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ORIGINAL ARTICLE

Analysis of an industrial system under uncertain environment by using different types of fuzzy numbers

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Abstract Uncertainties play a dominant role in the performance analysis of the system. For managing it, fuzzy set theory and its corresponding triangular fuzzy numbers have been utilized by most of the researchers for quantifying the data. However, in this manuscript, this hypothesis has been calmed by defining the different types of numbers, namely gamma, normal, Cauchy and triangular for uncertainties. Based on it, behavior, performance and sensitivity analysis of the system have been investigated at different levels of confidence and the preferences as provided by the decision makers towards the data. Based on it, various expressions of the system such as failure rate, repair time, reliability, availability etc., are obtained corresponding to these different types of the numbers. From the computed results, it is concluded that these indices are reduced range of prediction as compared to the existing approaches. A numerical example has been taken for demonstrating the approach.

Keywords Uncertainty analysis · Reliability · Fuzzy set theory · Repairable system · Fuzzy numbers

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1 Introduction

Today in the real-world decision, an uncertainty plays a dominant role in the analysis and hence a billion of dollars are being spent in maintaining the performance of the system by designing a reliable system and/or products. The primary objective of the system analyst is to increase the reliability and/or availability of the system by choosing the proper maintenance actions (Rani et al. 2016). But the uncertainties, which are occurring in most of the systems during data handling, are playing the dominant role. On the other hand, if the system analyst has used the collected data, without considering the uncertainties, during the analysis then their corresponding results do not give the exact information about the system behavior and consequently, the performance of the system will lay down (Garg 2014; Garg et al. 2014c).

To deal with such uncertainties, fuzzy set (FS) theory (Zadeh 1965) is one of the powerful tools to handle the impreciseness and vagueness in the data. Since their existence, various researchers have used the FS theory into the reliability analysis. For instance, Singer (1990) estimated the reliability of the system by using L–R (left–right) type fuzzy numbers. Later on, Cheng and Mon (1993) proposed confidence interval method for analyzing the system reliability under the fuzzy environment. Chen (1994) analyzed the reliability of the system using fuzzy arithmetic operations instead of ordinary arithmetic operations while Chen (1996) analyzed the system behavior by using α-cut arithmetic operations of the fuzzy number. In continuation with these, Mon and Cheng (1994) analyzed the reliability of the system by using different types of membership functions rather than triangular function. Utkin and Gurov (1998) measuring the system performance in terms of steady-state

availability and unavailability by considering the failure and repair process of the system. Jiang and Chen (2003) presented a method to solve some engineering problems with random general stress. Huang et al. (2006) presented a method based on the artificial neural network to estimate the parameters of the reliability and hence analyzed the performance of the system. Lei et al. (2005) presented the network equivalent approach for reliability evaluation of power distribution systems. Gupta and Bhattacharya (2007) proposed a methodology, by using statistical analysis, for estimating the failure probability of an industrial system.

From the above study, it is evident that most of the researchers have considered only the reliability parameter during the analysis. But, practically only the reliability index will not give the correct information about the system behavior, as there are other indices also with affect the performance of the system directly. So, it is necessary to modify these existing theories by considering all those indices of the system which affects the performance directly or indirectly. To overwhelmed this and to incorporate the preference of the decision makers into the analysis, Knezevic and Odoom (2001) presented a theory, called as fuzzy lambda-tau (FLT), by using Petri net (PN) and FS theory for analyzing the behavior of the system in terms of several parameters. Further, these theory has been used by the researchers and apply them to investigate the behavior of the different industrial systems (Garg and Sharma 2012a; Sharma and Garg 2011). But from their studies, it has been analyzed that it contains a wide range of spread (or support) when applied it to the large structural system. In order to resolve this issue, Garg and Sharma (2012b) and Garg et al. (2014a) presented a nonlinear optimization model for optimizing the spread corresponding to each reliability index and then solved it by using soft computing techniques. The major advantage of their approach is to (1) formulate an optimization model by using ordinary arithmetic operations instead of the fuzzy operations, (2) reduce the overall spread, so as to save the money and time. Also, Garg (2015b) presented a hybridized technique named as GA-GSA algorithm to optimize the parameter of the system so as to maximize the overall performance of the system. Garg (2015a) formulated a fuzzy Kolmogorov's differential equations corresponding to the system behavior of the system. Garg (2016) investigated the reliability of the series-parallel system using the credibility theory. Apart from that, there are some other types of the problems developed by the researchers using the concept of the artificial intelligence in the fields which are summarized in (Garg 2013, 2017; Garg and Sharma 2013; Garg et al. 2014b; Valipour 2016; Valipour et al. 2013).

Generally, the behavior of the system has been analyzed through various qualitative and quantitative techniques which require complete information. But due to various uncertain conditions, it is difficult to the plant personnel to collect all these information accurately. If somehow they can be collected then it has a wide range of uncertainties. On the other hand, to handle these uncertainties, researchers have expressed the obtained data in terms of fuzzy numbers which usually follows an exponential distribution along with the same type of the numbers. However, in real-life situations, these types of situation are rarely occurring because failure is an unavoidable phenomenon. Therefore, there is a need to investigate these issues and to include some other types of membership function into the analysis so that the plant personnel may use the technique for the variable rate also.

The gap in the research motivates us to extend the theory by taking some different kinds of the membership functions instead of only the exponential membership function. The main contributions of this work are summarized below:

1. To define and analyze the behavior of the system by using different types of linear or nonlinear membership functions viz., Gamma, Cauchy, normal, triangular etc., to measures the vagueness in the data.
2. To incorporate the preferences of the plant personnel towards the data into the analysis in the form of the spread (or support).
3. To measure the various reliability indices corresponding to the different types of the membership functions.
4. The sensitivity, as well as the performance analysis for each component of the system, has been investigated in detail for showing the impact of it onto the behavior of the system.
5. The most critical component of the system has been found by investigating the impact of the rates of the components on to the system availability.
6. In contrast to the existing approaches, the proposed method will depicts not only the behavior of the individual components but also depicts the effect of individual component behavior and their corresponding change in spreads. Further, a comparative study with existing studies has been established to show its advantages.

The rest of this paper is organized as follows. In Sect. 2, we briefly reviews some basic concepts of fuzzy sets and their corresponding membership functions. Section 3 presents a methodology for conducting the behavior analysis by using different types of fuzzy numbers. In Sect. 4, proposed approach has been illustrated with a case study of repairable system. Section 5 summarizes the results, discussion and the advantages of the proposed approach with respect to the existing studies. A concrete conclusion is drawn in Sect. 6.

2 Preliminaries

In this section, some basic concepts about the fuzzy sets and membership functions have been summarized.

