

# Hybrid algorithm DE–TLBO for optimal $H_\infty$ and PID control for multi-machine power system

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Received: 5 January 2016 / Revised: 23 July 2016 / Published online: 19 December 2016

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**Abstract** This paper propose, a robust excitation controller designed by a coordination of the optimal  $H_\infty$  tracking control and the proportional integral derivative (PID) controller optimized by the hybrid differential evolution and teaching–learning based optimization algorithm (DE–TLBO). These two controllers are used in order to guarantee the transient stability during a change in the operating conditions and the uncertainties in parameters. We have applied a method based on the modified tracking error by using the optimized exponential function, to avoid the compromise between the high gain in the control input and the  $H_\infty$  tracking performance with the variation in the system parameter. A new hybrid algorithm (DE–TLBO) is employed in this study to adjust optimally the parameters of the (PID–PSS) controller and the exponential form of the tracking error modified. The purpose of the suggested approach is to ensure a good tracking accuracy and to enhance the level of the oscillations damping in the multi-machine power system with an optimal choice of the parameters of all proposed controllers. The results of simulation demonstrate the efficient, and the robustness of the proposed approach ( $H_\infty$  and DE–TLBO–PID–PSS) under the different operation conditions.

**Keywords** Differential evolution · Teaching learning based optimization ·  $H_\infty$  tracking control · Multi-machine power system · Proportional integral derivative

## 1 Introduction

The stability of the electrical power systems in the modern power industry has become an important and urgent problem due to the increasing complexity electric power grids as well as the growing energy demand with the configurations and parameters which vary over time (Wan et al. 2014; Alizadeh et al. 2013). It has led to dynamic problems of low frequency oscillations in the system and the instability that needs to be detected and damped out quickly and adequately (Tripathy and Mishra 2015). The power system stabilizer (PSS) is widely used as a complementary controller in the system of excitation in order to improve the oscillation damping (Ali and Abd-Elazim 2012). However, when the operating point and the configurations of the power systems frequently change, the CPSS cannot ensure the best performance (Khodabakhshian and Hemmati 2012). Therefore, it is required to use an approach which can take into account parameter uncertainties and the change of the operating condition in the power system. Various intelligent methods have been proposed, to deal with the problems of PSS, such as the artificial neural network based on PSS (Segal et al. 2004), genetic algorithm (Hassan et al. 2014) fuzzy logic control (Touil and Attous 2015), and bio-inspired algorithms (Peres et al. 2015). Proportional–integral–derivative PID and Proportional Integral PI, have also been applied as a substitute of PSS (Jaleel and Thanvy 2013). The PID controller is usually applied in industry control due to its

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simple structure (Li and Liu 2012). It is difficult to properly adjust the gains of PID controller. Different algorithms of optimization have been used in the literature such as hybrid differential evolution (DE) and particle swarm optimization (PSO) (Sahu et al. 2014). In Chaib et al. (2015) the authors are used the bat algorithm to optimize the fractional order PID–PSS. The genetic algorithm based on PID–PSS is used in Duman and Öztürk (2010). The DE algorithm is applied in Dib and Boumhidi (2015). In this study, DE and TLBO algorithms are hybridized (DE–TLBO) in order to improve the mechanism of global search and increase the speed of the convergence of the all algorithms.

In this paper, our contribution consists in combining the design of the optimal  $H_\infty$  control and the PID–PSS controller optimized by the new hybrid DE and teaching–learning based optimization algorithm (DE–TLBO), in order to ensure a robust controller, to take into account a large parametric uncertainty and to guarantee the system stability, which leads to a flexible controller device.

Generally, TLBO and DE algorithms have a higher capacity at the beginning of the run for global searching at the beginning and a local search near the end of the run (Ghasemi et al. 2014). To balance the global and local search capacity, a modified learning strategy is integrated into the teacher phase. In this technique, the hybrid algorithm (DE–TLBO) uses the learning strategy based on neighborly search in the teacher phase in the TLBO so as to generate a new mutation vector, while integrating the differential learning to create another new mutation vector. The crossover operator is used to create new solutions in order that the population diversity will be increased (Zou et al. 2015).

The main objective of the new hybrid DE and teaching learning based optimization (DE–TLBO) is to adjust the parameter  $\beta$  of the general exponential form of the tracking error and the gains of PID–PSS controller for the optimal tuning of all controller parameters in order to ensure a robust performance and efficient results of tracking.

The approach of  $H_\infty$  control has been extensively applied to treat efficiently the robust stabilization of the nonlinear system owing to its capacity of disturbance attenuation and its effective robustness (Chen et al. 1996; Chang 2000). The optimal  $H_\infty$  tracking control is applied in this study for the multi-machine power system in order to attenuate the effects due to the approximate errors, disturbances and unmodeled dynamics, it is characterized by a simple structure designed to regulate the amplitude of the output of the angular speed deviation and the angle rotor so as to attenuate the amplitude of the oscillations and to track the desired operating point. In the presence of the uncertainties wide enough in the system parameters, the

high gain in the input of  $H_\infty$  control is required to achieve the intended attenuation level and guarantee the efficient performance of tracking (Miao et al. 2008; Pan et al. 2011). To avoid the compromise between a high control signal and the high attenuation level, different approaches have been used in the literature (Yilmaz and Hurmuzlu 2000; Chang and Hurmuzlu 1998). In this study, we have used a technique based on the tracking error modified by integrating the exponential function optimized by the new hybrid algorithm DE–TLBO to eliminate significantly the reaching phase.

The principal purpose of designing the proposed approach ( $H_\infty$  and DE–TLBO–PID) is to eliminate sufficiently the reaching phase and to enhance the level of oscillation damping which significantly affects the tracking errors and the robustness of the multi-machine power system under the change in operating point. The results of the simulations test demonstrates the validity of the proposed method is efficiently enhanced compared with the optimal controller proposed in Dib and Boumhidi (2015) and other controllers, with the presence of the parametric variation and the change in operating point, the proposed method provide an effective damping in the oscillation of the power system and good tracking to the desired values with faster convergence.

This paper is categorized in four major parts, described as follows: the mathematical model of the nonlinear multi-machine power system is explained in Sect. 2. The new hybrid DE algorithm with teaching learning based optimization DE–TLBO is described in Sect. 3. The design of the proposed approach is detailed in Sect. 4. The simulation results which conduct to the performance analysis and comparison of the applied approaches to the multi-machine power system are presented in Sect. 5. Finally, conclusion is explained in Sect. 6.

