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ORIGINAL ARTICLE

MAGDM-oriented dual hesitant fuzzy multigranulation probabilistic models based on MULTIMOORA

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Abstract
In real world, multi-attribute group decision making (MAGDM) is a complicated cognitive process that involves expression, fusion and analysis of multi-source uncertain information. Among diverse soft computing tools for addressing MAGDM, the ones from granular computing (GrC) frameworks perform excellently via efficient strategies for multi-source uncertain information. However, they usually lack convincing semantic interpretations for MAGDM due to extreme information fusion rules and instabilities of information analysis mechanisms. This work adopts a typical GrC framework named multigranulation probabilistic models to enrich semantic interpretations for GrC-based MAGDM approaches, and constructs MAGDM-oriented multigranulation probabilistic models with dual hesitant fuzzy (DHF) information in light of the MULTIMOORA (Multi-Objective Optimization by Ratio Analysis plus the full MULTIplicative form) method. After reviewing several basic knowledge, we first put forward four types of DHF multigranulation probabilistic models. Then, according to the MULTIMOORA method, a DHF MAGDM algorithm is designed via the proposed theoretical models in the context of person-job (P-J) fit. Finally, an illustrative case study for P-J fit is investigated, and corresponding validity tests and comparative analysis are conducted as well to demonstrate the rationality of the presented models.

Keywords Granular computing · MAGDM · Multigranulation probabilistic models · Dual hesitant fuzzy information · MULTIMOORA

1 Introduction

MAGDM consists of several decision matrices provided by a panel of decision makers, and each of them involves a set of finite alternatives that are depicted by finite attributes [40]. Until now, plenty of traditional approaches have been advised to handle MAGDM problems, and it is recognized that GrC-based methods act as quite effective representatives among them [9, 25, 38, 46, 48]. Compared with classic MAGDM methods, GrC-based methods excel in simulating human thinking processes and intelligent behaviors by using approximate reasonings rather than precise reasonings. In addition, GrC-based methods are able to divide a complicated problem into several fundamental components, then corresponding processing strategies for each component are employed to efficiently address the complicated problem. In the framework of GrC, lots of concrete theories were put forward in succession, such as fuzzy sets, rough sets, three-way decisions (3WD), formal concept analysis, quotient spaces, cloud models, and so forth [13, 35, 41].

Recently, with the rapid progress of data acquisition techniques and the arrival of big data era, individuals are convenient to have a quick access to vast amounts of data for decision making. In the meantime, some inevitable limitations can be found in MAGDM that may affect the acquisition of high-quality decision making results, such

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as various uncertainties existed in decision making data, individual subjectivity of decision makers, the absent of robust decision analysis tools, and so forth. Moreover, the addressing of MAGDM problems can be divided into three stages, i.e., information depiction, information fusion and information analysis [47, 49]. More specifically, the limitation of various uncertainties existed in decision making data is usually processed in information depiction stage, the limitation of individual subjectivity of decision makers is usually processed in information fusion stage, and the limitation of lacking robust decision analysis tools is usually processed in information analysis stage. However, it is noteworthy that existing MAGDM approaches are likely to confront with great challenges when coping with the above-mentioned limitations in three stages, thus it is imperative to construct novel GrC-based MAGDM approaches. In specific, the above-stated three challenges are listed as follows:

(1) In information depiction stage, the acquisited data for decision making often presents various uncertainties such as fuzziness, imprecision, hesitation, and so forth, which enhance the complexity of problem solving to some extent. The establishment of classic fuzzy sets not only enables experts to describe lots of fuzzy concepts, but also promotes the study of fuzzy MAGDM to the depth direction. Afterwards, many kinds of generalized fuzzy sets are designed in succession to handle a wider variety of uncertainties [3]. Hence, it is significant to select suitable generalized fuzzy sets to depict corresponding uncertainties and integrate them in the construction of MAGDM models.
(2) In information fusion stage, how to integrate individual preferences to group preferences is one of the core difficulties in this stage [24, 39, 42, 60]. The majority of current information fusion methods concentrate on efficient integrations of individual preferences but overlook the influence triggered by individual subjectivity of decision makers. Hence, it is significant to design novel MAGDM information fusion schemes that can avoid the above-mentioned negative impacts.
(3) In information analysis stage, utilizing the presented theoretical models to choose, rank or sort different alternatives plays a vital role in this stage [8, 23, 31, 32]. It is noted that if decision analysis tools lack strong stabilities, the final decision making results may vary according to decision risks, fault tolerance of models, differences of various decision making methods, etc. Hence, it is significant to construct robust decision analysis tools for analyzing MAGDM information.

For the sake of addressing the above-stated three challenges, the concept of DHF sets (DHFSs) [61, 62], multigranulation probabilistic rough sets (MGPRSs) [15, 59] and MULTIMOORA [1, 2] is scheduled to investigate MAGDM-oriented DHF multigranulation probabilistic models based on MULTIMOORA. The detailed motivations of utilizing them to establish novel GrC-based MAGDM approaches can be summarized below:

(1) DHFSs: DHFSs encompass several DHF elements (DHFEs) for corresponding alternatives, it is noted that each DHFE is equipped with a membership degree and a non-membership degree of an alternative to a provided set, and both of them own several finite values. For instance, a human resource expert may consume the value of membership degrees is 0.6 or 0.7 when evaluating the level of computer skills for an employee, and the value of non-membership degrees is 0.1 or 0.2 when performing the same task. Then, a DHFE $(\{0.6, 0.7\}, \{0.1, 0.2\})$ can handle the above-stated situation when the human resource expert shows a hesitating attitude between some values. Thus, DHFSs can depict fuzziness, imprecision and hesitation of uncertain information simultaneously for MAGDM. More recently, many scholars have developed DHF MAGDM approaches from the aspect of information measures [5, 6, 30], aggregation operators [33], traditional MAGDM tools [18, 22], rough set theories [52, 54, 55], 3WD [10, 11, 14], and so forth.
(2) MGPRSs: MGPRSs take advantages of multigranulation rough sets (MGRSs) [19–21, 26] and probabilistic rough sets (PRSs) [36, 43, 44] in information fusion and analysis, respectively. For one thing, multiple binary relations are arranged to be processed at the same time in light of the idea of parallel computing, and each binary relation can be regarded as a decision matrix offered by each decision maker. If we can process multiple binary relations together, it will be beneficial to study valid approaches to MAGDM with enhanced information fusion efficiencies. Moreover, MGRSs are also used to address risk-based MAGDM with the support of optimistic, pessimistic, adjustable and other explainable versions [28, 37, 45, 50, 51, 53, 56–58]. For instance, Xu and Guo [37] put forward the idea of double-quantitative 3WD in generalized multigranulation spaces. Zhang et al. [50, 51, 53] explored MGRSs in several generalized hesitant fuzzy (HF) information systems along with related GrC-based MAGDM approaches. Zhan et al. [45, 56–58] developed a series of covering-based rough set models and further discussed their applications in decision making. Sun et al. [28] considered a novel MGRS with diversified binary relations for solving fuzzy MAGDM problems. For another, noticing that the structure of rough approximations is rather strict in original rough sets, PRSs-based MAGDM methods own the ability

of fault tolerance by easing the strict limitations and setting suitable thresholds [12, 16, 17, 25, 27, 29]. For instance, Sun et al. [25, 27] proposed novel 3WD-based MAGDM approaches in fuzzy and linguistic information systems. Liang et al. [12] studied a method for 3WD by using traditional pythagorean fuzzy decision making tools. Mandal and Ranadive [16, 17] discussed MGPRSs in bipolar-valued fuzzy and interval-valued fuzzy information systems along with their 3WD-based MAGDM approaches. Wang and Liang [29] presented the preference measure of multi-granularity probabilistic linguistic term sets for addressing large-scale MAGDM problems.

(3) MULTIMOORA: For the sake of establishing a robust MAGDM method that is free from the influence of different MAGDM results, MULTIMOORA can generate the final ranking for alternatives by virtue of three subordinate rankings, i.e., ratio systems, reference point approaches and full multiplicative forms, and it has been confirmed that the final ranking by using MULTIMOORA is more robust than traditional MAGDM approaches in which a single ranking is concluded. In the past few years, many scholars have utilized MULTIMOORA in numerous realistic fields such as materials selections [7], type 2 diabetes selections [4], and other typical ones.

Considering that DHFSs, MGPRSs and MULTIMOORA can address related challenges in information depiction, information fusion and information analysis of MAGDM respectively, and there lack adequate supports for a comprehensive MAGDM approach that combines DHFSs, MGPRSs with MULTIMOORA simultaneously. Therefore, it is necessary to propose MGPRSs with MULTIMOORA in the DHF background and further construct a corresponding MAGDM approach. In addition, the feasibility of the constructed MAGDM approach is verified by a systematic case study for P-J fit. It is expected that the proposed DHF MAGDM approach owns the following advantages: (1) depicting various uncertain MAGDM information; (2) fusing multi-source MAGDM information that is incorporated into decision risks and fault tolerance abilities; (3) providing robust MAGDM results for complicated DHF MAGDM problems. In light of the above study motivations, we sum up the critical contributions of the current paper as follows:

(1) Four types of DHF membership degrees are put forward via optimistic/pessimistic tactics, DHF hybrid average (DHFHA) operators, DHF weighted Euclidean distances along with DHF hybrid geometric (DHFHG) operators, and related MAGDM-oriented DHF multigranulation probabilistic models are developed.
(2) A DHF MAGDM approach is constructed based on the developed theoretical models with MULTIMOORA.
(3) A P-J fit case study is studied along with validity tests and comparative analysis, the final conclusion shows the proposed MAGDM-oriented DHF multigranulation probabilistic models based on MULTIMOORA excel in providing reasonable and explainable results.

The remainder of the work is structured as follows. The next section plans to briefly revisit necessary preliminaries of DHFSs, DHF MGRSs, and PRSs. In Sect. 3, four types of MAGDM-oriented DHF multigranulation probabilistic models are put forward. In the following section, a DHF MAGDM approach based on MULTIMOORA is constructed. In Sect. 5, a realistic case study for P-J fit is explored together with validity tests and comparative analysis. The last section concludes the paper with future research directions.

2 Preliminaries

The current section aims to present the basic knowledge of DHFSs, DHF MGRSs, and PRSs in a brief way.

2.1 DHFSs

Definition 2.1 [62] Suppose U is a finite universe of discourse, a DHFS $\mathbb{D}$ over U is defined as follows:

$$\mathbb{D} = \{\langle x, h_{\mathbb{D}}(x), g_{\mathbb{D}}(x)\rangle | x \in U\}, \tag{1}$$

where $h_{\mathbb{D}}(x)$ and $g_{\mathbb{D}}(x)$ are two independent sets that contain several possible values in [0, 1], representing the membership degrees and non-membership degrees of $x \in U$ to $\mathbb{D}$, respectively, and $d(x) = (h_{\mathbb{D}}(x), g_{\mathbb{D}}(x))$ is named a DHFE. In an arbitrary DHFS $\mathbb{D}$, for each $x \in U$, let $\gamma \in h_{\mathbb{D}}(x), \eta \in g_{\mathbb{D}}(x)$, $\gamma^+ = \max\{\gamma | \gamma \in h_{\mathbb{D}}(x)\}$ and $\eta^+ = \max\{\eta | \eta \in g_{\mathbb{D}}(x)\}$, then we have $0 \le \gamma, \eta \le 1$ and $0 \le \gamma^+, \eta^+ \le 1$. For the sake of convenience, a set that contains all DHFSs over U is represented by $DHF(U)$.

In order to compare the magnitude of several DHFEs for MAGDM, Zhu et al. [62] developed the notion of score functions and accuracy functions for DHFEs.

Definition 2.2 [62] Given a DHFE $d(x) = \left(h_{\mathbb{D}}(x), g_{\mathbb{D}}(x)\right)$, the score function and the accuracy function of $d(x)$ are defined as follows:

$$s(d(x)) = \frac{\sum_{\gamma \in h_{\mathbb{D}}(x)} \gamma(x)}{\# h_{\mathbb{D}}(x)} - \frac{\sum_{\eta \in g_{\mathbb{D}}(x)} \eta(x)}{\# g_{\mathbb{D}}(x)}; \tag{2}$$

$$t(d(x)) = \frac{\sum_{\gamma \in h_{\mathbb{D}}(x)} \gamma(x)}{\# h_{\mathbb{D}}(x)} + \frac{\sum_{\eta \in g_{\mathbb{D}}(x)} \eta(x)}{\# g_{\mathbb{D}}(x)}, \tag{3}$$

where $\#h_{\mathbb{D}}(x)$ and $\#g_{\mathbb{D}}(x)$ represent the numbers of the values in $h_{\mathbb{D}}(x)$ and $g_{\mathbb{D}}(x)$, respectively. For any two DHFEs $d(x)$ and $d'(x)$, the concrete comparison laws are listed as follows:

(1) If $s(d(x)) > s\left(d'(x)\right)$, then $d(x) \succ d'(x)$;
(2) If $s(d(x)) < s\left(d'(x)\right)$, then $d(x) \prec d'(x)$;
(3) If $s(d(x)) = s\left(d'(x)\right)$ and $t(d(x)) > t\left(d'(x)\right)$, then $d(x) \succ d'(x)$;
(4) If $s(d(x)) = s\left(d'(x)\right)$ and $t(d(x)) < t\left(d'(x)\right)$, then $d(x) \prec d'(x)$;
(5) If $s(d(x)) = s\left(d'(x)\right)$ and $t(d(x)) = t\left(d'(x)\right)$, then $d(x) \sim d'(x)$.