Definition 1 (Zadeh 1965) A fuzzy set 'A' over the universal set X is defined as

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\} \tag{1}$$

where $\mu_A : X \to [0, 1]$ represent the degree of the membership of element x to A such that $\mu_A(x) \in [0, 1]$ for all $x \in X$.

Definition 2 (Klir and Yuan 2005) A fuzzy number is a normal, convex membership function, i.e., its membership function is piecewise continuous and there exist at least one $x_0 \in \mathbb{R}$ such that $\mu_A(x_0) = 1$. The membership function defined on $a_1, a_2, a_3, a_4 \in \mathbb{R}$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$ is given as

$$\mu_A(x) = \begin{cases} f(x); & x \in (a_1, a_2) \\ 1; & x \in [a_2, a_3] \\ g(x); & x \in (a_3, a_4) \\ 0; & \text{otherwise} \end{cases} \tag{2}$$

where $f, g : \mathbb{R} \to [0, 1]$ are the monotonic, continuous from the right and left, nondecreasing and nonincreasing functions respectively. We denote this fuzzy number as $A = (a_1, a_2, a_3, a_4)$.

Some of the popular distribution for handling the uncertainties in the data are summarized as follows:

(a) fuzzy normal distribution:

$$\mu_A(x) = e^{-k(x-a)^2}, \qquad \text{where } k > 0 \quad \text{and } a \in \mathbb{R} \tag{3}$$

(b) fuzzy sharp gamma distribution:

$$\mu_A(x) = \begin{cases} e^{k(x-a)^2}; & x \leq a \\ e^{-k(x-a)^2}; & x > a \end{cases} \tag{4}$$

where $k > 0$ and $a \in \mathbb{R}$.

(c) fuzzy Cauchy distribution:

$$\mu_A(x) = \frac{1}{1 + \alpha(x-a)^\beta}, \qquad \text{where } \alpha > 0, \tag{5}$$

β is a positive even

(d) fuzzy triangular distribution:

$$\mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \leq x < a_2 \\ 1 & \text{if } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{if } a_2 \leq x < a_3 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$
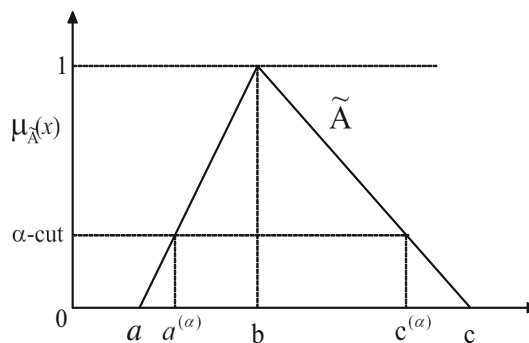

Fig. 1 α-Cut of the fuzzy set A

Definition 3 (Klir and Yuan 2005) An α-cut of a fuzzy set A, denoted by A_α is defined as

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}; \alpha \in [0, 1] \tag{7}$$

For instance, for a triangular fuzzy number $A = (a, b, c)$, their corresponding α-cut is defined below and shown graphically in Fig. 1 whose interval of confidence are defined are

$$A_\alpha = \left[a^{(\alpha)}, c^{(\alpha)}\right] = [a + \alpha(b - a), c - \alpha(c - b)] \tag{8}$$

Definition 4 (Ebeling 2001) Reliability of a system is expressed as a probability that that the system will perform its required function under given conditions for a stated time period. Mathematically, for a continuous random variable T, the reliability function $R(t)$ is defined as

$$R(t) = P(T \geq t) = 1 - \int_0^t f(u)du \tag{9}$$

where $R(t) \geq 0$, $R(0) = 1$, and $\lim_{t \to \infty} R(t) = 0$ and $f(t)$ is the failure probability density function.

In addition, the conditional probability of a failure in the time interval from t to $t + \Delta t$, given that the system has survived to time t is

$$P(t \leq T \leq t + \Delta t | T \geq t) = \frac{R(t) - R(t + \Delta t)}{R(t)} \tag{10}$$

Therefore, the hazard rate of the system is defined as

$$\lambda(t) = \frac{R(t) - R(t + \Delta t)}{R(t)\Delta t} = -\frac{dR(t)}{dt} \cdot \frac{1}{R(t)} \tag{11}$$

and hence its corresponding reliability can be derived as

$$R(t) = \exp\left[-\int_0^t \lambda(u)du\right] \tag{12}$$

Definition 5 (Ebeling 2001) Availability of a system is expressed as a probability that a system will be available to function at the given time t. Mathematically, it is defined at any given time t as

$$A(t) = \frac{\xi_s}{\lambda_s + \xi_s} + \frac{\lambda_s}{\lambda_s + \xi_s} e^{-(\lambda_s + \xi_s)t} \qquad (13)$$

where λ_s and ξ_s are respectively, represents the failure and repair rates of the system.

3 Methodology

In this section, we have presented an approach to analyze the system performance by using different types, linear and nonlinear, of the membership functions. For it, the following assumptions have been taken as:

1. component parameters are independent;
2. component is assumed to be as new after repairs;
3. standby and active components are of same nature.

Based on these assumptions, the methodology for conducting the behavior analysis are summarized in the following four steps as follows:

Step 1   Data related to each component are extracted in the form of failure rates (λ_i's) and repair times (τ_i's) either from the sheets or from their personal experiences.

Step 2   The obtained information from step 1 are generally out of date or imprecise due to lack of proper update or by human errors. So in order to quantify the uncertainties in the data, it must be converted into fuzzy numbers with the help of the system analyst preferences. For instance, if the decision-maker gives $\pm 15\%$ spread towards the data, then their corresponding triangular fuzzy number becomes $(\lambda_{i1}, \lambda_{i2}, \lambda_{i3}) = (0.85\lambda_{i2}, \lambda_{i2}, 1.15\lambda_{i2})$ corresponding to ith component of the failure rate λ_i. Similarly for the repair times and their representations are shown graphically in Fig. 2.

Table 1 Basic expressions of Lambda Tau methodology

Gate	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expression	$\prod_{j=1}^{n} \lambda_j \left[\sum_{i=1}^{n} \prod_{\substack{j=1 \\ i \neq j}}^{n} \tau_j \right]$	$\dfrac{\prod_{i=1}^{n} \tau_i}{\sum_{j=1}^{n} \left[\prod_{\substack{i=1 \\ i \neq j}}^{n} \tau_i \right]}$	$\sum_{i=1}^{n} \lambda_i$	$\dfrac{\sum_{i=1}^{n} \lambda_i \tau_i}{\sum_{i=1}^{n} \lambda_i}$

Step 3   By using the operations of AND/OR expressions given in Table 1 at different levels of α-cuts, the indices of various reliability parameters, listed in Table 2, are computed.