## 2 Mathematical model of multi-machine power system

The dynamics of (n) generators interconnected by a transmission network is presented by the third order model (Colbia-Vega et al. 2008):

The equation of the mechanical part:

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_s \\ \dot{\omega}_i = \frac{\omega_s}{2H_i} (P_{m_i} - D_i(\omega_i - \omega_s) - P_{e_i}) \end{cases} \quad (1)$$

The equation of the generator electrical part:

$$\dot{E}'_{qi} = \frac{1}{T'_{di}} (E_{fi} - E'_{qi} - (X_{di} - X'_{di})I_{di}) \quad (2)$$

The electrical power is written as:

$$P_{ei} = E'_{qi} I_{qi} \tag{3}$$

$I_{qi}$  and  $I_{di}$  are the currents in direct and quadrature reference for each generator which are expressed by:

$$\begin{cases} I_{qi} = G_{ii} E'_{qi} + \sum_{j=1, j \neq i}^n E'_{qi} \{ G_{ij} \cos(\delta_j - \delta_i) - B_{ij} \sin(\delta_j - \delta_i) \} \\ I_{di} = -B_{ii} E'_{qi} - \sum_{j=1, j \neq i}^n E'_{qi} \{ G_{ij} \sin(\delta_j - \delta_i) + B_{ij} \cos(\delta_j - \delta_i) \} \end{cases} \tag{4}$$

$P_{mi}$  is the mechanical input power assumed to be constant, we consider  $E_{fi}(t)$  as the input of the system.

The power system used in this study consists of three generators,  $i$ th generator is considered as a subsystem of the multi-machine power system. The subsystem is presented by the following states equations:

$$\begin{cases} \dot{x}_{i1} = \omega_i - \omega_s \\ \dot{x}_{i2} = \frac{\omega_s}{2H_i} (P_{mi} - D_i(\omega_i - \omega_s) - x_{i3} I_{qi}) \\ \dot{x}_{i3} = \frac{1}{T'_{di}} (E_{fi} - x_{i3} - (X_{di} - X'_{di}) I_{di}) \end{cases} \tag{5}$$

With  $x_i = [x_{i1}, x_{i2}, x_{i3}]^T = [\delta_i, \omega_i, E'_{qi}]^T$  denotes the state vector for  $i$ th subsystem.

The purpose of this study is controlling the rotor angle to track the desired value for each machine, for this reason, we choose the output is the rotor angle to calculate the relative degree, let define:  $z_{i1} = \delta_i$ ,  $z_{i2} = \omega_i$ ,  $z_{i3} = \dot{\omega}_i$  then the vector of state variables of the power system can be chosen to be:

$$z = [\delta_1, \omega_1, \dot{\omega}_1, \dots, \delta_n, \omega_n, \dot{\omega}_n]$$

This new state vector allows one to transform the system model described by (5) into the form given by:

$$\begin{cases} \dot{z}_{i1} = z_{i2} \\ \dot{z}_{i2} = z_{i3} \\ \dot{z}_{i3} = \frac{-1}{2H_i} \left( \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} [-E'_{qj} + (X'_{dj} - X_{dj}) I_{dj}] + D_i \dot{\omega}_i + \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial \delta_j} \omega_j + \sum_{j=1, j \neq i}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} E_{jdi} + \frac{\partial P_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} E_{fdi} \right) \end{cases} \tag{6}$$

We consider  $u = E_{fd}$ , the canonical form of the power system can be written as:

$$\begin{cases} \dot{z}_{i1} = z_{i2} \\ \dot{z}_{i2} = z_{i3} \\ \dot{z}_{i3} = f_i(x) + g_i(x)u_i \end{cases} \tag{7}$$

With

$$f_i(x) = \frac{-1}{2H_i} \left\{ \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} [-E'_{qj} + (X'_{dj} - X_{dj}) I_{dj}] + \sum_{j=1}^{NG} \frac{\partial P_{ei}}{\partial \delta_j} \omega_j + D_i \dot{\omega}_i + \sum_{j=1, j \neq i}^{NG} \frac{\partial P_{ei}}{\partial E'_{qj}} \frac{1}{T'_{doj}} E_{jdi} \right\} \tag{8}$$

$$g_i(x) = -\frac{1}{2H_i} \frac{\partial P_{ei}}{\partial E'_{qi}} \frac{1}{T'_{doi}} \tag{9}$$

### 3 Proposed designing of hybrid algorithm DE-TLBO

#### 3.1 Differential evolution algorithm

The DE algorithm mainly is characterized by three advantages; the fast convergence, the use of a few control parameters which makes the DE algorithm simple and easy to use (Cuevas et al. 2013). The optimization process is composed by three main steps: the mutation, the crossover and the selection.

- *Initialization*

The initial parameter values (at  $G = 0$ ) should better cover as much as possible all the search space by randomizing the individuals in the interval limited by the lower and the upper bounds:

$$x_{j,i,0} = x_{j,\min} + rand_j(0, 1) \times (x_{j,\max} - x_{j,\min}) \tag{10}$$

- *Mutation*

A donor vector  $X_{i,G}$  is created by combining the three target vectors ( $X_{r1,G}, X_{r2,G}, X_{r3,G}$ ) through a mutation strategy it can be written as:

$$V_{i,G} = X_{r1^i,G} + F \cdot (X_{r2^i,G} - X_{r3^i,G}) \tag{11}$$

$F$  is a constant from  $[0, 2]$ , the indices  $i, r_1, r_2$  and  $r_3$  are distinct.

- *Crossover*

The trial vector  $U_{i,G+1}$  is obtained from the target vector  $X_{i,G}$  and the donor vector  $V_{i,G}$  as follow:

$$U_{j,i,G} = \begin{cases} V_{j,i,G} & \text{if } rand_j \leq CR \text{ or } j = j_{rand} \\ X_{j,i,G} & \text{if } rand_j > CR \text{ or } j \neq j_{rand} \end{cases} \tag{12}$$

$CR$  is the crossover probability,  $rand_j \in [0, 1]$  is the  $j$ th random number index.

- *Selection*

The trial vector  $U_{i,G+1}$  is compared with the target vector  $X_{i,G}$  the vector which have the best fitness value is chosen to the next generation. The selection operation may be represented by:

$$X_{i,G+1} = \begin{cases} U_{i,G} & \text{if } J(U_{i,G}) < J(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \quad (13)$$

where  $i \in [1, N_p]$  and  $J(X)$  is the function to be minimized.

### 3.2 Teaching learning based optimization algorithm

TLBO algorithm is composed by two phases, the teacher and learner phases.

- *Teacher phase*

In this first phase, the learners (students) aim to improve their knowledge by the teacher. The learner who has the minimum value of the objective function is considered as the teacher which tries to increase the existing mean result (Mean) of the group of learners (class) (Kanwar et al. 2015).

$$U_i = X_i + r \times (X_{Teacher} - TF \times Mean) \quad (14)$$

TF is the teaching factor is randomly determined by the equation:

$$TF = round[1 + rand(0, 1)] \quad (15)$$

- *Learner phase*

During the second stage, a learner improves their knowledge by a random interaction with the other learners. The learner process can be expressed as follow:

Two learners are randomly selected  $x_i$  and  $x_j$  such that  $i \neq j$

$$newX_i = \begin{cases} X_i + r \times (X_i - X_j) & \text{if } f(X_i) < f(X_j) \\ X_i + r \times (X_j - X_i) & \text{if } f(X_j) < f(X_i) \end{cases} \quad (16)$$

The flowchart showing the operation of the DE–TLBO algorithm is illustrated in Fig. 1.

In the proposed DE–TLBO algorithm, during the teacher phase which is hybridized by the DE, two mutant vectors ( $U_i, V_i$ ) are associated to each learner  $X_i$ . The first mutant vector  $U_i$  is generated by the Eq. (14), and the second mutant vector  $V_i$  is generated by the mutation operator in the DE algorithm given by the Eq. (11).

