



Fuzzy goal programming approach to solve fully fuzzy multi-objective quadratic programming problem

Admasu Tadesse¹ · M. M. Acharya¹ · Srikumar Acharya¹ · Manoranjan Sahoo¹

Received: 5 February 2022 / Revised: 8 June 2022 / Accepted: 27 June 2022 / Published online: 24 July 2022

© The Author(s) under exclusive licence to The Society for Reliability Engineering, Quality and Operations Management (SREQOM), India and The Division of Operation and Maintenance, Lulea University of Technology, Sweden 2022

Abstract The focus of this paper is to suggest a solution methodology for a fully fuzzy multi-objective quadratic programming problem. Solving the provided mathematical model using known classical methods is extremely difficult. To solve the present mathematical programming problem, three major approaches are suggested. In first step, we used arithmetic operations between two fuzzy parameters and variables. The importance is given in the next step to handle fuzzy part of objective functions by ranking functions and after the completion of the second step, the fuzzy part of the fuzzy constraints tackled by the inequality property of between two triangular fuzzy numbers. Finally, the transformed multi-objective quadratic mathematical programming problem is solved using a weighted fuzzy goal programming approach. The final solution of the suggested model is derived using existing methodology and softwares. The working procedures of the proposed method is further discussed using numerical example.

Keywords Multi-objective programming · Triangular fuzzy number · Weighted fuzzy goal programming · Fuzzy decision variables · Quadratic programming · Fuzzy quadratic programming

1 Introduction

Real life problems such as production planning, water resource management etc., involve multiple conflicting objectives subject to constraints. The optimization problem with two or greater than two objective functions which are conflicting in nature is called multi-objective optimization problem. Thus the model under these multiple objective is called multi-objective programming model. The concept of multi-objective decision making problems was adopted and modified by different researchers namely; Zimmermann (1978); Yeager (1979); Henan (1979); Rommelfanger (1989); Chakraborty and Gupta (2002) and etc.

A quadratic programming (QP) problem is a optimization problem in which a quadratic function must be maximized or minimized subject to linear constraints (Rout et al. 2019). Unlike the linear programming problem the solutions procedure for QP problem is slightly complex. Some economic and business problems can be expressed mathematically and are directly applicable to the QP. Several algorithms have been devised to solve QP problems, including Powell (1985); Lawson and Hanson (1995); Goldfarb and Idnani (1983), and others. A QP problem involving two or more than two objectives functions are termed as multi-objective quadratic programming (MOQP) problem. The uncertainties in the parameters of a programming problems are due to fuzziness or randomness. The one arising from randomness can be incorporated through a probability distribution and other from fuzziness can be characterized by fuzzy numbers. A MOQP problem with all or some of its input parameters are assumed to be fuzzy number is called multi-objective fuzzy quadratic programming problem. Authors, like Sert et al. (2015); Yue et al. (2019) and Yang et al. (2020) suggested MOQP problem under the environment of fuzziness. Most of the

✉ Admasu Tadesse
jigina84@gmail.com

M. M. Acharya
mitali.acharya@kiit.ac.in

Srikumar Acharya
sacharyafma@kiit.ac.in; srikumar.kgp@gmail.com

Manoranjan Sahoo
mrshoofma@kiit.ac.in; msahoo@gmail.com

¹ Department of Mathematics, School of Applied Sciences, KIIT University, Bhubaneswar, India

time fuzziness of the mathematical programming problem is due to fuzziness appearing in different parts of a programming problem for example the right hand side part, constraint coefficient or the objective function coefficients or involved as decision variables. Taghi-Nezhad and Taleshian (2018) suggested a solution approach for solving fuzzy QP problem where the cost coefficients, the matrix of the quadratic form, constraints coefficients, and the righthand sides are all fuzzy numbers. Mansoori et al. (2018) introduced fuzzy QP problem with fuzzy parameters coefficients. Fathy (2019) introduced multi-objective and multi-level integer fuzzy QP problems, where all the parameters and decision variables of the programming problem are fuzzy number. Nasserri et al. (2019) suggested a fuzzy multi-objective quadratic model with variable constraints for optimal water resource allocation among different water-user sectors. Mussafi and Ismail (2021) exploited the QP problem to get the best risk level for risk-adjusted Islamic portfolios recently.

In majority of the literatures, authors assumed the decision parameters as fuzzy numbers while the decision variables as crisp ones. Since the variables are crisp, the solution obtained as the crisp, which is a real number. The solution is exact value in the fuzzy programming problems with fuzzy parameters. The fuzzy aspect of the decision is partly lost in this case so, it is reasonable and important to consider fuzzy mathematical programming problem with fuzzy decision variables. A fully fuzzy multi-objective quadratic programming (FFMOQP) problem is proposed in this paper where all the input parameters and decision variables of the programming problems are assumed to be triangular fuzzy number and triangular fuzzy decision variables respectively.

