

# Solving matrix games with linguistic payoffs

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**Abstract** In this paper, we introduced a matrix game with linguistic payoffs. In order to solve the matrix game with linguistic payoffs a solution methodology has been proposed. This proposed methodology is based on fuzzy representation of linguistic payoffs and defuzzification of fuzzy payoffs. In this method, the linguistic variable has been represented by fuzzy number. Here, widely known Yager's Ranking Index method has been utilized for defuzzification of fuzzy number. Finally, to illustrate the solution procedure of the proposed game problem, an example has been provided.

**Keywords** Matrix games · Linguistic payoff · Fuzzy number · Fuzzy payoff · Yager ranking index

## 1 Introduction

Modern description of game theory is generally considered to have started with the most pioneering research work "Theory of Games and Economic Behavior" (Neumann and Morgenstern 1944). In fact game theory has grown rapidly well, after the most influential and significant results provided by Nash (1950). Since then several types of mathematical game and their solution methodologies have been proposed. A game is modeled when two or more groups are in conflict over some issues and action of one group depends upon the action taken by the opponent. The

group members in a game are called the players. The possible actions taken by the players are called strategies. When each player selects a strategy, it will provide an outcome to the game; and the payoffs to all players, whenever reaches to maximizing his/her own payoff. Game theory is mathematical way out for obtaining the strategic interactions among multiple players or decision makers (DM) who select several strategies from the set of admissible strategies. It provides a vital role in managerial decision making and industrial management system. It is related with different types of competitive situations viz. economics, finance, banking, business sector, public voting, advertisements, marketing, politics etc. In the past, several researchers formulated and solved the matrix game considering precise payoff. This means that every probable situation to select the payoff involved in the matrix game is exactly determinable. And in this matrix game it is generally assumed that all information about payoffs of a game is known exactly by players. However, in reality, the players are often not able to evaluate the game exactly due to lack of proper information. As a result, payoffs of the game are not prescribed precisely. This implies that payoffs of the game are not precise. Harsanyi (1967) introduced impreciseness in games with a probabilistic method and developed the theory of Bayesian games. This theory could not perfectly solve the game problem with imprecise payoffs, as it was confined to only one possible kind of imprecision. But, in reality, impreciseness is of several types and can be handled by fuzzy set theory (Zadeh 1965). The fuzzy set theory was first introduced by Zadeh (1965). The fuzziness occurring in the game problems is treated as the fuzzy game problems. Fuzziness in game problems has had much more influence from an extensive range of research (Bector and Chandra 2002; Bector et al. 2004a, b;

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Vijay et al. 2005). Therefore, fuzzy games theory has got immensely studied in several areas, viz. economics, engineering and management science. In the last few years, several attempts have been made in the existing literature for solving game problems with fuzzy payoff. Compos (1989) formulated fuzzy linear programming model to solve fuzzy matrix game. Sakawa and Nishizaki (1994) solved multi-objective fuzzy games by introducing the theory of maxmin value. Chen and Larbani (2006) found that the equilibrium solution of multi-objective matrix games based on fuzzy payoffs is equivalent to the solution of the fuzzy multi-objective attribute decision-making problem. Çevikel and Ahlatcıoglu (2010) described new concepts of solutions for multi-objective two-person zero-sum games with fuzzy goals and fuzzy payoffs using linear membership functions. Li and Hong (2012) gave an approach for solving constrained matrix games with fuzzy payoffs considering the triangular fuzzy numbers. For, recent research directions on fuzzy matrix games one may refer to the works of Nayak and Pal (Nayak and Pal 2006, 2009), Seikh et al. (2015) and Sahoo (2015, 2016, 2017). In this paper, we have considered a matrix game problem with linguistic payoffs which is more realistic and has not been generally considered in the existing research. A linguistic variable (Zadeh 1975) is a variable whose values are represented by words or sentences in a natural or artificial language. In order to represent the linguistic variables fuzzy representation is the most appropriate representation as linguistic data is converted into quantitative data using the same. Therefore, the notion of fuzzy game theory furnishes an efficient scheme which solves real-life problems with linguistic payoffs. To solve the linguistic game problem, we have used fuzzy representation of linguistic payoffs and defuzzification of fuzzy payoffs. Here, a popularly known Yager's ranking index method (Yager 1981) has been used for defuzzification of fuzzy number. Then the fuzzy matrix game problem has been converted into a matrix game problem whose payoffs are precise valued and solved using the linear programming method. Finally, to illustrate the solution methodology, a numerical example has been solved and the computed results have been presented.

