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ORIGINAL ARTICLE

Approximation of MTTF calculation of a non-stationary gamma wear process

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Abstract Gamma distribution is known as the most suitable distribution to model the monotonically increasing wear or deterioration**.** Examples of wear or deterioration can be found in many of the mechanical components, such as in wear of the bearings, corrosion or erosion wear of pump casings and impellers etc. The component wearing gradually can be considered to have failed when it reaches a pre-specified, optimal wear limit. Calculation of MTTF (mean time to failure or to reach a wear or deterioration limit) is however cumbersome and time consuming as it has to be integrated numerically. The author presents an approximation formula to calculate the MTTF for a gamma wear process with temporal variability which can be easily applied to most of the mechanical components undergoing wear. Error in calculation of MTTF using the theoretical and the approximation formula have been displayed in a graphical form as a ratio of the theoretical MTTF.

Keywords Gamma wear process · MTTF · Wear limit · Maintenance effectiveness

1 Introduction

The gamma process is suitable to model gradual damage monotonically accumulating over time in a sequence of tiny increments, such as wear, fatigue, corrosion crack growth etc (Van Noortwijk 2007). Abdel Hameed (2010) was the first to propose the gamma process as a proper model for deterioration occurring random in time. Over the years, gamma process have been satisfactorily fitted to data on creep of concrete (Cinlar et al. 1977), fatigue crack growth (Lawless and Crowder 2004), corroded steel gates, thinning due to corrosion (Frangpool et al. 2004) and many more. Optimisation of maintenance intervals and wear limits of components deteriorating in accordance with a gamma wear process have also been adequately studied.

What makes the gamma wear process attractive to a naval maintenance engineer like the author is the fact that most of the ships borne equipment have a well kept record of the wear or deterioration over a period of time. This record is generally available in terms of monitored parameters, such as clearances of a plain bearing, or power output of an engine under certain specific conditions. What may however not be available would be data on failures of this equipment. Furthermore, most of the maintenance models available in literature make the maintenance decisions against the backdrop of uncertainty in time to failure. However, when it comes to deterioration or ageing, these models may not be able to model the different stages of deterioration as these are mostly adept in distinguishing the equipment as operational or failed (Van Noortwijk 2007). In order to represent deterioration on the basis of lifetime distributions, the failure rate function can be applied. However, failure rates can generally be ascertained in cases where we have a large number of sample components. For a case where we have only time based recording of deterioration or say wear of a single component, the lifetime distributions are clearly unsuitable for much use. A gamma wear process which also takes into account a temporal

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variability in deterioration is the most apt to a maintenance engineer to evaluate his equipment for optimal maintenance decisions. The author has demonstrated the use of time variant gamma wear process for arriving at optimal maintenance interval of a marine steam turbine (Verma et al. 2011).

A second advantage of using the gamma wear process to describe the wear or deterioration of equipment is that it provides a basis to arrive at the effectiveness of the maintenance carried out on the equipment. Since the deterioration or wear is measured in terms of a given parameter or a group of parameters (Verma et al. 2011), the measure of the parameters immediately after a maintenance carried out on an equipment can help us arrive at the effectiveness of the maintenance action as given by the equation below:

$$\text{Maintenance effectiveness} = \frac{\text{Wearparameter}_{\text{Prior maintenance}} - \text{Wearparameter}_{\text{Post maintenace}}}{\text{Wearparameter}_{\text{Prior maintenance}} - \text{Wearparameter}_{\text{Base value}}} \tag{1}$$

The $\text{Wearparameter}_{\text{Base value}}$ in Eq. 1 is the one which is recorded when the equipment was newly installed.

