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ORIGINAL ARTICLE

Multi-expert multi-criteria decision making based on the likelihoods of interval type-2 trapezoidal fuzzy preference relations

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Abstract

Interval type-2 trapezoidal fuzzy sets, as a particular form of interval type-2 fuzzy sets, can precisely characterize the subjective assessments and qualitative evaluations of a group of experts. In this paper, a novel likelihood-based interval type-2 trapezoidal fuzzy multi-expert multi-criteria decision-making approach is proposed. To do so, the concepts of likelihood-based performance index, likelihood-based comprehensive evaluation value, and signed distance-based evaluation value are adopted. The interval type-2 trapezoidal fuzzy Bonferroni aggregation operator is utilized to construct the likelihood-based interval type-2 trapezoidal fuzzy preference relations. Then, the consistent lower and upper likelihoods are adopted to enhance the efficiency of the group decision making framework. The proposed multi-expert decision making approach works well when there is high degree of fluctuations in the number of criteria and experts. The practicability and feasibility of the proposed approach are validated by applications to four cases. Several comparative analyses are conducted to authenticate the dominancy of the proposed method over conventional interval type-2 trapezoidal fuzzy multi-criteria decision-making approaches.

Keywords Multi-expert multi-criteria decision making · Preference relations · Interval type-2 trapezoidal fuzzy sets · Bonferroni aggregation operator

1 Introduction

Multi-criteria decision making refers to selecting, from a set of alternatives, the one that best performs the criteria [1]. Uncertain and vague assessments of data frequently arise in practical multi-criteria decision-making problems [2], particularly following a lack of experience and knowledge, duality of opinions, intangible and imperceptible criteria, or a multifaceted environment [3, 4]. Consequently, how to model the uncertainty in subjective decision making of experts becomes increasingly imperative [2, 5, 6]. Hence, the development of models which are able to model uncertainty based on the subjective preference of experts becomes a challenging issue.

The fuzzy set theory was introduced [7] and extended [8–10] owing to the unpredictability and fuzziness of data during years and with respect to the quiddity of different ambiguities. The fuzzy set theory can organize a systematic calculus by which experts are able to address linguistic information that improves the consistency of uncertain decision making. Type-2 fuzzy sets were generalized [11] as an extension of conventional fuzzy sets in which the membership function falls into a fuzzy set on the interval [0,1] [2]. The type-2 fuzzy sets were elaborated because of the incapability of conventional fuzzy sets in terms of covering all the imprecisions that currently exist in decision-making problems [12]. They are better than conventional type-1 fuzzy sets especially in terms of handling uncertainties that are manipulated by linguistic terms [13]. Even though the type-2 fuzzy sets are prominent in organizing the uncertainties in the cases with a high degree of ambiguity, these types of sets may involve inevitable large amounts of sophisticated computation [2, 14]. Given that the interval type-2 trapezoidal fuzzy sets (IT2TrFS) can express complex information, they have been widely utilized in decision-making problems [14–16].

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A multi-expert decision making problem refers to selecting the alternative that best satisfies the criteria based on the judgements of more than one expert. It is reasonable that the alternatives chosen by group decision making approaches are more in line with real world's optimal solutions because the group decision making approaches may remove some deficiencies of single person decision-making, including the lack of knowledge, lack of experience and limited cognition on criteria. The likelihoods of interval type-2 trapezoidal fuzzy preference relations was first presented by Chen [2] to develop a possibility-based approach for multiple criteria decision analysis under the IT2TrF environment. Even though there were various studies with the background of likelihood-based approaches under the IT2TrF environment [17–19], Chen [2] proposed an extended likelihood-based approach based on the possibilities and preference relations of IT2TrFSs with applications to multiple criteria decision analysis by specifying the consistent lower and upper likelihoods which were able to derive the range of possibilities of IT2TrF preference relations and mirror the whole uncertain information of IT2TrFSs as a crisp value. The proposed likelihood-based approach is useful since it reduces the computational complexity of conventional IT2TrF multi-criteria decision-making approaches such as the TOPSIS (technique for order preference by similarity to an ideal solution), TODIM (tomada de decisao interativa e multicrit'erio), ELECTRE (elicitation and choice translating reality), and AHP (analytic hierarchy process). Despite the dominancy of the likelihood-based approach over the conventional ones, as far as we know, it was only organized to handle single decision-making problems following the judgements of a single expert.

Overall, there are three challenges in the interval type-2 fuzzy multi-criteria group decision making that motivate the authors for conducting this research, which are listed as follows:

1. The complex computational process of IT2TrFSs. The IT2TrFSs cover more uncertainty than the conventional fuzzy set [2, 20]. However, dealing with these types of fuzzy sets involves complex calculation, which reduces the desire of using IT2TrFSs in practice.
2. The existing MCGDM approaches with IT2TrFSs are almost vulnerable when facing a huge amount of evaluation criteria. Because there are some operations such as pairwise comparisons, calculation of distances to ideal solutions, and aggregation of experts' judgements, the efficiency of the existing approaches is low as a result of the complex computations process of IT2TrFSs. Also, the conventional IT2TrF MCGDM approaches are vulnerable to encounter the cases with high fluctuations in the number of experts and opinions.
3. The likelihood-based approach proposed by Chen [2] receives the judgments of a single expert for the whole process, which makes it undesirable for solving real world problems.

There might be still room for a multi-expert approach considering the above deficiencies. Given these three gaps, we utilize the concept of Bonferroni mean operator [21] to aggregate the judgements of a group of decision-makers whom are involved in determining the linguistic performance ratings and the weights of criteria under the IT2TrF environment. The proposed interval type-2 fuzzy Bonferroni aggregation operator (IT2FBAO) is unique in the sense that it combines the judgements of decision-makers based on their importance whilst capturing the interrelationships between evaluation criteria [22]. Once the aggregated values are all obtained, we utilize the likelihoods of IT2TrF preference relations approach to rank the alternatives with respect to their performances on criteria. The likelihoods of IT2TrF contribute to establish a more facilitated approach which reduces the computational complexities involving with previous IT2TrF decision making approaches. In addition, due to an agile process, the likelihoods of IT2TrF preference relations enables decision makers to tackle the situations in which there is a high degree of fluctuation in terms of the number of evaluation criteria. The validation of the proposed MCGDM approach is confirmed by the applications to four cases including the supplier selection problem [15, 23, 24], food production problem [25–27], facility location selection problem [28], and facility site selection problem [29]. In the light of the proposed approach, decision-makers are able to: (1) determine the IT2TrF performance ratings and the weights of criteria based on the objectives of organization in format of linguistic variables, (2) aggregate the judgements of decision-makers concerning the performance ratings with the weights of criteria using the IT2FBM operator, (3) obtain the lower, upper and mean likelihoods of IT2TrF preference relations [2] for each alternative with respect to each criteria, (4) obtain the likelihood-based performance index for each alternative based on the obtained likelihoods, (5) calculate the likelihood-based comprehensive evaluation value to reveal a IT2TrF ranking level for each alternative and finally, (6) rank alternatives in descending order of the signed distance-based evaluation values [2, 30].

