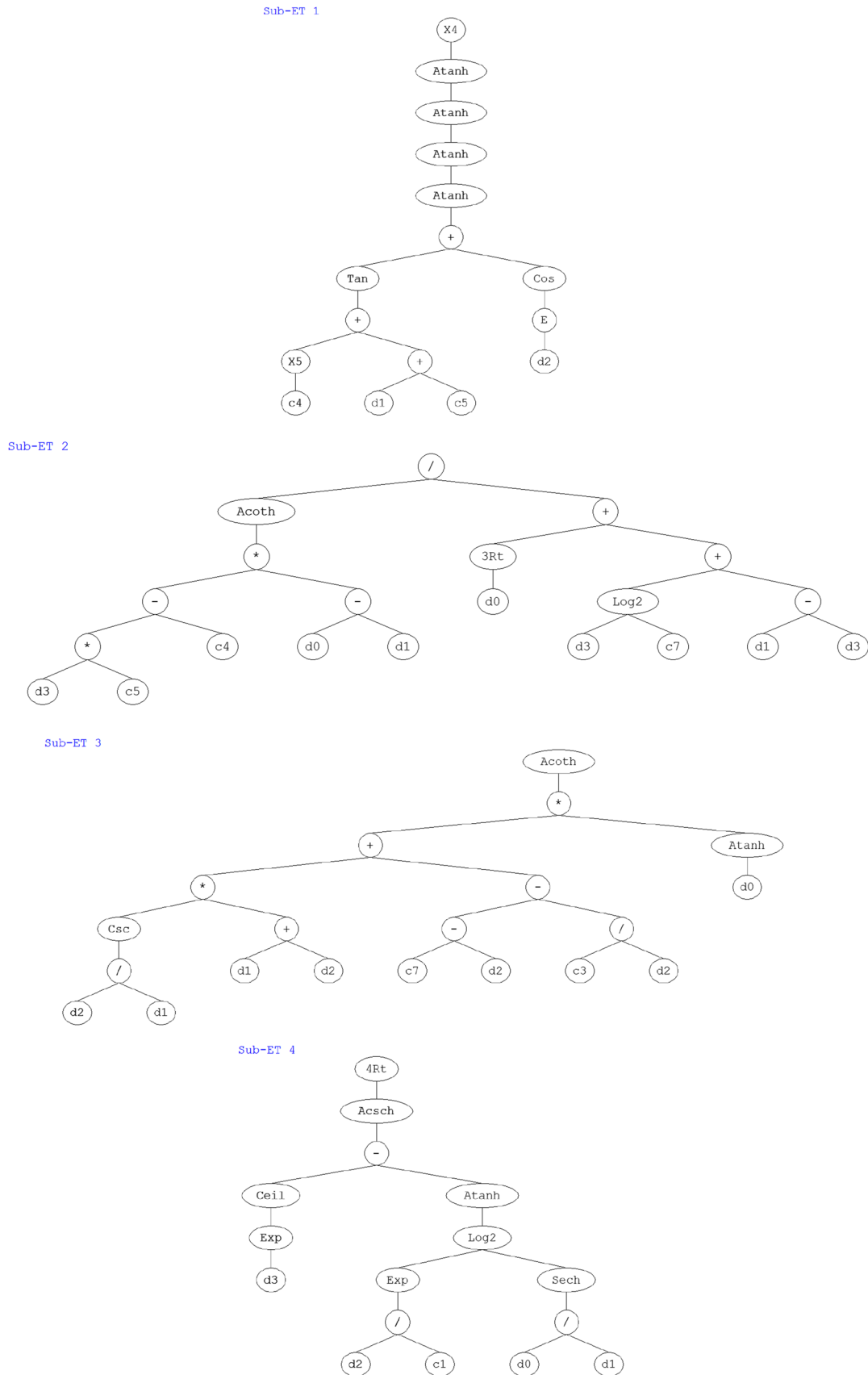


Fig. 4 Expression tree according to GEP equation

Sub-ET 5

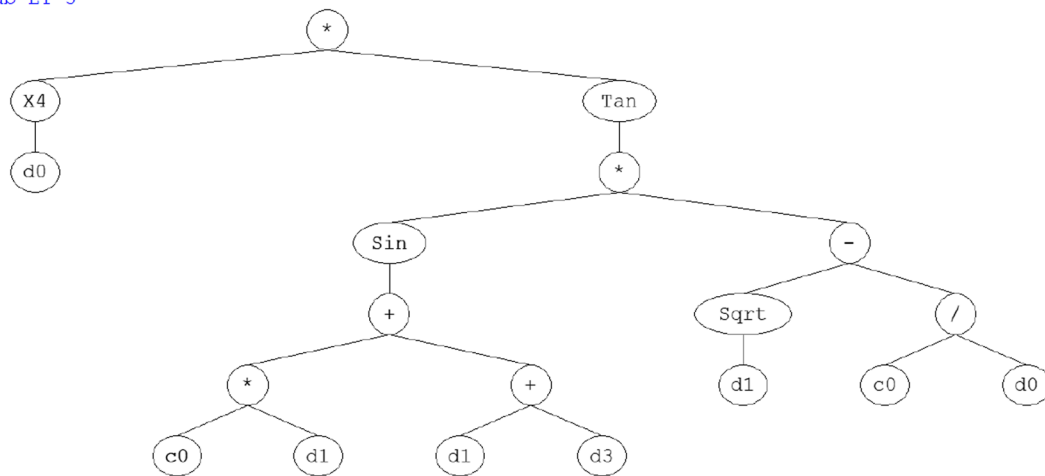


Fig. 4 (continued)

Table 1 GEP model parameters setting

Parameter	Setting
Chromosomes number	36
Genes number	5
Head size	12
Tail size	13
Dc size	13
Gene size	38
Linking function	Addition

Table 2 Statistics obtained from GEP run

Statistics	Value
R^2	0.99388966
Max fitness	1000
Best fitness	995.368337
Correlation coefficient (CC)	0.99690149
Mean squared error (MSE)	0.00002165
Root-mean-square error (RMSE)	0.00465322
Relative absolute error (RAE)	0.07969808
Mean absolute error (MAE)	0.00385506
Relative squared error (RSE)	0.00613077
Root relative squared error (RRSE)	0.07829926
Up/down accuracy	83.33%

Results and discussion

The genetic operation parameters setting is presented in Table 1, while Table 2 represents the statistics obtained from GEP after testing more than 1000 equation models and

running more than 11 h. The comparison between the results of the GEP and MLR presented in this study as well as MLR for previous studies illustrated in Table 3 is presented in terms of coefficient of determination (R^2), root-mean-square error (RMSE), Akaike information criteria (AIC) (AICs—which AIC with a correction for small sample sizes), mean absolute relative error (MARE) and scatter index (SI). These values are presented in Table 4, and the equations are defined below:

$$R^2 = \left[\frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \right]^2 \tag{10}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - y_i)^2} \tag{11}$$

$$AIC = n \times \log \left(\frac{\sum_{i=1}^n (Fr_{Exp_i} - Fr_{Eq_i})^2}{n} \right) + 2 \times k \tag{12}$$

$$AICs = AIC + \frac{2k^2 + 2k}{n - k - 1} \tag{13}$$

$$MARE = \frac{1}{n} \sum_{i=1}^n \frac{|x_i - y_i|}{x_i} \tag{14}$$

$$SI = \frac{RMSE}{\bar{x}} \tag{15}$$

Table 3 Equations for calculating side weir discharge coefficient in previous lecturer

Authors	Equations
Subramanya and Awasthy (1972)	$C_d = 0.864 \times \sqrt{\frac{1-F_r^2}{2+F_r^2}}$
Ranga Raju et al. (1979)	$C_d = 0.54 - 0.4 \times F_r$
Cheong (1991)	$C_d = 0.45 - 0.22 \times F_r^2$
Hager (1987)	$C_d = 0.485 \left(\frac{2+F_r^2}{2+3F_r^2} \right)^{0.5}$
Singh et al. (1994)	$C_d = \frac{1}{3} - 0.18F_r + 0.49\frac{P}{y}$
Jalili and Borghei (1996)	$C_d = 0.71 - 0.41F_r - 0.22\frac{P}{y}$
Borghei et al. (1999)	$C_d = 0.7 - 0.48F_r - 0.3\frac{P}{y} + 0.06\frac{L}{b}$
Ali et al. (2018)	$C_d = \left\{ \begin{aligned} &1.1308 - 1.5396\left(\frac{P}{L}\right)^{0.0394} - 0.1492(F_1)^{0.8292} \\ &+ 0.0105\left(\frac{y_1}{L}\right)^{3.6295} + 0.487\left(\frac{B}{L}\right)^{-0.0357} \end{aligned} \right\}^{0.2322}$
Borghei et al. (2003)	$C_d = 0.82 - 0.38F_r - 0.22\frac{P}{y} + 0.08\frac{L}{b}$

