IJEC (2017) 49:229–243
DOI 10.1007/s13158-017-0189-1

ORIGINAL ARTICLE

Preschool Teachers' Pedagogical Content Knowledge in Mathematics

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Published online: 19 June 2017

Abstract The study investigated preschool teachers' pedagogical content knowledge (PCK) in mathematics. The construct of PCK for teaching mathematics in preschool involves three components: (1) noticing mathematical situations in which children engage; (2) interpreting the nature of children's math activity; and (3) enhancing children's mathematical thinking and understanding. The research participants were 30 preschool teachers from a large city in South Korea. Teachers identified mathematical situations in a children's play scenario, interpreted the nature of the mathematical situations, and identified how children's mathematical thinking in the reported situation could be enhanced. The teachers' responses were quantitatively scored. The results indicate that participants possessed higher levels of PCK about number sense, measurement, and classification than for patterning, operations, shapes, and spatial relationships. Teachers need knowledge to interpret mathematical situations in order to identify ways to enhance children's mathematical thinking. Teachers with more teaching experience were more likely to have higher scores on the overall PCK measure. Teachers require greater knowledge to recognize specific mathematical concepts in use in children's play in order to be able to extend and enhance preschool children's mathematical thinking.

Keywords Preschool mathematics · Preschool teachers · Pedagogical content knowledge · Mathematical thinking · Teacher knowledge

Résumé L'étude a porté sur la connaissance du contenu pédagogique (CCP) des enseignants du préscolaire en mathématiques. La construction de la CCP pour l'enseignement des mathématiques au niveau préscolaire comporte trois volets: (1)

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remarquer les situations mathématiques dans lesquelles les enfants s'engagent; (2) interpréter la nature de l'activité mathématique des enfants; et (3) accroître la pensée et la compréhension mathématiques des enfants. Les participants à la recherche étaient 30 enseignants du préscolaire d'une grande ville de Corée du Sud. Les enseignants identifiaient les situations mathématiques dans un scénario de jeu d'enfants, interprétaient la nature des situations mathématiques et identifiaient comment la pensée mathématique des enfants dans la situation rapportée pourrait être améliorée. Les réponses des enseignants étaient quantitativement notées. Les résultats indiquent que les participants possédaient des niveaux plus élevés de CCP sur le sens du nombre, la mesure et la classification que sur la modélisation, les opérations, les formes et les relations spatiales. Les enseignants ont besoin de connaissances pour interpréter les situations mathématiques afin d'identifier les moyens d'améliorer la pensée mathématique des enfants. Les enseignants plus expérimentés étaient plus susceptibles d'avoir des scores plus élevés à la mesure globale de la CCP. Les enseignants ont besoin de plus de connaissance pour reconnaître les concepts mathématiques spécifiques utilisés dans le jeu des enfants afin d'être en mesure d'accroître et d'améliorer la pensée mathématique des enfants d'âge préscolaire

Resumen El estudió investigó el conocimiento de contenido pedagógico de profesores preescolares en matemática (PCK). La construcción de PCK para la enseñanza de matemática en el preescolar involucra tres componentes: (1) advertir situaciones matemáticas en las que los niños se involucran; (2) interpretar la naturaleza de la actividad matemática de los niños; y (3) fomentar el pensamiento matemático y el entendimiento de los niños y niñas. Los participantes de la investigación fueron 30 profesores preescolares de una ciudad grande de Corea del Sur. Los profesores identificaron situaciones matemáticas en un escenario de juego infantil; interpretaron la naturaleza de las situaciones matemáticas; e identificaron cómo el pensamiento matemático de los niños en la situación reportada podría ser fomentado. Las respuestas de los profesores fueron puntuadas cuantitativamente. Los resultados indicaron que los participantes que poseían niveles de PCK más altos acerca del sentido numérico, mediciones y clasificaciones no los tenían para patrones, operaciones numéricas, formas y relaciones espaciales. Los profesores necesitan conocimiento para interpretar las situaciones matemáticas para identificar formas de potenciar el pensamiento matemático en los niños. Los profesores con más experiencia pedagógica poseían mayores posibilidades de obtener puntajes más altos en el PCK general. Los profesores requieren mayores niveles de conocimiento para reconocer conceptos matemáticos específicos para su utilización en los juegos de los niños, para ser capaces de extender y potenciar el pensamiento matemático de niños de preescolar.