Step 4   The obtained fuzzified data need to be defuzzified in order to implement their decision in real world. For it, center of gravity method (Ross 2004) is used in the interval $[x_1, x_2]$ and is given by

$$\bar{x} = \frac{\int_{x_1}^{x_2} x \cdot \mu_A(x) dx}{\int_{x_1}^{x_2} \mu_A(x) dx} \qquad (14)$$

where $\bar{x}$ represent the defuzzified value.

The flowchart of the proposed approach has been summarized in Fig. 3.

4 Case study

The above mentioned methodology is illustrated with a case study of the the decomposition unit of a complex repairable urea fertilizer plant (Sharma and Garg 2011).

4.1 System description

Decomposition unit comprised of the four major components denoted by A, B, C and E which are connected in the series configuration and the brief description of each component associated with them is explained as follows (Sharma and Garg 2011).

1. First component A consists of two absorbers, namely as reboiler (A_1) and falling film heater (A_2) for absorbing

Fig. 2 A triangular fuzzy number for failure rate λ and repair time τ. **a** Triangular membership functions of λ_{i2}, **b** triangular membership functions of τ_{i2}

Table 2 Some reliability parameters

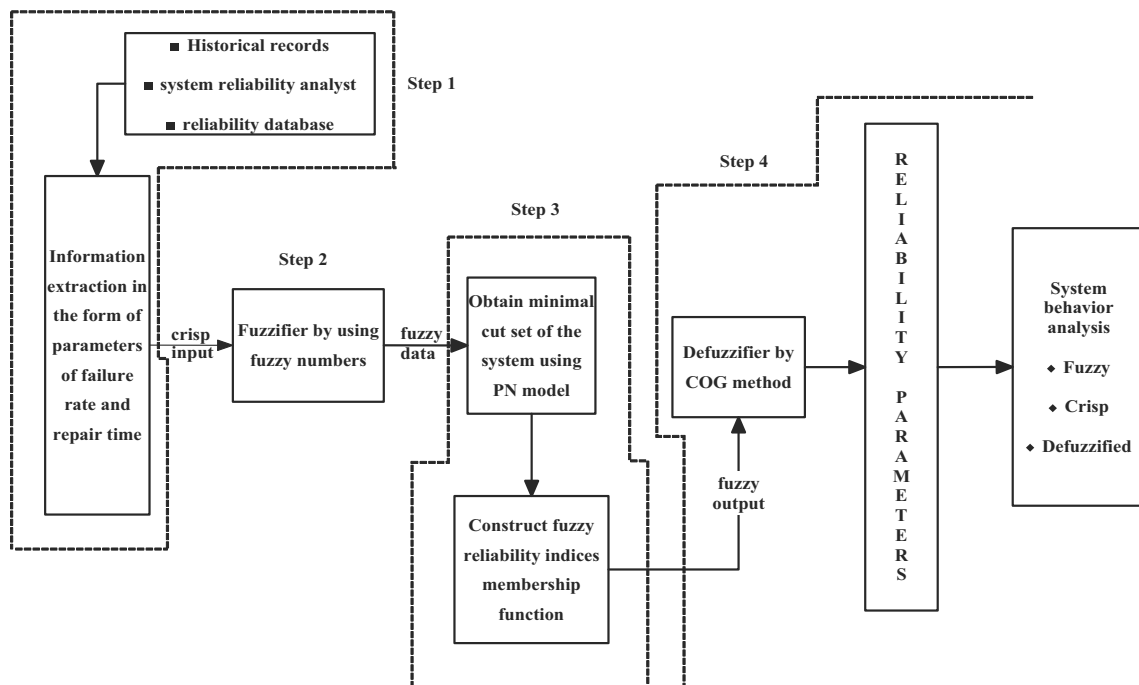| Parameters	Expressions
Failure rate	$\mathrm{MTTF}_s = \dfrac{1}{\lambda_s}$
Repair time	$\mathrm{MTTR}_s = \dfrac{1}{\xi_s} = \tau_s$
Mean time between failures (MTBF)	$\mathrm{MTBF}_s = \mathrm{MTTF}_s + \mathrm{MTTR}_s$
Reliability	$\mathrm{R}_s = e^{-\lambda_s t}$
Availability	$\mathrm{A}_s = \dfrac{\xi_s}{\lambda_s + \xi_s} + \dfrac{\lambda_s}{\lambda_s + \xi_s} e^{-(\lambda_s + \xi_s)t}$
Expected numbers of failures (ENOF)	$\mathrm{W}_s(0,t) = \dfrac{\lambda_s \xi_s t}{\lambda_s + \xi_s} + \dfrac{\lambda_s^2}{(\lambda_s + \xi_s)^2}\left[1 - e^{-(\lambda_s + \xi_s)t}\right]$

Fig. 3 Flow chart of the proposed approach

the high and low-pressure respectively. Failure of either of them will cause system failure.

2. Second component B consists of high-pressure B_1 and low-pressure B_2 absorber connected in series.

3. Third component called as the gas separator C used for separating the gases obtained from absorbers. Failure of it will cause system failure.

4. Fourth component consist of two units namely low-pressure E_1 and high-pressure heat exchanger E_2 with standby unit for recovering the heat of the gases. The entire system will fail if both the system fails to work.

The symmetric diagram and its equivalent PN model of the system for describing the working of the major component of the unit are shown in Fig. 4a, b. The data, in terms of its

failure rates and repair times, corresponding to these major components are summarized in Table 3 (Sharma and Garg 2011) while the minimal cut sets are obtained as $\{A_1\}$, $\{A_2\}$, $\{B_1\}$, $\{B_2\}$, $\{C\}$ and $\{E_1 E_2\}$.

4.2 Analysis by proposed approach

In this section, an analysis has been conducted to compute the system reliability parameters by considering the different level of uncertainties. For this, firstly the collective data, given in Table 3, has been fuzzified into the different types of fuzzy numbers namely triangular, normal, gamma and cauchy. Based on the Fig. 4b, mathematical expression of the system is represented as follows:

Fig. 4 Schematic diagram and Petri net model of the decomposition unit. **a** Schematic diagram, **b** Petri net model
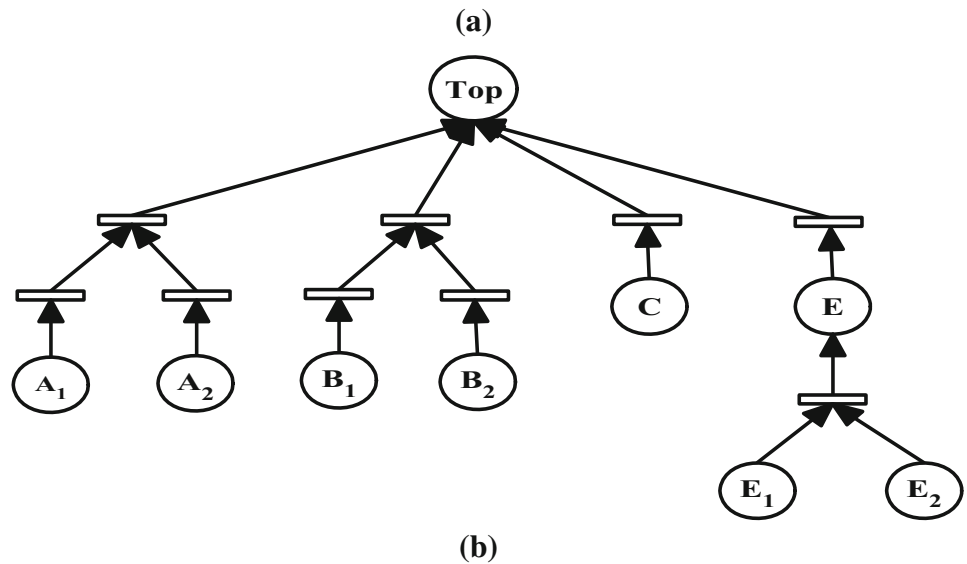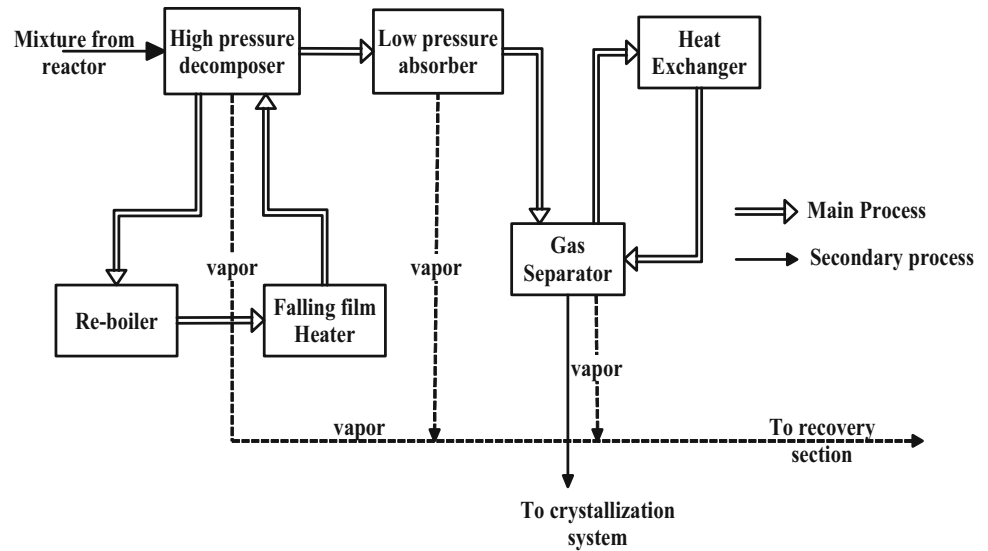

(a)

(b)

Table 3 Input data for decomposition system

Components→	Reboiler	Falling film pressure	Absorber	
	$(i = 1)$	$(i = 2)$	High pressure $(i = 3)$	Low pressure $(i = 4)$
Failure rate λ_i (h^{-1})	4.154×10^{-4}	3.952×10^{-4}	1.592×10^{-4}	4.783×10^{-4}
Repair time τ_i (h)	3.1746	2.6421	3.3323	4.7619
		Heat exchanger		
	Gas seperator	Low pressure	High pressure	
	$(i = 5)$	$(i = 6)$	$(i = 7)$	
Failure rate λ_i (h^{-1})	2.612×10^{-4}	6.956×10^{-4}	6.264×10^{-4}	
Repair time τ_i (h)	4.899	4.6831	6.2310	

$$S = A \cup B \cup C \cup E$$
$$= (A_1 \cup A_2) \cup (B_1 \cup B_2) \cup C \cup (E_1 \cap E_2) \quad (15)$$

$$\lambda_s = \sum_{i=1}^{5} \lambda_i + \lambda_6 \lambda_7 (\tau_6 + \tau_7) \quad \text{and} \quad \tau_s = \frac{\sum_{i=1}^{5} \lambda_i \tau_i + \lambda_6 \lambda_7 \tau_6 \tau_7}{\lambda_s}$$
$$(16)$$

Based on Eq. (15), the systems failure rate and repair time is computed by using Table 1 and get

The following steps of the proposed approach have been executed for the considered system as follows:

Step 1 The data related to main components A, B, C and E of the system is summarized in Table 3.

Step 2 Data are fuzzified into different fuzzy numbers by taking $\pm 15\%$ spread, as suggested by the system analyst, and hence their corresponding fuzzy numbers becomes $(0.85\lambda_i, \lambda_i, 1.15\lambda_i)$ and $(0.85\tau_i, \tau_i, 1.15\tau_i)$ for the failure rate and repair time of each component.

Step 3 Based on these inputs the various reliability parameters, as given in Table 2, are computed at the different level of uncertainties ranging from 0 to 1 corresponding to $t = 10$ (h) by using the different types of fuzzy numbers namely triangular, normal, gamma and Cauchy. The results corresponding to these are depicted graphically in Fig. 5 along with the existing techniques. From this figure, it has been observed that Cauchy distribution have the lesser range of uncertainties for each reliability indices than the other, which means that the decision-maker may analyze the behavior of the system in which failure rate and repair time follows a Cauchy distribution rather than the other types of distribution.

Step 4 From the computed results, given in Fig. 5, it has been seen that they are fuzzified in nature. But in order to get it implement into the real-life situation, it is necessary to convert this into the crisp number so that decision maker or system analyst may take their decision accordingly. In order to do so, the centre of gravity method has been used and the defuzzification value corresponding to failure rate at $\pm 15\%$ spread is given as 1.714209×10^{-4}, 1.714465×10^{-4}, 1.714157×10^{-4} and 1.714058×10^{-4} for the triangular, normal, gamma and Cauchy distribution, respectively. On the other hand, the crisp value for the failure rate is 1.714055×10^{-4} and hence we conclude that the defuzzified value corresponding to Cauchy distribution is very close to the crisp value and hence system analyst may take a decision more confidently as compared to other distributions. Similar observations have been seen from the other reliability indices.

4.3 Comparative study

To validate the proposed results, a comparative analysis has been conducted with some of the existing approaches (Chen 1994, 1996; Huang et al. 2004; Knezevic and Odoom 2001; Kumar and Aggarwal 1993; Sharma and Garg 2011) and their corresponding results along with the proposed approach results are summarized in Table 4. From this table, it is concluded that

(a) If we utilize the traditional method (Kumar and Aggarwal 1993) which used the probability theory to deal with the uncertainty involved in the collected data then the failure rate and repair time of the overall system have been computed by using Eq. (16) as $\lambda_s = 0.001714055$ and $\tau_s = 3.770787$. Therefore, based on these system failure rates and repair time, the overall expression of the system reliability, availability and mean time between failures are 0.98301, 0.994023 and 587.1824 respectively. Thus, the reliability of the considered unit is 98.301%. From their results, it has been concluded that it did not consider the degree of the uncertainties level and hence their results are suitable only where data are precise in nature.