In this study, we try to make the following contributions and innovations:

1. A novel likelihood-based MCGDM approach is proposed under the interval type-2 fuzzy environment. By adopting consistent lower and upper likelihoods of IT2TrF preference relations, the novel approach reduces the sophisticated computations of the previous multi-criteria decision-making approaches.

2. A systematic approach is formed by the properties of likelihoods of IT2TrF preference relations. The utilization of IT2TrF preference relations gives the systematic approach the ability to cover more uncertainty than conventional fuzzy sets. Meanwhile, the systematic approach can handle the fluctuations regarding the number of criteria.
3. The IT2FBAO is employed to accumulate the preferences of decision-makers based on their importance and expertise. The Bonferroni aggregation operator is adopted because it has the privilege to consider interrelationships between the evaluation criteria whilst accumulating the subjective preference of experts.
4. The practical effectiveness of the proposed likelihood-based MCGDM method is validated by the applications to four different cases. The comparative analyses indicate that the results obtained by the proposed approach match with the results obtained by original approaches.

The remainder of this paper is structured as follows: Sect. 2 introduces the concept of IT2TrFS, the properties of likelihoods of IT2TrF preference relations, the Bonferroni mean operator, and the terminology that is utilized throughout the paper. In Sect. 3, we propose a novel likelihood-based MCGDM method. In Sect. 4, the applications of the proposed approach to four case studies are explored. This study ends with conclusions in Sect. 5.

2 Basic concepts

In this section, the terminology and basic concepts which are utilized to develop the proposed group decision making method are reviewed, including the definitions of interval type-2 fuzzy sets, the lower and upper likelihoods of IT2TrF preference relations, and the Bonferroni aggregation operator.

Definition 1 [31]. Let X be the universe of discourse. A type-2 fuzzy set (T2FS) $\tilde{\tilde{A}}$ defined on X can be denoted as:

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_A(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \subseteq [0,1] \right\} \tag{1}$$

or

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} \mu_A(x,u)/(x,u) = \int_{x\in X} \left(\int_{u\in J_x} \mu_A(x,u)/u \right) \Bigg/ x \tag{2}$$

where x is the primary variable, $J_x \subseteq [0,1]$ is the primary membership function, and u is the secondary variable. $\int_{u\in J_x} \mu_A(x,u)/u$ is the second membership at x. If $\mu_A(x,u) = 1$ is true, then $\tilde{\tilde{A}}$ is called an interval type-2 fuzzy set (IT2FS), which is defined as:

$$\tilde{\tilde{A}} = \int_{x\in X} \int_{u\in J_x} 1/(x,u) = \int_{x\in X} \left(\int_{u\in J_x} 1/u \right) \Bigg/ x \tag{3}$$

The footprint of uncertainty (FOU) is defined as the area enclosed by the primary membership function of the IT2FS $\tilde{\tilde{A}}$, which can be mathematically defined as:

$$\mathrm{FOU}\left(\tilde{A}\right) = \bigcup_{x\in X} J_x = \bigcup_{x\in X} \left\{ (x,u) | u \in J_x \subseteq [0,1] \right\} \tag{4}$$

The FOU is the area enclosed by the lower membership function $\mu^L_{\tilde{A}}(x)$ (LMF) and the upper membership function $\mu^U_{\tilde{A}}(x)$ (UMF) as Fig. 1 indicates [20].

Definition 2 [32]. Let $a_1^-, a_2^-, a_3^-, a_4^-, a_1^+, a_2^+, a_3^+$, and a_4^+ be non-negative real values where $0 \le a_1^- \le a_2^- \le a_3^- \le a_4^-$, $0 \le a_1^+ \le a_2^+ \le a_3^+ \le a_4^+$, $a_1^+ \le a_1^-$, and $a_4^- \le a_4^+$. Also let h_A^- and h_A^+ denote the heights of A^- and A^+, respectively, and $0 \le h_A^- \le h_A^+ \le 1$. The lower and upper membership functions A^- and A^+ of $\tilde{\tilde{A}}$ can be defined as follows:

$$A^- = \begin{cases} \frac{h_A^-(x-a_1^-)}{a_2^- - a_1^-}, & \text{if} \quad a_1^- < x < a_2^- \\ h_A^-, & \text{if} \quad a_1^- \le x \le a_2^- \\ \frac{h_A^-(a_4^- - x)}{a_4^- - a_3^-}, & \text{if} \quad a_1^- < x < a_2^- \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$A^+ = \begin{cases} \frac{h_A^+(x-a_1^+)}{a_2^+ - a_1^+}, & \text{if} \quad a_1^+ < x < a_2^+ \\ h_A^+, & \text{if} \quad a_1^+ \le x \le a_2^+ \\ \frac{h_A^+(a_4^+ - x)}{a_4^+ - a_3^+}, & \text{if} \quad a_1^+ < x < a_2^+ \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

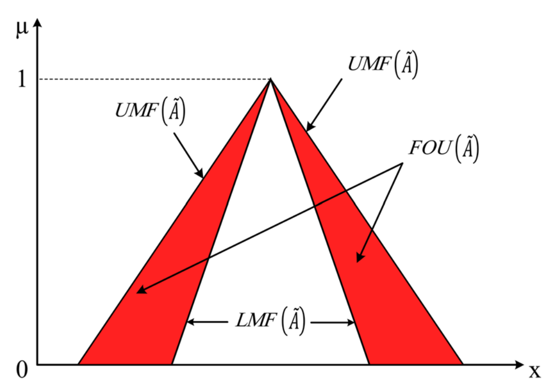

Fig. 1 The FOU of an IT2FS A

For two non-negative IT2TrF numbers $\boldsymbol{\Phi}_\rho = \left[\left(\varphi^-_{1\rho}, \varphi^-_{2\rho}, \varphi^-_{3\rho}, \varphi^-_{4\rho}; h^-_{\boldsymbol{\Phi}_\rho}\right), \left(\varphi^+_{1\rho}, \varphi^+_{2\rho}, \varphi^+_{3\rho}, \varphi^+_{4\rho}; h^+_{\boldsymbol{\Phi}_\rho}\right)\right]$ and $\boldsymbol{\Phi}_\beta = \left[\left(\varphi^-_{1\beta}, \varphi^-_{2\beta}, \varphi^-_{3\beta}, \varphi^-_{4\beta}; h^-_{\boldsymbol{\Phi}_\beta}\right), \left(\varphi^+_{1\beta}, \varphi^+_{2\beta}, \varphi^+_{3\beta}, \varphi^+_{4\beta}; h^+_{\boldsymbol{\Phi}_\beta}\right)\right]$, the operations are defined as follows [2]:

$$(1)\quad \boldsymbol{\Phi}_\rho \oplus \boldsymbol{\Phi}_\beta = \begin{bmatrix} \left(\varphi^-_{1\rho} + \varphi^-_{1\beta}, \varphi^-_{2\rho} + \varphi^-_{2\beta}, \varphi^-_{3\rho} + \varphi^-_{3\beta}, \varphi^-_{4\rho} + \varphi^-_{4\beta};\ \min\left\{h^-_{\boldsymbol{\Phi}_\rho}, h^-_{\boldsymbol{\Phi}_\beta}\right\}\right), \\ \left(\varphi^+_{1\rho} + \varphi^+_{1\beta}, \varphi^+_{2\rho} + \varphi^+_{2\beta}, \varphi^+_{3\rho} + \varphi^+_{3\beta}, \varphi^+_{4\rho} + \varphi^+_{4\beta};\ \min\left\{h^+_{\boldsymbol{\Phi}_\rho}, h^+_{\boldsymbol{\Phi}_\beta}\right\}\right) \end{bmatrix};$$

$$(2)\quad \boldsymbol{\Phi}_\rho \ominus \boldsymbol{\Phi}_\beta = \begin{bmatrix} \left(\varphi^-_{1\rho} - \varphi^-_{4\beta}, \varphi^-_{2\rho} - \varphi^-_{3\beta}, \varphi^-_{3\rho} - \varphi^-_{2\beta}, \varphi^-_{4\rho} - \varphi^-_{1\beta};\ \min\left\{h^-_{\boldsymbol{\Phi}_\rho}, h^-_{\boldsymbol{\Phi}_\beta}\right\}\right), \\ \left(\varphi^+_{1\rho} - \varphi^+_{4\beta}, \varphi^+_{2\rho} - \varphi^+_{3\beta}, \varphi^+_{3\rho} - \varphi^+_{2\beta}, \varphi^+_{4\rho} - \varphi^+_{1\beta};\ \min\left\{h^+_{\boldsymbol{\Phi}_\rho}, h^+_{\boldsymbol{\Phi}_\beta}\right\}\right) \end{bmatrix};$$

$$(3)\quad \boldsymbol{\Phi}_\rho \otimes \boldsymbol{\Phi}_\beta = \begin{bmatrix} \left(\varphi^-_{1\rho}.\varphi^-_{1\beta}, \varphi^-_{2\rho}.\varphi^-_{2\beta}, \varphi^-_{3\rho}.\varphi^-_{3\beta}, \varphi^-_{4\rho}.\varphi^-_{4\beta};\ \min\left\{h^-_{\boldsymbol{\Phi}_\rho}, h^-_{\boldsymbol{\Phi}_\beta}\right\}\right), \\ \left(\varphi^+_{1\rho}.\varphi^+_{1\beta}, \varphi^+_{2\rho}.\varphi^+_{2\beta}, \varphi^+_{3\rho}.\varphi^+_{3\beta}, \varphi^+_{4\rho}.\varphi^+_{4\beta};\ \min\left\{h^+_{\boldsymbol{\Phi}_\rho}, h^+_{\boldsymbol{\Phi}_\beta}\right\}\right) \end{bmatrix};$$

$$(4)\quad \frac{\boldsymbol{\Phi}_\rho}{\boldsymbol{\Phi}_\beta} = \begin{bmatrix} \left(\frac{\varphi^-_{1\rho}}{\varphi^-_{4\beta}}, \frac{\varphi^-_{2\rho}}{\varphi^-_{3\beta}}, \frac{\varphi^-_{3\rho}}{\varphi^-_{2\beta}}, \frac{\varphi^-_{4\rho}}{\varphi^-_{1\beta}};\ \min\left\{h^-_{\boldsymbol{\Phi}_\rho}, h^-_{\boldsymbol{\Phi}_\beta}\right\}\right), \\ \left(\frac{\varphi^+_{1\rho}}{\varphi^+_{4\beta}}, \frac{\varphi^+_{2\rho}}{\varphi^+_{3\beta}}, \frac{\varphi^+_{3\rho}}{\varphi^+_{2\beta}}, \frac{\varphi^+_{4\rho}}{\varphi^+_{1\beta}};\ \min\left\{h^+_{\boldsymbol{\Phi}_\rho}, h^+_{\boldsymbol{\Phi}_\beta}\right\}\right) \end{bmatrix};$$

$$(5)\quad \lambda\boldsymbol{\Phi}_\rho = \left[\left(\lambda\varphi^-_{1\rho}, \lambda\varphi^-_{2\rho}, \lambda\varphi^-_{3\rho}, \lambda\varphi^-_{4\rho};\ h^-_{\boldsymbol{\Phi}_\rho}\right), \left(\lambda\varphi^+_{1\rho}, \lambda\varphi^+_{2\rho}, \lambda\varphi^+_{3\rho}, \lambda\varphi^+_{4\rho};\ h^+_{\boldsymbol{\Phi}_\rho}\right)\right],$$

The likelihood $L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right)$ indicates the possibility that $\boldsymbol{\Phi}_\beta$ is not larger than $\boldsymbol{\Phi}_\rho$ and is computed by [2]

$$L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = \frac{L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) + L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right)}{2} \tag{7}$$

where

$$L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = \max\left\{1 - \max\left[\frac{\sum_{\xi=1}^4 \max\left(\varphi^+_{\xi\beta} - \varphi^-_{\xi\rho}, 0\right) + \left(\varphi^+_{4\beta} - \varphi^-_{1\rho}\right) + 2\max\left(h^+_{\boldsymbol{\Phi}_\beta} - h^-_{\boldsymbol{\Phi}_\rho}, 0\right)}{\sum_{\xi=1}^4 \left|\varphi^+_{\xi\beta} - \varphi^-_{\xi\rho}\right| + \left(\varphi^-_{4\rho} - \varphi^-_{1\rho}\right) + \left(\varphi^+_{4\beta} - \varphi^+_{1\beta}\right) + 2\left|h^+_{\boldsymbol{\Phi}_\beta} - h^-_{\boldsymbol{\Phi}_\rho}\right|}, 0\right], 0\right\} \tag{8}$$

$$L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = \max\left\{1 - \max\left[\frac{\sum_{\xi=1}^4 \max\left(\varphi^-_{\xi\beta} - \varphi^+_{\xi\rho}, 0\right) + \left(\varphi^-_{4\beta} - \varphi^+_{1\rho}\right) + 2\max\left(h^-_{\boldsymbol{\Phi}_\beta} - h^+_{\boldsymbol{\Phi}_\rho}, 0\right)}{\sum_{\xi=1}^4 \left|\varphi^-_{\xi\beta} - \varphi^+_{\xi\rho}\right| + \left(\varphi^+_{4\rho} - \varphi^+_{1\rho}\right) + \left(\varphi^-_{4\beta} - \varphi^-_{1\beta}\right) + 2\left|h^-_{\boldsymbol{\Phi}_\beta} - h^+_{\boldsymbol{\Phi}_\rho}\right|}, 0\right], 0\right\} \tag{9}$$

The lower and upper likelihoods $L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right)$ and $L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right)$ of the preference relation $\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta$ fulfills the following properties: (1) $0 \le L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) \le 1$; (2) $0 \le L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) \le 1$; (3) $L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) + L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = 1$; (4) If $\varphi^+_{4\rho} \le \varphi^+_{1\beta}$ and $h^-_{\boldsymbol{\Phi}_\beta} \le h^+_{\boldsymbol{\Phi}_\beta}$, then, $L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = 0$ and $L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = 1$; (5) If $\varphi^-_{1\beta} - \varphi^+_{4\rho} \ge 2\max\left(h^+_{\boldsymbol{\Phi}_\beta} - h^-_{\boldsymbol{\Phi}_\beta}, 0\right)$, then, $L^+\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = 0$ and $L^-\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = 1$. The likelihood $L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right)$ of the preference relation $\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta$ satisfies the following properties: (1) $0 \le L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) \le 1$; (2) $L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) + L\left(\boldsymbol{\Phi}_\beta \ge \boldsymbol{\Phi}_\rho\right) = 1$; (3) If $L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = L\left(\boldsymbol{\Phi}_\beta \ge \boldsymbol{\Phi}_\rho\right)$, then, $L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\beta\right) = L\left(\boldsymbol{\Phi}_\beta \ge \boldsymbol{\Phi}_\rho\right) = 0.5$; (4) $L\left(\boldsymbol{\Phi}_\rho \ge \boldsymbol{\Phi}_\rho\right) = 0.5$.

