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ORIGINAL ARTICLE

# Developing environmental indices using fuzzy numbers power average (FN-PA) operator

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**Abstract** Environment assessment is a complex multiple attribute decision-making problem, which is an important component of environmental management. Environmental indices (EI) are very useful for environmental decision-making by policy makers and for maintaining well-informed public. Due to the uncertainty in process of the determination of EI, it is necessary to use a mathematical operator to aggregate various non-commensurate input parameters in a reasonable manner. In this paper, a new mathematic model of environmental indices based on fuzzy numbers power average operator is proposed. Firstly, the data collected from different sources are modeled as fuzzy numbers. Secondly, a method to transform fuzzy variables into basic probability assignments is developed based on the similarity measure between generalized fuzzy numbers. Then conflict data is efficiently combined based on the power average operator. At last, a real application to determine water quality indices is used to illustrate the efficiency of the proposed method.

**Keywords** Power average operator · Environmental indices · Similarity measures · Fuzzy numbers · Data fusion

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## 1 Introduction

Environment assessment is an important component of environmental management, it can help people to study, to protect, and to renovate environment. It is a complex multiple attribute decision-making problem (Pischke and Cashmore 2006; Deng et al. 2014). Environmental indices (EI) can be used as a communication tool to describe the overall status of the environmental systems (including land, air and water) and to study the impact of regulatory policies on various environmental management practices (Sadiq et al. 2005; Pusatli et al. 2009). EI provide a condensed description of multi-dimensional environmental states by aggregating several variables (or indicators) into a single quantity. EI can also help in selecting appropriate decision actions for the improvement of environmental quality by considering various conflicting factors (Sadiq and Tesfamariam 2007; Deng et al. 2011). EI have also been used in life cycle assessment (Weiss et al. 2007; Khan et al. 2004) and characterize different types of environmental damages (Pan and Kao 2009). Recently years, an increasing number of environmental indices had been proposed in the literature (Debels et al. 2005; Kang 2002). Siracusa et al. (2004) proposed a pollutant interaction matrix method to calculate the global environmental protection index. Ebert and Welsch (2004) provided the characterization of meaningful environmental indices. Khanna (2000) developed an index of pollution based on the epidemiological dose-response function associated with each pollutant, and the welfare losses due to exposure to pollution. Gunasekera and Edwards (2003) proposed an index called the atmospheric hazard index (AHI), which can be used to assess the potential impact of airborne by releasing from a chemical production plant.

Two of the main problems should be taken into consideration in the determination of EI. One problem is the information should be well represented and efficiently handled in a flexible way. It is well known that fuzzy set theory is widely used in many uncertain decision makings and strategy selections. Fuzzy set theory is an useful model of uncertain information of linguistic variables, often represented by fuzzy numbers (Chan 2005; Chan et al. 2006; Sadiq et al. 2010; Chan and Chan 2011; Li 2010; Deng and Chen 2011; Deng et al. 2011, 2012, 2014; Gharibi et al. 2012; Meliadou et al. 2012; Sattler et al. 2012; Liu et al. 2012; Zhang et al. 2013; Wei et al. 2013; Chen et al. 2013; Laala et al. 2013; Guettaf et al. 2013; Yuxian et al. 2014). Chan and Kumar (2007) introduced a fuzzy extended AHP (FEAHP) which uses triangular fuzzy numbers to represent decision makers' comparison judgements and fuzzy synthetic extent analysis method to decide the final priority of different decision criteria. Chiadamrong (1999) adopted the concept of fuzzy set theory to overcome the precision-based evaluation for manufacturing strategies selection. Deng and Liu (2005) applied a topsis-based centroid-index ranking method of fuzzy numbers and its application in decision-making. Cheng and Qian (2010) established the index system for emergency plan, and analyzed and processed the index system by the fuzzy comprehensive evaluation method, therefore they provided a quantitative basis for the decision-making. Recently, a large number of fuzzy-based applications for developing environmental indices have been reported in the literature.

Lu and Lo (2002) used self-organizing maps and fuzzy theory to diagnose reservoir water quality. Arunraj and Maiti (2009) proposed a new methodology for the development of environmental consequence index (ECI) by using the fuzzy composite programming (FCP). Nasiri and Huang (2008) proposed a fuzzy multiple attribute analysis approach for the environmental performance assessment of waste recycling programs.

The other problem is the data, which often conflicts with each other in a high degree, should be combined into a reasonable way with a mathematic base. With the development of the environmental indices, some mathematical models were derived, as the abstraction of information and data, some issues and problems are arising. The problems are referred as characteristic properties including ambiguity, eclipsing, compensation and rigidity. Aggregation is a model which is defined as a mathematical tool to reduce a set of numbers to an unique representative or a meaningful number (Peneva and Popchev 2003).

These data aggregation methods generally include logical operators (and, or), averaging or compromising operators (arithmetic average, weighted average, geometric mean, weighted product), and others such as simple addition, root sum power, root sum square, and multiplicative forms (Smolkov and Wachowiak 2002; Silvert 2000).

Over the past few decades, significant literatures referenced on using statistical and mathematical aggregation methods to develop indices for the environmental indices (including air, water, and sediment quality). Ott (1978) introduced the weighted arithmetic mean in his paper on Environmental indices: theory and practice. Sadiq et al. (2010) used penalty functions to evaluate aggregation models for environmental indices. Sadiq and Tesfamariam (2007) proposed a new approach for generating OWA weight distribution by using probability density function (PDFs), they also developed environmental indices using fuzzy numbers ordered weighted averaging (FN-OWA) operators (Sadiq and Tesfamariam 2008). Bellenger and Herlihy (2009) used the directional output distance function from economic productivity theory as an alternative approach to environmental index construction, and provided a nonparametric way to aggregate individual characteristics.

Environmental indices is an important communication tool which is often used to describe the overall status of environmental systems. Environmental indices has been used for a long time and derived from mathematical models. Due to the diverse types and incomplete of input data, it's difficult to aggregate diverse data properly. Generally, aggregating information by means of techniques such as the average is a task common in many information fusion processes. The power average operator (Yager 2001) is provide a kind of empowerment as it allows groups of values close to each and reinforce each. This operator is particularly useful in group decision making and information fusion applications.

In this paper, a fuzzy evidential methodology to determine EI is proposed to take advantage of the desired properties of fuzzy set theory and power average operator. This paper is arranged as follows: Sect. 2 gives some basic introductions of fuzzy set theory and power average operator. The proposed method of power average operator to determine EI is detailed in Sect. 3 step by step. A real example to determine water quality indices shows in some published works (Swamee and Tyagi 2000) is will be used to illustrate the efficiency of the proposed method in Sects. 3 and 4 which ends with conclusions.

