Laplacian whale optimization algorithm

Amarjeet Singh[1]

Abstract Whale optimization algorithm is a new member of nature inspired optimization algorithm which is inspired from foraging behaviour of humpback whales. Similar to other heuristic algorithms, Whale optimization algorithm suffers with immature convergence and stagnation problems while solving optimization problems. In this paper, Whale optimization algorithm is hybridized with Laplace Crossover operator and a new algorithm, Laplacian whale optimization algorithm (LXWOA), has been proposed. It has been used to solve a set of 23 classical benchmark functions which consists of scalable unimodal functions, scalable multimodal functions and low dimensional multimodal functions and the results are compared with original whale optimization algorithm, particle swarm optimization, differential evolution, gravitational search algorithm and Laplacian gravitational search algorithm. In this paper, LXWOA and WOA have also been used to solve the problem of extraction of compounds from gardenia.

Keywords Whale optimization algorithm · Laplace crossover · Numerical optimization · Heuristic algorithm · Gardenia problem

✉ Amarjeet Singh
amarjeetiitr@gmail.com

1 Department of Mathematics, Janki Devi Memorial College, University of Delhi, Sir Ganga Ram Hospital Marg, New Delhi, Delhi 110060, India

1 Introduction

It is a great challenge for the research community to solve real life nonlinear optimization problems arising from various branches of scientific engineering. Researchers have developed various techniques to provide the best result for such problems but still there are many optimization problems to be solved. Therefore, researchers are working in this research area, hence, the literature available for these methods are becoming rich. For simplicity, the available literature of these methods to solve the given problem can be classified into deterministic methods and probabilistic methods. Deterministic methods follow a set of rules and for a particular input, they always produce the same output for a given problem. Deterministic methods are applicable only to a restricted class of problems. However, probabilistic methods are more general and they are applicable to a wide range of problems. The inspiration of mostly probabilistic methods is natural laws. Therefore, they are also known as Nature Inspired Algorithms.

From the most recent couple of decades, Nature Inspired Algorithms (NIA) are becoming more and more popular in engineering application problems since they are based on simple ideas and are easy to execute. The beauty of these algorithms is that they bypass local optima. Classical optimization methods require some information about the functions like gradient etc. but NIA does not require any such information. Therefore, it can be used in an extensive variety of problems covering diverse disciplines.

Nature Inspired optimization algorithms mimic biological or physical phenomena. The available literature of NIA is vast. Genetic Algorithms (GA) (Holland 1992), Evolution Strategy (Rechenberg 1978), Genetic Programing (Koza 1992), Probability-based Incremental Learning

(Dasgupta and Zbigniew 2013), Biogeography-Based Optimization (Simon 2008), Simulated Annealing (Kirkpatrick et al. 1983), Gravitational Search Algorithms (Rashedi et al. 2009; Singh and Deep 2015, 2017a, b), Central Force Optimization (Formato 2007), Curved Space Optimization (Moghaddam et al. 2012), Big-Bang Big-Crunch (Erol and Eksin 2006), Small-World Optimization Algorithm (Du et al. 2006), Particle Swarm Optimization (Kennedy 2011), Ant Colony Optimization (Dorigo et al. 2006), Marriage in Honey Bees Optimization (Abbass 2001), Cuckoo Search (Yang and Deb 2009), Artificial Bee Colony (Basturk and Karaboga 2006), Bat-inspired Algorithm (Yang 2010), Monkey Search (Mucherino and Seref 2007), Firefly Algorithm (Yang 2010), Teaching Learning Based Optimization (Rao et al. 2011), Tabu Search (Glover 1989, 1990), Harmony Search Algorithm (Geem et al. 2001), Firework Algorithm (Tan and Zhu 2010) etc. are the popular nature inspired algorithms.

All NIA starts with a random population and share a common feature and their search process have an exploration phase and an exploitation phase. The exploration phase corresponds to global search *i.e.* algorithm is capable of exploring the search space globally. Whereas, exploitation corresponds to local search *i.e.* algorithm is capable of detailed investigation in the promising area(s) of the search space. It is extremely difficult to maintain proper balance between exploration and exploitation due to the stochastic nature of the optimization procedure.

The literature to solve optimization problems is very rich, but there is no single technique which can solve all the problems (Wolpert and Macready 1997). Therefore new algorithms are being introduced and existing algorithms are being developed with the hope that they have some advantage over the existing ones. Consequently, searching a new heuristic algorithm is an open issue (Wolpert and Macready 1997). The algorithms are being hybridized with the operator which has the capability to search locally (to improve solution quality) or search globally search (to skip premature convergence) or both.

Whale Optimization Algorithm (WOA) (Mirjalili and Lewis 2016) is a newly developed algorithm. It has been tested on various optimization problems with different difficulty level, but similar to other heuristic algorithms, WOA suffers with immature convergence i.e. it may rapidly converge towards a local optima instead of global optima and stagnation in local solutions while solving optimization problems. Hence, the quality of the final solution may decrease dramatically. These issues may happen when the optimizer could not have a fine balance between its exploration and exploitation.

WOA has been hybridized with simulated annealing (SA) to tackle the feature selection problems (Mafarja and Mirjalili 2017). Kaveh and Ghazaan (Kaveh and Ghazaan

2017) have proposed a modified whale optimization algorithm for sizing optimization of skeletal structures. Oliva et al. (Oliva et al. 2017) utilized a chaos-embedded WOA to deal with parameter estimation of photovoltaic cells. WOA has been used to solve multilevel thresholding image segmentation (El Aziz et al. 2017), the optimal renewable resources placement problem in distribution networks (Reddy et al. 2017) and to find the optimal weights in neural networks (Aljarah et al. 2018). It also has been used to attain the best parameters of SVM classifier (Tharwat et al. 2017) for predicting the drug toxicity. Yan et al. (2018) proposed ameliorative whale optimization algorithm to solve multi-objective water resource allocation optimization models. Nasiri and Khiyabani (2018) proposed whale clustering optimization algorithm for clustering. Hasanien (2018) used whale optimization algorithm to improve the performance of photovoltaic power systems. Kaur and Arora (2018) proposed chaotic whale optimization algorithm. Jadhav and Gomathi (2018) proposed a technique for data clustering using whale optimization algorithm. Elaziz and Oliva (2018) used opposition-based learning to enhance the exploration phase of whale optimization algorithm and used it to estimate the parameters of solar cells using three different diode models. Algabalawy et al. (2018) employed whale algorithm to find the optimal design of the system for minimizing the total annual cost and system emissions. Mehne and Mirjalili (2018) used whale optimization algorithm to solve optimal control problems. Xiong et al. (2018) proposed improved whale optimization algorithm to extract the parameters of different solar photovoltaic models accurately. Ala'm et al. (2018) proposed a hybrid machine learning model based on support vector machines and whale optimization algorithm for the task of identifying spammers in online social networks. Saidala and Devarakonda (2018) proposed improved whale optimization algorithm and applied to a clinical dataset of an anaemic pregnant woman and obtained optimized clusters and cluster heads to secure a clear comprehension and meaningful insights in the clinical decision-making process. Horng et al. (Horng et al. 2017) proposed a novel multi-objective method for an optimal vehicle traveling based on Whale optimization algorithm. Hassan and Hassanien (2018) presented a novel automated approach for extracting the vasculature of retinal fundus images using whale optimization algorithm. Mostafa et al. (2017) proposed a technique for liver segmentation in MRI images based on whale optimization algorithm. Abdel-Basset et al. (2019) used modified whale optimization algorithm for solving 0–1 knapsack problem. El Aziz et al. (2018) proposed a new method for determining the multi-level thresholding values for image segmentation using whale optimization algorithm. El Aziz et al. (2018) proposed non-dominated sorting technique based on multi-

objective whale optimization algorithm for content-based image retrieval. Nazari-Heris et al. (2017) used whale optimization to solve combined heat and power economic dispatch problem. Sun et al. (2018) proposed a modified whale optimization algorithm for large-scale global optimization problems. Abdel-Basset et al. (2018) proposed a modified whale optimization algorithm for cryptanalysis of Merkle-Hellman Knapsack Cryptosystem. Luo and Shi (2018) proposed hybrid whale optimization algorithm based on modified differential evolution for global optimization problems. Whale Optimization Algorithm has been used in many application problems like feature selection and land pattern classification (Bui et al. 2019). Sreenu and Sreelatha (2017) proposed the task scheduling algorithm based on the multi-objective model and the whale optimization algorithm for task scheduling in cloud computing. Eid (2018) proposed binary whale optimisation algorithm for feature selection. Ghahremani-Nahr et al. (2019) used whale optimization algorithm to minimize the total costs of closed-loop supply chain network. Hussien et al. (2019) proposed a binary whale optimization algorithm to select the optimal feature subset for dimensionality reduction and classifications problem using a sigmoid transfer function. Laskar et al. (2019) proposed hybrid whale-particle swarm optimization algorithm for solving complex optimization problems. Yousri et al. (2019) proposed four chaotic whale optimization variants. Elhosseini et al. (2019) proposed A-C WOA and applied on a biped robot to find the optimal settings of the hip parameters. The more literature may be study in Mirjalili et al. (2020).

In this paper, the performance of WOA is improved by hybridizing it with Laplace Crossover, which is a well-known Real Coded Genetic Algorithm operator. The rest of the paper is written as follows: In Sect. 2, the Whale Optimization Algorithm is clarified. In Sect. 3, Laplace Crossover is reproduced. In Sect. 4, LXWOA is described. In Sect. 5, the numerical results are investigated. In Sect. 6, the problem of extraction of compounds from gardenia is discussed and solved using LXWO and WOA. At last, in Sect. 7, the conclusions are drawn.

