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ORIGINAL ARTICLE

MTSF (mean time to system failure) and profit analysis of a single-unit system with inspection for feasibility of repair beyond warranty

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Abstract This paper discussed the MTSF and profit analysis of a single-unit system with inspection for feasibility of repair beyond warranty subject to a single repair facility. Any failure during warranty is rectified by the manufacturer free of cost to the users provided failures are not due to the negligence of users. Beyond warranty, unit goes under inspection after failure for feasibility of its repair or replacement. The failure time of the system follows negative exponential distribution while repair and inspection time distributions are taken as arbitrary. The expressions for reliability, MTSF, availability of the system and profit function have been determined by using supplementary variable technique. Using Abel's lemma steady state behavior of the system has been derived. The numerical results for reliability and profit function are also obtained by taking particular values of various parameters and repair cost.

Keywords MTSF · Profit analysis · Inspection · Warranty

1 Introduction

Reliability plays a key role in the design of engineering systems. Achieving a high or required level of reliability is often an essential requisite. Several authors including Uematsu and Nishida (1987), Jin et al. (2009), Kharoufeh et al. (2010), Kaur et al. (2013) and Kadyan et al. (2013) studied single unit reliability models under the common assumptions:

(i) No warranty is provided for system.
(ii) Repair of the failed unit is always possible.

But, market survey shows that a large number of products are now being sold with a warranty which indicates the high popularity of warranty among customers. The warranty assures the users that a faulty item will either be repaired or replaced at no cost and that may increase sales.

Further, repair of the failed unit does not always feasible. Sometime replacement of the failed unit is cheaper than to continue its repair. In such cases, the failed unit may be replaced by new unit after getting necessary inspection for feasibility of repair in order to avoid unnecessary expenses on repair.

While considering the above facts, here we developed a single unit system with the concept of inspection for feasibility of repair beyond warranty subject to single repair facility. The following reliability measures are obtained by using supplementary variable technique.

(i) MTSF and Reliability;
(ii) Steady-state availability of the system; and
(iii) Profit analysis of the user.

2 Assumptions

(1) The system has a single unit.
(2) There is single repairman, who is always available with the system to do repair, inspection and replacement of the unit.

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(3) The cost of repair of the failed unit within warranty is borne by the manufacturer provided failures are not due to the negligence of users such as cracked screen, accident, misuse, physical damage, damage due to liquid and unauthorized modifications etc.
(4) Beyond warranty, unit goes for inspection after failure.
(5) Repairman inspects the failed unit to see the feasibility of repair or replacement.
(6) The distribution of failure time is taken as negative exponential while the inspection, and repair times are considered as arbitrary.

3 State-specification

S_0/ S_1	The unit is operative under warranty/beyond warranty
S_2/ S_4	The unit is in failed state under warranty/beyond warranty
S_3	The failed unit is under inspection

4 Notations

λ/λ_1	Constant failure rate of the unit within warranty/beyond warranty
α	Constant rate of completion of warranty
p/q	Probability that repair is feasible/not feasible
$\mu(x), S(x)/\mu_1(x), S_1(x)$	Repair rate of the unit and probability density function, for the elapsed repair time 'x' in warranty/beyond warranty
$h(y), S_2(y)$	Inspection rate of the failed unit and probability density function, for the elapsed inspection time 'y'
$p_0(t)/p_1(t)$	The Probability that at time 't' the system is in good state within warranty/beyond warranty
$p_2(x,t)\Delta/p_4(x,t)\Delta$	The Probability that at time 't' the system is in failed state within warranty/beyond warranty, the elapsed repair time lies in the interval [x,x + Δ)
$p_3(y,t)\Delta$	The Probability that at time 't' the failed unit is under inspection, the elapsed inspection time lies in the interval [y,y + Δ)
$p(s)$	Laplace transform of function $p(t)$
$S(x)$	$\mu(x)\exp[-\int_0^x \mu(x)dx]$
$S_1(x)$	$\mu_1(x)\exp[-\int_0^x \mu_1(x)dx]$
$S_2(x)$	$h(y)\exp[-\int_0^y h(y)dy]$