(b) If we apply the posbit fault tree model (Huang et al. 2004) to the considered system in order to compute the system failure rate and repair time then the results corresponding to its have been calculated as $\lambda_s = 6.264 \times 10^{-4}$, $\tau_s = 4.8990$. Thus, it remains constants at the different level of the uncertainties ranging from 0 to 1 and therefore, their approach is limited.

(c) Chen (1994) presented an arithmetic operations based method for computing the reliability of the system under the fuzzy environment. If we utilize their approach to the considered system then we get triangular fuzzy numbers corresponding to system failure rate as (0.00145582, 0.00171405, 0.00197292) and repair time as (3.205887, 3.770787, 4.335277). The variation of it for different values of the uncertainties ranging from 0 to 1 are summarized in Table 4 and conclude that they have a wide range of uncertainty.

(d) Chen (1996) presented the α-cut based approach for computing the fuzzy probability of the top event. By applying their approach to the considered system, we get the ranges of the failure rate and repair time are (0.00145582, 0.00171405, 0.00197292) and (2.365628, 3.770787, 5.875147) respectively. The complete variation of their ranges with respect to each level of uncertainties is given in Table 4.

(e) If we apply Knezevic and Odoom (2001) and Sharma and Garg (2011) approaches corresponding to the considered data, then the range corresponding to the repair time is being summarized in Table 4 for a different level of uncertainties.

From this comparison table, it has been observed that the variation of their values at the different level of uncertainties is less in the case of the Cauchy distribution as compared to the other existing approaches.

Fig. 5 Various reliability plot with depict the systems' behavior at ± 15% spread. a Failure rate, b repair time, c MTBF, d ENOF, e reliability, f availability
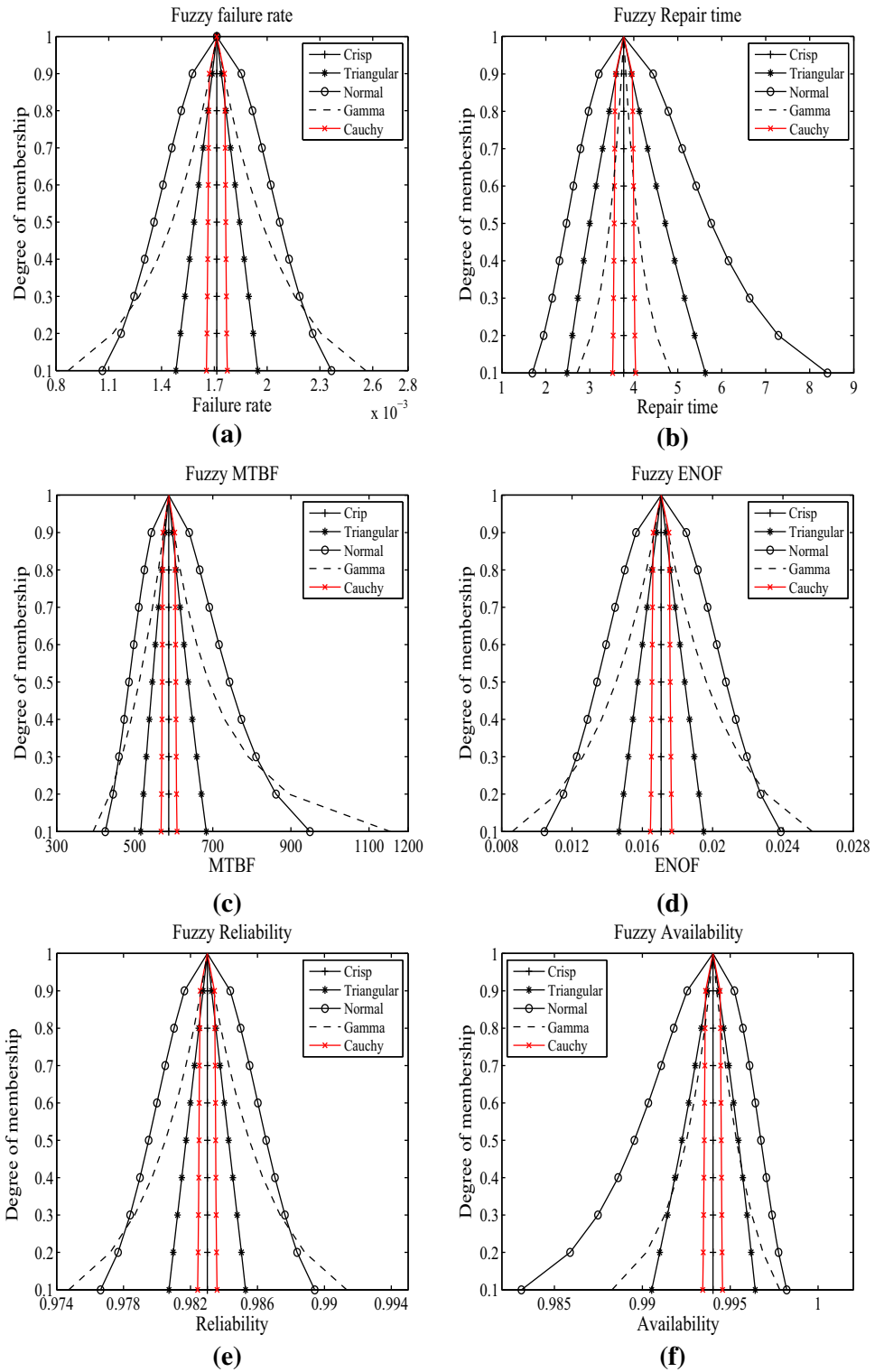
Table 4 Comparative analysis with the existing approaches