In what follows, we introduce DHF operational laws that occupy a central role in constructing DHF MAGDM approaches. It is noteworthy that the values in a DHFE are usually provided in disorder and the number of values in diverse DHFEs may be different. In order to reduce the computational complexity of DHF MAGDM, Zhang et al. [55] put forward the following two assumptions for a DHFE $d(x) = (h_{\mathbb{D}}(x), g_{\mathbb{D}}(x))$:

(1) Suppose the values in $h_{\mathbb{D}}(x)$ and $g_{\mathbb{D}}(x)$ are placed in an increasing order, then the σth value in $h_{\mathbb{D}}(x)$ is represented by $\gamma^{\tau(\sigma)}(x)$, whereas the σth value in $g_{\mathbb{D}}(x)$ is represented by $\eta^{\tau(\sigma)}(x)$.
(2) For any two DHFEs, if the numbers of the values in $h_{\mathbb{D}}(x)$ and $g_{\mathbb{D}}(x)$ are different, we can make them own the same number by supplementing some largest ones to $h_{\mathbb{D}}(x)$ and $g_{\mathbb{D}}(x)$ with less number of values.

In light of the above-stated assumptions, some common updated DHF operational laws are presented below.

Definition 2.3 [55] For any two DHFSs $\mathbb{D}_1$ and $\mathbb{D}_2$, the corresponding DHFEs for $\mathbb{D}_1$ and $\mathbb{D}_2$ are $d_1(x)$ and $d_2(x)$, respectively. For any $x \in U$, the following updated DHF operational laws are valid:

(1) $d_1(x) \boxplus d_2(x) = \{\{\gamma_1^{\tau(\sigma)}(x) + \gamma_2^{\tau(\sigma)}(x) - \gamma_1^{\tau(\sigma)}(x)\gamma_2^{\tau(\sigma)}(x)\}, \{\eta_1^{\tau(\sigma)}(x)\eta_2^{\tau(\sigma)}(x)\}\}$;
(2) $d_1(x) \boxtimes d_2(x) = \{\{\gamma_1^{\tau(\sigma)}(x)\gamma_2^{\tau(\sigma)}(x)\}, \{\eta_1^{\tau(\sigma)}(x)+\eta_2^{\tau(\sigma)}(x) -\eta_1^{\tau(\sigma)}(x)\eta_2^{\tau(\sigma)}(x)\}\}$;
(3) $d_1(x) \boxminus d_2(x) = \{\max\{0, \frac{\gamma_1^{\tau(\sigma)}(x)-\gamma_2^{\tau(\sigma)}(x)}{1-\gamma_2^{\tau(\sigma)}(x)}\}, \min\{1, \frac{\eta_1^{\tau(\sigma)}(x)}{\eta_2^{\tau(\sigma)}(x)}\}\}$;
(4) $d_1(x) \boxdot d_2(x) = \{\min\{1, \frac{\gamma_1^{\tau(\sigma)}(x)}{\gamma_2^{\tau(\sigma)}(x)}\}, \max\{0, \frac{\eta_1^{\tau(\sigma)}(x)-\eta_2^{\tau(\sigma)}(x)}{1-\eta_2^{\tau(\sigma)}(x)}\}\}$;
(5) $\left(d_1(x)\right)^{\lambda} = \{(\gamma_1^{\tau(\sigma)}(x))^{\lambda}, 1 - (1 - \eta_1^{\tau(\sigma)}(x))^{\lambda}\}, \lambda > 0$;
(6) $\lambda\left(d_1(x)\right) = \{1 - (1 - \gamma_1^{\tau(\sigma)}(x))^{\lambda}, (\eta_1^{\tau(\sigma)}(x))^{\lambda}\}, \lambda > 0$;
(7) the complement of $\mathbb{D}_1$, represented by ${\mathbb{D}_1}^{c}$, is provided by $\sim d_1(x) = \{\{\eta_1^{\tau(\sigma)}(x)\}, \{\gamma_1^{\tau(\sigma)}(x)\}\}$;
(8) the intersection of $\mathbb{D}_1$ and $\mathbb{D}_2$, represented by $\mathbb{D}_1 \Cap \mathbb{D}_2$, is provided by $d_1(x)\overline{\wedge}d_2(x) = \{\{\gamma_1^{\tau(\sigma)}(x) \wedge \gamma_2^{\tau(\sigma)}(x)\}, \{\eta_1^{\tau(\sigma)}(x) \vee \eta_2^{\tau(\sigma)}(x)\}\}$;
(9) the union of $\mathbb{D}_1$ and $\mathbb{D}_2$, represented by $\mathbb{D}_1 \Cup \mathbb{D}_2$, is provided by $d_1(x)\underline{\vee}d_2(x) = \{\{\gamma_1^{\tau(\sigma)}(x) \vee \gamma_2^{\tau(\sigma)}(x)\}, \{\eta_1^{\tau(\sigma)}(x) \wedge \eta_2^{\tau(\sigma)}(x)\}\}$.

2.2 DHF MGRSs

MGRSs are known as a very useful tool in fusing traditional binary relations. For enlarging the application range of MGRSs, Zhang et al. [52] explored the concept of DHF MGRSs. Prior to the formal presentation of DHF MGRSs, the following concept of DHF relations (DHFRs) is presented.

Definition 2.4 [52] Suppose U and V are two finite universes of discourse, a DHFR $\mathbb{R}$ over $U \times V$ is defined as follows:

$$\mathbb{R} = \{\langle (x, y), h_{\mathbb{R}}(x, y), g_{\mathbb{R}}(x, y)\rangle | (x, y) \in U \times V\}, \tag{4}$$

where $h_{\mathbb{R}}(x, y)$ and $g_{\mathbb{R}}(x, y)$ are two independent sets that contain several possible values in [0, 1], representing the membership degrees and non-membership degrees of $(x, y) \in U \times V$ to $\mathbb{R}$, respectively In an arbitrary DHFR $\mathbb{R}$, for each $(x, y) \in U \times V$, let $\gamma \in h_{\mathbb{R}}(x, y)$, $\eta \in g_{\mathbb{R}}(x, y)$, $\gamma^+ = \max\{\gamma | \gamma \in h_{\mathbb{R}}(x, y)\}$ and $\eta^+ = \max\{\eta | \eta \in g_{\mathbb{R}}(x, y)\}$, then we have $0 \le \gamma, \eta \le 1$ and $0 \le \gamma^+, \eta^+ \le 1$. For the sake of convenience, a set that contains all DHFRs over $U \times V$ is represented by $DHFR(U \times V)$.

In what follows, the concept of DHF MGRSs can be constructed via the above-stated DHFRs.

Definition 2.5 [52] Suppose U and V are two finite universes of discourse, $\mathbb{R}_i \in DHFR(U \times V)$ are m DHFRs over $U \times V$. For any $\mathbb{D} \in DHF(V)$, the optimistic DHF lower and upper approximations of $\mathbb{D}$ are defined as follows:

$$\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{O}(\mathbb{D}) = \left\{ \left\langle x, h_{\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{O}(\mathbb{D})}(x), g_{\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{O}(\mathbb{D})}(x) \right\rangle | x \in U \right\}; \tag{5}$$

$$\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D}) = \left\{ \left\langle x, h_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D})}(x), g_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D})}(x) \right\rangle \middle| x \in U \right\}, \tag{6}$$

where

$$h_{\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D})}(x) = \underline{\bigvee}_{i=1}^{m} \overline{\wedge}_{y\in V}\{g_{\mathbb{R}_i}(x,y)\underline{\vee}h_{\mathbb{D}}(y)\};$$

$$g_{\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D})}(x) = \overline{\bigwedge}_{i=1}^{m} \underline{\vee}_{y\in V}\{h_{\mathbb{R}_i}(x,y)\overline{\wedge}g_{\mathbb{D}}(y)\};$$

$$h_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D})}(x) = \overline{\bigwedge}_{i=1}^{m} \underline{\vee}_{y\in V}\{h_{\mathbb{R}_i}(x,y)\overline{\wedge}h_{\mathbb{D}}(y)\};$$

$$g_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D})}(x) = \underline{\bigvee}_{i=1}^{m} \overline{\wedge}_{y\in V}\{g_{\mathbb{R}_i}(x,y)\underline{\vee}g_{\mathbb{D}}(y)\}.$$

In a similar manner, the pessimistic DHF lower and upper approximations of $\mathbb{D}$ are defined as follows:

$$\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D}) = \left\{ \left\langle x, h_{\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x), g_{\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x) \right\rangle \middle| x \in U \right\}; \tag{7}$$

$$\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D}) = \left\{ \left\langle x, h_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x), g_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x) \right\rangle \middle| x \in U \right\}, \tag{8}$$

where

$$h_{\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x) = \overline{\bigwedge}_{i=1}^{m} \overline{\wedge}_{y\in V}\{g_{\mathbb{R}_i}(x,y)\underline{\vee}h_{\mathbb{D}}(y)\};$$

$$g_{\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x) = \underline{\bigvee}_{i=1}^{m} \underline{\vee}_{y\in V}\{h_{\mathbb{R}_i}(x,y)\overline{\wedge}g_{\mathbb{D}}(y)\};$$

$$h_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x) = \underline{\bigvee}_{i=1}^{m} \underline{\vee}_{y\in V}\{h_{\mathbb{R}_i}(x,y)\overline{\wedge}h_{\mathbb{D}}(y)\};$$

$$g_{\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D})}(x) = \overline{\bigwedge}_{i=1}^{m} \overline{\wedge}_{y\in V}\{g_{\mathbb{R}_i}(x,y)\underline{\vee}g_{\mathbb{D}}(y)\},$$

then the pairs $(\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D}), \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{O}(\mathbb{D}))$ and $(\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D}), \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{P}(\mathbb{D}))$ are named an optimistic DHF MGRS and a pessimistic DHF MGRS of $\mathbb{D}$, respectively.

2.3 PRSs

The notion of PRSs [36, 43, 44] was explored towards a gradual loosening of strict restrictions for original rough sets, thereby application scopes of the rough set community were expanded.

Definition 2.6 [36] Let U, R, P be a finite universe of discourse, an equivalence relation and the probabilistic measures, respectively, and α and β be the thresholds. For any $x \in U$ with $0 \le \beta < \alpha \le 1$, the lower and upper approximations of X are defined as follows:

$$\underline{R}_{\alpha}(X) = \{P(X|[x]_R) \ge \alpha | x \in U\}; \tag{9}$$

$$\overline{R}_{\beta}(X) = \{P(X|[x]_R) > \beta | x \in U\}, \tag{10}$$

then the pair $(\underline{R}_{\alpha}(X), \overline{R}_{\beta}(X))$ is named a PRS of X.

3 MAGDM-oriented DHF multigranulation probabilistic models

In light of unique merits owned by DHF MGRSs and PRSs, it is necessary to design hybrid models by integrating DHF MGRSs with PRSs. The core challenges of this section are to adopt reasonable semantic interpretations in information fusion schemes for the developed DHF multigranulation probabilistic models, and some common information fusion methods from MAGDM are selected to establish different types of MAGDM-oriented DHF multigranulation probabilistic models. Specifically, four MAGDM-oriented information fusion schemes are used in constructing the following theoretical models:

(1) According to the original definition of MGRSs [19, 20], two extreme information fusion tactics are designed to form MGRSs that correspond to the risk-based MAGDM with optimistic and pessimistic risk attitudes. In this case, it is necessary to explore type-I DHF MGPRSs by virtue of optimistic and pessimistic information fusion tactics.

(2) One of the most common DHF MAGDM methods is the DHFHA operator that concentrates on accepting groups' main views [33]. In this case, it is necessary to explore type-II DHF MGPRSs by virtue of DHFHA operators.
(3) Distance measures play a significant role in addressing various MAGDM problems from the standpoint of information measures [6]. In this case, it is necessary to explore type-III DHF MGPRSs by virtue of DHF weighted Euclidean distances.
(4) Another representative DHF aggregation operator is the DHFHG operator that gives attention to individuals' main views [33]. In this case, it is necessary to explore type-IV DHF MGPRSs by virtue of DHFHG operators.

