

# Optimal allocation of public water supply to the urban sectors of Enugu, Nigeria: a linear programming approach

Emma E. Ezenwaji · Raymond N. C. Anyadike ·  
Nnaemeka I. Igu

Received: 23 March 2013 / Accepted: 30 September 2013 / Published online: 16 October 2013  
© The Author(s) 2013. This article is published with open access at Springerlink.com

**Abstract** Recent studies in water supply in Enugu urban area have observed that there is a persistent water supply shortage relative to demand. One of the strategies for achieving a good water supply under the circumstance is through efficient water allocation to consumers. The existing allocation system by the Enugu State Water Corporation is not achieving the desired goal, because it is not based on any scientific criteria. In this study, we have employed the linear programming modelling technique to optimise the allocation of 35,000,000 L of water produced daily by the State Water Corporation and supplied to the four sectors of the town. The result shows that the model allocated 27,470,000 L to the residential sector, 3,360,000 L to commercial, 3,120,000 L to industrial and 882,000 L to public institutions sectors leaving a balance of 168,000 L to be utilised in emergency situations. This allocation pattern departs sharply from the present management technique adopted by the corporation. It is then suggested that for urban water supply to be sustainable in the town, the corporation should rely on this technique for water supply.

**Keywords** Allocation · Management · Linear programming · Sector · Utilised

## Introduction

Recent studies on urban water demand and supply in Nigeria are replete with findings of the wide gap between water demand and supply (Obeta 1997; Nnodu and Ilo 2001; Ezenwaji 2003; Uzoagbala 2006). Some of these and other works have also suggested that one of the best ways to solve this problem of water scarcity in urban areas is to devise a better strategy for allocating the available supply to consumers. For example Ajavi (2007) suggested the use of revenue returns method which implies that those who do not pay their water bills should not be supplied. Odubo and Ikio (2008) recommended the use of per-capital water demand of each household or establishment as a basis for water allocation to such consumers. In Enugu Urban area, the State Water Corporation employs what it terms the administrative method approach in the allocation of water supply to consumers. This is the estimation of water demand based on past consumption data, i.e. on established records of water consumption of each area or sector. The thrust of arguments of the supporters of this outlined allocation method is that each method should be used to address the problem based on the peculiarity of an area provided that such a method should not result in generous assumptions about demand growth with the possible effect of chronically overestimating the supply.

These arguments as sound as they seem, lack the rational basis which constitutes the most objective criteria for water allocation to consumers. We believe, however, that to achieve a rational water allocation system, a way of determining an optimal share of the available quantity of water to each sector should be a necessary proposition. This solution is achieved in this study by the use of the linear programming (LP) mathematical modelling technique which is designed to optimise the usage of limited

E. E. Ezenwaji (✉) · N. I. Igu  
Department of Geography and Meteorology,  
Nnamdi Azikiwe University, Awka, Nigeria  
e-mail: emmaezenwaji@gmail.com

R. N. C. Anyadike  
Department of Geography, University of Nigeria,  
Nsukka, Nigeria

resources which in this study is the limited quantity of water supplied to consumers. Linear programming has been used variously in business, energy and transportation studies by researchers in the field to determine the best method of combining resources for optimal benefits. For example, Wu (1989) employed the technique in the study of development projects as well as their potential contribution to decision makers in arriving at sensible and sound decisions so that scarce urban resources can be utilised efficiently in parts of South East Asia. Also, Oyebanji and Ogunlowo (1999) used linear programming technique to examine whether resources for production in a bread industry in southwest Nigeria are efficiently and effectively combined. Furthermore, Dragičević and Bojić (2009) used the technique to study how the total cost for energy used for net cost in steam condensing systems could be minimised. Finally, Mokebe and Joubert (2013) used the technique to optimise pulp stock production process in South Africa. However, the use of this technique has been very minimal in geographical studies except perhaps that of Abro (1986) that employed the method in the study of the possible combination of water resources for the efficient operation of Aswan dam in Egypt. The use of LP in geographical research in Nigeria has been undertaken by authors such as Onokala (1982) who employed the technique in the study of the optimal pattern in the evacuation of cocoa in Nigeria, and Ayadu (1988) who used the technique in the allocation of land for the cultivation of varieties of cassava in Agbor area of Delta State, Nigeria. In water supply studies no work known to us has been carried out on the allocation of municipal water supply in Nigeria through the use of this technique.