2 Whale optimization algorithm

Whale Optimization Algorithm (WOA) is proposed by Mirjalili and Lewis in 2016 (2016). It is inspired from the foraging behaviour of humpback whales. This foraging mechanism is known by bubble-net feeding method. In this mechanism, particular bubbles are created in a circle or '9'-shaped path as shown in Fig. 1. Goldbogen et al. (2013) utilized tag sensors to investigate the foraging behaviour of humpback whale and found two maneuvers (1) upward-spirals: humpback whales jump around 12 m down and

after that begin to rise in a winding shape around the prey and swim towards the surface, and (2) double-loops: it incorporates three distinct stages: coral loop, lobtail, and capture loop. More details can be found in Goldbogen et al. (2013).

Humpback whales use three strategies namely, searching of prey, encircling prey and bubble-net feeding method during the foraging behaviour. In the searching of prey, humpback whales search according to the positions of each other and explore new solutions randomly. In the encircling prey, humpback whales perceive the area of prey and recognize the location of prey after which they encircle them. Since there is no priori information about the position of the optimal solution in the search space, therefore, WOA algorithm expects that the current best solution is the target prey, or is close to the optimum. After defining it, other search agents update their positions towards the best search agent. In the bubble-net attacking, humpback whales swim around the prey within a shrinking circle and along a spiral-shaped path simultaneously. The working procedure of whale optimization algorithm is as follows:

Let N_p is the size of population and M_{itr} is the maximum number of iterations. The algorithm has two parameters a and b. The parameter a decreases linearly from 2 to 0 and the parameter b decreases from -1 to -2 as the iteration increases from 1 to M_{itr}. Then the coefficients A_i, C_i and a random number l_i for agent i are evaluated as follows:

$$A_i = a(2r_1 - 1)$$
$$C_i = 2r_2 \tag{1}$$

$$l_i = (b - 1)\, r_3 + 1 \tag{2}$$

where r_1, r_2 and r_3 are three uniformly distributed random numbers in [0, 1]. Then a random number p_r is generated if $p_r < 0.5$ and $|A_i| \geq 1$ then ith agent updates the dth component of next position $i.e.$ $x_i^d(t+1)$ with the random agent by the following equations:

$$D_i^d = \left| C_i \cdot x_{rand}^d(t) - x_i^d(t) \right| \tag{3}$$

$$x_i^d(t+1) = x_{rand}^d(t) - A_i \cdot D_i^d \tag{4}$$

where $x_{rand}^d(t)$ is the position of a random agent in dth dimension chosen from the current population. If $p_r < 0.5$ and $|A_i| < 1$ then ith agent updates the dth component of next position $i.e.$ $x_i^d(t+1)$ with the x_{best}^d agent by the following equations:

$$D_i^d = \left| C_i \cdot x_{best}^d(t) - x_i^d(t) \right| \tag{5}$$

$$x_i^d(t+1) = x_{best}^d(t) - A_i \cdot D_i^d \tag{6}$$

if $p_r \geq 0.5$ then, the distance $D_i^{'d} = \left| x_{best}^d(t) - x_i^d(t) \right|$ between the whale located at $x_i(t)$ and prey located at $x_{best}(t)$ in dth dimension is evaluated and is used to mimic
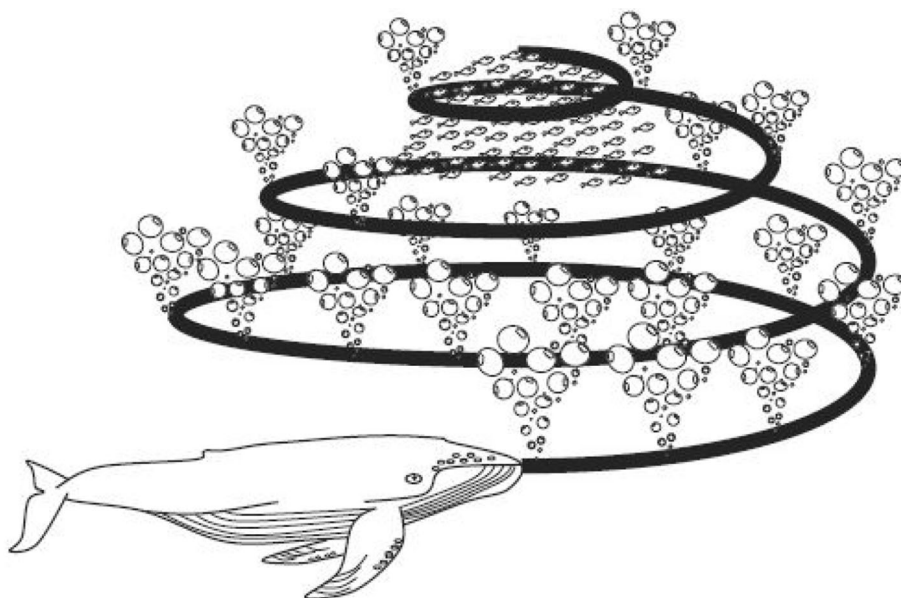
the helix-shaped movement of humpback whales in a spiral equation.

$$x_i^d(t+1) = D_i'^d \cdot e^{bl_i} \cdot \cos(2\pi l_i) + x_{best}^d(t) \tag{7}$$

The constant b controls the shape of spiral.

3 Laplace crossover

Deep and Thakur (2007) proposed Laplace Crossover (LX) to generate a pair of offspring $y_1 = (y_1^1, y_1^2, \ldots, y_1^m)$ and $y_2 = (y_2^1, y_2^2, \ldots, y_2^m)$ from a pair of parents $x_1 = (x_1^1, x_1^2, \ldots, x_1^m)$ and $x_2 = (x_2^1, x_2^2, \ldots, x_2^m)$. LX produces a pair of offspring in such a manner that both the offspring are symmetric with respect to the position of the parents. A Laplacian distributed random number l_i is created by the following rule:

$$l_i = \begin{cases} p - q \log_e(u_i), & v_i \leq 1/2 \\ p + q \log_e(u_i), & v_i > 1/2 \end{cases} \tag{8}$$

where u_i, v_i are two uniformly distributed random numbers in $[0,1]$. $p \in R$ is the location parameter and $q > 0$ is the scale parameter. The offspring are created by the following rules:

$$\begin{aligned} y_1^i &= x_1^i + l_i|x_1^i - x_2^i|, \\ y_2^i &= x_2^i + l_i|x_1^i - x_2^i|, \end{aligned} \tag{9}$$

If generated offspring does not belong to search space i.e. $y^i < y_{low}^i$ or $y^i > y_{up}^i$ for some i, then y^i is set a random number from the interval $[y_{low}^i, y_{up}^i]$.

When q is a small value, then it is likely that LX generates the offspring near to the parents and when q is a great value, then it is likely that it generates the offspring far from the parents. When p and q are fixed values, then LX deploys offspring proportional to the spread of parents (Deep and Thakur 2007). In literature, LX has been used to improve the performance of GA (Deep and Thakur 2007), GSA (Singh and Deep 2015), BBO (Garg and Deep 2016), PSO (Deep and Bansal 2009), etc.

4 Proposed algorithm

In the present study, WOA has been hybridized with the above defined Laplace crossover, which is a real coded crossover operator for real coded genetic algorithm and LXWOA is proposed.

In LXWOA algorithm, first a randomly distributed population of size N_p is initialized. At each iteration, LXWOA follows the WOA procedure first, then two agents are selected in which first one is the best agent and the second one is selected randomly from the current population. Laplace crossover is applied to the best and randomly selected agents. It generates two offspring. The fitness of both the offspring are tested with the worst agent in the current population one by one. If offspring has better fitness then it is replaced with the worst particle of the current population. Best is updated and iteration is incremented. Algorithm follows this procedure till termination criteria is satisfied. The pseudo code of LXWOA algorithm is shown in Fig. 2.

Fig. 2 Pseudo code of LXWOA algorithm

Set population size $= N_p$

Set problem dimension $=$ m and maximum iteration$= M_{itr}$

Initialize the whale population $x_i \left(i = 1, 2, ..., N_p \right)$

Evaluate the fitness

Set x_{best} *i.e.* the best search agent

while $(t < M_{itr})$

   for i = 1 to N_p

   Update a, A_i, C_i, l_i and p_r

     if1 ($p_r < 0.5$)

       if2 $(|A_i| < 1)$

$$D_i^d = \left| C_i \cdot x_{best}^d(t) - x_i^d(t) \right|$$

$$x_i^d(t+1) = x_{best}^d(t) - A_i \cdot D_i^d$$

       elseif2 $(|A_i| \geq 1)$

$$D_i^d = \left| C_i \cdot x_{rand}^d(t) - x_i^d(t) \right|$$

$$x_i^d(t+1) = x_{rand}^d(t) - A_i \cdot D_i^d$$

      endif2

     elseif1 ($p_r \geq 0.5$)

$$D_i'^d = \left| x_{best}^d(t) - x_i^d(t) \right|$$

$$x_i^d(t+1) = D_i'^d \cdot e^{bl_i} \cdot \cos(2\pi l_i) + x_{best}^d(t)$$

    endif1

   endfor

Check if any search agent goes beyond search space and amend it

Evaluate the fitness

Update x_{best} if there is a better solution

% Apply Laplace Crossover

Select and $x_1 = x_{best}$ and $x_2 =$ a random particle and apply Laplace Crossover