3.1 Type-I DHF MGPRSs

The initial works in terms of MGRSs include two subordinate editions, i.e., the optimistic edition that is likely to utilize at least one granular composition for meeting the demands of inclusion relations, the pessimistic counterpart that adopts all granular compositions for accomplishing the identical mission. Based on the above-stated two classic information fusion rules, the notion of type-I DHF MGPRSs is intended to construct with optimistic and pessimistic editions. In the context of typical MAGDM situations, the notion of single DHF membership degrees is designed at first that plays a significant role in formulating various multiple DHF membership degrees.

Suppose m decision makers in a group intend to make DHF MAGDM, and each of them is obliged to provide a decision matrix that is composed of a set of alternatives, a set of attributes along with a set of weights for attributes and decision makers. In specific, let $U = \{x_1, x_2, \cdots, x_p\}$ be the set of alternatives, $V = \{y_1, y_2, \cdots, y_q\}$ be the set of attributes, $\boldsymbol{\mu} = \left\{\mu_1, \mu_2, \cdots, \mu_q\right\}^T$ be the set of weights for attributes with $\mu_k \in [0,1]$ and $\sum\limits_{k=1}^{q} \mu_k = 1 (k = 1, 2, \cdots, q)$, $\boldsymbol{\omega} = \left\{\omega_1, \omega_2, \cdots, \omega_m\right\}^T$ be the set of weights for decision makers with $\omega_i \in [0,1]$ and $\sum\limits_{i=1}^{m} \omega_i = 1 (i = 1, 2, \cdots, m)$. Based on the above illustrations, all decision makers should offer assessed values for all alternatives via all attributes, and the offered assessed values are characterized by DHFEs. Then, each constructed decision matrix can be viewed as DHFRs $\mathbb{R}_i \in DHFR(U \times V) (i = 1, 2, \cdots, m)$ over $U \times V$. Next, a standard evaluation set $\mathbb{D}$ over V is established by means of all attributes, and $\mathbb{D}$ is characterized by DHFSs. At last, in accordance with the presented U, V, $\mathbb{R}_i$ and $\mathbb{D}$, a DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$ is obtained for developing MAGDM-oriented DHF multigranulation probabilistic models and related MAGDM approaches with MULTIMOORA. In what follows, the notion of single DHF membership degrees is proposed based on $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$.

Definition 3.1 Given a DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$. For any $x_j \in U (j = 1, 2, \cdots, p)$, $y_k \in V (k = 1, 2, \cdots, q)$, the single DHF membership degree of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)$, is defined as follows:

$$\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j) = \left(\frac{\sum_{y_k \in V} \mu_k \mathbb{R}_i(x_j, y_k)(\mathbb{D}(y_k))^c}{\sum_{y_k \in V} \mu_k \mathbb{R}_i(x_j, y_k)} \right)^c. \tag{11}$$

Based on the notion of single DHF membership degrees, it is necessary to expand single DHF membership degrees to the multigranulation context, thus the following maximum and minimum multiple DHF membership degrees are put forward.

Definition 3.2 Given a DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$. For any $x_j \in U (j = 1, 2, \cdots, p)$, the maximum and minimum multiple DHF membership degree of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\vartheta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$ and $\rho_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$, are defined as follows:

$$\vartheta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) = \max_{i=1}^{m} \{\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)\}; \tag{12}$$

$$\rho_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) = \min_{i=1}^{m} \{\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)\}. \tag{13}$$

With the support of maximum and minimum multiple DHF membership degrees, optimistic and pessimistic type-I DHF MGPRSs can be established conveniently.

Definition 3.3 Given a DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$, and let α and β be the thresholds that are characterized by DHFEs. For any $x_j \in U (j = 1, 2, \cdots, p)$ and $\beta < \alpha$, the optimistic type-I DHF multigranulation probabilistic rough approximations of $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\vartheta, \alpha}(\mathbb{D})$ and $\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\vartheta, \beta}(\mathbb{D})$, are defined as follows:

$$\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\vartheta, \alpha}(\mathbb{D}) = \left\{ \vartheta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) \geq \alpha | x_j \in U \right\}; \tag{14}$$

$$\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\vartheta, \beta}(\mathbb{D}) = \left\{ \vartheta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) > \beta | x_j \in U \right\}. \tag{15}$$

In a similar way, the pessimistic type-I DHF multigranulation probabilistic rough approximations of $\mathbb{D}$ in terms of $\mathbb{R}_i$, denoted by $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\rho,\alpha}(\mathbb{D})$ and $\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\rho,\beta}(\mathbb{D})$, are defined as follows:

$$\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\rho,\alpha}(\mathbb{D}) = \left\{ \rho_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j) \geq \alpha | x_j \in U \right\}; \tag{16}$$

$$\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\rho,\beta}(\mathbb{D}) = \left\{ \rho_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j) > \beta | x_j \in U \right\}, \tag{17}$$

then we name the pairs $(\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\vartheta,\alpha}(\mathbb{D}), \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\vartheta,\beta}(\mathbb{D}))$ and $(\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\rho,\alpha}(\mathbb{D}), \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\rho,\beta}(\mathbb{D}))$ an optimistic type-I DHF MGPRS and a pessimistic type-I DHF MGPRS of $\mathbb{D}$, respectively.

3.2 Type-II DHF MGPRSs

It is noteworthy that optimistic and pessimistic type-I DHF MGPRSs act as two extreme information fusion strategies for DHF MAGDM, proposing more risk-neutral methods for completing the same task is necessary. In what follows, by means of DHFHA operators, we aim to put forward the corresponding multiple DHF membership degrees at first.

Definition 3.4 Given a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$, let ω_i be the weight of $\mathbb{R}_i$, the σth value of membership degrees and non-membership degrees in $\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)$ be $\gamma_i^{\tau(\sigma)}(x_j)$ and $\eta_i^{\tau(\sigma)}(x_j)$, respectively. For any $x_j \in U (j = 1, 2, \cdots, p)$, the notion of multiple DHF membership degrees by means of DHFHA operators of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$, is defined as follows:

$$\begin{aligned}
\xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j) &= DHFWA(\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_j), \theta_{\mathbb{D}}^{\mathbb{R}_2}(x_j), \cdots, \theta_{\mathbb{D}}^{\mathbb{R}_m}(x_j)) \\
&= \boxplus_{i=1}^{m}(\omega_i \theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)) \\
&= \left\{ \left\{ 1 - \prod_{i=1}^{m}(1 - \gamma_i^{\tau(\sigma)}(x_j))^{\omega_i} \right\}, \right. \\
&\quad \left. \left\{ \prod_{i=1}^{m}(\eta_i^{\tau(\sigma)}(x_j))^{\omega_i} \right\} \right\}.
\end{aligned} \tag{18}$$

Afterwards, the following formulation of type-II DHF MGPRSs can be obtained via multiple DHF membership degrees by means of DHFHA operators.

Definition 3.5 Given a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$, and let α and β be the thresholds that are characterized by DHFEs. For any $x_j \in U (j = 1, 2, \cdots, p)$ and $\beta < \alpha$, the type-II DHF multigranulation probabilistic rough approximations of $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\xi,\alpha}(\mathbb{D})$ and $\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\xi,\beta}(\mathbb{D})$, are defined as follows:

$$\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\xi,\alpha}(\mathbb{D}) = \left\{ \xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j) \geq \alpha | x_j \in U \right\}; \tag{19}$$

$$\overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\xi,\beta}(\mathbb{D}) = \left\{ \xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j) > \beta | x_j \in U \right\}, \tag{20}$$

then the pair $(\underline{\sum_{i=1}^{m}\mathbb{R}_i}^{\xi,\alpha}(\mathbb{D}), \overline{\sum_{i=1}^{m}\mathbb{R}_i}^{\xi,\beta}(\mathbb{D}))$ is named a type-II DHF MGPRS of $\mathbb{D}$.

3.3 Type-III DHF MGPRSs

DHF distance measures are quite essential in fusing various discrete DHF information systems, hence it is necessary to develop corresponding multiple DHF membership degrees. In what follows, we aim to put forward the multiple DHF membership degrees by virtue of DHF weighted Euclidean distances at first.

Definition 3.6 Given a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$, let ω_i be the weight of $\mathbb{R}_i$, the σth value of membership degrees and non-membership degrees in $\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)$ be $\gamma_i^{\tau(\sigma)}(x_j)$ and $\eta_i^{\tau(\sigma)}(x_j)$, respectively, $(\{1\}, \{0\})$ be the positive ideal point. For any $x_j \in U (j = 1, 2, \cdots, p)$, the notion of multiple DHF membership degrees by means of DHF weighted Euclidean distances of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$, is defined as follows:

$$\begin{aligned}
\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j) = &\left[\sum_{i=1}^{m} \omega_i \left(\frac{1}{2} \left(\frac{1}{\# h_{\mathbb{D}_i}(x_j)} \sum_{\sigma=1}^{\# h_{\mathbb{D}_i}(x_j)} |\gamma_i^{\tau(\sigma)}(x_j) - 1|^2 \right. \right. \right. \\
&\left. \left. \left. + \frac{1}{\# g_{\mathbb{D}_i}(x_j)} \sum_{\sigma=1}^{\# g_{\mathbb{D}_i}(x_j)} |\eta_i^{\tau(\sigma)}(x_j)|^2 \right) \right) \right]^{\frac{1}{2}}.
\end{aligned} \tag{21}$$

Definition 3.7 Given a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$, and let α and β be the thresholds that are characterized by DHFEs. For any $x_j \in U(j = 1, 2, \cdots, p)$ and $\beta < \alpha$, the type-III DHF multigranulation probabilistic rough approximations of $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\psi,\alpha}(\mathbb{D})$ and $\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\psi,\beta}(\mathbb{D})$, are defined as follows:

$$\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\psi,\alpha}(\mathbb{D}) = \left\{ \psi_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) \geq \alpha | x_j \in U \right\}; \tag{22}$$

$$\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\psi,\beta}(\mathbb{D}) = \left\{ \psi_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) > \beta | x_j \in U \right\}, \tag{23}$$

then the pair $(\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\psi,\alpha}(\mathbb{D}), \overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\psi,\beta}(\mathbb{D}))$ is named a type-III DHF MGPRS of $\mathbb{D}$.

3.4 Type-IV DHF MGPRSs

Different from DHFHA operators that give high priorities on groups' main views, as another typical DHF aggregation operators, DHFHG operators emphasis on individuals' main views, hence developing multiple DHF membership degrees based on DHFHG operators is a useful supplement of DHF multigranulation probabilistic models. In what follows, by means of DHFHG operators, the corresponding multiple DHF membership degrees are presented at first.

Definition 3.8 Given a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$, let ω_i be the weight of $\mathbb{R}_i$, the σth value of membership degrees and non-membership degrees in $\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)$ be $\gamma_i^{\tau(\sigma)}(x_j)$ and $\eta_i^{\tau(\sigma)}(x_j)$, respectively. For any $x_j \in U(j = 1, 2, \cdots, p)$, the notion of multiple DHF membership degrees by means of DHFHG operators of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$, is defined as follows:

$$\begin{aligned} \zeta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) &= DHFWG(\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_j), \theta_{\mathbb{D}}^{\mathbb{R}_2}(x_j), \cdots, \theta_{\mathbb{D}}^{\mathbb{R}_m}(x_j)) \\ &= \boxtimes_{i=1}^{m}((\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j))^{\omega_i}) \\ &= \left\{ \left\{ \prod_{i=1}^{m} (\gamma_i^{\tau(\sigma)}(x_j))^{\omega_i} \right\}, \right. \\ &\quad \left. \left\{ 1 - \prod_{i=1}^{m} (1 - \eta_i^{\tau(\sigma)}(x_j))^{\omega_i} \right\} \right\}. \end{aligned} \tag{24}$$

Next, we formulate the notion of type-IV DHF MGPRSs by using multiple DHF membership degrees by means of DHFHG operators.

Definition 3.9 Given a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$, and let α and β be the thresholds that are characterized by DHFEs. For any $x_j \in U(j = 1, 2, \cdots, p)$ and $\beta < \alpha$, the type-IV DHF multigranulation probabilistic rough approximations of $\mathbb{D}$ with respect to $\mathbb{R}_i$, denoted by $\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\zeta,\alpha}(\mathbb{D})$ and $\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\zeta,\beta}(\mathbb{D})$, are defined as follows:

$$\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\zeta,\alpha}(\mathbb{D}) = \left\{ \zeta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) \geq \alpha | x_j \in U \right\}; \tag{25}$$

$$\overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\zeta,\beta}(\mathbb{D}) = \left\{ \zeta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j) > \beta | x_j \in U \right\}, \tag{26}$$

then the pair $(\underline{\sum_{i=1}^{m} \mathbb{R}_i}^{\zeta,\alpha}(\mathbb{D}), \overline{\sum_{i=1}^{m} \mathbb{R}_i}^{\zeta,\beta}(\mathbb{D}))$ is named a type-IV DHF MGPRS of $\mathbb{D}$.

