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Multi-objective linear fractional inventory problem under intuitionistic fuzzy environment

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Abstract In this paper, we have assumed an inventory multi-objective optimization model under intuitionistic fuzziness. In modelling, we have considered the situations where triangular intuitionistic fuzzy numbers used to express some of the input information which associated with decision variables. Further, a ranking function approach by considering linear and the nonlinear degree of membership functions have been used to obtain the crisp form of the fuzzy parameters. Finally, the fuzzy goal programming approach has been used to solve the resultant model to obtain the optimal ordering quantity. Also, a comparative study of the formulated problem under intuitionistic fuzziness has been done with a deterministic model of inventory. The concept of the paper is explained through a numerical example.

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1 Introduction

In industries, the top management personnel's, who are also the decision maker (DM), are not able to accurately estimate the business-oriented input information's. Since due to complex situations in business nobody can judge the actual parameters. These parameters are also known as the independent factors. However, the DM has some rough idea about these independent factors. The DM can roughly express these types of uncertainties by using fuzzy numbers either with non-negative trapezoidal or triangular IFN. Zadeh (1965) gave the concept of well known fuzzy set theory to tackle the uncertainty. Later on, with some advancement the concept of intuitionistic fuzzy set (IFS) theory was specified by Atanassov (1986, 1989), to tackle the intricate ambiguity in decision-making problems. Some important applicationsof IFS theory have been discussed by the following authors: Nagoorgani and Ponnalagu (2012), Mahapatra and Roy (2013), Garg and Rani (2013), Garg et al. (2013, 2014a, b), Wu and liu (2013), Chakraborty et al. (2014), De and Sana (2014), Garg (2014, 2015a, b, 2016a, b, 2017, 2018), Singh and Garg (2017), Garg and Arora (2017), and many others.

Today multi-objective optimization model has been recurrently formulated by the practitioners or researchers in almost every field of the research study. The best part of the multi-criterion decision making is to optimize all objective functions simultaneously in a single frame. The decision maker can also set up an aspiration level to each of the objective function which is considered as their goal values. In particular, most of the models formulated for supply chains management includes transportation problem, production planning problem and inventory problem based on a deterministic view of the decision maker. In the same

manner, a multi-criteria decision-making model with fuzzy parameters is found to be more realistic than the deterministic model. However, while modelling the optimization model, we have found that data are not often precisely known to the decision maker due to incomplete and inadequate information. Therefore in such situation first role of the DM is to study the pattern of data after then make it convenient for modelling. This paper deals with IFS concepts in inventory management. Inventory management is the lifeblood of every supply chain, and if it accurately managed then it plays an essential role in reducing the total cost, brings productivity for companies; otherwise, it serves as legal responsibility for the concerned authority. Moreover, as the prerequisite of the business changes, the inventory management needs to be modified as accordingly. Inventory management is concerned with the flow of materials from vendors (suppliers) to production and the subsequent movement of products through distribution centres to the customers at the right time to maintain the demand and supply ratio with the minimum cost of the product. The primary objective of inventory management is specifying the quantity of items when it should be ordered from the vendors (suppliers) and how much it is? Interested readers can read the research of Tersine (1994), Pentico and Drake (2011), for getting a proper overview on the inventory models.

From the last decade or with the commencement of the twentieth era, optimization technique has been recurrently used by the researchers in the field of inventory management. The researchers have developed many models related to the inventory control, but the conventional economic order quantity (EOQ) model proposed by Harris (1913) is still considered the most popular inventory model. Due to its ease of use and simplicity, many researchers tried to extend it under different realistic assumptions. Hariri and Abou-El-Ata (1997) formulated inventory model as a geometric programming problem with the assumption of varying deterministic ordering cost for a multi-item lot size problem. Sabri and Beamon (2000) formulated the most significant part of a supply chain management as a mathematical programming problem and minimize the cost related to production cost and delivery time of strategic and operational planning respectively. Fung et al. (2003) formulated multi production planning problem with ambiguous demands and capacities with the purpose to obtain the optimal order quantities. The fractional programming problem formulation of inventory management first proposed by Sadjadi et al. (2005). They considered a deterministic model with the primary aim was to optimise the holding cost and entirely ordered quantities simultaneously. Similarly, Chen (2005) used LFPP approach under the stochastic environment in inventory problem. Later on, two different types of inventory model proposed by Khanra et al. (2010), Valliathal and Uthayakumar (2010) with reliant demand rate. Yadav et al.

(2010) established a model with small storage volume with deterioration rate under some reliability and flexibility assumptions. Banerjee and Roy (2010) formulated stochastic inventory model by considering fuzzy cost function and uniform, exponential, and normal lead time demand. The developed model is solved by fuzzy intuitionistic optimization technique and Zimmermann (1978) fuzzy optimization technique respectively. Kundu and Chakrabarti (2011) discussed the circumstances of ambiguous demand and deterioration rate under the tolerable delay in disbursements for a lot-size model. Das and Maiti (2012) formulated an inventory model for production problem and used the simulated methodology to obtain the optimal quantity under uncertainty. Rafiei et al. (2013) constructed supply chain network design as a mixed integer programming problem and minimizes the expected cost of transhipment. Roy et al. (2013) considered a deterministic situation in EOQ model with the assumption of tolerable postponement in expenses. Chakrabortty et al. (2013) discussed an inventory model in which different inventory costs and demand quantity considered as fuzzy numbers. They used intuitionistic fuzzy programming technique with different types of membership functions to obtain the Pareto optimal solution. The same work has been extended by Bhaya et al. (2014) by considering scrap and reworkable types of deficient quality items. De et al. (2014) developed an intuitionistic fuzzy optimization algorithm for solving certain and uncertain inventory model where all the parameters of an uncertain model considered as fuzzy triangular numbers. Fattahi et al. (2015) determined the reorder level and ordering quantity by optimising total cost and service level simultaneously using Metaheuristic approach. Du et al. (2015) developed a multi-objective optimisation technique based neural network model to optimise inventory costs, and the non-dominated algorithm has been used to solve it. The most notable work using multi-criterion decision making in inventory management was given by Dutta and Kumar (2013, 2015). They considered multi-products without shortages and formulated the problem as a multi-objective inventory model with conflicting linear and fractional objective function respectively and obtained the optimal order quantity of products. Some other remarkable works related to inventory control management are as follows: Xu and Zhao (2008, 2010), Wee et al. (2009), Shah and Soni (2011), Pando et al. (2012, 2013), Min et al. (2012), Bera et al. (2012), Taleizadeh et al. (2013), Tripathi (2013), Mondal et al. (2013), Mahata and Goswami (2013), Yang (2014), Srivastav and Agrawal (2016), Garai et al. (2018) and the contribution has been summarized in Table 1.

De and Sana (2014) developed different optimization models for inventory management, the general optimization model for inventory and intuitionistic fuzzy optimization model for inventory. The primary purpose of the

Table 1 Summary of related literature

Author(s) and year	Item	Objective	Demand rate	Holding cost	Shortages	Parameters
Xu and Zhao (2008)	Multi	Multi	Constant	Constant	No	Fuzzy-rough
Wee et al. (2009)	Multi	Multi	Stock-dependent (linear function)	Constant	Completely backlogged	Crisp
Xu and Zhao (2010)	Multi	Multi	Constant	Constant	No	Fuzzy-rough
Shah and Soni (2011)	Single	Multi	Imprecise	Constant	Completely backlogged	Fuzzy-rough
Pando et al. (2012)	Single	Single	Stock-dependent (power function)	Nonlinear-stock dependent	No	Crisp
Min et al. (2012)	Single	Single	Stock-dependent (power function)	Constant	No	Crisp
Bera et al. (2012)	Multi	Multi	Time-dependent	Nonlinear-stock dependent	Partially backlogged	Crisp
Taleizadeh et al. (2013)	Single	Single	Constant	Constant	Completely backlogged	Crisp
Tripathi (2013)	Single	Single	Time-dependent (power function)	Linear-time dependent	No	Crisp
Pando et al. (2013)	Multi	Single	Price-dependent (power function)	Linear-time dependent	No	Crisp
Mondal et al. (2013)	Multi	Single	Stock-dependent (linear function)	Constant	No	Fuzzy-rough
Mahata and Goswami (2013)	Single	Single	Constant	Constant	Completely back ordered	Fuzzy-rough
Yang (2014)	Single	Single	Stock-dependent (power function)	Nonlinear-stock dependent	Partially backlogged	Crisp
Srivastav and Agrawal (2016)	Single	Multi	Stochastic demand	Nonlinear-time dependent	Completely backlogged	Crisp
Garai et al. (2018)	Multi	Multi	Stock-dependent (power function)	Nonlinear-stock dependent	Partially backlogged	Fuzzy-rough

study is to control the regular and overtime production quantity lot under some capacitated restrictions by minimizing inventory cost function. Garg (2015a, b) formulated an inventory model to optimize the annual inventory cost of the system under a fuzzy environment in which the reorder level follows the different types of probability distributions namely uniform, exponential and Laplace distribution. Garai et al. (2015) introduced a non-linear inventory model of geometric programming with the main objective to optimize the total cost of the system with budget restriction under fuzzy intuitionistic environment. Rani et al. (2016) developed an algorithm for obtaining the Pareto optimal solution for parabolic multi-objective nonlinear transportation and manufacturing problem. The main contribution of this study was the development of optimistic and pessimistic models in the intuitionistic fuzzy set environment by considering membership as well as non-membership function for these problems.