The crossover operator is applied to the mutant vectors ( $U_i, V_i$ ) to improve the potential diversity of population, this is the adapted formula in the teacher phase for the learner  $X_i$ , that can be described by Eq. (12).

The Selection operator is applied at the end of the teaching phase by comparing the parent  $X_i$  and the trial

vector  $newX_i$ , the vector which have the best fitness value is chosen for the next phase, this operation is described by the Eq. (13).

Finally, the original learner phase in the TLBO algorithm is still applied in the hybrid DE–TLBO algorithm, the learning process is described in Eq. (16).

## 4 Proposed control design

The optimal  $H_\infty$  control is characterized by high ability for the disturbance attenuation. Therefore, the combination between the  $H_\infty$  control theory and the nominal control can reduce the effects of the parameter uncertainties, external disturbances and the errors of the approximation (Lin 2009).

We consider the dynamical equations of the multi-machine power system which are represented by the canonical form described by the Eq. (7). We formulate the output tracking error of the power system in order to avoid the high control input gain; one introduces the following modified output tracking error as follows (Pan et al. 2012):

$$E(t) = e(t) - \eta(t) \quad (17)$$

We define the tracking error as followings:

$$e = \delta - \delta_r = z_1 - \delta_r \quad (18)$$

We can define the error vector by:

$$\underline{e} = [e_1, e_2, e_3] = [e, \dot{e}, \ddot{e}].$$

Where  $\eta(t)$  is designed in order to satisfy the following conditions (Yilmaz and Hurmuzlu 2000):

1. To make the modified error  $E$  small enough in the beginning of the movement  $t = 0$ .
2. Should rapidly disappear as the movement evolves at  $t > 0$ .

In this study  $\eta_i(t)$  is described by the exponential form, can written as:

$$\eta_i(t) = \gamma_i(t) \exp(\psi_i(t)) \quad (19)$$

$$\psi_i(t) = -\beta_i t \quad (20)$$

$$\gamma_i(t) = (q_{0i} + q_{1i}t + \dots + q_{n-1}t^{n-1}) \quad (21)$$

$$\eta_i(t) = (q_{0i} + q_{1i}t + \dots + q_{n-1}t^{n-1}) \exp(-\beta_i t) \quad (22)$$

For  $j = 0, 1, \dots, n - 1$ , where  $\beta_i$  a positive constant,  $q_i$  is chosen to satisfy condition (1) and  $\psi_i(t)$  is selected to satisfy condition (2).

In this study, a new method DE–TLBO algorithm is applied to adjust optimally the value of the parameter  $\beta_i$ .

Expanding (17) by the Taylor's series leads to

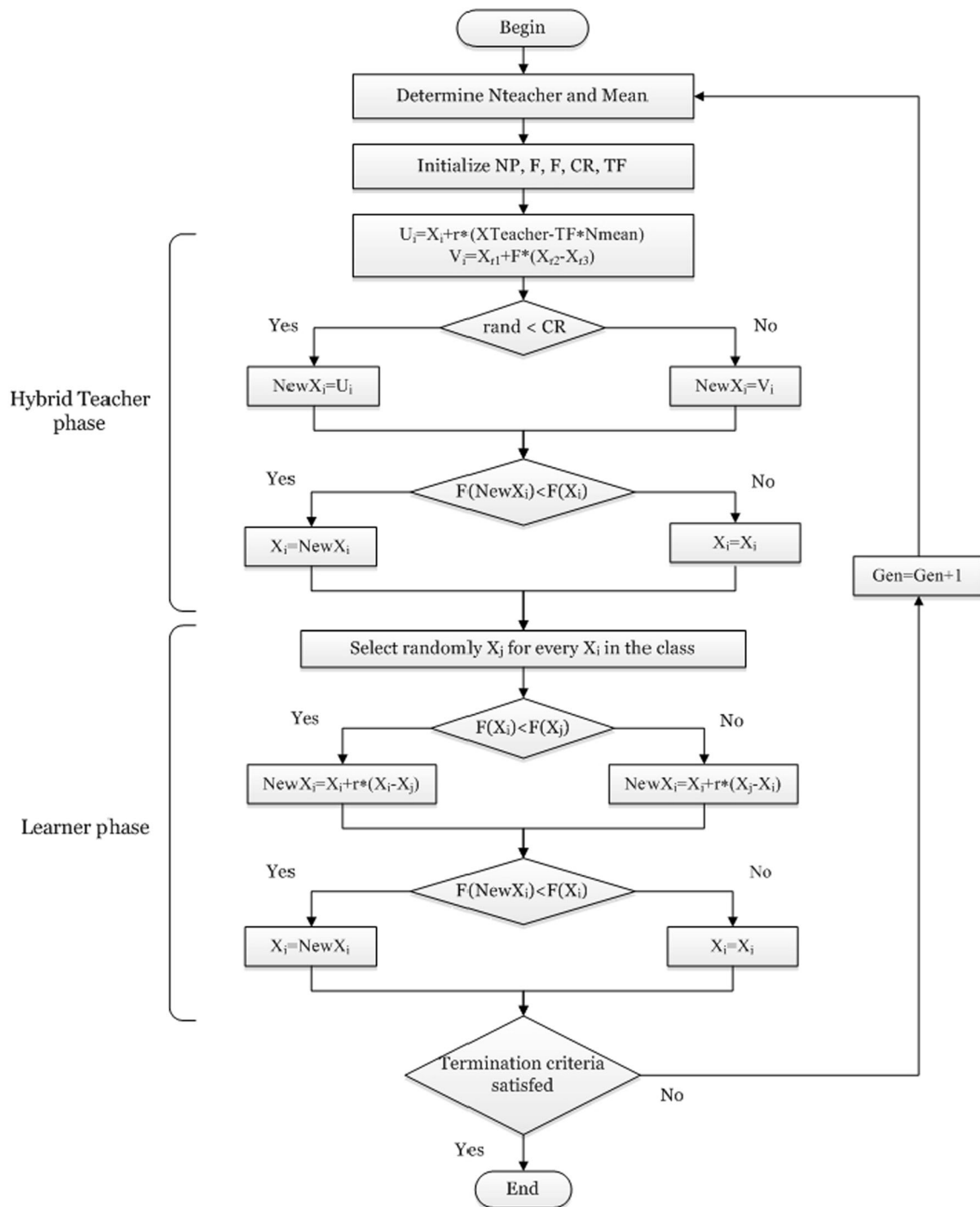


Fig. 1 Flowchart of the hybrid DE-TLBO algorithm

$$E_i(t) = \sum_{j=1}^{n-1} \frac{1}{j!} \left( (e_i^{(j)}(0) - \eta_i^{(j)}(0)) t^j \right) + o(t^{n-1}) \quad (23)$$

where  $o(t^{n-1})$  is an infinitesimal of higher order of  $t^{n-1}$ .

$$\eta_i^{(j)}(0) = e_i^{(j)}(0) \quad (24)$$

Then (20) becomes  $o(t^{n-1})$ , i.e., Condition 1 is satisfied. By solving the equation set in (22) we can obtain the values of  $q_i$ .

