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ORIGINAL ARTICLE

An improved inventory model with random review period and temporary price discount for deteriorating items

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Abstract This paper deals with a stochastic periodic review inventory system, wherein temporary price discount offer is taken into the account. The review period (time interval between two consecutive reviews) is considered as a random variable. In the real life, whereas, in one side, supplier stimulates the sale and/or rise cash flow by offering price discount/quantity discount, on the other side extra purchasing impels to deterioration of the product. To become the part of this, we translate some real-life situations such as deterioration, temporary price discount, partial backlogging into the mathematical model. This paper prudently studies the joint effect of deterioration and special sale offer. Furthermore, shortages are permissible in retailer's inventory system and partially backlogged. The model is mathematically rigorously analyzed and concavity of saving function is also shown. Illustration of the proposed model is exposed through suitable numerical examples, sensitivity analysis and graphical representation.

Keywords Inventory · Deterioration · Temporary price discount · Stochastic review period · Partial backlogging

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1 Introduction

Any process that prevents an item from use for its intended original form is termed as deterioration (Ghare and Schrader 1963). It may be: (a) spoilage, as in perishable foodstuffs, dairy products, fruits and vegetables; (b) physical depletion, as in pilferage or evaporation of volatile liquids such as gasoline and alcohol; (c) decay, as in radioactive substances; degradation, as in electronic components, or loss of potency as in photographic films and pharmaceutical drugs (Wee and Yu 1997). However, such type of products will not be useful after a finite time period. Due to finite shelf life, quick transaction of such type of products is indispensable. Price discount is a way to encourages the consumption that finally encourages the flow of products. A large pile of inventory models deal with deterioration and temporary price discount in deterministic framework (for detail see Sarkar et al. 2012; Shah 2012). Moreover, the literature of inventory control problem is highly devoted to the periodic review system wherein review period is taken as constant. Karimi-Nasab and Konstantaras (2013) state that due to many reasons such as delay in transportation, variation in supply time, in consumption, etc., the constant review period does not reflect the real life situation. A random scheduling period may coincide with random review period, when the replenishment is made at the time of review. In deterministic framework, concept of deterioration and price discount is being matured with inventory modeling. Despite it, no inventory model exist that simultaneously considered deterioration, price discount and stochastic review period. In this paper we try to fill this research gap.

Now, a chronological but brief review of the literature is desirable. Goyal (1990) established an economic order policy for a dynamic inventory system, wherein special discount period is offered occasionally by the supplier. Tersine and Barman (1995) addressed a composite economic order quantity (EOQ) model with one time temporary special discount offer. A composite EOQ model can be disaggregated into numerous deterministic EOQ models that can be applied to variety of different operational situations. Rhonda (1996) included special sale offer for restricted time period in a continuous review inventory system. Abad (2003) addressed an inventory problem, wherein the manufacturer offers price discount over a finite time-span. Moreover, he assumed that demand as price sensitive, and derived an optimum selling price. Sarker and Al Kindi (2006) developed an optimal ordering policy by considering discount offer. They have discussed many different cases such as sale period is longer than a cycle, coincidence/non-coincidence of sale period with replenishment, dependency of discounted price on order quantity etc. Yang et al. (2010) investigated the effect of a temporary price discount offered by supplier on retailers replenishment policy with lead time linked to order quantity (i.e., lead time depends upon the order quantity). Shah (2012) included a temporary price discount on the retailer's ordering policy wherein demand is stock dependent and price discount rate is linked to special order quantity. Then, she quantified the difference between the special order and regular order in cost saving scale.

Bose et al. (1995) developed an EOQ model for deteriorating items, wherein demand rate is linear increasing function of time. Gayen and Pal (2009) addressed a two warehouse inventory model for deteriorating item and stock dependent demand rate. Thangam and Uthayakumar (2010) included partial trade credit finance in two warehouse inventory storage system for deteriorating items. Wee and Yu (1997) developed an inventory model for deteriorating items, and considered two types of temporary discount purchasing facility: (1) Temporary discount purchasing made at regular replenishment time and (2) Temporary discount purchasing made at non-regular replenishment time. Arcelus et al. (2003) established the retailer's profit-maximization retail promotion strategy against the vendor's price discount on the deteriorating products. They determined the three main elements of the retailer's promotion strategy in response to the vendor's special one-time-only trade promotion: (a) the size of the special order to be placed from the vendor, under the different types of possible trade incentives offered; (b) the portions of the benefits to be passed on to its own customers, in the form of new price and/or credit terms incentives intended to stimulate demand on a temporary basis; and (c) the quantity to be sold under these one-time-only conditions. Dye et al. (2007) considered two situations in which the replenishment takes place, during a regularly scheduled replenishment period or it does not for a deteriorating item when a temporary price discount occurs. Sarkar et al. (2012) determined an optimal replenishment policy to the retailer for a deteriorating item and time-quadratic demand rate, wherein shortages are permitted and time dependent partial backlogging has been considered. In recent years many authors have addressed inventory models for deteriorating items under different assumptions such as Pal et al. (2014a, b, 2015), Palanivel and Uthayakumar (2015) and Jaggi et al. (2012), etc. Recently, Taleizadeh et al. (2013) developed an EOQ model for perishable item with special sale offer in deterministic environment.

In the practice, it seems that the supply disruption influences the inventory control system. In general, the supplier is relatively in more powerful position in a supply chain and he decides when to visit and replenish the retailer's order quantity (Ertogral and Rahim 2005). Moreover, it may be possible that the retailer's shop is situated in disadvantageous remote location, and regular replenishment and visit process may not be accomplished. In such situations, the review period may be a random variable (Chiang 2008). Ertogral and Rahim (2005) addressed a multi-period periodic review inventory control problem, wherein time interval between two consecutive reviews is taken as random variable. Furthermore, shortage quantity is considered as mixture of partial backlogging and lost sales. Chiang (2008) extended the periodic review inventory model by considering stochastic review interval, and used dynamic programming approach to solve the model. Arcelus et al. (2009) determined the retailer's decision making criterion in contrast of anticipating of vendor's temporary price discount offer. They also considered time period of price discount offer is uncertain and is taken as a random variable. Liu et al. (2009) determined a replenishment policy for items with deterioration in stochastic framework by considering review interval as a random variable. Recently, Karimi-Nasab and Konstantaras (2013) extended the periodic review model with temporary price discount in stochastic framework. They considered review period as a random variable, and encapsulated with two different distribution function namely, exponential and uniform distributions.