In this paper, we are formulating inventory management problem as multi-objective linear fractional inventory problem (MOLFIP) with conflicting objective functions,

where our aim to optimize the profit per-ordered quantity and holding cost per-ordered quantity respectively. Also, we consider a situation in which somehow the information about some of the parameters, i.e., holding cost, purchasing price, selling price, demand and ordering cost are not precisely known but available with vagueness. After studying the pattern of vagueness, IFN used to present all these parameters in the formulated problem. The ranking method is used for the conversion of intuitionistic fuzzy parameters into an equivalent crisp form. Finally, the FGP technique is used to solve the MOLFIP to obtain best compromise order quantity. Whereas, the fuzzy goals of MOLFIP determine the individual optimum solution. These goals categorised by their accompanying membership functions which renovated into fuzzy flexible membership goals using over and under deviational variable. The main contribution of this paper can be summarized below:

- Inventory management problem has been formulated as a multi-criterion decision-making problem with conflicting fractional objective functions.

- The problem has been formulated in the intuitionistic fuzzy environment.
- Two separate models have been presented and discussed with linear and non-linear membership functions.
- Different solution sets have been generated at different discrete values of α and β.

The continuing part of the article includes the formulation of MOLFIP with IFN in Sect. 2. The FGP methodology with some advancement developed in Sect. 3. Section 3 presents a numerical case study which is given to demonstrate the models. Finally, Sect. 5 is devoted to a conclusion.

2 Multi-objective linear fractional inventory problem

Inventories deal with maintaining sufficient stock of goods that will ensure a smooth operation of a production system or a business activity. Traditionally, inventory has been viewed by business and industry as a necessary evil. Too little inventory may cause a costly interruption in the operation of the system, and too much of inventory can ruin the competitive edge and profitability of the business. Dutta and Kumar (2013) formulated multi-objective linear fractional inventory model, wherein they considered a hypothetical system for the inventory model with deterministic parameters. Here, we have examined the modelling and optimisation of a multi-objective linear fractional inventory model, as advanced with some imprecise information considered on it, which is represented by an IFN.

The following notations and assumptions which have used in the formulation of the problem taken from Dutta and Kumar (2013):

Nomenclature

n	Number of items $i = 1, 2, 3, \ldots, n$
k	Stable cost per demand
B	Total offered budget for all items
F	Total accessible space for all items
Q_i	Ordering quantity of item i
h_i	Holding cost per item per unit time for ith item
P_i	Purchasing price of ith item
S_i	Selling Price of ith item
D_i	Demand quantity per unit time of ith item
f_i	Space required per unit for the ith item
OC_i	Ordering cost of ith item

The following assumptions which are essential for the inventory problem have been considered as:

1. An inventory model with multi-items.
2. Infinite time horizon with one period of the cycle time.
3. Constant demand rate.
4. Lead time is zero.
5. Holding cost and purchase price is supposed to be known and persistent.
6. No discount is offered.
7. Shortages are not permitted.
8. No deterioration is permissible.

Based on the above assumptions the mathematical model for the multi-objective linear fractional inventory model for one period of the cycle time is formulated as follows:

MODEL (1)

$$\text{Maximize } Z_1 = \frac{\sum_{i=1}^{n} (S_i - P_i) Q_i}{\sum_{i=1}^{n} (D_i - Q_i)}$$

$$\text{Minimize } Z_2 = \frac{\sum_{i=1}^{n} \frac{h_i Q_i}{2}}{\sum_{i=1}^{n} Q_i}$$

Subject to the constraint

$$\sum_{i=1}^{n} P_i Q_i \leq B$$

$$\sum_{i=1}^{n} f_i Q_i \leq F$$

$$kD_n - (OC_n)Q_n \leq 0$$

$$Q_n \geq 0 \quad and \quad OC_n > 0, \quad \forall i = 1, 2, 3, \ldots, n$$

where

$\sum_{i=1}^{n} (S_i - P_i) Q_i$	denotes the profit related to ordering quantity
$\sum_{i=1}^{n} \frac{h_i Q_i}{2}$	denotes the holding cost
$\sum_{i=1}^{n} (D_i - Q_i)$	denotes the back ordered quantity
$\sum_{i=1}^{n} Q_i$	denotes the total ordering quantity
$\sum_{i=1}^{n} \frac{kD_n}{Q_n}$	denotes the ordering cost

and

Constraint I denotes the upper limit of the total investment.

Constraint II denotes restriction on the warehouse spacing.

Constraint III denotes the budgetary constraint on ordering cost. Ordering cost for the nth item can express as:

1st Item $\frac{kD_1}{Q_1} \leq OC_1 = > kD_1 - (OC_1)Q_1 \leq 0$

2nd Item $\frac{kD_2}{Q_2} \leq OC_2 = > kD_2 - (OC_2)Q_2 \leq 0$

Similarly, for the nth item

$$\frac{kD_n}{Q_n} \le OC_n = > kD_n - (OC_n)Q_n \le 0$$

The above-formulated model (1) extended by making some more additional assumptions, which are as follows:

- Holding Cost cannot be fixed in advance, and it may vary over time-period.
- Purchasing Price and Selling price cannot be uniform respectively for the different zones and time periods. It can vary from these two factors.
- The demand for the items cannot prefix in advanced. It may always vary due to the behaviours of customers' requirements.
- The ordering cost cannot also be prefixed. It is also an independent factor of human behaviour. So it can also vary from the predetermined fixed cost.

Therefore, all these parameters that are holding cost, purchasing price, selling price, demand and ordering cost which are not precisely known to the decision maker. However, somehow the information about all these parameters is available with vagueness. We have considered that IFN's can present this vagueness in the parameters. Hence model (1), can be rewritten after amendments all these above assumptions as follows:

MODEL (2)

Maximize $\quad \tilde{Z}_1 = \dfrac{\sum_{i=1}^{n} (\tilde{S}_i^I - \tilde{P}_i^I)\, Q_i}{\sum_{i=1}^{n} (\tilde{D}_i^I - Q_i)}$

Minimize $\quad \tilde{Z}_2 = \dfrac{\sum_{i=1}^{n} \frac{\tilde{h}_i^I Q_i}{2}}{\sum_{i=1}^{n} Q_i}$

Subject to the constraints

$$\sum_{i=1}^{n} \tilde{P}_i^I Q_i \le B,$$

$$\sum_{i=1}^{n} f_i Q_i \le F,$$

$kD_n - (\widetilde{OC_n})^I \le 0 \;$ (for each n),

and all $Q_n \ge 0$

We assume that $\tilde{S}_i^I, \tilde{P}_i^I, \tilde{h}_i^I, \tilde{D}_i^I$ and $(\widetilde{OC_n})^I$ are IFN for each $i = 1, 2, \ldots, n$.

The preliminaries of IFN's and their ranking function has been studied from Atanassov (1986, 1989), Singh and Yadav (2016). In Model (2), the following input information namely selling price, purchasing price, holding cost, ordering cost and demand of items have been assuming as IFN and follows the following definitions, which are given below.

Definition 1 (*IFN*) An intuitionistic fuzzy set $\tilde{S}_i^I = \{<x, \mu_{\tilde{S}_i^I}(x), \gamma_{\tilde{S}_i^I}(x)> \; : x \in X\}$ called an IFN if the following situation holds:

1. If there exists $m \in R$ such that $\mu_{\tilde{S}_i^I}(m) = 1$ and $\gamma_{\tilde{S}_i^I}(m) = 0$ (m is known as the mean value of $\tilde{S}_i^I$).
2. If $\mu_{\tilde{S}_i^I}$ and $\gamma_{\tilde{S}_i^I}$ area continuous function from R to the closed $[0, \; 1]$ and $0 \le \mu_{\tilde{S}_i^I}(x) + \gamma_{\tilde{S}_i^I}(x) \le 1,\; \forall x \in R$, where

$$\mu_{\tilde{S}_i^I}(x) = \begin{cases} g_1(x), & m - a_{S_i} \le x < m \\ 1, & x = m \\ h_1(x), & m < x \le m + b_{S_i} \\ 0, & otherwise \end{cases}$$

and

$$\gamma_{\tilde{S}_i^I}(x) = \begin{cases} g_2(x), & m - a'_{S_i} \le x < m; \; 0 \le g_1(x) + g_2(x) \le 1 \\ 1, & x = m \\ h_2(x), & m < x \le m + b'_{S_i}; \; 0 \le h_1(x) + h_2(x) \le 1 \\ 0, & otherwise \end{cases}$$

Here m is the average value of IFN $\tilde{S}_i^I$; (a_{S_i}, b_{S_i}) and (a'_{S_i}, b'_{S_i}) are the left and right spreads of the linear membership function (LMF) $\mu_{\tilde{S}_i^I}(x)$ and non-linear membership function NLMF $\gamma_{\tilde{S}_i^I}(x)$, respectively. The symbols g_1 and h_1 are piecewise continuous, strictly increasing, and strictly decreasing functions in $[m - a_{S_i}, m)$ and $(m, m + b_{S_i}]$, strictly decreasing, and strictly increasing functions in $[m - a'_{S_i}, m]$ and $[m, m + b'_{S_i}]$, respectively. Therefore, the IFN can define as $\tilde{S}_i^I = (m; a_{S_i}, b_{S_i}; a'_{S_i}, b'_{S_i})$.