One of the many mathematical programming techniques that can be utilized to handle the problem of multi-objective optimization is called goal programming (GP) method. More importantly, fuzzy goal programming (FGP), has been shown to be an effective method for handling future aspirations for decision makers even when those making the decisions do not have precise knowledge about the future goals. In other word FGP is an extension of conventional GP used to solving multi-objective decision problems with imprecisely defined model parameters. FGP approach is used by a number of researchers to solve multi objective optimization problem. Narasimhan (1980); Ignizio (1982) and Aouni et al. (2009) are researchers who took initiative on solving problems employing FGP method. Lachhwani (2012) suggested a FGP procedure for fuzzy multi-objective quadratic programming problem. Pramanik and Dey (2011) proposed a priority based FGP approach to multi-objective quadratic programming problem. Some the recent work on this aspects are Dalman and Bayram (2017); Ambad and

Kulkarni (2017); Hossain and Hossain (2018); Ali et al. (2019); Kundu and Islam (2019); Rivaz et al. (2020); Rani et al. (2021), etc.

The different FGP formulations can be classified as: weighted FGP, lexicographic FGP, fuzzy MIN-MAX Goal Programming and interactive FGP. Weighted FGP is used in this paper. Generally, the decision makers appreciation of the positive and the negative deviations from the fuzzy goals can be different according to the relative importance of the objective, which can be revealed through the weights of each objective under considerations.

In this paper computation between two fuzzy numbers and variables, is done by using fuzzy arithmetic operations and we have used ranking function and property of fuzzy inequality for fuzzy objective functions and fuzzy constraints respectively to remove fuzziness. Weighted FGP (WFGP) approximation is used to obtain efficient solutions of crisp multi-objective programming problem.

The rest of the work is categorized as follows: basic preliminary is discussed in Sect. 2, after the introduction section. The mathematical model and the the solution procedures respectively are defined in Sect. 3 and in Sect. 4. Numerical example is provided for illustration of the methodology in Sect. 5. Result and discussion are included in Sect. 6. The paper is concluded in Sect. 7 followed by supportive references.

2 Basic preliminaries

Definition 2.1 For $(a^{(p)}, a, a^{(o)}) \in \mathbb{R}$, the membership function $\mu_{\tilde{a}}(x)$ of triangular fuzzy(TF) number $(a^{(p)}, a, a^{(o)}) = \tilde{a}$

$$\text{is defined as: } \mu_{\tilde{a}}(x) = \begin{cases} 0, & x \leq a^{(p)} \\ \frac{x-a^{(p)}}{a-a^{(p)}}, & a^{(p)} \leq x \leq a \\ \frac{a^{(o)}-x}{a^{(o)}-a}, & a \leq x \leq a^{(o)} \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.2 For two TF numbers $(a^{(p)}, a, a^{(o)}) \approx \tilde{a}$ and $(b^{(p)}, b, b^{(o)}) \approx \tilde{b}$, the following fuzzy operations between two fuzzy numbers hold true:

- (i) $(a^{(p)}, a, a^{(o)}) \oplus (b^{(p)}, b, b^{(o)}) \approx (a^{(p)} + b^{(p)}, a + b, a^{(o)} + b^{(o)})$
- (ii) $(a^{(p)}, a, a^{(o)}) \ominus (b^{(p)}, b, b^{(o)}) \approx (a^{(p)} - b^{(p)}, a - b, a^{(o)} - b^{(o)})$
- (iii) $k(a^{(p)}, a, a^{(o)}) \approx (ka^{(p)}, ka, ka^{(o)}), k \geq 0$
- (iv) $k(a^{(p)}, a, a^{(o)}) \approx (ka^{(p)}, ka, ka^{(o)}), k \leq 0$
- (v) $(a^{(p)}, a, a^{(o)}) \otimes (b^{(p)}, b, b^{(o)}) \approx (a^{(p)}b^{(p)}, ab, a^{(o)}b^{(o)}),$ if $a^{(p)} \geq 0$ and $b^{(p)} \geq 0$
- (vi) $(a^{(p)}, a, a^{(o)}) \otimes (b^{(p)}, b, b^{(o)}) \approx (a^{(p)}b^{(o)}, ab, a^{(o)}b^{(o)}),$ if $a^{(p)} < 0, a^{(o)} \geq 0, b^{(p)} \geq 0$
- (vii) $(a^{(p)}, a, a^{(o)}) \otimes (b^{(p)}, b, b^{(o)}) \approx (a^{(p)}b^{(o)}, ab, a^{(p)}b^{(o)}),$ if $a^{(o)} \leq 0$ and $b^{(p)} \geq 0$