The extraction of above discussion is that modeling of real-life business situation is keen area of research. Moreover no inventory model exists that simultaneously deal with deterioration and temporary price discount in stochastic framework. In this paper, we cover these factors that reflect a real-life business situation. In this connection we consider an integrated inventory modeling problem that simultaneously consider deterioration, temporary price discount, shortages that are mixture of partial backlogging and lost sales, and stochastic review period. To ease in mathematical formulation, order-level replenishment policy has been adhered. Moreover, the order-level systems are best fit for price discount problems (Ertogral and Rahim 2005). According to Karimi-Nasab and Konstantaras (2013), when the randomly review of inventory position is done at the time of replenishment then it is termed as random replenishment. However, in this paper random review period meant time interval between two consecutive replenishment is a random variable. We analytically show that the previous models are the special case of our model. Moreover, we rigorously mathematically analyze the model in order to find the unique global optimal solution. To the best of our knowledge, and as an evidence of the literature survey, this problem is not deliberated earlier. Overall these setting can be confronted in many real world inventory cases. Rest of the paper organized as follow: Sect. 2 lists the assumptions and notations. Section 3 provides mathematical derivation of the model, and shows the concavity of saving function. In Sect. 4, we have verified the mathematical formulation by the way of numerical example and its sensitivity for changes of parameters. Finally, in Sect. 5, we mention the concluding remarks and provide the direction of potential future researches.

2 Notations and assumptions

The following notations are used throughout the article.

2.1 Notations

r	Demand rate per unit time
x	Random review period ($x_{\min} \le x \le x_{\max}$)
$f(x)$	Probability density function of x
$x_{\min}$	Minimum value of the random variable x
$x_{\max}$	Maximum value of the random variable x
θ	The deterioration rate
α	Fraction of the shortage that is backorder ($0 < \alpha < 1$)
μ	Time when inventory level reaches to zero in regular review period
μ_s	Time when inventory level reaches to zero in the case of temporary price discount
v	Selling price per product
p	Purchasing price per product
p_d	Decrement in unit purchasing price during temporary special sale offer
p_s	Purchasing price per product in temporary price discount ($p_s = p - p_d$)
h	Inventory holding cost per product per unit time
h_s	Inventory holding cost per product per unit time in temporary price discount ($h_s = \frac{hp_s}{p}$)
d	Disposal cost per product for deteriorating items ($d = 5\,\%\,\text{of}\,p$)
b	Back-ordering cost per product
l	Loss of sale cost per product
$\bar{D}$	Expected deteriorating items in each regular review period
$\bar{D}_s$	Expected deteriorating items in temporary price discount
$\bar{Q}$	Expected order quantity in each regular review period
$\bar{Q}_s$	Expected order quantity in temporary price discount
$\bar{V}_s$	Expected selling quantity in temporary price discount
$\bar{V}$	Expected selling quantity in each regular review period
$\bar{I}$	Expected inventory during review period
$\bar{I}_s$	Expected inventory during temporary price discount
$\bar{B}$	Expected shortage during review period
$\bar{B}_s$	Expected shortage during temporary price discount
$\bar{L}$	Expected lost sale during review period
$\bar{L}_s$	Expected lost sale during temporary price discount
q	Maximum inventory level in regular period
q_s	Maximum inventory level in the case temporary price discount
ε	Expected profit during regular review period
ε_s	Expected profit during temporary price discount
T_p	Expected total saving amount.

2.2 Assumptions

While the following assumptions are made in development of the model.

1. Demand rate is uniform and constant throughout the horizon.
2. Shortage is allowed, and is partially backlogged, i.e., a fraction of shortage between two consecutive deliveries is backlogged and remaining is lost sales.
3. The inter arrival time between replenished are identically and independent distribution (i.i.d).
4. The rate of deterioration is constant.
5. Repairing or replacement for the deteriorated items are not permissible.
6. To motivate a retailer for ordering more quantity, a unique temporary price discount is offered.

3 Mathematical formulation of the model

In this section we construct the model, mathematically, and analyze it in order to obtain the global optimal solution. As discussed earlier and shown in Fig. 1, a unique temporary special sale offer is provided by the supplier to boost the sale. Actually, this is a saving-maximize problem over the

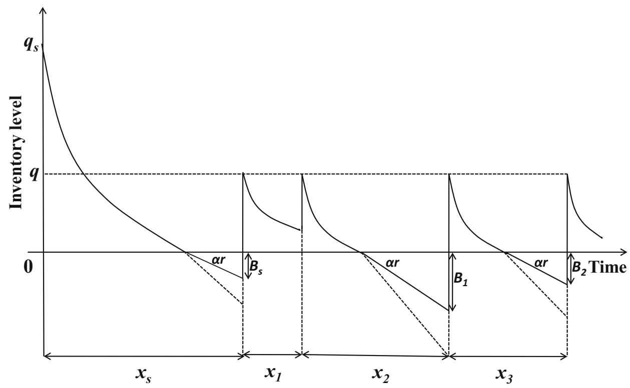

Fig. 1 Inventory variation over random review period

stochastic review period. Thus, we evaluate the expected maximum saving, which is difference between expected profit during special sale offer and regular time period. Therefore, expected total profit for two cases are obtained: (a) temporary price discount is installed and (b) temporary price discount is not installed. As Fig. 1 delineate, inventory level continuously decreases due to demand and deterioration up to time μ (μ_s in case of special sale offer). Then shortage occurs and is continue up to the end of cycle. The shortage quantity is partially backlogged in next arrival.

Case (i) All unit temporary price discount is installed.

At the beginning of the period, inventory starts with maximum sale q_s, and continuously depletes due to demand and deterioration. Variation of inventory level with respect to time t can be expressed by the following differential equation.

$$\frac{di_s}{dt} + \theta i_s = -r \tag{3.1}$$

with initial condition

$$i_s(0) = q_s \tag{3.2}$$

and boundary condition

$$i_s(\mu_s) = 0. \tag{3.3}$$

Equations (3.1) and (3.2) give inventory level at any time t as

$$i_s(t) = -\frac{r}{\theta} + \left(q_s + \frac{r}{\theta}\right)e^{-\theta t}; \quad t \geq 0. \tag{3.4}$$

Boundary condition (3.3) gives

$$\mu_s = \frac{1}{\theta} ln\left(1 + \frac{\theta q_s}{r}\right). \tag{3.5}$$

Therefore, the total accumulated amount of inventory over the random interval [0, x] is:

$$I_s(x) = I_s = \begin{cases} \int_0^x i_s(t)dt; & 0 \leq x < \mu_s \\ \int_0^{\mu_s} i_s(t)dt; & x \geq \mu_s. \end{cases} \tag{3.6}$$