Motivated by the classical Bonferroni mean (BM) operator, the IT2 Trapezoidal Fuzzy Bonferroni Aggregation Operator (IT2TrFBAO) is defined as follows:

Definition 3 [33]. Let $\tilde{\tilde{\Phi}}_i = [\tilde{\Phi}_i^-, \tilde{\Phi}_i^+] = [(\varphi_{i1}^-, \varphi_{i2}^-, \varphi_{i3}^-, \varphi_{i4}^-; h_i^-), (\varphi_{i1}^+, \varphi_{i2}^+, \varphi_{i3}^+, \varphi_{i4}^+; h_i^+)], (i = 1, 2, \ldots, m)$ be a set of interval type-2 trapezoidal fuzzy variables and $p, q \geq 0$. An IT2TrFBAO is defined as:

$$\mathrm{IT2TFBAO}^{p,q}\left(\tilde{\tilde{\Phi}}_1, \tilde{\tilde{\Phi}}_2, \ldots, \tilde{\tilde{\Phi}}_m\right) = \left(\frac{1}{m(m-1)} \sum_{i,j=1, i\neq j}^{m} \Phi_i^p \Phi_j^q\right)^{\frac{1}{p+q}} = \left[\tilde{\Phi}^-, \tilde{\Phi}^+\right] \tag{10}$$

where

$$\tilde{\Phi}^- = \left[\left(\begin{array}{l} \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i1}^-\right)^p \left(\varphi_{j1}^-\right)^q\right)^{\frac{1}{p+q}}, \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i2}^-\right)^p \left(\varphi_{j2}^-\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i3}^-\right)^p \left(\varphi_{j3}^-\right)^q\right)^{\frac{1}{p+q}}, \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i4}^-\right)^p \left(\varphi_{j4}^-\right)^q\right)^{\frac{1}{p+q}}; \\ \min\left\{h_1^-, h_2^-, \ldots, h_m^-\right\} \end{array}\right)\right] \tag{11}$$

$$\tilde{\Phi}^+ = \left[\left(\begin{array}{l} \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i1}^+\right)^p \left(\varphi_{j1}^+\right)^q\right)^{\frac{1}{p+q}}, \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i2}^+\right)^p \left(\varphi_{j2}^+\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i3}^+\right)^p \left(\varphi_{j3}^+\right)^q\right)^{\frac{1}{p+q}}, \left(\frac{1}{m(m-1)} \sum_{i,j=1,i\neq j}^{m} \left(\varphi_{i4}^+\right)^p \left(\varphi_{j4}^+\right)^q\right)^{\frac{1}{p+q}}; \\ \min\left\{h_1^+, h_2^+, \ldots, h_m^+\right\} \end{array}\right)\right] \tag{12}$$

3 A likelihood-based MCGDM method using IT2TrFBAO and IT2TrF preference relations

This section presents an effective method that utilizes the IT2TrFBAO to aggregate the judgements in group decision making. The likelihoods of IT2TrF preference relations are then employed to facilitate the complex computations of the previous approaches.

Consider an MCGDM problem in which the performance ratings of alternatives and the weights of criteria are specified as IT2TrFSs. Let the set of alternatives be $Z = \{z_1, z_2, \ldots, z_n\}$ where n is the number of alternatives. The set of criteria is $C = \{c_1, c_2, \ldots, c_m\}$, where m is the number of criteria. The set of criteria C consists of two subsets including C_b and C_c, where C_b denotes the collection of benefit criteria, and C_c denotes the collection of cost criteria. It is noted that $C_b \cap C_c = \emptyset$ and $C_b \cup C_c = C$. Then, the set of experts $E = \{e_1, e_2, \ldots, e_l\}$ are responsible for stating the performance of alternatives $Z = \{z_1, z_2, \ldots, z_n\}$ with respect to each criterion in $C = \{c_1, c_2, \ldots, c_m\}$ in IT2TrFSs. The importance of experts might not be equal in all cases. The weight vector of experts is defined as $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)$, where $\lambda_k \in [0, 1]$ and $\sum_{k=1}^{l} \lambda_k = 1$.

Let a non-negative IT2TrFS $\tilde{\tilde{\Phi}}_{ij}^k$ be the performance rating of an alternative $z_j \in Z$ regarding a specific criterion $c_j \in C$ provided by $e_k \in E$. The lower and upper membership functions of $\tilde{\tilde{\Phi}}_{ij}^k$ are denoted as the IT2TrFSs $\Phi_i^{k-}(x_j)$ and $\Phi_i^{k+}(x_j)$, respectively. Then, $\tilde{\tilde{\Phi}}_{ij}^k$ is characterized as follows [2]:

$$\tilde{\tilde{\Phi}}_{ij}^k = \left[\Phi_{ij}^{k-}, \Phi_{ij}^{k+}\right] = \left[\left(\varphi_{ij1}^{k-}, \varphi_{ij2}^{k-}, \varphi_{ij3}^{k-}, \varphi_{ij4}^{k-}; h_{\Phi_{ij}}^{k-}\right), \left(\varphi_{ij1}^{k+}, \varphi_{ij2}^{k+}, \varphi_{ij3}^{k+}, \varphi_{ij4}^{k+}; h_{\Phi_{ij}}^{k+}\right)\right] \tag{13}$$

where $0 \leq \varphi_{ij1}^{k-} \leq \varphi_{ij2}^{k-} \leq \varphi_{ij3}^{k-} \leq \varphi_{ij4}^{k-}$, $0 \leq \varphi_{ij1}^{k+} \leq \varphi_{ij2}^{k+} \leq \varphi_{ij3}^{k+} \leq \varphi_{ij4}^{k+}$, $\varphi_{ij1}^{k+} \leq \varphi_{ij1}^{k-}$, $\varphi_{ij4}^{k-} \leq \varphi_{ij4}^{k+}$, $0 \leq h_{\Phi_{ij}}^{k-} \leq h_{\Phi_{ij}}^{k+} \leq 1$ and $\Phi_{ij}^{k-} \subset \Phi_{ij}^{k+}$.

