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ORIGINAL ARTICLE

Fuzzy rule-based reliability analysis using NSGA-II

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Abstract Practically, reliability-based system designs are modeled in various kinds of uncertainty such as expert's information character, qualitative statements, vagueness, etc. Fuzzy set theory is suitable for tackling such types of uncertainty effectively. In most of the practical situations, where reliability enhancement is an essential requirement, decision-making is a complicated task due to the presence of several mutually conflicting objectives such as system's cost, weight, and volume. To solve such problems, multi-objective evolutionary algorithms (MOEAs) are efficient techniques for finding multiple Pareto-optimal solutions in a single simulation run. This paper applies an elitist MOEA, namely, NSGA-II to fuzzy multi-objective reliability optimization problem consisting of conflicting objectives such as system reliability and its cost. Linguistic hedges (or modifiers) are used to modify the Pareto-optimal solution set obtained by NSGA-II in terms of the membership grades of the objective values. The max–min composition of the membership grades gives the maximum satisfaction level to each possible combination of the linguistic hedges. After that, fuzzy rule-based system (FRBS) is proposed for evaluating the system efficiency to each case which is used in the decision-making of reliability. A numerical example is given to illustrate the method. Finally, the proposed approach is comparatively studied with the existing approach.

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1 Introduction

In the broadest sense, reliability is defined as a "measure of performance of the systems". A design engineer is usually asked to maximize the system reliability and reduce its cost simultaneously. These conflicting objectives affect system efficiency. A system is considered more efficient if it achieves its optimum goals simultaneously. Niwas and Garg (2018) proposed an approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty policy. However, multi-objective reliability models of the system design provide a better interpretation in such a situation. Moreover, various kinds of uncertainty such as vagueness, incompleteness, and unreliability of input information are found in the decision making of reliability. So, the models of real-world application should be more flexible and adaptable to the human judgment and decision-making process (Garg and Sharma 2013). Fuzzy set theory (Zimmermann 1996) deals with the kind of uncertainty that arises due to imprecision associated with the complexity of the system as well as vagueness of human judgement (Chen 1994; Utkin and Gurov 1996; Bing et al. 2000; Mohanta et al. 2004; Bag et al. 2009; Mahapatra and Roy (2009); Garg and Sharma 2012; Garg 2016, 2017; Mahata et al. 2018; Hussain et al. 2018; Salahshour et al. 2018; Mondal 2018).

The fuzzy optimization techniques to multi-objective reliability problems can be viewed in Park (1987), Dhingra

(1992), Rao and Dhingra (1992), Huang (1997), Ravi et al. (2000), Mahapatra and Roy (2006), Huang et al. (2005, 2006), Kishor et al. (2009), Kumar and Yadav (2017, 2019), and Muhuri et al. (2018). Generally, achieving the optimal reliability design is considered quite difficult due to its NP-hard character (Chern 1992). To resolve it, various heuristic approaches for solving reliability optimization can be found in the literature such as Huang et al. (2009), Ayyoub and El-Sheikh (2009), Ebrahimipour and Sheikhalishahi (2011), Damghani et al. (2013), Mutingi and Mbohwa (2014), Garg and Sharma (2013), Garg (2014), Garg et al. (2014a; 2014b), Pant et al. (2015), Garg (2015a, b, c), Kim and Kim (2017) etc.

The preference-based approach (Deb 2001) suggests converting the multi-objective optimization problem (MOOP) to a single-objective optimization problem (SOOP) by emphasizing one Pareto-optimal solution at a time. In a practical situation, such a method needs to be applied many times for getting multiple Pareto-optimal solutions. The other drawback of such method is the dependency on a number of user-defined parameters, which are difficult to set in the arbitrary problem. To fix up these issues, a number of MOEAs have been suggested. The primary reason for their developments is to find multiple Pareto-optimal solutions in a single simulation run. MOEAs work with a population of solutions and generate a well-diverse set of solutions near the true Pareto-optimal region. Over the past decades, many generations of MOEAs emerged in the literature such as MOGA-Multi-Objective Genetic Algorithm (Fonseca and Fleming 1993), NPGA-Niched Pareto Genetic Algorithm (Horn et al. 1994), NSGA-Non-dominated Sorting Genetic Algorithm (Srinivas and Deb 1994), SPEA-Strength Pareto Evolutionary Algorithm (Zitzler and Thiele 1998), PAES-Pareto Archived Evolution Strategy (Knowles and Corne 1999), PESA-Pareto Envelop-based Selection Algorithm (Corne et al. 2000), MOMGA-Multiobjective Optimization with Messy Genetic Algorithm (Veldhuizen and Lamont 2000), PESA-II (Corne et al. 2001), SPEA2 (Zitzler et al. 2002), NSGA-II (Deb et al. 2002), MOEA/D-Multi-objective Evolutionary Algorithm based on Decomposition (Zhang and Li 2007), AGE-Approximation Guided Evolutionary-II (Wagner and Neumann 2013), NSGA-III (Jain and Deb 2013; Deb and Jain 2014) etc. NSGA-II is a very popular second-generation elitist MOEA which has significant applications in the engineering design problems. It is known of its some of the features like parameter-less sharing, elitist strategy, classifying the solutions into the fronts, and less computational complexity. Simulation results on difficult test problems show that NSGA-II gives much better convergence and diversity near the true Pareto-optimal front (Deb et al. 2002) compared to especially elitists MOEAs like SPEA, PAES. Many MOEAs and their solution approaches can be viewed in Deb (2001), Konak et al. (2006), Coello et al. (2007), Das and Panigrahi (2009), and Zhang and Xing (2017). MOEA approach is well-suited to multi-objective reliability problems which are addressed by some researchers such as Salazar et al. (2006), Taboada et al. (2007), Wang et al. (2009), Kishor et al. (2009), and Kumar and Yadav (2017).

In this paper, a methodology is proposed to find the best optimal system design in fuzzy multi-objective reliability optimization problem. However, Kishor et al. (2009) proposed interactive fuzzy multi-objective reliability optimization problem as per the preference of the Decision-Maker (DM) using convex-concave, concave-convex, and sigmoidal shapes of the membership functions, but it does not give the best trade-off optimal solution which is a demand of the system design for a practical purpose. This leads to the motivation for finding the best trade-off or compromise solution using various combination of linguistic modifiers such as *very, more or less and indeed* and FRBS. This is an ideal approach as suggested by Deb (2001), where an effort is made to find multiple trade-offs optimal solutions with a wide range of values for the objectives and then one of the obtained solutions is chosen using higher-level information. The advantage of using NSGA-II is to find a well distributive set of Pareto-optimal solutions in a single simulation run. The proposed approach shows the efficacy over the existing approach (Ravi et al. 2000), where various kinds of aggregate operators are used to determine the optimal system design. The proposed approach is comparatively studied with the existing approach using box-plot comparison. The rest of the paper is organized as follows. Section 2 gives some preliminaries such as basic definitions, fuzzy rule-based system and the mathematical model of the problem. Section 3 gives a brief description of the NSGA-II algorithm. Section 4 gives the proposed methodology with an illustrative example. Section 5 gives the results with its discussion and Sect. 6 gives the conclusions.

2 Preliminaries

Definition 1 In general, an MOOP is defined as follows (Miettinen 2001):

$$\text{Minimize} \quad F(X) = [f_1(X), f_2(X), \ldots, f_k(X)]^T \tag{1}$$

$$\text{subject to} \quad X \in \Omega,$$

involving $(k \geq 2)$ conflicting objective functions $f_i : \mathbb{R}^n \to \mathbb{R}$ need to be minimized simultaneously. The decision (variable) vector $X = [x_1, x_2, \ldots, x_n]^T \in \Omega \subset \mathbb{R}^n$, where Ω is the feasible region formed by *constraint functions*. The image of the feasible region denoted by $Z \subset \mathbb{R}^k$ and it is called a feasible objective region. The elements of Z are called objective vectors denoted by $z = [f_1(X), f_2(X), \ldots, f_k(X)]^T$ consisting of objective functions values. If f_i is to be maximized, it is equivalent to minimize $-f_i$. When all the objective and the constraint functions are linear then the problem is called multi-objective linear programming problem (MOLPP). If at least one of the functions is nonlinear then the problem is called a multi-objective nonlinear programming problem (MONLPP). Correspondingly, if all the objective functions and the feasible region are convex then the problem is convex and if some of the functions are non-convex then the problem is non-convex. The concept of optimality in the MOOP is studied in terms of Pareto terminology, which is defined as follows.

Definition 2 Pareto dominance (Ngatchou et al. 2006; Coello et al. 2007): A vector $X \in \Omega$ is said to dominate another $Y \in \Omega$ denoted by $X \preceq Y$ iff $f_i(X) \leq f_i(Y) \forall i = 1, 2, \ldots, k$, and there exists at least one $f_j(X) < f_j(Y), j \in \{1, 2, \ldots, k\}, j \neq i$.

Definition 3 Pareto-optimal solution (Jimenez and Bilbao 2009; Garg and Sharma 2013): A solution vector $X \in \Omega$ is said to be Pareto-optimal solution (Pareto optimal) iff there does not exist another vector $X' \in \Omega$ which dominates $X \in \Omega$.

Definition 4 Pareto-optimal set (Coello et al. 2007; Garg and Sharma 2013): The Pareto-optimal set is defined as

$$PS := \{X \in \Omega | \neg \exists X' \in \Omega : X' \preceq X\}$$

Definition 5 Pareto-optimal front (Garg and Sharma 2013; Kumar and Yadav 2019): The Pareto-optimal front is defined as

$$PF := \left\{F(X) = [f_1(X), f_2(X), \ldots, f_k(X)]^T \in Z | X \in PS\right\}.$$

Definition 6 Fuzzy set (Zimmermann 1996): Let X be a collection of objects generically denoted by x. A fuzzy set $\tilde{A}$ in X is a set of ordered pair defined in the form as

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right) : x \in X\right\}$$

where $\mu_{\tilde{A}} : X \rightarrow [0, 1]$ is called the membership function and its function value is called grade of membership of x in $\tilde{A}$.

Definition 7 Linguistic hedge (or modifier) (Zimmermann 1996; Kerre and De Cock 1999): A linguistic hedge (or a linguistic modifier) is an operation that modifies the meaning of the term. Suppose $\tilde{H}$ is a fuzzy set in X, then the modifier m generates the composite term $\tilde{M} = m(\tilde{H})$. Modifiers are frequently used in mathematical models as follows.