The entire paper is arranged in five sections as follows. Some basic concept of linguistic variables, fuzzy sets, defuzzification and two-person zero sum game is described in Sect. 2. Transformation of game problem into a linear programming problem is discussed in Sect. 3. Numerical example and computational results are reported in Sect. 4. Finally, conclusions have been made in Sect. 5.

## 2 Some basic concept of linguistic variables, fuzzy sets, defuzzification and two-person zero sum game

In this section, several basic concepts about linguistic variables, fuzzy sets, defuzzification and two-person zero sum game. Some basic definitions of linguistic variables and fuzzy sets introduced by Zadeh (1975) is also discussed briefly.

### 2.1 Linguistic variables

Linguistic variable is a variable whose values are words or sentence in a natural language (Zadeh 1975). Generally, a linguistic variable is characterized by  $(\chi, T(\chi), U, R, M)$  where  $\chi$  is the name of variable;  $T(\chi)$  is the set of natural languages called linguistic values;  $U$  is the universal set;  $R$  is a context free grammar for generating linguistic terms of  $T(\chi)$ ;  $M$  is a mapping from  $T$  to the fuzzy subset of  $U$ .

For example, payoffs are linguistic variables which can take the values as “extremely low”, “very low”, “low”, “high”, “very high” and “extremely high” i.e.,  $\chi = \text{Payoffs}$ . The term set  $T(\chi = \text{Payoffs})$  are represented as follows:  $T(\chi = \text{Payoffs}) = \{\text{very low, low, high, very high, extremely high}\}$ , where the universe of discourse for payoffs may be taken from the interval  $U = [\$0, \$50]$ .

### 2.2 Fuzzy set

A fuzzy set  $\tilde{A}$  in  $X$  is defined as the set of pairs  $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$  where  $\mu_{\tilde{A}}(x) : x \rightarrow [0, 1]$  is mapping, called the membership function of the fuzzy set  $\tilde{A}$ . Here  $X$  be a collection of objects and  $x$  be an element of  $X$ .

### 2.3 $\alpha$ -cut of a fuzzy set

The  $\alpha$ -cut of a fuzzy set  $\tilde{A}$  is a crisp set denoted by  $\tilde{A}_\alpha$  and defined by  $\tilde{A}_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1\}$ .

### 2.4 Convex fuzzy set

A fuzzy set  $\tilde{A}$  is convex fuzzy set iff for all  $x_1, x_2 \in X$  and  $0 \leq \theta \leq 1$ ,  $\mu_{\tilde{A}}(x)$  satisfy the relation  $\mu_{\tilde{A}}(\theta x_1 + (1 - \theta)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$ .

### 2.5 Normal fuzzy set

A fuzzy set  $\tilde{A}$  is normal fuzzy set if there exists at least one value of  $x \in X$  such that  $\mu_{\tilde{A}}(x) = 1$ .

### 2.6 Fuzzy number

A normalized convex fuzzy set is called fuzzy number i.e., a fuzzy number  $\tilde{A}$  is a fuzzy set if  $\tilde{A}$  is both convex and normal.

### 2.7 Triangular fuzzy number (TFN)

A fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  is said to be triangular fuzzy number if its membership function  $\mu_{\tilde{A}}(x)$  is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & \text{for } a_1 \leq x < a_2 \\ \frac{a_3 - x}{a_3 - a_2} & \text{for } a_2 < x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

where  $a_1, a_2$  and  $a_3$  are real numbers.

### 2.8 Parabolic fuzzy number (PFN)

A fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  is said to be parabolic fuzzy number if its membership function  $\mu_{\tilde{A}}(x)$  is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \left(\frac{a_2 - x}{a_2 - a_1}\right)^2 & \text{for } a_1 \leq x < a_2 \\ 1 - \left(\frac{x - a_2}{a_3 - a_2}\right)^2 & \text{for } a_2 < x \leq a_3 \\ 0 & \text{otherwise} \end{cases}$$

where  $a_1, a_2$  and  $a_3$  are real numbers.

### 2.9 Yager’s ranking index

Yager’s ranking index (Yager 1981) for a fuzzy number  $\tilde{A}$  is denoted by  $Y(\tilde{A})$  and is defined by the formula  $Y(\tilde{A}) = 0.5 \int_0^1 (\underline{A}_\alpha + \bar{A}_\alpha) d\alpha$  where  $\underline{A}_\alpha$  and  $\bar{A}_\alpha$  are the lower and upper bounds of the  $\alpha$ -level interval of the fuzzy number  $\tilde{A}$ .