Definition 2.3 For two TF numbers $\tilde{a} \approx (a^{(p)}, a, a^{(o)})$ and $\tilde{b} \approx (b^{(p)}, b, b^{(o)})$, the following properties are true

- (i) $\tilde{a} \approx \tilde{b}$ iff $a^{(p)} \approx b^{(p)}, a \approx b$ and $a^{(o)} \approx b^{(o)}$
- (ii) $\tilde{a} \approx (a^{(p)}, a, a^{(o)}) \geq 0$ iff $a^{(p)} \geq 0$

Definition 2.4 A ranking function is a function $\mathbb{R} : F(R) \rightarrow R$ that maps each fuzzy number into the real line. $F(R)$ is a set of fuzzy numbers defined on set of real numbers. Let $(a^{(p)}, a, a^{(o)}) \approx \tilde{a}$ be a TF number, then $\mathbb{R}(\tilde{a}) \approx \frac{a^{(p)} + 2a + a^{(o)}}{4}$

Definition 2.5 [Pareto-optimal solution (Abebaw Gessesse et al. 2020)]: For a multi-objective programming problem, a feasible solution x^* is said to be pareto-optimal or optimal compromise solution if and only if there does not exist any other feasible solution x such that $Z^{k_1}(x) \leq Z^{k_1}(x^*)$ for $\forall k_1 \in 1, 2, \dots, K$ and $Z^{k_2}(x) < Z^{k_2}(x^*)$ for $\forall k_2 \in 1, 2, \dots, K$. The collection of all the Pareto optimal solutions in the decision space is called the Pareto optimal set (PS), and the projection of PS in the objective space is called the Pareto optimal front (PF).

Definition 2.6 [Multi-objective optimization problem (Abebaw Gessesse et al. 2020)]: Many real-world optimization problems involve multiple conflicting objectives, known as multi-objective optimization problems, which can be mathematically formulated as follows:

$$\min : Z(x) = (Z^1(x), Z^2(x), \dots, Z^k(x))$$

subject to

$$x \in X,$$

where X is the search space of decision variable with $X = x_1, x_2, \dots, x_k$ denoting the decision vector.

Definition 2.7 When each of the objectives is given an imprecise aspiration level, these objectives are referred to be fuzzy goals. The fuzzy goals are now defined in the area of fuzzy programming by their related membership functions. According to Gupta and Chakraborty (1997) the membership function for the i th fuzzy goal can be defined as:

$$\mu_i(d_i(x)) = \begin{cases} 0, & \text{for } d_i \geq p \\ \frac{p-d_i(x)}{p}, & \text{for } 0 \leq d_i(x) \leq p \\ 1, & \text{for } d_i(x) \leq 0, \end{cases}$$

where the distance function: $d_i(x) = |\bar{Z}_i - Z_i(x)|$, $i = 1, 2, 3, \dots, k$. This distance depends up on x . At $x = \bar{x}$ (ideal point in x -space) $d_i = 0$ and at $x = \underline{x}$ implies $Z_i(x) = \underline{Z}_i$. The maximum value of $d_i(x)$ can be calculated as $\bar{d}_i = |\bar{Z}_i - \underline{Z}_i|$ for $p = \sup\{\bar{d}_i\}$

3 Mathematical model

FFMOQP problem with fuzzy decision variables is of the form:

$$\max : \tilde{Z}_k \approx \sum_{j=1}^n \tilde{c}_j^{(k)} \otimes \tilde{x}_j \oplus \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \tilde{q}_{ij}^{(k)} \tilde{x}_i \tilde{x}_j, (k = 1, 2, \dots, K) \tag{3.1}$$

Subject to

$$\sum_{j=1}^n \tilde{a}_{ij} \tilde{x}_j \leq \tilde{b}_i, i \in \{1, 2, \dots, m\} \tag{3.2}$$

$$\tilde{x}_j \geq 0, j \in [1, 2, \dots, n] \tag{3.3}$$

where $\tilde{q}_{ij} = (q_{ij}^p, q_{ij}, q_{ij}^o)$ is constant symmetric matrix assumed to be triangular fuzzy number,

$\tilde{c}_j = (c_j^p, c_j, c_j^o)$ is cost coefficient assumed to be triangular fuzzy number,

$(a_{ij}^p, a_{ij}, a_{ij}^o) = \tilde{a}$ and $(b_i^p, b_i, b_i^o) = \tilde{b}_i$ are triangular fuzzy constraint coefficients and

$\tilde{x}_1 = (x_1^p, x_1, x_1^o) = (x_1, x_2, x_3)$ and $\tilde{x}_2 = (x_2^p, x_2, x_2^o) = (x_4, x_5, x_6)$ are triangular fuzzy decision variables and their representations.