Then our design objective is to impose  $H_\infty$  control so that the following asymptotically stable tracking:

$$\ddot{E}_i(t) + k_{2i}\dot{E}_i(t) + k_{1i}E_i(t) = 0 \tag{25}$$

$$\eta_i(t) = (q_{0i} + q_{1i}t + q_{2i}t^2) \exp(-\beta_i t) \tag{26}$$

With  $k_i = [k_{1i}, k_{2i}, 1]^T$  are the coefficients of the Hurwitz polynomial:

$$h_i(\lambda) = \lambda^2 + k_{2i}\lambda + k_{1i} \tag{27}$$

$$k_{1i}\dot{E}_i(t) + k_{2i}\ddot{E}_i(t) - \ddot{\eta}_i(t) + f_i(x) + g_i(x)u_i = 0 \tag{28}$$

If  $f_i(x)$  and  $g_i(x)$  are known, we can construct the nominal control:

$$u_{eq_i} = \frac{-1}{g_i(x)} (k_{1i}\dot{E}_i(t) + k_{2i}\ddot{E}_i(t) - \ddot{\eta}_i(t) + f_i(x)) \tag{29}$$

The dynamic equation of the output tracking error of the nonlinear system (28) is described by:

$$\dot{E}_i = A_i E_i + B_i [g_i(x) \cdot u_{h_i}] \tag{30}$$

where

$$A_i = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_{1i} & -K_{2i} & -K_{3i} \end{pmatrix} \text{ and}$$

$$B_i = [0 \quad 0 \quad 1]$$

where  $u_{h_i}$  is a H $_{\infty}$  compensator, defined as:

$$u_{h_i} = -\frac{1}{g_i(x)r_i} E_i^T P_i B_i \tag{31}$$

where  $r$  is a positive scalar value and  $P = P^T > 0$  is the P solution of the Riccati equation (Chen et al. 1996).

$$PA + A^T P + Q - \frac{2}{r} PBB^T P + \frac{1}{\rho^2} PBB^T P = 0 \tag{32}$$

*Remark* The solvability of H $_{\infty}$  tracking performance is on the existence of positive semi definite and symmetric solution P of which can be rewritten as (Chen et al. 1996):

$$PA + A^T P + Q - PB \left( \frac{2}{r} - \frac{1}{\rho^2} \right) B^T P = 0 \tag{33}$$

where  $Q > 0$ ,  $\rho$  is prescribed the level of attenuation and  $r$  is positive constant.

The above Riccati equation has a solution semi-definite positive  $P = P^T > 0$  if and only if:

$$\frac{2}{r} - \frac{1}{\rho^2} \geq 0 \quad \text{or} \quad 2\rho^2 \geq r \tag{34}$$

The design of the control strategy applied in this study consists of the combining the three terms the nominal control  $u_{eq_i}$ , the robust term designed by the optimal H $_{\infty}$  tracking control without reaching phase  $u_{h_i}$  and the PID-

PSS optimized by the new hybrid algorithm DE–TLBO  $u_{DE-TLBO-PID}$  is used for damping the oscillations in multi-machine power systems

$$u_i = u_{eq_i} + u_{h_i} + u_{(DE-TLBO-PID)_i} \tag{35}$$

$$u_i = \frac{-1}{g_i(x)} \left( \begin{matrix} k_{1i}\dot{E}_i(t) + k_{2i}\ddot{E}_i(t) \\ -\ddot{\eta}_i(t) + f_i(x) + \frac{1}{r_i} E_i^T P_i B_i \end{matrix} \right) + \tilde{k}_{p_i} \Delta\omega_i + \tilde{k}_{i_i} \int_0^t \Delta\omega_i dt + \tilde{k}_{d_i} \frac{d\Delta\omega_i}{dt} \tag{36}$$

where  $\tilde{k}_p, \tilde{k}_I, \tilde{k}_d$  are the optimal value of proportional gain, integral gain and derivative gain, respectively, adjusted optimally by the hybrid algorithm DE–TLBO.

The combination between the optimal H $_{\infty}$  control by modifying the output tracking error and the PID–PSS controller optimized by the hybrid algorithm (DE–TLBO), ensures the optimal tracking by eliminating completely the reaching phase with a minimal effort of control, and improve the oscillation damping under variation in operating point.

### 5 Simulation of multi-machine power system

We validate the robustness and the performance of the proposed approach by the simulations in MATLAB for the three machine nine-bus power system the data of the system under study are shown in Table 1 (Colbia-Vega et al. 2008).

The conductance nodal matrix G and the susceptance nodal matrix B which represent the topology of the network are defined as:

$$G = [G_{ij}] = \begin{bmatrix} 0.8453 & 0.2870 & 0.2095 \\ 0.2870 & 0.4199 & 0.2132 \\ 0.2095 & 0.2132 & 0.2770 \end{bmatrix}$$

$$B = [B_{ij}] = \begin{bmatrix} -2.9882 & 1.5130 & 1.2256 \\ 1.5130 & -2.7238 & 1.0879 \\ 1.2256 & 1.0879 & -2.3681 \end{bmatrix}$$

**Table 1** The values of the nominal parameters

Parameters	Gen 1	Gen 2	Gen 3
H	23.64	6.4	3.01
X <sub>d</sub>	0.146	0.8958	1.3125
X' <sub>d</sub>	0.0608	0.7798	0.1813
D	0.3100	0.5350	0.6000
P <sub>m</sub>	0.7157	1.6295	0.8502
T' <sub>do</sub>	8.96	6.0	5.89



• **Objective function**

The primary goal is to minimize the objective function to improve the stability of the power system. In this paper, an integral time absolute error (ITAE) of the speed deviation  $\Delta\omega_i$  is chosen as the objective function.

$$J = \int_0^T t(|\Delta\omega_1| + |\Delta\omega_2| + |\Delta\omega_3|)dt$$

Minimize J

Subject to

$$\begin{cases} K_{p_i}^{\min} \leq K_{p_i} \leq K_{p_i}^{\max} \\ K_{i_i}^{\min} \leq K_{i_i} \leq K_{i_i}^{\max} \\ K_{d_i}^{\min} \leq K_{d_i} \leq K_{d_i}^{\max} \\ \beta_i^{\min} \leq \beta_i \leq \beta \end{cases} \quad \text{for } i = 1, 2, 3 \tag{37}$$

The typical ranges of the optimized parameters of the PID controller are [0 120] for  $K_p$ , [0 10] for  $K_i$  and  $K_d$ , and The parameter of the exponential function  $\beta_i$  is [0 10].

The values of the parameters optimized by the DE algorithm and the hybrid algorithm DE–TLBO are given in the Table 2. The parameters of the conventional PID controller are given in the Table 3.

The objective of this section is to compare the performance of the proposed control (optimal  $H_\infty$  tracking control without reaching phase and DE–TLBO–PID) with ( $H_\infty$  and DE–PID–PSS) proposed by Dib and Boumhidi (2015), with ( $H_\infty$  and PID–PSS), the (PID–PSS) and the PSS (Naresh et al. 2013).