**Lemma 2.1** *If  $\tilde{A} = (a_1, a_2, a_3)$  is TFN then  $Y(\tilde{A}) = \frac{1}{4}(a_1 + 2a_2 + a_3)$ .*

*Proof* Let  $\tilde{A} = (a_1, a_2, a_3)$  be a TFN, then  $\underline{A}_\alpha = a_1 + (a_2 - a_1)\alpha$  and  $\bar{A}_\alpha = a_3 - (a_3 - a_2)\alpha$  and hence  $Y(\tilde{A}) =$

$$0.5 \int_0^1 [a_1 + (a_2 - a_1)\alpha + a_3 - (a_3 - a_2)\alpha] d\alpha = \frac{1}{4} (a_1 + 2a_2 + a_3).$$

**Lemma 2.2** *If  $\tilde{A} = (a_1, a_2, a_3)$  is PFN then  $Y(\tilde{A}) = \frac{1}{3}(a_1 + a_2 + a_3)$ .*

*Proof* Let  $\tilde{A} = (a_1, a_2, a_3)$  be a PFN, then  $\underline{A}_\alpha = a_2 - (a_2 - a_1)\sqrt{1 - \alpha}$  and  $\bar{A}_\alpha = a_2 + (a_3 - a_2)\sqrt{1 - \alpha}$  and hence  $Y(\tilde{A}) = 0.5 \int_0^1 [a_2 - (a_2 - a_1)\sqrt{1 - \alpha} + a_2 + (a_3 - a_2)\sqrt{1 - \alpha}] d\alpha = \frac{1}{3}(a_1 + a_2 + a_3)$ .

**Definition 2.1** Let  $Y(\tilde{A})$  and  $Y(\tilde{B})$  are Yager’s ranking index of two fuzzy numbers  $\tilde{A} = (a_1, a_2, a_3)$  and  $\tilde{B} = (b_1, b_2, b_3)$  respectively. Then ranking or order relations of two fuzzy numbers are as follows:

1. If  $Y(\tilde{A}) > Y(\tilde{B})$  then  $\tilde{A} > \tilde{B}$ .
2. If  $Y(\tilde{A}) < Y(\tilde{B})$  then  $\tilde{A} < \tilde{B}$ .
3. If  $(Y(\tilde{A}) = Y(\tilde{B}), a_3 = b_3)$  and  $(a_2 > b_2)$  then  $\tilde{A} > \tilde{B}$ .
4. If  $(Y(\tilde{A}) = Y(\tilde{B}), a_3 = b_3)$  and  $(a_2 < b_2)$  then  $\tilde{A} < \tilde{B}$ .
5. If  $(Y(\tilde{A}) = Y(\tilde{B}), a_3 = b_3)$  and  $(a_2 = b_2)$  then  $\tilde{A} = \tilde{B}$ .

*Example 2.1* We assumed that the payoff varies from \$ 0 to \$ 50 and linguistic values are “extremely low”, “very low”, “low”, “high”, “very high” and “extremely high”. If  $\tilde{A}$  is a fuzzy set for the variable “extremely high”, then membership function  $\mu_{\tilde{A}}(x)$  is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x \leq 44 \\ \frac{x - 44}{6} & \text{if } 44 < x \leq 50 \\ 1 & \text{if } 50 < x \end{cases}$$

Again, “extremely high” also be represented by fuzzy number. If  $\tilde{A}$  is represented by triangular fuzzy number  $\tilde{A} = (44, 48, 50)$ , then membership function  $\mu_{\tilde{A}}(x)$  of the fuzzy number  $\tilde{A} = (44, 48, 50)$  is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - 44}{4} & \text{if } 44 \leq x \leq 48 \\ \frac{50 - x}{2} & \text{if } 48 \leq x \leq 50 \end{cases}$$

**Definition 2.2** Two-person zero-sum matrix game.

The two-person zero-sum game  $G$  can be defined in the form of rectangular matrix  $P = (a_{ij})_{m \times n}$  and represented by

$$P = (a_{ij})_{m \times n} = \begin{matrix} & B_1 & B_2 & \cdots & B_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{matrix}$$

where  $A_i \in \{x_1, x_2, \dots, x_m\}$  be a pure strategy for player  $A$  and  $B_j \in \{y_1, y_2, \dots, y_n\}$  be a pure strategy for player  $B$ . From matrix  $P$  it is clear that, if player  $A$  chooses a pure strategy  $x_i$  and the player  $B$  chooses a pure strategy  $y_j$ , then  $a_{ij}$  be the payoff for player  $A$  and  $-a_{ij}$  be a payoff for player  $B$ . Also, payoff matrix of player  $B$  is  $-P$ .