## Materials and methods

### Area of study

Enugu urban area consists of three local government areas (LGAs), namely: Enugu North, Enugu South and Enugu East (Fig. 1). It is located approximately between latitudes  $06^{\circ}30'$  and  $06^{\circ}4'N$  and longitudes  $07^{\circ}20'$  and  $07^{\circ}35'E$ . The urban area has a total area of 145.8sqkm (Ezenwaji, 2009) and bound in the northeast by Isi Uzo LGA, northwest by Igbo-Etiti LGA, in the northwest by Udi LGA, and south by Nkanu West and Nkanu East LGAs.

Geologically, it lies in the eastern Nigeria sedimentary basin which is underlain by Enugu shales, the lower coal measure (Mamu formation) and false bedded sandstone (Ajalli formation). Its topographical features can be classified into two, namely; the escarpment zone of the west and the plains in the east. The climate of the area is the tropical wet and dry according to Koppen's climatic

classification system. The mean daily maximum temperature is usually above  $27^{\circ}C$  throughout the year and mean rainfall totals of about 1,800 mm is common. Rainy season starts in March and ends in November.

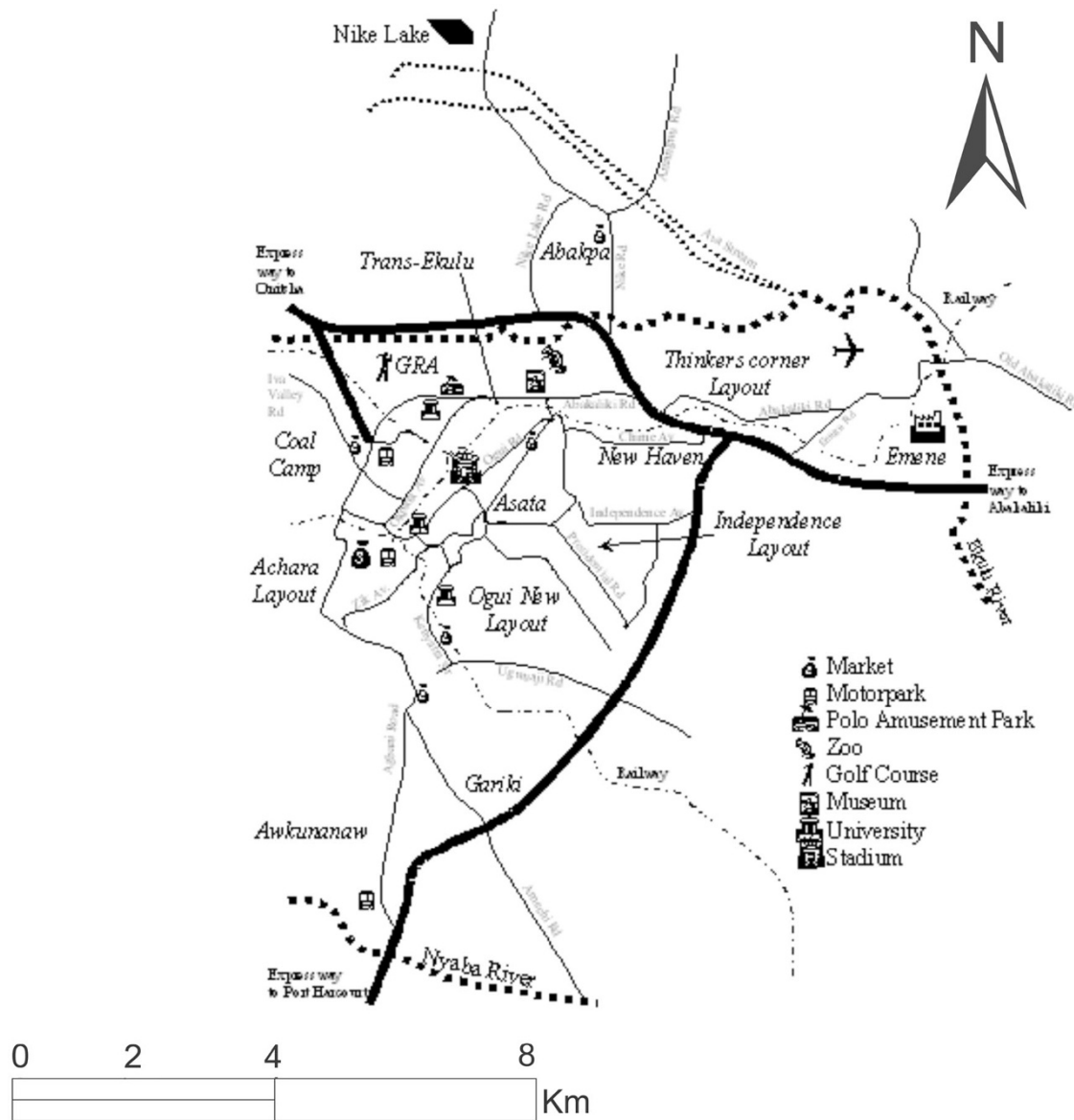
The town is drained by two main rivers namely Ekulu and Nyaba. The population of the urban area was 772,664 in 2006 according to that year's national population census but has now grown to 834,219 in 2011 when the 2006 figure was projected with 3 % annual growth rate recommended by the National Population Commission (NPC 2010). Enugu like other urban areas in the developing countries has a number of sectors. Four of such sectors namely Residential, Commercial, Industrial and the Public institutions are prominent in Enugu. The residential structure of Enugu show 41 wards with Emene (12.1 sqkm) being the largest ward while, Nkwo Nike (0.6 sqkm) being the least, in terms of area of land. Also the commercial establishments are well established, among them are