Table 4 Present work statistics comparison with previous studies

Authors	AIC	AICs	RMSE	MARE	R ²	SI
Present (GEP) results	-216.51	-918.51	0.004653	0.005234	0.99389	0.006231
Present (MLR) results	-210.723	-207.566	0.012101	0.013075	0.958565	0.016204
Subramanya and Awasthy (1972)	-85.7575	-82.5996	0.15989	0.192468	0.097021	0.213696
Ranga Raju et al. (1979)	-60.0829	-56.925	0.277577	0.357514	0.100804	0.371685
Cheong (1991)	-50.3328	-47.175	0.337342	0.440455	0.100804	0.451714
Hager (1987)	-52.6433	-49.4854	0.274483	0.354513	0.097244	0.367543
Singh et al.(1994)	-89.1471	-85.9892	0.149129	0.171664	0.065501	0.19969
Jalili and Borghei (1996)	-63.6206	-60.4627	0.248475	0.318192	0.147583	0.332717
Borghei et al. (1999)	-63.9808	-60.8228	0.237019	0.310161	0.816017	0.317376
Ali et al. (2018)	-60.5438	-61.7628	0.256712	0.356521	0.713012	0.319743
Borghei et al.(2003)	-69.9232	-69.8343	0.275439	0.363231	0.816562	0.335476

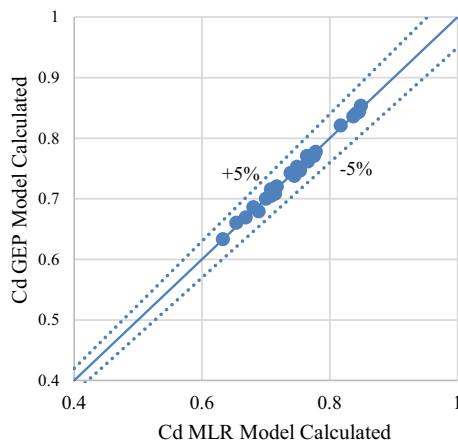


Fig. 5 Comparison between C_d calculated using MLR and GEP\

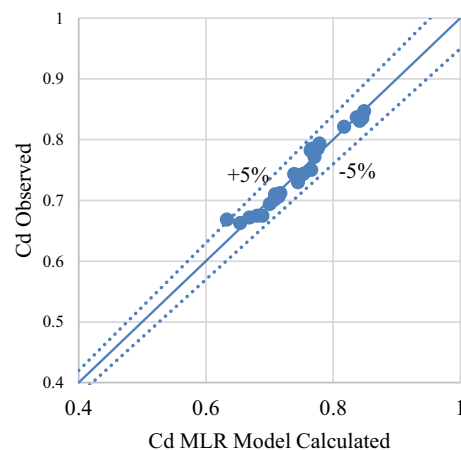


Fig. 6 Comparison between C_d calculated using MLR and C_d observed

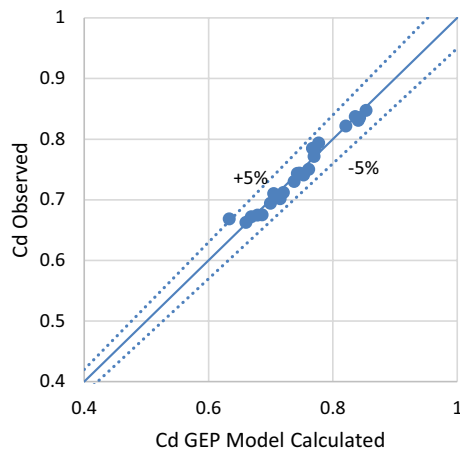


Fig. 7 Comparison between C_d calculated using GEP and C_d observed

where x_i and y_i are the actual and modeled C_d values, respectively, \bar{x} and \bar{y} are the mean actual and modeled C_d values, respectively. k is the number of estimated parameters.