## 2 Basic theory

### 2.1 Fuzzy set theory

The nation of fuzzy sets was firstly proposed in 1965 by Zadeh (1965), providing a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership. Fuzzy set theory is an efficient and effective tool to deal with the uncertain information. The fuzzy set theory was used widely in many areas such as business (Ragone et al. 2009; Zhang et al. 2013), medical and related health sciences (Kharal 2009), natural sciences (Feng and Wang 2007; Prato 2009), in which the information is incomplete or imprecise, to assess risks or make decisions and so on (Deng 2006; Fenton and Wang 2006; Liu et al. 2008; Wei and Chen 2009; Lee and Chen 2008; Lee 2008; Chen and Chen 2009; Padma and Balasubramanie 2009; Lee et al. 2009; Chen et al. 2010; Deng et al. 2011; Liu et al. 2012; Kang et al. 2012; Zhang et al. 2012).

Fuzzy set theory is based on the idea that each element in a certain system can get one value within the interval 0 to 1. Mathematically, it can be expressed as below. Let be X a classical set which generates a space, and its elements let be marked x. A fuzzy set A is defined on an universe X may be given as:

$$A = \{(x, u_A(x)) | x \in X\} \tag{1}$$

where $u_A: \ X \to [0,1]$ is the membership function A. The membership value $u_A(x)$ describes the degree of belongingness of $x \in X$ in A.

A fuzzy number describes the relationship between an uncertain quantity x and a membership function $u_A$, which ranges between 0 and 1. A trapezoidal fuzzy number can be represented by five vertices (a, b, c, d, w),where $a \le b \le c \le d$, and $0 < w \le 1$,

The membership function is defined by:

$$u_A = \begin{cases} 0, & x < a \\ \dfrac{(x-a)}{b-a}, & a \le x \le b \\ w, & b \le x \le c \\ \dfrac{(x-c)}{d-c}, & c \le x \le d \\ 0, & x > d \end{cases} \tag{2}$$

If w = 1, then the fuzzy number A is called a normal trapezoidal fuzzy number denote A = (a, b, c, d). If a = b, and c = d, then the fuzzy number A is called a crisp interval. If b = c then the fuzzy number A is called a generalized triangular fuzzy number. If a = b = c = d, then the fuzzy number A is called a real number.

Example: There are three different generalized fuzzy numbers as:

$$A = (0.1, 0.2, 0.4, 0.6; 1.0)$$
$$B = (0.1, 0.2, 0.4, 0.6, 0.7)$$
$$C = (0.6, 0.8, 0.8, 1.0, 1.0)$$

A/B/C shows the three generalized fuzzy numbers in Fig. 1

Compared with normal fuzzy numbers, the generalized fuzzy numbers can be dealt with uncertain information in a more flexible way. For example, in decision making situation, the value $w_1, w_2$ represents the confidence degree of decision-maker A and B's opinion respectively ($w_1 = 1.0, w_2 = 0.7$), and the fuzzy number C is the triangular fuzzy number.

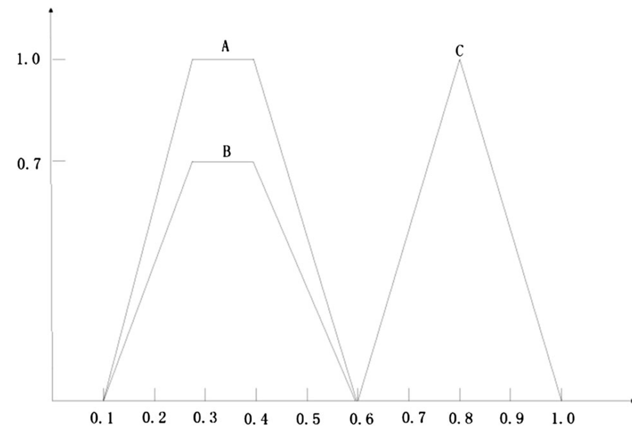


**Fig. 1** Three generalized trapezoidal fuzzy number A, B and C

## 2.2 Linguistic variable

A linguistic or a qualitative scale should be assigned to estimate EI to guide informed decision-making. Linguistic variables are represented in words or sentences or artificial languages, which can also be defined as trapezoidal fuzzy numbers as $(a_k, b_k, c_k, d_k)$. In this paper there five linguistic constants (k = $1, 2, \ldots, 5$) to express the environmental indices over the universe of discourse, namely, very poor (VP), poor (P), fair (F), and good (G) to very good (VG), and expressed in positive trapezoidal fuzzy number in Table 1, which shows graphically in Fig. 2. A variety of different methods can represent the linguistic items, it depends on the real application systems and the domain experts' opinions to select method.

## 2.3 Power average operator

Power average operator is a aggregation information technique which commonly used in many information

**Table 1** Five linguistic constants

| Linguistic constants (k) | $a_k$ | $b_k$ | $c_k$ | $d_k$ |
|---|---|---|---|---|
| Very poor (VP) | 0.0 | 0.0 | 0.05 | 0.25 |
| Poor (P) | 0.05 | 0.25 | 0.35 | 0.45 |
| Fair (F) | 0.35 | 0.45 | 0.55 | 0.65 |
| Good (G) | 0.55 | 0.65 | 0.70 | 0.95 |
| Very good (VG) | 0.70 | 0.95 | 1.00 | 1.00 |

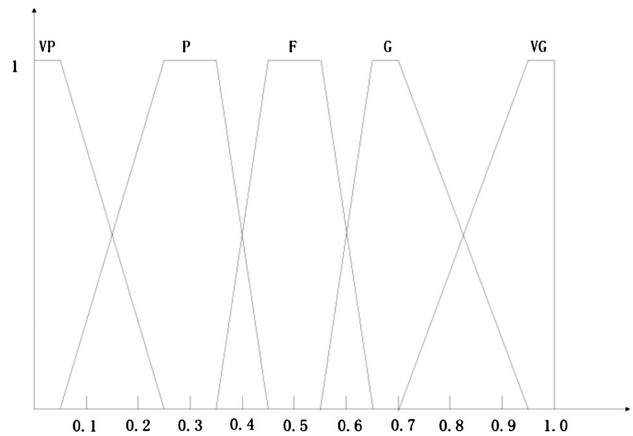


**Fig. 2** The five linguistic constants

fusion process. It was introduced by Yager (2001) and developed in 2010 (Yager 2010) in order to allow values being aggregated to support and reinforce each other. This operator is particularly useful in group decision making. A verity of literatures have been proposed in the recent years. Deng and Shi (2003) presented a new method to fusion sensor data based on power average operator. Yejun and Wang (2011) developed some new linguistic aggregation, such as 2-tuple linguistic power average operator (2TLPA) operator, 2-tuple linguistic weighted PA operator, 2TLP-OWA operator which are based on power average operator. Xu (2011) established various properties of power aggregation operator (including power average operator) and applied them to develop some approaches to multiple attribute group decision making with Atanassov's intuitionistic fuzzy information.