*Case 1: Control response with nominal loading* The operating point for the first case is given as:

$$X_{ir} = (x_{i1r}, x_{i2r}, x_{i3r}) = [\delta_i \quad \Delta\omega_i \quad E'_{qi}]$$

For  $i = 1, 2, 3$  of the three-machine system are considered as:

$$\delta_{1r} = 0.0396, \quad 3\Delta\omega_{1r} = 0, \quad E'_{q1r} = 1.0566$$

$$\delta_{2r} = 0.3444, \quad \Delta\omega_{1r} = 0, \quad E'_{q2r} = 1.0502$$

$$\delta_{3r} = 0.2300, \quad \Delta\omega_{1r} = 0, \quad E'_{q3r} = 1.017$$

To validate the robustness and the performance of the proposed method, we use two performance indices: the

**Table 3** The parameters of the conventional PID

Parameters	$K_p$	$K_i$	$K_d$
PID-G1	60	1	5
PID-G2	30	0.7	2
PID-G3	27	3.1	1.3

integral of time absolute value of error (ITAE) and the integral of time squared error (ITSE):

$$ITAE = \int_0^T t(|\Delta\omega_1| + |\Delta\omega_2| + |\Delta\omega_3|)dt \tag{38}$$

$$ITSE = \int_0^T (\Delta\omega_1^2(t) + \Delta\omega_2^2(t) + \Delta\omega_3^2(t)) \cdot t \cdot dt \tag{39}$$

The numerical results of these indices for all controllers are given in Table 4.

It is clear from Table 4 that minimum ITAE and ITSE values are obtained with the proposed method and therefore the performance of  $H_\infty$  and DE–TLBO–PID controllers are superior to the other controllers.

It is observed from Fig. 2 that the convergence rate of the hybrid DE–TLBO algorithm is considerably faster and better than the other algorithms.

We define the tracking error for the three generators by the following equations:

$$\begin{cases} e_1 = \delta_1 - \delta_{1r} = z_{11} - \delta_{1r} \\ e_2 = \delta_2 - \delta_{2r} = z_{21} - \delta_{2r} \\ e_3 = \delta_3 - \delta_{3r} = z_{31} - \delta_{3r} \end{cases}$$

The tracking errors of the rotor angle for each generator are illustrated in the Figs. 3, 4 and 5; these results show that the proposed method significantly reduces the deviation of the rotor angle comparing with the other control devices. We can deduce that the proposed controller device has a better ability to maintain the system to follow the desired values as well as to reach the point of operation in a reduced time.

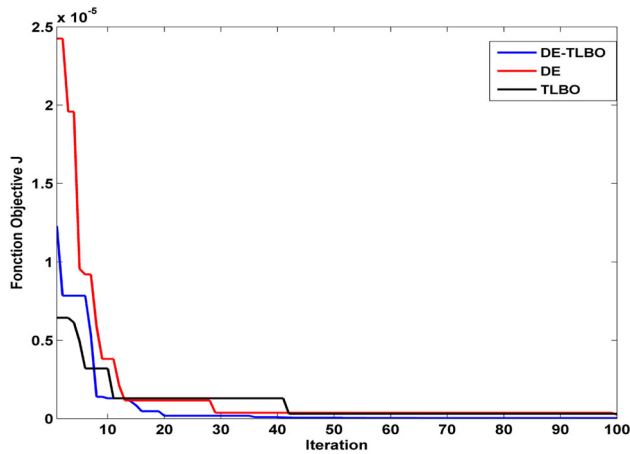
Simulations results in the first case have shown the superior performance of the proposed method ( $H_\infty$  and DE–TLBO–PID) in terms of the elimination the reaching phase and the reduction of the oscillation.

**Table 2** The optimal parameters

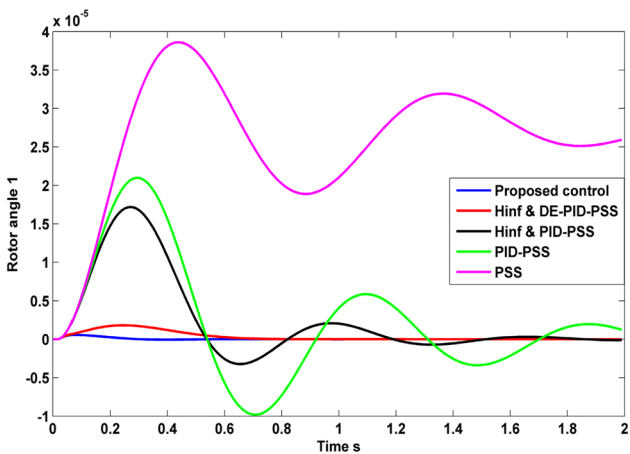
	DE algorithm			DE–TLBO algorithm		
	Gen 1	Gen 2	Gen 3	Gen 1	Gen 2	Gen 3
$K_p$	104.7243	71.3457	93.3432	116.0630	54.3091	95.7425
$K_i$	4.5194	4.3171	7.4675	5.4608	4.6651	7.4183
$K_d$	7.0650	2.4945	3.3584	8.3049	1.7646	2.156
$\beta$	3.1693	1.3739	5.0133	1.1036	2.8665	3.056

**Table 4** The values of the nominal parameters

	ITAE	ITSE
Proposed control	0.0101	1.7992e−007
$H_\infty$ and DE–PID–PSS	0.0265	2.0010e−007
$H_\infty$ and PID–PSS	0.5895	1.5702e−005
PID–PSS	0.7641	3.8008e−005
PSS	2.2672	1.9390e−004