Definition 2 (*Triangular IFN*) Let us assume that $\tilde{S}_i^I = \{(a_{S_i}, b_{S_i}, c_{S_i}) ; (e_{S_i}, b_{S_i}, f_{S_i})\}, \forall\, e_{S_i} \le a_{S_i} \le b_{S_i} \le c_{S_i} \le f_{S_i}$ be a triangular IFN with LMF $\mu_{\tilde{S}_i^I}$ and NLMF $\gamma_{\tilde{S}_i^I}$ which is given as:

$$\mu_{\tilde{S}_i^I}(x) = \begin{cases} \dfrac{x - a_{S_i}}{b_{S_i} - a_{S_i}}, & a_{S_i} < x \le b_{S_i} \\ 1, & x = b_{S_i} \\ \dfrac{c_{S_i} - x}{c_{S_i} - b_{S_i}}, & b_{S_i} \le x \le c_{S_i} \\ 0, & otherwise \end{cases} \quad (1)$$

and

$$\gamma_{\tilde{S}_i^I}(x) = \begin{cases} \dfrac{b_{S_i} - x}{b_{S_i} - e_{S_i}}, & e_{S_i} < x \le b_{S_i} \\ 1, & x = b_{S_i} \\ \dfrac{x - b_{S_i}}{f_{S_i} - b_{S_i}}, & b_{S_i} \le x < f_{S_i} \\ 0, & otherwise \end{cases} \quad (2)$$

The left and right bound of the triangular IFN at (α, β) set with its LMF and NLMF can express as:

$$L_{\mu_{S_i}}(x) = \frac{x - a_{S_i}}{b_{S_i} - a_{S_i}}, \quad a_{S_i} \le x \le b_{S_i} \quad \text{and}$$

$$R_{\mu_{S_i}}(x) = \frac{c_{S_i} - x}{c_{S_i} - b_{S_i}}, \quad b_{S_i} \le x \le c_{S_i}$$

$$L_{\gamma_{S_i}}(x) = \frac{b_{S_i} - x}{b_{S_i} - e_{S_i}}, \quad e_{S_i} \le x \le b_{S_i}$$

$$\text{and} \quad R_{\gamma_{S_i}}(x) = \frac{x - b_{S_i}}{f_{S_i} - b_{S_i}}, \quad b_{S_i} \le x \le f_{S_i}$$

The inverse function L^{-1} and R^{-1} can define analytically as

$$L_{\mu_{S_i}}^{-1}(h) = a_{S_i} + (b_{S_i} - a_{S_i})h, \quad R_{\mu_{S_i}}^{-1}(h) = c_{S_i} - (c_{S_i} - b_{S_i})h$$

$$L_{\gamma_{S_i}}^{-1}(h) = b_{S_i} - (b_{S_i} - e_{S_i})h, \quad R_{\gamma_{S_i}}^{-1}(h) = b_{S_i} + (f_{S_i} - b_{S_i})h$$

The magnitude of LMF for $\tilde{S}_i^I$ at α level can express as:

$$
\begin{aligned}
P_{\mu_{S_i}}(\tilde{S}_i^I) &= \frac{\int_0^1 h[\alpha(a_{S_i} + (b_{S_i} - a_{S_i})h) + (1-\alpha)(c_{S_i} - (c_{S_i} - b_{S_i})h)]dh}{\int_0^1 h\,dh} \\
&= \frac{\left[\frac{a_{S_i}\alpha h^2}{2} + \frac{\alpha(b_{S_i} - a_{S_i})h^3}{3} + \frac{(1-\alpha)c_{S_i}h^2}{2} - \frac{(1-\alpha)(c_{S_i} - b_{S_i})h^3}{3}\right]_0^1}{\left[\frac{h^2}{2}\right]_0^1} \\
&= \frac{\alpha(a_{S_i} - c_{S_i}) + c_{S_i} + 2b_{S_i}}{3}
\end{aligned}
\tag{3}
$$

The magnitude of NLMF for $\tilde{S}_i^I$ at β level can express as:

$$
\begin{aligned}
P_{\gamma_{S_i}}(\tilde{S}_i^I) &= \frac{\int_0^1 h[\beta(b_{S_i} - (b_{S_i} - e_{S_i})h) + (1-\beta)(b_{S_i} + (f_{S_i} - b_{S_i})h)]dh}{\int_0^1 h\,dh} \\
&= \frac{\left[\frac{\beta b_{S_i}h^2}{2} - \frac{\beta(b_{S_i} - e_{S_i})h^3}{3} + \frac{(1-\beta)b_{S_i}h^2}{2} + \frac{(1-\beta)(f_{S_i} - b_{S_i})h^3}{3}\right]_0^1}{\left[\frac{h^2}{2}\right]_0^1} \\
&= \frac{2\beta(e_{S_i} - f_{S_i}) + b_{S_i} + 2f_{S_i}}{3}
\end{aligned}
\tag{4}
$$

The magnitude of LMF and NLMF for $\tilde{S}_i^I$ at (α, β) level can express as:

$$P(\tilde{S}^I) = \left[\frac{\alpha(a_{S_i} - c_{S_i}) + c_{S_i} + 2b_{S_i}}{3}, \frac{2\beta(e_{S_i} - f_{S_i}) + b_{S_i} + 2f_{S_i}}{3}\right]$$

Similarly, the same assumption followed for the other intuitionistic fuzzy parameters, i.e., $(\tilde{P}_i^I, \tilde{h}_i^I, \tilde{D}_i^I, (\widetilde{OC_n})^I)$ respectively. Based on the above conversion procedures of IFN; the model (2) has been rewritten as:

MODEL (2A)

$$\text{Maximize} \quad \tilde{Z}_1 = \frac{\sum_{i=1}^n \left(\frac{\alpha(a_{S_i} - c_{S_i}) + c_{S_i} + 2b_{S_i}}{3} - \frac{\alpha(a_{P_i} - c_{P_i}) + c_{P_i} + 2b_{P_i}}{3}\right) Q_i}{\sum_{i=1}^n \left(\frac{\alpha(a_{D_i} - c_{D_i}) + c_{D_i} + 2b_{D_i}}{3} - Q_i\right)}$$

$$\text{Minimize} \quad \tilde{Z}_2 = \frac{\sum_{i=1}^n \frac{(\alpha(a_{h_i} - c_{h_i}) + c_{h_i} + 2b_{h_i})Q_i}{6}}{\sum_{i=1}^n Q_i}$$

Subject to the constraints

$$\sum_{i=1}^n \left(\frac{\alpha(a_{P_i} - c_{P_i}) + c_{P_i} + 2b_{P_i}}{3}\right)Q_i \le B, \tag{5}$$

$$\sum_{i=1}^n f_i Q_i \le F, \tag{6}$$

$$k\,D_n - \left(\frac{\alpha(a_{OC_n} - c_{OC_n}) + c_{OC_n} + 2b_{OC_n}}{3}\right) \le 0 \tag{7}$$
$$\text{(for each } n\text{),}$$

$$\text{and all} \quad Q_n \ge 0 \tag{8}$$

Similarly, an equivalent crisp form of **MODEL (2)** with NLMF expressed as follows:

MODEL (2B)

$$\text{Maximize} \quad \tilde{Z}_1 = \frac{\sum_{i=1}^n \left(\frac{2\beta(e_{S_i} - f_{S_i}) + b_{S_i} + 2f_{S_i}}{3} - \frac{2\beta(e_{P_i} - f_{P_i}) + b_{P_i} + 2f_{P_i}}{3}\right) Q_i}{\sum_{i=1}^n \left(\frac{2\beta(e_{D_i} - f_{D_i}) + b_{D_i} + 2f_{D_i}}{3} - Q_i\right)}$$

$$\text{Minimize} \quad \tilde{Z}_2 = \frac{\sum_{i=1}^n \frac{(2\beta(e_{h_i} - f_{h_i}) + b_{h_i} + 2f_{h_i})Q_i}{6}}{\sum_{i=1}^n Q_i}$$

Subject to the constraints

$$\sum_{i=1}^n \left(\frac{2\beta(e_{P_i} - f_{P_i}) + b_{P_i} + 2f_{P_i}}{3}\right)Q_i \le B, \tag{9}$$

$$\sum_{i=1}^n f_i Q_i \le F, \tag{10}$$

$$k\,D_n - \frac{2\beta(e_{OC_n} - f_{OC_n}) + b_{OC_n} + 2f_{OC_n}}{3} \quad \text{(for each } n\text{),} \tag{11}$$

$$\text{and all} \quad Q_n \ge 0 \tag{12}$$

3 Fuzzy goal programming approach

FGP is a powerful and flexible technique that can apply to a variety of decision-making problems involving multiple objectives. Several contributions have been reported in the literature on FGP approach. In all these literature efficient methodologies has been developed for solving multi-objective programming problems employing the FGP approach. After doing some manipulations in FGP

approach, we have given a stepwise solution procedure for solving the formulated MOLFIP.