The power average (P-A) operator takes a collection of values and provides a single value. It can be defined as follows:

$$P - A(a_1, \ldots, a_n) = \frac{\sum_{i=1}^{n} (1 + T(a_i))a_i}{\sum_{i=1}^{n} (1 + T(a_i))} \tag{3}$$

where

$$T(a_i) = \sum_{\substack{j=1 \\ j \neq i}}^{n} Sup(a_i, a_j) \tag{4}$$

is denoted the support for a from b.

In most situations we assume the Sup(a,b) satisfies the following three properties:

1. $Sup(a, b) \in [0, 1]$
2. $Sup(a, b) = Sup(b, a)$
3. $Sup(a, b) \geq Sup(x, y) \quad$ if $|a - b| \leq |x - y|$

Under the condition of three above, the closer the two values are, the more they support each other.

Let us to present some properties of the power average. First, this operator provides a generalization of the simple average. If Sup(a,b) = 0 for all a and b, then

$$P - A(a_1, a_2, \ldots, a_n) = \frac{1}{n} \sum_i a_i \tag{5}$$

Thus, without support elements, the power average is a simple average. Generally, if $Sup(a, b) = k$ for all a and b, then $T(a_i) = k(n - 1)$ for all i, and hence $P - A(a_1, a_2, \ldots, a_n) = (1/n) \sum_i a_i$. Thus, when all the supports are the same, the power average reduces to the simple average.

It is easy to denote $V_i = 1 + T(a_i)$ and $w_i = V_i / \sum_{i=1}^{n} V_i V_i$. Here, $w_i$ is a proper set of weights, $w_i \geq 0$ and $\sum_i w_i = 1$. And

$$\begin{gathered} P - A(a_1, a_2, \ldots, a_n) = \sum_i w_i a_i \\ \mathrm{Min}[a_i] \leq \mathrm{P - A(a_1, a_2, \ldots, a_n)} \leq \mathrm{Max_i}[a_i] \end{gathered} \tag{6}$$

Monotonicity is not the property of the power average operator. As we all know, if the power average operator processes the property of monotonicity, it will satisfy that if $a_i > b_i$ for all i, then $P - A(a_i, a_i, \ldots, a_n) \geq P - A(b_1, b_2, \ldots, b_n)$. The following example demonstrates that the increase of one argument can cause the decrease in power average.

Example: Assume the support function Sup(a,b) in such that

$$\begin{aligned} &Sup(2, 4) = 0.5 \\ &Sup(2, 10) = 0.3 \\ &Sup(4, 10) = 0.4 \\ &Sup(2, 11) = 0 \\ &Sup(4, 11) = 0 \end{aligned}$$

$Sup(a, b) = Sup(b, a)$ for all the values.

Consider $P - A(2, 4, 10)$, in this case

$$\begin{aligned} &T(2) = Sup(2, 4) + Sup(2, 10) = 0.5 + 0.3 = 0.8 \\ &T(4) = Sup(4, 2) + Sup(4, 10) = 0.5 + 0.4 = 0.9 \\ &T(10) = Sup(10, 2) + Sup(10, 4) = 0.3 + 0.4 = 0.7 \end{aligned}$$

and therefore

$$\begin{aligned} &P - A(2, 4, 10) \\ &\quad = \frac{(1 + 0.8)2 + (1 + 0.9)4 + (1 + 0.7)10}{(1 + 0.8) + (1 + 0.9) + (1 + 0.7)} = 5.22 \end{aligned}$$

Consider $P - A(2, 4, 11)$, in this case

$$\begin{aligned} &T(2) = Sup(2, 4) + Sup(2, 11) = 0.5 + 0 = 0.5 \\ &T(4) = Sup(4, 2) + Sup(4, 11) = 0.5 + 0 = 0.5 \\ &T(11) = Sup(11, 2) + Sup(11, 4) = 0 + 0 = 0 \end{aligned}$$

and therefore

$$P-A(2,4,11)=\frac{(1+0.5)2+(1+0.5)4+(1+0)11}{(1+0.5)+(1+0.5)+(1+0)}=5$$

Thus, we can see that $P-A(2,4,10)>P-A(2,4,11)$, so the power average operator doesn't monotony.

### 2.4 The proposed method

The purpose of the study is to construct a mathematic model to merge together the fuzzy numbers and determine environmental indices. Several problems should be solved in this paper. Firstly, how to transform raw quality to quality data. Secondly, how to construct the function of Sup(a,b). Thirdly, is how to combine conflict data in the determination of environmental indices. Last, to compare the result of the indices with the linguistic value and make decisions. These problems will be detailed.

### 2.5 Transforming raw quality data to quality data

In real data processing, some raw data can not be directly used in most existing methods, the numbers must be interpreted on some sort of scale. Therefore, a method is needed to transform the row quality data to quality data. In the real data processing, the generalized fuzzy number is widely used. For more detailed information, please refer to the previous work.

### 2.6 Similarity between generalized fuzzy numbers

Similarity is fundamentally important in almost every scientific field, and widely used in diverse fields like decision-making, pattern recognition, machine learning and market prediction, etc. (Mitra and Pal 2005; Pedrycz 1997). Similarity measure between two fuzzy numbers is related to their commonality. The greater the commonality between a pair of objects, the more similar they have. A variety of methods have been proposed to calculate the degree of similarity between fuzzy numbers (Hejazi et al. 2011). Chen and Chen (2003) proposed a novel similarity measure based on center-of-gravity (COG) points. Deng et al. (2004) proposed the similarity measure based on radius of gyration (ROG) points in 2004. In this paper a new similarity measure is proposed, this method is more directly to show the similarity between the fuzzy numbers.

Assume that they are two trapezoidal fuzzy numbers, where $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$, then the degree of similarity $S(A,B)$ between the trapezoidal fuzzy numbers A and B can be calculated as:

$$S(A,B)=\frac{|A\cap B|}{|A\cup B|} \tag{7}$$

where $|A\cap B|$ denotes the intersection area of A and B, while $|A\cup B|$ denotes the union area of A and B. The result of S(A,B) is an interval of [0,1], the higher similarity they have, the result is closer of 1.

### 2.7 Construct the function of Sup(a,b)

In power average operator, it is very important to construct the function of $Sup(A,B)$. The function of $Sup(A,B)$ must satisfy the three properties mentioned.

$$0\le S(A,B)=\frac{|A\cap B|}{|A\cup B|}\le 1$$

$$S(A,B)=\frac{|A\cap B|}{|A\cup B|}=S(B,A)$$

$$S(A,B)\ge S(X,Y)\quad \text{when } |\mathrm{A}-\mathrm{B}|\le|\mathrm{X}-\mathrm{Y}|$$

So Sup(A,B) $= S(A,B)$ can be defined.