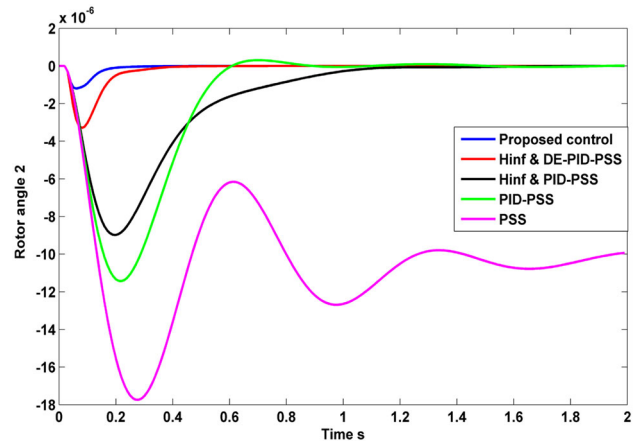


**Fig. 2** Convergence characteristics of DE–TLBO, DE and TLBO

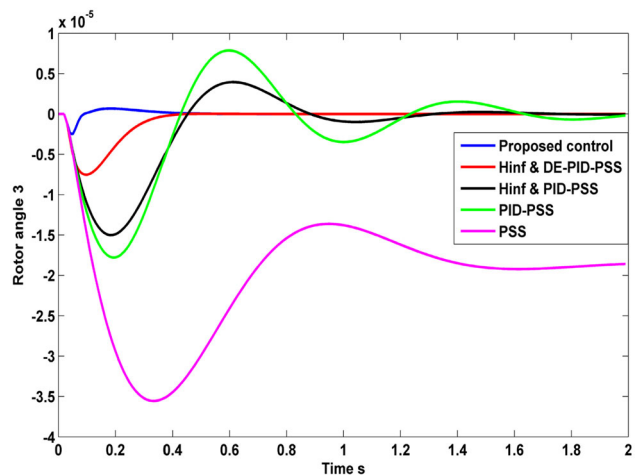


**Fig. 3** Response of the tracking error  $e_1$

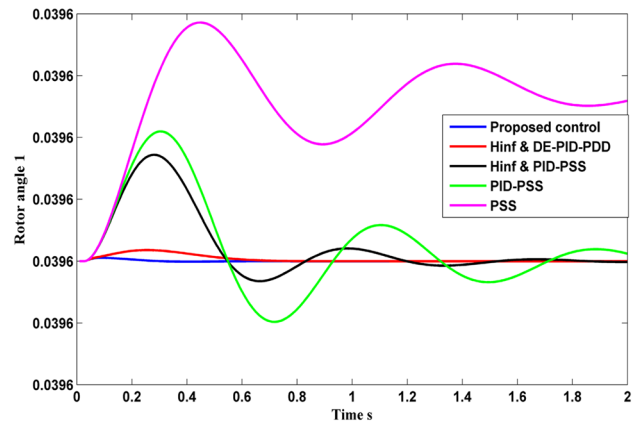
From the simulation results seen in Figs. 6, 7, 8, 9, 10 and 11 shows that the proposed method ( $H_\infty$  and DE–TLBO–PID) permit to reduce significantly the deviation of the power angle, speed deviation comparing with ( $H_\infty$  and DE–PID–PSS), we can deduce that the proposed controllers are always effective and has the best ability to keep the system track the desired values and helps the system to achieve the operating point very quickly. The (DE–TLBO)



**Fig. 4** Response of the tracking error  $e_2$



**Fig. 5** Response of the tracking error  $e_3$



**Fig. 6** Response of the rotor angle  $\delta_1$

algorithm has a good robustness and a much reduced time convergence.

*Case 2: Control response including the parameter variations* In practice, a third-order model of power system



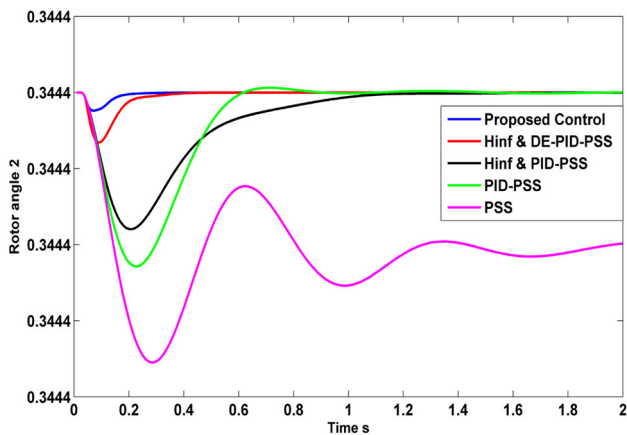


Fig. 7 Response of the rotor angle  $\delta_2$

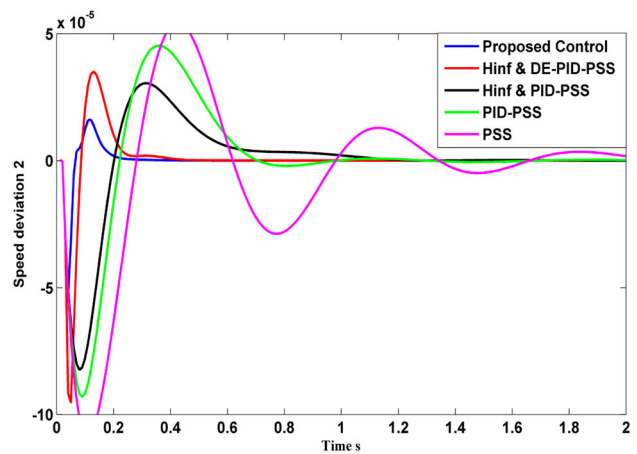


Fig. 10 Response of the speed deviation  $\Delta w_2$

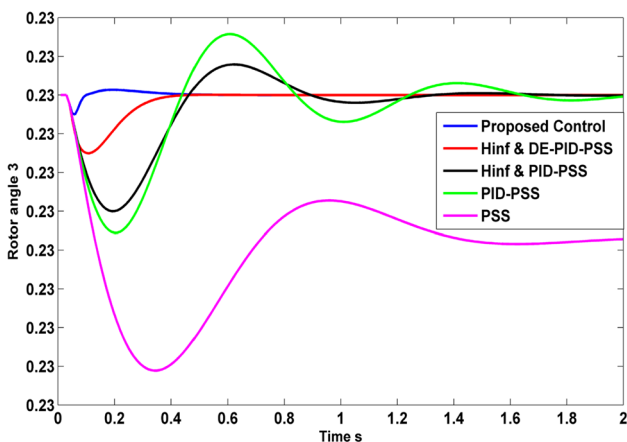


Fig. 8 Response of the rotor angle  $\delta_3$

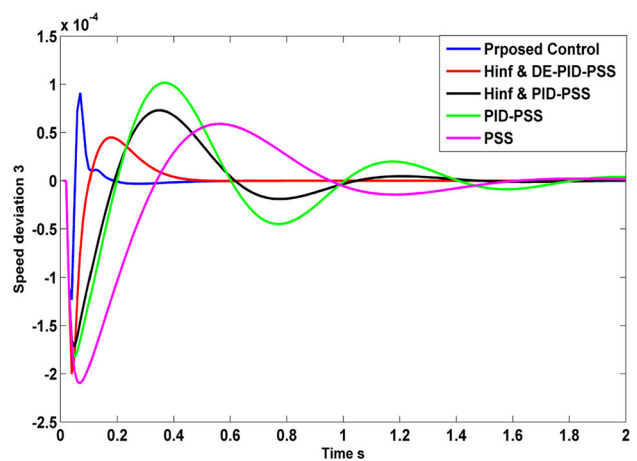


Fig. 11 Response of the speed deviation  $\Delta w_3$

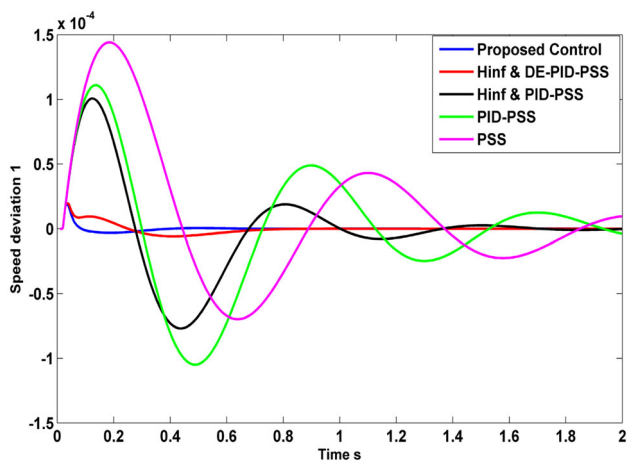


Fig. 9 Response of the speed deviation  $\Delta w_1$

could not represent accurately the generator unit and the exact model is unavailable. Therefore, it is required to test the performance and the robustness of the proposed approach in the presence of the variation in the system parameters and model errors. We consider the change in

Table 5 Parameter variation of the nominal values

Parameters	H	$T'_{do}$
Generator 1	18.4393 [−22%]	7.4368 [−17%]
Generator 2	8.128 [+27%]	7.5 [+25%]
Generator 3	3.45 [+15%]	6.5968 [+12%]

the inertia constant  $H_i$  and the time constant  $T'_{do}$  for each generator see Table 5.