**Definition 2.3** Two-person zero-sum linguistic matrix game.

In most of the cases, it is observed that the payoffs are not precise valued. For this reason, let payoff  $a_{ij}$  be the imprecise valued for the player  $A$  and hence  $-a_{ij}$  be the imprecise payoff for the player  $B$ . Here, we have considered imprecise payoffs  $a_{ij}$  as linguistic variables. If we denote linguistic payoff  $a_{ij}$  as  $T(\chi = a_{ij})$  then linguistic payoff matrix of player  $A$  is  $\tilde{P} = (T(\chi = a_{ij}))_{m \times n}$ .

As linguistic variables are represented by fuzzy set and/ or fuzzy numbers then we have defined two-person zero sum fuzzy matrix game as follows:

**Definition 2.4** Two-person zero-sum fuzzy matrix game.

If player  $A$  chooses strategy  $x_i$  where as player  $B$  chooses  $y_j$ , then the fuzzy matrix game of player  $A$  is given by  $\tilde{P} = (\tilde{a}_{ij})_{m \times n}$  where  $\tilde{a}_{ij}$  be the fuzzy valued for linguistic variable  $T(\chi = a_{ij})$ .

**Definition 2.5** Let  $\tilde{P}$  be a fuzzy matrix game whose payoff matrix is  $(\tilde{a}_{ij})_{m \times n}$ . The set of mixed strategies for the player  $A$  is denoted by  $S_m = \{x = (x_1, x_2, \dots, x_m) : \sum_{i=1}^m x_i = 1 \text{ and } x_i \geq 0\}$ . Similarly, the set of mixed strategies for the player  $B$  is denoted by  $S_n = \{y = (y_1, y_2, \dots, y_n) : \sum_{j=1}^n y_j = 1 \text{ and } y_j \geq 0\}$ .

**Definition 2.6** The expected payoff of a fuzzy matrix game  $\tilde{P} = (\tilde{a}_{ij})_{m \times n}$  is denoted  $E(x, y)$  and is defined by  $E(x, y) = \sum_{i=1}^m \sum_{j=1}^n \tilde{a}_{ij} x_i y_j$ .

**Definition 2.7** In two person zero-sum game, player  $A$ 's mixed strategy  $x^*$  and player  $B$ 's mixed strategy  $y^*$  are said to be optimal strategies if  $E(x, y^*) \leq E(x^*, y^*)$  and  $E(x^*, y) \leq E(x^*, y)$  for any mixed strategies  $x$  and  $y$ .

**Definition 2.8** In two person zero-sum fuzzy matrix game  $(x^*, y^*)$  is called a fuzzy expected maximum equilibrium strategy if  $E(x, y^*) \leq E(x^*, y^*) \leq E(x^*, y)$  for any mixed strategies  $x$  and  $y$ .

**Definition 2.9** If  $E(x, y)$  be the expected payoff of a fuzzy matrix game  $\tilde{P} = (\tilde{a}_{ij})_{m \times n}$  then the common value of  $\max_x \min_y E(x, y)$  and  $\min_y \max_x E(x, y)$  is called the value of the fuzzy matrix game denoted by  $V(\tilde{P})$  or  $v$ .

### 3 Conversion of a fuzzy matrix game into a linear programming problem (LPP)

For a fuzzy matrix game  $\tilde{P} = (\tilde{a}_{ij})_{m \times n}$  one can formulated the game into a LPP is as follows:

Minimize  $z$   
 subject to

$$z - \sum_{i=1}^m \tilde{a}_{ij} x_i \leq 0, \text{ for all } j = 1, 2, \dots, n \tag{1}$$

$$\sum_{i=1}^m x_i = 1 \quad x_i \geq 0 \text{ for all } i = 1, 2, \dots, m$$

Here, problem (1) is a LPP with respect to the player  $A$ . Now, the dual of the problem (1) is as follows:

Maximize  $w$   
 subject to

$$w - \sum_{i=1}^m \tilde{a}_{ij} x_i \geq 0, \text{ for all } i = 1, 2, \dots, m \tag{2}$$

$$\sum_{j=1}^n y_j = 1 \quad y_j \geq 0 \text{ for all } j = 1, 2, \dots, n$$

Problem (2) is a LPP with respect to the player  $B$ .