Example: Assume two triangular fuzzy number transformed from the row data A = (0.04,0.23,0.48), B = (0.41,0.62,0.83), as shown in Fig. 3. Then the similarity measure based on the area method can be applied to generate Sup(A,B) using Eq. (7). Therefore, Sup(A,B) = Sup(B,A) can be derived:

$$|A\cap B|=0.005327$$

$$|A\cup B|=0.424628$$

$$S(A,B)=\frac{|A\cap B|}{|A\cup B|}=0.01233$$

$$Sup(A,B)=S(A,B)=0.01233$$

### 2.8 Combine conflict data

As the Sup(A,B) is calculated above, the fuzzy data can be combined together using Eq.(3) The power of average

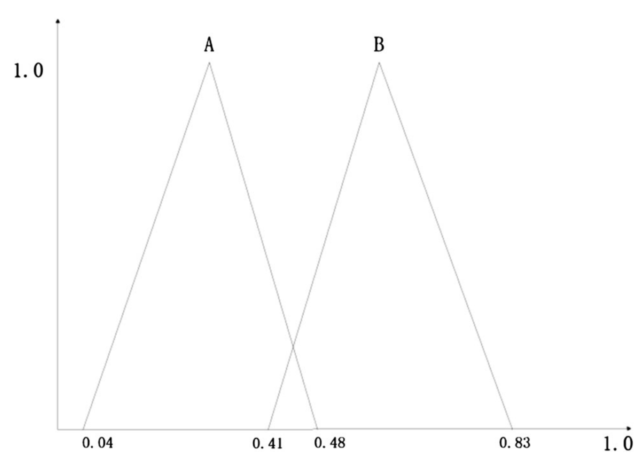


**Fig. 3** The example of how to get Sup(A,B)

operator needs a normal step to combine the conflict data. In this paper Sup(A,B) is calculated by using the area method, then use Eq. (4) to calculate $T(a_i)$. With these work done, power average can be calculated by Eq. (3).

### 2.9 Interpreting fuzzy numbers power average to get result for decision making

After calculated the result of power average operator,a linguistic value can be used to guide informed decision-making. In this step, the area method Eq. (7) to calculate the similarity between the result of data fusion and the linguistic value defined in this section. Then the fuzzy numbers of five items will be figure out, the larger the data is, the closer it is corresponding to indicators. Using these data, decision will be easy to make.

### 2.10 The sum of the proposed algorithm

In summary, the proposed algorithm can be represented step by step:

Step 1: To recorder the input parameters and transform the raw quality data to quality data. In this step the raw quality data was translated the actual values into an interval of [0,1] through transformation functions.
Step 2: To construct the function of Sup(a,b). In this step area method is used to construct the similarity of the two fuzzy numbers. Since the area method satisfied all the conditions of the Sup(a,b), the function of Sup(a,b) defined.
Step 3: To combine the conflict data. Firstly, to calculate the support function of Sup(a,b) between any two fuzzy numbers. Then using the power average operator to combine the conflict data.
Step 4: To interpret fuzzy numbers power average to get result for decision making. Using the area method to acquire similarity between the result of the power average operator and linguistic value, then make decision.

## 3 Numerical example

In recent years, water resource management has raised a large concern, and a multitude of literatures have been proposed (Lu et al. 2010; Harmel et al. 2009). Water quality index (WQI) is a communication tool used to describe the status of water by translating a large amount of non-commensurate data into a single value (Ott 1978). It is useful in establishing background levels of water quality for a given aquatic system for implementing regulatory policies and evaluating decision actions planned for the improvement and the rehabilitation of an aquatic system (Silvert 2000). Numerous literatures about WQI have been addressed in recent years. Song and Kim (2009) introduced a water quality index termed QUAL2E water quality loading index, which is specifically used for simulated water quality to mainly reflect pollutant loading levels. Lermontov et al. (2009) proposed the criterion of a new water quality index based on fuzzy logic, the fuzzy water quality index (FWQI).

In this section the example of the water quality was listed to demonstrate the fuzzy number power average method. The raw quality data used in this paper was calculated in precious works in Table 2, which consists of nine water quality indicators (sub-indices) including $BOD_5$, fecal coliforms, dissolved oxygen DO (proportion) respected to saturation, nitrates, PH, phosphates, temperature, total solid and turbidity. After applied the transformation function, the raw water quality data is transformed into triangle fuzzy numbers through four vertices (a, b, c, w) where w $= 1$, $a \leq b \leq c$, which represents the minimum, most likely and maximum values respectively. The transformation of raw water quality data into fuzzy sub-indices is show in Table 2.

In the Table 2, as the water quality indicators are non-commensurate, transformation functions are used to translate the actual values into an interval of [0,1], where '0' corresponds to the worst value and '1' corresponds to the best value. Therefore, an appropriate transformation function is needed to transform the actual values over a normalized interval [0,1]. In Table 2, Swamee and Tyagi (2000) proposed various of transformation functions, including uniform decreasing sub-indices(UDS) and uni-modal sub-indices (US). Using these functions the actual value of a specific water quality indicator can be translated into a normalized interval [0,1]. Only the final data calculated in this Table is need.

After that, those triangle fuzzy number can be applied to construct the function of Sup(A,B). Taking the first evidence $BOD_5$ for example. Firstly, calculate the support for $BOD_5$ form the other eight water quality indicators as below.

$$BOD_5 = (0.014, 0.13, 0.34)$$
$$Fecal\ coliforms = (0.25, 0.43, 0.59)$$
$$DO(proportion) = (0.41, 0.62, 0.83)$$
$$Nitrates = (0.04, 0.23, 0.48)$$
$$PH = (0.79, 0.87, 0.94)$$
$$Phosphates = (0.08, 0.25, 0.47)$$
$$Temperature = (0.25, 0.40, 0.63)$$
$$Total\ solids = (0.10, 0.17, 0.37)$$
$$Turbidity = (0.07, 0.27, 0.50)$$

Then use the similarity measure based on the area method of the Eq. (7) in this paper. Obtain the calculated results