1.1 Gamma process

The gamma process is parameterized by α and β which can be estimated from the deterioration data. If W_t (deteriorating state) is a gamma process then for all $0 \leq s < t$ the random variable $W_t - W_s$ (increments of deterioration between s and t) has a gamma pdf with shape parameter $\alpha(t-s)$ and a scale parameter β, given by:

$$f_{\alpha(t-s),\beta}(w) = \frac{\beta^{\alpha(t-s)}}{\Gamma(\alpha(t-s))} \cdot w^{\alpha(t-s)-1} \cdot e^{-w\cdot\beta} I_{\{x\geq 0\}} \tag{2}$$

The gamma process has a non-negative independent increment property. The mean and variance of its degradation rate can be expressed as α/β and α/β^2. For such a process the deteriorating state starting from w_0, the associated failure time distribution, CDF for a given failure threshold, W_{limit} can be expressed as

$$F_{\alpha,\beta}(w) = 1 - \frac{1}{\Gamma(\alpha\cdot(t))} \cdot \int_0^{(W_{\text{limit}}-w_0)\cdot\beta} e^{-u} \cdot u^{(\alpha(t)-1)} \cdot du \tag{3}$$

For a gamma wear process that also accounts for the temporal variability associated with the deterioration process the shape parameter is a function of time ‘t’ (Pandey and Yuan 2006) given as:

$$\alpha(t) = \lambda \cdot t^{\zeta} \tag{4}$$

The values of ‘ζ’ reflects the shape of the expected deterioration. Its values could be one depicting degradation of say, concrete due to corrosion of reinforcement, two for sulphate attack etc. (Ellingwood and Mori 1993). The gamma process is stationary if the value of $\zeta = 1$.

Wear is usually recorded in various measurements of a specific parameter for e.g. in bearings it could be measured in ‘mm’. The deterioration of a steam turbine performance could be measured in terms of power output per kg of steam flow at a designated pressure and temperature. Instead of measuring in terms of real parameters it is better to designate it on a scale of say 0–10 where ten is the wear

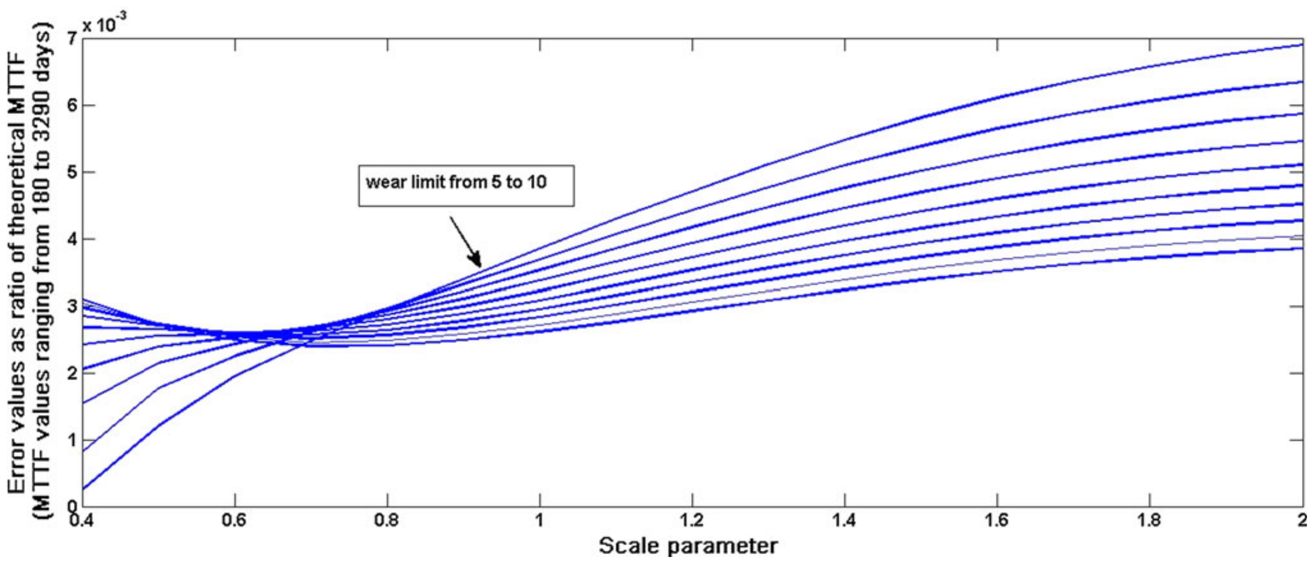

Fig. 1 Error ratio values vs. variation in scale parameter β ($\lambda = 0.02$; $\zeta = 1.2$)