Introduction

Mathematical thinking and reasoning are increasingly important in modern society as a result of technological changes. In order to adapt to the information-based society, mathematical literacy and skills are indispensable (Yore et al. 2007).

Mathematical literacy and skills are key factors in children's educational success. Early development of mathematical skills can enhance school achievement as well as increasing career opportunities in adulthood. It is important to recognize the significance of quality mathematics education in early childhood because such education can lay the foundation for successful mathematics experiences through school (Anders and Rossbach 2015).

Many studies have emphasized the importance of teachers'pedagogical content knowledge (PCK) in mathematics in order to provide high-quality early mathematics education to children (Macdonald et al. 2012; Gervasoni et al. 2012). PCK in mathematics correlates highly with abilities to teach mathematics, teaching efficacy, students' positive attitude toward mathematics, and mathematics achievement (Empson and Junk 2004; Hill et al. 2005). Ginsburg and Amit (2008) conducted a case study on the teaching of mathematics by a preschool teacher and concluded that teachers with a deep knowledge of the subject, as well as pedagogical content knowledge in mathematics, can provide mathematics education that helps children think mathematically in their daily activities. This study explores the mathematical knowledge base of early childhood teachers. It addresses the level and extent of preschool teachers' PCK in mathematics and discusses directions for teacher education in order to raise the level of preschool teachers' abilities to teach mathematics to young children.

Preschool Teachers' PCK in Mathematics

This article focuses on three facets of professional competence related to teaching mathematics: (1) awareness of opportunities for mathematical learning in children's play; (2) interpreting children's math activities based on preschool mathematical concepts; and (3) enhancing children's in-depth mathematical thinking. The theoretical framework stems from the work of Ball et al. (2008) and uses Shulman's (1986) constructs of content knowledge and pedagogical content knowledge. A central contribution of Shulman's work has been to provide a conceptual framework about teacher knowledge (Grossman 1990; Ball et al. 2008). However, Ball et al. (2008) contend that Shulman's original constructs of teacher knowledge are too broad and inclusive to specify the critical professional knowledge needed for teaching, particularly for math instruction. For example, how should a researcher operationalize the term, pedagogical content knowledge for teaching mathematics? Ball et al. (2008) suggested that it is necessary to explore teachers' use of mathematical concepts and focus on what teachers actually do during math lessons. This premise emphasizes that teachers' knowledge is situated in their actual practice. Ball et al. explored how math was treated, taught, and incorporated into the regular day-to-day demands of teaching. Their analyses of the mathematical demands of teaching showed that special and unique mathematical knowledge was required, described in Shulman's (1986) term as PCK, and includes the everyday teaching tasks that include commenting on students' work, listening to students' talk, and assigning homework (Ball et al. 2008). Their research identified the types of content and pedagogy that might constitute good pedagogical mathematical knowledge that provided clear directions for elementary teacher education.

Unfortunately, Ball's work on PCK offers little information to those of us wishing to better understand effective teaching of preschool mathematics. Indeed, Ball's work is related to PCK for teaching elementary mathematics and which is somewhat different from the knowledge base required to teach mathematics in preschool (Lee 2010; McCray and Chen 2012; Anders and Rossbach 2015). Ball's interview and survey instrument related to PCK specifically gauged 'proficiency at providing students with mathematical explanations and representation and working with unusual solution methods' (Hill et al. 2005, p. 387). Elementary mathematics takes place at a fixed time and uses written notation to support students' meaningful understanding of arithmetic. In preschool, mathematics is implemented in play-based mathematical activities, which include real-world representations.

Preschool children are active mathematics learners, possessing informal mathematical knowledge (Clements et al. 2004; Baroody et al. 2006; Ginsburg and Ertle 2008). This informal mathematical knowledge is developed when children engage in problem-solving processes which often occur during children's free play time (Edo et al. 2009; Brandt 2013). Nonetheless, young children are not yet ready to understand, let alone use, written arithmetic symbols. Instead, preschool children are primed to explore and recognize mathematical concepts through their daily activities by engaging in various mathematical situations, and then later beginning to learn formal mathematics using mathematical symbols or letters. For example, preschool children can see division when they see cookies split into a given number of equally sized groups or into groups of cookies of a given size (Lee 2014). Situation of simply separating a quantity into two or more parts and sharing does not represent a formal understanding of division. However, through continuous experiences on mathematizing in informal math-related situations, children begin to understand formal mathematical knowledge, including the principles and symbols of division (Empson and Junk 2004; Lee 2014). The important basis for such learning is that children have had appropriate mathematical interactions with their teachers in such situational contexts as sharing cookies. These types of mathematical communications help children associate informal mathematical knowledge with later formal mathematical knowledge more easily. And such particular preschool pedagogical tasks require a knowledge base that is distinct from the knowledge that Ball's previous work identified for elementary mathematics education.