The random review period results random inventory level, order quantity, backorder quantity, etc. Similar to Karimi-Nasab and Konstantaras (2013), we obtain the expected order quantity, expected inventory, expected deterioration, expected sale quantity, expected backlogged and lost sale, respectively, as follows:

$$\begin{aligned} \bar{Q_s} &= E(Q_s) \\ &= \int_{x_{\min}}^{\mu_s} (q_s - i_s(x))f(x)dx \\ &\quad + \int_{\mu_s}^{x_{\max}} [q_s + \alpha r(x - \mu_s)]f(x)dx, \end{aligned} \tag{3.7}$$

$$\begin{aligned} \bar{I_s} &= E(I_s) \\ &= \int_{x_{\min}}^{\mu_s} \left(\int_0^x i_s(t)dt\right)f(x)dx \\ &\quad + \int_{\mu_s}^{x_{\max}} \left(\int_0^{\mu_s} i_s(t)dt\right)f(x)dx, \end{aligned} \tag{3.8}$$

$$\begin{aligned} \bar{D_s} &= E(D_s) \\ &= \int_{x_{min}}^{\mu_s} (q_s - i_s(x) - rx)f(x)dx + \int_{\mu_s}^{x_{\max}} (q_s - r\mu_s)f(x)dx, \end{aligned} \tag{3.9}$$

$$\begin{aligned} \bar{V_s} &= \bar{Q_s} - \bar{D_s} \\ &= \int_{x_{\min}}^{\mu_s} rxf(x)dx + r\int_{\mu_s}^{x_{\max}} [\alpha x + (1-\alpha)\mu_s]f(x)dx, \end{aligned} \tag{3.10}$$

$$\bar{B_s} = E(B_s) = \int_{\mu_s}^{x_{\max}} \alpha r(x - \mu_s)f(x)dx, \tag{3.11}$$

$$\bar{L_s} = E(L_s) = \int_{\mu_s}^{x_{\max}} (1-\alpha)r(x - \mu_s)f(x)dx. \tag{3.12}$$

The expected profit ε_s per cycle is

$$\varepsilon_s = v\bar{V_s} - (p - p_d)\bar{Q_s} - h\bar{I_s} - b\bar{B_s} - l\bar{L_s} - p(\bar{B_s} - \bar{B}) - d\bar{D_s}. \tag{3.13}$$

Case (ii) All unit temporary price discount is not installed.

Suppose a supplier gives special sale offer, but retailer does not procure more quantity, i.e., temporary price discount is not installed. The procurement is made as a regular cycle. Similar to case (i), the expected order quantity, expected inventory, expected deterioration, etc. are obtained as follows:

$$\begin{aligned} \bar{Q} &= E(Q) \\ &= \int_{x_{\min}}^{\mu} (q - i(x))f(x)dx + \int_{\mu}^{x_{\max}} [q + \alpha r(x - \mu)]f(x)dx, \end{aligned} \tag{3.14}$$

$$\bar{I} = E(I) = \int_{x_{\min}}^{\mu}\left(\int_0^x i(t)dt\right)f(x)dx + \int_{\mu}^{x_{\max}}\left(\int_0^{\mu} i(t)dt\right)f(x)dx, \tag{3.15}$$

$$\bar{D} = E(D) = \int_{x_{\min}}^{\mu}(q - i(x) - rx)f(x)dx + \int_{\mu}^{x_{\max}}(q - r\mu)f(x)dx, \tag{3.16}$$

$$\bar{V} = \bar{Q} - \bar{D} = \int_{x_{\min}}^{\mu} rxf(x)dx + r\int_{\mu}^{x_{\max}}[\alpha x + (1-\alpha)\mu]f(x)dx, \tag{3.17}$$

$$\bar{B} = E(B) = \int_{\mu}^{x_{\max}} \alpha r(x-\mu)f(x)dx, \tag{3.18}$$

$$\bar{L} = E(L) = \int_{\mu}^{x_{\max}}(1-\alpha)r(x-\mu)f(x)dx. \tag{3.19}$$

The expected profit ε for this case is

$$\varepsilon = v\bar{V} + v\left(\frac{\bar{Q}_s}{\bar{Q}} - 1\right)\bar{V} - (p - p_d)\bar{Q} - p\left(\frac{\bar{Q}_s}{\bar{Q}} - 1\right)\bar{Q} - h_s\bar{I} - h\bar{I}\left(\frac{\bar{Q}_s}{\bar{Q}} - 1\right) - b\frac{\bar{Q}_s}{\bar{Q}}\bar{B} - (l + p_d)\bar{L} - l\left(\frac{\bar{Q}_s}{\bar{Q}} - 1\right)\bar{L} - d\frac{\bar{Q}_s}{\bar{Q}}\bar{D}. \tag{3.20}$$

As discussed earlier, this is a saving-maximization problem in stochastic framework. Thus, we find the expected saving T_p when special sale offer is installed. This is as follows:

$$T_p = \varepsilon_s - \varepsilon = \kappa_1\bar{Q}_s - h\bar{I}_s - (v+d)\bar{D}_s - (p+b)\bar{B}_s - l\bar{L}_s - \kappa_2, \tag{3.21}$$

where

$$\kappa_1 = \frac{1}{\bar{Q}}\{p_d\bar{Q} + h\bar{I} + b\bar{B} + l\bar{L} + (v+d)\bar{D}\} \geq 0,$$
$$\kappa_2 = p_d\bar{Q} + (h - h_s)\bar{I} - p\bar{B} - p_d\bar{L}.$$

Remark 1 If we take $\theta \to 0$, then $\mu_s = q_s/r, \bar{D}_s = 0 = \bar{D}, I_s, B_s, L_s$ and $\bar{Q}_s$ are same as Karimi-Nasab and Konstantaras (2013). This validates our formulation.