α	Kumar and Aggarwal (1993) [L, R]	Huang et al. (2004) [L, R]	Chen (1994) [L, R]	Chen (1996) [L, R]
0.0	[3.770787, 3.770787]	[4.8990, 4.8890]	[3.205887, 4.335277]	[2.365628, 5.875147]
0.2	[3.770787, 3.770787]	[4.8990, 4.8890]	[3.318867, 4.222379]	[2.604234, 5.381144]
0.4	[3.770787, 3.770787]	[4.8990, 4.8890]	[3.431847, 4.109481]	[2.862298, 4.927328]
0.6	[3.770787, 3.770787]	[4.8990, 4.8890]	[3.544827, 3.996583]	[3.141474, 4.509846]
0.8	[3.770787, 3.770787]	[4.8990, 4.8890]	[3.657807, 3.883685]	[3.443613, 4.125323]
1.0	[3.770787, 3.770787]	[4.8990, 4.8890]	[3.770787, 3.770787]	[3.770787, 3.770787]
α	Sharma and Garg (2011)	By proposed approach		
	Triangular	Normal	Gamma	Cauchy
	[L, R]	[L, R]	[L, R]	[L, R]
0.0	[2.365628, 5.875147]	[1.694014, 8.401902]	[4.841168, 2.710806]	[3.503791, 4.058135]
0.2	[2.603384, 5.384763]	[1.952032, 7.288923]	[4.518084, 3.027993]	[3.518313, 4.041385]
0.4	[2.861527, 4.929605]	[2.311771, 6.152826]	[4.195775, 3.347160]	[3.534860, 4.022466]
0.6	[3.140856, 4.511119]	[2.625069, 5.417609]	[4.007564, 3.534425]	[3.547744, 4.007857]
0.8	[3.443242, 4.125855]	[2.972304, 4.784175]	[3.874170, 3.667483]	[3.562417, 3.991348]
1.0	[3.770787, 3.770787]	[3.770787, 3.770787]	[3.770787, 3.770787]	[3.770787, 3.770787]

L, Left membership; R, right membership values

5 Results, discussion and advantages of the approach

In this section, the results corresponding to the considered system have been summarized in terms of their behavior, sensitivity as well as the performance analysis.

5.1 Behavior analysis

The following observations have been computed from the behavior analysis as.

1. From the Table 4, it is seen that the existing approaches are restricted to their domain and are unable to handle the uncertainties in a precise way. Also, their theories are applicable in all those circumstances where the data related to the system components are precise in nature.

2. From Fig. 5, it is seen that the range of uncertainties varies, in the form of support, with respect to the distribution wise. For depicting the decrease in spread (or support in %) by the Cauchy distribution over the others, a support has been computed based on their behavior plots and the results are summarized in Table 5. From this table, for instance, in the case of repair time, the level of uncertainties is reduced by 82.3771, 91.7359, and 73.9789% from triangular, normal and gamma distribution respectively, when compared with Cauchy distribution results. Also, from this table, it is observed that the largest and smallest spread reduced corresponding to repair time and failure rate, respectively, when it is measured from cauchy results to triangular and normal results while MTBF

Table 5 Data related to spread of reliability indices

	Failure rate	Repair time	ENOF	MTBF	Reliability	Availability
Computed spread for reliability indices						
I	0.000465	3.145579	0.004828	168.0353	0.004574	0.005890
II	0.001301	6.707887	0.013449	524.2541	0.012795	0.015094
III	0.001691	2.130361	0.017087	762.1275	0.016626	0.009551
IV	0.000121	0.554343	0.001248	42.03197	0.001196	0.001185
Decrease in spread (in %) from						
I–IV	73.9784	82.3771	74.1507	74.9867	73.8522	79.8811
II–IV	90.6994	91.7359	90.7204	91.9825	90.6525	92.1492
III–IV	92.8444	73.9789	92.6962	94.4849	92.8064	87.5929

I, Triangular; II, normal; III, Gamma; IV, Cauchy

Table 6 Defuzzified value of the reliability parameters of decomposition Unit

Spreads (%)	Technique	Failure rate ($\times 10^{-3}$)	Repair time	MTBF	ENOF	Reliability	Availability
± 0	Crisp	1.714055	3.770787	587.1824	0.017068	0.983005	0.994023
± 15		Defuzzified values at different spreads					
	I	1.714209	3.938953	594.3054	0.017077	0.983005	0.993699
	II	1.714465	3.773389	687.7066	0.017103	0.983019	0.993511
	III	1.714157	4.459679	640.8231	0.017112	0.983015	0.992216
	IV	1.714056	3.777679	587.7229	0.017069	0.983005	0.994004
± 25	I	1.714483	4.254475	606.9019	0.017093	0.983006	0.993067
	II	1.715194	3.783575	694.2676	0.017164	0.983045	0.992593
	III	1.714225	5.057561	687.3688	0.017153	0.983022	0.990554
	IV	1.714056	3.777804	587.7327	0.017154	0.983005	0.994005
± 50	I	1.715767	6.090886	678.0131	0.017202	0.983006	0.988921
	II	1.718614	3.780774	727.0733	0.017465	0.983164	0.988065
	III	1.714396	7.525587	879.4595	0.017333	0.983039	0.983291
	IV	1.714057	3.778223	587.7656	0.017069	0.983006	0.994004

I, triangular; II, normal; III, gamma; IV, Cauchy

and repair time, respectively, from gamma results and hence prediction range of all the indices decreases. This type of analysis will benefit the system analyst to depict the effect of individual component behavior and their corresponding change in spreads in lesser time.

3. In order to maintain the supremacy of the approach, the reliability parameters, given in Table 2, are computed by the proposed approach at different levels of the

uncertainties such as ± 25 and ± 50%. The defuzzified values at different spreads say ± 15, ± 25 and ± 50% are summarized in Table 6 with different types of fuzzy numbers. From this table, it has been observed that values obtained by taking Cauchy distribution are closer to the crisp values as compared to the other existing distributions. Also, it is evident that defuzzified values changes with the change of spread. For

Table 7 Change in MTBF for Various Combinations of Reliability Indices

S. no.	Reliability, failure rate ($\times 10^{-3}$), availability	Mean time between failures			
		I	II	III	IV
1	[0.962, 1.475, 0.975]	Min: 1350.3006	Min: 1025.4848	Min: 1101.6768	Min: 1489.3671
		Max: 1800.1226	Max: 3071.5394	Max: 2538.3120	Max: 1603.3020
2	[0.962, 1.711, 0.975]	Min: 1164.4911	Min: 884.4026	Min: 949.9660	Min: 1284.6200
		Max: 1553.1527	Max: 2649.8220	Max: 2190.9777	Max: 1383.0078
3	[0.962, 2.032, 0.975]	Min: 981.0363	Min: 745.1083	Min: 800.1777	Min: 1082.4675
		Max: 1309.3124	Max: 2233.4488	Max: 1848.0449	Max: 1165.5052
4	[0.983, 1.475, 0.990]	Min: 597.4890	Min: 453.7531	Min: 487.5115	Min: 658.9626
		Max: 796.3011	Max: 1358.8212	Max: 1122.5644	Max: 709.3367
5	[0.983, 1.711, 0.990]	Min: 515.2523	Min: 391.3121	Min: 420.3664	Min: 568.3443
		Max: 686.9956	Max: 1172.1749	Max: 968.8390	Max: 611.8375
6	[0.983, 2.032, 0.990]	Min: 434.0577	Min: 329.6623	Min: 354.0722	Min: 478.8744
		Max: 579.0753	Max: 987.8938	Max: 817.0617	Max: 515.5739
7	[0.992, 1.475, 0.999]	Min: 279.4266	Min: 212.1731	Min: 228.1149	Min: 307.9635
		Max: 371.6163	Max: 634.4676	Max: 522.9008	Max: 331.3818
8	[0.992, 1.711, 0.999]	Min: 240.9025	Min: 182.9224	Min: 196.6606	Min: 265.5131
		Max: 320.4118	Max: 547.0326	Max: 450.8876	Max: 285.7080
9	[0.992, 2.032, 0.999]	Min: 202.8667	Min: 154.0424	Min: 165.6048	Min: 223.6006
		Max: 269.8561	Max: 460.7055	Max: 379.7871	Max: 240.6130