From simulation results shown in Figs. 12, 13, 14, 15, 16 and 17 it can be clearly seen that the proposed approach can still ensure an efficient control performance even with the change in the system parameters and results a satisfactory tracking performance and achieves a good level in the oscillation damping.

*Case 3: Control response with change in the operation point* In this section, we present the simulation results when the variation in the operating point (EP) occurs. In

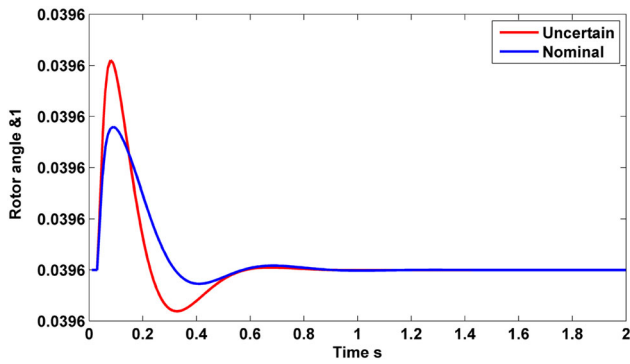


Fig. 12 Response of the rotor angle  $\delta_1$  under parameter variations

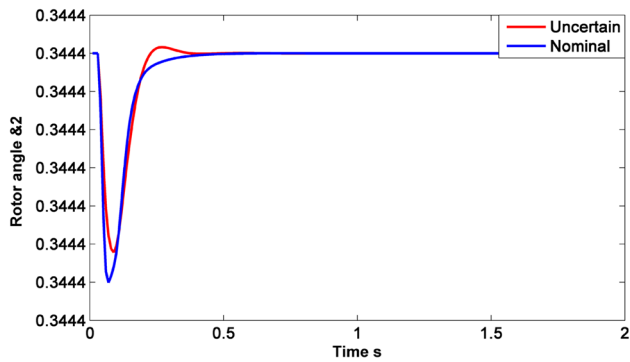


Fig. 13 Response of the rotor angle  $\delta_2$  under parameter variations

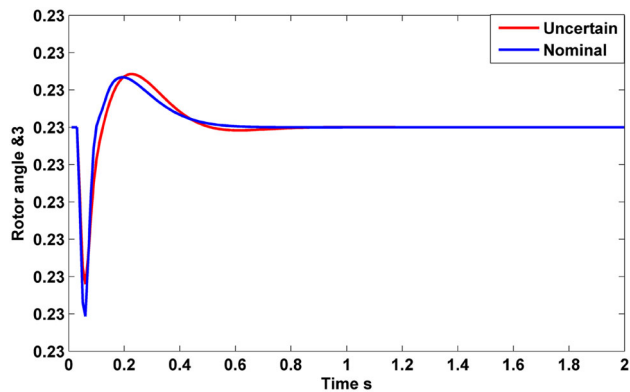


Fig. 14 Response of the rotor angle  $\delta_3$  under parameter variations

this case, the operating point EP1 changes to the following value EP2.

$$X_{ir}^* = (x_{i1r}^*, x_{i2r}^*, x_{i3r}^*) = [\Delta\omega_i \quad \Delta\omega_i \quad E'_{qi}]$$

For  $i = 1, 2, 3$ :

$$EP2: \begin{cases} x_{11r}^* = 0.0377, & x_{12r}^* = 0, & x_{13r}^* = 1.0768 \\ x_{21r}^* = 0.0376, & x_{22r}^* = 0, & x_{23r}^* = 0.9833 \\ x_{31r}^* = 0.2187, & x_{32r}^* = 0, & x_{33r}^* = 1.0713 \end{cases}$$

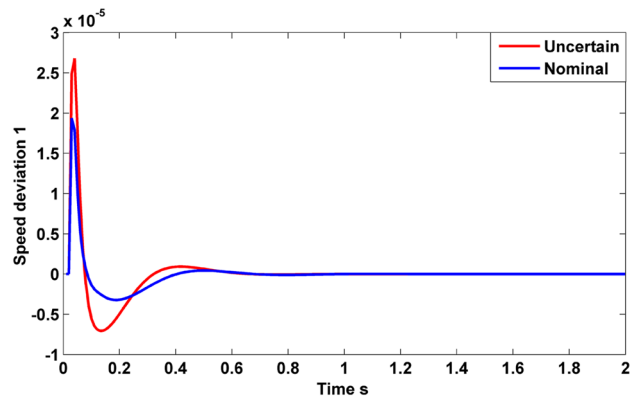


Fig. 15 Response of the speed deviation  $\Delta w_1$  under parameter variations

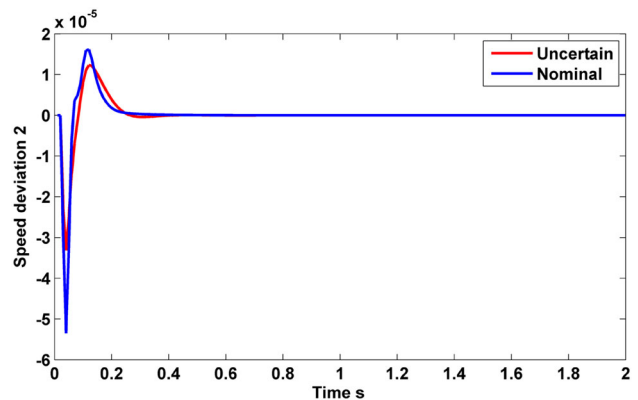


Fig. 16 Response of the speed deviation  $\Delta w_2$  under parameter variations