Hotel establishments, car washing centres, laundry and dry cleaning services, mini and supermarkets, banks, etc. The public institutions sector has Fire Fighting establishments, army and police barracks, public hospitals and schools among its prominent establishments and finally the industrial sector has a wide range of industries including the processing and the manufacturing ones.

### Sample selection and collection

Our sample selection for the four major sectors was done as follows: In the residential sector, a total of 9,020 out of 86,970 or 10.4 % households were purposively selected. The same method was employed in the commercial sector where 426 out of 806 commercial establishments or 53 % was selected. In the public institutions sector, their small number of 21 made us to sample all of them while out of 95 industries 75 or 78.9 % were sampled. The large percentage of establishments sampled in the commercial public institutions and industries were expected because of the small sizes of their target population. However, the relatively low percentage sampled in the residential sector was occasioned by cost implication, poor access to the households and our inability to recruit about 400 field research assistants required.

Primary data were, therefore, collected from each of the four sectors through questionnaire, interviews and direct observations, while secondary data were obtained from published materials. Questionnaires were designed in five sets to collect detailed and relevant data. Four out of these five were meant for each of the four sectors, while the fifth was served on the State Water Corporation. Two sampling techniques were utilised, namely the stratified and simple random sampling. For our stratified sampling, each sector was divided into strata in which every stratum was allocated a population of the total sample size (the percentage



**Fig. 1** Map of Enugu urban area of Nigeria

used in the sampling has been stated earlier). Then the simple random was undertaken in each ward with the use of random numbers.

The data generated from the questionnaire and other sources on the causes of water supply shortage were used to model the allocation of water supply in the town. A total of 11 variables accounted for urban water supply shortages in the study area, many of which were in line with the variables identified by some researchers in the field (Chima 1989; Obeta 2003). Our fieldwork experience shows that many of these factors are among the fundamental causes of water supply shortages in the study area. The variables (factors) which are mainly physical, economic and infrastructural were labelled and described (Table 1).