Let y_1 and y_2 be the offsprings

Replace y_1 and y_2 with worst particle if they have better fitness

t=t+1

endwhile

Table 1 Scalable unimodal functions

No.	Name	Definition	Domain	Known optimum
1.	Sphere	$F_1(x) = \sum_{i=1}^{n} x_i^2$	$[-100, 100]^n$	0
2.	Schwefel 2.22	$F_2(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i|$	$[-10, 10]^n$	0
3.	Schwefel 1.2	$F_3(x) = \sum_{j=1}^{i} \left(\sum_{j=1}^{i} x_j \right)^2$	$[-100, 100]^n$	0
4.	Schwefel 2.21	$F_4(x) = \max\{|x_i|, 1 \leq i \leq 30\}$	$[-100, 100]^n$	0
5.	Rosenbrock	$F_5(x) = \sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$	$[-30, 30]^n$	0
6.	Step	$F_6(x) = \sum_{i=1}^{n} (\lfloor x_i + 0.5 \rfloor)^2$	$[-100, 100]^n$	0
7.	Noise	$F_7(x) = \sum_{i=1}^{n} i x_i^4 + random\,[0, 1)$	$[-1.28, 1.28]^n$	0

In this paper dimension = 30 is considered

5 Results and discussion

The performance of LXWOA is tested on a set of 23 classical benchmark functions (Mirjalili and Lewis 2016) which consists of three types of problems (1) Scalable unimodal function (F_1 to F_7) are described in Table 1 (2) Scalable multimodal function (F_8 to F_{13}) are in Table 2 (3) low dimensional multimodal function with fixed dimension (F_{14} to F_{23}) are in Table 3. The experiments are performed on the processor: Intel (R) Core (TM) i3-2350 M CPU @ 2.30 GHz, RAM: 4.00 GB, Operating System: Window 10, Integrated Development Environment: MATLAB 2013. The parameter of Laplace crossover p is set to zero and the value of q is finely tuned by conducting several experiments. In these experiments, q varies from 0.05 to 0.30 and LXWOA algorithm is run 30 times with population size $N_p = 30$ and the termination criteria is set at 500 iterations. For $q = 0.05, 0.10, 0.15, 0.20, 0.25$ and 0.30, iteration wise average best so far is plotted for the function F_1, F_8, F_{14} and F_{16} and shown in the Fig. 3. From these figures, it is observed that the performance of LXWOA on F_1 is best and approximately same when $q = 0.05$ and $q = 0.10$ and it is worst when $q = 0.30$. The performance of LXWOA on F_8 is best when $q = 0.10$ and it is worst when $q = 0.15$. The performance of LXWOA on F_{14} is best when $q = 0.10$ and it is worst when $q = 0.05$. The performance of LXWOA on F_{16} is worst when $q = 0.25$ and it is best approximately same when $q = 0.05$ and $q = 0.10$. From these figures, it is concluded that the performance of LXWOA is best when $q = 0.10$.

LXWOA, WOA, GSA and LXGSA are run 30 times each with population size $N_p = 30$, $q = 0.10$ and the termination criteria is set at 500 iterations for all the functions. For a fair comparison among WOA and LXWOA the first randomly generated population is used for the first run of WOA and LXWOA, second randomly generated population is used for the second run of WOA and LXWOA, and so on. Best, Average, Median and Standard deviation (SD) of the objective function values obtained from LXWOA, WOA GSA and LXGSA are calculated over 30 runs and shown in Table 4 for scalable unimodal functions with dimension 30, Table 5 for scalable multimodal functions with dimension 30 and Table 6 for low dimensional multimodal functions. The results of Particle Swarm Optimization (PSO), and Differential Evolution (DE) are taken from (Mirjalili and Lewis 2016) to compare with LXWOA and WOA. In (Mirjalili and Lewis 2016), results are given in terms of Average and SD.

From the Table 4, it is observed that out of 7 problems, there are 3 problems, namely F_1, F_2 and F_7 in which the performance of LXWOA is better than WOA, PSO, DE, GSA and LXGSA and there are 4 problems, namely F_3, F_4,

Table 2 Scalable multimodal functions

No.	Name	Definition	Domain	Known optimum
8.	Schwefel 2.26	$F_8(x) = \sum_{i=1}^{n} \left(-x_i \sin\sqrt{|x_i|}\right)$	$[-500, 500]^n$	$-12{,}569.48$
9.	Rastrigin	$F_9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	$[-5.12, 5.12]^n$	0
10.	Ackley	$F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos 2\pi x_i\right) + 20 + e$	$[-32, 32]^n$	0
11.	Griewank	$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	$[-600, 600]^n$	0
12.	Penalize 1	$F_{12}(x) = \frac{\pi}{n}\left\{10\sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$	$[-50, 50]^n$	0
13.	Penalize 2	$F_{13}(x) = 0.1\left(\sin^2(3\pi x_1) + \sum_{i=1}^{n-1}(x_i - 1)^2\left[1 + \sin^2(3\pi x_{i+1})\right] + (x_n - 1)^2\left[1 + \sin^2(2\pi x_n)\right]\right) + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1), \quad u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$	$[-50, 50]^n$	0

In this paper dimension = 30 is considered

Table 3 Low dimensional multimodal test functions with fixed dimension

No.	Name	Definition	Dim.	Domain	Known optimum
14.	De Jong	$F_{14}(x) = \left\{ 0.002 + \sum_{j=1}^{25} \left[j + (x_1 - a_{1j})^6 + (x_2 - a_{2j})^6 \right]^{-1} \right\}^{-1}$	2	$[-65.53, 65.53]^2$	1
15.	Kowalik	$F_{15}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4} \right]^2$	4	$[-5, 5]^4$	0.0003075
16.	Camel Back-6 Hump	$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	$[-5, 5]^2$	−1.0316285
17.	Branin	$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6 \right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	2	$[-5, 10] \times [0, 15]$	$\frac{5}{4\pi}$
18.	Goldstein-price	$F_{18}(x) = \left[1 + (x_1 + x_2 + 1)^2 \left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2 \right) \right]$ $\left[30 + (2x_1 - 3x_2)^2 \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2 \right) \right]$	2	$[-5, 5]^2$	3
19.	Hartman 3	$F_{19}(x) = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2 \right]$	3	$[0, 1]^3$	−3.862747
20.	Hartman 6	$F_{20}(x) = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2 \right]$	6	$[0, 1]^6$	−3.322368
21.	Shekel 1	$F_{21}(x) = -\sum_{i=1}^{5} \left[(x - a_i)(x - a_i)^T + c_i \right]^{-1}$	4	$[0, 10]^4$	−10.1532
22.	Shekel 2	$F_{22}(x) = -\sum_{i=1}^{7} \left[(x - a_i)(x - a_i)^T + c_i \right]^{-1}$	4	$[0, 10]^4$	−10.4028
23.	Shekel 3	$F_{23}(x) = -\sum_{i=1}^{10} \left[(x - a_i)(x - a_i)^T + c_i \right]^{-1}$	4	$[0, 10]^4$	−10.5363
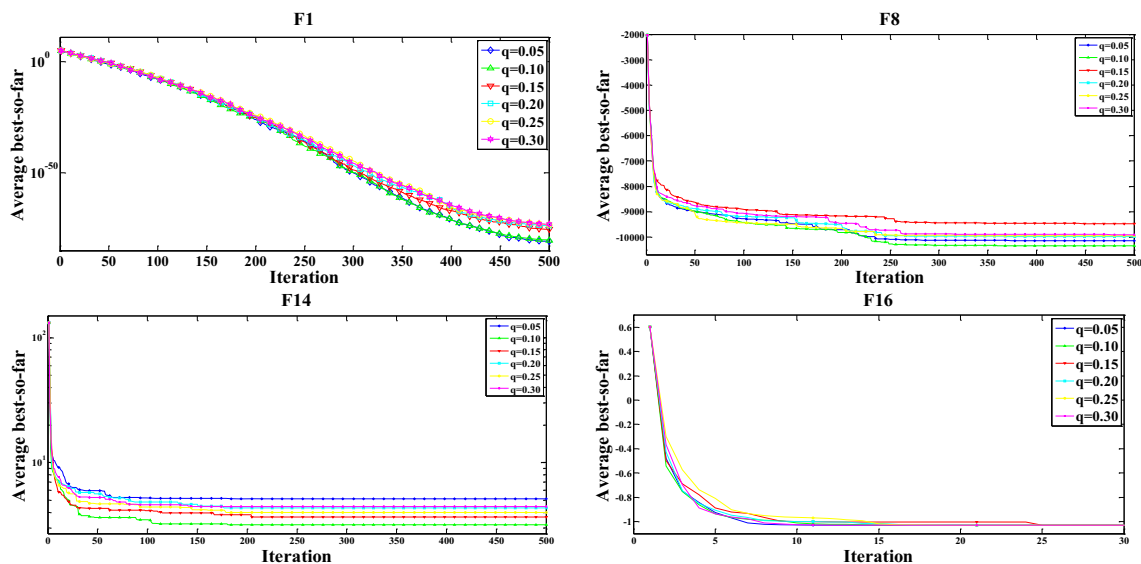
Fig. 3 Convergence plot of LXWOA at different values of q

F_5 and F_6 DE has better Average and SD. If LXWOA and WOA are compared together, then LXWOA has better performance on all 7 problems. Hence, from the Table 4, it is concluded that the performance of LXWOA is better than WOA on scalable unimodal functions when dimension is 30.