4 MAGDM based on DHF MGPRSs with MULTIMOORA

The key point of interest of the current section is to design a comprehensive MAGDM method in light of the presented four types of DHF MGPRSs with MULTIMOORA. In addition, for the sake of better illustrating the applicability of the designed MAGDM method, the context of MAGDM in this section is scheduled to adopt P-J fit situations, and the specific problem statement for P-J fit is shown in the text below.

4.1 Problem statement

It is widely recognized that human resource is the most precious resource for the survival and the development of enterprises, and acts as the heart of competitions in the modern society. In the area of human resource management, managers have paid close attention to the core issue that how to fully mobilize the enthusiasm of the staff to achieve their talent and play the role of employees better for making the best use of the staff. Hence, only by letting individuals fully display their talents, will it be possible for business organizations to cultivate an increasing number of excellent innovation employees.

As a significant research branch of human resource management, P-J fit has gradually become the new hotspot in the enterprise management research that is subjected to more and more concerns of scholars. P-J fit aims to arrange appropriate talents to appropriate posts according to the relationship between posts and capabilities of talents. In a word, talents and posts are two fundamental factors of P-J fit, the key issue of P-J fit is to solve reasonable person and post matching. In view of the great significance of P-J fit, plenty of scholars have put forward a series of P-J fit methods. However, some challenges, such as reasonable depiction of uncertain information and integration of experts' preferences without individual subjectivity, still remain in the majority of current P-J fit methods. Thus, in light of several merits of the proposed theoretical models mentioned in previous sections, it is meaningful to explore efficient P-J fit methods by taking advantages of them.

4.2 Application model

For the sake of effectively utilizing the proposed four types of DHF MGPRS to address a real-world P-J fit situation, it is noted that the conclusion of P-J fit is largely dependent on the relationships between posts and capabilities of talents provided by human resource experts, thus transforming a complicated P-J fit situation into a typical MAGDM is conducive to solving of a practical P-J fit problem. Similar to the general DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$ presented in the last section, a DHF information system in the background of P-J fit should be given in the text below.

Suppose m human resource experts in an advisory panel intend to conduct P-J fit for an employee via DHF MAGDM, each of them is obliged to provide the relationship between posts and capabilities of the assessed employee, and each provided relationship is composed of a set of posts, a set of capabilities along with a set of weights for capabilities and human resource experts. In specific, let $U = \{x_1, x_2, \cdots, x_p\}$ be the set of posts, $V = \{y_1, y_2, \cdots, y_q\}$ be the set of capabilities, $\boldsymbol{\mu} = \{\mu_1, \mu_2, \cdots, \mu_q\}^T$ be the set of weights for capabilities with $\mu_k \in [0, 1]$ and $\sum\limits_{k=1}^{q} \mu_k = 1(k = 1, 2, \cdots, q)$, $\boldsymbol{\omega} = \left\{\omega_1, \omega_2, \cdots, \omega_m\right\}^T$ be the set of weights for human resource experts with $\omega_i \in [0, 1]$ and $\sum\limits_{i=1}^{m} \omega_i = 1(i = 1, 2, \cdots, m)$. Based on the above illustrations, all human resource experts should offer assessed values for each post via all capabilities, and the offered assessed values are characterized by DHFEs. Then, each constructed relationship can be viewed as DHFRs $\mathbb{R}_i \in DHFR(U \times V)(i = 1, 2, \cdots, m)$ over $U \times V$. Next, the assessed employee will be evaluated by virtue of all capabilities, and the evaluation set $\mathbb{D}$ over V is established that is characterized by DHFSs. At last, in accordance with the above-stated U, V, $\mathbb{R}_i$ and $\mathbb{D}$, a DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$ for P-J fit is prepared to handle P-J fit via the developed theoretical models with MULTIMOORA. In what follows, we present the specific scheme of the DHF MAGDM procedure.

Since the conclusion of P-J fit is usually obtained from the final ranking result of different posts for the assessed employee, and MULTIMOORA can generate a robust ranking result from the aspect of information fusion, it is necessary to integrate the developed theoretical models with MULTIMOORA to acquire a robust final ranking result for P-J fit. In specific, MULTIMOORA adopts three subordinate rankings, i.e., ratio systems, reference point approaches and full multiplicative forms, to complete the task of information fusion. Moreover, it is noted that the proposed DHF membership degrees by means of DHFHA operators, DHF weighted Euclidean distances and DHFHG operators are in line with the notion of ratio systems, reference point approaches along with full multiplicative forms in the context of DHF MAGDM, hence it is convenient for us to develop a MAGDM method based on DHF MGPRSs with MULTIMOORA.

Given a DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$ for P-J fit, for any post $x_j \in U(j = 1, 2, \cdots, p)$, multiple DHF membership degrees by means of DHFHA operators of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$ can be calculated according to Definition 3.4, which is denoted by $\xi_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$. Then, we can acquire the first ranking result of different posts via the value of $\xi_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$, and the value of $\xi_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$ is larger, the x_j is better.

In a similar manner, multiple DHF membership degrees by means of DHF weighted Euclidean distances and DHFHG operators of x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$ can be calculated according to Definitions 3.6 and 3.8, which are denoted by $\psi_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$ and $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m} \mathbb{R}_i}(x_j)$ respectively. Then, we can acquire the second ranking result of different posts via the value of $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$, and the value of $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ is smaller, the x_j is better. Finally, the third ranking result of different posts via the value of $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ is further acquired, and the value of $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ is larger, the x_j is better.

In light of the MULTIMOORA method, the final ranking, also named as the MULTIMOORA ranking, is acquired from three previous subordinate rankings based on the dominance theory. Afterwards, we aim to propose the concept of ranking order functions to find the optimal post for the assessed employee. In specific, for any post $x_j \in U(j = 1, 2, \cdots, p)$, three types of ranking results of different posts have been obtained via the value of $\xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$, $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ and $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$, and we arrange different posts from the best one to the worst one, then we denote ranking order functions for $\xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$, $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ and $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ as $Ind(\xi_j)$, $Ind(\psi_j)$ and $Ind(\zeta_j)$. Suppose $Ind(\max_{j=1}^{p} \xi_j) = 1$ and $Ind(\min_{j=1}^{p} \xi_j) = p$ are fulfilled, the MULTIMOORA ranking is acquired via the value of $Ind(\xi_j) + Ind(\psi_j) + Ind(\zeta_j)$, then we name $x^* = \min_{j=1}^{p} \left(Ind\left(\xi_j\right) + Ind\left(\psi_j\right) + Ind\left(\zeta_j\right)\right)$ the optimal post for the assessed employee.

Remark 4.1 In the construction of MAGDM based on DHF MGPRSs, the utilization of MULTIMOORA provides a reasonable information fusion scheme for effectively integrating type-II, type-III with type-IV DHF MGPRSs. Moreover, the information fusion conclusion is characterized by the MULTIMOORA ranking, which is more robust than each of the subordinate rankings as presented above. In specific, the information fusion result obtained from type-II, type-III and type-IV DHF MGPRSs can be seen as three independent single-source results that may cause potential differences of decision conclusions. With the support of MULTIMOORA, a comprehensive decision conclusion can be obtained by taking advantages of the above three types of DHF MGPRSs, it is expected that the comprehensive decision conclusion will be immune to the volatility of different information fusion schemes. Hence, it is meaningful to sum up the following algorithm for MAGDM based on DHF MGPRSs with MULTIMOORA, and it is noteworthy that the summarized algorithm offers a one-stop scheme from the DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$ for P-J fit to the optimal post x^*.

4.3 Algorithm for MAGDM based on DHF MGPRSs with MULTIMOORA

In what follows, a P-J fit algorithm for MAGDM based on DHF MGPRSs with MULTIMOORA is summed up.

Input A DHF information system $\left(U, V, \mathbb{R}_i, \mathbb{D}\right)$ for P-J fit.

Output The optimal post x^*.

Step 1 Determine the weight value of each capability and the weight value of each human resource expert.

Step 2 Calculate single DHF membership degrees $\theta_{\mathbb{D}}^{\mathbb{R}_i}\left(x_j\right)$ for each post x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$.

Step 3 Calculate multiple DHF membership degrees $\xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ and obtain the first ranking result of different posts via the value of $\xi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$.

Step 4 Calculate multiple DHF membership degrees $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ and obtain the second ranking result of different posts via the value of $\psi_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$.

Step 5 Calculate multiple DHF membership degrees $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$ and obtain the third ranking result of different posts via the value of $\zeta_{\mathbb{D}}^{\sum_{i=1}^{m}\mathbb{R}_i}(x_j)$.

Step 6 Obtain the MULTIMOORA ranking according to ranking order functions $Ind(\xi_j)$, $Ind(\psi_j)$ and $Ind(\zeta_j)$.

Step 7 Determine the optimal post x^* in light of $x^* = \min_{j=1}^{p}(Ind(\xi_j) + Ind(\psi_j) + Ind(\zeta_j))$.

The flow diagram of the above-presented P-J fit algorithm via MAGDM based on DHF MGPRSs with MULTIMOORA is summed up in Fig. 1.

5 An illustrative case study for P-J fit

This section plans to prove the applicability of the P-J fit algorithm via an illustrative case study at first. Moreover, corresponding validity tests and comparative analysis are arranged for better demonstrate the validity of the P-J fit algorithm as well.

5.1 Case description

Suppose a software development enterprise intends to organize a P-J fit process for a new employee, and an advisory panel is set up that is composed of three human resource

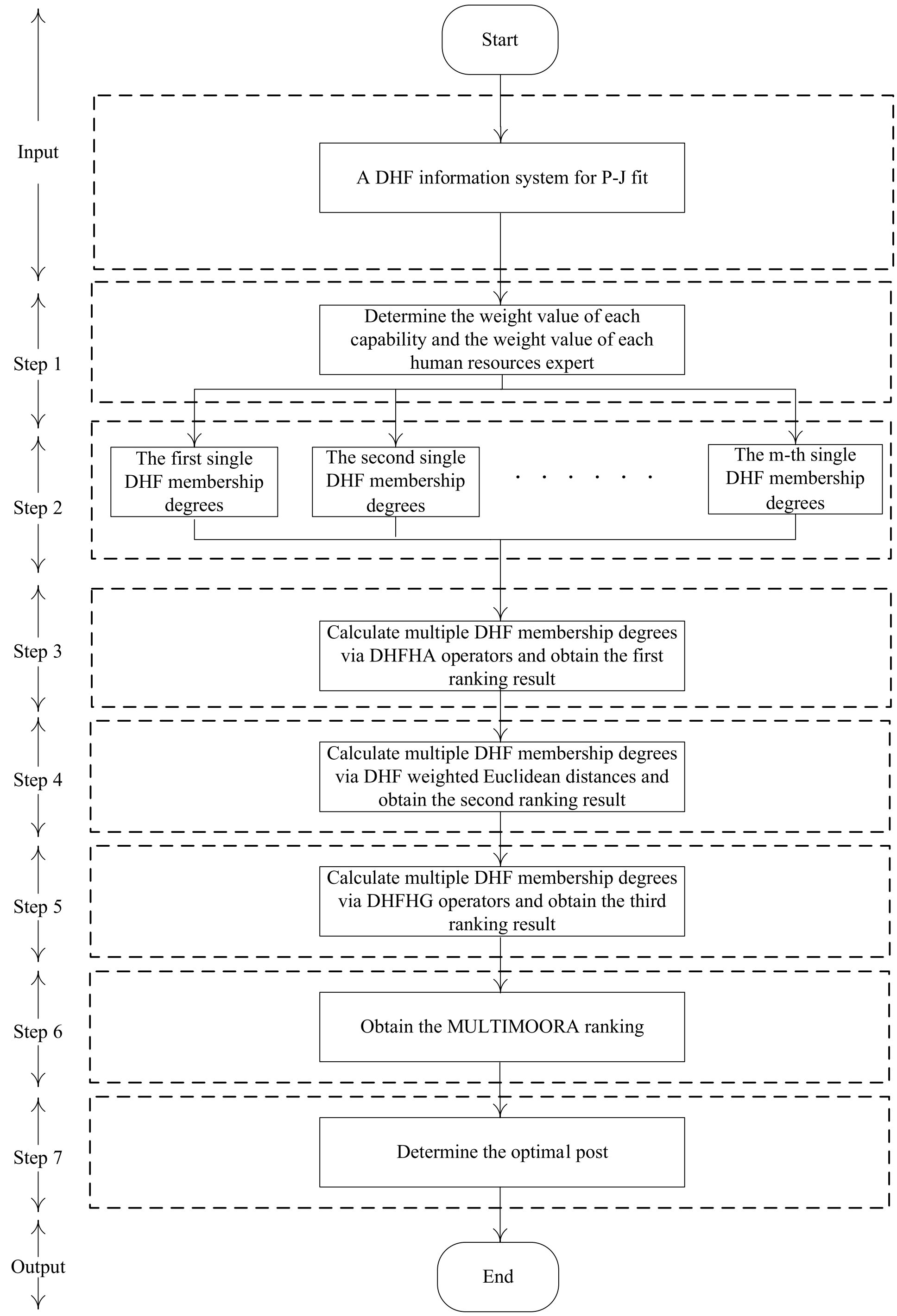

Fig. 1 The summary of the P-J fit algorithm via MAGDM based on DHF MGPRSs with MULTIMOORA