3.1 Fuzzy goal programming formulation

From the defined membership function 2.7 above, the flexible goals with aspired level 1 is defined, thus the fuzzy goal programming(FGP) problem becomes: Lachhwani (2012)

$$\frac{p - d_i}{p} + D_i^- - D_i^+ = 1, i = 1, 2, \dots \tag{3.4}$$

(or)

$$-\bar{Z}_i + Z_i(x) + pD_i^- + pD_i^+ = 0, \forall i = 1, 2, \dots \tag{3.5}$$

The i th membership goal with aspired level 1 can presented as:

$$C_i x + x^T H_i x - \bar{Z}_i + pD_i^- - pD_i^+ = 0 \tag{3.6}$$

where $D_i^- (\geq 0)$ and $D_i^+ (\geq 0)$ with $D_i^- D_i^+ = 0$ represent under or over deviational variables respectively form the aspired levels.

3.2 Weighted fuzzy goal programming formulation

In conventional GP, the under and/or over deviational variables are included in the achievement function for minimizing them and that depends upon the type of the

objective functions to be optimized. In this work, only the under deviational variables D^- is required to be minimized to achieve the aspired levels of the fuzzy goals. It may be noted that when a membership goal is fully achieved, the we get $D^- = 0$ and $D^+ = 1$. The weighted WFGP approach becomes, find x so as to minimize:

$$\sum_{i=1}^K \omega_i D_i^- \tag{3.7}$$

Subject to

$$C_i x + x^T H_i x - \bar{Z}_i + p D_i^- - p D^+ = 0 \tag{3.8}$$

$$x \in S = \{x \in R^n : Ax \leq b, x \geq 0\}, \tag{3.9}$$

In the above model C_i, H_i are row vectors with n -components and x and b are column vectors with n and m -components. $x^T H_i x$ is a strictly concave quadratic function on the convex set S of all its possible solutions, with a global unique maximum of $Z_i(x)$ for every $i = 1, 2, \dots, k$ over S . The relative importance of obtaining the sought level of the different fuzzy objectives subject to the constraint established in the decision-making situation is represented by the $\omega_i \geq 0$ ($i = 1, 2, \dots$).

4 Solution procedures

This section presents an approach for solving a multi-objective fully fuzzy QP problem (Table 1).

The solution methodologies are discussed in depth as follows:

Step 1. Ranking function method is applied on Eq. (3.1) and to handle fuzzy QP programming problem and moreover, the property of inequality between two fuzzy numbers are used for defuzzification of fuzzy constraints, (3.2)–(3.3). Thus the programming problem becomes:

$$\begin{aligned} \max : Z_k = & \frac{1}{4} \sum_{j=1}^n ((q_{ij}^k)^p x_j^p + 2q_{ij}^k x_j + (q_j^k)^o x_j^o) + \frac{1}{8} \sum_{i=1}^n \sum_{j=1}^n ((q_{ij}^k)^p x_i^p x_j^p \\ & + 2q_{ij}^k x_i x_j + (q_{ij}^k)^o x_i^o x_j^o), k \in \{1, 2, \dots, K\} \end{aligned} \tag{4.1}$$

Subject to

$$\sum_{j=1}^n a_{ij}^p x_j^p \leq b_i^p, i \in \{1, 2, \dots, m\} \tag{4.2}$$

$$\sum_{j=1}^n a_{ij} x_j \leq b_i, i \in \{1, 2, \dots, m\} \tag{4.3}$$

$$\sum_{j=1}^n a_{ij}^o x_j^o \leq b_i^o, i \in \{1, 2, \dots, m\} \tag{4.4}$$

$$x_j^o - x_j \geq 0, j \in \{1, 2, \dots, n\} \tag{4.5}$$

$$x_j - x_j^p \geq 0, j \in \{1, 2, \dots, n\} \tag{4.6}$$

$$x_j^p \geq 0, j \in \{1, 2, \dots, n\} \tag{4.7}$$

Step 2 Use any commercially available optimization packages such as LINDO, LINGO, CPLEX and etc. to solve the defined problem in step 1 individually to obtain the ideal solutions.

Step 3. Create payoff matrix.

Step 4. Find the upper (\bar{Z}_i) and lower (Z_i) bound of each objective function at each ideal solution resulted from optimizing single objective.

Step 5. Compute the distance $\bar{d}_i = |\bar{Z}_i - Z_i|$.

Step 6. Find $p = \sup\{\bar{d}\}$

Step 7. For each objective function, define a membership function in light of definition 2.7.

Step 8. Formulate the WFGP from all the above steps.

Step 9. Solve the WFGP model with the purpose of minimizing the weighted sum of deviations from the goal.