3.1 Procedure for solving molfip

This stepwise solution procedure of FGP has been used to obtain the optimal order quantity in an intuitionistic fuzzy environment, which are as follows:

Step 1 Formulate the inventory problem as MOLFIP with certain and uncertain parameters (holding cost, purchasing price, selling price, demand and ordering cost).

Step 2 As explained in Sect. 2, the first step is to convert the intuitionistic fuzzy parameters of MOLFIP into the equivalent crisp form using the ranking procedure. The equivalent crisp form for MODEL (2A) of MOLFIP in which IFN is in the shape of LMF has obtained by using Eq. (3). Similarly, the case when the IFN is in the form of the NLMF, the resultant equivalent crisp form for the MODEL (2B) of MOLFIP has obtained by using Eq. (4).

Step 3 Here we have two resultant crisp form models of MOLFIP, i.e., MODEL (2A) and MODEL (2B). Firstly the MODEL (2A) is solved as a solitary objective problem using only one objective at a time for different values of $\alpha \in [0, 1]$ and ignoring the other objective functions.

Similarly, MODEL (2B) is solved as a solitary objective problem using only one objective at a time for the different values of $\beta \in [0, 1]$ and ignoring the other objective functions. The solutions thus obtained of these models are considered to be ideal solutions and these solutions help to the decision maker for setting the aspiration level to each of the objective function.

Step 4 The aspiration level to each of the objective functions, $\tilde{Z}_j(x)$, $j = 1, 2$ of the models can define in such a way.

Now MOLFIP (2A) is defined as follows:

$$\tilde{Z}_1(X) \succeq g_1, \quad (\text{fuzzy} - \max)$$
$$\tilde{Z}_2(X) \preceq g_2, \quad (\text{fuzzy} - \min) \tag{13}$$

Subject to Constraint (5)–(8)

Where $(g_1)_\alpha = Max\ (Z_1(x))_\alpha$ and $(g_2)_\alpha = Min\ (Z_2(x))_\alpha$ $\forall \alpha \in [0, 1]$.

Solved the Eq. (13) by achieving all its defined fuzzy goals and for the optimal values $Q_{i*} = (Q_1, Q_2, \ldots, Q_n)$.

Similarly, MOLFIP (2B) is defined as follows:

$$\tilde{Z}_1(X) \succeq g_1, \quad (\text{fuzzy} - \max)$$
$$\tilde{Z}_2(X) \preceq g_2, \quad (\text{fuzzy} - \min) \tag{14}$$

Subject to Constraint (9)–(12)

Where $(g_1)_\beta = Max\ (Z_1(x))_\beta$ and $(g_2)_\beta = Min\ (Z_2(x))_\beta$ $\forall \beta \in [0, 1]$.

Similarly, Solved the Eq. (14) by achieving all its defined fuzzy goals and for the optimal values $Q_* = (Q_1, Q_2, \ldots, Q_n)$.

Step 5 Construct a fuzzy linear membership function for the Model (2A) for both the defined objective functions. Hence, the membership function of the fuzzy goal $\tilde{Z}_1(X) \succeq g_1$ (i.e., fuzzy-max) is defined as:

$$\mu_1(\tilde{Z}_1(X)) = \begin{cases} 1, & if\ \tilde{Z}_1(X) \geq g_1 \\ \dfrac{\tilde{Z}_1(X) - L_1}{g_1 - L_1}, & if\ L_1 \leq \tilde{Z}_1(X) \leq g_1 \\ 0, & if\ \tilde{Z}_1(X) \leq L_1 \end{cases}$$

where L_1 is the lower tolerance limit of the fuzzy goal, $\tilde{Z}_1(x)$ (Fig. 1).

Moreover, similarly, for the fuzzy goal $\tilde{Z}_2 \preceq g_2$ (i.e., fuzzy-min), the membership function is defined as:

$$\mu_2(\tilde{Z}_2(X)) = \begin{cases} 1, & if\ \tilde{Z}_2(X) \leq g_2 \\ \dfrac{U_2 - \tilde{Z}_2(X)}{U_2 - g_2}, & if\ g_2 \leq \tilde{Z}_2(X) \leq U_2 \\ 0, & if\ \tilde{Z}_2(X) \geq U_2 \end{cases}$$

where U_2 is the upper tolerance limit of the fuzzy goal $\tilde{Z}_2(x)$ (Fig. 2).

It has been known that the possible highest degree of membership function can only be unity in fuzzy programming. Therefore, the flexible membership goals with the aspiration level can define as:

$$\frac{\tilde{Z}_1(X) - L_1}{g_1 - L_1} + d_1^- - d_1^+ = 1,$$
$$\frac{U_2 - \tilde{Z}_2(X)}{U_2 - g_2} + d_2^- - d_2^+ = 1,$$

where $d_j^- \geq 0$, $d_j^+ \geq 0$, $j = 1, 2$ with $d_j^+ d_j^- = 0$, $j = 1, 2$ are respectively under- and over-deviations from the targeted goal. Similarly, the same procedure has been
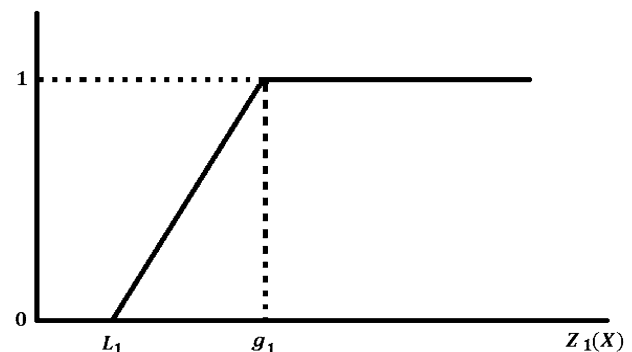

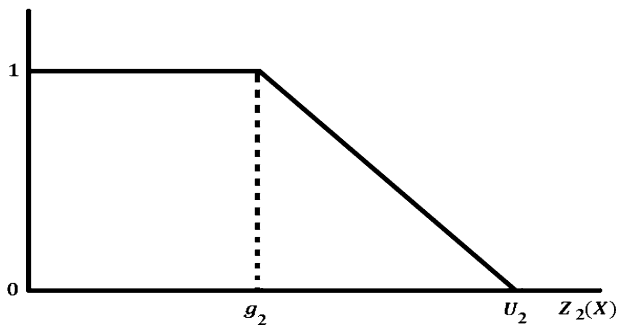**Fig. 1** Linear membership function $(\tilde{Z}_1(X) \succeq g_1)$

Fig. 2 Linear membership function $(\tilde{Z}_2(X) \preceq g_2)$

followed for construction of membership function for Model (2B).

Step 6 Calculate the weight which is attached to both the objective function:

$$w_1 = \frac{1}{g_1 - l_1} \quad for \ \max - fuzzy \ goal$$

$$w_2 = \frac{1}{u_2 - g_2} \quad for \ \min - fuzzy \ goal$$

Similarly, the same weighted criterion has been used for the Model (2B).

Step 7 By following all the above-given steps, FGP model for MOLFIP is formulated as follows:

$$Min \quad \sum_{j=1}^{2} w_j d_j^-$$

subject to the constraints (9) *to* (12) of model 2(A)

$$and \quad \frac{\tilde{Z}_1(X) - L_1}{g_1 - L_1} + d_1^- - d_1^+ = 1,$$

$$\frac{U_2 - \tilde{Z}_2(X)}{U_2 - g_2} + d_2^- - d_2^+ = 1,$$

$$d_j^-, d_j^+ \geq 0, \ with \quad d_j^- \times d_j^+ = 0, j = 1, 2$$

$$Q_n \geq 0$$

$$(15)$$

A similar model has been formulated for Model (2B) with some others additional set of constraints (13)–(16) in place of the set of constraints (9)–(12). Finally, the above-formulated problem has been solving to obtain the optimal compromise solution Q^* by using the optimising software LINGO 16.0. If the decision maker is not satisfied with the solution, then proceed to step 8.