Concentration: It decreases the membership grades of all the members of $\tilde{H}$ by spreading in the curve. It is defined as:

$$\mu_{con(\tilde{H})}(x) = \left(\mu_{(\tilde{H})}(x)\right)^2 \quad \text{for all } x \in X$$

Dilation: It increases the membership grades of all members by spreading out the curve. It is defined as:

$$\mu_{dil(\tilde{H})}(x) = \left(\mu_{(\tilde{H})}(x)\right)^{1/2} \quad \text{for all } x \in X$$

Contrast intensification: It affects an increase of the membership grades greater than or equal to 0.5 and a decrease of the membership grades smaller than 0.5. It is defined as:

$$\mu_{int(\tilde{H})}(x) = \begin{cases} 2\left(\left(\mu_{(\tilde{H})}(x)\right)^2\right), & \mu_{(\tilde{H})}(x) \in [0, 0.5] \\ 1 - 2\left(1 - \mu_{(\tilde{H})}(x)\right)^2, & \text{otherwise} \end{cases}$$

Therefore, in general, strong and weak modifiers are given as: $m_\delta\left(\mu_{(\tilde{H})}(x)\right) = \left(\mu_{(\tilde{H})}(x)\right)^\delta$ = a strong modifier or concentrator, if $\delta > 1$ and a weak modifier or dilator if $\delta < 1$.

The following linguistic hedges are associated with above mathematical operators: *very* $\tilde{H} = con(\tilde{H})$; *more or less* $\tilde{H} = dil(\tilde{H})$; *Indeed* $\tilde{H} = Int(\tilde{H})$; *plus* $\tilde{H} = \tilde{H}^{1.25}$; *slightly* $\tilde{H}$= *int[plus* $\tilde{H}$ *and not (very* $\tilde{H}$*].*

2.1 Fuzzy rule-based system (FRBS)

An FRBS (Cordon 2011) is one of the major applications of fuzzy set theory. In a broad sense, fuzzy rule-based system is a system of rule-based, where fuzzy sets and fuzzy logic are used as tools for representing different forms of knowledge, modeling the interactions, and relationships between its variables. The architecture of FRBS is as shown in Fig. 1. It consists of four principal units: the fuzzifier module, fuzzy inference engine, knowledge base, and the defuzzifier module. These units are described as follows.

- Fuzzifier module: The inputs of the system are usually given by the crisp values. Since the data manipulation in FRBS is based on fuzzy set theory, so fuzzification is necessary. In this process, the numerical values of each crisp input are converted into a set of the membership grades defined by the membership functions of the linguistic values. A knowledge base expert or an optimization algorithm usually determines the shape

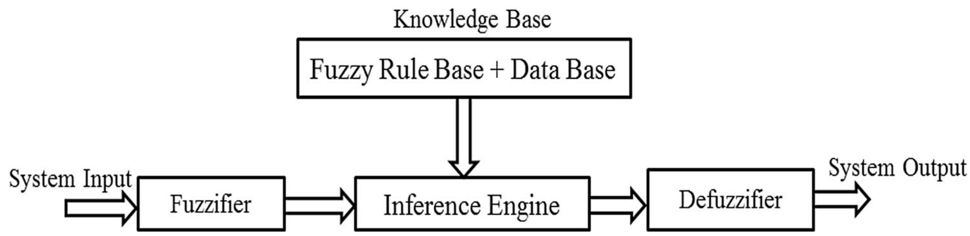


Fig. 1 Architecture of fuzzy rule-based system

and distribution of the membership functions on the universe of discourse.

- Knowledge-base: It contains the knowledge specific to the domain of application. An FRBS is characterized by a set of linguistic statements derived by a domain expert to map inputs to outputs. Domain knowledge is usually represented in the form of a set of "IF–THEN" rules, also known as production rules, and it is expressed as:

IF (a set of conditions are satisfied) THEN (a set of actions can be inferred).

- Inference engine: To deal with the fuzzy information described above, the fuzzy inference engine employs the fuzzy knowledge-based methods such as Mamdani, Sugeno, etc., (Zimmermann 1996; Cordon 2011; Suptami et al. 2019) to simulate human decision-making and infer outputs.
- Defuzzifier module: This module defuzzifies the processed fuzzy data into the crisp data which suits to real-world applications.

2.2 Mathematical model of the problem

Reliability is one of the crucial design parameters that affect the system's performance significantly. Practically, the problem of system reliability is constructed as a typical nonlinear programming problem with nonlinear cost functions. Suppose the system consists of m components, the reliability of each component is given by r_j; $j = 1, 2, \ldots, m$ and their corresponding costs are denoted by $C_j(r_j)$. Moreover, in reliability optimization, we need to be optimized several mutually conflicting objectives subject to several design constraints. For instance, a design engineer is asked to improve the system reliability (R_S) with the reduction of system cost (C_S) simultaneously. Therefore, multiple objectives have become an essential part of the reliability-based design of the engineering systems. In addition, the cost of reliability is assumed to be a monotonically increasing function of reliability (Aggarwal and Gupta 1975; Huang et al. 2005). Therefore, a suitable multi-objective reliability optimization model (Garg et al. 2014b) of the system design by considering the system reliability and the system cost as objectives is given as follows:

$$\text{Maximize } R_S(r_1, r_2, \ldots, r_m) = \begin{cases} \prod_{j=1}^{m} r_j \text{ for series system} \\ \text{or} \\ 1 - \prod_{i=1}^{m}\left(1 - r_j\right) \text{ for parallel system} \\ \text{or} \\ \text{combination of series and parallel system} \end{cases} \tag{2}$$

$$\text{Minimize } C_S(r_1, r_2, \ldots, r_m) = \sum_{j=1}^{m} C_j\left(r_j\right)$$

Or, Minimize $\left(-R_S, C_S\right)$

subject to $r_{j,min} \le r_j \le 1$, $R_{S,min} \le R_S \le 1$, for $j = 1, 2, \ldots, m$.

3 Elitist non-dominated sorting genetic algorithm (NSGA-II)

Non-dominated sorting genetic algorithm (NSGA) was initially developed by Srinivas and Deb (1994). NSGA uses Goldberg's domination criterion (1989) to rank the solutions and utilizes fitness sharing approach to maintain the diversity in the solution set. It has been criticizing especially for non-elitist approach, high computational complexity and specifying the sharing parameter. To cope up these issues, Deb et al. (2002) developed an improved version of NSGA and called it as NSGA-II by introducing some new features such as fast non-dominated sorting algorithm, crowding distance, crowded-comparison operator.

A fast-non-dominated sorting approach gives the worst-case computational complexity as $O\left(k(2N)^2\right)$, where k is the number of objectives and N is the population size. This approach searches iteratively non-dominated solutions into different fronts. First, for each solution i in the population, the algorithm calculates two entities:

(i) n_i, the number of solutions dominating i,
(ii) S_i, a set of solutions dominated by i.

The solutions for which $n_i = 0$ belong to the first front. Second, for each member j in the set S_i, the value of n_j is reduced by one. If any n_j is reduced to zero during this stage, the corresponding member j is put in the second front. The above process is continued with each member in the second front to identify the third front and so on. Furthermore, NSGA-II applies the concept of crowding-distance assignment with the worst-case computational complexity as $O(k(2N)\log(2N))$. The introduction of crowding-distance replaces the fitness sharing approach that requires a sharing parameter to be set by the user. The crowding-distance of the i^{th} solution in the p^{th} objective function is denoted by d_{ip} and its overall crowding-distance value is denoted by CD_i (See Fig. 2). These values are calculated as follows:

$$d_{ip} = \frac{f_p^{i+1} - f_p^{i-1}}{f_p^{max} - f_p^{min}} \tag{3}$$

$$CD_i = \sum_{p=1}^{k} d_{ip} \tag{4}$$

where f_p^{i+1} and f_p^{i-1} denote the p^{th} objective function of the $(i+1)^{th}$ and $(i-1)^{th}$ individual (solution) respectively, and f_p^{max} and f_p^{min} represent the maximum and minimum values of the p^{th} objective function.

A higher value of crowding-distance gives the lesser crowded region and vice versa (Deb et al. 2002). So, the crowding-distance selects the solutions located in less-crowded regions which are extended up to the entire front for making diversity in the solution set. Finally, NSGA-II introduces an elitist strategy with the worst-case computational complexity $O(2N\log(2N))$. The elitist strategy (Zitzler et al. 2000; Laumanns et al. 2002) is used to enhance the convergence of an MOEA and avoid the loss of optimal solutions after getting it. In Fig. 3, an evaluation cycle of the NSGA-II is shown. First, an offspring Q_t of size N is obtained by using the genetic operators such as selection, recombination, and mutation. A combined population R_t of size 2 N is then formed which consists of the current population P_t and the offspring population Q_t. By using fast non-dominated sorting, R_t is divided into different fronts $PF_1, PF_2, \ldots, PF_l$. Let the number of solutions in each front PF_i be N_i. Next, we choose members for the new population P_{t+1} from the front PF_1 to PF_{t-1}, noting that $N_1 + N_2 + \cdots + N_t > N$ and $N_1 + N_2 + \cdots + N_{t-1} \le N$. Afterwards, to get the exactly N population members in P_{t+1}, we sort the solutions in front PF_t using the crowding distance sorting procedure and choose the best solutions to fill any empty slot in the new population P_{t+1}. This process is continued until the termination condition is satisfied. The pseudo code of the NSGA-II algorithm is given as follows.

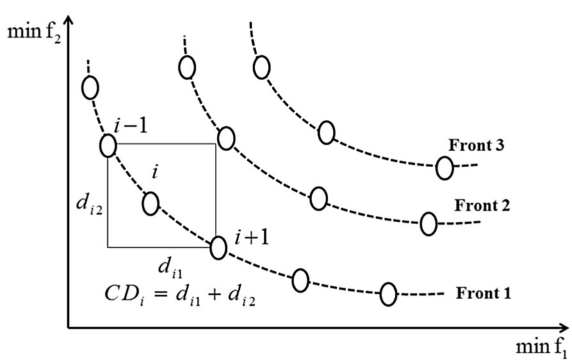


Fig. 2 Fitness evaluation and individual crowding distance estimation of NSGA-II

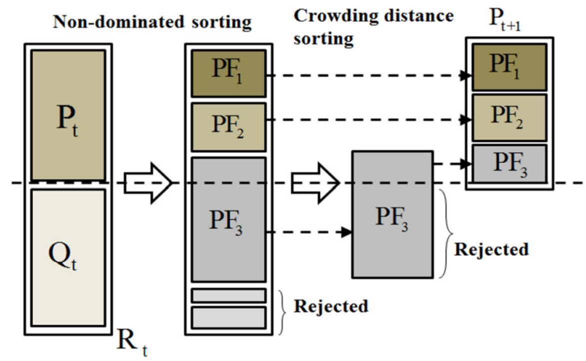


Fig. 3 Evaluation cycle of the NSGA-II algorithm (Deb et al. 2002)

Step 1. Initialize randomly a parent population P_0 of size N. Set $t = 0$.

Step 2. Assign fitness (rank) according to non-domination level and crowded-comparison operator.