**Table 2** Transformation of raw water quality data into fuzzy sub-indices (modified after Swamee and Tyagi 2000) UDS uniform decreasing sub-indices, US unimodal sub-indices

| i | WQ indicators | Obs. values ($q_i$) | Transformation *function*[a] | Parameters for the transformation *function*[b] | Fuzzy sub-indices (si) |
|---|---|---|---|---|---|
| 1 | $BOD_5$ (mg/L) | 20 | UDS $(\bar{m}^1, \bar{q}_c^1)$ | $\bar{m}^1 = (2.1, 3, 3.9); \bar{q}_c^1 = (10, 20, 30)$ | (0.014, 0.13, 0.34) |
| 2 | Fecal coliforms (MPN/100 mL) | 66 | UDS$(\bar{m}^2, \bar{q}_c^2)$ | $\bar{m}^2 = (0.21, 0.3, 0.39); \bar{q}_c^2 = (2, 4, 6)$ | (0.25, 0.43, 0.59) |
| 3 | DO (proportion) | 0.6 | US$(q*^3, \bar{n}^3, \bar{p}^3, r^3)$ | $\bar{n}^3 = (1.5, 3, 4.5); \bar{p}^3 = (0.9, 1, 1.1)$ $q*^3 = 1; r^3 = 0$ (used as crisp values) | (0.41, 0.62, 0.83) |
| 4 | Nitrates (mg/L) | 25 | UDS$(\bar{m}^4, \bar{q}_c^4)$ | $\bar{m}^4 = (2.1, 3, 3.9); \bar{q}_c^4 = (20, 44, 60)$ | (0.04, 0.23, 0.48) |
| 5 | PH | 7.8 | US$(q*^5, \bar{n}^5, \bar{p}^5, r^5)$ | $\bar{n}^5 = (1.6, 4, 6.4); \bar{p}^5 = (5.4, 6, 6.6)$ $q*^5 = 7; r^5 = 0$ (used as crisp values | (0.79, 0.87, 0.94) |
| 6 | Phosphates (mg/L) | 2 | UDS$(\bar{m}^6, \bar{q}_c^6)$ | $\bar{m}^6 = (0.7, 1, 1.3); \bar{q}_c^6 = (0.34, 0.67, 1.01)$ | (0.08, 0.25, 0.47) |
| 7 | Temperature (°C) | 32 | US$(q*^7, \bar{n}^7, \bar{p}^7, r^7)$ | $\bar{n}^7 = (0.25, 0.5, 0.75); \bar{p}^7 = (6.3, 7, 7.7)$ $q*^7 = 20; r^7 = 0$ (used as crisp values) | (0.25, 0.4, 0.63) |
| 8 | Total solids (mg/L) | 1,000 | US$(q*^8, \bar{n}^8, \bar{p}^8, r^8)$ | $\bar{n}^8 = (0.5, 1, 1.5); \bar{p}^8 = (0.9, 1, 1.1)$ $q*^8 = 75; r^8 = 0.8$ (used as crisp values) | (0.10, 0.17, 0.37) |
| 9 | Turbidity (JTU) | 70 | UDS$(\bar{m}^9, \bar{q}_c^9)$ | $\bar{m}^9 = (1.05, 1.5, 1.95); \bar{q}_c^9 = (25, 50, 75)$ | (0.07, 0.27, 0.50) |

[a] Two types of transformation functions are used $UDS(\bar{m}^i, \bar{q}_c^i) = \left(1 + \frac{q_i}{\bar{q}_c^i}\right)^{-\bar{m}}$; $US(q*^i, \bar{n}^i, \bar{p}^i, r^i) = \frac{\bar{p}^i r^i + (\bar{n}^i + \bar{p}^i)(1 - r^i)\left(\frac{q^i}{q*^i}\right)^{\bar{n}i}}{\bar{p}^i + \bar{n}^i(1 - r^i)\left(\frac{q^i}{q*^i}\right)^{\bar{n}i + \bar{p}i}}$

[b] The bar over the transformation parameters represents triangular fuzzy numbers

and generate the result to get T($a_i$). Therefore, the result of the function Sup(A,B) is the support for $BOD_5$ form the other eight water quality indicators as following:

$$Sup(a_1, a_2) = 0.0322$$
$$Sup(a_1, a_3) = 0$$
$$Sup(a_1, a_4) = 0.4159$$
$$Sup(a_1, a_5) = 0$$
$$Sup(a_1, a_6) = 0.3305$$
$$Sup(a_1, a_7) = 0.0329$$
$$Sup(a_1, a_8) = 0.4031$$
$$Sup(a_1, a_9) = 0.3075$$

$$T(a_i) = \sum_{j=2}^{9} Sup(a_1, a_j) = Sup(a_1, a_2) + Sup(a_1, a_3) + \cdots + Sup(a_1, a_9) = 1.5221$$

where $Sup(a_1, a_2)$ represents the support between $BOD_5$ and Fecal coliforms, $Sup(a_1, a_3)$ represents the support between $BOD_5$ and DO (proportion), $Sup(a_1, a_4)$ represents the support between $BOD_5$ and Nitrates, $Sup(a_1, a_5)$ represents the support between $BOD_5$ and PH, $Sup(a_1, a_6)$ represents the support between $BOD_5$ and Phosphates, $Sup(a_1, a_7)$ represents the support between $BOD_5$ and Temperature, $Sup(a_1, a_8)$ represents the support between $BOD_5$ and Total solids, $Sup(a_1, a_9)$ represents the support between $BOD_5$ and Turbidity. With the method mentioned above, another eight water quality can be calculated and support each other. The result of support for each other is calculated in the Table 3.

Then calculate the $\sum_{i=1}^{n}(1 + T(a_i))a_i$. As in this paper, $a_i$ stands for triangular fuzzy number, here some arithmetic operations are need.

$$A + B = [a_1 + b_1, a_2 + b_2, a_3 + b_3] \tag{8}$$

$$Q \cdot A = [Q \cdot a_1, Q \cdot a_2, Q \cdot a_3] \tag{9}$$

where A and B stand for triangular fuzzy number (a,b,c) where $a \le b \le c$, and Q comes to a specific number. Example:

$$A = (0.1, 0.2, 0.3)$$
$$B = (0.2, 0.3, 0.4)$$
$$Q = 2$$

then calculate as below

$$A + B = [0.1 + 0.2, 0.2 + 0.3, 0.3 + 0.4] = [0.3, 0.5, 0.7]$$
$$Q \cdot A = [2 \times 0.1, 2 \times 0.2, 2 \times 0.3] = [0.2, 0.4, 0.6]$$

Following, power average operator can be applied to combine the water quality indicators. In the example, each indictor should be combine with balance indicators and get support. First using Eq. (4) to sum them and get $T(a_i)$, then combine all data to get the result by Eq. (3). The result of power average operator can be calculated as:

**Table 3** The result of support for each other $Sup(a_i, a_j) = S(a_i, a_j)$, $a_1 - a_9$ is denoted the nine water quality indicators, $a_i$ correspond to the table of longitudinal and transverse form corresponding to the $a_j$

| $S(a_i, a_j)$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ | $a_8$ | $a_9$ | $T(a_i)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_1$ | – | 0.0322 | 0 | 0.4159 | 0 | 0.3305 | 0.0329 | 0.4031 | 0.3075 | 1.5221 |
| $a_2$ | 0.0322 | – | 0.1302 | 0.1872 | 0 | 0.1987 | 0.8429 | 0.0632 | 0.2468 | 1.7012 |
| $a_3$ | 0 | 0.1302 | – | 0.0123 | 0.0098 | 0.0104 | 0.1594 | 0 | 0.0221 | 0.3442 |
| $a_4$ | 0.4159 | 0.1873 | 0.0123 | – | 0 | 0.7969 | 0.1923 | 0.5409 | 0.7524 | 2.8980 |
| $a_5$ | 0 | 0 | 0.0098 | 0 | – | 0 | 0 | 0 | 0 | 0.0098 |
| $a_6$ | 0.3306 | 0.1987 | 0.0104 | 0.7969 | 0 | – | 0.2046 | 0.5113 | 0.8536 | 2.9061 |
| $a_7$ | 0.0329 | 0.8429 | 0.1594 | 0.1923 | 0 | 0.2046 | – | 0.0675 | 0.2548 | 1.7544 |
| $a_8$ | 0.4031 | 0.6324 | 0 | 0.5409 | 0 | 0.5113 | 0.0676 | – | 0.4525 | 2.6078 |
| $a_9$ | 0.3075 | 0.2468 | 0.0221 | 0.7524 | 0 | 0.8536 | 0.2548 | 0.4525 | – | 2.8897 |

$$(1+T(a_1))a_1 = (0.0353, 0.3278, 0.8575)$$
$$(1+T(a_2))a_2 = (0.6753, 1.1615, 1.5937)$$
$$(1+T(a_3))a_3 = (0.5511, 0.8334, 1.1157)$$
$$(1+T(a_4))a_4 = (0.1559, 0.8965, 1.8710)$$
$$(1+T(a_5))a_5 = (0.7977, 0.8785, 0.9492)$$
$$(1+T(a_6))a_6 = (0.3125, 0.9765, 1.8359)$$
$$(1+T(a_7))a_7 = (0.6886, 1.1018, 1.7353)$$
$$(1+T(a_8))a_8 = (0.3607, 0.6133, 1.2627)$$
$$(1+T(a_9))a_9 = (0.2722, 1.0502, 1.9448)$$
$$\sum_{i=1}^{9}(1+T(a_i)) = 25.6337$$
$$\sum_{i=1}^{9}(1+T(a_i))a_i = (3.8495, 7.8418, 13.1661)$$
$$P-A(a_1, a_2, \ldots, a_n) = \frac{\sum_{i=1}^{9}(1+T(a_i))a_i}{\sum_{i=1}^{9}(1+T(a_i))} = (0.1501, 0.3060, 0.5136)$$

where $a_i$ represents the the triangle fuzzy set of the nine water quality, then calculate the similarity between indicators and linguistic variables based on the area method by Eq. (4). The final result as below:

$$S(VP, P, F, G, VG) = (0.0381, 0.6143, 0.2976, 0, 0)$$

therefore, the water quality index is classified as poor.

## 3.1 Discussion

The power average operator is applied in a wide range of situations, such as statistics, economics, engineering and decision-making. It provides a more versatility in the information-aggregation process, which take into account the information about the relationship between the fused values, and allow values being aggregated to support and reinforce each other. In this paper we proposed a new similarity method of the area method, this method provide a more directly and reasonable way to calculate the similarity between the fuzzy numbers. Using this method we construct the support function of the power average operator. In Fig. 4 it is easily to know the WQI is classified as poor. Sadiq and Tesfamariam (2008) studied the EI using the fuzzy numbers ordered weighted averaging (FN-OWA) operators. The result of the WQI is based on FN-OWA and FN-PA clearly in Table 4.

In Table 4 when $\delta$ is different the result of the WQI based on the FN-OWA will be different. The $\delta$ is a degree

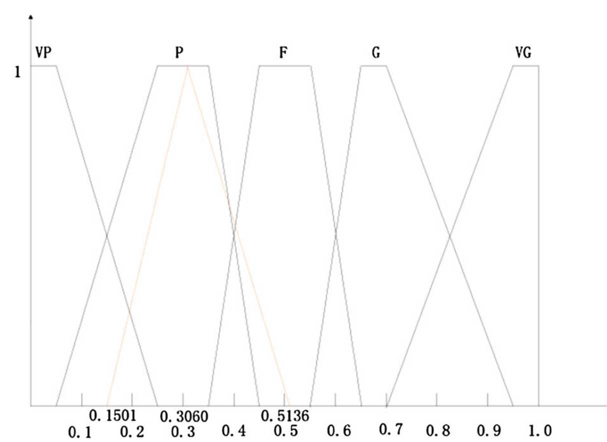


**Fig. 4** Similarity measure between the result of power average operator and linguistic constants

**Table 4** Comparison of proposed method with the previous work Sadiq and Tesfamariam (2008)

| Method | The degree of a polynomial function $\delta$ | The result of WQI |
|---|---|---|
| FN-OWA | 1/3 | F |
| | 1 | P |
| | 3 | P |
| FN-PA | – | P |

of a polynomial function, for $\delta = 1/3$ represent the decision-maker's attitude is or-type, while $\delta = 1$ represents the decision-maker's attitude is neutral. When $\delta = 3$ represents the decision-maker's attitude is and-type. The OWA (Yager 1988) operator is a technique for aggregating information, providing a parameterized family of aggregation operators which included the maximum, the minimum and the average. The OWA operator aggregates the information according to the attitudinal character (or degree of orness) of the decision-maker. The decision-makers' different interests may lead to different results. The more interest of the data of the decision-maker, the more the data will influence the result. While the power average operator is based on the support measure, in this paper the support measure is based on the new similarity measure of the area method. The more support (or closer) between the fuzzy numbers, the more its can reflect the result of the data fusion. Therefore, the developed power average operator can relieve the influence of the unfair arguments on the aggregated results, and make it more reasonable.

## 4 Conclusion

Environmental indices are used as a communication tool to describe overall status of environmental system, which provides a condensed description of multi-dimensional environmental states by aggregating several indicators into single quantity. According to the problems that mentioned above, the uncertain information should be well represented and efficiently handled in a flexible way, data often conflicts with each other in a high degree, a fuzzy evidential method is proposed to determine the environmental indices. Based on the fuzzy set theory and the power average operator, the proposed method is believed to be efficient in the determination of environmental indices. The power average operator provide an aggregation operator which allows argument values to support each other in the aggregation process. In this paper a new environmental indices based on fuzzy numbers power average operator is proposed. It uses a new similarity measure which is based on the area method to calculate the similarity between the data, and it provides a direct way of the similarity calculation. With the aid of the power average the values are allowed to be aggregated to support each other. Then the operator can be applied to evaluate the water quality index and make decisions. Comparing with the previous work of OWA, the fuzzy numbers power average operator provide a more simple and reasonable way.