3.1 Uniform distribution function

Consider a case in which random review period is equally likely in the interval $[x_{\min}, x_{\max}]$. This situation is contemplated here by considering review period as uniformly distributed where $f(x) = 1/(x_{\max} - x_{\min}), x_{\min} \leq x \leq x_{\max}$. Then, Eqs. (3.7)–(3.12) can be rewritten as:

$$\bar{Q}_s = \frac{1}{x_{\max} - x_{\min}}\left\{\frac{\alpha r}{2}(x_{\max} - \mu_s)^2 - \left(q_s + \frac{r}{\theta}\right) \times \left(x_{\min} + \frac{e^{-\theta x_{\min}}}{\theta}\right) + x_{\max}q_s + \frac{r}{\theta}\mu_s + \frac{r}{\theta^2}\right\},$$

$$\bar{I}_s = \frac{\frac{1}{\theta}}{x_{\max} - x_{\min}}\left\{\frac{r}{2}(\mu_s^2 + x_{\min}^2) - rx_{\max}\mu_s - \left(q_s + \frac{r}{\theta}\right)\left(x_{\min} + \frac{e^{-\theta x_{\min}}}{\theta}\right) + x_{\max}q_s + \frac{r}{\theta}\mu_s + \frac{r}{\theta^2}\right\},$$

$$\bar{D}_s = \frac{1}{x_{\max} - x_{\min}}\left\{\frac{r}{2}(\mu_s^2 + x_{\min}^2) - rx_{\max}\mu_s - \left(q_s + \frac{r}{\theta}\right)\left(x_{\min} + \frac{e^{-\theta x_{\min}}}{\theta}\right) + x_{\max}q_s + \frac{r}{\theta}\mu_s + \frac{r}{\theta^2}\right\},$$

$$\bar{B}_s = \frac{1}{x_{\max} - x_{\min}}\frac{\alpha r}{2}(x_{\max} - \mu_s)^2,$$

$$\bar{L}_s = \frac{1}{x_{\max} - x_{\min}}\frac{(1-\alpha)r}{2}(x_{\max} - \mu_s)^2.$$

Now, we use gradient method to maximize the cost function (3.21). For this

$$\frac{dT_p}{dq_s} = \frac{1}{x_{\max} - x_{\min}}\Bigg[\frac{1}{(1 + \frac{\theta}{r}q_s)}\left\{\frac{\kappa_1}{\theta} - \frac{1}{\theta}\left(\frac{h}{\theta} + d + v\right) + (x_{\max} - \mu_s) \times \left(\frac{h}{\theta} + d + v + \alpha(p+b) + (1-\alpha)l - \alpha\kappa_l\right)\right\} + \left(x_{\max} - x_{\min} - \frac{e^{-\theta x_{\min}}}{\theta}\right) \times \left(\kappa_1 - \frac{h}{\theta} - d - v\right)\Bigg]$$
$$= \frac{1}{x_{\max} - x_{\min}}\left[\frac{1}{(1 + \frac{\theta}{r}q_s)}\left\{(a_1 + a_2)(x_{\max} - \mu_s) - \frac{a_1}{\theta}\right\} - a_1\left(x_{\max} - x_{\min} - \frac{e^{-\theta x_{\min}}}{\theta}\right)\right], \tag{3.22}$$

where

$$a_1 = \frac{h}{\theta} + d + v - \kappa_1,$$
$$a_2 = (1-\alpha)\kappa_1 + \alpha(p+b) + (1-\alpha)l \geq 0.$$

Theorem 1 (Uniqueness of optimal solution) *If $a_1 \geq 0$ or $a_1 < 0$ and $a_2 \geq -a_1$, then inventory problem always has a unique optimal solution if and only if $q_s < \frac{r}{\theta}\left(e^{\frac{a_2}{a_1+a_2} + \theta x_{\max}} - 1\right)$. Moreover, if $a_1 < 0$ and $a_2 < -a_1$, then uniqueness is assured if and only if $q_s > \frac{r}{\theta}\left(e^{\frac{a_2}{a_1+a_2} + \theta x_{\max}} - 1\right)$.*

Proof See Appendix 1 for proof.

The optimal q_s^* can be obtained from equation

$$\frac{dT_p}{dq_s} = 0 \Rightarrow \frac{1}{(1+\frac{\theta}{r}q_s)}\left\{(a_1+a_2)(x_{\max}-\mu_s) - \frac{a_1}{\theta}\right\}$$
$$= a_1\left(x_{\max} - x_{\min} - \frac{e^{-\theta x_{\min}}}{\theta}\right).$$

Although Theorem 1 insured the uniqueness of optimal solution, but it is difficult to find q_s^* in closed form. So, one can use any iteration method such as Newton–Raphson etc. and find q_s^*.

3.2 Truncated normal distribution function

Normal distribution is one of the most useful distribution in modeling of real life situations. According to cental limit theorem any identically distributed random variables tends to approximately normal distribution. However, in dealing with normal distribution, an unrealistic situation incur regarding its range that is $-\infty$ to ∞. But it is impossible to take review period as negative or ∞. The general idea about area of normal curve is that the interval $[\rho - 3\sigma, \rho + 3\sigma]$ cover 99.73 % of total area, where ρ is mean and σ is standard deviation. Hence, for convenience we truncated normal distribution to a interval $[x_{\min}, x_{\max}]$, where $0 \le x_{\min} \le \rho - 3\sigma$ and $x_{\max} \ge \rho + 3\sigma$. Thus, we take review period is truncated normally distributed as, $f(x) = \frac{1}{\Phi(x_{\max})-\Phi(x_{\min})}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(\frac{x-\rho}{\sigma})^2}, x_{\min} \le x \le x_{\max}$, where Φ c.d.f. of standard normal distribution. For this distribution function, the cost function become as:

$$\bar{Q_s} = \frac{1}{\Phi(x_{\max})-\Phi(x_{\min})}\Bigg[\left(q_s+\frac{r}{\theta}\right)\left\{\frac{1}{2}Erf\left[\frac{\rho-x_{\min}}{\sqrt{2}\sigma}\right] - e^{\theta(\theta\sigma^2-2\rho)} \times \frac{1}{2}\left(Erf\left[\frac{\theta\sigma^2-\rho+\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\theta\sigma^2-\rho+x_{\min}}{\sqrt{2}\sigma}\right]\right)\right\}$$
$$-\frac{1}{2}Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - \frac{1}{2}Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]$$
$$+\alpha r(\rho-\mu_s)\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]\right).$$
$$+\alpha r\frac{\sigma}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}(\frac{\mu_s-\rho}{\sigma})^2} - e^{-\frac{1}{2}(\frac{x_{\max}-\rho}{\sigma})^2}\right)\Bigg],$$

$$\bar{I_s} = \frac{\frac{1}{\theta}}{\Phi(x_{\max})-\Phi(x_{\min})}$$
$$\times\Bigg\{r\left(\rho-\frac{1}{\theta}\right)\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\min}}{\sqrt{2}\sigma}\right]\right)$$
$$- r\mu_s\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]\right)$$
$$- q_s\frac{1}{2}\left(Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\min}}{\sqrt{2}\sigma}\right]\right)$$
$$-\left(q_s+\frac{r}{\theta}\right)e^{\theta(\theta\sigma^2-2\rho)}\frac{1}{2}\left(Erf\left[\frac{\theta\sigma^2-\rho+\mu_s}{\sqrt{2}\sigma}\right]\right.$$
$$\left.-Erf\left[\frac{\theta\sigma^2-\rho+x_{\min}}{\sqrt{2}\sigma}\right]\right) + \frac{r\sigma}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}(\frac{\mu_s-\rho}{\sigma})^2} - e^{-\frac{1}{2}(\frac{x_{\min}-\rho}{\sigma})^2}\right)\Bigg\},$$