Step 3. **while** $t <$ number of maximum generations **do**

(i) Create an offspring population Q_t of size N applying selection, crossover, and mutation.
(ii) Combine via $R_t = P_t \cup Q_t$.
(iii) Sort on R_t and classifying them into non-dominated fronts PF_i, $i = 1, 2, \ldots,$ etc.
(iv) Set a new population $P_{t+1} = \emptyset$ and set a counter $i = 1$.

while Parent population size $|P_{t+1}| + |PF_i| < N$ **do**

(i) Calculate the crowding distance of PF_i.
(ii) Add the i^{th} non-dominated front PF_i to the parent population P_{t+1}.
(iii) $i = i + 1$.

end while

(v) Sort the PF_i using the crowding distance-based comparison operator.
(vi) Fill the parent population P_{t+1} with the first $N - |P_{t+1}|$ solutions of PF_i.
(vii) Generate the offspring population Q_{t+1}.
(viii) Set $t = t + 1$.

end while

Step 4. Collect the non-dominated solutions in vector P.

4 Proposed methodology

The proposed methodology is given step by step as follows.

Step 1. Fuzzification of the given model of the problem.

The values of the first objective R_S in (2) belong to [0, 1]. The fuzzy set $\tilde{R}_S$ is created as per the degree of satisfaction α_R with values of R_S. Similarly, the second objective in (2) is C_S and its possible value lies in $[0, \infty)$. The fuzzy set $\tilde{C}_S$ is created as per the degree of satisfaction α_C with values of C_S. The membership functions of $\tilde{R}_S$ and $\tilde{C}_S$ are respectively, defined as:

$$\mu_{\tilde{R}_S} = \begin{cases} 0, R_S \le R_S^l \\ h_1(R_S), R_S^l \le R_S \le R_S^u \\ 1, R_S \ge R_S^u \end{cases} \tag{5}$$

$$\mu_{\tilde{C}_S} = \begin{cases} 1, C_S \le C_S^l \\ h_2(C_S), C_S^l \le C_S \le C_S^u \\ 0, C_S \ge C_S^u \end{cases} \tag{6}$$

where R_S^l and R_S^u are the lower and upper limits on R_S; C_S^l and C_S^u are the lower and upper limits on C_S; $h_1(R_S)$ is a monotonically increasing function of R_S; $h_2(C_S)$ is a monotonically decreasing function of C_S. These values are determined by the DM according to the actual situation. Each membership function is required to be maximized so that it could achieve the maximum degree of satisfaction (Kumar and Yadav 2017). The shapes of $\mu_{\tilde{R}_S}$ and $\mu_{\tilde{C}_S}$ are shown in Figs. 4 and 5 respectively. The mathematical model of the problem given in (2) is reformulated as follows.

$$\text{Maximize } (\mu_{\tilde{R}_S}, \mu_{\tilde{C}_S}) \tag{7}$$

$$\text{or, Minimize } (-\mu_{\tilde{R}_S}, -\mu_{\tilde{C}_S})$$

$$\text{subject to } r_{j,min} \le r_j \le 1, j = 1, 2, \ldots, m$$

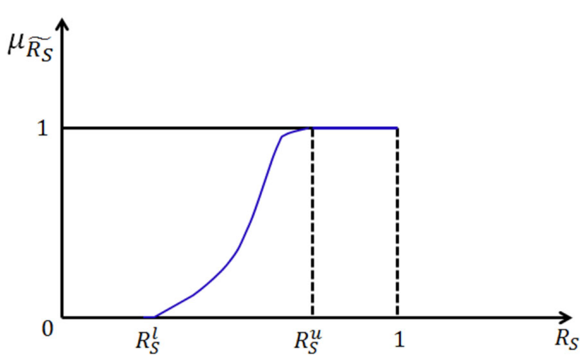


Fig. 4 Monotonically increasing membership function for system reliability

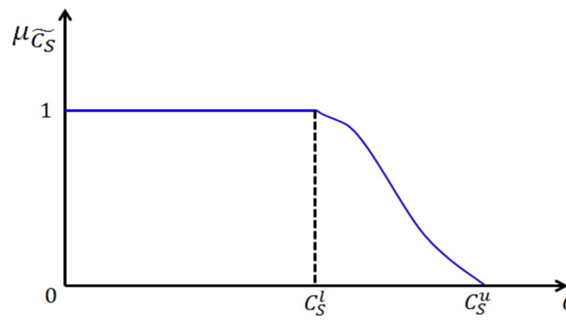


Fig. 5 Monotonically decreasing membership function for system cost

Theorem 1 *The Pareto-optimal solutions of the FMOOP (7) satisfy the MOOP (2).*

Proof Let R^* be a Pareto-optimal solution vector of (7). Then by definition of Pareto-optimal solution,

$\nexists R \in \Omega$ (feasible region) such that $-\mu_{\tilde{R}_S}(R) \le -\mu_{\tilde{R}_S}(R^*)$ and $-\mu_{\tilde{C}_S}(R) < -\mu_{\tilde{C}_S}(R^*)$.

$\Leftrightarrow \nexists R \in \Omega$ such that $-h_1[R_S(R)] \le -h_1[R_S(R^*)]$ and $-h_2[C_S(R)] < -h_2[C_S(R^*)]$

$\Leftrightarrow \nexists R \in \Omega$ such that $h_1[R_S(R)] \ge h_1[R_S(R^*)]$ and $h_2[C_S(R)] > h_2[C_S(R^*)]$

$\Leftrightarrow \nexists R \in \Omega$ such that $R_S(R) \ge R_S(R^*)$ and $C_S(R) < C_S(R^*)$ (since h_1 is a monotonically increasing and h_2 is a monotonically decreasing function).

$\Leftrightarrow \nexists R \in \Omega$ such that $-R_S(R) \le -R_S(R^*)$ and $C_S(R) < C_S(R^*)$

$\Leftrightarrow R^* \in \Omega$ is a Pareto-optimal solution of the MOOP given by (2). □

Step 2. Find the Pareto-optimal solutions (POF) of the fuzzy multi-objective reliability problem in terms of the membership grades.

NSGA-II is applied to the FMOOP (7). The POF is obtained by settings the parameters of NSGA-II such as crossover probability (p_c), mutation probability (p_m), maximum number of generations (t_{max}), distribution indices for crossover (η_c) and mutation (η_m). On the basis of rigorous experiments and tuning of the parameters, the best POF is obtained.

Step 3. Modify the Pareto-optimal solutions (POF) in various forms of linguistic hedges.

Linguistic hedges modify the membership function values of R_S and C_S as (Indeed high, Indeed low), (Indeed high, Very low), (Indeed high, Low), (Indeed high, More or less low), (Very high, Indeed low), (Very high, Very low), (Very high, Low), (Very high, More or less low), (High, Indeed low), (High, Very low), (High, low), (High, More or less low), (More or less high, Indeed low), (More or less high, Very low), (More or less high, low), (More or less high, More or less low). It can be defined as follows.

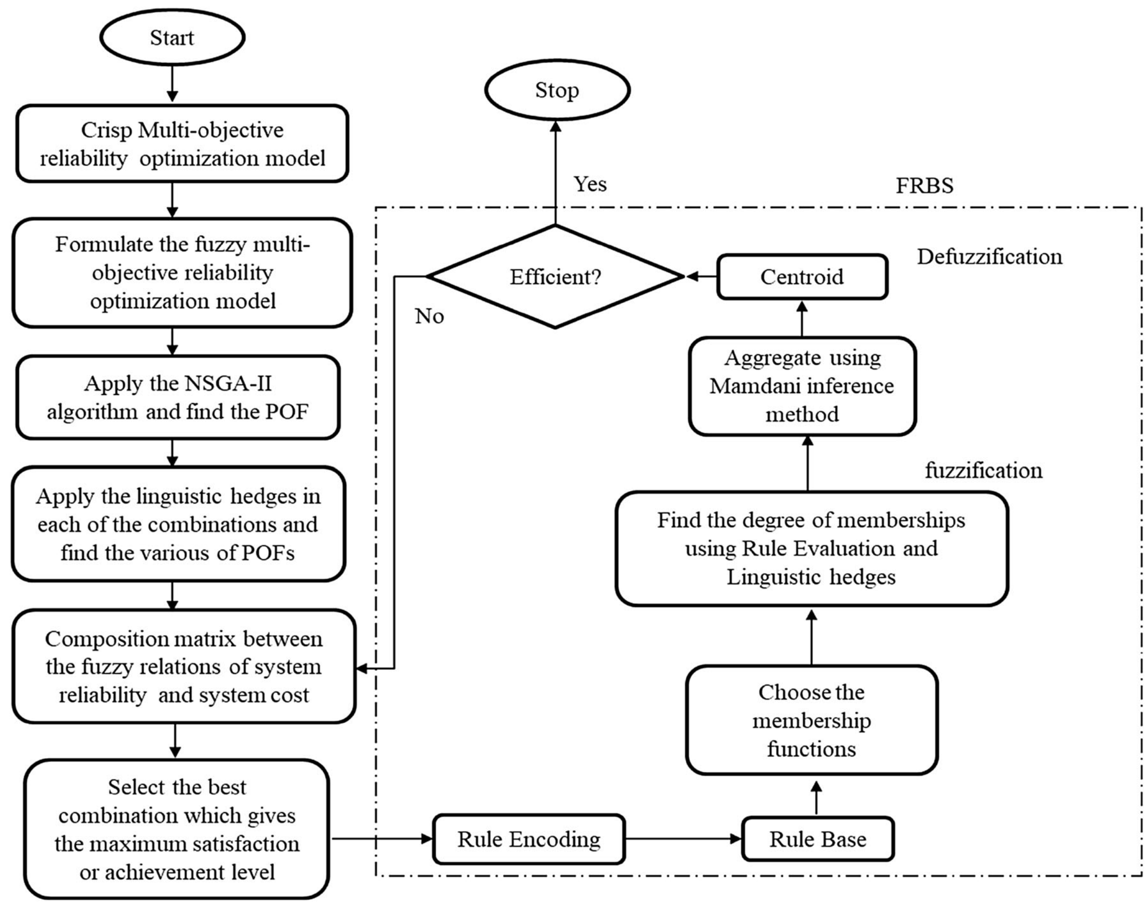


Fig. 6 Flow diagram of the proposed methodology

Indeed High (IDH): It is a linguistic form of modifier "intensification" applied to $\tilde{R}_S$ defined by

$$\mu_{\tilde{R}_S} = \begin{cases} 0, & R_S \le R_S^l \\ 2\left(\dfrac{R_S - R_S^l}{R_S^u - R_S^l}\right)^2, & R_S^l \le R_S \le (R_S^u + R_S^l)/2 \\ 1 - 2\left(1 - \left(\dfrac{R_S - R_S^l}{R_S^u - R_S^l}\right)\right)^2, & (R_S^u + R_S^l)/2 \le R_S \le R_S^u \\ 1, & R_S \ge R_S^u \end{cases}$$

Very High (VH): It is a linguistic form of modifier "concentration" applied to $\tilde{R}_S$ defined by

$$\mu_{\tilde{R}_S} = \begin{cases} 0, & R_S \le R_S^l \\ \left(\dfrac{R_S - R_S^l}{R_S^u - R_S^l}\right)^2, & R_S^l \le R_S \le R_S^u \\ 1, & R_S \ge R_S^u \end{cases}$$