Step 10. The resultant crisp non-linear mathematical model is solved by LINGO software and fuzzy optimal solution $\tilde{x}_i = (x_1, x_2, x_3)$ is obtained.

Note: $(a^p, a, a^o) \approx \tilde{a}$ and $(b^p, b, b^o) \approx \tilde{b}$ and $(x^p, x, x^o) \approx \tilde{x} \approx (x_1, x_2, x_3)$

The general solution procedure of FFMOQP problem is depicted by the following flowchart (Fig. 1):

5 Numerical example

$$\begin{aligned} \max : \tilde{Z}_1 \approx & (10, 14, 18) \otimes \tilde{x}_1 \oplus (4, 10, 14) \\ & \otimes \tilde{x}_2 \ominus (0, 2, 3) \otimes \tilde{x}_1^2 \ominus (1, 3, 5) \otimes \tilde{x}_2^2 \end{aligned} \tag{5.1}$$

$$\begin{aligned} \max : \tilde{Z}_2 \approx & (5, 15, 25) \otimes \tilde{x}_1 \oplus (5, 15, 25) \\ & \otimes \tilde{x}_2 \ominus (0.5, 1, 1.5) \otimes \tilde{x}_1^2 \ominus (0.5, 1, 1.5) \otimes \tilde{x}_2^2 \end{aligned} \tag{5.2}$$

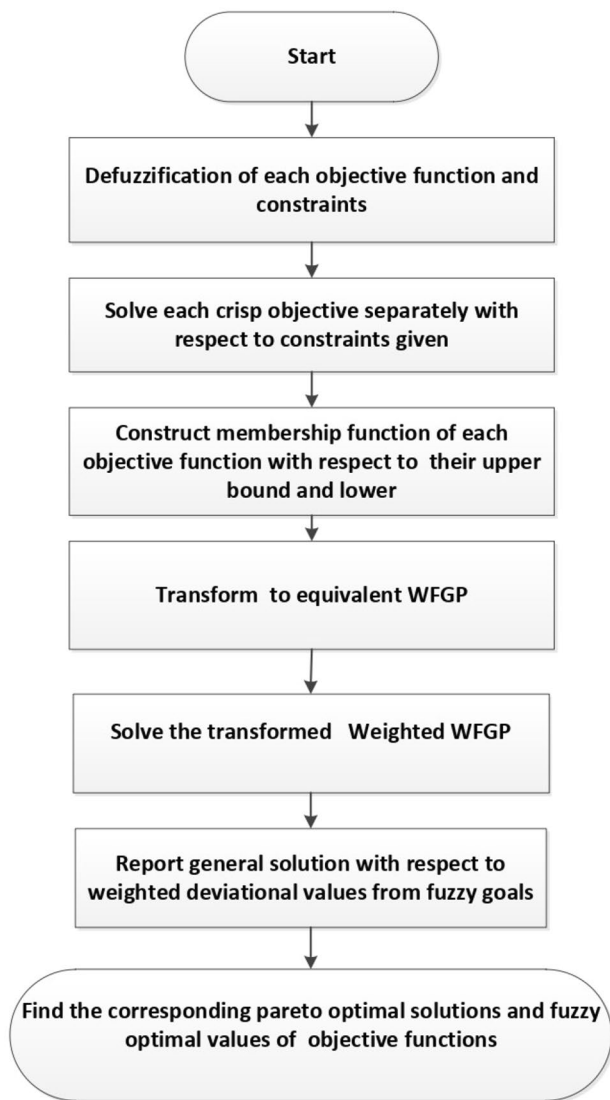


Fig. 1 Flowchart on general solution procedures of multi-objective fuzzy QP problem

$$\max : \tilde{Z}_3 \approx (8, 24, 40) \otimes \tilde{x}_1 \oplus (5, 15, 25) \otimes \tilde{x}_2 \ominus (0.5, 1, 1.5) \otimes \tilde{x}_1^2 \ominus (0.5, 1, 1.5) \otimes \tilde{x}_2^2 \tag{5.3}$$