From the Table 5, it is observed that out of 6 problems, there are 3 problems, namely F_9, F_{10}, and F_{11} in which the performance of LXWOA is better than PSO, DE, GSA and LXGSA. On the problem F_8 and F_{12}, LXWOA has better

Best and Median in comparison to WOA, GSA and LXGSA. On F_{13}, LXGSA has better Best and LXWOA has better Median. DE has better Average and SD on F_8, F_{12} and F_{13}. When only LXWOA and WOA are considered then the performance of LXWOA is better on 3 problems, namely F_8, F_{11} and F_{12}. On F_9 and F_{10}, LXWOA and WOA have the same solution. On F_{13}, WOA has better performance in comparison to LXWOA. Hence, from the Table 5, it is concluded that the performance of LXWOA is

Table 4 Best, average, median and standard deviation (SD) of objective function value obtained from LXWOA, WOA, GSA and LXGSA for scalable unimodal functions with dimension = 30

Function	Algorithm	Best	Average	Median	SD
F_1	LXWOA	**4.22E−88**	**6.54E−77**	**1.21E−83**	**2.83E−76**
	WOA	5.53E−85	4.72E−73	5.53E−78	2.50E−72
	PSO	N/A	0.000136	N/A	0.000202
	DE	N/A	8.20E−14	N/A	5.90E−14
	GSA	1.089E−16	3.432E−16	2.503E−16	2.163E−16
	LXGSA	9.442E−17	4.678E−16	4.213E−16	3.158E−16
F_2	LXWOA	4.49E−57	**6.09E−53**	**7.21E−55**	**2.18E−52**
	WOA	**1.12E−57**	8.09E−51	1.09E−53	4.03E−50
	PSO	N/A	0.042144	N/A	0.045421
	DE	N/A	1.50E−09	N/A	9.90E−10
	GSA	8.419E−08	1.131E+00	3.015E−01	1.601E+00
	LXGSA	5.073E−08	2.028E−03	1.075E−07	1.053E−02
F_3	LXWOA	7197.08909	17,117.043	16,659.0959	6207.8913
	WOA	16,818.8775	41,342.7506	42,770.1954	12,947.7679
	PSO	N/A	70.12562	N/A	22.11924
	DE	N/A	**6.80E−11**	N/A	**7.40E−11**
	GSA	367.93177	851.94434	860.50975	291.56749
	LXGSA	**278.80713**	659.92181	**643.82142**	222.06728
F_4	LXWOA	**0.00989078**	18.999404	17.2163919	14.6545992
	WOA	0.84749337	48.3902289	45.238951	30.4122189
	PSO	N/A	1.086481	N/A	0.317039
	DE	N/A	**0**	N/A	**0**
	GSA	3.2110645	7.0670614	7.1700443	2.5648859
	LXGSA	1.2997698	5.4481264	**5.0522147**	2.2729241
F_5	LXWOA	26.6614903	27.4849083	**27.138808**	0.70597056
	WOA	27.3642746	28.1622818	27.9251976	0.44873917
	PSO	N/A	96.71832	N/A	60.11559
	DE	N/A	**0**	N/A	**0**
	GSA	25.895515	99.315747	64.999447	105.010271
	LXGSA	**20.522718**	37.352214	27.183192	31.030327
F_6	LXWOA	0.0033804	0.30085325	**0.26085737**	0.22717279
	WOA	0.04787168	0.43382843	0.41799493	0.25929252
	PSO	N/A	0.000102	N/A	8.28E−05
	DE	N/A	**0**	N/A	**0**
	GSA	**0**	21.5	6.5	44.67179161
	LXGSA	**0**	9.433333333	4	18.81430735
F_7	LXWOA	**3.72E−05**	**0.00228876**	**0.00164173**	**0.00219575**
	WOA	8.35E−05	0.00435107	0.00183437	0.00640528
	PSO	N/A	1.23E−01	N/A	4.50E−02
	DE	N/A	0.00463	N/A	0.0012
	GSA	0.08620274	1.58261390	0.37907712	2.87342855
	LXGSA	0.02591328	0.07831479	0.07434209	0.03488764

Best values are shown by boldface

better than WOA on scalable multimodal function when dimension is 30.

From the Table 6, it is observed that out of 10 problems, there are 2 problems, namely F_{19} and F_{20} in which LXGSA has better performance in comparison to others. On F_{16} and F_{17} (up to 4 decimal), all algorithms have approximately same solution. On F_{18}, LXGSA, PSO, DE found optimal solution and the solution found by LXWOA and WOA are very close to optimal solution. On F_{14}, F_{15}, F_{21}, F_{22} and F_{23}, DE has least Average and SD in comparison to other algorithms consider in this study which means that DE performs better on these problems. Hence, it is concluded

Table 5 Best, average, median and standard deviation (SD) of objective function value obtained from LXWOA, WOA, GSA and LXGSA for scalable multimodal functions with dimension = 30

Function	Algorithm	Best	Average	Median	SD
F_8	LXWOA	**− 12569.247**	− 10,482.949	**− 10,894.579**	1734.3735
	WOA	**− 12,569.459**	− 9959.5446	− 9010.1823	1872.8263
	PSO	N/A	− 4.84E+03	N/A	1.15E+03
	DE	N/A	**− 11,080.1**	N/A	**574.7**
	GSA	− 3685.888241	− 2586.921938	− 2454.983462	458.2062508
	LXGSA	− 6991.255853	− 4848.031367	− 4733.80066	710.4445225
F_9	LXWOA	**0**	**0**	**0**	**0**
	WOA	**0**	**0**	**0**	**0**
	PSO	N/A	4.67E+01	N/A	1.16E+01
	DE	N/A	69.2	N/A	38.8
	GSA	20.8941352	40.0636143	38.8033680	10.9427193
	LXGSA	22.8840482	40.3952822	38.8033528	11.3700881
F_{10}	LXWOA	**8.88E−16**	**4.44E−15**	**4.44E−15**	**2.47E−15**
	WOA	**8.88E−16**	**4.32E−15**	**4.44E−15**	**2.38E−15**
	PSO	N/A	0.276015	N/A	0.50901
	DE	N/A	9.70E−08	N/A	4.20E−08
	GSA	8.3007E−09	1.0059E−01	1.3897E−08	3.0881E−01
	LXGSA	6.1245E−09	1.5157E−08	1.3959E−08	5.3883E−09
F_{11}	LXWOA	**0**	**0**	**0**	**0**
	WOA	**0**	0.00613605	**0**	0.03360851
	PSO	N/A	0.009215	N/A	0.007724
	DE	N/A	0	N/A	0
	GSA	17.547450	27.744648	28.462734	5.551119
	LXGSA	4.696050	16.194750	14.287670	8.629175
F_{12}	LXWOA	**0.00048444**	0.0170252	**0.01903697**	0.00926204
	WOA	0.00611412	0.02893606	0.02315188	0.01955525
	PSO	N/A	0.006917	N/A	0.026301
	DE	N/A	**7.90E−15**	N/A	**8.00E−15**
	GSA	0.4637620	2.1842792	2.2820715	1.0693484
	LXGSA	0.1194997	1.3342209	0.9579515	0.9352277
F_{13}	LXWOA	0.35753337	0.86269746	0.86454	0.32827913
	WOA	0.19079886	0.5617074	**0.53102885**	0.31825744
	PSO	N/A	6.68E−03	N/A	8.91E−03
	DE	N/A	**5.10E−14**	N/A	**4.80E−14**
	GSA	0.6010870	11.6421777	11.3545666	6.4108955
	LXGSA	**0.0000000**	2.9707697	1.0381519	4.9797020

Best values are shown by boldface

that the performance of DE is better than others on low dimensional multimodal problems. When only LXWOA and WOA are considered then it is found that the performance of LXWOA is better than WOA.

In order to observe the behaviour of the objective function value with a passage of iterations the convergence plots of the WOA and LXWOA are plotted and shown in Figs. 4 and 5. On the horizontal axis the iterations are shown, whereas on the vertical axis the average best-so-far is shown. Average best-so-far is the average value of objective function in each iteration over 30 runs. From the plots it is concluded that LXWOA is converging fast towards optima in comparison to WOA algorithm.

t test the performance of LXWOA has been compared with WOA, GSA and LXGSA using a pairwise one tailed *t*-test with 29° of freedom at 0.05 level of significance over the objective function value of all the problems considered. The null hypothesis is assumed that "there is no difference between algorithms" and alternative hypothesis is "there is difference". The *p*-value and conclusions are shown in Table 7. The following criterion is used to conclude the results: A+ shows that *p*-value < 0.01 and LXWOA is highly significant than algorithm 1, A shows that *p*-value

Table 6 Best, average, median and standard deviation (SD) of objective function value obtained from LXWOA, WOA, GSA and LXGSA for low dimensional multimodal functions