High (H): It is a linguistic form but no modification is applied to $\tilde{R}_S$ which is the linear membership function as:

$$\mu_{\tilde{R}_S} = \begin{cases} 0, & R_S \le R_S^l \\ \left(\dfrac{R_S - R_S^l}{R_S^u - R_S^l}\right), & R_S^l \le R_S \le R_S^u \\ 1, & R_S \ge R_S^u \end{cases}$$

More or Less High (MOLH): It is a linguistic form of modifier "dilation" applied to $\tilde{R}_S$ defined by

$$\mu_{\tilde{R}_S} = \begin{cases} 0, & R_S \le R_S^l \\ \left(\dfrac{R_S - R_S^l}{R_S^u - R_S^l}\right)^{1/2}, & R_S^l \le R_S \le R_S^u \\ 1, & R_S \ge R_S^u \end{cases}$$

Indeed Low (IDL): Similarly, it is a linguistic form of modifier "intensification" applied to $\tilde{C}_S$ defined by $\mu_{\tilde{C}_S} =$

$$\begin{cases} 1, & C_S \le C_S^l \\ 2 * \left(\dfrac{C_S^u - C_S}{C_S^u - C_S^l}\right)^2, & C_S^l \le C_S \le (C_S^u + C_S^l)/2 \\ 1 - 2 * \left(1 - \left(\dfrac{C_S^u - C_S}{C_S^u - C_S^l}\right)\right)^2, & (C_S^u + C_S^l)/2 \le C_S \le C_S^u \\ 0, & C_S \ge C_S^u \end{cases}$$

Very Low (VL): It is a modifier "concentration" applied to $\tilde{C}_S$ defined by

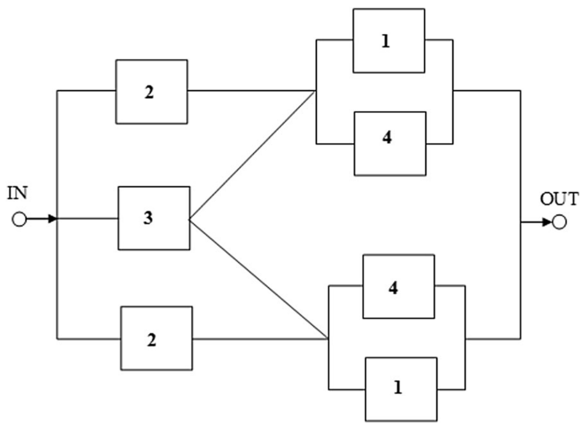


Fig. 7 Block diagram of the life-support system

Table 1 Parameter settings for NSGA-II

Population size (N)	60
Maximum number of generations (t_{max})	100
Crossover probability (p_c)	0.8
Mutation probability (p_m)	0.2
Distribution index for crossover (η_c)	20
Distribution index for mutation (η_m)	80
Random seed	0.9876
Number of design variables	4

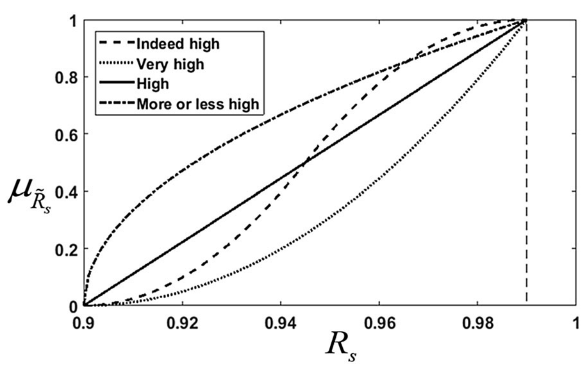


Fig. 8 Linguistic hedges applied to $\tilde{R}_S$

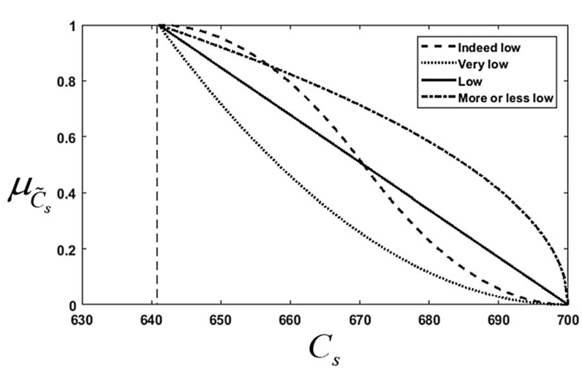


Fig. 9 Linguistic hedges applied to $\tilde{C}_S$

Table 2 Linguistic hedges applied to the system reliability $R_S = \tilde{R}_1$

MOLH	H	VH	IDH
0.1313	0.0172	0.0003	0.0006
0.2076	0.0431	0.0019	0.0037
0.2565	0.0658	0.0043	0.0087
0.3006	0.0903	0.0082	0.0163
0.3501	0.1226	0.0150	0.0300
0.3831	0.1468	0.0215	0.0431
0.4118	0.1696	0.0288	0.0575
0.4373	0.1913	0.0366	0.0732
0.4623	0.2137	0.0457	0.0913
0.4924	0.2425	0.0588	0.1176
0.5100	0.2601	0.0676	0.1353
0.5258	0.2764	0.0764	0.1528
0.5476	0.2998	0.0899	0.1798
0.5641	0.3183	0.1013	0.2025
0.5836	0.3406	0.1160	0.2321
0.6034	0.3641	0.1325	0.2651
0.6235	0.3888	0.1512	0.3024
0.6408	0.4106	0.1686	0.3372
0.6640	0.4409	0.1944	0.3887
0.6820	0.4652	0.2164	0.4328
0.6984	0.4877	0.2379	0.4758
0.7117	0.5065	0.2566	0.5130
0.7321	0.5359	0.2872	0.5693
0.7504	0.5632	0.3172	0.6184
0.7643	0.5842	0.3413	0.6542
0.7871	0.6196	0.3839	0.7106
0.8062	0.6513	0.4225	0.7550
0.8200	0.6723	0.4520	0.7853
0.8335	0.6948	0.4828	0.8137
0.8536	0.7286	0.5309	0.8527
0.8729	0.7620	0.5806	0.8867
0.8856	0.7843	0.6152	0.9070
0.8971	0.8048	0.6477	0.9238
0.9089	0.8261	0.6824	0.9395
0.9241	0.8540	0.7292	0.9573
0.9348	0.8739	0.7638	0.9682
0.9495	0.9015	0.8127	0.9806
0.9566	0.9150	0.8373	0.9856
0.9680	0.9370	0.8780	0.9921
0.9771	0.9547	0.9115	0.9959
0.9813	0.9631	0.9275	0.9973
0.9919	0.9838	0.9678	0.9995
0.9982	0.9964	0.9927	0.9998

$$\mu_{\tilde{C}_S} = \begin{cases} 1, & C_S \leq C_S^l \\ \left(\dfrac{C_S^u - C_S}{C_S^u - C_S^l}\right)^2, & C_S^l \leq C_S \leq C_S^u \\ 0, & C_S \geq C_S^u \end{cases}$$

Low (L): It is a linguistic form but no modification is applied to $\tilde{C}_S$ which is the linear membership function as:

$$\mu_{\tilde{C}_S} = \begin{cases} 1, & C_S \leq C_S^l \\ \left(\dfrac{C_S^u - C_S}{C_S^u - C_S^l}\right), & C_S^l \leq C_S \leq C_S^u \\ 0, & C_S \geq C_S^u \end{cases}$$

More or Less Low (MOLL): It is a linguistic form of modifier "dilation" applied to $\tilde{C}_S$ defined by

$$\mu_{\tilde{C}_S} = \begin{cases} 1, & C_S \leq C_S^l \\ \left(\dfrac{C_S^u - C_S}{C_S^u - C_S^l}\right)^{1/2}, & C_S^l \leq C_S \leq C_S^u \\ 0, & C_S \geq C_S^u \end{cases}$$

Step 4. Construct the composition of fuzzy relations.

Let $\tilde{R}_1(x,y),(x,y) \in X \times Y$ and $\tilde{R}_2(y,z),(y,z) \in Y \times Z$ be two fuzzy relations. These are constructed as:

$\tilde{R}_1$ = [MOLH H VH IDH]; $\tilde{R}_2$ = [MOLL L VL IDL]; The composition of $\tilde{R}_1$ and $\tilde{R}_2$ is defined by "max–min" as:

$\tilde{R} = \tilde{R}_1 \circ \tilde{R}_2 = \{[(x,z), max_y\{min\{\mu_{\tilde{R}_1}(x,y), \mu_{\tilde{R}_2}(y,z)\}\}] | x \in X, y \in Y, z \in Z\}$, $\mu_{\tilde{R}_1 \circ \tilde{R}_2}$ is the membership function of a fuzzy relation on fuzzy sets $\tilde{R}_S$ and $\tilde{C}_S$.

The fuzzy relation $\tilde{R}$ is 4 by 4 composition matrix called the matrix of maximum satisfaction level achieved by each combination of fuzzy relation.

Step 5. Induce the FRBS in each combination.

In this Step, FRBS is induced to infer the output called the efficiency of the system for each element of the fuzzy relation $\tilde{R}$. Input variables R_S and C_S are categorized as LOW, HIGH, MEDIUM and EXTREME while output E_S (system efficiency) as POOR, BELOW AVERAGE, GOOD, VERY GOOD, EXCELLENT and OUTSTANDING. Triangular membership functions are used to define the membership grades for each variable. Mamdani type fuzzy inference and centroid defuzzification methods are adopted in this model. The efficiency matrix of the system is denoted by E_S. The corresponding values of R_S and C_S to each element in $\tilde{R}$ is substituted in rule-viewer of MATLAB Fuzzy Logic Toolbox. This value gives the system efficiency to each relation. The flow diagram of the proposed methodology is as shown in Fig. 6.

Step 6. Find the best trade-off or compromise solution.