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## References

Arunraj NS, Maiti J (2009) Development of environmental consequence index (ECI) using fuzzy composite programming. J Hazard Mater 162:29–43

Bellenger MJ, Herlihy AT (2009) An economic approach to environmental indices. Ecol Econ 68:2216–2223

Chan FTS (2005) Application of a hybrid case-based reasoning approach in electroplating industry. Expert Syst Appl 29(1):121–130

Chan FTS, Au KC, Chan PLY (2006) A decision support system for production scheduling in an ion plating cell. Expert Syst Appl 30(4):727–738

Chan FTS, Kumar N (2007) Global supplier development considering risk factors using fuzzy extended AHP-based approach. Int J Syst Assur Eng Manag 35:417–431

Chan FTS, Chan HK (2011) Improving the productivity of order picking of a manual-pick and multi-level rack distribution warehouse through the implementation of class-based storage. Expert Syst Appl 383:2686–2700

Chen SJ, Chen SM (2003) Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. IEEE Trans Fuzzy Syst 11(1):45–56

Chen SM, Chen JH (2009) Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Expert Syst Appl 36(3):6833–6842

Chen Z, Zhao L, Lee K (2010) Environmental risk assessment of offshore produced water discharges using a hybrid fuzzy-stochastic modeling approach. Environ Model Softw 25:782–792

Chen S, Deng Y, Wu J (2013) Fuzzy sensor fusion based on evidence theory and its application. Appl Artif Intell 27(3):235–248

Cheng CY, Qian X (2010) Evaluation of emergency planning for water pollution incidents in reservoir based on fuzzy comprehensive assessment. Procedia Environ Sci 2:566–670

Chiadamrong N (1999) An integrated fuzzy multi-criteria decision making method for manufacturing strategies selection. Comput Ind Eng 37(1):433–436

Debels P, Figueroa R, Urrutia R, Barra R, Niell X (2005) Evaluation of water quality in the Chillan River (Central Chile) using physicochemical parameters and a modified water quality index. Environ Monit Assess 110:301–322

Deng Y, Shi W (2003) Fuzzy sensor data fusion based on power average operator. J Shaihai Jiao Tong Univ 37(8):1279–1287

Deng Y, Shi W, Liu Q (2004) A new similarity measure of generalized fuzzy numbers and its application to pattern recognition. Pattern Recogn Lett 25:875–883

Deng Y, Liu Q (2005) A TOPSIS-based centroid-index ranking method of fuzzy numbers and its application in decision-making. Cybern Syst 36(6):581–595

Deng Y (2006) Plant location selection based on fuzzy TOPSIS. Int J Adv Manuf Technol 28(7–8):839–844

Deng Y, Chen FTS (2011) A new fuzzy dempster MCDM method and its application in supplier selection. Expert Syst Appl 38:9854–9861

Deng Y, Chen FTS, Wu Y, Wang D (2011) A new linguistic MCDM method based on multiple-criterion data fusion. Expert Syst Appl 38:6985–6993

Deng Y, Jiang W, Sadiq R (2011) Modeling contaminant intrusion in water distribution networks: a new similarity-based DST method. Expert Syst Appl 38(1):571–578

Deng Y, Chen Y, Zhang Y, Mahadevan S (2011) Fuzzy dijkstra algorithm for shortest path problem under uncertain environment. Appl Soft Comput 12(3):1231–1237

Deng Y, Chen Y, Zhang Y, Mahadevan S (2012) Fuzzy dijkstra algorithm for shortest path problem under uncertain environment. Appl Soft Comput 12(3):1231–1237

Deng X, Yong H, Deng Y (2014) Environmental impact assessment based on d numbers. Expert Syst Appl 41(2):635–643

Deng X, Yong H, Deng Y (2014) Supplier selection using ahp methodology extended by d numbers. Expert Syst Appl 41(1):156–167

Ebert U, Welsch H (2004) Meaningful environmental indices: a social choice approach. J Environ Econ Manag 47:270–283

Feng Z, Wang Q (2007) Research on health evaluation system of liquid-propellant rocket engine ground-testing bed based on fuzzy theory. Acta Astronaut 61(10):840–853

Fenton N, Wang W (2006) Risk and confidence analysis for fuzzy multicriteria decision making. Knowl-Based Syst 19(6):430–437

Gharibi H, Mahvi AH, Nabizadeh R, Arabalibeik H, Yunesian M, Sowlat MH (2012) A novel approach in water quality assessment based on fuzzy logic. J Environ Manag 112:87–95

Guettaf A, Benchabane F, Bahri M, Bennis O (2013) Torque ripple minimization in switched reluctance motor using the fuzzy logic control technique. Int J Syst Assur Eng Manag 1–7. doi:10.1007/s13198-014-0232-7

Gunasekera MY, Edwards DW (2003) Estimating the environmental impact of catastrophic chemical releases to the atmosphere: an index method for ranking alternative chemical process routes. Process Saf Environ Prot 81(6):463–474

Harmel RD, Smith DR, King KW, Slade RM (2009) Estimating storm discharge and water quality data uncertainty: a software tool for monitoring and modeling applications. Environ Model Softw 24:832–842

Hejazi SR, Doostparast A, Hosseini SM (2011) An improved fuzzy risk analysis based on a new similarity measures of generalized fuzzy numbers. Expert Syst Appl 38:9179–9185

Kang SM (2002) A sensitivity analysis of the Korean composite environmental index. Ecol Econ 43:159–174

Kang B, Deng Y, Sadiq R, Mahadevan S (2012) Evidential cognitive maps. Knowl-Based Syst 35:77–86

Khan FI, Sadiq R, Veitch B (2004) Life cycle iNdeX (LInX): a new indexing procedure for process and product design and decision-making. J Clean Prod 12(1):59–76

Khanna N (2000) Measuring environmental quality: an index of pollution. Ecol Econ 35:191–202

Kharal A (2009) Homeopathic drug selection using intuitionistic fuzzy sets. Homeopathy 98(1):35–39

Laala W, Zouzou S-E, Guedidi S (2013) Induction motor broken rotor bars detection using fuzzy logic: experimental research. Int J Syst Assur Eng Manag 1–8. doi:10.1007/s13198-013-0171-8

Lee LW, Chen SM (2008) Fuzzy risk analysis based on fuzzy numbers with different shapes and different deviations. Expert Syst Appl 34(4):2763–2771

Lee GH (2008) Rule-based and case-based reasoning approach for internal audit of bank. Knowl-Based Syst 21(2):140–147

Lee E, Park Y, Shin JG (2009) Large engineering project risk management using a bayesian belief network. Expert Syst Appl 36(3):5880–5887

Lermontov A, Yokoyama L, Lermontov M, Machado MAS (2009) River quality analysis using fuzzy water quality index: Ribeira do Iguape river watershed, Brazil. Ecol Indicat 9:1188–1197