limit and 0 is the zero wear or 'new' condition state of the subject component. Using the time dependent condition of the shape parameter $\alpha(t)$ we can rewrite the equations for gamma pdf and cdf as shown below in Eqs. 5 and 6:

$$f_{W(t)}(w) = \frac{\beta^{\lambda \cdot t^{\zeta}}}{\Gamma(\lambda \cdot t^{\zeta})} w^{\lambda \cdot t^{\zeta} - 1} \cdot \mathrm{e}^{-\beta w} = Ga(w|\alpha(t), \beta) \tag{5}$$

$$F_{W(t)}(w) = \int_{0}^{(W_{\text{limit}} - W_0)} \frac{\beta^{\lambda \cdot t^{\zeta}}}{\Gamma(\lambda \cdot t^{\zeta})} w^{\lambda \cdot t^{\zeta} - 1} \cdot \mathrm{e}^{-\beta x} \cdot \mathrm{d}x \tag{6}$$

The above equations are however given in terms of wear or deteriorated state of the subject component or system. To evaluate the MTTF or the mean time to reach the wear limit it is essential that we use the equations in terms of time. These equations are shown below.

$$f(t) = \frac{\zeta \cdot t^{\zeta - 1}}{\Gamma(\lambda \cdot t^{\zeta})} \int_{(W_{\text{limit}} - W_0)\beta}^{\infty} (\log(u) - digam(\lambda \cdot t^{\zeta})) u^{\lambda \cdot t^{\zeta} - 1} \cdot \mathrm{e}^{-u} \mathrm{d}u \tag{7}$$

$$F(t) = 1 - \frac{1}{\Gamma(\lambda \cdot t^{\zeta})} \int_{0}^{(W_{\text{limit}} - W_0)\beta} \mathrm{e}^{-u} \cdot u^{(\lambda \cdot t^{\zeta} - 1) \cdot \mathrm{d}u} \tag{8}$$

where digam(x) is the digama fucntion given by $\frac{\partial \log \Gamma(x)}{\partial a}$.

Solution for optimality equations often require MTTF calculations multiple times using Eqs. 8 or 7 numerically

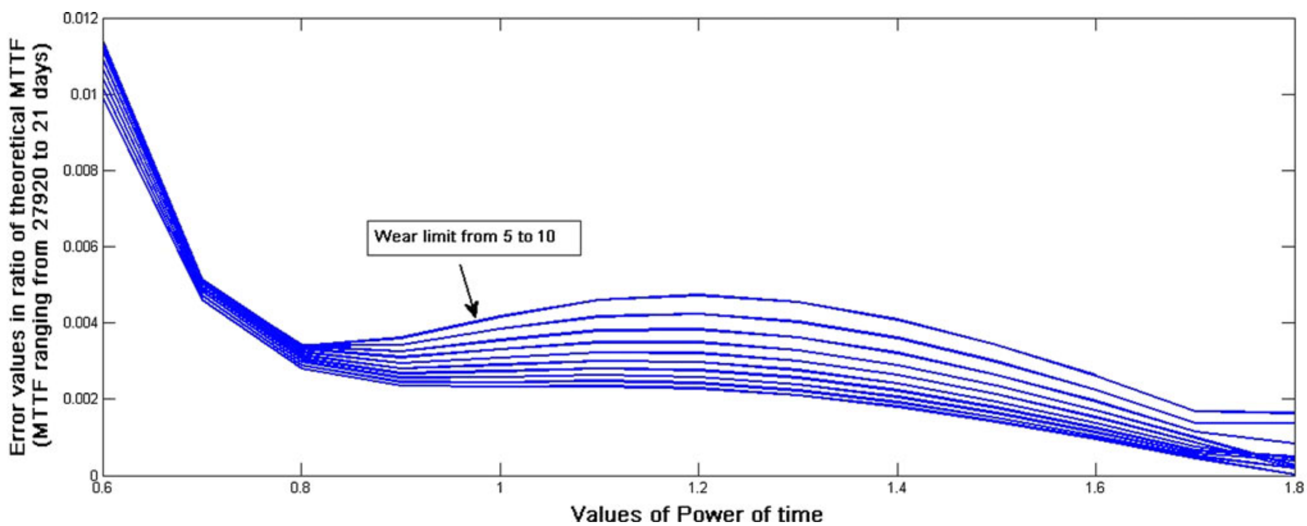

Fig. 2 Error ratio values vs. variation in 'ζ' ($\lambda = 0.02$; $\beta = 0.9$)