The best compromise solution in the fuzzy relation $\tilde{R}$ or find the best relation in $\tilde{R}$ which gives the maximum satisfaction level as well as maximum efficiency in the system design. It is obtained by max–min composition as well. The optimal system design is proposed as:

$\mu_{S,max} = \tilde{R} \circ \tilde{E}_S$

Table 3 Linguistic hedges applied to the system cost $C_S = \tilde{R}_2$

MOLL	L	VL	IDL
0.9998	0.9995	0.9990	0.9999
0.9964	0.9928	0.9856	0.9999
0.9888	0.9777	0.9559	0.9990
0.9811	0.9626	0.9265	0.9972
0.9732	0.9471	0.8970	0.9944
0.9655	0.9321	0.8688	0.9908
0.9553	0.9127	0.8330	0.9847
0.9471	0.8970	0.8045	0.9788
0.9406	0.8847	0.7827	0.9734
0.9288	0.8627	0.7442	0.9623
0.9233	0.8524	0.7266	0.9564
0.9140	0.8354	0.6980	0.9458
0.9028	0.8151	0.6643	0.9316
0.8953	0.8015	0.6424	0.9212
0.8896	0.7913	0.6262	0.9129
0.8770	0.7692	0.5917	0.8935
0.8663	0.7505	0.5632	0.8755
0.8546	0.7303	0.5334	0.8546
0.8419	0.7088	0.5024	0.8304
0.8264	0.6830	0.4665	0.7990
0.8132	0.6613	0.4374	0.7706
0.8036	0.6457	0.4170	0.7490
0.7901	0.6243	0.3898	0.7177
0.7737	0.5986	0.3583	0.6778
0.7600	0.5770	0.3329	0.6422
0.7385	0.5454	0.2975	0.5868
0.7141	0.5099	0.2600	0.5196
0.6991	0.4887	0.2388	0.4776
0.6766	0.4578	0.2096	0.4192
0.6485	0.4205	0.1768	0.3537
0.6194	0.3836	0.1472	0.2943
0.6012	0.3615	0.1307	0.2613
0.5807	0.3373	0.1137	0.2275
0.5536	0.3064	0.0940	0.1878
0.5201	0.2705	0.0732	0.1463
0.4879	0.2381	0.0567	0.1134
0.4430	0.1962	0.0385	0.0770
0.4250	0.1806	0.0326	0.0653
0.3879	0.1504	0.0226	0.0453
0.3449	0.1189	0.0141	0.0283
0.3034	0.0921	0.0085	0.0170
0.2361	0.0557	0.0031	0.0062
0.1078	0.0116	0.0001	0.0003

Table 4 Composition matrix $\tilde{R} = \tilde{R}_1 \circ \tilde{R}_2$

	MOLH	H	VH	IDH
MOLL	0.7596	0.6820	0.5917	0.7177
L	0.6766	0.5770	0.4652	0.5868
VL	0.6012	0.4578	0.3329	0.4520
IDL	0.7141	0.0006	0.4374	0.6422

where '$\circ$' is the composition defined in Step 4.

4.1 Illustration

Let us consider a life support system in a space capsule (Ravi et al. 2000). This system needs a single path to its success which contains two redundant subsystems. Each subsystem connects with two redundant components 1 and 4, and each of the redundant subsystems connects in series with component 2 and the resultant pair of series–parallel arrangement forms two equal paths. In order to back up for the pair, component 3 enters as a third path. This problem forms a continuous nonlinear optimization problem and consists of four components, each having component reliability $r_j, j = 1, 2, 3, 4$. The Mathematical model of the life-support system using block diagram (see Fig. 7) is given as follows.

$$\text{Maximize } R_S = 1 - r_3[(1-r_1)(1-r_4)]^2 - (1-r_3)[1 - r_2\{1-(1-r_1)(1-r_4)\}]^2 \tag{8}$$

$$\text{Minimize } C_S = 2\sum_{j=1}^{4} K_j r_j^{\alpha_j}$$

subject to $0.5 \le r_j \le 1, \quad j = 1, 2, 3, 4$

where different parameters values of K_j, as $K_1 = 100, K_2 = 100, K_3 = 200, K_4 = 150$ and α_j as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0.6$.

The problem is posed as "Maximize system reliability as close as possible to 1 with approximate system cost of 641.8 (cost units)".

The MOOP given in (8) is reformulated as FMOOP:

$$\text{Maximize } \left(\mu_{\tilde{R}_S}, \mu_{\tilde{C}_S}\right) \tag{9}$$

subject to $0.5 \le r_j \le 1, \quad j = 1, 2, 3, 4;$

where the linear membership functions of R_S and C_S are given by

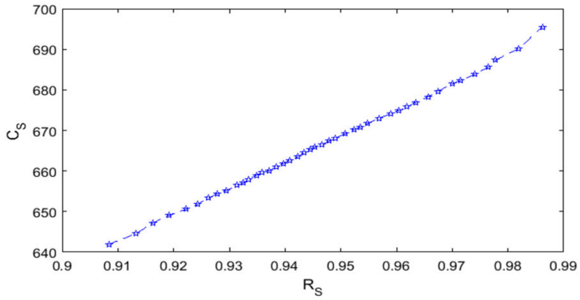
Fig. 10 POFs obtained on the basis of all possible combinations of linguistic hedges

$$\mu_{\tilde{R}_S} = \begin{cases} 0, & R_S \le 0.9 \\ \dfrac{R_S - 0.9}{0.99 - 0.9}, & 0.9 \le R_S \le 0.99 \\ 1, & R_S \ge 0.99 \end{cases} \tag{10}$$

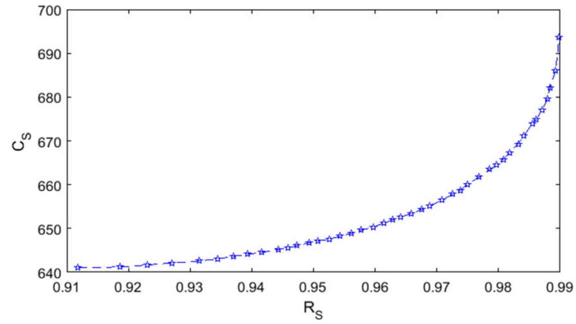
$$\mu_{\tilde{C}_S} = \begin{cases} 1, & C_S \le 641 \\ \dfrac{700 - C_S}{700 - 641}, & 641 \le C_S \le 700 \\ 0, & C_S \ge 700 \end{cases} \tag{11}$$

5 Results and discussion

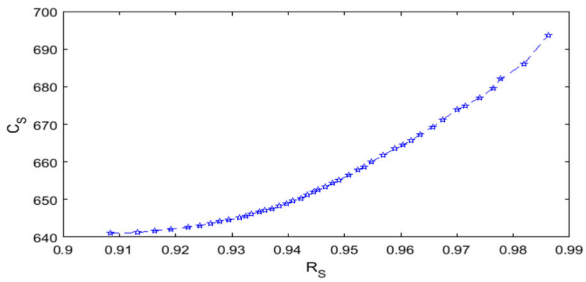
After applying the proposed approach, the results are graphically shown. The parameter settings for the NSGA-II algorithm are given in Table 1. NSGA-II is applied to the FMOOP (9). Here, Population size is taken as 60, out of which 43 solutions are found non-dominated. Linguistic hedges applied to the given problem are as shown in Figs. 8 and 9. Tables 2 and 3 give the list of linguistic hedges values applied to a set of optimal values. The POFs for all possible cases are demonstrated in Fig. 10. Table 4 gives a composite relation (or maximum satisfaction level) to each possible combination of linguistic hedges. The proposed FRBS model (see Fig. 11) is then invoked in each possible case. Table 5 gives encoding rules for the FRBS. In Fig. 12, the domain of input variables changes dynamically in each case and its output shows as a surface plot correspondingly. On the basis of maximum satisfaction level (μ_{max}), the optimal values of input variables R_S and C_S are obtained. These values are put in the rule-viewer of MATLAB Fuzzy Logic Toolbox (Coleman et al. 1999) of FRBS for getting the efficiency of the system. Finally, Table 6 gives the list of the optimal solutions and their efficiencies towards the system. The DM can use this information of his/her own perspectives in the decision-making. From Table 6, it is observed that (MOLH, MOLL) achieves the maximum satisfaction level highest at 0.75962, while (IDH, L) achieves the maximum satisfaction level lowest at 0.00059. From the efficiency point of view, (VH, L) reaches the highest efficiency at 59% and (MOLH, MOLL) reaches the lowest efficiency at 53.9%. It is also observed that (VH, MOLL) attains the highest $\mu_{S,max} = 0.582$ with maximum satisfaction level at 0.60123 and its efficiency at 58.2%. This value is obtained as the best optimal value in all the possible cases by the proposed



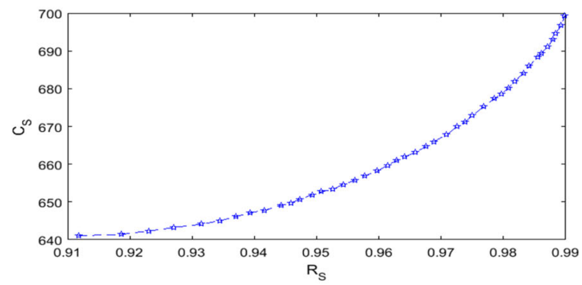
(IDH, IDL)



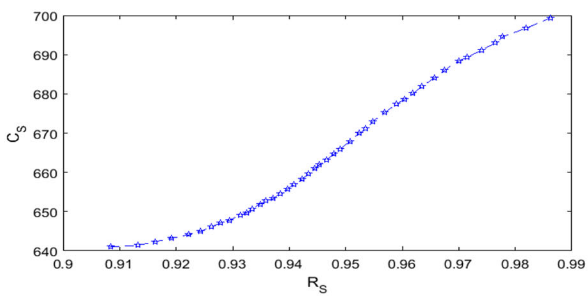
(VH, VL)



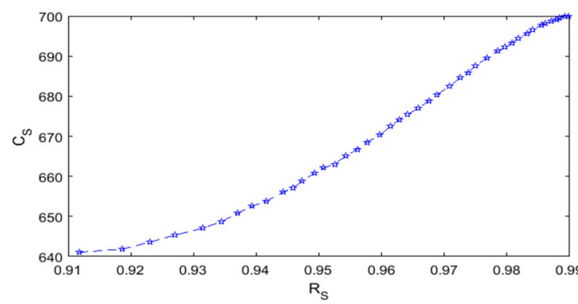
(IDH, VL)



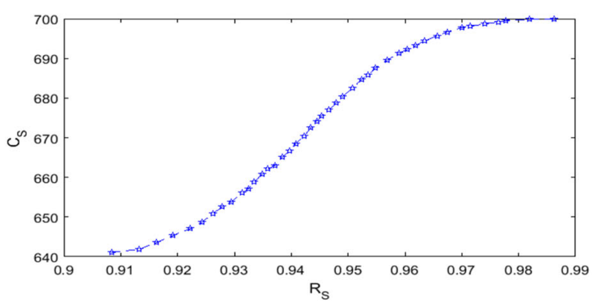
(VH, L)



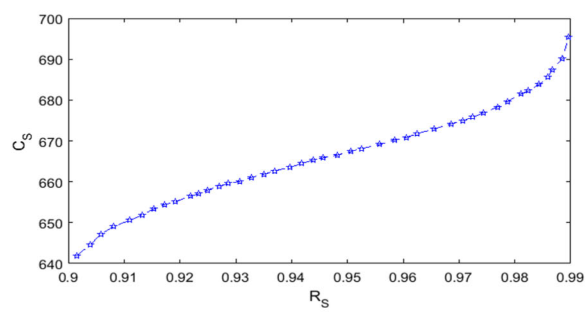
(IDH, L)