Subject to

$$(0.5, 1, 1.5)\tilde{x}_1 \oplus (0.5, 1, 1.5)\tilde{x}_2 \leq (1, 3, 5) \tag{5.4}$$

$$(2, 4, 6)\tilde{x}_1 \oplus (0.5, 1, 1.5)\tilde{x}_2 \leq (6, 7, 8) \tag{5.5}$$

$$\tilde{x}_i \geq 0, \quad i = 1, 2 \tag{5.6}$$

Using ranking function for the defuzzification of fuzzy objective function and the property of inequality between fuzzy numbers for defuzzification of fuzzy constraints, we get the following crisp multi-objective QP problem:

$$\begin{aligned} \max : Z_1 \\ = \frac{10x_1 + 4x_4 - 3x_3^2 - 5x_6^2 + 28x_2 + 20x_5 - 4x_2^2 - 6x_5^2 + 18x_3 + 14x_6 - x_4^2}{4} \end{aligned} \tag{5.7}$$

$$\begin{aligned} \max : Z_2 \\ = \frac{15x_1 + 5x_4 - 1.5x_3^2 - 1.5x_6^2 + 30x_2 + 30x_5 - 2x_2^2 - 2x_5^2 + 25x_3 + 25x_6 - 0.5x_1^2 - 0.5x_4^2}{4} \end{aligned} \tag{5.8}$$

$$\begin{aligned} \max : Z_3 \\ = \frac{8x_1 + 4x_4 - 1.5x_3^2 - 1.5x_6^2 + 24x_2 + 30x_5 - 2x_2^2 - 2x_5^2 + 40x_3 + 25x_6 - 0.5x_1^2 - 0.5x_4^2}{4} \end{aligned} \tag{5.9}$$

Subject to

$$0.5x_1 + 0.5x_4 \leq 1 \tag{5.10}$$

$$x_2 + x_5 \leq 3 \tag{5.11}$$

$$1.5x_4 + 1.5x_6 \leq 5 \tag{5.12}$$

$$2x_1 + 0.5x_4 \leq 6 \tag{5.13}$$

$$4x_2 + x_5 \leq 7 \tag{5.14}$$

$$6x_3 + 6x_6 \leq 8 \tag{5.15}$$

$$0 \leq x_1 \leq x_2 \leq x_3 \tag{5.16}$$

$$0 \leq x_4 \leq x_5 \leq x_6 \tag{5.17}$$

The following steps are taken to formulate fuzzy membership function:

- i. Each objective function is separately maximized with respect to the given constraints.
- ii. Then payoff matrix is formulated.
- iii. Lower and Upper bound of the objective functions are identified at all the ideal solutions resulted from step 1.

From the payoff matrix 1, above we can calculate $\bar{d}_1 = |Z_1^U - Z_1^L| = |19.599 - 7.465| = 12.134$, $\bar{d}_2 = |Z_2^U - Z_2^L| = |39.542 - 22.499| = 17.043$, $\bar{d}_3 = |Z_3^U - Z_3^L| = |41.147 - 32.499| = 9.096$. Then $p = \sup\{\bar{d}_i\} = 17.043$ for $i = 1, 2, 3$. The formulation of equivalent WFGP is give as follows:

Table 1 Payoff matrix

	Z_1	Z_2	Z_3
$X^{(1)}$	19.59903	22.49930	32.05057
$X^{(2)}$	7.46529	39.54177	41.04147
$X^{(3)}$	11.17273	39.17848	41.14693

$$\min : \omega_1 D_1^- + \omega_2 D_2^- + \omega_3 D_3^- \tag{5.18} \quad 0 \leq x_4 \leq x_5 \leq x_6 \tag{5.29}$$

Subject to The crisp nonlinear-programming model from 5.18 to 5.29

$$\frac{10x_1 + 4x_4 - 3x_3^2 - 5x_6^2 + 28x_2 + 20x_5 - 4x_2^2 - 6x_5^2 + 18x_3 + 14x_6 - x_4^2}{4} - 19.599 + 17.043D_1^- - 17.043D_1^+ = 0 \frac{15x_1 + 5x_4 - 1.5x_3^2 - 1.5x_6^2 + 30x_2 + 30x_5 - 2x_2^2 - 2x_5^2 + 25x_3 + 25x_6 - 0.5x_1^2 - 0.5x_4^2}{4} \tag{5.19}$$

$$- 39.542 + 17.043D_2^- - 17.043D_2^+ = 0 \frac{8x_1 + 4x_4 - 1.5x_3^2 - 1.5x_6^2 + 24x_2 + 30x_5 - 2x_2^2 - 2x_5^2 + 40x_3 + 25x_6 - 0.5x_1^2 - 0.5x_4^2}{4} \tag{5.20}$$

$$- 41.147 + 17.043D_3^- - 17.043D_3^+ = 0 \tag{5.21}$$

$$0.5x_1 + 0.5x_4 \leq 1 \tag{5.22}$$

$$x_2 + x_5 \leq 3 \tag{5.23}$$

$$1.5x_4 + 1.5x_6 \leq 5 \tag{5.24}$$

$$2x_1 + 0.5x_4 \leq 6 \tag{5.25}$$

$$4x_2 + x_5 \leq 7 \tag{5.26}$$

$$6x_3 + 6x_6 \leq 8 \tag{5.27}$$

$$0 \leq x_1 \leq x_2 \leq x_3 \tag{5.28}$$

is solved in light of three cases shown as below. Case 1: $\omega_1=0.5, \omega_2=0.3, \omega_3=0.2$
 Case 2: $\omega_1=0.5, \omega_2=0.2, \omega_3=0.3$
 Case 3: $\omega_1=0.3, \omega_2=0.5, \omega_3=0.2$
 The following Tables 2 and 3 recapitulate all the results of solved model from 5.18 to 5.29