#### Data analysis

In calculating the quantity of water demand and supply in each sector, we employed objective units of enquiry to organise the data. For example, in the residential sector, our unit of water demand investigation is the head of household which according to Darr et al. (1975; 1976) is a married person with a family or an unmarried person of at least 20 years of age, living alone and earning income. We calculated the total quantity of water requirements in this sector by ascertaining the daily demand and supply amounts of each household and used it to multiply for the number of days in the year to arrive at the annual supply and demand of each household. The summation of the total

**Table 1** Variables responsible for water supply shortages in Enugu urban area

Label	Variable description
DIST	Distance of the source of municipal water supply to consumers
HOUR	Number of hours of electricity supply per day
MAIN	Average monthly recovery of operation and maintenance costs by the State Water Corporation
NUMB	Numbers of consumers in residential commercial public institutions and industrial sectors without water pipe lines
PRESE	Number of storage reservoir that serve the four sectors in the town
VEHL	Number of service vehicles (cars and tankers) at the disposal of the water corporation in 2008
FUND	Funds released to the State Water Corporation by the State Government in 2007
DIES	Average monthly diesel purchased per day by the State Water Corporation
PUMP	Number of faulty and unrepaired pumps recorded by the State Water Corporation
STREM	Number of streams with poor access roads from where the State Water Corporation tankers obtain water supply

supply and water demand of all households gave us requirements of the residential sector figures. This same method was employed in the calculation of water demand and supply in other sectors. In the commercial sector various units of the subsectors were employed. For example for the restaurants we used the number of persons patronizing the establishment per day and at the airport we employed, the number of daily passengers. In the public institutions establishment same was the case, and in the industrial establishments, we estimated water demand and supply based on the physical output by units of each of the industries. The results of these computations are shown in Table 2.

However, the major analytical tool employed was the Linear Programming mathematical model performed with the Lindo 7.0 computer software. This technique is designed to optimise the usage of limited resources which in our own case is water supply. The number of units utilised under each sector are residential (41), commercial (14), public institutions (6) and the industrial (13). These and the variables responsible for water shortages in Enugu urban area were utilised by model.

## Result and discussion

The optimum allocation of water supply to the sector in Enugu urban area was achieved by the model already described which is the simplex algorithm that works through an efficient procedure that searches various subsets of the problem to achieve a feasible solution. Table 3 shows the construction of the model, indicating the objective function and the number of constraints that exist.

The table indicates that there are eleven constraints and the level of usage of individual variables as are shown in the left-hand side (LHS), while the limit of available resources is shown at the last column which is called right-hand side (RHS). With these, we can construct the

objective function of water supply in Enugu urban area. It could be stated at this point that the main objective of Enugu State Water Corporation is to allocate as much water as possible (i.e. maximise) to each of the four sectors. With this in mind, we can state the objective function as;

$$\text{Max } Z = 41x_1 + 41x_2 + 6x_3 + 13x_4 \quad (1)$$

Where  $Z$  = the maximum water to be allocated (in litres)

$x_1, x_2, x_3, x_4$  = the various sectors

The algebraic expression in Table 3 is written as follows:

$$\begin{aligned} 7.5x_1 + 27.5x_2 + 31.2x_3 + 29x_4 &\leq 27.5 \\ 4.0x_1 + 3.0x_2 + 2.0x_3 + 3.0x_4 &\leq 4.00 \\ 1.37x_1 + 1.47x_2 + 1.5x_3 + 2.2x_4 &\leq 5.00 \\ 0.35x_1 + 0.54x_2 + 0.27x_3 + 3 + 1.38x_4 &\leq 1.40 \\ 0.24x_1 + 2.67x_2 + 1.63x_3 + 1.67x_4 &\leq 4.00 \\ 1.28x_1 + 1.44x_2 + 1.47x_3 + 1.19x_4 &\leq 4.00 \\ 0.68x_1 + 1.23x_2 + 1.25x_3 + 1.10x_4 &\leq 4.00 \\ 8.2x_1 + 2.8x_2 + 2.3x_3 + 1.20x_4 &\leq 10.4 \\ 0.3x_1 + 3.1x_2 + 8.1x_3 + 7.20x_4 &\leq 14.00 \\ 2.28x_1 + 1.78x_2 + 1.83x_3 + 2.33x_4 &\leq 4.50 \\ 0.08x_1 + 0.06x_2 + 2.07x_3 + 3.38x_4 &\leq 0.08 \end{aligned} \quad (2)$$

From the above equation, the maximum allocation of the 35,000,000 L of water to the sectors as worked out by the model is achieved (Table 4).