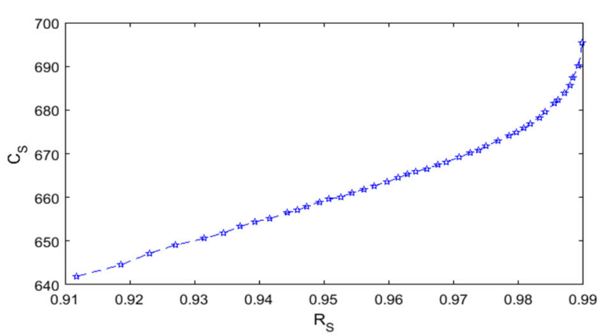
(VH, MOLL)



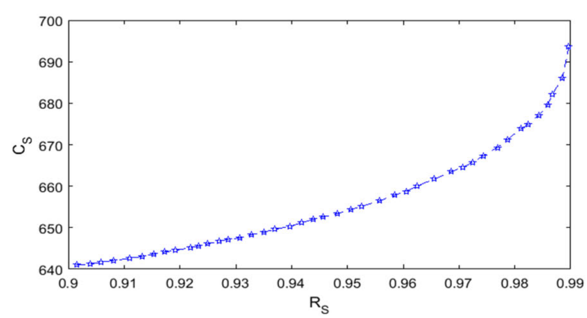
(IDH, MOLL)



(H, IDL)



(VH, IDL)



(H, VL)

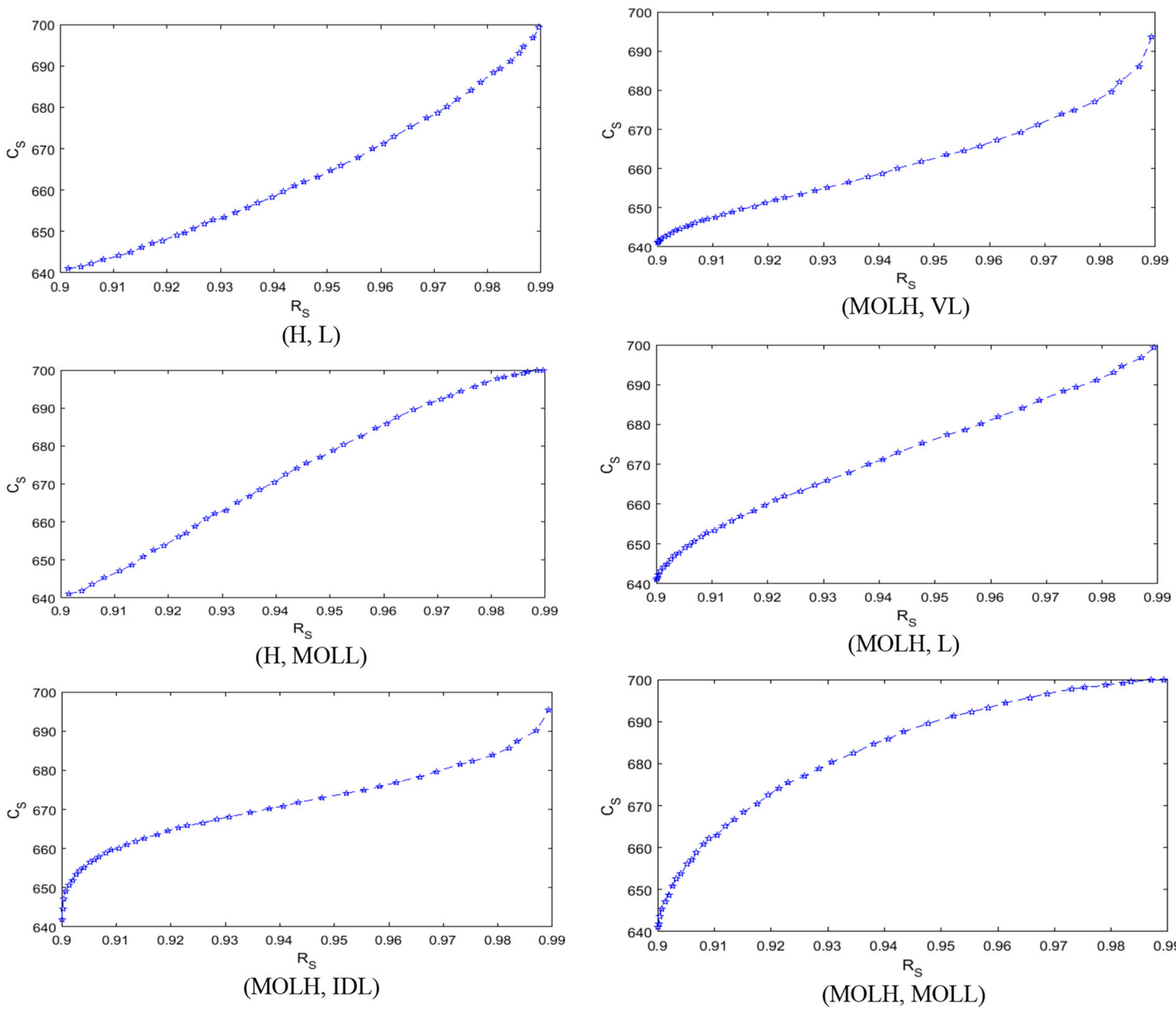


Fig. 10 continued

approach. Ravi et al. (2000) solved this problem using various kinds of aggregate operators to look into the impacts of system design and different optimal designs are found. The proposed approach does not need any kinds of aggregators in the formulation of the FMOOP and solves the problem in purely multi-objective manner as suggested by Deb (2001). Figure 13 shows a box-plot comparison between the optimal values obtained by Ravi et al. (2000) and the proposed approach.

$$\begin{array}{cc} & \textit{System Efficiency} \\ E_S = \begin{array}{c} \textit{Low} \\ \textit{Medium} \\ \textit{High} \\ \textit{Extreme} \end{array} & \left[\begin{array}{cccc} \textit{Good} & \textit{Average} & \textit{Below average} & \textit{Poor} \\ \textit{Very good} & \textit{Good} & \textit{Average} & \textit{Below average} \\ \textit{Excellent} & \textit{Very good} & \textit{Good} & \textit{Average} \\ \textit{Outstanding} & \textit{Excellent} & \textit{Very good} & \textit{Good} \end{array}\right] \\ R_S \uparrow C_S \rightarrow & \begin{array}{cccc} \textit{Low} & \textit{Medium} & \textit{High} & \textit{Extreme} \end{array} \end{array}$$

6 Conclusions

In this paper, a methodology is developed to provide the best optimal system design in the fuzzy multi-objective reliability optimization problem. FMOOP is solved by an elitist MOEA, namely, NSGA-II and the Pareto-optimal solution set is obtained in terms of the membership grades. After that, linguistic hedges are used to modify the solution set in various cases and FRBS is invoked effectively to find the system efficiency in each case. The conclusions of the proposed approach are drawn as follows:

- The proposed approach does not require any kind of aggregator operators and deals with the problem in a purely multi-objective manner.
- The advantage of using NSGA-II is to get a well-distributive solution set in one simulation run, where

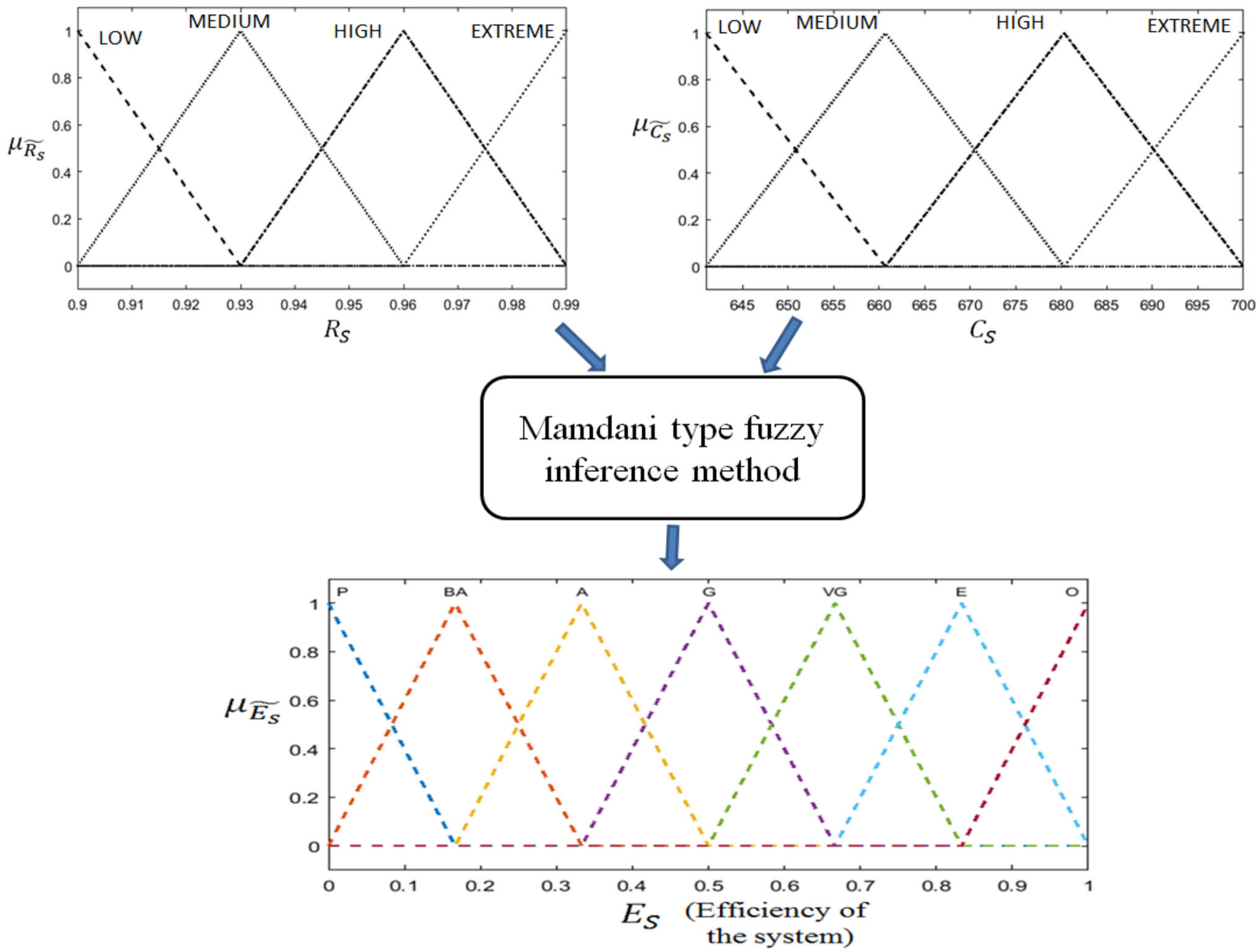


Fig. 11 The proposed FRBS model

Table 5 Encoded-rules for the FRBS

1. If (R_S is LOW) and (C_S is LOW) then (EFFICIENCY is G) (1)
2. If (R_S is LOW) and (C_S is MEDIUM) then (EFFICIENCY is A) (1)
3. If (R_S is LOW) and (C_S is HIGH) then (EFFICIENCY is BA) (1)
4. If (R_S is LOW) and (C_S is EXTREME) then (EFFICIENCY is P) (1)
5. If (R_S is MEDIUM) and (C_S is LOW) then (EFFICIENCY is VG) (1)
6. If (R_S is MEDIUM) and (C_S is MEDIUM) then (EFFICIENCY is G) (1)
7. If (R_S is MEDIUM) and (C_S is HIGH) then (EFFICIENCY is A) (1)
8. If (R_S is MEDIUM) and (C_S is EXTREME) then (EFFICIENCY is BA) (1)
9. If (R_S is HIGH) and (C_S is LOW) then (EFFICIENCY is E) (1)
10. If (R_S is HIGH) and (C_S is MEDIUM) then (EFFICIENCY is VG) (1)
11. If (R_S is HIGH) and (C_S is HIGH) then (EFFICIENCY is G) (1)
12. If (R_S is HIGH) and (C_S is EXTREME) then (EFFICIENCY is A) (1)
13. If (R_S is EXTREME) and (C_S is LOW) then (EFFICIENCY is O) (1)
14. If (R_S is EXTREME) and (C_S is MEDIUM) then (EFFICIENCY is E) (1)
15. If (R_S is EXTREME) and (C_S is HIGH) then (EFFICIENCY is VG) (1)
16. If (R_S is EXTREME) and (C_S is EXTREME) then (EFFICIENCY is G) (1)