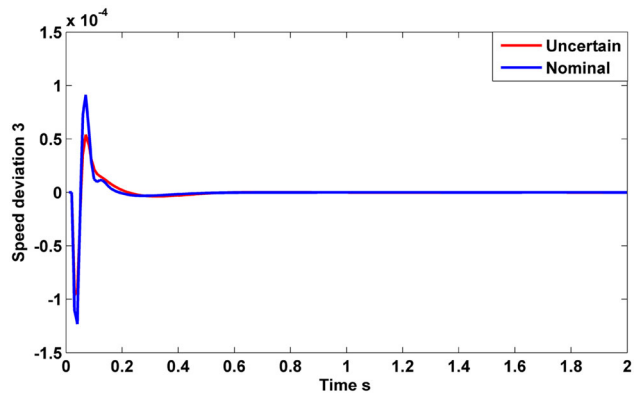
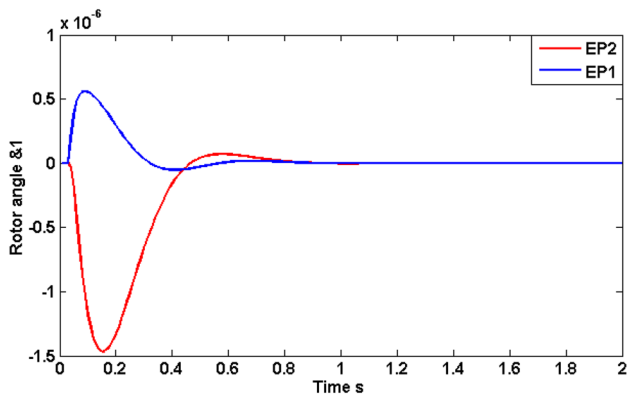
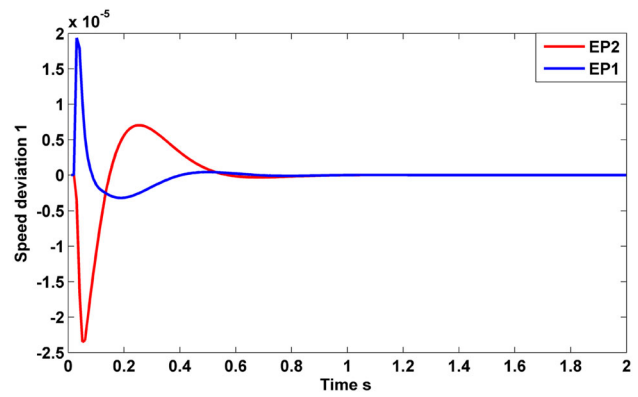


Fig. 17 Response of the speed deviation  $\Delta w_3$  under parameter variations

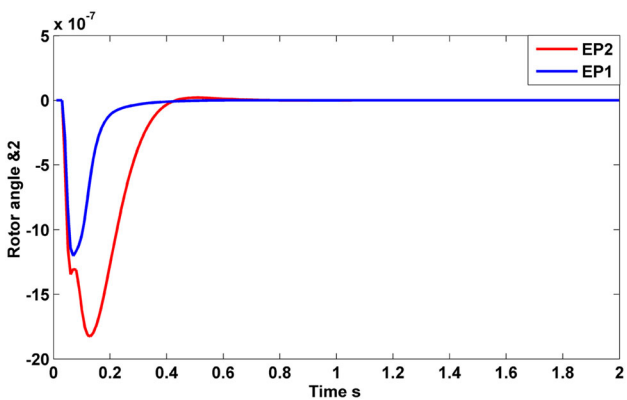
The results of simulation illustrated in Figs. 18, 19, 20, 21, 22 and 23 demonstrate that the proposed method ( $H_\infty$  and DE–TLBO–PID) stabilizes the power system with the



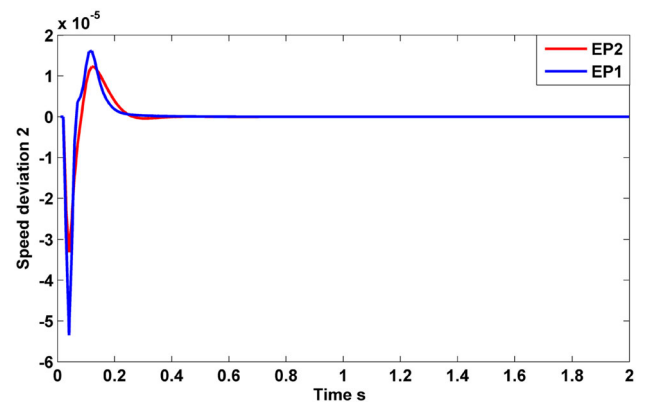
**Fig. 18** The variation of the rotor angle  $\delta_1$  under changes in the operation point



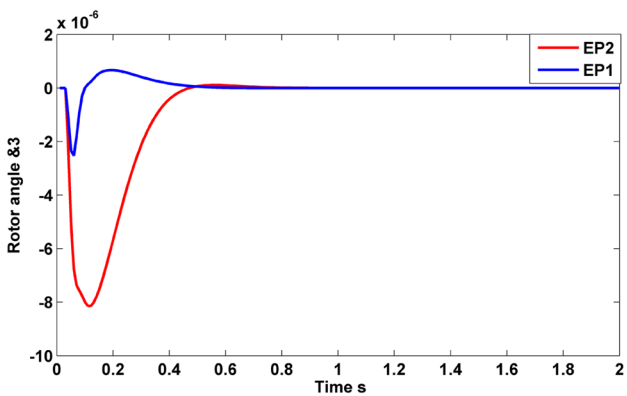
**Fig. 21** Response of the speed deviation  $\Delta w_1$  under changes in the operation point



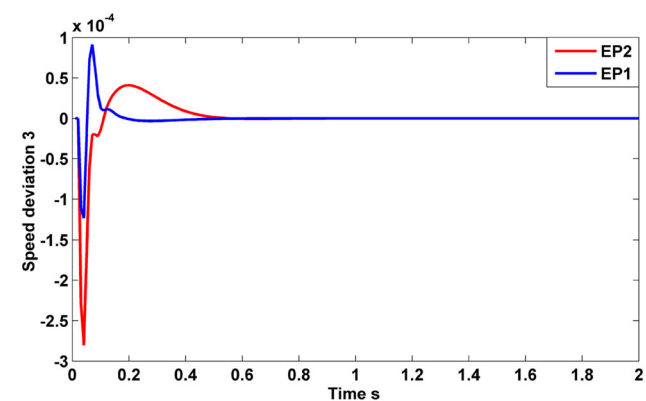
**Fig. 19** The variation of the rotor angle  $\delta_2$  under changes in the operation point



**Fig. 22** Response of the speed deviation  $\Delta w_2$  under changes in the operation point



**Fig. 20** The variation of the rotor angle  $\delta_3$  under changes in the operation point



**Fig. 23** Response of the speed deviation  $\Delta w_3$  under changes in the operation point

new equilibrium point EP2 and the performance of the tracking is achieved efficiently.

We can therefore conclude that the proposed controllers ( $H_\infty$  and DE-TLBO-PID) is characterized by a stable performance and can guarantee high performance of the tracking and a good level in the oscillation damping in

a very reduced time, the controllers have demonstrated the robustness even when to changes in the operating point and the system parameter variations are occurred.

## 6 Conclusion

In this paper, the optimized PID–PSS using the new hybrid algorithm (DE–TLBO) combined with the optimal  $H_\infty$  tracking control provides an effective solution to eliminate significantly the reaching phase and damp the oscillations under the variation in the system parameters and the operation point in the multi-machine power system. The hybrid algorithm (DE–TLBO) has been employed to tune optimally the parameter  $\beta$  of the exponential function which is an important factor for the rapid convergence of the tracking error, and also to adjust the parameters of the PID–PSS in order to guarantee the dynamic stability. The comparison performed by the simulations show the robust performance of the proposed approach in terms of damping the oscillations, in terms of the best tracking to the desired values optimally and in terms of the rapid convergence even in the presence of the parameter variations.

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