$$\bar{D_s} = \frac{1}{\Phi(x_{\max})-\Phi(x_{\min})}$$
$$\times\Bigg\{r\left(\rho-\frac{1}{\theta}\right)\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\min}}{\sqrt{2}\sigma}\right]\right)$$
$$- r\mu_s\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]\right)$$
$$- q_s\frac{1}{2}\left(Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\min}}{\sqrt{2}\sigma}\right]\right)$$
$$-\left(q_s+\frac{r}{\theta}\right)e^{\theta(\theta\sigma^2-2\rho)}\frac{1}{2}\left(Erf\left[\frac{\theta\sigma^2-\rho+\mu_s}{\sqrt{2}\sigma}\right]\right.$$
$$\left.-Erf\left[\frac{\theta\sigma^2-\rho+x_{\min}}{\sqrt{2}\sigma}\right]\right) + \frac{r\sigma}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}(\frac{\mu_s-\rho}{\sigma})^2} - e^{-\frac{1}{2}(\frac{x_{\min}-\rho}{\sigma})^2}\right)\Bigg\},$$

$$\bar{B_s} = \frac{1}{\Phi(x_{\max})-\Phi(x_{\min})}$$
$$\times\Bigg\{\alpha r(\rho-\mu_s)\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]\right)$$
$$+\alpha r\frac{\sigma}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}(\frac{\mu_s-\rho}{\sigma})^2} - e^{-\frac{1}{2}(\frac{x_{\max}-\rho}{\sigma})^2}\right)\Bigg\},$$

$$\bar{L_s} = \frac{1}{\Phi(x_{\max})-\Phi(x_{\min})}$$
$$\times\Bigg\{(1-\alpha)r(\rho-\mu_s)\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]\right)$$
$$+(1-\alpha)r\frac{\sigma}{\sqrt{2\pi}}\left(e^{-\frac{1}{2}(\frac{\mu_s-\rho}{\sigma})^2} - e^{-\frac{1}{2}(\frac{x_{\max}-\rho}{\sigma})^2}\right)\Bigg\},$$

where

$$Erf(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-x^2}dx. \tag{3.23}$$

Now, we further use gradient method to determine the optimal policy.

$$\frac{dT_p}{dq_s} = \frac{1}{\Phi(x_{\max})-\Phi(x_{\min})}$$
$$\times\Bigg[a_1\left\{a_3\left(\frac{1}{2}Erf\left[\frac{\theta\sigma^2-\rho+\mu_s}{\sqrt{2}\sigma}\right] - \frac{1}{2}Erf\left[\frac{\theta\sigma^2-\rho+x_{\min}}{\sqrt{2}\sigma}\right]\right)\right.$$
$$\left.+\frac{1}{2}Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right] - \frac{1}{2}Erf\left[\frac{\rho-x_{\min}}{\sqrt{2}\sigma}\right]\right\}$$
$$+\frac{a_1+a_2}{\frac{\theta}{r}q_s+1}\frac{1}{2}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] - Erf\left[\frac{\rho-x_{\max}}{\sqrt{2}\sigma}\right]\right)\Bigg], \tag{3.24}$$

where

$$a_3 = e^{\frac{1}{2}\theta(\theta\sigma^2-2\rho)}. \tag{3.25}$$

Theorem 2 (Uniqueness of optimal solution) *If* $a_1 \ge 0$ *or* $a_1 < 0$ *and* $a_2 \ge -a_1$ *then uniqueness is assured if and only if* $\frac{a_2}{\theta\sigma(a_1+a_2)}e^{-\frac{1}{2}(\frac{\rho-\mu_s}{\sigma})^2} + \sqrt{\frac{\pi}{2}}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] + Erf\left[\frac{x_{\max}-\rho}{\sqrt{2}\sigma}\right]\right) > 0.$ *Moreover, if* $a_1 < 0$ *and* $a_2 < -a_1$ *then uniqueness is assured if and only if* $\frac{a_2}{\theta\sigma(a_1+a_2)}e^{-\frac{1}{2}(\frac{\rho-\mu_s}{\sigma})^2} + \sqrt{\frac{\pi}{2}}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] + Erf\left[\frac{x_{\max}-\rho}{\sqrt{2}\sigma}\right]\right) < 0.$

Proof See Appendix 2 for proof.

Uniqueness of optimal solution is proved in above theorem. Similar to Sect. 3.1, it is difficult to find the optimal value of q_s from $dT_p/dq_s = 0$ of Eq. (3.24). Thus we use Mathematica software to find its value.

4 Numerical example and its sensitivity analysis

In this section we consider some numerical examples for illustration purpose of forgoing discussion.

4.1 Uniform distribution function

Example 1 Let us consider a grocery retailer procures rice from a nearby rice mill (supplier) at the cost of $10 per bag. The retailer's demand of rice is five bags per month. Furthermore, let us consider that a fraction 0.01 of total inventory deteriorates, and for which disposal cost incurs of 5 % of purchasing cost. Maintenance and carrying cost of rice in the retailer's warehouse is $1 per bag per month. The retailer sells it $25 per bag. The supplier review the retailer stocks randomly and is uniformly distributed over the periods: (a) [1, 5] and (ii) [3, 8]. The retailer's inventory cannot exceed maximum capacity 10 bags for regular purchasing (when temporary price discount does not implement). Shortage is permissible at the retailer level, but all customers are not willing to wait. We here consider that 5 % customers leave the system. Backlogging cost of shortage is $2 per bag and per unit lost sale cost is $2. Let us consider that the supplier offers once a temporary price discount to encourage the purchasing. Different discounted price is tested here, e.g., ratio of discounted price and actual price are {0.50, 0.55, 0.60, 0.65, 0.70}.

The input parameters for the above problem are as follow: $r = 5, q = 10, \theta = 0.01$, $\alpha = 0.05$, $p = 10$, $v = 25$, $h = 1$, $d = 0.5, b = l = 2, \frac{p_d}{p} = [0.50, 0.55, 0.60, 0.65, 0.70]$. The parameters of random review period are: (a) $x_{\min} = 1$, $x_{\max} = 5$ and (b) $x_{\min} = 3, x_{\max} = 8$. The optimum solution is obtained in Table 1 for both cases of uniform distribution. Concavity of the cost function for this input parameter is shown in Fig. 2.