**Procedure 3.1** (Conversion of game problem into a linear programming problem (LPP) whose payoff elements are linguistic variables) The procedural steps are as follows:

- Step 1* A game with linguistic payoff, replace all the linguistic payoff elements by fuzzy numbers. The resulting payoff matrix is fuzzy payoff matrix
- Step 2* Find Yager's ranking index of all the elements of fuzzy payoff matrix
- Step 3* In fuzzy payoff matrix, replace all the payoff elements with fuzzy numbers by their corresponding Yager's ranking indices
- Step 4* Using problem (1), construct a linear programming problem with respect to the player  $A$  and using problem (2), construct a linear programming problem with respect to the player  $B$

**Procedure 3.2** (solution procedure) The procedural steps to solve fuzzy matrix game are as follows:

*Step 1* Solve

Minimize  $z$ ; subject to  $z$

$$- \sum_{i=1}^m Y(\tilde{a}_{ij}) x_i \leq 0, \sum_{i=1}^m x_i = 1 \text{ and } x_i \geq 0.$$

Step 2 Solve

$$\begin{aligned} &\text{Maximize } w; \text{ subject to } w \\ &-\sum_{i=1}^m Y(\tilde{a}_{ij})x_i \geq 0, \sum_{j=1}^n y_j = 1 \text{ and } y_j \geq 0 \end{aligned}$$

Step 3 For player A, write  $z$  and  $x_1, x_2, \dots, x_m$

Step 4 For player B, write  $w$  and  $y_1, y_2, \dots, y_n$

### 4 Numerical example and computational results

In order to illustrate the solution procedure, we have considered a numerical example of a matrix game with linguistic payoffs. The payoff matrix is given below:

		Player B		
		Television	Radio	Press
Player A	Television	Very high	Very low	Extremely high
	Radio	Extremely low	Low	Very low
	Press	High	Very high	Low

In this payoff matrix, we assumed that the payoff varies from \$ 0 to \$ 50. Here, we have replaced linguistic variables by fuzzy numbers and these linguistic variables can be characterized as fuzzy sets whose membership functions are given below:

$$\begin{aligned} \mu_{\text{Extremely Low ("Around 2")}}(x) &= \begin{cases} \frac{x}{2} & \text{if } 0 \leq x \leq 2 \\ \frac{5-x}{3} & \text{if } 2 \leq x \leq 5 \end{cases}, \\ \mu_{\text{Very Low ("Around 4")}}(x) &= \begin{cases} \frac{x-2}{2} & \text{if } 2 \leq x \leq 4 \\ \frac{7-x}{3} & \text{if } 4 \leq x \leq 7 \end{cases}, \\ \mu_{\text{Low ("Around 8")}}(x) &= \begin{cases} \frac{x-4}{4} & \text{if } 4 \leq x \leq 8 \\ \frac{12-x}{4} & \text{if } 8 \leq x \leq 12 \end{cases}, \\ \mu_{\text{High ("Around 36")}}(x) &= \begin{cases} \frac{x-33}{3} & \text{if } 33 \leq x \leq 36 \\ \frac{38-x}{2} & \text{if } 36 \leq x \leq 38 \end{cases}, \end{aligned}$$

**Table 1** Linguistic term and their corresponding fuzzy data

Linguistic term (qualitative data)	Extremely low	Very low	Low	High	Very high	Extremely high
Fuzzy data (quantitative data)	(0, 2, 5)	(2, 4, 7)	(4, 8, 12)	(33, 36, 38)	(37, 40, 42)	(44, 48, 50)

$$\begin{aligned} \mu_{\text{Very High ("Around 40")}}(x) &= \begin{cases} \frac{x-37}{3} & \text{if } 37 \leq x \leq 40 \\ \frac{42-x}{2} & \text{if } 40 \leq x \leq 42 \end{cases}, \\ \mu_{\text{Extremely High ("Around 48")}}(x) &= \begin{cases} \frac{x-44}{4} & \text{if } 44 \leq x \leq 48 \\ \frac{50-x}{2} & \text{if } 48 \leq x \leq 50 \end{cases} \end{aligned}$$

Now, in payoff matrix, the linguistic data is converted into the quantitative data using the Table 1. Hence, the fuzzy payoff matrix is as follows:

		Player B		
		Television	Radio	Press
Player A	Television	(37, 40, 42)	(2, 4, 7)	(44, 48, 50)
	Radio	(0, 2, 5)	(4, 8, 12)	(2, 4, 7)
	Press	(33, 36, 38)	(37, 40, 42)	(4, 8, 12)

The Yager’s ranking indices of all the elements of payoff matrix are given in Table 2. Using Procedure 3.1 and Procedure 3.2 we have solved the matrix game and finally computed results have been presented in Table 3. Further, it is mentioned that, the computational work has been done using LINGO 10.