5 Formulation of mathematical model of system

Using the probabilistic arguments and limiting transitions (shown in Appendix), we have the following difference-differential equations (Cox 1962):

$$\left[\frac{d}{dt}+\lambda+\alpha\right]p_0(t)=\int_0^\infty \mu(x)p_2(x,t)dx \tag{1}$$

$$\left[\frac{d}{dt}+\lambda_1\right]p_1(t)=\alpha p_0(t)+\int_0^\infty \mu_1(x)p_4(x,t)dx + \int_0^\infty qh(y)p_3(y,t)dy \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu(x)\right]p_2(x,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+h(y)\right]p_3(y,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_1(x)\right]p_4(t)=0 \tag{5}$$

5.1 Boundary conditions

$$p_2(0,t)=\lambda p_0(t) \tag{6}$$

$$p_3(0,t)=\lambda_1 p_1(t) \tag{7}$$

$$p_4(0,t)=\int_0^\infty ph(y)p_3(y,t)dy \tag{8}$$

5.2 Initial conditions

$$\begin{aligned} p_i(0)&=0;\ \text{when } i\neq 0\\ p_i(0)&=1;\ \text{when } i=0 \end{aligned} \tag{9}$$

6 Model analysis

6.1 Solution of the equations

Taking Laplace transforms of Eqs. (1)–(8) and using (9), we obtain

$$[s+\lambda+\alpha]p_0(s)=1+\int_0^\infty \mu(x)p_2(x,s)dx \tag{10}$$

$$[s+\lambda_1]p_1(s) = \alpha p_0(s) + \int_0^\infty \mu_1(x)p_4(x,s)dx + \int_0^\infty qh(y)p_3(y,s)dy \quad (11)$$

$$\left[\frac{\partial}{\partial x} + s + \mu(x)\right]p_2(x,s) = 0 \quad (12)$$

$$\left[\frac{\partial}{\partial x} + s + h(y)\right]p_3(y,s) = 0 \quad (13)$$

$$\left[\frac{\partial}{\partial x} + s + \mu_1(x)\right]p_4(x,s) = 0 \quad (14)$$

$$p_2(0,s) = \lambda p_0(s) \quad (15)$$

$$p_3(0,s) = \lambda_1 p_1(s) \quad (16)$$

$$p_4(0,s) = \int_0^\infty ph(y)p_3(y,s)dy \quad (17)$$

Taking integration of Eqs. (12), (13) and (14), we get the following equations

$$p_2(x,s) = p_2(0,t)\exp(-(sx + \int_0^x \mu(x)dx)) \quad (18)$$

$$p_3(y,s) = p_3(0,t)\exp(-(sy + \int_0^y h(y)dy)) \quad (19)$$

and

$$p_4(x,s) = p_4(0,t)\exp(-(sx + \int_0^x \mu_1(x)dx)) \quad (20)$$

Using Eqs. (15) and (18), Eq. (10) yields

$$[s+\lambda+\alpha]p_0(s) = 1 + p_2(0,s)\int_0^\infty \mu(x) \times \exp\left(-\left(\int_0^x \mu(x)dx\right)\right)dx = 1 + \lambda p_0(s)S(s)$$

$$p_0(s) = \frac{1}{T(s)} \quad (21)$$

where

$$T(s) = s + \lambda + \alpha - \lambda S(s) \quad (22)$$

$$p_4(0,s) = p_3(0,t)\int_0^\infty ph(y)\exp(-(sy + \int_0^y h(y)dy))dy$$

$$p_4(0,s) = p\lambda_1 p_1(s)S_h(s) \quad (23)$$

Using Eq. (23), Eq. (20) yields

$$p_4(x,s) = p\lambda_1 p_1(s)S_h(s)\exp\left(-\left(sx + \int_0^x \mu_1(x)dx\right)\right) \quad (24)$$

Using Eqs. (16), (19), (23) and (24), Eq. (11) yields

$$[s+\lambda_1]p_1(s) = \alpha p_0(s) + p_4(0,s)\int_0^\infty \mu_1(x) \times \exp(-(sx + \int_0^x \mu_1(x)dx)dx + p_3(0,s)\int_0^\infty qh(y) \times \exp(-(sy + \int_0^y h(y)dy)dy = \alpha p_0(s) + p\lambda_1 p_1(s)S_1(s)S_h(s) + q\lambda_1 p_1(s)S_h(s)$$

$$p_1(s) = \frac{A(s)}{T(s)} \quad (25)$$

where

$$A(s) = \frac{\alpha}{(s + \lambda_1 - p\lambda_1 S_1(s)S_h(s) + q\lambda_1 S_h(s))} \quad (26)$$