Function	Algorithm	Best	Average	Median	SD
F_{14}	LXWOA	**0.99800384**	3.58484502	2.98210516	3.36422006
	WOA	**0.99800384**	3.97013194	2.48706803	3.96689046
	PSO	N/A	3.627168	N/A	2.560828
	DE	N/A	**0.998004**	N/A	**3.30E−16**
	GSA	**0.99800384**	5.46887718	5.22136507	2.82023499
	LXGSA	**0.99800384**	3.23197220	**1.99203091**	2.62260919
F_{15}	LXWOA	**0.00030847**	0.00085229	0.00061099	0.00058168
	WOA	**0.00030787**	0.00117425	**0.00059669**	0.00260336
	PSO	N/A	0.000577	N/A	**0.000222**
	DE	N/A	**3.30E−04**	N/A	**4.50E−14**
	GSA	0.00199107	0.00851930	0.00708096	0.00521632
	LXGSA	0.00058737	0.00272881	0.00140278	0.00487748
F_{16}	LXWOA	**− 1.0316285**	**− 1.0316285**	**− 1.0316285**	**2.61E−11**
	WOA	**− 1.0316285**	**− 1.0316285**	**− 1.0316285**	**6.18E−09**
	PSO	N/A	− 1.03163	N/A	6.25E−16
	DE	N/A	− 1.03E+00	N/A	3.10E−13
	GSA	**− 1.03162845**	**− 1.03162845**	**− 1.03162845**	**4.610E−16**
	LXGSA	**− 1.03162845**	**− 1.03162845**	**− 1.03162845**	**4.879E−16**
F_{17}	LXWOA	**0.397887358**	0.397887358	0.397887358	**1.78E−08**
	WOA	**0.397887358**	0.39789209	0.39788887	6.36E−06
	PSO	N/A	0.397887	N/A	0.00E+00
	DE	N/A	0.397887	N/A	9.90E−09
	GSA	**0.397887358**	**0.397887358**	**0.397887358**	**0**
	LXGSA	**0.397887358**	**0.397887358**	**0.397887358**	**0**
F_{18}	LXWOA	**3**	**3.00008507**	3.00001885	0.00015186
	WOA	3.00000008	3.00009761	3.00001321	0.00023369
	PSO	N/A	**3.00E+00**	N/A	1.33E−15
	DE	N/A	**3.00E+00**	N/A	2.00E−15
	GSA	**3**	3.035995876	**3**	0.197157535
	LXGSA	**3**	**3**	**3**	4.37999E−15
F_{19}	LXWOA	**− 3.8627819**	− 3.8621387	− 3.8623976	0.00066689
	WOA	**− 3.8627818**	− 3.8546655	− 3.8609132	0.01384994
	PSO	N/A	− 3.86E+00	N/A	2.58E−15
	GSA	**− 3.86277979**	− 3.85542699	− 3.8549018	0.001998711
	LXGSA	**− 3.86277979**	**− 3.86277979**	**− 3.86277979**	3.16177E−15
F_{20}	LXWOA	− 3.3219951	− 3.2637595	− 3.3219571	0.08503449
	WOA	− 3.3219195	− 3.2307532	− 3.3133816	0.10565286
	PSO	N/A	− 3.27E+00	N/A	**6.05E−02**
	GSA	− 2.6501605	− 1.6328745	− 1.5338478	0.4589163
	LXGSA	**− 3.3223680**	**− 3.2764764**	**− 3.3223680**	0.0624305
F_{21}	LXWOA	**− 10.1532**	− 7.6041983	**− 7.6041974**	2.59257641
	WOA	− 10.150525	− 7.0942847	− 5.0551419	2.7691968
	PSO	N/A	− 6.8651	N/A	3.019644
	DE	N/A	**− 10.1532**	N/A	**0.0000025**
	GSA	**− 10.153200**	− 4.970431	− 5.055198	1.451668
	LXGSA	**− 10.153200**	− 5.191435	− 5.055198	1.965018

Table 6 continued

Function	Algorithm	Best	Average	Median	SD
F_{22}	LXWOA	**− 10.402941**	− 7.267452	− 7.7453047	3.27955326
	WOA	− 10.401231	− 6.5671551	− 5.0875108	3.28048397
	PSO	N/A	− 8.45653	N/A	3.087094
	DE	N/A	**− 10.4029**	N/A	**3.90E−07**
	GSA	**− 10.4029406**	− 7.77108567	− 8.13199829	2.680440355
	LXGSA	**− 10.4029406**	− 8.59176558	**− 10.4029406**	2.922939069
F_{23}	LXWOA	**− 10.53641**	− 7.2052629	− 5.1284808	3.04449842
	WOA	− 10.534427	− 6.822795	− 5.1283876	3.16109578
	PSO	N/A	− 9.95291	N/A	1.782786
	DE	N/A	**− 10.5364**	N/A	**1.90E−07**
	GSA	**− 10.5364098**	− 8.9471948	**− 10.5364098**	2.727929868
	LXGSA	**− 10.5364098**	− 9.81535261	**− 10.5364098**	1.869769309

Best values are shown by boldface

< 0.05 and LXWOA is significantly better than algorithm 1, B shows that p-value = 0.05 and LXWOA is alike algorithm 1, C shows that p-value < 0.1 and LXWOA is marginally significant than algorithm 1, D shows that p-value > 0.1 and LXWOA is not significant.

On observing the results shown in Table 7, it can be concluded that if WOA versus LXWOA is considered then 8 out of the 20 problems show that LXWOA is highly significant than WOA. If GSA versus LXWOA is considered then 15 out of the 23 problems show that LXWOA is highly significant than GSA. If LXGSA versus LXWOA is considered then 12 out of the 22 problems show that LXWOA is highly significant than LXGSA. There are two problems, namely F_1 and F_{16} in which p-value cannot be computed because the standard error of the difference is 0.

In the above experiment I, the maximum number of iterations (i.e. 500) are fixed. In one iteration, WOA requires 30 number of function evaluations and LXWOA requires 32 number of function evaluations for a population of size 30. Therefore, WOA and LXWOA evaluates 15,000 and 16,000 number of functions respectively in a run of 500 iteration. This shows that the cost of LXWOA is higher than the cost of WOA. Keeping in this mind, experiment II is conducted in which termination criteria is set to be "maximum number of function evaluation less than 5000". The available results of PSO and DE are taken for 500 iteration, hence, experiment I has been conducted by fixing the termination criteria to be "maximum iteration = 500". But in experiment II, author wants to utilize maximum exploration and exploitation capability of the algorithm to find the best quality of solution. So, the maximum number of function evaluation is set to 5000 for both the algorithms. The Best, Worst, Average, Median and Standard deviation (SD) of the objective function values obtained from LXWOA and WOA reported in Table 8 for all problems listed in Tables 1, 2 and 3.

The results in Table 8 shows that the solutions obtained from LXWOA are better than WOA on scalable unimodal functions (F_1 to F_7). Out of 6 problems of scalable multimodal functions (F_8 to F_{13}), there are 2 problems, namely F_9 and F_{10} in which LXWOA and WOA have the same set of solutions. There is one problem, namely F_{13} in which WOA turns to have better solution and there is one problem, namely F_{11} in which LXWOA has better solution. On F_8 and F_{12}, LXWOA has better Best but WOA has better Worst, Average and Median. Out of 10 problems of low dimensional multimodal functions (F_{14} to F_{23}), there are 7 problems, namely F_{19}, F_{20}, F_{21}, F_{22} and F_{23} in which the performance of LXWOA is better than WOA. There are 4 problems, namely F_{14}, F_{16}, F_{17} and F_{18} in which LXWOA and WOA showed approximately the same solutions. On F_{15}, LXWOA has better Worst and Median but better Best and Average has been found by WOA.

From the above analysis, it can be concluded that the performance of LXWOA has improved in comparison to WOA on Scalable unimodal function, Scalable multimodal function and low dimensional multimodal functions for a fixed dimensions.