Table 1 The first DHFR between posts and capabilities

$\mathbb{R}_1$	y_1	y_2	y_3	y_4	y_5
x_1	$(\{0.6, 0.7\}, \{0.2, 0.3\})$	$(\{0.7, 0.8\}, \{0.1, 0.2\})$	$(\{0.5, 0.6\}, \{0.1, 0.2\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.4, 0.6\}, \{0.2, 0.3\})$
x_2	$(\{0.2, 0.4\}, \{0.5, 0.6\})$	$(\{0.5, 0.7\}, \{0.2, 0.3\})$	$(\{0.3, 0.5\}, \{0.4, 0.5\})$	$(\{0.4, 0.5\}, \{0.2, 0.3\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$
x_3	$(\{0.3, 0.4\}, \{0.4, 0.5\})$	$(\{0.6, 0.7\}, \{0.2, 0.3\})$	$(\{0.3, 0.5\}, \{0.4, 0.5\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.5, 0.6\}, \{0.1, 0.2\})$
x_4	$(\{0.2, 0.3\}, \{0.6, 0.7\})$	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.7, 0.8\}, \{0.1, 0.2\})$	$(\{0.2, 0.3\}, \{0.5, 0.6\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$
x_5	$(\{0.7, 0.8\}, \{0.1, 0.2\})$	$(\{0.5, 0.7\}, \{0.1, 0.3\})$	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.3, 0.5\}, \{0.4, 0.5\})$	$(\{0.3, 0.4\}, \{0.4, 0.5\})$

Table 2 The second DHFR between posts and capabilities

$\mathbb{R}_2$	y_1	y_2	y_3	y_4	y_5
x_1	$(\{0.4, 0.6\}, \{0.2, 0.4\})$	$(\{0.6, 0.8\}, \{0.1, 0.2\})$	$(\{0.4, 0.6\}, \{0.2, 0.3\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.5, 0.7\}, \{0.2, 0.3\})$
x_2	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.5, 0.7\}, \{0.1, 0.2\})$	$(\{0.2, 0.4\}, \{0.4, 0.5\})$	$(\{0.3, 0.5\}, \{0.2, 0.4\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$
x_3	$(\{0.3, 0.4\}, \{0.4, 0.5\})$	$(\{0.5, 0.7\}, \{0.2, 0.3\})$	$(\{0.4, 0.5\}, \{0.3, 0.4\})$	$(\{0.3, 0.4\}, \{0.4, 0.6\})$	$(\{0.4, 0.6\}, \{0.1, 0.3\})$
x_4	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.7, 0.8\}, \{0.1, 0.2\})$	$(\{0.1, 0.2\}, \{0.5, 0.6\})$	$(\{0.3, 0.4\}, \{0.4, 0.6\})$
x_5	$(\{0.6, 0.8\}, \{0.1, 0.2\})$	$(\{0.4, 0.7\}, \{0.2, 0.3\})$	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.2, 0.4\}, \{0.3, 0.4\})$	$(\{0.3, 0.4\}, \{0.4, 0.5\})$

Table 3 The third DHFR between posts and capabilities

$\mathbb{R}_3$	y_1	y_2	y_3	y_4	y_5
x_1	$(\{0.6, 0.7\}, \{0.2, 0.3\})$	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.5, 0.6\}, \{0.1, 0.3\})$	$(\{0.2, 0.3\}, \{0.5, 0.6\})$	$(\{0.4, 0.6\}, \{0.2, 0.4\})$
x_2	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.5, 0.7\}, \{0.2, 0.3\})$	$(\{0.5, 0.6\}, \{0.3, 0.4\})$	$(\{0.4, 0.6\}, \{0.2, 0.3\})$	$(\{0.4, 0.5\}, \{0.2, 0.4\})$
x_3	$(\{0.3, 0.5\}, \{0.4, 0.5\})$	$(\{0.6, 0.7\}, \{0.2, 0.3\})$	$(\{0.2, 0.5\}, \{0.4, 0.5\})$	$(\{0.3, 0.4\}, \{0.5, 0.6\})$	$(\{0.5, 0.6\}, \{0.2, 0.4\})$
x_4	$(\{0.1, 0.3\}, \{0.6, 0.7\})$	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.7, 0.8\}, \{0.1, 0.2\})$	$(\{0.1, 0.3\}, \{0.5, 0.6\})$	$(\{0.2, 0.3\}, \{0.4, 0.6\})$
x_5	$(\{0.6, 0.7\}, \{0.1, 0.2\})$	$(\{0.5, 0.7\}, \{0.1, 0.3\})$	$(\{0.6, 0.7\}, \{0.1, 0.3\})$	$(\{0.3, 0.5\}, \{0.4, 0.5\})$	$(\{0.2, 0.4\}, \{0.4, 0.5\})$

experts. In order to overcome the challenges from various uncertain information, a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})(i = 1, 2, 3)$ for P-J fit is finally provided. In specific, let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be the set of posts, where $x_j (j = 1, 2, 3, 4, 5)$ represents algorithm engineer, administrative assistant, marketing specialist, sales representative, and finance analyst. Moreover, let $V = \{y_1, y_2, y_3, y_4, y_5\}$ be the set of capabilities, where $y_k (k = 1, 2, 3, 4, 5)$ represents mathematical skills, computer skills, foreign language skills, writing skills, and management skills. Additionally, let $\boldsymbol{\mu} = \{\mu_1, \mu_2, \mu_3, \mu_4, \mu_5\}^T$ be the set of weights for capabilities with $\mu_k \in [0, 1]$ and $\sum_{k=1}^{q} \mu_k = 1 (k = 1, 2, 3, 4, 5)$, $\boldsymbol{\omega} = \{\omega_1, \omega_2, \omega_3\}^T$ be the set of weights for human resource experts with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{m} \omega_i = 1 (i = 1, 2, 3)$. Based on the above illustrations, all human resource experts offer assessed values for each post via all capabilities, and the offered assessed values are characterized by DHFEs. Then, each constructed relationship can be viewed as DHFRs $\mathbb{R}_i \in DHFR(U \times V)$ over $U \times V$ that are presented in Tables 1, 2, 3 below. Afterwards, the assessed new employee will be evaluated by virtue of all capabilities, and the evaluation set $\mathbb{D}$ over V is represented by $\mathbb{D} = \{\langle y_1, \{0.3\}, \{0.6\}\rangle, \langle y_2, \{0.7\}, \{0.2\}\rangle, \langle y_3, \{0.8\}, \{0.1\}\rangle, \langle y_4, \{0.2\}, \{0.5\}\rangle, \langle y_5, \{0.4\}, \{0.6\}\rangle\}$. At last, in accordance with the above-stated U, V, $\mathbb{R}_i$ and $\mathbb{D}$, a DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$ for P-J fit is established.

5.2 The detailed P-J fit process

Given the above-mentioned DHF information system $(U, V, \mathbb{R}_i, \mathbb{D})$ for P-J fit, the detailed P-J fit process is shown as the following steps.

At first, for the sake of effectively conducting comparative analysis with other similar GrC-based MAGDM approaches, suppose five capabilities and three human resource experts share an identical weight, that is $\mu_k = \frac{1}{5}$ and $\omega_i = \frac{1}{3}$.

Then, the following single DHF membership degrees $\theta_{\mathbb{D}}^{\mathbb{R}_i}(x_j)$ for each post x_j in $\mathbb{D}$ with respect to $\mathbb{R}_i$ is calculated.

$$\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_1) = \left(\frac{\sum_{y_k \in V} \mu_k R_1(x_1, y_k) (D(y_k))^c}{\sum_{y_k \in V} \mu_k R_1(x_1, y_k)} \right)^c$$
$$= (\{0.082, 0.1265\}, \{0.6793, 0.77\}).$$

In an identical manner, we can further obtain the remaining single DHF membership degrees as follows:

$$\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_2) =(\{0.1059, 0.1576\}, \{0.5621, 0.664\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_3) =(\{0.1057, 0.1558\}, \{0.6265, 0.6972\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_4) =(\{0.1798, 0.2531\}, \{0.4951, 0.5913\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_5) =(\{0.0736, 0.12\}, \{0.678, 0.767\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_2}(x_1) =(\{0.0838, 0.1458\}, \{0.6508, 0.77\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_2}(x_2) =(\{0.1034, 0.1828\}, \{0.5749, 0.6639\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_2}(x_3) =(\{0.0898, 0.1692\}, \{0.5948, 0.6972\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_2}(x_4) =(\{0.1485, 0.2309\}, \{0.5039, 0.5991\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_2}(x_5) =(\{0.0647, 0.1042\}, \{0.6251, 0.752\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_3}(x_1) =(\{0.082, 0.1414\}, \{0.6593, 0.7525\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_3}(x_2) =(\{0.0803, 0.1367\}, \{0.6123, 0.7092\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_3}(x_3) =(\{0.1155, 0.182\}, \{0.6285, 0.7202\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_3}(x_4) =(\{0.1645, 0.2531\}, \{0.387, 0.5593\}),$$
$$\theta_{\mathbb{D}}^{\mathbb{R}_3}(x_5) =(\{0.0736, 0.1216\}, \{0.6229, 0.7416\}).$$

In light of the above results, the following multiple DHF membership degrees $\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ are calculated.

$$\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_1) =DHFWA(\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_1), \theta_{\mathbb{D}}^{\mathbb{R}_2}(x_1), \theta_{\mathbb{D}}^{\mathbb{R}_3}(x_1))$$
$$=(\{0.0826, 0.1379\}, \{0.6631, 0.7641\}).$$

In an identical manner, we can further obtain the remaining multiple DHF membership degrees $\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ as follows:

$$\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_2) =(\{0.0966, 0.1592\}, \{0.5827, 0.6787\}),$$
$$\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_3) =(\{0.1037, 0.1691\}, \{0.6164, 0.7048\}),$$
$$\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_4) =(\{0.1644, 0.2458\}, \{0.4588, 0.583\}),$$
$$\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5) =(\{0.0706, 0.1153\}, \{0.6415, 0.7535\}).$$

Afterwards, the score functions of $\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ can be obtained, that is $s(\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_1)) = -0.6034$, $s(\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_2)) = -0.5028$, $s(\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_3)) = -0.5242$, $s(\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_4)) = -0.3158$ and $s(\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5)) = -0.6046$. Thus, the first ranking result of different posts shows $\xi_4 \succ \xi_2 \succ \xi_3 \succ \xi_1 \succ \xi_5$.

Next, the following multiple DHF membership degrees $\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ are further calculated.

$$\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_1) = \left[\sum_{i=1}^{3}\frac{1}{3}\left(\frac{1}{2}\left(\frac{1}{2}\sum_{\sigma=1}^{2}|\gamma_i^{\tau(\sigma)}(x_1)-1|^2 + \frac{1}{2}\sum_{\sigma=1}^{2}|\eta_i^{\tau(\sigma)}(x_1)|^2\right)\right)\right]^{\frac{1}{2}}$$
$$=0.8076.$$

In an identical way, we can further obtain $\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_2) = 0.7626$, $\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_3) = 0.77$, $\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_4) = 0.6754$ and $\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5) = 0.8105$. Thus, the second ranking result of different posts shows $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1 \succ \psi_5$.

Similar with the above steps, the following multiple DHF membership degrees $\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ are further calculated.

$$\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_1) =DHFWG(\theta_{\mathbb{D}}^{\mathbb{R}_1}(x_1), \theta_{\mathbb{D}}^{\mathbb{R}_2}(x_1), \theta_{\mathbb{D}}^{\mathbb{R}_3}(x_1))$$
$$=(\{0.0826, 0.1376\}, \{0.6633, 0.7643\}).$$

In an identical way, we can further obtain the remaining multiple DHF membership degrees $\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ as follows:

$$\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_2) =(\{0.0958, 0.1579\}, \{0.5837, 0.6798\}),$$
$$\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_3) =(\{0.1031, 0.1687\}, \{0.6169, 0.7051\}),$$
$$\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_4) =(\{0.1638, 0.2455\}, \{0.4645, 0.5836\}),$$
$$\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5) =(\{0.0705, 0.115\}, \{0.6429, 0.7538\}).$$

Afterwards, the score functions of $\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_j)$ can be obtained, that is $s(\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_1)) = -0.6037$, $s(\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_2)) = -0.5049$, $s(\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_3)) = -0.5251$, $s(\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_4)) = -0.3194$ and $s(\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5)) = -0.6056$. Thus, the third ranking result of different posts shows $\zeta_4 \succ \zeta_2 \succ \zeta_3 \succ \zeta_1 \succ \zeta_5$.