Li DF (2010) Multiattribute decision making method based on generalized owa operators with intuitionistic fuzzy sets. Expert Syst Appl 37(12):8673–8678

Liu J, Chan FTS, Li Y, Zhang Y, Deng Y (2012) A new optimal consensus method with minimum cost in fuzzy group decision. Knowl-Based Syst 35:357–360

Liu Q, Xiong J, Deng Y (2008) A subjective methodology for risk quantification based on generalized fuzzy numbers. Int J Gen Syst 37(2):149–165

Lu RS, Lo SL (2002) Diagnosing reservoir water quality using self-organizing maps and fuzzy theory. Water Res 36(9):2265–2274

Lu HW, Huang GH, He L (2010) Development of an interval-valued fuzzy linear-programming method based on infinite $\alpha$-cuts for water resources management. Environ Model Softw 25:354–361

Meliadou A, Santoro F, Nader MR, Dagher MA, Al Indary S, Salloum BA (2012) Prioritising coastal zone management issues through fuzzy cognitive mapping approach. J Environ Manag 97:56–68

Mitra S, Pal SK (2005) Fuzzy sets in pattern recognition and machine intelligence. Fuzzy Sets Syst 156(3):381–386

Nasiri F, Huang G (2008) A fuzzy decision aid model for environmental performance assessment in waste recycling. Environ Model Softw 23:677–689

Ott WR (1978) Environmental indices: theory and practice. Ann Arbor Science Publishers, Ann Arbor

Padma T, Balasubramanie P (2009) Knowledge based decision support system to assist work-related risk analysis in musculo-skeletal disorder. Knowl-Based Syst 22(1):72–78

Pan TC, Kao JJ (2009) Inter-generation equity for assessing environmental sustainablity: an example on global warming. Ecol Indic 9(4):725–731

Pedrycz W (1997) Fuzzy sets in pattern recognition: accomplishments and challenges. Fuzzy Sets Syst 90(2):171–176

Peneva V, Popchev I (2003) Properties of the aggregation operators related with fuzzy relations. Fuzzy Sets Syst 139:615–633

Pischke F, Cashmore M (2006) Decision-oriented environmental assessment: an empirical study of its theory and methods. Environ Impact Assess Rev 26(7):643–662

Prato T (2009) Adaptive management of natural systems using fuzzy logic. Environ Model Softw 24:940–944

Pusatli OT, Camur MZ, Yazicigil H (2009) Susceptibility indexing method for irrigation water management planning: applications to K. Menderes river basin, Turkey. J Environ Manag 90(1):341–347

Ragone A, Straccia U, Di Noia T, Di Sciascio E, Donini FM (2009) Fuzzy matchmaking in e-marketplaces of peer entities using Datalog. Fuzzy Sets Syst 160(2):251–268

Sadiq R, Khan FI, Veitch B (2005) Evaluating offshore technologies for produced water management using GreenPro-I a risk-based life cycle analysis for green and clean process selection and design. Comput Chem Eng 29:1023–1039

Sadiq R, Tesfamariam S (2007) Probability density functions based weights for ordered weighted averaging (OWA) operators: an example of water quality indices. Eur J Oper Res 182(3):1350–1368

Sadiq R, Tesfamariam S (2007) Ordered weighted averaging (OWA) operators for developing water quality indices using probabilistic density function. Eur J Oper Res 182:1350–1368

Sadiq R, Tesfamariam S (2008) Deveping environmental using fuzzy numbers ordered weighted averaging (FN-OWA) operators. Stoch Environ Res Risk Assess 22(4):495–505

Sadiq R, Rodríguez MJ, Tesfamariam S (2010) Integrating indicators for performance assessment of small water utilities using ordered weighted averaging (owa) operators. Expert Syst Appl 37(7):4881–4891

Sadiq R, Haji SA, Cool G, Rodriguez MJ (2010) Using penalty function to evaluate aggregation models for environmental indices. J Environ Manag 91:706–716

Sattler C, Stachow U, Berger G (2012) Expert knowledge-based assessment of farming practices for different biotic indicators using fuzzy logic. J Environ Manag 95(1):132–143

Silvert W (2000) Fuzzy indices of environmental conditions. Ecol Model 130(1):111–119

Silvert W (2000) Fuzzy indices of environmental conditions. Ecol Model 130:111–119

Siracusa G, La Rosa AD, Sterlini SE (2004) A new methodology to calculate the environmental index (Ep). A case study applied to a company producing composite materials. J Environ Manag 73: 275–284

Smolkov R, Wachowiak MP (2002) Aggregation operators for selection problems. Fuzzy Sets Syst 131(1):23–34

Song T, Kim K (2009) Development of a water quality loading index based on water quality modeling. J Environ Manag 90:1534–1543

Swamee PK, Tyagi A (2000) Describing water quality with aggregation index. ASCE J Environ Eng 126(5):451–455

Wei SH, Chen SM (2009) A new approach for fuzzy risk analysis based on similarity measures of generalized fuzzy numbers. Expert Syst Appl 36(1):589–598

Wei D, Deng X, Zhang X, Deng Y, Mahadevan S (2013) Identifying influential nodes in weighted networks based on evidence theory. Phys A 392(10):2564–2575

Weiss M, Patel M, Heilmeier H, Bringezu S (2007) Applying distance-to-target weighing methodology to evaluate the environmental performance of bio-based energy, fuels, and materials. Resour Conserv Recycl 50(3):260–281

Xu Z (2011) Approaches to multiple attribute group decision making based on intuitionistic fuzzy power aggregation operators. Knowl-Based Syst 24(6):749–760

Yager RR (1988) On ordered weighted averaging aggregation in multicriteria decision making. IEEE Trans Syst Man Cybern B 18:183–190

Yager RR (2001) The power average operator. IEEE Trans Syst Man Cybern A 31(6):724–731

Yager RR (2010) Information. Prefer Decis Stud Fuzziness Soft Comput 257:397–414

Yejun X, Wang H (2011) Approaches based on 2-tuple linguistic power aggregation operators for multiple attribute group decision making under linguistic environment. Appl Soft Comput 11:3988–3997

Yuxian D, Mo H, Deng X, Sadiq R, Deng Y (2014) A new method in failure mode and effects analysis based on evidential reasoning. Int J Syst Assur Eng Manag 5(1):1–10

Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353

Zhang Y, Deng X, Wei D, Deng Y (2012) Assessment of e-commerce security using ahp and evidential reasoning. Expert Syst Appl 39(3):3611–3623

Zhang X, Deng Y, Chan FTS, Xu P, Mahadevan S, Hu Y (2013) Ifsjsp: a novel methodology for the job-shop scheduling problem based on intuitionistic fuzzy sets. Int J Prod Res 51(17):5100–5119