4.1.1 Sensitivity of demand and deterioration

"How demand and deterioration make effects on decision policy?" is examined here. For this we change percentage value of demand rate r and deterioration rate θ from −40 to 40 %, and effectiveness is shown in Tables 2 and 3, respectively. Table 2 evinces that when demand rate increases then expected inventory during special sale offer rapidly increases, whereas during regular cycle it rapidly decreases. Moreover, profit saving T_p^* also rapidly increases. That indicates if market demand rate is high then retailer should procure more item during special sale offer. A little increment in backordered quantity and in lost sales is also significant. Table 3 indicates that when deterioration rate increases, the profit saving slightly decreases, whereas backordered quantities and lost sales during both special sale offer and regular cycle increase. A slight decrement in maximum inventory level q_s^* due to increasing deterioration rate is also signified. A common perceive that 'demand have high impact in an inventory system' is revealed here, also.

4.2 Truncated normal distribution function

Example 2 We now suppose that the review period is truncated in $[x_{\min}, x_{\max}]$ for normally distributed function. The value of the parameters are same as Example 1, that is $r = 5, q = 10, \theta = 0.01, \alpha = 0.05, p = 10, v = 25, h = 1$, $d = 0.5, b = l = 2, \frac{p_d}{p} = [0.50, 0.55, 0.60, 0.65, 0.70]$. Moreover, we consider two cases of truncated normal distribution as (a) $x_{\min} = 1, x_{\max} = 5, \rho = 3, \sigma = 2$ and (b) $x_{\min} = 3, x_{\max} = 8, \rho = 5.5, \sigma = 2$. The optimum solutions for both data sets are provided in Table 4.

4.2.1 Sensitivity analysis for truncated normal distribution function

Similar to Example 1, we change percentage value of r and θ from −40 to 40 %. The effect of changes of values for r and θ on decision parameters are shown in Tables 5 and 6, respectively. Table 5 evinces that when demand rate r increases, then expected inventory during special sale offer is increased rapidly, whereas during regular cycle it decreases. Moreover, maximum inventory level q_s^* during special sale offer increases rapidly. That mean, more procurement during special sale offer generate more saving. This observation is similar as in Example 1, but magnitude of earlier is less than later. Table 6 evinces the case of increment in deterioration rate. When deterioration rate increases, then profit save, maximum inventory during special sale offer increase slightly, whereas backordered quantities and lost sales during both special sale offer and regular cycle increase slightly.

The sensitivity analysis of both examples for different distribution functions show that the tendency of decision variables for changes values of demand rate and deterioration rate is almost same, but in case of uniform distribution magnitude of profit saving is more. Throughout this numerical experiment we observe that uniform distributed random review period gives more profit compare to truncated normal distribution.

Table 1 Optimal solution under variation of various cost parameter for uniform distribution

α	$x_{\min}$	$x_{\max}$	$\frac{p_d}{p}$	q_s^*	T_p^*	$\bar{I}_s$	$\bar{Q}_s$	$\bar{D}_s$	μ_s	$\bar{B}_s$	$\bar{L}_s$
0.05	1	5	0.50	18.3722	42.0604	29.227	14.1427	0.292270	3.60855	0.0605045	1.14958
			0.55	18.6883	46.5414	30.089	14.2498	0.300888	3.6695	0.0553197	1.05107
			0.60	18.9820	51.0730	30.895	14.3454	0.308952	3.72611	0.050125	0.963537
			0.65	19.2555	55.6498	31.651	14.4310	0.316506	3.77879	0.046651	0.885497
			0.70	19.5105	60.2672	32.359	14.5079	0.323591	3.8279	0.0429318	0.815704
	3	8	0.50	28.3430	182.188	74.756	25.3114	0.747561	5.51376	0.154535	2.93617
			0.55	28.7944	197.810	76.914	25.5312	0.769144	5.59916	0.144101	2.73791
			0.60	29.2223	213.537	78.976	25.7332	0.789764	5.68004	0.134555	2.55655
			0.65	29.6282	229.361	80.946	25.9191	0.809464	5.75671	0.125809	2.23037
			0.70	30.0135	245.274	82.829	26.0904	0.828290	5.82944	0.117783	2.23788
0.10	1	5	0.50	18.4104	40.9183	29.331	14.2157	0.293307	3.61591	0.119732	1.07759
			0.55	18.7088	45.1479	30.145	14.3116	0.301450	3.67345	0.109983	0.989845
			0.60	18.9873	49.4230	30.910	14.3977	0.309097	3.72712	0.101264	0.911372
			0.65	19.2475	53.7390	31.629	14.4753	0.316287	3.77726	0.093443	0.840987
			0.70	19.4912	58.0918	32.305	14.5454	0.323059	3.82419	0.0864086	0.777678
	3	8	0.50	28.0169	172.656	73.208	25.3106	0.723082	5.45202	0.324609	2.92149
			0.55	28.4638	187.278	75.332	25.5226	0.753318	5.53661	0.303413	2.73072
			0.60	28.8883	202.093	77.366	25.7181	0.773655	5.61691	0.283955	2.55559
			0.65	29.2919	217.002	79.313	25.8985	0.793134	5.6932	0.266066	2.39459
			0.70	29.6760	231.998	81.179	26.0654	0.811793	5.76574	0.249597	2.24637
0.15	1	5	0.50	18.4547	39.8239	29.451	14.2893	0.294512	3.62445	0.177387	1.0052
			0.55	18.7353	43.8026	30.217	14.3745	0.302175	3.67856	0.163706	0.927666
			0.60	18.9982	47.8219	30.940	14.4515	0.3094	3.72924	0.151391	0.857881
			0.65	19.2450	51.8778	31.622	14.5213	0.316216	3.77677	0.140277	0.794903
			0.70	19.4769	55.9669	32.265	14.5847	0.322654	3.82142	0.130223	0.737929
	3	8	0.50	27.7647	164.291	72.018	25.3566	0.720178	5.40425	0.505343	2.86361
			0.55	28.2017	178.118	74.084	25.5569	0.740843	5.48702	0.473632	2.68391
			0.60	28.6179	192.042	76.069	25.7424	0.760686	5.56578	0.444405	2.5183
			0.65	29.0147	206.055	77.974	25.9143	0.779739	5.6408	0.417435	2.36547
			0.70	29.3930	220.151	79.803	26.0738	0.798035	5.7123	0.392517	2.22426