In light of the above three ranking results of different posts, the following ranking order functions are determined.

$$Ind(\xi_4) = Ind(\psi_4) = Ind(\zeta_4) = 1,$$
$$Ind(\xi_2) = Ind(\psi_2) = Ind(\zeta_2) = 2,$$
$$Ind(\xi_3) = Ind(\psi_3) = Ind(\zeta_3) = 3,$$
$$Ind(\xi_1) = Ind(\psi_1) = Ind(\zeta_1) = 4,$$
$$Ind(\xi_5) = Ind(\psi_5) = Ind(\zeta_5) = 5.$$

Finally, the MULTIMOORA ranking shows $x_4 \succ x_2 \succ x_3 \succ x_1 \succ x_5$, i.e., the optimal post is x_4 (sales representative).

5.3 Validity tests

For the study of MAGDM, another significant topic is how to prove the validity of the newly constructed MAGDM method in the related background. In order to complete the task of validity tests for various MAGDM methods, Wang and Triantaphyllou [34] established an efficient standard that mainly involves two kinds of tests. Afterwards, we intend to demonstrate the validity of the constructed DHF MAGDM method by virtue of test 1 and test 2.

In test 1, the standard indicates a valid MAGDM method should not alter the optimal alternative when substituting a sub-optimal alternative for a worse one. In light of the standard, let alternative x_5 be the sub-optimal alternative, and $\mathbb{D}^c$ be the worse alternative. Then, we substitute x_5 for $\mathbb{D}^c$ in Tables 1, 2, 3. Next, we calculate the updated values for multiple DHF membership degrees $\xi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5)$, $\psi_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5)$ and $\zeta_{\mathbb{D}}^{\sum_{i=1}^{3}\mathbb{R}_i}(x_5)$, the results show $(\{0.0004\}, \{0.9796\})$, 0.9962 and $(\{0.0004\}, \{0.9796\})$, respectively, which indicate the ranking order of x_5 remains the same as the one obtained in Sect. 5.2. Thus, the proposed DHF MAGDM method passes test 1.

In test 2, the standard shows that we first decompose a MAGDM problem into smaller ones, then the same MAGDM method is applied on smaller ones. Afterwards, we integrate the ranking order of smaller ones on the premise that the transitive property exists. Finally, the integrated result should be the same as the original result of the undecomposed MAGDM problem. In light of this standard, we decompose the DHF MAGDM problem presented in previous sections into two smaller ones, i.e., $\{x_1, x_2, x_3, x_4\}$ and $\{x_2, x_3, x_4, x_5\}$. Then, we utilize the same DHF MAGDM method on them, and the MULTIMOORA ranking shows $x_4 \succ x_2 \succ x_3 \succ x_1$ and $x_4 \succ x_2 \succ x_3 \succ x_5$, respectively. At last, the integrated result $x_4 \succ x_2 \succ x_3 \succ x_1 \succ x_5$ can be obtained by integrate $x_4 \succ x_2 \succ x_3 \succ x_1$ with $x_4 \succ x_2 \succ x_3 \succ x_5$, which is the same as the original result obtained in Sect. 5.2. Thus, the proposed DHF MAGDM method passes test 2.

Remark 5.1 According to Literature [34], four classic models were examined to compare the decision results by using them respectively, i.e., the weighted sum model, the weighted product model, the analytic hierarchy process (AHP), and the revised AHP. Then, two tests were designed to seek the best method. In specific, test 1 aims to check the stability of a method in yielding the identical decision result when a sub-optimal alternative is replaced by a worse one, whereas test 2 aims to compare the decision result when using multi-dimensional situations and single-dimensional situations respectively. Finally, the two tests were conducted by using simulated decision problems with random numbers, and the usefulness of the two tests were proven to be effective.

5.4 Comparative analysis and discussions

In this section, we intend to conduct the following comparative analysis from two aspects, i.e., the one from classic DHF MAGDM methods via the case study presented in Sect. 5.1; another one from a real-world application example in the background of bank credit ratings (take Industrial and Commercial Bank of China (ICBC) as an example). At first, we present the first comparative analysis from the aspect of classic DHF MAGDM methods below.

By utilizing optimistic, pessimistic and neutral information fusion tactics, Literature [52] designed a DHF MAGDM method via the model of several DHF MGRSs. In what follows, the DHF MAGDM method proposed in Literature [52] is employed to address the same P-J fit situation expressed in Sect. 5.1. According to the formulation of DHF MGRSs presented in Definition 2.5, the optimistic and pessimistic DHF multigranulation rough approximations with respect to $(U, V, \mathbb{R}_i, \mathbb{D})$ are calculated.

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{D}) = \{\langle x_1, \{0.3, 0.4\}, \{0.5, 0.6\}\rangle,$$
$$\langle x_2, \{0.2, 0.4\}, \{0.3, 0.5\}\rangle, \langle x_3, \{0.4\}, \{0.4, 0.6\}\rangle,$$
$$\langle x_4, \{0.5, 0.6\}, \{0.2, 0.3\}\rangle, \langle x_5, \{0.3\}, \{0.6\}\rangle\},$$

$$\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{D}) = \{\langle x_1, \{0.6, 0.7\}, \{0.2\}\rangle,$$
$$\langle x_2, \{0.5, 0.7\}, \{0.2, 0.3\}\rangle, \langle x_3, \{0.5, 0.7\}, \{0.2, 0.3\}\rangle,$$
$$\langle x_4, \{0.7, 0.8\}, \{0.1, 0.2\}\rangle, \langle x_5, \{0.6, 0.7\}, \{0.1, 0.3\}\rangle\},$$

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{D}) = \{\langle x_1, \{0.3\}, \{0.6\}\rangle,$$
$$\langle x_2, \{0.2, 0.3\}, \{0.4, 0.5\}\rangle, \langle x_3, \{0.4\}, \{0.5, 0.6\}\rangle,$$
$$\langle x_4, \{0.4, 0.6\}, \{0.3, 0.4\}\rangle, \langle x_5, \{0.3\}, \{0.6\}\rangle\},$$

$$\overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{D}) = \{\langle x_1, \{0.7\}, \{0.1, 0.2\}\rangle,$$
$$\langle x_2, \{0.5, 0.7\}, \{0.2\}\rangle, \langle x_3, \{0.6, 0.7\}, \{0.2, 0.3\}\rangle.$$
$$\langle x_4, \{0.7, 0.8\}, \{0.1, 0.2\}\rangle, \langle x_5, \{0.6, 0.7\}, \{0.1, 0.2\}\rangle\}.$$

Then, three comprehensive synthesized sets by virtue of optimistic, pessimistic and neutral information fusion tactics are further calculated.

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{D}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{D}) =$$
$$\{\langle x_1, \{0.72, 0.82\}, \{0.1, 0.12\}\rangle,$$
$$\langle x_2, \{0.6, 0.82\}, \{0.06, 0.15\}\rangle,$$
$$\langle x_3, \{0.7, 0.82\}, \{0.08, 0.18\}\rangle,$$
$$\langle x_4, \{0.85, 0.92\}, \{0.02, 0.06\}\rangle,$$
$$\langle x_5, \{0.72, 0.79\}, \{0.06, 0.18\}\rangle\},$$

$$\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{D}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{D}) =$$
$$\{\langle x_1, \{0.79\}, \{0.06, 0.12\}\rangle,$$
$$\langle x_2, \{0.6, 0.79\}, \{0.08, 0.1\}\rangle,$$
$$\langle x_3, \{0.76, 0.82\}, \{0.1, 0.18\}\rangle,$$
$$\langle x_4, \{0.82, 0.92\}, \{0.03, 0.08\}\rangle,$$
$$\langle x_5, \{0.72, 0.79\}, \{0.06, 0.12\}\rangle\},$$

$$(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{D}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{O}(\mathbb{D})) \boxplus$$
$$(\underline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{D}) \boxplus \overline{\sum_{i=1}^{3}\mathbb{R}_i}^{P}(\mathbb{D})) =$$
$$\{\langle x_1, \{0.9412, 0.9622\}, \{0.006, 0.0144\}\rangle,$$
$$\langle x_2, \{0.84, 0.9622\}, \{0.0048, 0.015\}\rangle,$$
$$\langle x_3, \{0.928, 0.9676\}, \{0.008, 0.0324\}\rangle,$$
$$\langle x_4, \{0.973, 0.9936\}, \{0.0006, 0.0048\}\rangle,$$
$$\langle x_5, \{0.9216, 0.9559\}, \{0.0036, 0.0216\}\rangle\}.$$

At last, in light of the above three comprehensive synthesized sets, three kinds of ranking results can be obtained by calculating corresponding score functions. In specific, optimistic synthesized sets indicate the ranking result is $x_4 \succ x_1 \succ x_5 \succ x_3 \succ x_2$, pessimistic synthesized sets indicate the ranking result is $x_4 \succ x_1 \succ x_5 \succ x_3 \succ x_2$, neutral synthesized sets indicate the ranking result is $x_4 \succ x_1 \succ x_3 \succ x_5 \succ x_2$. Hence, all three kinds of ranking results show the optimal post is x_4 (sales representative).

Remark 5.2 It is noteworthy that the three ranking results obtained from optimistic, pessimistic and neutral synthesized sets show the optimal post is x_4, which is the same as the P-J fit result acquired from the MULTIMOORA ranking. Additionally, there exist differences between the order of x_1, x_2, x_3 and x_5. The reason for their differences comes from the adoption of different information fusion strategies, which shows the one from the aspect of DHF MGRSs utilizes the intersection and union operators to fuse multi-source

Table 4 The extracted client information for credit ratings

R	H_1	H_2	H_3	H_4	H_5	H_6
k_1	300200	6400	1.5	5.58	29000	3.54
k_2	168000	7600	1.74	5.82	50320	3.9
k_3	264000	10000	4.26	4.68	20500	4.56
k_4	198000	8300	7	4.02	26500	4.92
k_5	300000	4800	4.92	3.24	15500	5.04
k_6	117000	2600	5.7	6	500	5.76
k_7	138000	9500	4.38	7.2	7500	3.18
k_8	294000	6200	6	2.22	18000	3.06
k_9	300800	4600	3.48	0.96	6000	3.36
k_{10}	255000	5800	6	5.28	2000	3.66

uncertain information, and the final information fusion result is vulnerable to the impact of extreme or erroneous values. Different from DHF MGRSs, the developed DHF MAGDM method refers to three common DHF information fusion rules, it is accepted that the utilization of MULTIMOORA is conducive to obtaining a relatively robust decision making result. Moreover, it is noted that the proposed theoretical models consider both the weight of attributes and decision makers, whereas traditional GrC-based DHF models, such as DHF MGRSs, overlook the above two kinds of weights, hence the proposed theoretical models can better address real-world MAGDM problems.

In what follows, we present another real-world application example in the background of ICBC's credit ratings for illustrating the applicability of the proposed theoretical models. In order to address a realistic decision making problem for ICBC's credit ratings, Literature [56] explored a novel covering-based fuzzy rough set with classic decision making methods. By referring to the original data presented in Literature [56], we transform the original data to the following DHF information systems and solve the new DHF MAGDM problem by using our proposed theoretical models.

First, we present the case description for ICBC's credit ratings. It is noted that ICBC plans to launch diverse preferential services each year for better serving clients. In order to make an accurate assessment of different clients, ICBC usually uses several significant attributes to provide a ranking result for clients, hence it is conducive for banks to offer high-quality personalized services to different levels of clients by conducting credit ratings. In view of the above-mentioned facts, several realistic data from the client information database of ICBC in 2018 was extracted to explore a case study analysis in Literature [56]. In specific, let $W = \{k_j | j = 1, 2, \cdots, 10\}$ be the set of clients, another universe $H = \{H_l | l = 1, 2, \cdots, 6\}$ be 6 attributes, where $H_l (l = 1, 2, 3, 4, 5, 6)$ represents deposit situations, personal monthly incomes, degree of education, career situations, debt situations, and interview situations. In addition, $H_l(k_j)$ represents an actual assessment value of the alternative k_j in terms of the attribute H_l, where $H_l(k_j)$ is a rational number. Then based on the universes W and H, the extracted client information for credit ratings is presented in Table 4 below.

It is noteworthy that $H_l(k_j)$ in Table 4 is outside the range of DHFEs which prevents the usage of the proposed theoretical models, thus it is necessary to transform it to a DHFE by using the following normalization methods.