The IT2TrF characteristics of alternative $z_j \in Z$ for $e_k \in E$ can be denoted as follows:

$$\tilde{\tilde{\Phi}}_i^k = \left\{ \left\langle \left[\Phi_{ij}^{k-}, \Phi_{ij}^{k+}\right] \right\rangle \middle| j = 1, 2, \ldots, m \right\} \tag{14}$$

Let a non-negative IT2TrFS $\tilde{\tilde{\Phi}}_{ij}$ denote the aggregated performance rating of alternative $z_j \in Z$ with respect to criterion $c_j \in C$. The IT2TrFS $\tilde{\tilde{\Phi}}_{ij}$ can be obtained by utilizing the Bonferroni mean operator based on the IT2TrF judgements of experts. In the Bonferroni aggregation operator, the fusion of the judgements of experts depends on the importance of each expert. Under this perspective, the weighted performance rating is

$$\tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{k} = l\lambda_k * \tilde{\tilde{\boldsymbol{\Phi}}}_{i}^{k} \tag{15}$$

where l is the number of experts and is a positive integer which plays the role of a balancing coefficient. λ_k is the weight of the k th expert. Suppose that the weight vector of experts is $\lambda = (1/l, 1/l, \ldots, 1/l)$ where all experts have equal importance in decision making process. Then, the weighted performance vector $\left(\tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{1}, \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{2}, \ldots, \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{l}\right) = \left(l\lambda_1.\tilde{\tilde{\boldsymbol{\Phi}}}_{ij}^{1}, l\lambda_2.\tilde{\tilde{\boldsymbol{\Phi}}}_{ij}^{2}, \ldots, l\lambda_l.\tilde{\tilde{\boldsymbol{\Phi}}}_{ij}^{l}\right)$ is equal to $\left(\tilde{\tilde{\boldsymbol{\Phi}}}_{ij}^{1}, \tilde{\tilde{\boldsymbol{\Phi}}}_{ij}^{2}, \ldots, \tilde{\tilde{\boldsymbol{\Phi}}}_{ij}^{l}\right)$.

After that, these weighted performance ratings are aggregated by the IT2TrFBAO as:

$$\mathrm{IT2TFBM}^{p,q}\left(\tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{1}, \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{2}, \ldots, \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{l}\right) = \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{\text{aggregated}} = \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij} = \left[\tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{-}, \tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{+}\right] \tag{16}$$

By Eqs. (11) and (12), the above equation can be elaborated as follows:

$$\tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{-} = \left[\left(\begin{array}{l} \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij1}^{k-}\right)^p \left(l.\lambda_{k'}.\varphi_{ij1}^{k'-}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij2}^{k-}\right)^p \left(l.\lambda_{k'}.\varphi_{ij2}^{k'-}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij3}^{k-}\right)^p \left(\lambda_{k'}.\varphi_{ij3}^{k'-}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij4}^{k-}\right)^p \left(l.\lambda_{k'}.\varphi_{ij4}^{k'-}\right)^q\right)^{\frac{1}{p+q}}; \\ \min\left\{h_1^-, h_2^-, \ldots, h_l^-\right\} \end{array}\right)\right]$$

and

$$\tilde{\tilde{\hat{\boldsymbol{\Phi}}}}_{ij}^{+} = \left[\left(\begin{array}{l} \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij1}^{k+}\right)^p \left(l.\lambda_{k'}.\varphi_{ij1}^{k'+}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij2}^{k+}\right)^p \left(l.\lambda_{k'}.\varphi_{ij2}^{k'+}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij3}^{k+}\right)^p \left(l.\lambda_{k'}.\varphi_{ij3}^{k'+}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.\varphi_{ij4}^{k+}\right)^p \left(l.\lambda_{k'}.\varphi_{ij4}^{k'+}\right)^q\right)^{\frac{1}{p+q}}; \\ \min\left\{h_1^+, h_2^+, \ldots, h_l^+\right\} \end{array}\right)\right] \tag{18}$$

where $0 \le \varphi_{ij1}^{k-} \le \varphi_{ij2}^{k-} \le \varphi_{ij3}^{k-} \le \varphi_{ij4}^{k-}$, $0 \le \varphi_{ij1}^{k+} \le \varphi_{ij2}^{k+} \le \varphi_{ij3}^{k+} \le \varphi_{ij4}^{k+}$, $\varphi_{ij1}^{k+} \le \varphi_{ij1}^{k-}$, $\varphi_{ij4}^{k-} \le \varphi_{ij4}^{k+}$, $0 \le h_{\Phi_{ij}}^{-} \le h_{\Phi_{ij}}^{+} \le 1$ and $\boldsymbol{\Phi}_{ij}^{-} \subset \boldsymbol{\Phi}_{ij}^{+}$.

Let a non-negative IT2TrFS $\tilde{\tilde{W}}_j^k$ denote the weight of criterion $c_j \in C$ provided by expert $e_k \in E$. The lower and upper membership functions of $\tilde{\tilde{W}}_j^k$ are then designated as IT2TrFSs $W^{k-}(x_j)$ and $W^{k+}(x_j)$, respectively. Then, $\tilde{\tilde{W}}_j^k$ is characterized as follows [2]:

$$\tilde{\tilde{W}}_j^k = \left[W_j^{k-}, W_j^{k+}\right] = \left[\left(w_{j1}^{k-}, w_{j2}^{k-}, w_{j3}^{k-}, w_{j4}^{k-}; h_{W_j}^{k-}\right), \left(w_{j1}^{k+}, w_{j2}^{k+}, w_{j3}^{k+}, w_{j4}^{k+}; h_{W_j}^{k+}\right)\right] \tag{19}$$

where $0 \le w_{j1}^{k-} \le w_{j2}^{k-} \le w_{j3}^{k-} \le w_{j4}^{k-}$, $0 \le w_{j1}^{k+} \le w_{j2}^{k+} \le w_{j3}^{k+} \le w_{j4}^{k+}$, $w_{j1}^{k+} \le w_{j1}^{k-}$, $w_{j4}^{k-} \le w_{j4}^{k+}$, $0 \le h_{W_j}^{k-} \le h_{W_j}^{k+} \le 1$ and $W_j^{k-} \subset W_j^{k+}$

An IT2TrFS $\tilde{\tilde{W}}^k$ is a set of IT2TrF weights defined by $e_k \in E$. $\tilde{\tilde{W}}^k$ is characterized as:

$$\tilde{\tilde{W}}^k = \left\{\left[W_j^{k-}, W_j^{k+}\right] \middle| j = 1, 2, \ldots, m\right\} \text{ for } k = 1, 2, \ldots, l \tag{20}$$