Now, the Laplace transform of the probability that the system is in the failed state is given by

$$p_2(s) = \int_0^\infty p_2(x,s)dx = \lambda p_0(s)\left(\frac{1-S(s)}{s}\right)$$

$$p_2(s) = \frac{\lambda B(s)}{T(s)} \quad (27)$$

where

$$B(s) = \left(\frac{1-S(s)}{s}\right) \quad (28)$$

Similarly $p_3(s) = \int_0^\infty p_3(y,s)dy = \lambda_1 p_1(s)\left(\frac{1-S_h(s)}{s}\right)$

$$p_3(s) = \frac{(\lambda A(s)C(s))}{T(s)} \quad (29)$$

where

$$C(s) = \left(\frac{1-S_h(s)}{s}\right) \quad (30)$$

Similarly

$$p_4(s) = \int_0^\infty p_4(x,s)dx = p\lambda_1 p_1(s)S_h(s)\left(\frac{1-S_1(s)}{s}\right)$$

$$p_4(s) = \frac{(\lambda_1 A(s)S_h(s)D(s))}{T(s)} \quad (31)$$

where

$$D(s) = \left(\frac{1-S_1(s)}{s}\right) \quad (32)$$

It is worth noticing that

$$p_0(s) + p_1(s) + p_2(s) + p_4(s) = \frac{1}{s} \tag{33}$$

6.2 Evaluation of Laplace transforms of up and down state probabilities

Let Av(t) is the probability that the system is operating satisfactorily at time '*t*'. The Laplace transforms of Av(t) or probabilities that the system is in up state ($P_{up}(t)$) (i.e. good state) and down state ($P_{down}(t)$) (i.e. failed state) at time '*t*' are as follows

$A_v(s)$ *or* $p_{up}(s) = p_0(s) + p_1(s)$

$$A_v(s) = \frac{(1 + A(s))}{T(s)} \tag{34}$$

$p_{down}(s) = p_2(s) + p_3(s) + p_4(s)$

$$p_{down}(s) = \frac{(\lambda B(s) + \lambda_1 A(s) C(s) + \lambda_1 p A(s) D(s) S_h(s))}{T(s)} \tag{35}$$

6.3 Steady-state behavior of the system

Using Abel's Lemma in Laplace transforms, viz. $\lim_{s\to 0} s(A_v(s)) = \lim_{t\to\infty} A_v(t) = A_v(say)$, Provided the limit on the right hand side exists, the following time independent probabilities have been obtained.

$$A_v = \frac{1}{(1 - p\lambda_1 S_1'(0) - \lambda_1 S_h'(0))} \tag{36}$$

$$P_{down} = \frac{-p\lambda_1 S_1'(0) - \lambda_1 S_h'(0)}{(1 - p\lambda_1 S_1'(0) - \lambda_1 S_h'(0))} \tag{37}$$

6.4 Reliability of the system (R(t))

Using the method similar to that in Sect. 5, the differential–difference equations for reliability are:

$$\left[\frac{d}{dt} + \lambda + \alpha\right] p_0(t) = 0 \tag{38}$$

$$\left[\frac{d}{dt} + \lambda_1\right] p_1(t) = \alpha p_0(t) \tag{39}$$

Theorem 1 The reliability of the system is given by

$$R(t) = \exp(-(\lambda+\alpha)t)\left[\frac{\lambda - \lambda_1}{\lambda - \lambda_1 + \alpha}\right] + \exp(-\lambda_1 t)\left[\frac{\alpha}{\lambda - \lambda_1 + \alpha}\right]$$

The proof of the Theorem-1 is given in the Appendix.