From Table 4, it could be seen that a total of 34,832,000 L out of 35,000,000 L was allocated by the model to various sectors. This leaves a balance of 168,000 L that could be used in emergency situation. The model shows that the residential sector was allocated 27,470,000 L representing 78.9 % of the total amount allocated to all sectors, this leaves the rest of the sectors (3 of them) with an allocation of 7,362,000 L representing 21.1 % of the allocated amounts.

**Table 2** Total daily water demand and supply for the four sectors of Enugu urban area

Sector	Total water demand (L)	Total water supply (L)	Deficiency	% Deficiency	Total supplied by the water corporation
Residential	119,120,551	61,981,756	57,138,795	48	22,219,428
Commercial	9,139,593	3,974,865	5,164,728	56	2,111,139
Public institution	19,084,074	11,976,136	7,107,938	37	4,829,621
Industrial	60,048,548	43,165,140	17,883,638	30	5,839,813

Source: field work (2009)

**Table 3** Modelling of optimum water supply in Enugu urban area

Constraints	Objective function: $\text{Max } Z = 41x_1 + 14x_2 + 6x_3 + 13x_4$				
	Level of usage of available resources (LHS)				Max availability of limiting factors (RHS)
	Residential	Commercial	Public institution	Industrial	
DIST (km)	7.50	27.50	31.20	29.00	27.50
HOUR (h)	4.00	3.00	2.00	3.00	4.00
MAIN (M)	1.37	1.47	1.50	2.20	5.00
NUMB (M)	0.35	0.54	0.27	1.38	1.40
PRESE (L)	0.24	0.67	1.63	1.67	4.00
VEHL (L)	1.28	1.44	1.47	1.19	4.00
DIES (Drums)	8.20	2.80	2.30	1.20	10.40
PIPE (M)	0.30	3.10	8.10	7.20	14.00
PUMP (No)	2.28	1.78	1.83	2.30	4.50
STREM (No)	0.08	0.06	2.07	2.38	0.80

**Table 4** Optimal allocation of water supply to the sectors

Total quantity supplied (L)	Sectors				Total quantity allocated (L)
	Residential $X_1$ (L)	Commercial $X_2$ (L)	Public institution $X_3$ (L)	Industrial $X_4$ (L)	
35,000,000	27,470,000	3,360,000	882,000	3,162,000	34,832,000

**Table 5** Comparison between model allocation and water supply to the sectors

Sector	Water supply by the water corporation (L)	Model allocation to the sectors (L)
Residential	22,219,428	27,470,000
Commercial	2,111,139	3,162,000
Public institution	4,829,621	882,000
Industrial	5,839,812	3,162,000
Total	35,000,000	34,832,000

Table 5 gives a comparison between the model allocation and the quantity of water supplied to the sectors by the water corporation.

It could be seen from Table 5 that the water corporation supplies all water in its possession while the model left a little (168,000 L) to handle very critical situations. Also it is now seen that the water corporation was undersupplying the

residential sector to the extent of 5,250,572 L. Same for commercial sector with 1,248,861 L, but in the public institution and industrial sectors the situation is different as they were over supplied by 3,947,621 L in the public institution and 2,677,812 L in the industries. This model allocation will provide a necessary guide for the planning of a realistic water supply system for the town. The application of this model technique to water supply is indeed unique as it has been variously used in transportation and agriculture. However, if the State Water Corporation adopts water supply management practices based on this technique then it will go a step in solving the age-long water supply problems in the town.