By Eqs. (15) to (18), the IT2TrF weights are obtained for every single criterion by aggregating the subjective preference of each expert regarding the corresponding criterion. The aggregated weight of each criterion is shown as follows:

$$\tilde{\tilde{W}}_j^{-} = \left[\left(\begin{array}{l} \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.w_{j1}^{k-}\right)^p \left(l.\lambda_{k'}.w_{j1}^{k'-}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.w_{j2}^{k-}\right)^p \left(l.\lambda_{k'}.w_{j2}^{k'-}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.w_{j3}^{k-}\right)^p \left(l.\lambda_{k'}.w_{j3}^{k'-}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)} \sum_{k,k'=1,k\neq k'}^{l} \left(l.\lambda_k.w_{j4}^{k-}\right)^p \left(l.\lambda_{k'}.w_{j4}^{k'-}\right)^q\right)^{\frac{1}{p+q}}; \\ \min\left\{h_1^-, h_2^-, \ldots, h_l^-\right\} \end{array}\right)\right] \tag{21}$$

and

$$\tilde{\tilde{W}}_j^+ = \left[\left(\begin{array}{l} \left(\frac{1}{l(l-1)}\sum_{k,k'=1,k\neq k'}^{l}\left(l.\lambda_k.w_{j1}^{k+}\right)^p\left(l.\lambda_{k'}.w_{j1}^{k'+}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)}\sum_{k,k'=1,k\neq k'}^{l}\left(l.\lambda_k.w_{j2}^{k+}\right)^p\left(l.\lambda_{k'}.w_{j2}^{k'+}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)}\sum_{k,k'=1,k\neq k'}^{l}\left(l.\lambda_k.w_{j3}^{k+}\right)^p\left(l.\lambda_{k'}.w_{j3}^{k'+}\right)^q\right)^{\frac{1}{p+q}}, \\ \left(\frac{1}{l(l-1)}\sum_{k,k'=1,k\neq k'}^{l}\left(l.\lambda_k.w_{j4}^{k+}\right)^p\left(l.\lambda_{k'}.w_{j4}^{k'+}\right)^q\right)^{\frac{1}{p+q}}; \\ \min\left\{h_1^+, h_2^+, \ldots, h_l^+\right\} \end{array}\right)\right] \tag{22}$$

where $0 \le w_{j1}^{k-} \le w_{j2}^{k-} \le w_{j3}^{k-} \le w_{j4}^{k-}, 0 \le w_{j1}^{k+} \le w_{j2}^{k+} \le w_{j3}^{k+} \le w_{j4}^{k+}$, $w_{j1}^{k+} \le w_{j1}^{k-}, w_{j4}^{k-} \le w_{j4}^{k+}, 0 \le h_{W_j}^{k-} \le h_{W_j}^{k+} \le 1$ and $W_j^{k-} \subset W_j^{k+}$.

Once the aggregated performance ratings and weights are obtained, the likelihoods of IT2TrF preference relations approach [2, 19, 34] can be adopted to determine the dominancy of each criterion over other criteria.

As mentioned prior, the likelihood $L\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right)$ indicates the possibility that $\tilde{\tilde{\Phi}}_{i'j}$ is not larger than $\tilde{\tilde{\Phi}}_{ij}$. For two aggregated performance ratings $\tilde{\tilde{\Phi}}_{ij}$ and $\tilde{\tilde{\Phi}}_{i'j}$ where $i, i' = 1, 2, \ldots, m$ and $i \neq i'$, we firstly calculate the lower likelihood $L^-\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right)$, upper likelihood $L^+\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right)$ and overall likelihood $L\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right)$ of an IT2TrF preference relation $\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}$ in terms of each criterion $c_j \in C$. The alternative $z_i \in Z$ performs better in a benefit criterion $c_j \in C_I$ if the IT2TrF performance rating $\tilde{\tilde{\Phi}}_{ij}$ has a high possibility of being greater than or equal to the IT2TrF performance rating $\tilde{\tilde{\Phi}}_{i'j}$ for other $m-1$ alternatives, i.e. $i' = 1, 2, \ldots, m-1$ and $i \neq i'$. In contrast, the alternative $z_i \in Z$ performs better in a cost criterion $c_j \in C_{II}$ if the IT2TrF performance rating $\tilde{\tilde{\Phi}}_{ij}$ has a high possibility of being less than or equal to the IT2TrF performance rating $\tilde{\tilde{\Phi}}_{i'j}$ for other $m-1$ alternatives. With respect to the justifications discussed, the likelihood-based performance index of $\tilde{\tilde{\Phi}}_{ij}$ is obtained by [2]:

$$P\left(\tilde{\tilde{\Phi}}_{ij}\right) = \begin{cases} \sum_{i'=1,i'\neq i}^{m} L\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right) & \text{if } c_j \in C_I \\ \sum_{i'=1,i'\neq i}^{m} L\left(\tilde{\tilde{\Phi}}_{i'j} \ge \tilde{\tilde{\Phi}}_{ij}\right) & \text{if } c_j \in C_{II} \end{cases} \tag{23}$$

Since $L\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right) + L\left(\tilde{\tilde{\Phi}}_{i'j} \ge \tilde{\tilde{\Phi}}_{ij}\right) = 1$, Eq. (23) can be rewritten as Eq. (24) [2]:

$$P\left(\tilde{\tilde{\Phi}}_{ij}\right) = \begin{cases} \sum_{i'=1,i'\neq i}^{m} L\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right) & \text{if } c_j \in C_I \\ m-1-\sum_{i'=1,i'\neq i}^{m} L\left(\tilde{\tilde{\Phi}}_{ij} \ge \tilde{\tilde{\Phi}}_{i'j}\right) & \text{if } c_j \in C_{II} \end{cases} \tag{24}$$

Due to the fact that each criterion has individual priority, the importance weight of each criteria is multiplied by its relative likelihood-based performance index $P\left(\tilde{\tilde{\Phi}}_{ij}\right)$ to obtain an overall evaluation of each alternative $z_i \in Z$. These importance weights are determined based on the objectives of decision making.

Let $\hat{Y}_i$ represent a likelihood-based overall evaluation value of each alternative $z_i \in Z$. Specifically, $\hat{Y}_i$ is obtained by multiplying the likelihood-based performance indices regarding each criterion by the corresponding weight assigned to the criterion. Then, $\hat{Y}_i$ is got by summing these products over all criteria, which is shown as [2]:

$$\hat{Y}_i = \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).\tilde{\tilde{W}}_j = \left[\left(\begin{array}{l} \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).w_{1j}^-, \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).w_{2j}^-, \\ \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).w_{3j}^-, \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).w_{4j}^-; \min h_{W_j}^- \end{array}\right), \ldots\right] \tag{25}$$

The obtained results are denoted as $h_{Y_i}^- = \min h_{W_j}^-$, $h_{Y_i}^+ = \min h_{W_j}^+$, $\upsilon_{\zeta i}^- = \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).w_{\zeta j}^-$ and $\upsilon_{\zeta i}^+ = \sum_{j=1}^{n} P\left(\tilde{\tilde{\Phi}}_{ij}\right).w_{\zeta j}^+$ for $\zeta = 1, 2, 3, 4$. Hence, the brief likelihood-based overall evaluation value of alternative $z_i \in Z$ can be defined as [2]:

$$\tilde{\tilde{Y}}_i = \left[Y_i^-, Y_i^+\right] = \left[\left(\upsilon_{1i}^-, \upsilon_{2i}^-, \upsilon_{3i}^-, \upsilon_{4i}^-; h_{Y_i}^-\right), \left(\upsilon_{1i}^+, \upsilon_{2i}^+, \upsilon_{3i}^+, \upsilon_{4i}^+; h_{Y_i}^+\right)\right] \tag{26}$$

where $0 \le \upsilon_{1i}^- \le \upsilon_{2i}^- \le \upsilon_{3i}^- \le \upsilon_{4i}^-, 0 \le \upsilon_{1i}^+ \le \upsilon_{2i}^+ \le \upsilon_{3i}^+ \le \upsilon_{4i}^+$, $\upsilon_{1i}^+ \le \upsilon_{1i}^-, \upsilon_{4i}^- \le \upsilon_{4i}^+, 0 \le h_{Y_i}^- \le h_{Y_i}^+ \le 1$ and $Y_i^- \subset Y_i^+$.