6 The problem of extraction of compounds from Gardenia

Three bioactive compounds, namely, crocin (Y_1), geniposide (Y_2) and total phenolic (Y_3) compounds are obtained from gardenia fruits, which are affected by three independent variables, concentrations of ethanol (X_1), extraction temperature (X_2) and extraction time (X_3). Young et al. (Yang et al. 2009) have presented a nonlinear multi-objective optimization problem using the method of least square fitting and has solved it using Response Surface Method. Shashi et al. (2010) used DDX-LLM algorithm and Garg and Deep (2016) used Laplacian biogeography
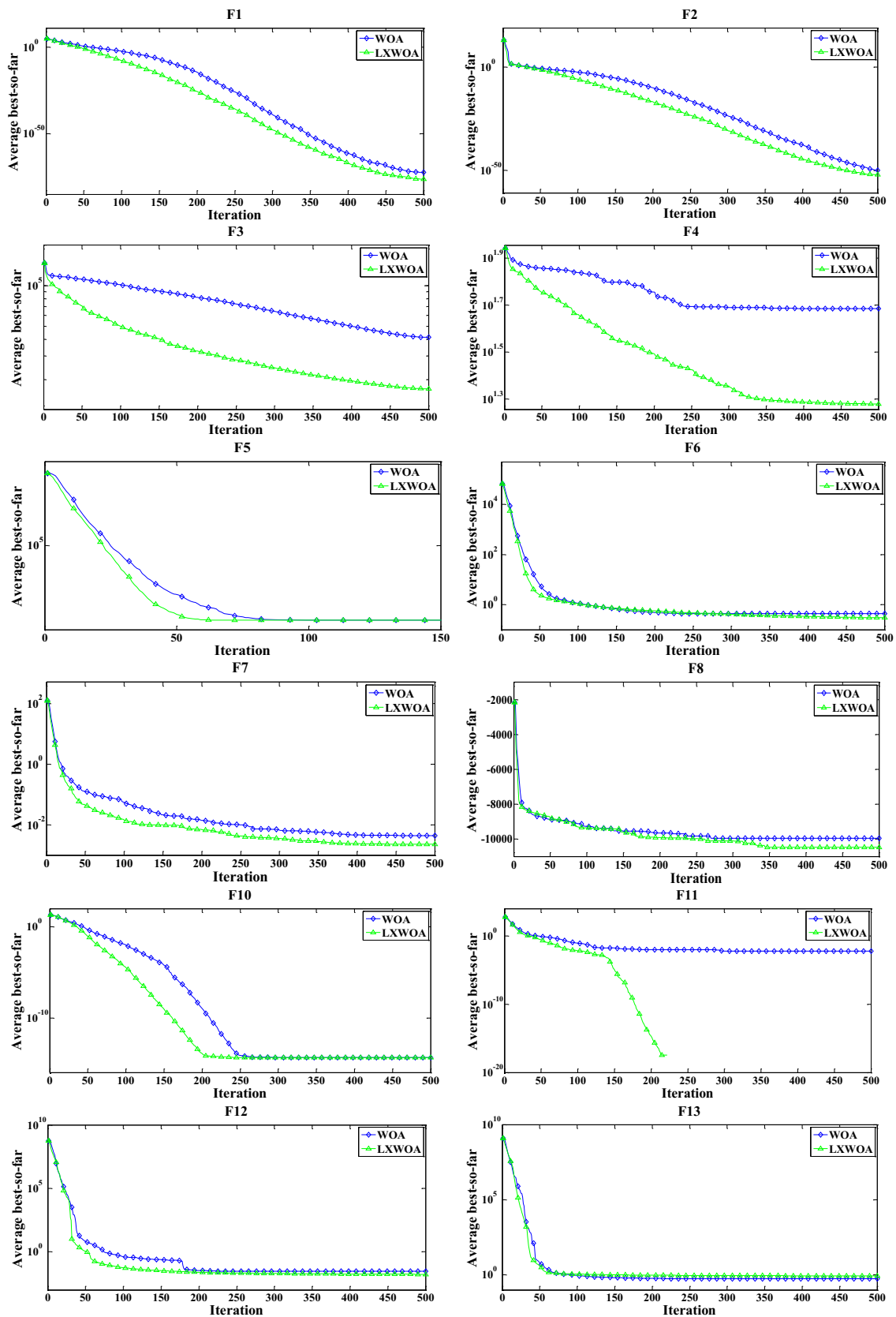
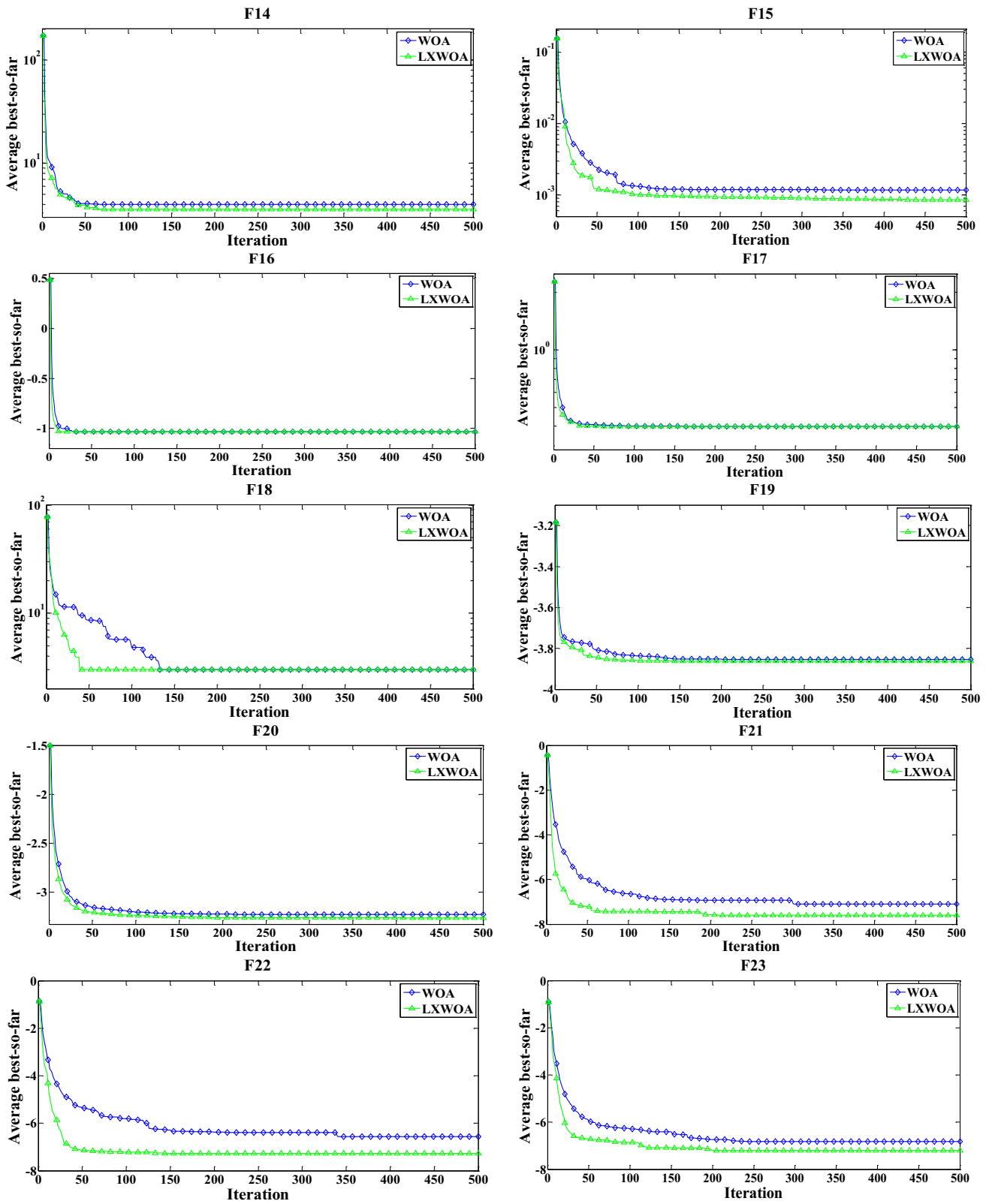**Fig. 4** Iteration wise convergence plot of WOA and LXWOA

Fig. 5 Iteration wise convergence plot of WOA and LXWOA

Table 7 A pairwise *t*-test results of objective function values with 95% confidence interval at 0.05 level of significance

Function	WOA versus LXWOA		GSA versus LXWOA		LXGSA versus LXWOA	
	p-value	Conclusion	*p*-value	Conclusion	*p*-value	Conclusion
F_2	–	–	0.000	A+	0.150	D
F_3	0.000	A+	0.000	A+	0.000	A+
F_4	0.000	A+	0.000	A+	0.000	A+
F_5	0.000	A+	0.000	A+	0.046	A
F_6	0.022	A	0.007	A+	0.006	A+
F_7	0.053	C	0.002	A+	0.000	A+
F_8	0.095	C	0.000	A+	0.000	A+
F_9	–	–	0.000	A+	0.000	A+
F_{10}	–	–	0.042	A	–	–
F_{11}	0.163	D	0.000	A+	0.000	A+
F_{12}	0.002	A+	0.000	A+	0.000	A+
F_{13}	0.000	A+	0.000	A+	0.014	A
F_{14}	0.334	D	0.014	A	0.319	D
F_{15}	0.260	D	0.000	A+	0.025	A
F_{17}	0.000	A+	0.056	C	0.056	C
F_{18}	0.355	D	0.163	D	0.002	A+
F_{19}	0.003	A+	0.000	A+	0.000	A+
F_{20}	0.002	A+	0.000	A+	0.254	D
F_{21}	0.234	D	0.000	A+	0.000	A+
F_{22}	0.145	D	0.242	D	0.042	A
F_{23}	0.280	D	0.016	A	0.000	A+

based optimization (LX-BBO) algorithm to solve it. In this paper, the above problem has been resolved with WOA and LXWOA. The mathematical model of the above problem, which is mentioned in Yang et al. (2009) and Shashi and Katiyar 2010), is as follows.

In formulation, each yield is in the form of a function of three independent variables. These functions are the second order polynomial equation which is as follows:

$$Y_k = b_0 + \sum_{i=1}^{3} b_i X_i + \sum_{i=1}^{3} b_{ii} X_i^2 + \sum_{i \neq j=1}^{3} b_{ij} X_i X_j \quad (10)$$

where, Y_k represents yield, b_0 represents a constant, b_i, b_{ii} and b_{ij} linear, quadratic and interactive coefficients respectively of the model. X_i and X_j are independent variables. The resulting equation of yield Y_1, Y_2 and Y_3 are as follows:

$$\begin{aligned} Y_1 &= 3.8384907903 + 0.0679672610 X_1 \\ &\quad + 0.0217802311 X_2 + .0376755412 X_3 \\ &\quad - 0.0012103181 X_1^2 + 0.0000953785 X_2^2 \\ &\quad - 0.0002819634 X_3^2 + 0.0005496524 X_1 X_2 \\ &\quad - 0.0009032316 X_2 X_3 + 0.0008033811 X_1 X_3 \quad (11) \end{aligned}$$

$$\begin{aligned} Y_2 &= 46.6564201287 + 0.6726057655 X_1 \\ &\quad + 0.4208752507 X_2 + 0.9999909858 X_3 \\ &\quad - 0.0161053654 X_1^2 - 0.0034210643 X_2^2 \\ &\quad - 0.0116458859 X_3^2 + 0.0122000907 X_1 X_2 \\ &\quad - 0.0095644212 X_2 X_3 + 0.0089464814 X_1 X_3 \quad (12) \end{aligned}$$

$$\begin{aligned} Y_3 &= -6.3629169281 + 0.4060552042 X_1 \\ &\quad + 0.3277005337 X_2 + 0.3411029105 X_3 \\ &\quad - 0.0053585731 X_1^2 - 0.0020487593 X_2^2 \\ &\quad - 0.0042291040 X_3^2 + 0.0017226318 X_1 X_2 \\ &\quad - 0.0011990977 X_2 X_3 + 0.0007814998 X_1 X_3 \quad (13) \end{aligned}$$

This problem demonstrates a multi-objective optimization problem in which all three yield is needed to be maximized.

Multi-objective optimization problem is a problem in which more than one objective function are optimized together. The weighted method approach is a simple and effective technique to handle multi-objective optimization problems. In this technique, the user converts these problems into the single objective problem by defining different or equal weight for each objective function. In this research paper, it has been solved with WOA and LXWOA by converting the problem into one objective problem by giving equal weight to all the yields.