6 Result and discussion

Average of the three cases (1, 2 and 3) of each fuzzy optimal value of objective function is taken and discussed as follows:
 From the results illustrated in the Tables 2 and 3 it is found that the maximized fuzzy values of objective functions \tilde{Z}_1, \tilde{Z}_2 and \tilde{Z}_3 respectively are (-9.5513, 17.4120, 41.4340), (2.8355, 36.6541, 62.6566) and (5.4192, 36.0772, 78.9210). It is observed that amount of highest membership value achieved by fuzzy objective function, \tilde{Z}_3 is 0.8960. The least amount of membership values achieved by fuzzy objective

Table 2 Fuzzy PO solutions of WFGP model

Fuzzy pareto optimal solutions	Case 1	Case 2	Case 3
\tilde{x}_1	(0.88137, 0.88137, 0.88138)	(0.77780, 0.7778, 0.78)	(0.86528, 0.86528, 0.86528)
\tilde{x}_2	(1.11863, 1.80784, 1.80784)	(1.1111, 2.2222, 2.2222)	(1.13472, 1.87221, 1.87221)

Table 3 Fuzzy optimal values of the three objective functions and the membership values achieved

Weights	\tilde{Z}_1	\tilde{Z}_2	\tilde{Z}_3	$\mu_1(d_1)$	$\mu_1(d_2)$	$\mu_3(d_1)$
Case 1	(- 5.3836, 19.0591, 39.9232)	(3.9323, 36.2931, 48.3173)	(6.5765, 36.9172, 75.2551)	0.91022	0.7863	0.8346
Case 2	(- 15.6122, 17.0869, 43.8799)	(1.131148, 39.4565, 72.2333)	(3.4650, 37.1234, 81.1156)	0.7863	0.8019	0.9016
Case 3	(- 7.6581, 16.0899, 40.4984)	(3.4430, 34.2127, 67.4192)	(6.2161, 34.1909, 80.3930)	0.8998	0.8018	0.9866

function, \tilde{Z}_1 is 0.8299. In general WFGP approach is powerful method to handle multi-objective problems where the aspiration levels are not known certainly.

7 Conclusion

A solution procedure of FFMOQP problem is proposed in this study. All the input parameters and decision variables are considered to be triangular fuzzy number and triangular fuzzy decision variables. Moreover triangular fuzzy number is used as decision solutions. Initially, the fuzzy objective functions are defuzzified by applying the ranking function for triangular fuzzy number on each individual objective function. The inequality constraints involved in the multi-objective QP model with fuzzy parameters and fuzzy decision variables are converted to their crisp equivalent by using the property of inequality among the fuzzy numbers. Finally WFGP approach is applied to handle the crisp multi-objective QP problem. The transformed model is solved by the existing software(LINGO). The numerical example's result has been found to be superior.

The proposed methods employed in this work are very important; to find the crisp equivalent of the fully fuzzy mathematical programming problems and finally to solve the equivalent crisp model accordingly. The study be applied to fully fuzzy real-world decision-making situations including fuzzy supply chain management, portfolio management, and so forth.

Acknowledgements We, the authors, would like to express our gratitude to the researchers whose findings we have mentioned in this study. We would like to extend our gratitude to reviewers and editorial teams for giving us opportunities to revise the manuscript that helps to improve the quality of the research work.

Funding No funding has been done in this paper.

Data availability In the submitted manuscript, all the models developed or used during the study paper are included.

Declarations

Conflict of interest There is no conflict of interest by the authors with anyone whosoever is connected with this research.

References

Abebaw Gessesse A, Mishra R, Acharya MM, Das KN (2020) Genetic algorithm based fuzzy programming approach for multi-objective linear fractional stochastic transportation problem involving four-parameter burr distribution. *Int J Syst Assur Eng Manag* 11(1):93–109