For H_1, this attribute refers to more amount of deposits will bring to higher scores. In addition, due to possible policy adjustments of ICBC, the upper limit of deposit situations may vary between 250,000 RMB, 300,000 RMB and 350,000 RMB at different times, then the following fuzzy assessment values of the client k in terms of H_1 can be obtained.

$$H_{1a}(k) = \begin{cases} \frac{E}{250000}, & E \in [0, 250000), \\ 1, & E \in [250000, \infty). \end{cases} \tag{27}$$

$$H_{1b}(k) = \begin{cases} \frac{E}{300000}, & E \in [0, 300000), \\ 1, & E \in [300000, \infty). \end{cases} \tag{28}$$

$$H_{1c}(k) = \begin{cases} \frac{E}{350000}, & E \in [0, 350000), \\ 1, & E \in [350000, \infty). \end{cases} \tag{29}$$

In above formulas, the symbol E refers to the assessment value of a client over a certain attribute.

For H_2, the upper limit of personal monthly incomes is set as 9,000 RMB, 10,000 RMB and 11,000 RMB respectively according to the change of policy adjustments at different times, then the following fuzzy assessment values of the client k in terms of H_2 can be obtained.

$$H_{2a}(k) = \begin{cases} \frac{E}{9000}, & E \in [0, 9000), \\ 1, & E \in [9000, \infty). \end{cases} \tag{30}$$

$$H_{2b}(k) = \begin{cases} \frac{E}{10000}, & E \in [0, 10000), \\ 1, & E \in [10000, \infty). \end{cases} \tag{31}$$

$$H_{2c}(k) = \begin{cases} \frac{E}{11000}, & E \in [0, 11000), \\ 1, & E \in [11000, \infty). \end{cases} \tag{32}$$

For H_3, we notice that the score interval of H_3 falls within [0, 10], and the upper limit of this attribute is usually set as 5, 6, and 7 respectively, then the following fuzzy assessment values of the client k in terms of H_3 can be obtained.

$$H_{3a}(k) = \begin{cases} \frac{E}{5}, & E \in [0, 5), \\ 1, & E \in [5, 10]. \end{cases} \tag{33}$$

$$H_{3b}(k) = \begin{cases} \frac{E}{6}, & E \in [0, 6), \\ 1, & E \in [6, 10]. \end{cases} \tag{34}$$

$$H_{3c}(k) = \begin{cases} \frac{E}{7}, & E \in [0, 7), \\ 1, & E \in [7, 10]. \end{cases} \tag{35}$$

For H_4, the setting of this attribute is similar with H_3, thus bank staffs are likely to determine the score interval of H_4 as [0, 10], and the upper limit of this attribute is also set as 5, 6, and 7 respectively, then the following fuzzy assessment values of the client k in terms of H_4 can be obtained.

$$H_{4a}(k) = \begin{cases} \frac{E}{5}, & E \in [0, 5), \\ 1, & E \in [5, 10]. \end{cases} \tag{36}$$

$$H_{4b}(k) = \begin{cases} \frac{E}{6}, & E \in [0, 6), \\ 1, & E \in [6, 10]. \end{cases} \tag{37}$$

$$H_{4c}(k) = \begin{cases} \frac{E}{7}, & E \in [0, 7), \\ 1, & E \in [7, 10]. \end{cases} \tag{38}$$

For H_5, this attribute refers to more amount of debts will bring to lower scores. In addition, the upper limit of debt situations may vary between 60,000 RMB, 50,000 RMB and 40,000 RMB at different times, then the following fuzzy assessment values of the client k in terms of H_5 can be obtained.

$$H_{5a}(k) = \begin{cases} 1 - \frac{E}{60000}, & E \in [0, 60000), \\ 0, & E \in [60000, \infty). \end{cases} \tag{39}$$

Table 5 The first transformed fuzzy client information for credit ratings

R_1	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.71	0.3	1	0.52	0.71
k_2	0.67	0.84	0.35	1	0.16	0.78
k_3	1	1	0.85	0.94	0.66	0.91
k_4	0.79	0.92	1	0.8	0.56	0.98
k_5	1	0.53	0.98	0.65	0.74	1
k_6	0.47	0.29	1	1	0.99	1
k_7	0.55	1	0.88	1	0.88	0.64
k_8	1	0.69	1	0.44	0.7	0.61
k_9	1	0.51	0.7	0.19	0.9	0.67
k_{10}	1	0.64	1	1	0.97	0.73

Table 6 The second transformed fuzzy client information for credit ratings

R_2	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.64	0.25	0.93	0.42	0.59
k_2	0.56	0.76	0.29	0.97	0	0.65
k_3	0.88	1	0.71	0.78	0.59	0.76
k_4	0.66	0.83	1	0.67	0.47	0.82
k_5	1	0.48	0.82	0.54	0.69	0.84
k_6	0.39	0.26	0.98	1	0.99	0.96
k_7	0.46	0.95	0.73	1	0.85	0.53
k_8	0.98	0.62	1	0.37	0.64	0.51
k_9	1	0.46	0.58	0.16	0.88	0.56
k_{10}	0.85	0.58	1	0.88	0.96	0.61

Table 7 The third transformed fuzzy client information for credit ratings

R_3	H_1	H_2	H_3	H_4	H_5	H_6
k_1	0.86	0.58	0.21	0.8	0.28	0.51
k_2	0.48	0.69	0.25	0.83	0	0.56
k_3	0.75	0.91	0.61	0.67	0.49	0.65
k_4	0.57	0.75	1	0.57	0.34	0.7
k_5	0.86	0.44	0.7	0.46	0.61	0.72
k_6	0.33	0.24	0.81	0.86	0.99	0.82
k_7	0.39	0.86	0.63	1	0.81	0.45
k_8	0.84	0.56	0.86	0.32	0.55	0.44
k_9	0.86	0.42	0.5	0.14	0.85	0.48
k_{10}	0.73	0.53	0.86	0.75	0.95	0.52

$$H_{5b}(k) = \begin{cases} 1 - \frac{E}{50000}, & E \in [0, 50000), \\ 0, & E \in [50000, \infty). \end{cases} \tag{40}$$

$$H_{5c}(k) = \begin{cases} 1 - \frac{E}{40000}, & E \in [0, 40000), \\ 0, & E \in [40000, \infty). \end{cases} \tag{41}$$

For H_6, the setting of this attribute is similar with H_3 and H_4, bank staffs will determine credit ratings of each clients via the collected personal information and on-site interviews. Thus bank staffs are likely to determine the score interval of H_6 as [0, 10] as well, and the upper limit of this attribute is also set as 5, 6, and 7 respectively, then the following fuzzy assessment values of the client k in terms of H_6 can be obtained.

$$H_{6a}(k) = \begin{cases} \frac{E}{5}, & E \in [0, 5), \\ 1, & E \in [5, 10]. \end{cases} \tag{42}$$

$$H_{6b}(k) = \begin{cases} \frac{E}{6}, & E \in [0, 6), \\ 1, & E \in [6, 10]. \end{cases} \tag{43}$$

$$H_{6c}(k) = \begin{cases} \frac{E}{7}, & E \in [0, 7), \\ 1, & E \in [7, 10]. \end{cases} \tag{44}$$

Afterwards, by using the normalization schemes $H_{ka}(k = 1, 2, 3, 4, 5, 6)$ in light of the formulas (27), (30), (33), (36), (39) and (42), the first transformed fuzzy client information for credit ratings is presented in Table 5 below.

In a similar way, by using the normalization schemes $H_{kb}(k = 1, 2, 3, 4, 5, 6)$ in light of the formulas (28), (31), (34), (37), (40) and (43), the second transformed fuzzy client information for credit ratings is presented in Table 6 below.

At last, by using the normalization schemes $H_{kc}(k = 1, 2, 3, 4, 5, 6)$ in light of the formulas (29), (32), (35), (38), (41) and (44), the third transformed fuzzy client information for credit ratings is presented in Table 7 below.

In real world, experts may choose different normalization schemes to obtain transformed fuzzy client information for credit ratings. If they consume more than one normalization scheme is reasonable, then a hesitation attitude will emerge. For instance, if an expert chooses the transformed result of Tables 5, 6, then several hesitant fuzzy elements (HFEs) can be formulated. Afterwards, in order to express the client information for credit ratings more accurately, we introduce non-membership degrees for each formulated HFEs by using the complement of an HFE h, where the complement of h is denoted by $h^c = \bigcup_{\xi \in h} \{1 - \xi\}$. Finally, by combining formulated HFEs with their non-membership degrees, the form of DHFEs can be eventually constructed. In what follows, by virtue of the transformed result of Table 5 and Table 6, the following Table 8 can be obtained; by virtue of the transformed result of Table 5 and Table 7, the following Table 9 can be obtained; by virtue of the transformed result of Table 6 and Table 7, the following Table 10 can be obtained.

In light of Tables 8, 9, 10, we use the notion of TOPSIS (Technique for Order Preference by Similarity to an

Table 8 The first transformed DHF client information for credit ratings

$\mathbb{R}_4$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	$(\{1\},\{0\})$	$(\{0.64,0.71\},\{0.29,0.36\})$	$(\{0.25,0.3\},\{0.7,0.75\})$	$(\{0.93,1\},\{0,0.07\})$	$(\{0.42,0.52\},\{0.48,0.58\})$	$(\{0.59,0.71\},\{0.29,0.41\})$
k_2	$(\{0.56,0.67\},\{0.33,0.44\})$	$(\{0.76,0.84\},\{0.16,0.24\})$	$(\{0.29,0.35\},\{0.65,0.71\})$	$(\{0.97,1\},\{0,0.03\})$	$(\{0,0.16\},\{0.84,1\})$	$(\{0.65,0.78\},\{0.22,0.35\})$
k_3	$(\{0.88,1\},\{0,0.12\})$	$(\{1\},\{0\})$	$(\{0.71,0.85\},\{0.15,0.29\})$	$(\{0.78,0.94\},\{0.06,0.22\})$	$(\{0.59,0.66\},\{0.34,0.41\})$	$(\{0.76,0.91\},\{0.09,0.24\})$
k_4	$(\{0.66,0.79\},\{0.21,0.34\})$	$(\{0.83,0.92\},\{0.08,0.17\})$	$(\{1\},\{0\})$	$(\{0.67,0.8\},\{0.2,0.33\})$	$(\{0.47,0.56\},\{0.44,0.53\})$	$(\{0.82,0.98\},\{0.02,0.18\})$
k_5	$(\{1\},\{0\})$	$(\{0.48,0.53\},\{0.47,0.52\})$	$(\{0.82,0.98\},\{0.02,0.18\})$	$(\{0.54,0.65\},\{0.35,0.46\})$	$(\{0.69,0.74\},\{0.26,0.31\})$	$(\{0.84,1\},\{0,0.16\})$
k_6	$(\{0.39,0.47\},\{0.53,0.61\})$	$(\{0.26,0.29\},\{0.71,0.74\})$	$(\{0.98,1\},\{0,0.02\})$	$(\{1\},\{0\})$	$(\{0.99\},\{0.01\})$	$(\{0.96,1\},\{0,0.04\})$
k_7	$(\{0.46,0.55\},\{0.45,0.54\})$	$(\{0.95,1\},\{0,0.05\})$	$(\{0.73,0.88\},\{0.12,0.27\})$	$(\{1\},\{0\})$	$(\{0.85,0.88\},\{0.12,0.15\})$	$(\{0.53,0.64\},\{0.36,0.47\})$
k_8	$(\{0.98,1\},\{0,0.02\})$	$(\{0.62,0.69\},\{0.31,0.38\})$	$(\{1\},\{0\})$	$(\{0.37,0.44\},\{0.56,0.63\})$	$(\{0.64,0.7\},\{0.3,0.36\})$	$(\{0.51,0.61\},\{0.39,0.49\})$
k_9	$(\{1\},\{0\})$	$(\{0.46,0.51\},\{0.49,0.54\})$	$(\{0.58,0.7\},\{0.3,0.42\})$	$(\{0.16,0.19\},\{0.81,0.84\})$	$(\{0.88,0.9\},\{0.1,0.12\})$	$(\{0.56,0.67\},\{0.33,0.44\})$
k_{10}	$(\{0.85,1\},\{0,0.15\})$	$(\{0.58,0.64\},\{0.36,0.42\})$	$(\{1\},\{0\})$	$(\{0.88,1\},\{0,0.12\})$	$(\{0.96,0.97\},\{0.03,0.04\})$	$(\{0.61,0.73\},\{0.27,0.39\})$