Corollary 1 The mean time to system failure (MTSF) is

$$MTSF = \left[\frac{\lambda - \lambda_1}{(\lambda - \lambda_1 + \alpha)(\lambda + \alpha)}\right] + \left[\frac{\alpha}{(\lambda - \lambda_1 + \alpha)(\lambda_1)}\right]$$

Proof Calculating $MTSF = \int_0^\infty R(t)dt$ implies the result '*' given in the Appendix.

7 Particular results

7.1 Availability of the system

If repair and inspection times follow exponential distribution i.e. $S(s) = \frac{\mu}{(s+\mu)}$, $S_1(s) = \frac{\mu_1}{(s+\mu_1)}$ *and* $S_h(s) = \frac{h}{(s+h)}$ where μ and μ_1 are constant repair rates and h is constant inspection rate. Putting these values in Eqs. (21)–(26), we get

$$p_0(s) = \frac{1}{I(s)} \tag{40}$$

where

$$I(s) = \frac{(s^2 + s(\lambda + \alpha + \mu) + \alpha\mu)}{(s + \mu)} \tag{41}$$

$$p_1(s) = \frac{E(s)}{I(s)} \tag{42}$$

where

$$E(s) = \left[\frac{\alpha(s+\mu_1)(s+h)}{(s+\mu_1)(s+h)(s+\lambda_1) - ph\lambda_1\mu_1 - qh\lambda_1(s+\mu_1)}\right] \tag{43}$$

$A_v(s)$ *or* $p_{up}(s) = p_0(s) + p_1(s)$

$$= \left[\frac{(s^4 + b_3 s^3 + b_2 s^2 + b_1 s)}{s(s^2 + s(\lambda + \alpha + \mu) + \alpha\mu)(s^2 + sa_1 + a_0)}\right] \tag{44}$$

where $b_3 = (\lambda_1 + h + \alpha + \mu_1 + \mu)$, $b_2 = (\lambda_1\mu + \mu\alpha + \alpha\mu_1 + \mu_1 h + \mu h + \mu\mu_1 + \lambda_1 h + h\alpha + \lambda_1\mu_1 - qh\lambda_1)$, $b_1 = (\mu\mu_1 h + \lambda_1\mu h + \lambda_1\mu\mu_1 - \lambda_1\mu q h + \alpha\mu\mu_1 + h\alpha\mu + h\alpha\mu_1)$, $b_0 = (\alpha\mu\mu_1 h) a_1 = (\mu_1 + h + \lambda_1)$ *and* $a_0 = (\lambda_1 h + \mu_1 h + \lambda_1\mu_1 - q\lambda_1 h)$

Taking inverse Laplace transforms of Eq. (44), we get

$$A_v(t) = \frac{b_0}{z_1 z_2 z_3 z_4} + \left\{\frac{z_1^4 + b_3 z_1^3 + b_2 z_1^2 + b_1 z_1 + b_0}{z_1(z_1 - z_2)(z_1 - z_3)(z_1 - z_4)}\right\} \exp(z_1 t) + \left\{\frac{z_2^4 + b_3 z_2^3 + b_2 z_2^2 + b_1 z_2 + b_0}{z_2(z_2 - z_1)(z_2 - z_3)(z_2 - z_4)}\right\} \exp(z_2 t)$$
$$+ \left\{\frac{z_3^4 + b_3 z_3^3 + b_2 z_3^2 + b_1 z_3 + b_0}{z_3(z_3 - z_1)(z_3 - z_2)(z_3 - z_4)}\right\} \exp(z_3 t) + \left\{\frac{z_4^4 + b_3 z_4^3 + b_2 z_4^2 + b_1 z_4 + b_0}{z_4(z_4 - z_1)(z_4 - z_2)(z_4 - z_3)}\right\} \exp(z_4 t) \tag{45}$$

z_1 *and* z_2 are the roots of the equation $(s^2 + s(\lambda + \alpha + \mu) + \alpha\mu) = 0$ and z_3, z_4 are roots of the equation $(s^2 + sa_1 + a_0) = 0$ (Balagurusamy 2009)

7.2 Profit analysis of the user

Suppose that the warranty period of the system is (0, w]. Since the repairman is always available with the system, therefore beyond warranty period, it remains busy during the interval (w, t]. Let K_1 be the revenue per unit time and K_2 be the repair cost per unit time, then the expected profit H(t) during the interval (0, t] is given by

$$H(t) = K_1 \int_0^t A_v(t)dt - K_2(t - w)$$

8 Numerical analysis

The research of numerical analysis is given in Tables 1, 2 and 3.