Thus, the definition and extent of PCK for preschool mathematics should be re-conceptualized to better reflect preschool mathematical content and specific pedagogical tasks. Preschool mathematics includes concepts of number sense, ordering, operations, patterns, shapes, spatial relationships, measurement, and classification (NCTM 2002). PCK for preschool mathematics is concerned with how to introduce these concepts effectively to young children.

Three Constructs of PCK in Preschool Mathematics

A reconceptualization of PCK in mathematics, as it pertains to teaching preschool children, should include the following three activities: noticing mathematical situations in children's everyday activities or play; interpreting such mathematical

situations based on preschool mathematical content; and enhancing children's in-depth mathematical thinking.

Noticing

This construct can be defined in a multitude of ways, but a connecting thread is identifying noteworthy aspects of classroom situations and making connections between the specific classroom events and broader principles of teaching and learning (van Es and Sherin 2002; Jacobs et al. 2010). This study focuses on a particular type of noticing the nature of children's mathematical thinking that occurs during children's free play. Preschool children mostly engage in mathematical thinking and problem solving during play (Edo et al. 2009; Brandt 2013; Vogel 2013). The more the teacher recognizes mathematical situations in children's play, the more opportunities can be provided to induce children's mathematical thinking. By identifying a focus on noticing, this study draws attention to a variety of mathematical situations that teachers may notice in children's play.

Interpreting

The second construct of PCK for preschool mathematics is *interpretin*g the nature of mathematical situations recognized in children's play, conversations, or everyday activities. According to Jacobs et al. (2010), teachers' ability to make an accurate interpretation of how children are enacting math in their play and what strategies they employ to solve a problem is an important indicator as to whether or not teachers can subsequently provide meaningful and enjoyable math activities. This study focuses on how, and the extent to which, teachers interpret mathematical situations in children's play based on important concepts of preschool mathematics.

Enhancing

The third construct of PCK for preschool mathematics is *enhancing* children's mathematical thinking. Good-quality mathematics education requires a teacher to extend and sustain children's thinking and experimentation with specific math content (Clements 2004). This can help young children construct their own strategies and develop efficient, generalizable methods to solve problems (Stipek et al. 2012). Thus, it is important for preschool teachers to have in-depth PCK on promoting children's mathematical thinking.

In summary, PCK for preschool mathematics can be conceptualized as a set of three interrelated skills. One is being able to notice the variety of mathematical situations that occur in the classroom; second is to interpret such situations based on specific preschool mathematical concepts; and third is to enhance children's mathematical thinking based on what teachers have noticed and interpreted.

The Present Study

Preschool teachers must be able to draw on PCK in mathematics (noticing, interpreting, and enhancing) in order to affect the quality of pedagogical interactions in the preschool classroom and children's mathematical thinking. Many research studies suggested that early childhood teachers' PCK for teaching mathematics is different for each teacher and, generally, inadequate (Copley 2010; Macdonald et al. 2012). According to Jae E Lee's work (2014) that examined US pre-kindergarten teachers' knowledge about children's mathematical thinking, teachers varied in their level of understanding of students' mathematical thinking. Other studies have reported that preschool teachers' PCK in mathematics is associated with various teacher characteristics such as emotional attitudes toward mathematics, joy and interest in mathematics, pedagogical beliefs, years of teaching experience, and the types of teaching qualifications (Lee 2010; Anders and Rossbach 2015). This indicates that the personal background of the teacher and contextual factors are important elements affecting the cultivation of PCK which leads to the provision of high-quality mathematics education for young children.