**Conclusion**

From our analysis, there is a clear difference in the quantity of water being allocated to the sectors by the State Water

Corporation and that achieved by the model. The basic lesson learnt from this revelation is that the prevailing allocation method by the State Water Corporation is inefficient to the extent that water is oversupplied to some sectors and seriously undersupplied to others. This calls to question the water supply criteria being used by the Water Corporation. Since Enugu town has the same social-economic character of other Nigerian towns and to a great extent with urban areas in developing countries, any plan dealing with the allocation of public water supply to the urban sectors within these regions should be based on the criteria specified by this model. Thus the allocation of water with this model technique assures of optimal supply of water to strategic consumers on the satisfaction of the prevalent constraints. The implication of this research which can be applied internationally is that water is a scarce resource in most urban areas especially in developing countries. The reoccurring water supply shortages relative to demand has been a source of worry to many public water supply managers in these urban areas. The situation as presently experienced is that water which is manufactured at a very great financial cost is often over supplied in some sectors and undersupplied in others. This undoubtedly gives rise to water wastages to the over supplied sectors which could be translated into unnecessary fund leakages in the system. The areas that are undersupplied very often resort to the consumption of water from doubtful sources which equally gives rise to the affliction of waterborne diseases and unnecessary expenditure of lean income of the sector to procure it. All these show that the supply of water in that manner in any urban area will lead to various shades of water supply unsustainability. Water managers especially in urban areas can take advantage of this finding to improve their water allocations to various urban sectors within their jurisdiction.

**Open Access** This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

## References

Abro JO (1986) Linear programming application in dam renovation. Lessons from Aswan Dam. *Hydrol Stud* 3(1):216–228

- Ajavi OO (2007) Domestic water use in parts of Akure, Ondo State. *J Water Supply* 4(2):60–78
- Ayadu KD (1988) An application of linear programming technique in the allocation of agricultural land in Agbor Area, Bendel State. *J Rural Dev* 2(2):1112–1121
- Chima GN (1989) Rural water supply in Isiala Ngwa L.G.A of Imo State, Nigeria. Unpublished M.Sc. Thesis, Department of Geography University Of Nigeria, Nsukka
- Darr P, Feldman S, Kamen C (1975) Socio-economic factors affecting domestic water demand in Israel. *Water Resour Res* 11(6):805–809
- Darr P, Fieldman S, Kamen C (1976) The demand for urban water: Akansas case study. *Land Econ* 39(3):204–210
- Dragičević S, Bojić M (2009) Application of linear programming in energy management. *Serbian J Manag* 4(2):227–238
- Ezenwaji EE (2003) Urgent water demand management in Nigeria. In: Paper Delivered at the 29th WEDC International Conference in Abuja, Nigeria
- Mokebe KD, Joubert JW (2013) Application of linear programming models to optimise pulp stock production process. *Tappsa J* 1:20–26
- National Population Commission (2010) Projection of 2006 Population Figure Mimeographed
- Nnodu VC, Ilo IC (2001) Comparative quality evaluation of sources of domestic water supply in Enugu urban area. *Environ Rev* 3(1):576–582
- Obeta MC (1997) The spatial pattern of residential water demand and supply in Nsukka, urban area of Enugu State Nigeria. Unpublished M.Sc. Thesis, University of Nigeria, Nsukka
- Obeta MC (2003) Patterns and problems of rural water supply in Enugu State, Nigeria. Unpublished Ph.D Thesis, University of Nigeria, Nsukka
- Odubo MO, Ikio FA (2008) Urban water supply in the 21st Century Africa. In: Donkor DO (ed) African regional development in the 21st Century. Pal Publishers, Accra, Ghana
- Onokala PC (1982) Optimal pattern in the evacuation of Cocoa in Nigeria. *Nigeria Geogr J* 25:3–6
- Oyebanji MO, Ogunlowo AS (1999) Application of linear programming technique in bakery: a case study of Omo-Oroki bakery. Unpublished M.Sc. Dissertation, Federal University of Technology, Akure
- Uzoagbala EM (2006) Water demand management and water scarcity problems in Nigeria Towns. *Sub Sahara Bull* 4(2):84–90
- Wu YM (1989) Application of linear programming—a case study. *Land Dev Stud* 6(3):201–216