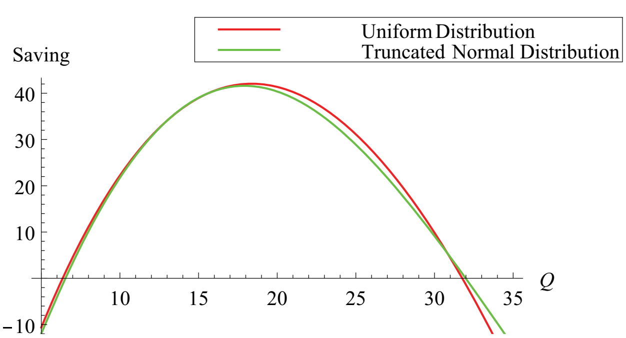

Fig. 2 Concavity of saving function

5 Conclusions

This paper presents an integrated inventory model that simultaneously deals with deterioration, temporary price discount and partial backlogging in stochastic framework. To capture the real world situations, we have considered review period as a random variable, and discussed two cases of random review period: (a) uniform distribution and (b) truncated normal distribution function. If we take deterioration rate as zero, then our model coincides with Karimi-Nasab and Konstantaras (2013), which validates the authenticity of the formulation. The model is mathematically analyzed to find the unique optimal policy. Furthermore, the competency of this model is elaborated by the way of numerical examples and its sensitivity analysis for changes of the key parameters. Through the numerical experiment, we can say that more purchasing during special sale offer significantly benefits the retailer. Furthermore, we find that if review period is truncated normally distributed then profit saving is less compared to uniform distribution. This model can be implemented in many industries such as foodstuffs, chemical industries,

Table 2 Sensitivity analysis of demand for uniform distribution

% change in r	r	q_s^*	T_p^*	μ_s	$\bar{I}_s$	$\bar{B}_s$	$\bar{L}_s$	$\bar{Q}_s$	$\bar{I}$	$\bar{B}$	$\bar{L}$	$\bar{Q}$
−40	3	11.0784	−1.609	3.62625	17.6859	0.03538	0.67231	8.50454	14.8173	0.055536	1.0552	8.0930
−20	4	14.6458	16.758	3.59602	23.2406	0.04928	0.93630	11.2961	11.7649	0.160116	3.0422	9.07545
0	5	18.3722	42.060	3.60855	29.2270	0.0605	1.14958	14.1427	9.6719	0.284963	5.4143	9.68242
20	6	22.2387	72.217	3.63941	35.5950	0.06942	1.31899	17.0370	8.1722	0.420108	7.9821	10.0997
40	7	26.2285	106.369	3.67844	42.3021	0.07641	1.45179	19.9712	7.0542	0.561199	10.663	10.4078

Table 3 Sensitivity analysis of deterioration for uniform distribution

% change in θ	θ	q_s^*	T_p^*	μ_s	$\bar{I}_s$	$\bar{B}_s$	$\bar{L}_s$	$\bar{Q}_s$	$\bar{I}$	$\bar{B}$	$\bar{L}$	$\bar{Q}$
−40	0.006	18.6005	42.8454	3.67918	30.0687	0.0545175	1.03583	14.1446	9.71943	0.283487	5.38625	9.67207
−20	0.008	18.4855	42.4463	3.64349	29.6434	0.0575039	1.09257	14.1446	9.69561	0.284226	5.4003	9.67727
0	0.010	18.3722	42.0604	3.60855	29.227	0.0604045	1.14958	14.1427	9.67191	0.284963	5.4143	9.68242
20	0.012	18.2605	41.6874	3.57435	28.8195	0.0635135	1.20679	14.1390	9.64834	0.285697	5.42824	9.68754
40	0.014	18.1505	41.3268	3.54087	28.4208	0.0665329	1.26413	14.1338	9.62489	0.286428	5.44212	9.69262

Table 4 Optimal solution under variation of various cost parameter for Truncated normal Distribution

σ	α	$x_{\min}$	$x_{\max}$	$\frac{p_d}{p}$	q_s^*	T_p^*	$\bar{I}_s$	$\bar{Q}_s$	$\bar{D}_s$	μ_s	$\bar{B}_s$	$\bar{L}_s$
2	0.05	1	5	0.50	17.9163	41.6034	28.3087	14.1481	0.283087	3.52056	0.0597373	1.13501
				0.55	18.2037	45.9807	29.0931	14.246	0.290931	3.57603	0.0549975	1.04495
				0.60	18.472	50.4045	29.8299	14.3337	0.298299	3.62778	0.0507696	0.964623
				0.65	18.723	54.8698	30.5234	14.4125	0.305234	3.67619	0.0469837	0.892691
				0.70	18.9585	59.3725	31.1771	14.4837	0.311771	3.72158	0.0435813	0.828044
		3	8	0.50	20.8038	37.946	19.5764	8.7916	0.195764	4.07652	0.0247296	0.469862
				0.55	21.0432	41.5938	19.9804	8.83976	0.199804	4.12248	0.0224076	0.425744
				0.60	21.2622	45.2643	20.3519	8.88207	0.203519	4.16451	0.0203762	0.387148
				0.65	21.4633	48.9546	20.6945	8.91942	0.206945	4.20308	0.0185905	0.35322
				0.70	21.6485	52.6625	21.0112	8.95255	0.210112	4.23859	0.0170138	0.323262
3	0.1	1	5	0.50	18.1951	40.7078	28.8942	14.2145	0.288942	3.57437	0.119381	1.07443
				0.55	18.4812	44.8905	29.6752	14.3066	0.296752	3.62957	0.110014	0.990123
				0.60	18.7489	49.1168	30.4102	14.3895	0.304102	3.68118	0.101619	0.914573
				0.65	18.9997	53.3825	31.1028	14.4644	0.311028	3.72951	0.0940729	0.846656
				0.70	19.2351	57.6839	31.7563	14.5321	0.317563	3.77487	0.0872684	0.785416
		3	8	0.50	19.707	35.1957	21.3236	9.89909	0.213236	3.86571	0.0685893	0.617304
				0.55	19.9724	38.6409	21.8271	9.95734	0.218271	3.91676	0.0626763	0.564087
				0.60	20.2179	42.1135	22.2955	10.0092	0.222955	3.96396	0.0574339	0.516905
				0.65	20.4456	45.6107	22.7321	10.0556	0.227321	4.00772	0.052769	0.474921
				0.70	20.6572	49.1299	23.1397	10.0971	0.231397	4.04838	0.0486039	0.437435
4	0.15	1	5	0.50	18.3308	39.7105	29.1992	14.2878	0.291992	3.60055	0.177209	1.00419
				0.55	18.6049	43.6635	29.9478	14.3711	0.299478	3.65341	0.163833	0.928385
				0.60	18.862	47.6561	30.6544	14.4465	0.306544	3.70299	0.151778	0.860073
				0.65	19.1037	51.6847	31.3219	14.5149	0.313219	3.74956	0.140885	0.798346
				0.70	19.3312	55.7459	31.9533	14.5771	0.319533	3.79338	0.131016	0.742424
		3	8	0.50	19.2274	33.3681	22.2233	10.4991	0.222233	3.77339	0.116882	0.662329
				0.55	19.4912	36.5942	22.7522	10.5574	0.227522	3.82417	0.107528	0.609323
				0.60	19.737	39.848	23.2481	10.6099	0.232481	3.87149	0.0991511	0.561856
				0.65	19.9667	43.1268	23.7137	10.6571	0.237137	3.91566	0.0916277	0.519224
				0.70	20.1816	46.428	24.1513	10.6999	0.241513	3.95699	0.0848515	0.480825