Preschool teachers' PCK has been debated as an essential prerequisite for effective teaching of mathematics for many years. Nevertheless, only a few studies exist that have empirically investigated preschool teachers' PCK, especially in relation to the abilities of the teacher to notice, interpret, and enhance children's mathematical thinking. This study addresses this gap by examining the extent to which preschool teachers possess understanding of these three constructs for PCK in mathematics. Using a scenario of children's play adapted from the *Preschool Mathematics PCK Interview* (McCray and Chen 2012), the study assesses preschool teachers' mathematical PCK through semi-structured interviews. Based on the quantitative analyses of teachers' interview scores, this article describes the mathematical knowledge base that preschool teachers hold and the relationships between their abilities to notice, interpret, and enhance children's mathematical thinking. The study also examines contextual factors related to preschool teachers' PCK in mathematics and how early childhood teacher education programs may help practicing teachers develop a deeper knowledge base for teaching mathematics.

Research questions for this study are:

1. What are the levels and extent of preschool teachers' PCK in mathematics in relation to noticing, interpreting, and enhancing mathematical situations in children's play?
2. How do the three constructs of noticing, interpreting, and enhancing relate to each other?
3. What teacher demographic factors are associated with preschool teachers' PCK in mathematics?

Research Method

Research Context: The Korean Preschool System

In Korea, compulsory schooling starts at age 6 years; participation in preschool programs or childcare is voluntary. However, attendance rates for children age 3 years and older are now high. In 2012, 85% of all children between age 3 and 6 years attended preschool (Korea Education Statistics Service 2015). Such high attendance rate relates to the current change of Korean early child education and care (ECEC) systems. The Korean government introduced the Nuri curriculum as a national curriculum in 2013 for all children aged 3–5 years in both kindergartens and childcare centers. The implementation of Nuri curriculum allows for the provision of a high-quality curriculum to all preschool children regardless of the type of ECEC service that they are attending (Chang 2013).

The Nuri curriculum consists of the basic qualities essential for children aged 3–5 years with a focus on child-centered and play-based curricula. The contents of the Nuri curriculum are composed of five areas: Physical Activity and Health, Communication which includes language art, Experience in Art, Social Relationships, and Nature and Discovery which relates to mathematical skills and scientific inquiries (Korean Ministry of Education & Ministry of Health and Welfare 2013). The vast majority of preschool teachers have completed a 3–4-year undergraduate degree program, and they usually participate in professional in-service training sessions which are supported by the Korean government.

Participants

Participants in this study were 30 Korean preschool teachers who taught 3–5 year-old children in either childcare centers or private preschools. The first step in recruiting participants was to meet with district representatives who helped the researcher connect to principals of childcare centers and preschools. The principals from six sites decided to participate in this research project and gave permissions to contact teachers who worked in their preschools or childcare centers. The researcher then sent emails to these teachers and gained informed consent from 30 preschool teachers who were willing to participate in this research.

Demographic information about the teachers was gathered for years of teaching, educational qualification, number of professional development courses in mathematics that teachers had received, and the type of teaching certificate. Seven teachers had fewer than 3 years of teaching; 14 had between 3 and 5 years of teaching; and 9 had more than 5 years of teaching. Eight teachers held Bachelor degrees from 3-year colleges, 14 held Bachelor degrees from 4-year colleges, and 8 held Masters degrees. Twenty-seven teachers held a state certificate in early childhood education, and three held an assistant director certificate in early childhood education. On average, the teachers had taken one or more mathematics courses in college; seven teachers reported having taken none. The number of professional development courses in mathematics that teachers had taken since

beginning teaching ranged from none (11 teachers) to 7 or more (4 teachers), with an overall average of 3 professional development courses.

Measurement Instrument

Preschool teachers' PCK in mathematics was assessed using a scenario-type approach (Ball et al. 2008). The participants were presented with a hypothetical scenario and teachers were asked questions that were adapted from the Preschool Mathematics PCK Interview (McCray and Chen 2012). First, teachers read the scenario to themselves and then the researcher read it to them. The researcher then asked the teacher a set of questions about the scenario. The scenario used is as follows:

> Brittaney and Jacob are playing in the dramatic play area and want to put their five babies to bed. There are no doll beds, so they make 'cribs' out of three shoeboxes. Jacob says, "But there aren't enough cribs." Brittany responds, "These babies are younger," picking out the three babies with no hair and setting them near the shoeboxes. She picks up the two babies with thick hair, says, "These babies don't need to nap anymore," and sets then aside. Jacob says, "OK, but this baby needs the most room," and puts the biggest bald baby into the biggest shoebox. Brittany watches him and then puts the medium-sized baby in the medium-sized shoebox and the smallest bald baby in the smallest shoebox. Jacob says, "Now go to sleep, babies."