Table 8 Best, worst, average, median and standard deviation (SD) of objective function value obtained from LXWOA and WOA, when number of function evaluation less than 5000

		Best	Worst	Average	Median	SD
F1	LXWOA	**1.69E−223**	**6.00E−201**	**2.25E−202**	**2.73E−209**	**0.00E+00**
	WOA	1.96E−209	2.73E−174	1.12E−175	6.60E−191	0.00E+00
F2	LXWOA	**1.27E−148**	**5.93E−136**	**2.00E−137**	**9.53E−143**	**1.08E−136**
	WOA	1.13E−136	4.13E−120	2.24E−121	5.16E−127	8.57E−121
F3	LXWOA	**886.9526**	**6829.8305**	**3448.6196**	**3524.5140**	**1469.5980**
	WOA	1842.3174	25077.8463	10,955.8780	8914.8447	6550.8715
F4	LXWOA	**0.003433**	**39.671743**	**10.627369**	**6.640341**	**11.763112**
	WOA	0.260888	84.753947	43.333357	41.448827	27.696969
F5	LXWOA	**24.441370**	**26.447747**	**25.032597**	**24.964978**	**0.448631**
	WOA	25.282108	27.575309	26.225782	26.074569	0.557266
F6	LXWOA	**1.00E−06**	2.00E−02	**2.66E−03**	**1.57E−04**	5.26E−03
	WOA	0.000062	**0.001971**	0.000388	0.000227	0.000398
F7	LXWOA	1.85E−05	**3.92E−03**	**1.09E−03**	**5.97E−04**	1.16E−03
	WOA	**1.72E−05**	9.57E−03	1.37E−03	6.74E−04	1.99E−03
F8	LXWOA	**− 12569.4825**	− 7239.2648	− 11,618.4949	− 12,280.5711	1451.3722
	WOA	− 12,569.4410	**− 9310.8193**	**− 12,023.5839**	**− 12,346.6143**	825.2364
F9	LXWOA	**0**	**0**	**0**	**0**	**0**
	WOA	**0**	**0**	**0**	**0**	**0**
F10	LXWOA	**8.88E−16**	**7.99E−15**	4.09E−15	**4.44E−15**	2.70E−15
	WOA	**8.88E−16**	**7.99E−15**	3.85E−15	**4.44E−15**	2.48E−15
F11	LXWOA	**0**	**0**	**0**	**0**	**0**
	WOA	0	0.02027139	0.00067571	0	0.00370103
F12	LXWOA	**4.8792E−07**	0.00581755	0.00066326	8.8584E−05	0.00121686
	WOA	5.2999E−06	**0.00058855**	**8.1236E−05**	**5.0201E−05**	0.00010877
F13	LXWOA	0.00154796	0.46860958	0.13555144	0.10809413	0.11967755
	WOA	**0.00024321**	**0.11739748**	**0.01600827**	**0.01126839**	**0.02322964**
F14	LXWOA	**0.99800384**	**5.92884513**	1.69093506	**0.99800384**	1.30186458
	WOA	**0.99800384**	**5.92884513**	**1.22850192**	**0.99800384**	0.95873431
F15	LXWOA	0.00030770	**0.00122881**	0.00058382	**0.00035067**	0.0003783
	WOA	**0.00030763**	0.00126439	**0.00056520**	0.00040233	0.00033303
F16	LXWOA	**− 1.03162845**	**− 1.0316285**	**− 1.03162845**	**− 1.03162845**	**5.5626E−16**
	WOA	**− 1.03162845**	**− 1.0316285**	**− 1.03162845**	**− 1.03162845**	8.9826E−15
F17	LXWOA	**0.39788736**	**0.39788736**	**0.39788736**	**0.39788736**	**1.9336E−13**
	WOA	**0.39788736**	**0.39788736**	**0.39788736**	**0.39788736**	5.9743E−10
F18	LXWOA	**3**	**3**	**3**	**3**	**3.5748E−10**
	WOA	**3**	**3**	**3**	**3**	1.0797E−11
F19	LXWOA	**− 3.86278215**	**− 3.8627703**	**− 3.86278109**	**− 3.86278201**	**2.3901E−06**
	WOA	− 3.86278214	− 3.8549006	− 3.86182228	− 3.8627739	0.0024764
F20	LXWOA	**− 3.32199517**	**− 3.1376166**	**− 3.2641707**	**− 3.32199516**	**0.06394486**
	WOA	− 3.32199514	− 3.081284	− 3.23832994	− 3.20210265	0.08651612
F21	LXWOA	**− 10.1531997**	**− 10.153196**	**− 10.1531992**	**− 10.1531997**	**1.1109E−06**
	WOA	**− 10.1531997**	− 10.093571	− 10.1510262	− 10.1531992	0.01086304
F22	LXWOA	**− 10.4029406**	**− 10.367541**	**− 10.4017598**	**− 10.4029406**	0.00646298
	WOA	**− 10.4029406**	− 3.7243003	− 9.77997585	− 10.4029395	1.91145172
F23	LXWOA	**− 10.5364098**	**− 10.536398**	**− 10.5364092**	**− 10.5364098**	2.322E−06
	WOA	**− 10.5364098**	− 2.421734	− 10.0425505	− 10.5364087	1.88857021

Best values are shown by boldface

Table 9 The yield obtained by different algorithm

Algorithm/yield	Crocin (Y_1)	Geniposide (Y_2)	Phenolic (Y_3)
WOA	**9.6418**	**117.7906**	**25.2781**
LXWOA	**9.6418**	**117.7906**	**25.2781**
LX-BBO (Garg and Deep 2016)	9.2504	113.924	25.2279
Response Surface Method (Yang et al. 2009)	8.36	108.5	24.5
DDL-LLM (Shashi and Katiyar 2010)	8.43	110.026	24.81

Mathematically, for the given yield function Y_1, Y_2 and Y_3, the objective is to solve the following function:

$$\text{Maximum } g = w_1 Y_1 + w_2 Y_2 + w_3 Y_3 \qquad (14)$$

where w_1, w_2 and w_3 are the weights given by the user. In this paper, $w_1 = 0.33$, $w_2 = 0.33$ and $w_3 = 0.34$ are kept.

Results In this section, the results of the problem of the extraction of compounds from gardenia by WOA and LXWOA have been presented and these results have been compared with the results available in the literature. The algorithms have been run to maximum 100 iterations by keeping the number of candidates in the population as 30. The results obtained on the basis of 30 independent experiments have been shown using boldface in Table 9. The range of X_1, X_2 and X_3 are taken in 0–100.

When WOA and LXWOA are used to solve the above problem, it is concluded that when the concentration of ethanol is $X_1 = 63.482211\%$, extraction temperature is $X_2 = 100\ °C$ and extraction time $X_3 = 27.290530$ min, then crocin ($Y_1 = 9.6418$), geniposide ($Y_2 = 117.7906$) and phenolic ($Y_3 = 25.2781$) are obtained in maximum amount.

7 Conclusions

In this paper, a novel Laplacian whale optimization algorithm (LXWOA) has been proposed and results are compared with Whale Optimization Algorithm, Particle Swarm Optimization, Differential Evolution, Gravitational Search Algorithm and Laplacian Gravitational Search Algorithm. From the above study, it is concluded that Laplacian whale optimization algorithm gives most promising results as compared to whale optimization algorithm, particle swarm optimization, differential evolution, gravitational search algorithm and Laplacian gravitational search algorithm on scalable unimodal and scalable multimodal functions when dimension is set 30. The performance of LXWOA has been improved as compared to WOA on Scalable unimodal function, Scalable multimodal function and low dimensional multimodal functions with fixed dimensions. The results obtained from LXWOA and WOA for the problem of extraction of compounds from gardenia are better as compared to the results available in the literature.

References

Abbass HA (2001) MBO: marriage in honey bees optimization-a haplometrosis polygynous swarming approach. In: Proceedings of the 2001 congress on evolutionary computation, pp 207–214

Abdel-Basset M, El-Shahat D, El-Henawy I, Sangaiah AK, Ahmed SH (2018) A novel whale optimization algorithm for cryptanalysis in Merkle-Hellman cryptosystem. Mobile Netw Appl 1–11

Abdel-Basset M, El-Shahat D, Sangaiah AK (2019) A modified nature inspired meta-heuristic whale optimization algorithm for solving 0–1 knapsack problem. Int J Mach Learn Cybern 1–20

Ala'm AZ, Faris H, Hassonah MA (2018) Evolving support vector machines using whale optimization algorithm for spam profiles detection on online social networks in different lingual contexts. Knowl Based Syst 153:91–104

Algabalawy MA, Abdelaziz AY, Mekhamer SF, Aleem SHA (2018) Considerations on optimal design of hybrid power generation systems using whale and sine cosine optimization algorithms. J Electr Syst Inf Technol 5(3):312–325

Aljarah I, Faris H, Mirjalili S (2018) Optimizing connection weights in neural networks using the whale optimization algorithm. Soft Comput 1–15

Basturk B, Karaboga D (2006) An artificial bee colony (ABC) algorithm for numeric function optimization. In: Proceedings of the IEEE swarm intelligence symposium, pp 12–14

Bui QT, Pham MV, Nguyen QH, Nguyen LX, Pham HM (2019) Whale optimization algorithm and adaptive neuro-fuzzy inference system: a hybrid method for feature selection and land pattern classification. Int J Remote Sens 1–16

Dasgupta D, Zbigniew M (2013) Evolutionary algorithms in engineering applications. Springer

Deep K, Bansal JC (2009) Optimization of directional over current relay times using Laplace Crossover Particle Swarm Optimization (LXPSO). In: 2009 World congress on nature & biologically inspired computing (NaBIC), pp 288–293

Deep K, Thakur M (2007) A new crossover operator for real coded genetic algorithms. Appl Math Comput 188(1):895–911

Dorigo M, Birattari M, Stutzle T (2006) Ant colony optimization. IEEE Comput Intell Mag 1(4):28–39

Du H, Wu X, Zhuang J (2006) Small-world optimization algorithm for function optimization. In: Jiao L, Wang L, Gao X, Liu J, Wu F (eds) Advances in natural computation. ICNC 2006. Lecture Notes in Computer Science, Springer, Berlin, Heidelberg, vol 4222, pp 264–273