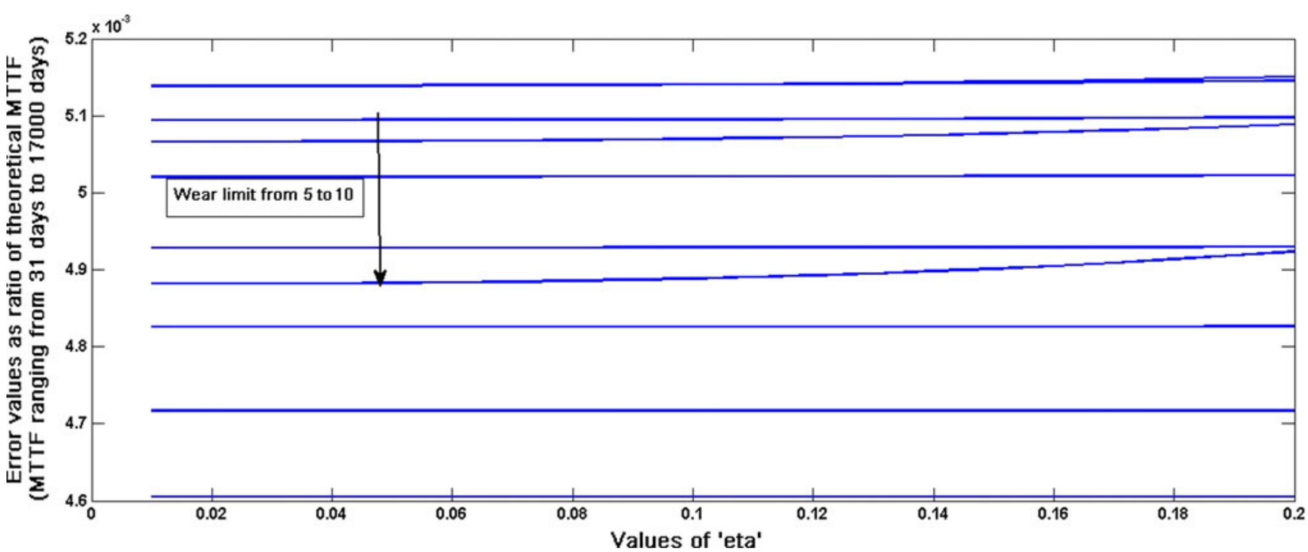

Fig. 3 Error ratio values vs. variation in 'λ' ($\zeta = 0.7$; $\beta = 0.9$)

integrated between 0 and infinity. This is a time consuming process. Instead the approximation equation given in Eq. 9 can be used to quickly arrive at the MTTF values, which for most of the mechanical wearing component would range from 4 months to around 7–8 years. Figures 1, 2 and 3 show the effect of variables on error values of MTTF shown as the ratio of theoretical MTTF values calculated using eq. 8.

$$\mathrm{MTTF}_{\mathrm{approx}} = \left(\frac{\mathrm{wearlimit} \cdot \beta}{\lambda} + \frac{0.479}{\lambda} \cdot e^{\left(\frac{1-\zeta}{\Gamma(\beta+\zeta)}\right)}\right)^{\frac{1}{\zeta}} \quad (9)$$

2 Conclusion

Wear or deterioration experienced in the mechanical components can be adequately mapped using the non-stationary gamma wear process. The search for optimal solutions for maintenance intervals or inspection intervals require multiple evaluations of MTTF or mean time to reach the designated wear limits. The search for the optimal solutions can be hastened up using the approximation Eq. 9 which keeps the error within less than 3% for mechanical components with a like span of 4 months to 7–8 years.

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