This instrument is designed to assess preschool teachers' ability to: (1) notice mathematical situations presented in children's play; (2) interpret why such situations are mathematical, and how children's activities are connected with mathematical concepts; (3) enhance children thinking about the mathematical concepts. The interview scenario covers a range of early mathematics concepts and skills (e.g., number, operations, shapes, spatial relationships, measurement, patterns, and classification). Summarized below (Table 1) are the meanings of each subtopic of early mathematics concepts and skills (NCTM 2002) that are embedded in the scenario.

Interview Stages

The first part of the interview was related to teachers' ability to notice mathematics within the scenario. Participants were prompted to identify parts of the scene which, according to their understanding, contained concepts of preschool mathematics as described in Table 2. The second part of the interview was related to teachers' ability to interpret these mathematical situations that teachers had identified in the scenario. Participants were asked to interpret the identified scene by describing the nature of the mathematical concepts underlying the identified sequence using the taxonomy (numeracy, counting, operations, compare, ordering, measuring, spatial relations, or classifying) originally proposed by McCray (2008), as well as to indicate which subdomains of preschool mathematics contents were covered.

Table 1 Content areas of early mathematical understanding and associated skills

Content areas of early mathematical understanding	Mathematical skills
Number sense	Counting, comparing/ordering, number use, 1-to-1 correspondence
Operations	Adding to/taking away
Shapes	Recognizing, identifying, and naming geometry figures
Spatial relationships	Knowing the relationship between objects and their location in a three-dimensional world
Measurement	Recognizing, comparing, and ordering attributes such as length, volume, weight, and area
Patterns	Recognizing and creating pattern
Classification	Sorting/grouping objects using one or more attributes

Table 2 Play scenario: mathematical situations, skills and content areas of early mathematical understanding in the play scenario

Mathematical situations	Mathematical skills in the situations	Mathematical content areas in the situations
Baby-to-shoebox by size order	Order by size/monotonic seriation	Measurement
	Rule that repeats	Patterns
Shoebox = crib	Shape/space match	Shape
Babies INSIDE shoebox	'Crib' encloses	Spatial relations
	Relationships of location/between objects	
Babies/shoeboxes differ in size	Comparison by size	Measurement
Sort by hair = age	Logic/similarity	Classification
	Grouping	
3 cribs hold 3 babies	Counting, number use	Number sense
	1-to-1 correspondence	
	Compare/see amount/number	
Put two babies aside	Take away right amount/number	Operations

Table 2 indicates the mathematical situations, concepts, and contents included in the scenario as suggested by McCray and Chen (2012).

Participants were encouraged to name, describe, and categorize as many mathematical elements as they could identify in the scene. The ability to analyze a play situation and identify mathematical concepts tied to the situation relied explicitly on teachers' in-depth understanding of mathematical content knowledge. The third part of the interview required teachers' to identify how to enhance children's mathematical thinking based on their interpretation. Teachers were asked

to suggest some follow-up activities to help the children's mathematical thinking in the scenario and asked about the form of strategies and interactions that they would have with the children. Teachers' responses were evaluated on how the responses reflected their knowledge of children's mathematical thinking within the content strand and about their strategies which they could use to enhance children's thinking (McCray and Chen 2012).

Scoring

The original scoring system (McCray 2008) was adapted and revised to examine the level of PCK that teachers held. In the first part of the interview, the participants scored one point when they correctly identify the mathematical situation in the scene. There were seven mathematical situations in the play scene presented to the teachers, so if the participants recognized all mathematical situations then they could score 7 points. In the second part of the interview, two points were awarded if the teacher' interpretation was concrete and accurate and related to a mathematical concept described in Table 2. If the participant interpreted the situation related to mathematical concept somewhat ambiguously only one point was awarded and if the teacher's interpretation was general and ambiguous, no points were awarded. Also, if the participant accurately identified the mathematical content related to the situations, 1 point was awarded. Since there are 12 interpretation items and 8 mathematical contents, the range of possible scores was 0–32. Finally, the participants' responses were assessed on their description on how to enhance children's mathematical thinking for the degree of a math-focus in the description (0–4 points) and/or the degree of a play-related focus in the description of their interaction with the children (0–4 points).