From Fig. 1, it has been observed that consideration of triangular fuzzy number (TFN) and parabolic fuzzy number (PFN) are both admissible in the sense that the choices of probability values for both cases against the player A are nearly the same. And it has been seen that, the probability values are not significantly greater than those of the other.

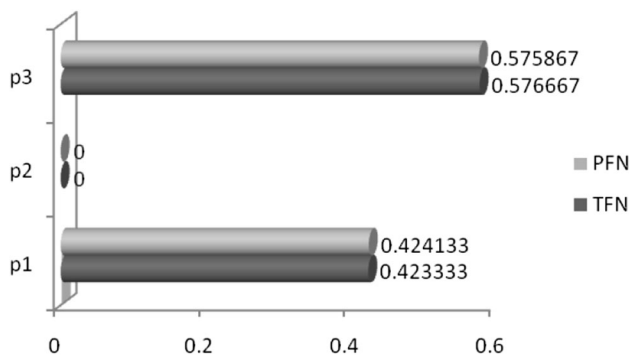
From Fig. 2, it has also been observed that the value of the game is slightly high in case of replacement of the entire payoff elements with linguistic variables by triangular fuzzy numbers than that of replacement of the entire payoff elements with linguistic variables by parabolic fuzzy numbers. But two values are admissible as a difference of 0.040521, a very small quantity. Similar results hold good for player B.

**Table 2** Yager’s ranking indices of all the payoff elements

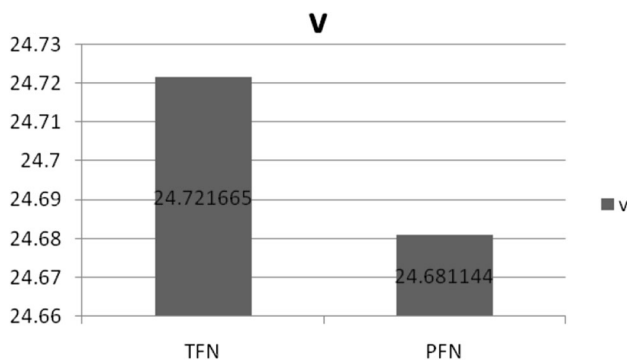
Fuzzy data $\tilde{A} = (a_1, a_2, a_3)$ (quantitative data)	If $\tilde{A}$ is triangular fuzzy number (TFN), then Yager’s ranking Index $Y(\tilde{A})$	If $\tilde{A}$ is parabolic fuzzy number (PFN), then Yager’s ranking Index $Y(\tilde{A})$
(0, 2, 5)	2.25	2.33
(2, 4, 7)	4.25	4.33
(4, 8, 12)	8.0	8.0
(33, 36, 38)	35.75	35.67
(37, 40, 42)	39.75	39.67
(44, 48, 50)	47.5	47.33

**Table 3** Computed results for player A and player B

Fuzzy Number	Player A		Player B		Value of the game
	Strategies $x$	Probabilities ( $x_1, x_2, x_3$ )	Strategies $y$	Probabilities ( $y_1, y_2, y_3$ )	
TFN	(Television, Radio, Press)	(0.423333, 0.0, 0.576667)	(Television, Radio, Press)	(0.0, 0.526667, 0.473333)	24.721665
PFN	(Television, Radio, Press)	(0.424133, 0.0, 0.575867)	(Television, Radio, Press)	(0.0, 0.526718, 0.473282)	24.681144



**Fig. 1** Choices of probabilities values for the maximizing player A considering TFN and PFN



**Fig. 2** Value of the game considering TFN and PFN

### 5 Conclusions

In this paper, we have solved matrix game with linguistic payoffs. Here, the elements of payoff matrix are considered as descriptive words that characterized linguistic variables. For this purpose, a method for solving matrix game is proposed to determine the solution. Also, the game with their strategies and value of the game have been presented and compared. The proposed solution methodology would be very much helpful for solving such complicated type game problems. Finally, it can be concluded that the proposed method established in this paper is capable of solving real-life decision making problems in the years to come having linguistic parameters.

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