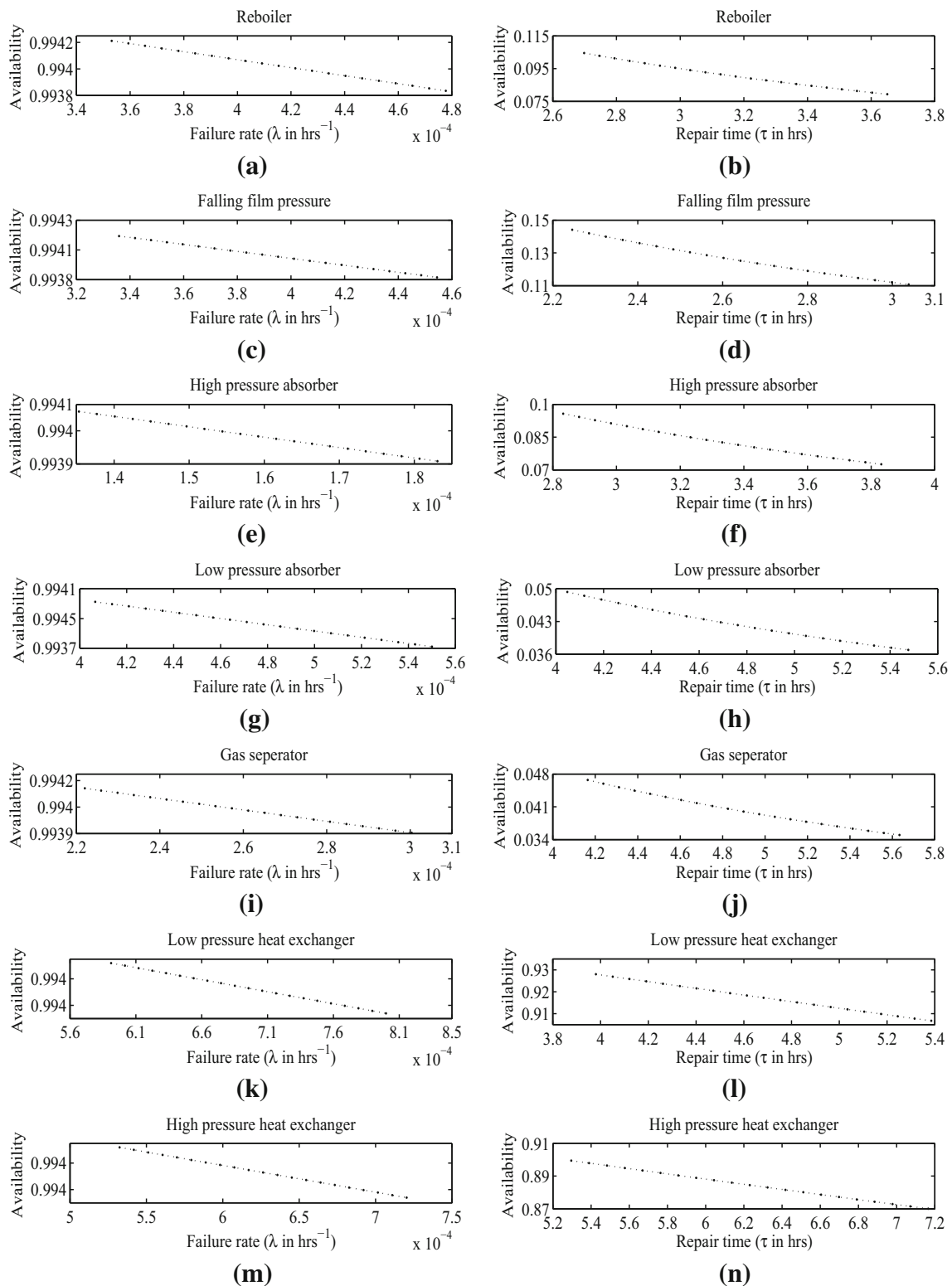I, Triangular; II, normal; III, gamma; IV, Cauchy

Fig. 6 Effect of the individual component failure rate and repair time on system availability

instance, the repair time of the system increases by 8.011, 0.269, 13.406 and 0.003% for triangular, normal, gamma and Cauchy distribution, respectively,

when spread changes from ± 15 to $\pm 25\%$, and it further increases by 43.164, 0.074, 48.798 and 0.011% when changes from ± 25 to $\pm 50\%$.