$$H(t) = K_1 \left[\frac{b_0 t}{z_1 z_2 z_3 z_4} + \left\{ \frac{z_1^4 + b_3 z_1^3 + b_2 z_1^2 + b_1 z_1 + b_0}{z_1^2 (z_1 - z_2)(z_1 - z_3)(z_1 - z_4)} \right\} \exp((z_1 t) - 1) + \left\{ \frac{z_2^4 + b_3 z_2^3 + b_2 z_2^2 + b_1 z_2 + b_0}{z_2^2 (z_2 - z_1)(z_2 - z_3)(z_2 - z_4)} \right\} \exp((z_2 t) - 1) \right. \\ \left. + \left\{ \frac{z_3^4 + b_3 z_3^3 + b_2 z_3^2 + b_1 z_3 + b_0}{z_3^2 (z_3 - z_1)(z_3 - z_2)(z_3 - z_4)} \right\} \exp((z_3 t) - 1) + \left\{ \frac{z_4^4 + b_3 z_4^3 + b_2 z_4^2 + b_1 z_4 + b_0}{z_4^2 (z_4 - z_1)(z_4 - z_2)(z_4 - z_3)} \right\} \exp((z_4 t) - 1) \right] \\ - K_2(t - w) \tag{46}$$

Table 1 Effect of failure rates (λ and λ_1) and rate of completion of warranty (α) on Reliability (R(t))

Time (t)	$\lambda_1 = 0.02$, $\alpha = 0.003$, R(t) (for $\lambda = 0.01$)	$\lambda_1 = 0.02$, $\alpha = 0.003$, R(t) (for $\lambda = 0.02$)	$\lambda = 0.01$, $\alpha = 0.003$, R(t) (for $\lambda_1 = 0.03$)	$\lambda = 0.01$, $\lambda_1 = 0.02$, R(t) (for $\alpha = 0.005$)
10	0.90353744	0.8187308	0.9023208	0.9026852
11	0.89428347	0.8025188	0.8928418	0.89326861
12	0.88510119	0.7866279	0.8834209	0.88391256
13	0.87599061	0.7710516	0.8740593	0.87461773
14	0.86695174	0.7557837	0.864758	0.86538475
15	0.85798456	0.7408182	0.8555182	0.85621422
16	0.84908904	0.726149	0.8463406	0.84710669
17	0.84026512	0.7117703	0.8372263	0.83806267

Table 2 Effect of repair cost (K_2) on expected profit (H(t))

Time (t)	$\lambda = 0.01$, $\lambda_1 = 0.02$, $h = 0.5$, $\alpha = 0.003$, $\mu = 0.2$, $\mu_1 = 0.1$, $q = 0.3$				
	K_1	W	H(t) (For $K_2 = 150$)	H(t) (For $K_2 = 100$)	H(t) (For $K_2 = 50$)
10	500	3	3809.47	4159.47	4509.47
11	500	3	4137.704	4537.704	4937.704
12	500	3	4465.341	4915.341	5365.341
13	500	3	4792.471	5292.471	5792.471
14	500	3	5119.167	5669.167	6219.167
15	500	3	5445.486	6045.486	6645.486
16	500	3	5771.475	6421.475	7071.475
17	500	3	6097.171	6797.171	7497.171

Table 3 Effect of inspection rate (h) on expected profit (H(t))

Time (t)	$\lambda = 0.01$, $\lambda_1 = 0.02$, $\alpha = 0.003$, $\mu = 0.2$, $\mu_1 = 0.1$, $K_1 = 500$, W = 3, $K_2 = 150$ H(t) (For h = 0.5)	$\lambda = 0.01$, $\lambda_1 = 0.02$, $\alpha = 0.003$, $\mu = 0.2$, $\mu_1 = 0.1$, $K_1 = 500$, W = 3, $K_2 = 150$ H(t) (For h = 0.6)
10	3809.47	3809.566
11	4137.704	4137.831
12	4465.341	4465.505
13	4792.471	4792.677
14	5119.167	5119.42
15	5445.486	5445.792
16	5771.475	5771.84
17	6097.171	6097.602