Since it is hard to compare the likelihood-based overall evaluation values in IT2TrF form, the signed distance-based method, introduced by Chen [2, 3, 16, 32] and expanded by Chen et al. [14], is utilized to obtain a comparable value for each alternative. The signed distance approaches including oriented distances and direct distances were also adopted to rank fuzzy numbers [32] and developed for IT2TrFSs with the applications in several studies [2, 3, 14, 16]. The signed distance method simultaneously considers both lower and upper intervals of an IT2TrFS which differs from conventional distance measures [2]. Since the IT2TrF signed distances satisfy the law of trichotomy and linear ordering, these types of distances are suggested for obtaining the comparable values of $\tilde{\tilde{Y}}_i$ for every single alternative $z_i \in Z$.

Let ε_i denote the signed distance correspond to $\hat{Y}_i$ for alternative $z_i \in Z$. ε_i can be obtained by [2, 3, 14, 16]:

$$\varepsilon_i = \frac{1}{8}\left[\upsilon_{1i}^- + \upsilon_{2i}^- + \upsilon_{3i}^- + \upsilon_{4i}^- + 4.\upsilon_{1i}^+ + 2.\upsilon_{2i}^+ + 2.\upsilon_{3i}^+ + 4.\upsilon_{4i}^+ + 3\left(\upsilon_{2i}^+ + \upsilon_{3i}^+ - \upsilon_{1i}^+ - \upsilon_{4i}^+\right)\frac{h_{Y_i}^-}{h_{Y_i}^+}\right] \tag{27}$$

The signed distance ε_i indicates a comprehensive performance of alternative $z_i \in Z$ that can be easily compared for ranking alternatives. A large value of ε_i specifies a good preference of the alternative $z_i \in Z$. Hence, the best alternative $z^* \in Z$ can be determined as [2]:

$$z^* = \left\{ z_i \in Z \middle| \max \varepsilon_i, i = 1, 2, \ldots, n \right\} \tag{28}$$

Besides, other alternatives can be prioritized according to their ε_i values. This study adopts the signed distance-based approach [2, 3, 15, 16, 30] to rank IT2TrFNs. This approach is unique in the sense that it considers both positive and negative values to determine the ranking of IT2TrFSs.

This study aims at proposing an MCGDM model which employs the Bonferroni mean to combine the subjective preferences of a group of experts. To fulfill this objective, we propose an integrated algorithm which is established considering the basic concepts that are defined in Sect. 2 and also the properties of IT2TrF preference relations. The proposed algorithm simplifies the complex computations of the previous approaches such as IT2F TOPSIS [35, 36], IT2F TODIM [37, 38], IT2F PROMETHEE [16, 39], and IT2F DEMATEL [40–42]. We hereby propose the steps of this algorithm as follows:

Step 1. Clarify the set of alternatives $Z = \left\{z_1, z_2, \ldots, z_n\right\}$, the set of criteria $C = \left\{c_1, c_2, \ldots, c_m\right\}$, and also the set of experts $E = \left\{e_1, e_2, \ldots, e_l\right\}$. The criteria are divided into the benefit set C_b and cost set C_c.

Step 2. Select suitable IT2F linguistic variables or other data collection tools to form the IT2TrFN rating $\tilde{\tilde{\Phi}}_{ij}$ for alternative $z_i \in Z$ with respect to criterion $c_j \in C$. The weight $\tilde{\tilde{W}}_j^k$ of criterion $c_j \in C$ is provided by expert $e_k \in E$.

Step 3. Use Eq. (15) to incorporate the weights $\lambda_k (k = 1, 2, \ldots, l)$ of experts. Then, obtain the weighted IT2TrFN rating $\dot{\tilde{\tilde{\Phi}}}_{ij}^k$ for each $z_i \in Z$, $c_j \in C$ and $e_k \in E$, and the weight $\dot{\tilde{\tilde{W}}}_j^k$ for each $c_j \in C$ and $e_k \in E$.

Step 4. Calculate the aggregated performance rating $\hat{\tilde{\tilde{\Phi}}}_{ij}$ using the IT2F Bonferroni aggregation operator that is shown in Eqs. (17) and (18) for each $z_i \in Z$, $c_j \in C$ and $e_k \in E$. At the same time, work out the aggregated importance weight $\hat{\tilde{\tilde{W}}}_j$ using Eqs. (21) and (22) for each $c_j \in C$ and $e_k \in E$.

Step 5. Calculate the lower, upper and mean likelihoods by Eqs. (7)–(9), respectively.

Step 6. Compute the likelihood-based performance index $P\left(\hat{\tilde{\tilde{\Phi}}}_{ij}\right)$ using Eq. (23) for each aggregated performance rating $\hat{\tilde{\tilde{\Phi}}}_{ij}$ for $z_i \in Z$ and $c_j \in C$. Calculate the likelihood-based comprehensive evaluation value Y_i by combining $P\left(\hat{\tilde{\tilde{\Phi}}}_{ij}\right)$ and corresponding aggregated importance weight $\hat{\tilde{\tilde{W}}}_j$ using Eq. (25) for each alternative $z_i \in Z$.

Step 7. Compute the signed distance-based evaluation value ε_i for each alternative $z_i \in Z$ using Eq. (27) and eventually, rank the alternatives in descending order of ε_i and determine the best alternative by Eq. (28).

In the above algorithm, Steps 1 and 2 are the problem formulation stage. Steps 3 and 4 are the Bonferroni aggregation stage. Steps 5 to 7 are the likelihood-based computation stage.

4 Applications

This section implements the proposed IT2TrF likelihood-based MCGDM approach to four pre-existent problems including a suitable material supplier problem [24], the food production company [25], the textile companies' facility location problem [28], and the facility site selection problem [29].

4.1 Application to the material supplier selection

This subsection exemplifies the effectiveness of the proposed approach by resolving the material supplier selection problem that has been studied by Chen et al. [24], Hatami-Marbini and Tavana [23], and Chen [15].