Procedure and Data Analysis

Interview data were collected across an 8-week period in 2015. During this time, the researcher made a visit, after the preschool day was over, to six sites. The interviews lasted for approximately 30 min, and all interviews were digitally recorded and transcribed. In order to establish the reliability of the researcher' scoring on the participants' responses to the scenario, the teachers' responses were also scored by a graduate student in early childhood education and a professor of early childhood mathematics education. Consensus agreement was established at 94% and after going through the process of re-discussing the inconsistencies based on the guidelines suggested by McCray and Chen (2012), final consensus agreement became 100%.

The Statistical Package for Social Sciences (SPSS) was used in the quantitative analyses. Descriptive statistics to describe teachers' scores on the three aspects of the scenario were developed, including percentages for teachers' correct identification of the seven subcategories of early mathematics concepts in the scenario. To investigate whether teacher demographic factors accounted for significant differences in scores, *t* tests were used. Correlations between the three aspects of teachers' PCK were also calculated.

Results

To assess participants' knowledge for teaching mathematics, the Preschool Mathematics PCK Interview was used (McCray and Chen 2012). The score for '*Noticing a mathematical situation'* ranged from 0 to 7; '*Interpreting math activ*ity' ranged from 0 to 32, and the score for '*Enhancing mathematical thinkin*g' ranged from 0 to 8. Total possible score across the three subscales was 47. The mean score on each MKT subscale was calculated.

The overall mean score for PCK obtained from 30 preschool teachers was 26.18 (SD = 5.48). The mean for each subcategories was 4.82 (SD = .91) for noticing; 15.23 (SD = 3.62) for interpreting; and 6.14 (SD = 2.12) for enhancing mathematical thinking. The frequencies in identifying the seven mathematical subtopics indicated that classification (90.0%) was the most frequently recognized mathematical subtopic identified, followed by number sense (86.7%), measurement (86.7%), pattern (63.3%), shape (46.7%), operation (46.7%), and spatial relationships (13.3%). The results suggested that more than 80% of participants could interpret mathematical concepts related to classification, number sense, and measurement which could be considered adequate knowledge in those mathematical domains only. In addition, there was a significant correlation between the scores for interpreting and enhancing children's mathematical thinking ($r = .59$, $p < .01$).

Differences in total scores for PCK by years of teaching, acquired degree, and mathematics professional development received were examined using t tests. It was found that that teachers with more than 5 years of teaching experience at the preschool level had overall higher PCK than those with fewer than 5 years of experience ($t = 2.60$, $p < .01$) while teachers who held a Bachelor degree had lower PCK scores than teachers who had acquired a Master's degree in early childhood education ($t = 2.49$, $p < .05$).

Discussion

Several studies have emphasized the importance of early childhood teachers' PCK for being able to implement high-quality mathematics education (Sarama and DiBiase 2004; Ginsburg and Ertle 2008). This study investigated preschool teachers' PCK in mathematics based on three constructs (noticing, interpreting, enhancing) and the relationships between their PCK and teachers' demographic characteristics. This is one of the few international studies and the first study in Korea that has investigated the PCK of preschool teachers. The mean scores for each of the three categories of PCK revealed that these teachers' ability to interpret and enhance children's mathematical thinking is relatively low. Other empirical studies on preschool teachers' PCK in mathematics have also reported relatively low levels of such knowledge (Lee 2010, 2014). This study found that mathematical concepts related to number sense, measurement, and classification were most commonly recognized by teachers. However, teachers' ability to notice mathematical situations was not related to their ability to interpret children's math activity.

This indicated that noticing mathematical situations is not necessarily associated with ability to interpret such a situation. These findings support Lee's (2014) study, which reported that preschool teachers' ability to notice does not necessarily translate into effective execution of the interpretation. On the other hand, teachers' ability to interpret was significantly associated with their ability to enhance children's mathematical thinking. This implies that the higher the ability to interpret children's mathematics within specific content domains, the more broadly these teachers might enhance children's mathematical thinking.

The findings from this study suggest that teacher education should focus on developing a stronger knowledge base to better interpret children's math activities and their mathematical thinking. This would enable early childhood teachers to provide mathematics education that helps children mathematically think and solve various problems that they experience. According to Aubrey et al. (2012), helping teachers teach children to think and reason is an important strategy to improve children's problem-solving skills as well as their math performance. Anders and Rossbach (2015) stated that this is crucial as the awareness of mathematical content seems to be a prerequisite for enhancing preschool teachers' sensitivity to mathematical content in play-based situations and offering meaningful pedagogical interactions with the child. Having such skills not only enables better interpretation of children's mathematical thinking but also enables teachers to decide what is most important to teach, what should be taught now rather than later, and what kind of problems could be posed to students that would most likely facilitate their understanding of particular mathematical ideas (Loughran 2010).