9 Interpretation and conclusion

Table 1 shows the behavior of system reliability and it indicates that reliability of the system decreases with the increase of failure rates (λ and λ_1) and rate of completion of warranty (α) with respect to time and for fixed values of other parameters. Table 2 interprets that expected profit increases with the decrease of repair cost (K_2) with respect to time. Table 3 shows the effect of inspection on expected profit and it is observed that the expected profit increases with the increase of inspection rate (h) with respect to time. Hence, on the basis of the numerical results obtained for particular values of various parameters, it is concluded that the system with inspection beyond warranty can be made profitable to use by decreasing the repair cost and increasing the inspection rate.

Appendix

The derivation of Eqs. (1)–(5)

Assuming failure rates of the system are constant and repair rates are arbitrary. By applying supplementary variable technique, we develop the following differential-difference equations associated with the state transition diagram (Fig. 1) of the system at times (t + Δt), (y + Δy) and (x + Δx).

$$p_0(t+\Delta t) = p_0(t)(1-(\alpha+\lambda)\Delta t) + \int_0^\infty \mu(x)p_2(x,t)\Delta t dx + o(\Delta t),$$

$p_1(t+\Delta t) = p_1(t)(1-\lambda_1\Delta t) + \alpha p_0(t) + \int_0^\infty \mu_1(x)p_4(x,t)\Delta t dx + \int_0^\infty qh(y)p_3(y,t)\Delta t dy + o(\Delta t)$, $p_2(x+\Delta x, t+\Delta t) = p_2(x,t)(1-\mu(x)\Delta x) + o(\Delta x, \Delta t)$,

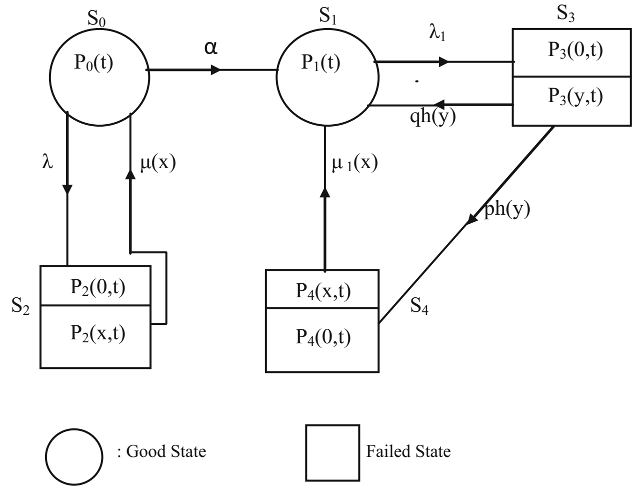

Fig. 1 State Transition Diagram of the Model

$$p_3(y+\Delta y, t+\Delta t) = p_3(y,t)(1-h(y)\Delta y) + o(\Delta y, \Delta t),$$

$$p_4(x+\Delta x, t+\Delta t) = p_4(x,t)(1-\mu_1(x)\Delta x) + o(\Delta x, \Delta t),$$

The proof of Theorem-1

Proof Taking Laplace transforms of Eqs. (38) and (39), using (9), we get

$[s+\lambda+\alpha]p_0(s) = 1$

$[s+\lambda_1]p_0(s) = \alpha p_0(s)$

The solution can be written as

$$p_0(s) = \frac{1}{(s+\lambda+\alpha)}$$

$$p_1(s) = \frac{\alpha}{(s+\lambda+\alpha)(s+\lambda_1)}$$

$$R(s) = p_0(s) + p_1(s)$$

$$= \frac{1}{(s+\lambda+\alpha)} + \frac{\alpha}{(s+\lambda+\alpha)(s+\lambda_1)}$$

Taking inverse Laplace transform, we get

$$R(t) = \exp(-(\lambda+\alpha)t)\left[\frac{\lambda-\lambda_1}{\lambda-\lambda_1+\alpha}\right] + \exp(-\lambda_1 t)\left[\frac{\alpha}{\lambda-\lambda_1+\alpha}\right] \quad (*)$$

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