- Ali I, Gupta S, Ahmed A (2019) Multi-objective linear fractional inventory problem under intuitionistic fuzzy environment. *Int J Syst Assur Eng Manag* 10(2):173–189
- Ambad PM, Kulkarni MS (2017) A goal programming approach for multi-objective warranty optimization. *Int J Syst Assur Eng Manag* 8(4):842–861
- Aouni B, Martel J-M, Hassaine A (2009) Fuzzy goal programming model: an overview of the current state-of-the art. *J Multi-Criteria Decis Anal* 16(5–6):149–161
- Chakraborty M, Gupta S (2002) Fuzzy mathematical programming for multi objective linear fractional programming problem. *Fuzzy Sets Syst* 125(3):335–342
- Dalman H, Bayram M (2017) Interactive fuzzy goal programming based on taylor series to solve multiobjective nonlinear programming problems with interval type-2 fuzzy numbers. *IEEE Trans Fuzzy Syst* 26(4):2434–2449
- Fathy E (2019) A modified fuzzy approach for the fully fuzzy multi-objective and multi-level integer quadratic programming problems based on a decomposition technique. *J Intell Fuzzy Syst* 37(2):2727–2739
- Goldfarb D, Idnani A (1983) A numerically stable dual method for solving strictly convex quadratic programs. *Math Program* 27(1):1–33
- Gupta S, Chakraborty M (1997) Multi objective linear programming: a fuzzy programming approach. *Int J Manag Syst* 13(2):207–214
- Henan E (1979) On the efficiency of the product operator in fuzzy programming with multiple objective. *Fuzzy Sets Syst* 2:259–263
- Hossain MS, Hossain MM (2018) Application of interactive fuzzy goal programming for multi-objective integrated production and distribution planning. *Int J Process Manag Benchmarking* 8(1):35–58
- Ignizio JP (1982) On the (re) discovery of fuzzy goal programming. *Decis Sci* 13(2):331–336
- Kundu T, Islam S (2019) An interactive weighted fuzzy goal programming technique to solve multi-objective reliability optimization problem. *J Ind Eng Int* 15(1):95–104
- Lachhwani K (2012) Fuzzy goal programming approach to multi objective quadratic programming problem. *Proc Nation Acad Sci, India Sect A: Phys Sci* 82(4):317–322
- Lawson CL, Hanson RJ (1995), *Solving least squares problems*, SIAM
- Mansoori A, Effati S, Eshaghnezhad M (2018) A neural network to solve quadratic programming problems with fuzzy parameters. *Fuzzy Opt Decis Mak* 17(1):75–101
- Mussafi NSM, Ismail Z (2021) Optimum risk-adjusted islamic stock portfolio using the quadratic programming model: An empirical study in indonesia. *J Asian Financ Econ Bus* 8(5):839–850
- Narasimhan R (1980) Goal programming in a fuzzy environment. *Decis Sci* 11(2):325–336
- Nasseri H, Baghban A, Mahdavi I (2019) A new approach for solving fuzzy multi-objective quadratic programming of water resource allocation problem. *J Ind Eng Manag Stud* 6(2):78–102
- Powell MJD (1985) *On the quadratic programming algorithm of goldfarb and idnani. Mathematical programming essays in honor of George B. Dantzig Part II*. Springer, Berlin, pp 46–61
- Pramanik S, Dey PP (2011) Multi-objective quadratic programming problem: a priority based fuzzy goal programming. *Int J Comp Appl* 26(10):30–35
- Rani N, Goyal V, Gupta D (2021) 'Multi-level multi-objective fully quadratic fractional optimization model with trapezoidal fuzzy numbers using Rouben ranking function and fuzzy goal programming', *Materials Today: Proceedings*
- Rivaz S, Nasseri S, Ziaseraji M (2020) A fuzzy goal programming approach to multiobjective transportation problems. *Fuzzy Inf Eng* 12(2):139–149

- Rommelfanger H (1989) Interactive decision making in fuzzy linear optimization problems. *Eur J Oper Res* 41(2):210–217
- Rout PK, Nanda S, Acharya S (2019) Multi-objective fuzzy probabilistic quadratic programming problem. *Int J Oper Res* 34(3):387–408
- Sert SA, Bagci H, Yazici A (2015) Mofca: Multi-objective fuzzy clustering algorithm for wireless sensor networks. *Appl Soft Comput* 30:151–165
- Taghi-Nezhad NA, Taleshian F (2018) A solution approach for solving fully fuzzy quadratic programming problems. *J Appl Res Ind Eng* 5(1):50–61
- Yang G, Li X, Huo L, Liu Q (2020) 'A solving approach for fuzzy multi-objective linear fractional programming and application to an agricultural planting structure optimization problem. *Chaos Solitons Fractals* 141:110352
- Yeager R (1979) 'Mathematical programming with fuzzy constraints and a preference on the objective', *Kybernetes*
- Yue W, Wang Y, Xuan H (2019) Fuzzy multi-objective portfolio model based on semi-variance-semi-absolute deviation risk measures. *Soft Comput* 23(17):8159–8179
- Zimmermann H.-J. (1978) Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets Syst* 1(1):45–55

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.