Results in Table 4 represent statistics comparison for the present work with previous studies, and it may be seen that the GEP model refers to highest value of R^2 (0.994) and the lowest value of MARE and RMSE (0.00523 and 0.00465), respectively, as well as the AIC refers to the best value (-216.51) compared with all others equations, and all that indicate that the execution of GEP is the best with respect to other previous equations; overall, all values refer to a good agreement of equation for the present work compared with MLR according to Ahmed (2015) and all other previous equations.

Figure 5 shows a comparison between the discharge coefficients estimated using MLR models and GEP models,

respectively, while Figs. 6 and 7 show the discharge coefficient estimated using an MLR model and GEP model, respectively. Compared with observed coefficient of discharge, Figs. 5, 6 and 7 show agreement between compared coefficients of discharge computed from different models and observed values having a relative error below 5%.

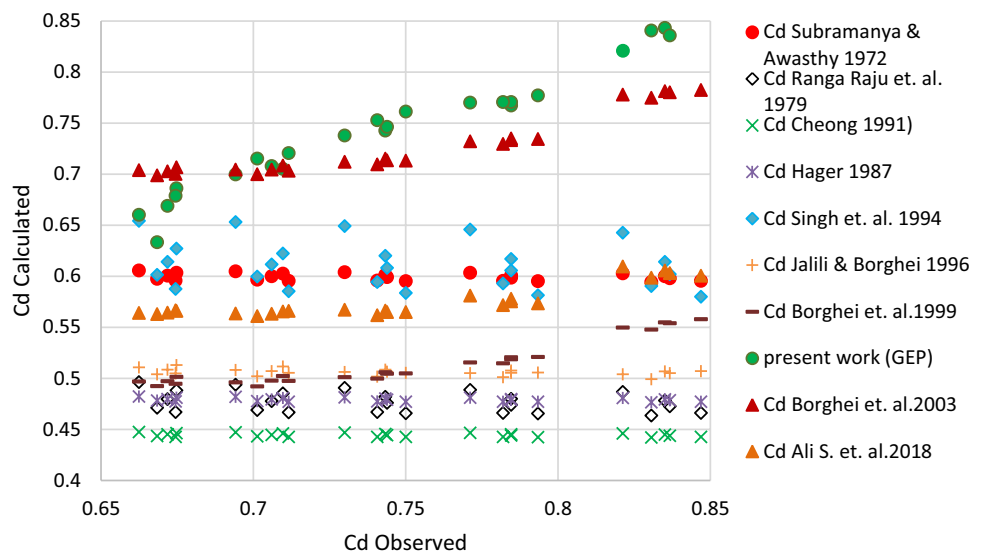
Figure 8 presents the discharge coefficient estimated from equations shown in Table 3 as well as that value estimated from the present model with the observed value. According to these results, all values estimated from previous equation range from 0.45 to 0.75 for vertical side weir, while in the present work, values range between 0.65 and 0.85 for the skew side weir.

These values increased when the side angle increased; this means the discharge coefficient for skew side weir is greater than its values for vertical side weirs and these values for skew side weir increased when the side angle increased.

Conclusion

In the present study, a Gene Expression Programming (GEP) was used to predict an equation to calculate coefficient of discharge in skew side weir in a rectangular channel; this equation was compared with equation predicted from Multiple Linear Regression (MLR) which estimated from statistical tools. The two methods give a good result compared with the observed one with absolute error not exceeding 5% for both methods with correlation coefficient 0.9 and 0.996 for MLR and GEP, respectively, as well as the root-mean-square errors (RMSE) 0.0123 and 0.0046 for MLR and GEP, respectively. The results presented in this method compared with others equation calculated show that the accuracy of modeling and fitting of GEP is better than other methods.

Fig. 8 Comparison of C_d calculated using GEP and that values calculated using equations in Table 3 from previous studies



This conclusion is made by considering the fact that the AIC for the number of parameters that fitted in model where its value (-216.51) is the best value compared with other equations, as well as the best value of the present work model for (AICs = -918.51 , MARE = 0.005234 and SI = 0.006231) compared with other values of equations is presented in this study. The values of C_d for skew side weir were greater than its values for straight vertical. Finally, the results refer to using GEP that gives more accuracy than MLR and other previous literature equations in discharge coefficient calculation and may be used as an improved alternative technique.

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