Table 5 Sensitivity analysis of demand for truncated normal distribution

% change in r	r	q_s^*	T_p^*	μ_s	$\bar{I}_s$	$\bar{B}_s$	$\bar{L}_s$	$\bar{Q}_s$	$\bar{I}$	$\bar{B}$	$\bar{L}$	$\bar{Q}$
−40	3	10.801	**−2.44057**	3.53705	17.1248	0.0349835	0.664686	8.50656	14.9787	0.0497585	0.945411	8.20437
−20	4	14.2815	15.7296	3.50811	22.5068	0.0486646	0.924627	10.1553	11.3004	0.153229	2.91135	9.20718
0	5	17.9163	41.6034	3.52056	28.3087	0.0597373	1.13501	14.1481	9.71239	0.27961	5.31259	9.78454
20	6	21.6831	72.5115	3.55009	34.4707	0.0686246	1.30387	17.0408	8.1885	0.416673	7.91679	10.1651
40	7	25.5639	107.394	3.58688	40.9461	0.0757338	1.43894	19.9705	7.0596	0.559346	10.6276	10.4430

Table 6 Sensitivity analysis of deterioration for truncated normal distribution

% change in θ	θ	q_s^*	T_p^*	μ_s	$\bar{I}_s$	$\bar{B}_s$	$\bar{L}_s$	$\bar{Q}_s$	$\bar{I}$	$\bar{B}$	$\bar{L}$	$\bar{Q}$
−40	0.006	18.1163	42.3036	3.58444	29.065	0.0542975	1.03165	14.1427	9.76072	0.278074	5.28340	9.77516
−20	0.008	18.0153	41.9476	3.55212	28.6823	0.0570142	1.08327	14.1462	9.73649	0.278843	5.29802	9.77987
0	0.010	17.9163	41.6034	3.52056	28.3087	0.0597373	1.13501	14.1481	9.71239	0.27961	5.31259	9.78454
20	0.012	17.8192	41.2704	3.48974	27.9442	0.0624642	1.18682	14.1485	9.68843	0.280373	5.32709	9.78917
40	0.014	17.724	40.9484	3.45964	27.5883	0.0651927	1.23866	14.1476	9.66459	0.281134	5.34154	9.79376

pharmaceutical drugs, photographic films, etc., where supplier visits the retailer in irregular interval, and offers the special sale to bust the purchasing. Furthermore, finite shelf life is a characteristic of these types of products, which is taken into the account in the formulation of the model.

This research work can be further extended to other cases like considering stochastic demand rate, to include preservation technology to prevent from deterioration or/and increasing deterioration rate.

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Appendix 1

$$\frac{d^2T_p}{dq_s{}^2} = \frac{-1}{r(x_{\max} - x_{\min})(\frac{\theta}{r}q_s + 1)^2}\{(a_1 + a_2)\theta(x_{\max} - \mu_s) + a_2\}.$$

If $a_1 \geq 0$ or $a_1 < 0$ and $a_2 \geq -a_1$, then $d^2T_p/dq_s^2 < 0$ if and only if $a_2 > -(a_1 + a_2)\theta(x_{\max} - \mu_s) \Rightarrow q_s < \left(e^{\frac{a_2}{a_1+a_2}+\theta x_{\max}} - 1\right)r/\theta$. Moreover, if $a_1 < 0$ and $a_2 < -a_1$, then $d^2T_p/dq_s^2 < 0$ if and only if $a_2 > -(a_1 + a_2)\theta(x_{\max} - \mu_s) \Rightarrow q_s > \left(e^{\frac{a_2}{a_1+a_2}+\theta x_{\max}} - 1\right)r/\theta$.

Appendix 2

$$\frac{d^2T_p}{dq_s{}^2} = \frac{-1}{\sqrt{2\pi}r\sigma(\Phi(x_{\max}) - \Phi(x_{\min}))(\frac{\theta}{r}q_s + 1)^2}\left\{a_2e^{-\frac{1}{2}(\frac{\rho-\mu_s}{\sigma})^2} + \theta\sigma(a_1 + a_2)\sqrt{\frac{\pi}{2}}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] + Erf\left[\frac{x_{\max}-\rho}{\sqrt{2}\sigma}\right]\right)\right\}.$$

If $a_1 \geq 0$ or $a_1 < 0$ and $a_2 \geq -a_1$, then $d^2T_p/dq_s^2 < 0$ if $a_2e^{-\frac{1}{2}(\frac{\rho-\mu_s}{\sigma})^2} + \theta\sigma(a_1 + a_2)\sqrt{\frac{\pi}{2}}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] + Erf\left[\frac{x_{\max}-\rho}{\sqrt{2}\sigma}\right]\right) > 0, \Rightarrow \frac{a_2}{\theta\sigma(a_1+a_2)}e^{-\frac{1}{2}(\frac{\rho-\mu_s}{\sigma})^2} + \sqrt{\frac{\pi}{2}}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] + Erf\left[\frac{x_{\max}-\rho}{\sqrt{2}\sigma}\right]\right) > 0.$. Moreover, if $a_1 < 0$ and $a_2 < -a_1$, then $d^2T_p/dq_s^2 < 0$ if and only if $\frac{a_2}{\theta\sigma(a_1+a_2)}e^{-\frac{1}{2}(\frac{\rho-\mu_s}{\sigma})^2} + \sqrt{\frac{\pi}{2}}\left(Erf\left[\frac{\rho-\mu_s}{\sqrt{2}\sigma}\right] + Erf\left[\frac{x_{\max}-\rho}{\sqrt{2}\sigma}\right]\right) < 0$.

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