Step 8 Construct the membership functions for the decision vector $Q^* = (Q_1, Q_2, \ldots, Q_n)$.

Let t^L and t^R are the maximum negative and maximum positive tolerance values, which are not necessarily to be the same. This tolerance value helps the DM to extend the feasible region to search for the optimal solution. In the case of no feasible region, this tolerance limits can increase

for searching the new feasible region in which the desired optimal solution can found (Fig. 3).

The linear membership function of controlled decision vector can be defined as follows:

$$\mu(Q_n) = \begin{cases} \dfrac{Q_n - (Q_n^* - t^L)}{t^L}, & if \ Q_n^* - t^L \leq Q_n \leq Q_n^* \\ \dfrac{(Q_n^* + t^R) - Q_n}{t^R}, & if \ Q_n^* \leq Q_n \leq Q_n^* + t^R \\ 0, & if \ otherwise \end{cases}$$

where Q^* is the most preferred solution. The satisfaction level of Q_n^* is linearly increasing in the interval $[Q_n^* - t^L, Q_n]$, and linearly decreasing in the interval $[Q_n, Q_n^* + t^L]$.

So, for the defined membership function of the decision vector, the flexible membership goals with aspiration levels are can be expressed as follows:

$$\mu(Q_n) + d_n^- - d_n^+ = 1,$$

Alternatively, equivalently as

$$\frac{Q_n - (Q_n^* - t^L)}{t^L} + d_n^{L-} - d_n^{L+} = 1$$

$$\frac{(Q_n^* + t^R) - Q_k}{t^R} + d_n^{R-} - d_n^{R+} = 1$$

where $d_n^- = (d_n^{L-}, d_n^{R-})$, $d_n^+ = (d_n^{L+}, d_n^{R+})$ and $d_n^{L-}, d_n^{R-}, d_n^{L+}, d_n^{R+} \geq 0$ with $d_n^{L-} \times d_n^{L+} = 0$ and $d_n^{R-} \times d_n^{R+} = 0$, respectively, represents the under-deviational and over-deviational from the targeted goals. Calculation of weights w_n^L and w_n^R attached to the decision variables is as follows:

$$w_n^L = \frac{1}{t^L} \quad and \quad w_n^R = \frac{1}{t^R}$$

Similarly, the same procedure will follow for construction of membership function for Model (2B).

Step 9 By following all the above steps, final FGP model for MOLFIP has been formulated as follows:
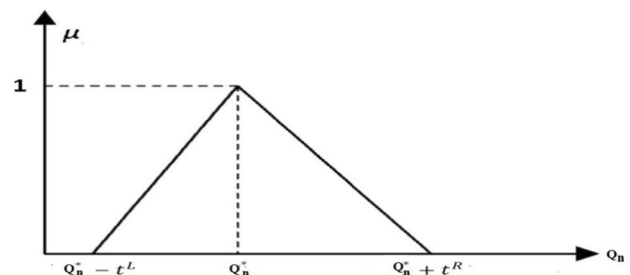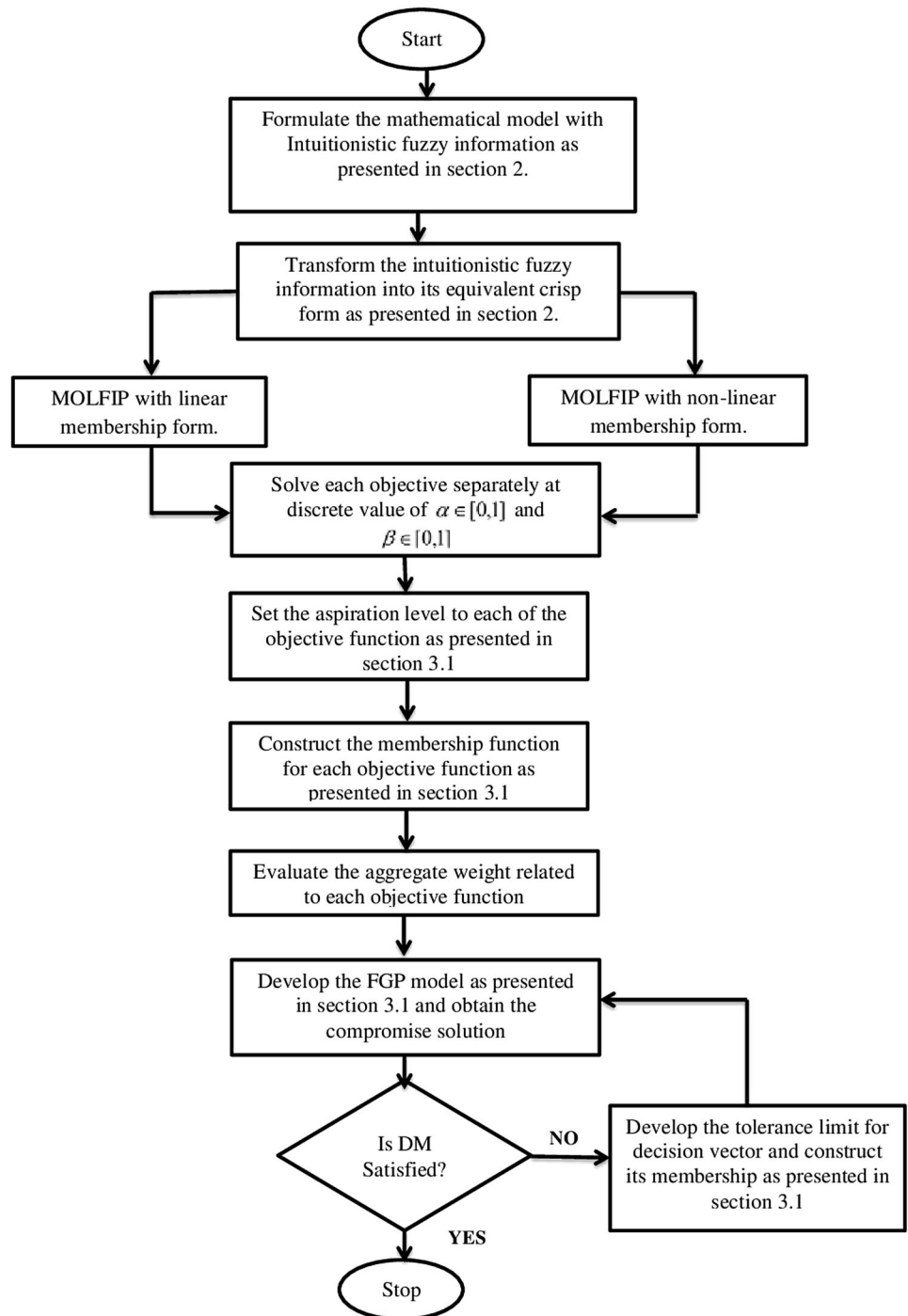

Fig. 3 Membership function of decision vector

Fig. 4 Flowchart of solution methodology

```
                          ( Start )
                              │
                              ▼
            ┌──────────────────────────────────┐
            │ Formulate the mathematical model  │
            │ with Intuitionistic fuzzy         │
            │ information as presented in       │
            │ section 2.                        │
            └──────────────────────────────────┘
                              │
                              ▼
            ┌──────────────────────────────────┐
            │ Transform the intuitionistic      │
            │ fuzzy information into its         │
            │ equivalent crisp form as          │
            │ presented in section 2.           │
            └──────────────────────────────────┘

  ┌──────────────────────┐          ┌──────────────────────┐
  │ MOLFIP with linear   │          │ MOLFIP with non-     │
  │ membership form.     │          │ linear membership    │
  │                      │          │ form.                │
  └──────────────────────┘          └──────────────────────┘

            ┌──────────────────────────────────┐
            │ Solve each objective separately   │
            │ at discrete value of α ∈ [0,1]    │
            │ and β ∈ [0,1]                     │
            └──────────────────────────────────┘
                              │
                              ▼
            ┌──────────────────────────────────┐
            │ Set the aspiration level to each  │
            │ of the objective function as      │
            │ presented in section 3.1          │
            └──────────────────────────────────┘
                              │
                              ▼
            ┌──────────────────────────────────┐
            │ Construct the membership function │
            │ for each objective function as    │
            │ presented in section 3.1          │
            └──────────────────────────────────┘
                              │
                              ▼
            ┌──────────────────────────────────┐
            │ Evaluate the aggregate weight     │
            │ related to each objective         │
            │ function                          │
            └──────────────────────────────────┘
                              │
                              ▼
            ┌──────────────────────────────────┐
            │ Develop the FGP model as          │
            │ presented in section 3.1 and      │
            │ obtain the compromise solution    │
            └──────────────────────────────────┘
                              │
                              ▼
                      ◇ Is DM ◇ ──NO──► ┌──────────────────────┐
                      ◇ Satisfied? ◇    │ Develop the tolerance │
                              │          │ limit for decision    │
                             YES         │ vector and construct  │
                              ▼          │ its membership as     │
                          ( Stop )       │ presented in          │
                                         │ section 3.1           │
                                         └──────────────────────┘
```