An important contribution of this study is that it provides evidence of the depth of preschool teachers' content knowledge in mathematics. In this study, more than 80% of participants noticed and interpreted mathematical content related to classification, number sense, and measurement. The mathematical category in which participants noticed and interpreted most poorly was spatial relationships. This finding was also suggested by findings from Lee (2010), which explored US kindergarten teachers' PCK in mathematics. Lee (2010) stated that the scores of kindergarten teachers' PCK of mathematics on number sense were highest, while scores obtained for spatial relationships were the lowest. When teachers do not possess the knowledge about how to teach certain mathematical content, meaningful interactions with children related to such content cannot be achieved, nor can follow-up activities related to the contents occur. This means that there is less opportunity for children to think deeply about the specific mathematical contents. For example, teachers with less knowledge about spatial relationships are less likely to provide math activities or interactions that extend such content. This finding informs teacher educators to provide preschool teachers with opportunities to develop a better understanding of ways to teach mathematics in general, but especially in geometry.

The results revealed that there were differences in teachers' overall PCK scores by years of teaching and level of qualification. Teachers with more than 5 years' teaching experience at the preschool scored higher in interpreting and enhancing children's mathematical thinking than those with fewer. Also, the study found a significant relationship between the total PCK score and level of teacher

qualification. This indicated that greater teaching experience at the preschool level and teachers' level of qualification translated into higher levels of skills in interpreting and enhancing children's mathematical thinking. These findings support Lee's work (2010) and have underscored the importance of teaching experience and the level of education as significant indicators of PCK in mathematics.

Other contextual factors such as teachers' participation in mathematics coursework and type of teaching certificate were not significantly related to teachers' PCK. The results of this study were obtained from only 30 preschool teachers so additional studies that examine differences in teachers' demographic variables and their PCK level could be conducted.

Limitations

The first limitation of this study identified is the small size of the sample. The sample was drawn from one large city in Korea, and the sample may not be representative of all preschool teachers in Korea. Although preschool settings were drawn randomly, the schools and the participants needed to give their consent to take part in the study. As a consequence, the sample might be selective in terms of possessing positive attitudes or more interest in mathematics teaching. A further limitation of this study is the measure of PCK for preschool mathematics which must be broadened and linked to observational measures for further validation in future studies. It would be worthwhile to try to expand the measure to include teacher knowledge that is used outside the free-play setting or create and validate additional free-play scenarios that reflect diverse preschool learning environment (McCray and Chen 2012).

Implications for Future Research and Practice

This study provides specific leverage points to support early childhood practitioners' learning. The ability to analyze play situations and identify opportunities for 'mathematization' relies explicitly on teachers' in-depth mathematical understanding (Clements et al. 2004; Brandt 2013). Teachers' abilities to interpret and enhance children's mathematical thinking is a product of their knowledge of children's development for understanding mathematical concepts as well as their own knowledge of strategies, which they can deploy, to challenge children to move to the next stage of complexity in this content area (McCray and Chen 2012). Strategies adopted by teachers to help children explore mathematical ideas in their daily routines in preschool are effective only when teachers have a depth of personal understanding about mathematical content. Thus, to support the development of PCK in mathematics for prospective and practicing teachers, professional development should go beyond the level of introducing mathematical content and simple or general methods of teaching mathematics. Instead, it should help teachers to identify specific mathematical situations that are evident in children's everyday activities and classroom routines, to interpret those situations accurately, and to find various teaching and learning strategies that 'mathematize' children's informal experience and enhance their thinking about mathematical concepts. This direction

requires effective professional development and powerful mechanisms in order to tap into teachers' content knowledge, to improve pedagogy and children's learning about mathematics in preschool classrooms.

This study has identified the levels of preschool teachers' expertise in noticing children's mathematics knowledge in play situations, as well as their abilities to interpret and enhance children's mathematical thinking. This study has underscored the important role for professional development to help preschool teachers gain more expertise for teaching mathematics. Specific content and implementation strategies for professional development programs that help both pre-service and in-service preschool teachers to improve their knowledge for teaching mathematics, specifically in noticing and interpreting mathematical situations that arise within children's everyday activities and play, are an important goal for teacher education.

Acknowledgements This study was supported by research Grants from the Baekseok University in 2016.

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