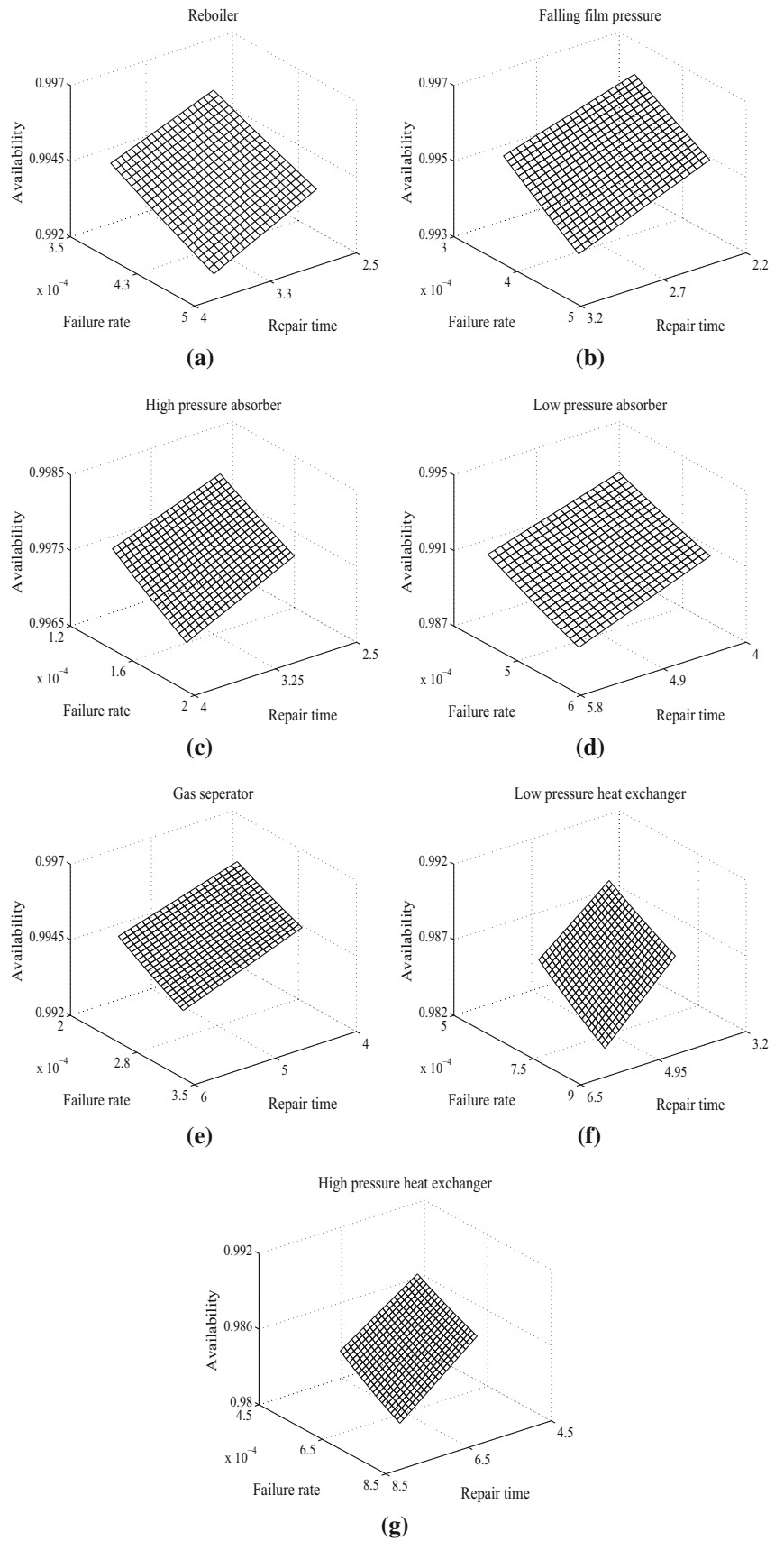**Fig. 7** Effect of the component parameters on system availability

Table 8 Ranges of the component failure rate and repair time on system availability

Component	Range of failure rate (10^{-3})	Range of repair time	Range of availability
Reboiler	0.3531–0.4777	2.6984–3.6508	Min: 0.99231
			Max: 0.99537
Falling film pressure	0.3359–0.4545	2.2458–3.0384	Min: 0.99373
			Max: 0.99628
High pressure absorber	0.1353–0.1831	2.8325–3.8321	Min: 0.99692
			Max: 0.99814
Low pressure absorber	0.4066–0.5500	4.0476–5.4762	Min: 0.98812
			Max: 0.99251
Gas seperator	0.2220–0.3004	4.1642–5.6338	Min: 0.99337
			Max: 0.99581
Low pressure heat exchanger	0.5913–0.7999	3.9806–5.3856	Min: 0.98206
			Max: 0.98927
High pressure heat exchanger	0.5324–0.7204	5.296–7.1656	Min: 0.98083
			Max: 0.98813

5.2 Sensitivity analysis

In this section, an analysis has been conducted on to the system MTBF by varying the other reliability parameters. For it, initially, ranges of the repair time and ENOF are fixed and have taken from Fig. 5b, d at cut level $\alpha = 0$ respectively. Then, for the different combinations of the other parameters have been taken and their impact on MTBF have been summarized Table 7. These results will be highly beneficial for the plant personnel to depict the effect of each component and hence change their strategy/target goals accordingly. For instance, in the first three combinations, we have taken reliability as 0.962 and availability as 0.975, while failure rate varies from 1.475×10^{-3} to 1.711×10^{-3} and further to 2.032×10^{-3} then ranges of MTBF is reduced almost by 74.671, 94.431 and 92.069% from triangular, gamma and normal distribution when observed from Cauchy distribution results. Similarly, for other pairs, we see their corresponding reductions.

5.3 Performance analysis

In order to decide the future strategy or to save the money or time, it is necessary to investigate the most critical component of the system on which more attentions should be given. For it, the variation of the failure rate and repair time of each component on to the system availability have been investigated and their respective variations are summarized in Fig. 6. For instance, in the case of reboiler component, if we increase their failure rate from 0.35309×10^{-3} to 0.47771×10^{-3} h^{-1} then their availability varies from 0.993834 to 0.994212. On the other hand, if we change their repair time from 2.69841 to 3.65079 h, then

their component availability varies from 0.079396 to 0.104476. The variations of this component are plotted in Fig. 6a, b respectively. Similarly, in the case of gas separator component, if we change their failure rate from 0.2220×10^{-3} to 0.30038×10^{-3} h^{-1} then availability changes from 0.99385 to 0.994191, and when we change their repair time from 4.16415 to 5.63385 h then the availability varies from 0.03495 to 0.04671. The variation of the availability corresponding to this component is shown in Fig. 6i, j.

However, in order to analyze their effect simultaneously, an analysis has been conducted and their impact on the system availability is depicted graphically in Fig. 7. It may be observed from Fig. 7a that the variation of the failure rate (0.3531×10^{-3} to 0.4777×10^{-3}) and repair time (2.6984–3.6508) of the "reboiler" component have less impact on system availability (up to 0.31%). On the other hand, the component "high-pressure absorber" have the large impact on the system availability up to 0.122%. The complete variations of the ranges are summarized in Table 8. Based on this analysis, we observe that the preferential order of the given components in accordance to high pressure absorber (B_1), Falling film pressure (A_2), Gas separator (C), Reboiler (A_1), low pressure absorber (B_1), low pressure heat exchanger (E_1) and high pressure heat exchanger (E_2) (Table 4).

6 Conclusion

In the present manuscript, an investigation has been done to analyze the system performance of an industrial system by utilizing the vague and uncertain data. For it, the collective information, from the various resources, has been fuzzified into the different form of the fuzzy numbers,

linear and nonlinear, instead of only linear triangular numbers. The proposed method with four different membership functions has been applied to analyze the decomposition unit of a fertilizer plant. From the computed results, it is concluded that Cauchy fuzzy numbers are the best fit for the system data than the other existing models as it reduced the level of the uncertainties in the form of the support spread at any level of confidence. Further, the effects of the various reliability parameters on the system MTBF have been investigated. Using availability index, the critical component of the system has been ranked for improving the performance of the system by proper maintenance actions. Based on results, experts may change their target goals and suggest some suitable actions for improving the quality of the industrial systems. In the future, the presented approach has been extended to solve some other types of problems such as mathematical programming, optimization theory, and uncertain data.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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