Solve each objective separately at discrete value of $\alpha \in [0,1]$ and $\beta \in [0,1]$

Table 2 Input data for a three-item inventory problem

Item (1)	1	2	3
Holding cost per item per year (Rs)	$\widetilde{12}^I = (10, 12, 14; 9, 12, 15)$	$\widetilde{16}^I = (14, 16, 18; 13, 16, 19)$	$\widetilde{18}^I = (16, 18, 20; 15, 18, 21)$
Purchasing price (Rs)	$\widetilde{625}^I = (623, 625, 627; 622, 625, 628)$	$\widetilde{730}^I = (728, 730, 732; 727, 730, 733)$	$\widetilde{440}^I = (438, 440, 442; 437, 440, 443)$
Selling price (Rs)	$\widetilde{650}^I = (648, 650, 652; 647, 650, 653)$	$\widetilde{750}^I = (748, 750, 752; 747, 750, 753)$	$\widetilde{450}^I = (448, 450, 452; 447, 450, 453)$
Demand quantity (units/year)	$\widetilde{1000}^I = (950, 1000, 1050; 900, 1000, 1100)$	$\widetilde{2000}^I = (1950, 2000, 2050; 1900, 2000, 2100)$	$\widetilde{1500}^I = (1450, 1500, 1550; 1400, 1500, 1600)$
Ordering cost per order (Rs)	$\widetilde{320}^I = (318, 320, 322; 317, 320, 323)$	$\widetilde{350}^I = (348, 350, 352; 347, 350, 353)$	$\widetilde{250}^I = (248, 250, 252; 247, 250, 253)$
Space required for item (m²)	2	4	2

Fixed cost per order, $k = 7$. Maximum available space, F = 13,000 m². Maximum inventory budget, B = Rs. 900,000

Table 3 Individual maximum solution (Z_1)

α	Z_1^{Max}	Q_1	Q_2	Q_3
0	11.35605	1361.623	40.25665	42.35372
0.1	11.39663	1362.040	40.20540	42.28313
0.2	11.43748	1362.458	40.15411	42.21246
0.3	11.47862	1362.876	40.10278	42.14172
0.4	11.52002	1363.294	40.05141	42.07090
0.5	11.56171	1363.712	40.00	42.00
0.6	11.60369	1364.131	39.94855	41.92903
0.7	11.64595	1364.549	39.89706	41.85798
0.8	11.68849	1364.968	39.84554	41.78686
0.9	11.73134	1365.388	39.79397	41.71566
1.0	11.77447	1365.807	39.74237	41.64439

Table 4 Individual minimum solution (Z_1)

α	Z_1^{Min}	Q_1	Q_2	Q_3
0	0.4012205	22.19335	40.25665	42.35372
0.1	0.4013598	22.12978	40.20540	42.28313
0.2	0.4014990	22.06617	40.15411	42.21246
0.3	0.4016382	22.00250	40.10278	42.14172
0.4	0.4017774	21.93878	40.05141	42.07090
0.5	0.4019165	21.87500	40.00	42.00
0.6	0.4020555	21.81117	39.94855	41.92903
0.7	0.4021945	21.74729	39.89706	41.85798
0.8	0.4023335	21.68335	39.84554	41.78686
0.9	0.4024724	21.61937	39.79397	41.71566
1.0	0.4026112	21.55532	39.74237	41.64439

Table 5 Individual minimum solution (Z_2)

α	Z_2^{Min}	Q_1	Q_2	Q_3
0	6.750770	1361.623	40.25665	42.35372
0.1	6.681393	1362.040	40.20540	42.28313
0.2	6.612023	1362.458	40.15411	42.21246
0.3	6.542659	1362.872	40.10278	42.14172
0.4	6.473303	1363.294	40.05141	42.07090
0.5	6.403953	1363.712	40.00	42.00
0.6	6.334611	1364.131	39.94855	41.92903
0.7	6.265275	1364.549	39.89706	41.85798
0.8	6.195946	1364.968	39.84554	41.78686
0.9	6.126624	1365.388	39.79397	41.71566
1.0	6.057310	1365.807	39.74237	41.64439

$$Min \quad \sum_{j=1}^{2} w_j d_j^- + \sum_{n=1}^{3} [w_n^L (d_n^{L-} + d_n^{L+}) + w_n^R (d_n^{R-} + d_n^{R+})]$$

subject to the constraint (9) to (12) of model 2(A)

and
$$\frac{\tilde{Z}_1(X) - L_1}{g_1 - L_1} + d_1^- - d_1^+ = 1,$$

$$\frac{U_2 - \tilde{Z}_2(X)}{U_2 - g_2} + d_2^- - d_2^+ = 1,$$

$$\frac{Q_n - (Q_n^* - t^L)}{t^L} + d_n^{L-} - d_n^{L+} = 1$$

$$\frac{(Q_n^* + t^R) - Q_k}{t^R} + d_n^{R-} - d_n^{R+} = 1$$

$$d_j^-, d_j^+ \geq 0, \text{ with } d_j^- \times d_j^+ = 0, j = 1, 2$$

$$d_n^{L-}, d_n^{L+} \geq 0, \text{ with } d_n^{L-} \times d_n^{L+} = 0$$

$$d_n^{R-}, d_n^{R+} \geq 0, \text{ with } d_n^{R-} \times d_n^{R+} = 0$$

$$Q_n \geq 0$$

(16)

A similar model has been formulated for Model (2B) with some others additional set of constraints (13)–(16) in place of the set of constraints (9)–(12). Finally, the above-formulated problem has been solved to obtain the optimal compromise solution Q^* by using the optimising software LINGO 16.0 (Fig. 4).

4 Numerical example

The following numerical example has been used to illustrate the proposed approach. Dutta and Kumar (2013) have presented their inventory model with fixed input information data set. Here we are considering the case of inventory optimisation problem in which the input information is available in the form of IFN. This available information is summarised in Table 2 and given below:

By using the above information of Table 2 in model 2, the MOLFIP with intuitionistic fuzzy input data is expressed as:

$$Max\,Z_1 = \frac{(\widetilde{650}^I - \widetilde{625}^I)Q_1 + \left(\widetilde{750}^I - \widetilde{730}^I\right)Q_2 + \left(\widetilde{450}^I - \widetilde{440}^I\right)Q_3}{\left(\widetilde{1000}^I - Q_1\right) + \left(\widetilde{2000}^I - Q_2\right) + \left(\widetilde{1500}^I - Q_3\right)}$$

$$Min\,Z_2 = \frac{\frac{\widetilde{12}^I Q_1}{2} + \frac{\widetilde{16}^I Q_1}{2} + \frac{\widetilde{18}^I Q_1}{2}}{Q_1 + Q_2 + Q_3}$$

Subject to the constraint

Table 6 Individual maximum solution (Z_2)

α	Z_2^{Max}	Q_1	Q_2	Q_3
0	18.32294	22.19335	40.25665	1944.100
0.1	18.19245	22.12978	40.20540	1944.883
0.2	18.06194	22.06617	40.15411	1945.666
0.3	17.93143	22.00250	40.10278	1946.450
0.4	17.80091	21.93878	40.05141	1947.234
0.5	17.67038	21.87500	40.00	1948.018
0.6	17.53984	21.81117	39.94855	1948.804
0.7	17.40929	21.74729	39.89706	1949.590
0.8	17.27874	21.68335	39.84554	1950.376
0.9	17.14817	61.61937	39.79397	1951.163
1.0	17.01760	21.55532	39.74237	1951.950

Table 7 Relative weights

α	w_1	w_2
0	0.091283940	0.086414216
0.1	0.090948197	0.086872995
0.2	0.090612697	0.087336878
0.3	0.090277299	0.087805787
0.4	0.089942272	0.088279899
0.5	0.089607393	0.088759285
0.6	0.089272686	0.089244048
0.7	0.088938256	0.089734265
0.8	0.088604123	0.090229955
0.9	0.088270076	0.090731373
1.0	0.087936371	0.091238462

Table 8 Optimal compromise objective values with order quantities

α	Z_1	Z_2	Q_1	Q_2	Q_3
0	7.498250	8.622942	22.19335	1187.256	42.41547
0.1	7.520145	8.578061	22.12978	1187.530	42.28313
0.2	7.544812	8.508621	22.06617	1187.855	42.21246
0.3	7.569633	8.439189	22.00250	1188.181	42.14172
0.4	7.594610	8.369765	21.93878	1188.507	42.0709
0.5	7.619745	8.300349	21.87500	1188.833	42.00
0.6	7.645038	8.230941	21.81117	1189.159	41.92903
0.7	7.660459	8.190292	21.74729	1189.639	41.81031
0.8	7.696106	8.092148	21.68335	1189.812	41.78686
0.9	7.721884	8.052039	21.61937	1190.139	41.71566
1.0	7.782631	8.005682	21.58310	1190.436	41.63432