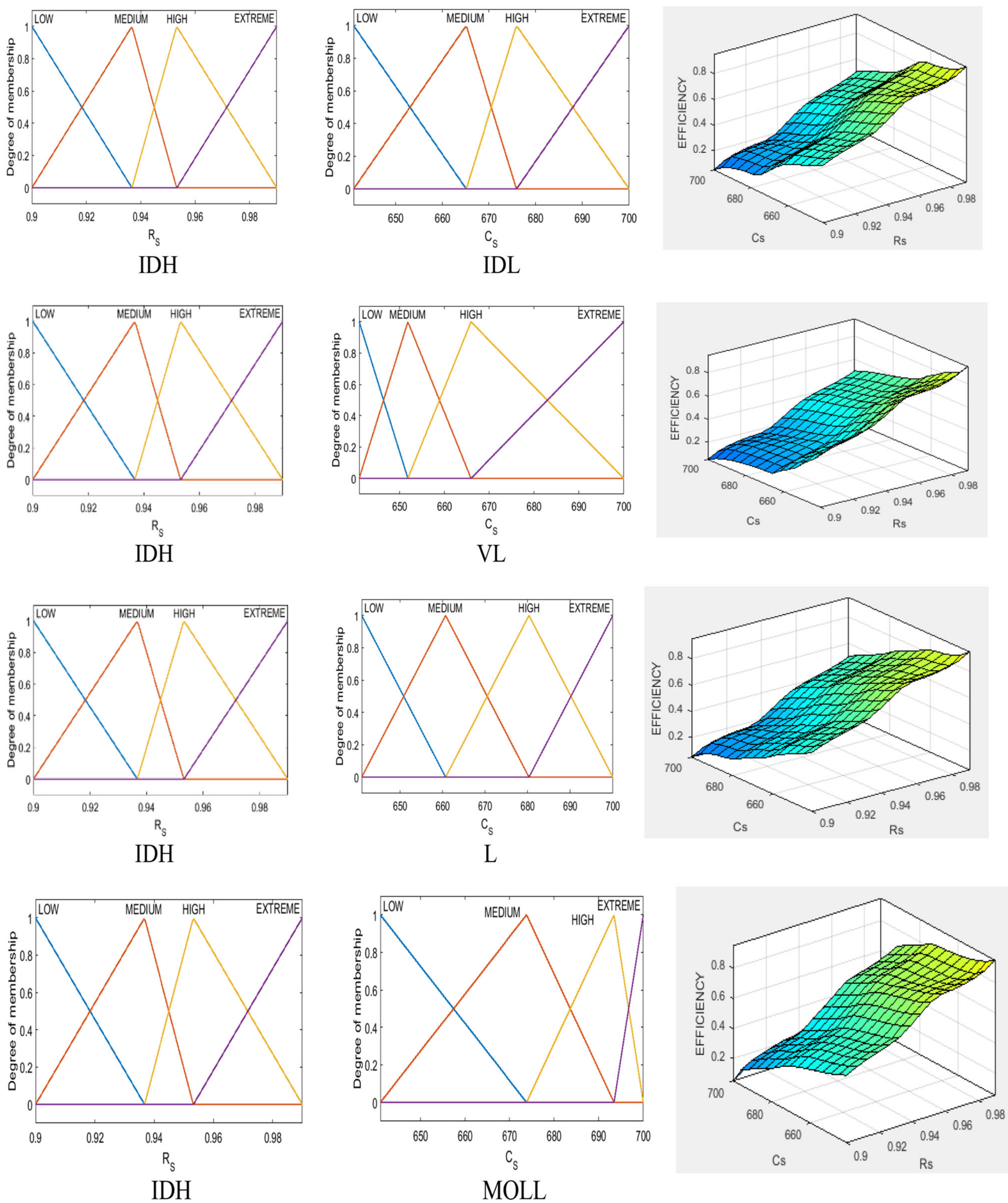


Fig. 12 Surface viewer plot for each combination of linguistic hedges

the DM gets more information such as non-dominated and their characteristics.

- Various combinations of linguistic hedges (or modifiers) are applied to determine the optimal system design and FRBS is used to evaluate its efficiency.