Eid HF (2018) Binary whale optimisation: an effective swarm algorithm for feature selection. Int J Metaheuristics 7(1):67–79

El Aziz MA, Ahmed Ewees AA, Hassanien AE (2017) Whale Optimization Algorithm and Moth-Flame Optimization for multilevel thresholding image segmentation. Expert Syst Appl 83:242–256

El Aziz MA, Ewees AA, Hassanien AE, Mudhsh M, Xiong S (2018) Multi-objective whale optimization algorithm for multilevel thresholding segmentation. In: Advances in soft computing and

machine learning in image processing. Springer, Cham, pp 23–39

El Aziz MA, Ewees AA, Hassanien AE (2018b) Multi-objective whale optimization algorithm for content-based image retrieval. Multimed Tools Appl 77(19):26135–26172

Elaziz MA, Oliva D (2018) Parameter estimation of solar cells diode models by an improved opposition-based whale optimization algorithm. Energy Convers Manag 171:1843–1859

Elhosseini MA, Haikal AY, Badawy M, Khashan N (2019) Biped robot stability based on an A-C parametric Whale Optimization Algorithm. J Comput Sci 31:17–32

Erol OK, Eksin I (2006) A new optimization method: big bang-big crunch. Adv Eng Softw 37(2):106–111

Formato RA (2007) Central force optimization: a new metaheuristic with applications in applied electromagnetics. Prog Electromagn Res 77:425–491

Garg V, Deep K (2016) Optimal extraction of bioactive compounds from Gardenia using Laplacian biogeography based optimization. In: Kim J, Jim Z (eds) Harmony search algorithm advances in intelligent systems and computing, vol 382. Springer, Berlin, pp 251–258

Geem ZW, Kim JH, Loganathan G (2001) A new heuristic optimization algorithm: harmony search. Simulation 76(2):60–68

Ghahremani-Nahr J, Kian R, Sabet E (2019) A robust fuzzy mathematical programming model for the closed-loop supply chain network design and a whale optimization solution algorithm. Expert Syst Appl 116:454–471

Glover F (1989) Tabu search—Part I. ORSA J Comput 1(3):190–206

Glover F (1990) Tabu search—Part II. ORSA J Comput 2(1):4–32

Goldbogen JA, Friedlaender AS, Calambokidis J, Mckenna MF, Simon M, Nowacek DP (2013) Integrative approaches to the study of baleen whale diving behavior, feeding performance, and foraging ecology. Bioscience 63(2):90–100

Hasanien HM (2018) Performance improvement of photovoltaic power systems using an optimal control strategy based on whale optimization algorithm. Electr Power Syst Res 157:168–176

Hassan G, Hassanien AE (2018) Retinal fundus vasculature multilevel segmentation using whale optimization algorithm. Signal Image Video Process 12(2):263–270

Holland JH (1992) Genetic algorithms. Sci Am 267(1):66–72

Horng MF, Dao TK, Shieh CS (2017) A multi-objective optimal vehicle fuel consumption based on whale optimization algorithm. In: Advances in intelligent information hiding and multimedia signal processing. Springer, Cham, pp 371–380

Hussien AG, Hassanien AE, Houssein EH, Bhattacharyya S, Amin M (2019) S-shaped binary whale optimization algorithm for feature selection. In: Recent trends in signal and image processing. Springer, Singapore, pp 79–87

Jadhav AN, Gomathi N (2018) WGC: hybridization of exponential grey wolf optimizer with whale optimization for data clustering. Alex Eng J 57(3):1569–1584

Kaur G, Arora S (2018) Chaotic whale optimization algorithm. J Comput Des Eng 5(3):275–284

Kaveh A, Ghazaan MI (2017) Enhanced whale optimization algorithm for sizing optimization of skeletal structures. Mech Based Des Struct Mach 45(3):345–362

Kennedy J (2011) Particle swarm optimization. In: Encyclopedia of machine learning. Springer US, pp 760–766

Kirkpatrick S, Gelatt CD, Vecchi MP (1983) Optimization by simulated annealing. Science 220:671–680

Koza JR (1992) Genetic programming

Laskar NM, Guha K, Chatterjee I, Chanda S, Baishnab KL, Paul PK (2019) HWPSO: a new hybrid whale-particle swarm optimization algorithm and its application in electronic design optimization problems. Appl Intell 49(1):265–291

Luo J, Shi B (2018) A hybrid whale optimization algorithm based on modified differential evolution for global optimization problems. Appl Intell 1–19

Mafarja MM, Mirjalili S (2017) Hybrid Whale Optimization Algorithm with simulated annealing for feature selection. Neurocomputing 260:302–312

Mehne HH, Mirjalili S (2018) A parallel numerical method for solving optimal control problems based on whale optimization algorithm. Knowl Based Syst 151:114–123

Mirjalili S, Lewis A (2016) The whale optimization algorithm. Adv Eng Softw 95:51–67

Mirjalili S, Mirjalili SM, Saremi S, Mirjalili S (2020) Whale Optimization Algorithm: theory, literature review, and application in designing photonic crystal filters. In: Nature-inspired optimizers. Springer, Cham, pp 219–238

Moghaddam FF, Moghaddam RF, Cheriet M (2012) Curved space optimization: a random search based on general relativity theory. arXiv: 1208.2214

Mostafa A, Hassanien AE, Houseni M, Hefny H (2017) Liver segmentation in MRI images based on whale optimization algorithm. Multimed Tools Appl 76(23):24931–24954

Mucherino A, Seref O (2007) Monkey search: a novel metaheuristic search for global optimization. In: AIP conference proceedings, pp 162–173

Nasiri J, Khiyabani FM (2018) A whale optimization algorithm (WOA) approach for clustering. Cogent Math Stat 5(1):1–13

Nazari-Heris M, Mehdinejad M, Mohammadi-Ivatloo B, Babamalek-Gharehpetian G (2017) Combined heat and power economic dispatch problem solution by implementation of whale optimization method. Neural Comput Appl 1–16

Oliva D, El Aziz MA, Hassanien AE (2017) Parameter estimation of photovoltaic cells using an improved chaotic whale optimization algorithm. Appl Energy 200:141–154

Rao RV, Savsani VJ, Vakharia DP (2011) Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. Comput Aided Des 43(3):303–315

Rashedi E, Nezamabadi-Pour H, Saryazdi S (2009) GSA: a gravitational search algorithm. Inf Sci 179(13):2232–2248

Rechenberg I (1978) Evolutionsstrategien. Springer, Berlin, pp 83–114

Reddy PDP, Reddy VCV, Manohar TG (2017) Optimal renewable resources placement in distribution networks by combined power loss index and whale optimization algorithms. J Electr Syst Inf Technol 175–191

Saidala RK, Devarakonda N (2018) Improved whale optimization algorithm case study: clinical data of anaemic pregnant woman. In: Data engineering and intelligent computing. Springer, Singapore, pp 271–281

Shashi DK, Katiyar VK (2010) Multi-objective extraction optimization of bioactive compounds from Gardenia using real coded genetic algorithm. In: 6th World congress of biomaconics, vol 31, pp 1436–1466

Simon D (2008) Biogeography-based optimization. IEEE Trans Evol Comput 12(6):702–713

Singh A, Deep K (2015) Real coded genetic algorithm operators embedded in gravitational search algorithm for continuous optimization. Int J Intell Syst Appl 7(12):1–22

Singh A, Deep K (2017a) Novel hybridized variants of gravitational search algorithm for constraint optimization. Int J Swarm Intell 3(1):1–22

Singh A, Deep K (2017b) Hybridized gravitational search algorithms with real coded genetic algorithms for integer and mixed integer optimization problems. In: Proceedings of sixth international conference on soft computing for problem solving. Springer, Singapore, pp 84–112

Sreenu K, Sreelatha M (2017) W-scheduler: whale optimization for task scheduling in cloud computing. Cluster Comput 1–12

Sun Y, Wang X, Chen Y, Liu Z (2018) A modified whale optimization algorithm for large-scale global optimization problems. Expert Syst Appl 114:563–577

Tan Y, Zhu Y (2010) Fireworks algorithm for optimization. In: Advances in swarm intelligence. Springer, pp 355–364

Tharwat A, Moemen YS, Hassanien AE (2017) Classication of toxicity effects of biotransformed hepatic drugs using whale optimized support vector machines. J Biomed Inform 68:132–149

Wolpert DH, Macready WG (1997) No free lunch theorems for optimization. IEEE Trans Evol Comput 1(1):67–82

Xiong G, Zhang J, Shi D, He Y (2018) Parameter extraction of solar photovoltaic models using an improved whale optimization algorithm. Energy Convers Manag 174:388–405

Yan Z, Sha J, Liu B, Tian W, Lu J (2018) An ameliorative whale optimization algorithm for multi-objective optimal allocation of water resources in Handan, China. Water 10(1):87–116

Yang X-S (2010) A new metaheuristic bat-inspired algorithm. In: Proceedings of the workshop on nature inspired cooperative strategies for optimization (NICSO 2010). Springer, pp 65–74

Yang X-S (2010b) Firefly algorithm, stochastic test functions and design optimisation. Int J Bio-Inspired Comput 2(2):78–84

Yang X-S, Deb S (2009) Cuckoo search via Lévy flights. In: Proceedings of the world congress on nature & biologically inspired computing, NaBIC 2009, pp 210–214

Yang B, Liu X, Gao Y (2009) Extraction optimization of bioactive compounds (crocin, geniposide and total phenolic compounds) from Gardenia (Gardenia jasminoides Ellis) fruits with response surface methodology. Innov Food Sci Emerg Technol 10:610–615

Yousri D, Allam D, Eteiba MB (2019) Chaotic whale optimizer variants for parameters estimation of the chaotic behavior in permanent magnet synchronous motor. Appl Soft Comput 74:479–503