Fig. 5 Different values of the objective function
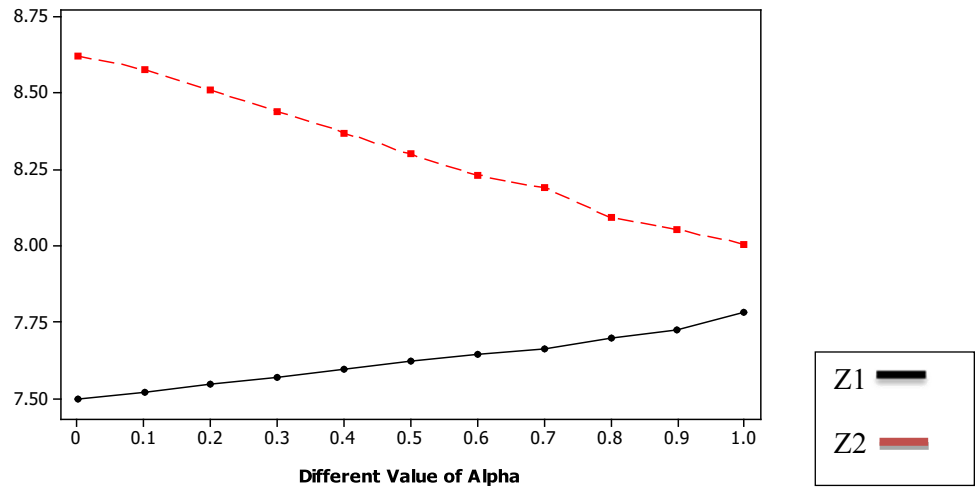
Table 9 Individual maximum solution (Z_1)

β	Z_1^{Max}	Q_1	Q_2	Q_3
0	10.79465	1356.748	41.09848	43.51852
0.1	10.94024	1358.136	40.87979	43.21675
0.2	11.08959	1359.526	40.66059	42.91401
0.3	11.24285	1360.919	40.44090	42.61031
0.4	11.40017	1362.314	40.22070	42.30564
0.5	11.56171	1363.712	40.00	42.00
0.6	11.72766	1365.112	39.77879	41.69338
0.7	11.89818	1366.514	39.55708	41.38577
0.8	12.07348	1367.919	39.33486	41.07717
0.9	12.25376	1369.327	39.11213	40.76758
1.0	12.43922	1370.736	38.8889	40.45699

Table 11 Individual minimum solution (Z_2)

β	Z_2^{Min}	Q_1	Q_2	Q_3
0	7.449531	1356.748	41.09848	43.51852
0.1	7.240244	1358.136	40.87979	43.21675
0.2	7.031042	1359.526	40.66059	42.91401
0.3	6.821927	1360.919	40.44090	42.61031
0.4	6.612897	1362.314	40.22070	42.30564
0.5	6.403953	1363.712	40.00	42.00
0.6	6.195096	1365.112	39.77879	41.69338
0.7	5.986325	1366.514	39.55708	41.38577
0.8	5.777641	1367.919	39.33486	41.07717
0.9	5.569044	1369.327	39.11213	40.76758
1.0	5.360534	1370.736	39.88889	40.45699

Table 10 Individual minimum solution (Z_1)

β	Z_1^{Min}	Q_1	Q_2	Q_3
0	0.3999972	23.18841	41.09848	43.51852
0.1	0.4003840	22.92703	40.87979	43.21675
0.2	0.4007694	22.66501	40.66059	42.91401
0.3	0.4011533	22.40233	40.44090	42.610391
0.4	0.4015357	22.13899	40.22070	42.30564
0.5	0.4019165	21.87500	40.00	42.00
0.6	0.4022955	21.61035	39.77879	41.69338
0.7	0.4026728	21.34503	39.55708	41.38577
0.8	0.4030482	21.07905	39.33486	41.07717
0.9	0.4034216	20.81240	39.11213	40.76758
1.0	0.4037930	20.54507	38.88889	40.45699

Table 12 Individual maximum solution (Z_2)

β	Z_2^{Max}	Q_1	Q_2	Q_3
0	19.62314	23.18841	41.09848	1935.242
0.1	19.23281	22.92703	40.87979	1937.786
0.2	18.84236	22.66501	40.66059	1940.336
0.3	18.45181	22.40233	40.44090	1942.891
0.4	18.06115	22.13899	40.22070	1945.452
0.5	17.67038	21.87500	40.00	1948.018
0.6	17.27950	21.61035	39.77879	1950.590
0.7	16.88852	21.34503	39.55708	1953.168
0.8	16.49742	21.07905	39.33486	1955.751
0.9	16.10622	20.81240	39.11213	1958.340
1.0	15.71491	20.54507	38.88889	1960.934

Table 13 Relative weights

β	w_1	w_2
0	0.096203309	0.082144909
0.1	0.094877957	0.083384990
0.2	0.093555691	0.084664556
0.3	0.092236485	0.085985388
0.4	0.090920379	0.087349572
0.5	0.089607393	0.088759285
0.6	0.088297379	0.090216849
0.7	0.086990507	0.091724648
0.8	0.085686632	0.093285505
0.9	0.084385776	0.094902088
1.0	0.086288037	0.096577524

Table 14 Optimal compromise objective values with order quantities

β	Z_1	Z_2	Q_1	Q_2	Q_3
0	7.685360	19.62314	23.18841	41.09848	1935.242
0.1	7.723917	19.03581	22.92703	40.87979	1938.154
0.2	7.83834	18.92735	22.66501	40.62648	1940.687
0.3	8.079115	18.45181	22.40233	40.44090	1942.891
0.4	8.219140	18.06115	22.13899	40.22070	1945.452
0.5	8.295876	17.68649	21.87500	39.98724	1948.071
0.6	8.513711	17.27950	21.61035	39.77879	1950.590
0.7	8.668768	16.88852	21.34503	39.55708	1953.168
0.8	8.829376	16.49742	21.07905	39.33486	1955.751
0.9	8.995837	16.10622	20.81240	39.11213	1958.340
1.0	9.168477	15.71491	20.54507	38.88889	1960.934

$$\widetilde{625}^I Q_1 + \widetilde{730}^I Q_2 + \widetilde{440}^I Q_3 \leq 900,000$$
$$2Q_1 + 4Q_2 + 2Q_3 \leq 13,000$$
$$7\left(\widetilde{1000}^I\right) - \widetilde{320}^I Q_1 \leq 0$$
$$7\left(\widetilde{2000}^I\right) - \widetilde{350}^I Q_2 \leq 0 \qquad (17)$$
$$7\left(\widetilde{1500}^I\right) - \widetilde{250}^I Q_3 \leq 0$$

All $Q_i \geq 0$, $i = 1, 2, 3$

The Eq. (16) is in the form of fuzzy numbers. We cannot solve it directly by any given methods. Therefore first we convert it to crisp form and then will be solved. As we have discussed the two models are made for this, Model (2A) and Model (2B). Here we are debating Model (2A) under the case 1 and Model (2B) under the case 2.

Case 1 for Model (2A) This model has been solved by followed the stepwise solution procedure as defined in Sect. 3.1. The maximum and minimum individual values of the objective function at the different value of $\alpha \in [0, 1]$ has been given in Tables 3, 4, 5 and 6.

The relative weights attached to each objective functions given in Table 7.

Table 8 has been obtained by using the table's values of 3–7. Table 8 is the final solution table for the Model (2A).