Table 9 The second transformed DHF client information for credit ratings

$\mathbb{R}_5$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	$(\{0.86,1\},\{0,0.14\})$	$(\{0.58,0.64\},\{0.36,0.42\})$	$(\{0.21,0.25\},\{0.75,0.79\})$	$(\{0.8,0.93\},\{0.07,0.2\})$	$(\{0.28,0.42\},\{0.58,0.72\})$	$(\{0.51,0.59\},\{0.41,0.49\})$
k_2	$(\{0.48,0.56\},\{0.44,0.52\})$	$(\{0.69,0.76\},\{0.24,0.31\})$	$(\{0.25,0.29\},\{0.71,0.75\})$	$(\{0.83,0.97\},\{0.03,0.17\})$	$(\{0\},\{1\})$	$(\{0.56,0.65\},\{0.35,0.44\})$
k_3	$(\{0.75,0.88\},\{0.12,0.25\})$	$(\{0.91,1\},\{0,0.09\})$	$(\{0.61,0.71\},\{0.29,0.39\})$	$(\{0.67,0.78\},\{0.22,0.33\})$	$(\{0.49,0.59\},\{0.41,0.51\})$	$(\{0.65,0.76\},\{0.24,0.35\})$
k_4	$(\{0.57,0.66\},\{0.34,0.43\})$	$(\{0.75,0.83\},\{0.17,0.25\})$	$(\{1\},\{0\})$	$(\{0.57,0.67\},\{0.33,0.43\})$	$(\{0.34,0.47\},\{0.53,0.66\})$	$(\{0.7,0.82\},\{0.18,0.3\})$
k_5	$(\{0.86,1\},\{0,0.14\})$	$(\{0.44,0.48\},\{0.52,0.56\})$	$(\{0.7,0.82\},\{0.18,0.3\})$	$(\{0.46,0.54\},\{0.46,0.54\})$	$(\{0.61,0.69\},\{0.31,0.39\})$	$(\{0.72,0.84\},\{0.16,0.28\})$
k_6	$(\{0.33,0.39\},\{0.61,0.67\})$	$(\{0.24,0.26\},\{0.74,0.76\})$	$(\{0.81,0.98\},\{0.02,0.19\})$	$(\{0.86,1\},\{0,0.14\})$	$(\{0.99\},\{0.01\})$	$(\{0.82,0.96\},\{0.04,0.18\})$
k_7	$(\{0.39,0.46\},\{0.54,0.61\})$	$(\{0.86,0.95\},\{0.05,0.14\})$	$(\{0.63,0.73\},\{0.27,0.37\})$	$(\{1\},\{0\})$	$(\{0.81,0.85\},\{0.15,0.19\})$	$(\{0.45,0.53\},\{0.47,0.55\})$
k_8	$(\{0.84,0.98\},\{0.02,0.16\})$	$(\{0.56,0.62\},\{0.38,0.44\})$	$(\{0.86,1\},\{0,0.14\})$	$(\{0.32,0.37\},\{0.63,0.68\})$	$(\{0.55,0.64\},\{0.36,0.45\})$	$(\{0.44,0.51\},\{0.49,0.56\})$
k_9	$(\{0.86,1\},\{0,0.14\})$	$(\{0.42,0.46\},\{0.54,0.58\})$	$(\{0.5,0.58\},\{0.42,0.5\})$	$(\{0.14,0.16\},\{0.84,0.86\})$	$(\{0.85,0.88\},\{0.12,0.15\})$	$(\{0.48,0.56\},\{0.44,0.52\})$
k_{10}	$(\{0.73,0.85\},\{0.15,0.27\})$	$(\{0.53,0.58\},\{0.42,0.47\})$	$(\{0.86,1\},\{0,0.14\})$	$(\{0.75,0.88\},\{0.12,0.25\})$	$(\{0.95,0.96\},\{0.04,0.05\})$	$(\{0.52,0.61\},\{0.39,0.48\})$

Table 10 The third transformed DHF client information for credit ratings

$\mathbb{R}_6$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.58, 0.71\}, \{0.29, 0.42\})$	$(\{0.21, 0.3\}, \{0.7, 0.79\})$	$(\{0.8, 1\}, \{0, 0.2\})$	$(\{0.28, 0.52\}, \{0.48, 0.72\})$	$(\{0.51, 0.71\}, \{0.29, 0.49\})$
k_2	$(\{0.48, 0.67\}, \{0.33, 0.52\})$	$(\{0.69, 0.84\}, \{0.16, 0.31\})$	$(\{0.25, 0.35\}, \{0.65, 0.75\})$	$(\{0.83, 1\}, \{0, 0.17\})$	$(\{0, 0.16\}, \{0.84, 1\})$	$(\{0.56, 0.78\}, \{0.22, 0.44\})$
k_3	$(\{0.75, 1\}, \{0, 0.25\})$	$(\{0.91, 1\}, \{0, 0.09\})$	$(\{0.61, 0.85\}, \{0.15, 0.39\})$	$(\{0.67, 0.94\}, \{0.06, 0.33\})$	$(\{0.49, 0.66\}, \{0.34, 0.51\})$	$(\{0.65, 0.91\}, \{0.09, 0.35\})$
k_4	$(\{0.57, 0.79\}, \{0.21, 0.43\})$	$(\{0.75, 0.92\}, \{0.08, 0.25\})$	$(\{1\}, \{0\})$	$(\{0.57, 0.8\}, \{0.2, 0.43\})$	$(\{0.34, 0.56\}, \{0.44, 0.66\})$	$(\{0.7, 0.98\}, \{0.02, 0.3\})$
k_5	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.44, 0.53\}, \{0.47, 0.56\})$	$(\{0.7, 0.98\}, \{0.02, 0.3\})$	$(\{0.46, 0.65\}, \{0.35, 0.54\})$	$(\{0.61, 0.74\}, \{0.26, 0.39\})$	$(\{0.72, 1\}, \{0, 0.28\})$
k_6	$(\{0.33, 0.47\}, \{0.53, 0.67\})$	$(\{0.24, 0.29\}, \{0.71, 0.76\})$	$(\{0.81, 1\}, \{0, 0.19\})$	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.99\}, \{0.01\})$	$(\{0.82, 1\}, \{0, 0.18\})$
k_7	$(\{0.39, 0.55\}, \{0.45, 0.61\})$	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.63, 0.88\}, \{0.12, 0.37\})$	$(\{1\}, \{0\})$	$(\{0.81, 0.88\}, \{0.12, 0.19\})$	$(\{0.45, 0.64\}, \{0.36, 0.55\})$
k_8	$(\{0.84, 1\}, \{0, 0.16\})$	$(\{0.56, 0.69\}, \{0.31, 0.44\})$	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.32, 0.44\}, \{0.56, 0.68\})$	$(\{0.55, 0.7\}, \{0.3, 0.45\})$	$(\{0.44, 0.61\}, \{0.39, 0.56\})$
k_9	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.42, 0.51\}, \{0.49, 0.58\})$	$(\{0.5, 0.7\}, \{0.3, 0.5\})$	$(\{0.14, 0.19\}, \{0.81, 0.86\})$	$(\{0.85, 0.9\}, \{0.1, 0.15\})$	$(\{0.48, 0.67\}, \{0.33, 0.52\})$
k_{10}	$(\{0.73, 1\}, \{0, 0.27\})$	$(\{0.53, 0.64\}, \{0.36, 0.47\})$	$(\{0.86, 1\}, \{0, 0.14\})$	$(\{0.75, 1\}, \{0, 0.25\})$	$(\{0.95, 0.97\}, \{0.03, 0.05\})$	$(\{0.52, 0.73\}, \{0.27, 0.48\})$

Ideal Solution) principles to obtain the best optimal DHF decision making objects and the worst optimal DHF decision making objects, i.e., $\mathbb{R}_i^+(H_l) = \max_{j=1}^{10}\{H_l(k_j)\}$ and $\mathbb{R}_i^-(H_l) = \min_{j=1}^{10}\{H_l(k_j)\}$, where $i = 4, 5, 6$. Afterwards, we obtain the following DHF decision making objects in terms of Tables 8, 9, 10 respectively.

$$T\left(\mathbb{R}_4^+(H_l), \mathbb{R}_4^-(H_l)\right) = \left\{\left\langle H_l, \left(\frac{\sum_{\gamma_1^{\tau(\sigma)} \in h_{\mathbb{R}_4^+}(H_l), \gamma_2^{\tau(\sigma)} \in h_{\mathbb{R}_4^-}(H_l)} (\gamma_1^{\tau(\sigma)} + \gamma_2^{\tau(\sigma)})}{2}\right), \left(\frac{\sum_{\eta_1^{\tau(\sigma)} \in g_{\mathbb{R}_4^+}(H_l), \eta_2^{\tau(\sigma)} \in g_{\mathbb{R}_4^-}(H_l)} (\eta_1^{\tau(\sigma)} + \eta_2^{\tau(\sigma)})}{2}\right)\right\rangle \middle| l = 1, 2, 3, 4, 5, 6\right\}. \tag{45}$$

Based on the above formula, $T\left(\mathbb{R}_5^+(H_l), \mathbb{R}_5^-(H_l)\right)$ and $T\left(\mathbb{R}_6^+(H_l), \mathbb{R}_6^-(H_l)\right)$ can be obtained in a similar way. Thus, we eventually form a DHF information system $\left(W, H, \mathbb{R}_i, T\left(\mathbb{R}_i^+(H_l), \mathbb{R}_i^-(H_l)\right)\right)(i = 4, 5, 6)$ for ICBC's credit ratings. Suppose six attributes and three experts share an identical weight, then by virtue of the algorithm for MAGDM based on DHF MGPRSs with MULTIMOORA, we obtain the ranking result as $k_3 \succ k_{10} \succ k_1 \succ k_7 \succ k_5 \succ k_4 \succ k_8 \succ k_6 \succ k_2 \succ k_9$. Moreover, we extract some ranking results by using different methods in light of Table 4 from Literature [56], which are presented in Table 11 below.

According to Table 11, it is noted that the optimal client remains the same by using diverse methods, i.e., k_3. The conclusion by using our method owns some similarities with the TOPSIS-WAA method based on covering-based fuzzy rough sets when ranking other non-optimal clients. Since our method can solve the problem of ICBC's credit ratings from the aspect of group decision making with a stable information fusion tactic, whereas other classic decision making methods can only address single-expert decision making. Therefore, the proposed theoretical models perform excellently when solving real-world application examples in the background of ICBC's credit ratings.

6 Conclusion

This paper concentrates on addressing three challenges in information depiction, fusion and analysis stages of a complicated MAGDM situation, and explores MAGDM-oriented DHF multigranulation probabilistic models in light of MULTIMOORA in detail. First, four types of DHF

Table 11 Ranking results by using different methods

Different methods	Ranking of all alternatives
The TOPSIS method	$k_3 \succ k_{10} \succ k_4 \succ k_7 \succ k_8 \succ k_1 \succ k_5 \succ k_2 \succ k_9 \succ k_6$
The weighted arithmetic average (WAA) operator method	$k_3 \succ k_{10} \succ k_4 \succ k_7 \succ k_5 \succ k_8 \succ k_1 \succ k_2 \succ k_6 \succ k_9$
The TOPSIS-WAA method based on covering-based fuzzy rough sets	$k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9$
Our method	$k_3 \succ k_{10} \succ k_1 \succ k_7 \succ k_5 \succ k_4 \succ k_8 \succ k_6 \succ k_2 \succ k_9$

MGPRSs are proposed via optimistic/pessimistic tactics, DHFHA operators, DHF weighted Euclidean distances and DHFHG operators. Afterwards, in light of the proposed theoretical models, a DHF MAGDM approach is established based on MULTIMOORA. Finally, a practical P-J fit case study is studied along with validity tests and comparative analysis for showing the practicality and efficiency of the established DHF MAGDM method. Overall, both the developed theoretical models and the established DHF MAGDM method show the following merits:

(1) The context of DHFSs enables decision makers to describe complicated uncertain information that involves fuzziness, imprecision and hesitation, thus the scheme of information depiction lays solid foundations for establishing DHF MAGDM methods.
(2) The utilization of MGRSs and PRSs not only increases the computational efficiency for the information fusion stage, but also offers the fault tolerance ability. In light of the above-stated features, this work explores four types of DHF MGPRSs from the aspect of practical MAGDM situations, which overcome limitations of traditional MGRSs and PRSs.
(3) MULTIMOORA is fused with MAGDM-oriented multigranulation probabilistic models in the context of DHFSs, thereby helping to provide reasonable and explainable results for DHF MAGDM. Moreover, validity tests and comparative analysis have shown the applicability and efficiency in solving a P-J fit situation.

In light of the above summarized merits, the proposed DHF MGPRSs act as an effective tool for fusing and analyzing multi-source DHF information according to different risks of decision makers, and a stable information fusion tactic can be obtained by using the constructed MAGDM based on DHF MGPRSs with MULTIMOORA, which can provide a robust decision result for complicated DHF MAGDM problems. In the future, there exist several interesting topics in the theoretical and realistic studies of MAGDM-oriented multigranulation probabilistic models. For one thing, the fusion of other generalized fuzzy contexts and decision-oriented soft computing tools can open a door for addressing more domain-specific MAGDM situations. For another, plenty of MAGDM situations emerge with features of large group of experts, high-dimensional attributes and dynamic decision contexts, hence it is necessary to study valid methods for these scenarios.

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