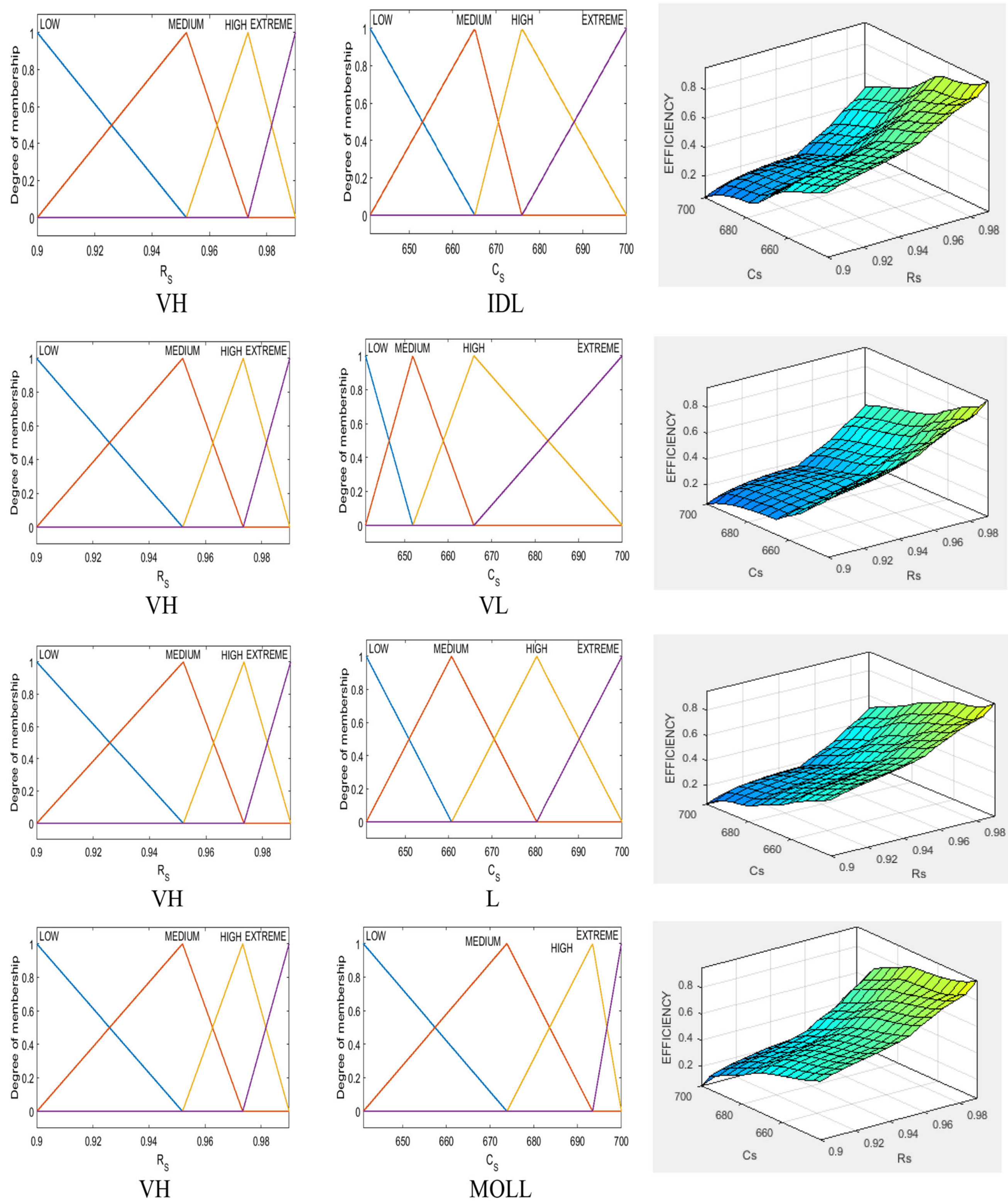


Fig. 12 continued

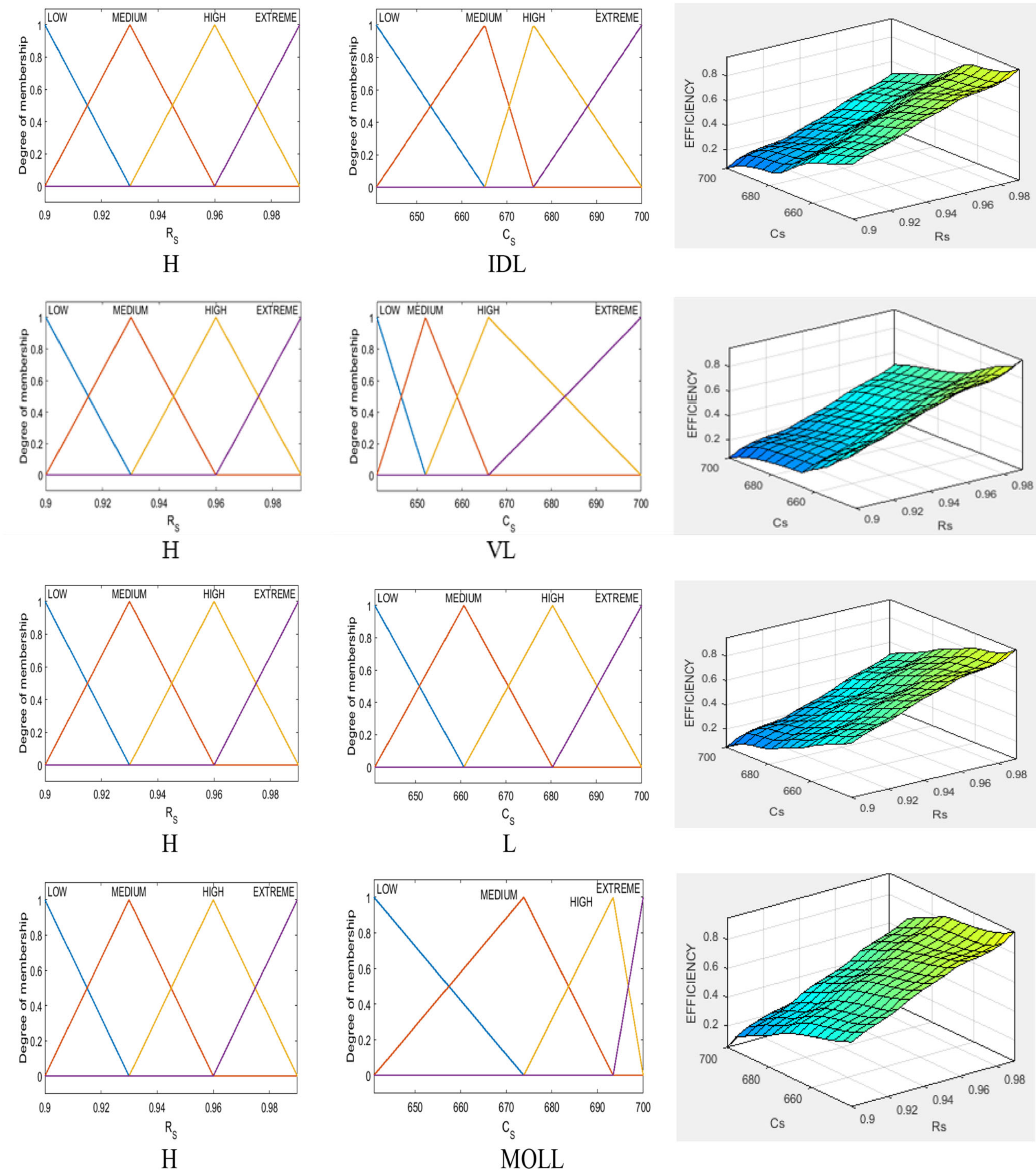


Fig. 12 continued

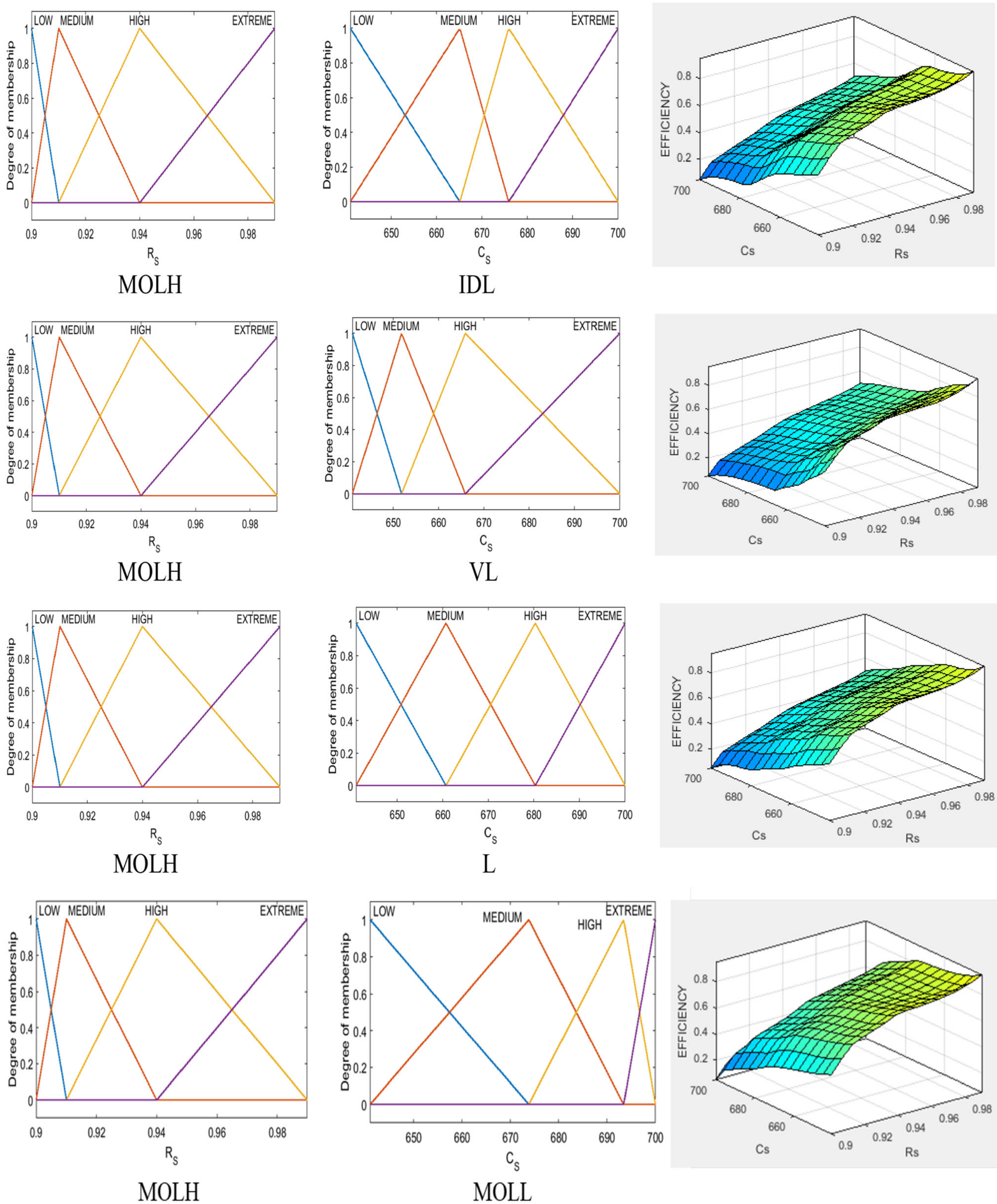


Fig. 12 continued

Table 6 The list of optimal system designs and their efficiencies for all possible combinations suggested by linguistic hedges and the significance of bold values is discussed in Sect. 5

Possible cases	r_1	r_2	r_3	r_4	R_S	C_S	μ_{max}	E_S (%)
(IDH, IDL)	0.55494	0.99149	0.5	0.5	0.94896	668.13	0.64218	55.3
(IDH, VL)	0.53456	0.99223	0.5	0.5	0.94444	652.20	0.43766	57.5
(IDH, L)	0.50142	0.86846	0.5	0.5	0.90836	641.03	**0.00059**	**54.4**
(IDH, MOLL)	0.51263	0.92531	0.5	0.5	0.95235	684.66	0.71406	56.3
(VH, IDL)	0.71264	0.96125	0.5	0.5	0.97379	670.84	0.45205	58.4
(VH, VL)	0.67654	0.96565	0.5	0.5	0.96879	655.18	0.33295	56.6
(VH, L)	0.71243	0.96815	0.5	0.5	0.97502	672.99	**0.45784**	**59**
(VH, MOLL)	0.73394	0.97183	0.5	0.5	0.97970	692.29	**0.60123**	**58.2**
(H, IDL)	0.65548	0.91464	0.5	0.5	0.95576	669.13	0.58678	56.6
(H, VL)	0.54589	0.96448	0.5	0.5	0.94187	651.24	0.46519	58.4
(H, L)	0.61773	0.93738	0.5	0.5	0.95258	665.96	0.57702	55.7
(H, MOLL)	0.6354	0.97314	0.5	0.5	0.96253	687.63	0.67664	57.8
(MOLH, IDL)	0.51273	0.92768	0.5	0.5	0.92585	666.57	0.71772	55.4
(MOLH, VL)	0.50576	0.87758	0.5	0.5	0.91193	648.25	0.59167	57.8
(MOLH, L)	0.51145	0.90155	0.5	0.5	0.91948	659.70	0.68205	57.2
(MOLH, MOLL)	0.53427	0.92478	0.5	0.5	0.93072	680.35	**0.75962**	**53.9**

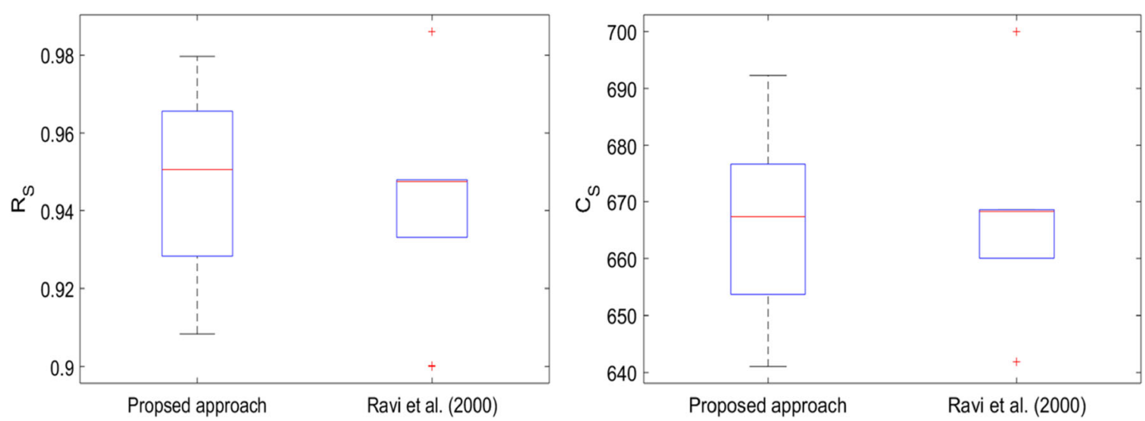


Fig. 13 Box-plot comparison of the objective values with the existing approach

- The optimal system design obtained by the combination (MOLH, MOLL) gives the highest maximum satisfaction (achievement) level, while (IDH, L) gives the lowest.
- From an efficiency point of view, (VH, L) gives the maximum system efficiency, while (MOLH, MOLL) gives the minimum.
- The combination (VH, MOLL) gives the best optimal system design in all possible cases.
- A box-plot comparison shows that the proposed approach gives a better spread of the optimal values in the entire search space compared to the existing approach.
- The proposed approach gives flexibility to the DM for choosing the best optimal system design of his/her own interests.
- The proposed approach can be extended to other high levels of uncertainty techniques such as type-2 fuzzy set, intuitionistic fuzzy set, LR type intuitionistic fuzzy set, interval-valued intuitionistic fuzzy set, etc.
- The proposed approach may be useful to determine the optimal design in an engineering system.

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