From Table 8 and Fig. 5, we have seen that at $\alpha = 0$, the optimal order quantities are as follows: $Q_1 = 22.19335$ units, $Q_2 = 1187.256$ units, $Q_3 = 42.41547$ units with optimal compromise value of objective functions are $Z_1 = 7.498250$, $Z_2 = 8.62294$. Similarly, at $\alpha = 1.0$, the optimal order quantities are as $Q_1 = 21.58310$ units, $Q_2 = 1190.436$ units, $Q_3 = 41.63432$ units with optimal compromise value of objective functions are $Z_1 = 7.782631$, $Z_2 = 8.005682$. The obtained result shows that proposed approach can generates more consistent results with efficient solutions. Hence, the solutions obtained at the different value of α helps the decision

Fig. 6 Different values of objective function
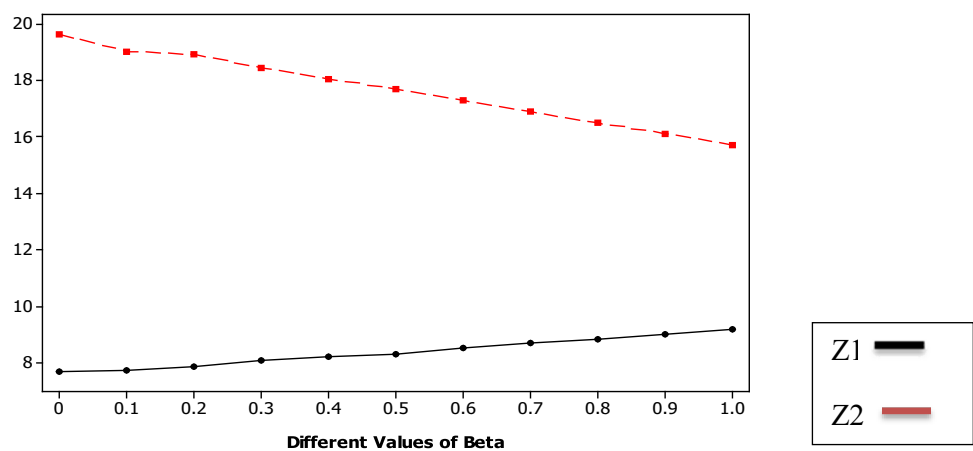
Table 15 Relative efficiency of solutions

	α (Z_1, Z_2)	R.E.	β (Z_1, Z_2)	R.E.
0	(7.498250, 8.622942)	(1.5419, 0.7123)	(7.685360, 19.62314)	(1.5044, 0.3130)
0.1	(7.520145, 8.578061)	(1.5374, 0.7161)	(7.723917, 19.03581)	(1.4969, 0.3227)
0.2	(7.544812, 8.508621)	(1.5324, 0.7219)	(7.838340, 18.92735)	(1.4750, 0.3245)
0.3	(7.569633, 8.439189)	(1.5274, 0.7278)	(8.079115, 18.45181)	(1.4311, 0.3329)
0.4	(7.594610, 8.369765)	(1.5224, 0.7339)	(8.219140, 18.06115)	(1.4067, 0.3401)
0.5	(7.619745, 8.300349)	(1.5173, 0.7400)	(8.295876, 17.68649)	(1.3937, 0.3473)
0.6	(7.645038, 8.230941	(1.5122, 0.7463)	(8.513711, 17.27950)	(1.3580, 0.3555)
0.7	(7.660459, 8.190292)	(1.5093, 0.7500)	(8.668768, 16.88852)	(1.3337, 0.3637)
0.8	(7.696106, 8.092148)	(1.5023, 0.7591)	(8.829376, 16.49742)	(1.3095, 0.3723)
0.9	(7.721884, 8.052039)	(1.4973, 0.7628)	(8.995837, 16.10622	(1.2852, 0.3814)
1.0	(7.782631, 8.005682)	(1.4849, 0.7673)	(9.168477, 15.71491)	(1.2610, 0.3909)

maker to choose the best solution which increases or decreases according to the preference as defined in terms of weight. Specifically, $\alpha = 0$ gives the widest lowest possibility, indicating that the objective value will never fall outside of this range while at the other extreme end of $\alpha = 1$, it provides the most optimistic value of the objective value. For this example, both the objective function is fuzzy intuitionistic and conflicting in nature, the most likely value of objective function are $Z_1 = 7.619745$, $Z_2 = 8.300349$, while the pessimistic value are $Z_1 = 7.498250$, $Z_2 = 8.622942$ and the optimistic value of objective function are $Z_1 = 7.782631$, $Z_2 = 8.005682$.

Case 2 for Model (2B) In similar to Model (2A), this model has also been solved by following the stepwise solution procedure as defined in Sect. 3.1. The maximum and minimum individual values of the objective function at the different value of $\beta \in [0, 1]$ has been given in Tables 9, 10, 11 and 12.

The relative weights attached to each objective functions given in Table 13.

Table 14 has been obtained by using the Table's values of 9–13. Table 14 is the final solution Table for the Model (2B).

From the Table 14 and Fig. 6, we have seen that at $\beta = 0$, the optimal order quantity quantities are as follows: $Q_1 = 23.18841$ units, $Q_2 = 41.09848$ units, $Q_3 = 1935.242$ units with optimal compromise value of objective functions are $Z_1 = 7.685360$, $Z_2 = 19.62314$. Similarly, at $\beta = 1.0$, the optimal order quantities are as $Q_1 = 20.54507$ units, $Q_2 = 38.88889$ units, $Q_3 = 1960.934$ units with optimal compromise value of objective functions are $Z_1 = 9.168477$, $Z_2 = 15.71491$. The obtained result shows that proposed approach can generates more consistent results with efficient solutions. Hence, the solutions obtained at a different value of β helps the decision maker

to choose the best solution which increases or decreases according to the preference as defined in terms of weight. Specifically, $\beta = 0$ gives the widest lowest possibility, indicating that the objective value will never fall outside of this range while at the other extreme end of $\beta = 1.0$, it provides the most optimistic value of the objective value. For this example, both the objective function is fuzzy intuitionistic and conflicting in nature, the most likely value of objective function are $Z_1 = 8.295876$, $Z_2 = 17.68649$, while the pessimistic value are $Z_1 = 7.685360$, $Z_2 = 19.62314$ and the optimistic value of objective function are $Z_1 = 9.168477$, $Z_2 = 15.71491$. LMF focuses more on Q_2 ordered quantity while in case of NLMF focus shifted more to Q_3 ordered quantity.

The above example has been compared with Dutta and Kumar (2013). They formulated the problem in deterministic environment and obtained the following results $Q_1 = 1363.712$, $Q_2 = 40$, $Q_3 = 42$ with $Z_1 = 11.5617$, $Z_2 = 6.1424$.

The relative effectiveness (R.E.) of an obtained compromise solution with respect to the Dutta and Kumar (2013) approach is defined as:

$$R.E. = T_{Dutta\,and\,Kumar(2013)} / T_{ProposedApproach}$$

where, $T_{Dutta\,and\,Kumar(2013)}$ represents the trace value of a compromise solution obtained by Dutta and Kumar (2013) and $T_{ProposedApproach}$ represents the trace value of the compromise solution obtained by using the proposed approach (Table 15).

From these results, it has been found that, due to the existence of conflicting objective function in the formulated MOLFIP, they cannot optimized sufficiently. In the same manner, the R.E. related to the first objective function decreases for the different levels of α and β, while the R.E. related to the second objective function increases for the different levels of α and β.

5 Managerial perceptions, contributions and limitations

5.1 Managerial perceptions

The following visions are drawn from the proposed work which is supportive of any operational managers in the manufacturing unit:

- The problem formulated in this paper is helpful to demonstrate a simulation study based on the realistic situation of inventory management, which is continuously faced by the operational managers in manufacturing and production unit, where DM want to optimize the different costs, which are directly related to the inventory management.
- The model considered holding cost, purchasing price, selling price, demand and ordering cost of the items with some ambiguity which is a natural phenomenon rather than an exact value, and hence, provides an elastic view to the operational managers to optimize the different inventory costs under ambiguity.

5.2 Contributions

The proposed work makes following contribution:

- A mathematical model for inventory management has been formulated with the primary objective to maximize the profit and minimize the holding cost of pre-ordered quantity.
- The proposed work gives the concept of intuitionistic fuzzy set theory in inventory management.
- Developed an FGP model for inventory management, where DM can control the quantity of ordered items.
- Different solution sets have been generated at some discrete values of α and β.

5.3 Limitations

There are some limitations related to our model which is given below:

- A hypothetical case study has been used to illustrate the proposed work.
- The proposed model limited to vagueness only, but in real-world problems decision maker has to face randomness and multi-choices situations.
- The role of deterioration during different production cycle is yet to be explored in this model.
- Fuzzy goal programming model is limited to linear membership function only. Hence, the role of other membership function in this model is yet to be explored.

6 Conclusion

In this paper, we have considered inventory modelling under IFS and maximize the profit and minimize the holding cost related to per-ordered quantity with some realistic set of constraints. The formulated problem cannot be solved unless it converted into an equivalent deterministic form, which has been done by using the ranking function approach by defining the membership and non-membership function respectively, which gives decision maker an optimistic and pessimistic view in multi-criterion decision making. The formulated models have been solved by using an FGP technique for attaining the most acceptable solution of the models by controlling the decision vectors at different levels of α and β. Tables 7 and 13 gives an optimum compromise solution with an optimum ordered quantity and in Table 14, the relative efficiency of the obtained solution has been compared with the deterministic model of inventory management. The procedure given in this manuscript can also be used for solving all those problems where the input information is in the form of an intuitionistic parameter. Such problems may be related to transportation problems, assignment problems, production planning problems, problems related to workforce management and many more. In a future study, the model can be further extended with different types of inventory and production-related costs and can also be solve with different types of linear and non-linear membership functions such exponential, hyperbolic, parabolic, piecewise linear, gaussian, and singleton membership function respectively.

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