In this case, a high-technology manufacturing company has to select a reliable supplier to provide the basic material components of new products. After an initial screening, five potential suppliers $Z = \left\{z_1, z_2, \ldots, z_5\right\}$ are selected for further assessments. The evaluation process is undergoing with three experts $E = \left\{e_1, e_2, e_3\right\}$ who have different opinions about the performance of suppliers and the weight of criteria. There are also five benefit criteria considered for the further evaluations, which includes the profitability of the supplier $\left(c_1\right)$, relationship closeness $\left(c_2\right)$, technological capability $\left(c_3\right)$, conformance quality $\left(c_4\right)$, and conflict resolution $\left(c_5\right)$.

The seven-point linguistic rating scale was utilized by Chen et al. [24] and Hatami-Marbini and Tavana [23]. Both studies utilized type-1 trapezoidal fuzzy membership function to arrange the decision matrix and to determine the weights of criteria. However, in Ref. [15], the transformation standard of the studies [18, 36, 43] was adopted to convert linguistic terms into interval type-2 fuzzy sets. We employ the seven-point IT2TrF linguistic scales proposed by Chen [15] to prepare the inputs for the likelihood-based group decision making approach. The computational steps of the proposed approach are summarized as follows:

Step 1. Following Refs. [24], [23], and [15], we define the set of experts as $E = \{e_1, e_2, e_3\}$ and the set of suppliers (alternatives) $Z = \{z_1, z_2, \dots, z_5\}$. Also, the opinions of three experts are equally important [15, 23, 24]. Given this, the weight vector of experts is $\lambda = (\lambda_1, \lambda_2, \lambda_3) = (1/3, 1/3, 1/3)$. The set of criteria is denoted by $C_b = \{c_1, c_2, \dots, c_5\}$ and $C_c = \emptyset$.

Step 2. The seven-point linguistic scale utilized for this case include the following terms for determining the performance ratings: Very Poor (VP), Poor (P), Medium Poor (MP), Fair (F), Medium Good (MG), Good (G), and Very Good (VG). The importance weights of criteria are also characterized using the following seven-point linguistic scale: Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H), and Very High (VH). Three experts determine the performance ratings and importance weights of criteria using the IT2TrF linguistic scale mentioned above, which are shown in Table 18 of "Appendix A" [15].

Step 3. The weighted performance rating $\hat{\tilde{\tilde{\Phi}}}^k_{ij}$ and importance weight $\hat{\tilde{\tilde{W}}}^k_j$ are obtained according to the importance of each expert. Because the weight vector of experts $\lambda = (1/3, 1/3, 1/3)$, we have $\hat{\tilde{\tilde{\Phi}}}^k_{ij} = 3(1/3)\dot{\tilde{\tilde{\Phi}}}^k_{ij}$ where $\dot{\tilde{\tilde{\Phi}}}^k_{ij} = \tilde{\tilde{\Phi}}^k_{ij}$ for each $z_i \in Z$, $c_j \in C$ and $e_k \in E$ based on Eq. (15). Similarly, the importance weights are $\hat{\tilde{\tilde{W}}}^k_j = 3(1/3)\dot{\tilde{\tilde{W}}}^k_j$ where $\dot{\tilde{\tilde{W}}}^k_j = \tilde{\tilde{W}}^k_j$ for each $c_j \in C$ and $e_k \in E$.

Step 4. Let $p, q = 1$. Then, we obtain the aggregated performance rate $\hat{\tilde{\tilde{\Phi}}}_{ij}$ by Bonferroni mean operator using Eqs. (17) and (18) for each $z_i \in \{z_1, z_2, \dots, z_5\}$, $c_j \in \{c_1, c_2, \dots, c_5\}$ and $e_k \in \{e_1, e_2, e_3\}$. Aggregated the weights $\hat{\tilde{\tilde{W}}}_j$ by Eqs. (21) and (22) for each $c_j \in C$ and $e_k \in \{e_1, e_2, e_3\}$. The values of these aggregated variables are shown in Table 1.

Step 5. In this step, the lower, upper and mean likelihoods $L^-\left(\hat{\tilde{\tilde{\Phi}}}_{ij} \geq \hat{\tilde{\tilde{\Phi}}}_{i'j}\right)$, $L^+\left(\hat{\tilde{\tilde{\Phi}}}_{ij} \geq \hat{\tilde{\tilde{\Phi}}}_{i'j}\right)$, and $L\left(\hat{\tilde{\tilde{\Phi}}}_{ij} \geq \hat{\tilde{\tilde{\Phi}}}_{i'j}\right)$, respectively, are calculated following Eqs. (7) to (9). The obtained results of the likelihoods are shown in Table 2.

Step 6. We calculate the likelihood-based performance index $P\left(\hat{\tilde{\tilde{\Phi}}}_{ij}\right)$ for each aggregated performance rating $\hat{\tilde{\tilde{\Phi}}}_{ij}$ for $z_i \in \{z_1, z_2, \dots, z_5\}$, $c_j \in \{c_1, c_2, \dots, c_5\}$ by Eq. (23). The results are shown in Table 3.

As the likelihood-based performance indices are all obtained, the likelihood-based comprehensive evaluation value Y_i can be computed by combining $P\left(\hat{\tilde{\tilde{\Phi}}}_{ij}\right)$ and the corresponding weight $\hat{\tilde{\tilde{W}}}_j$ based on Eq. (25) for each alternative $z_i \in \{z_1, z_2, \dots, z_5\}$. The results are shown in Table 4.

Step 7. We compute signed distance-based evaluation value ε_i for each alternative $z_i \in \{z_1, z_2, \dots, z_5\}$ following Eq. (27). The best supplier is determined by ranking the alternatives in descending order of ε_i using Eq. (28). The results are shown in Table 4, which indicate $\varepsilon_2 > \varepsilon_3 > \varepsilon_4 > \varepsilon_1 > \varepsilon_5$ and thus $z_2 \succ z_3 \succ z_4 \succ z_1 \succ z_5$. In this case, the best alternative is $z^* = z_2$.

4.2 Application to the food production company

In this subsection, we utilize the proposed IT2TrF likelihood-based MCGDM approach for choosing the best food product, presented by Chen [25], Li [26], and Chen and Niou [27].

Assume that a company tends to produce a new food product based on the considerations of three experts $E = \{e_1, e_2, e_3\}$ whom are invited to choose the best food product among three types of food that are the alternatives of this case $Z = \{z_1, z_2, z_3\}$. Three experts are reach an agreement to consider five benefit criteria including colorful (c_1), taste (c_2), smell (c_3), profit (c_4), and expiration date (c_5). The weight of these criteria and the performance of each alternative under the criteria are determined by these three experts.

The seven-point linguistic rating scale with type-1 trapezoidal fuzzy membership functions were utilized to establish decision matrix and the weights of criteria in previous studies [25–27] which are generalized to IT2TrFSs in this study. A type-1 trapezoidal fuzzy number $\tilde{A} = [a_1, a_2, a_3, a_4; h^-_A, h^+_A]$ can be characterized as an interval type-2 trapezoidal fuzzy set $\tilde{\tilde{A}} = \left[\left(a^-_1, a^-_2, a^-_3, a^-_4; h^-_A, h^+_A\right), \left(a^+_1, a^+_2, a^+_3, a^+_4; h^-_A, h^+_A\right)\right]$ in which $a^-_i = a^+_i = a_i$ $(i = 1, 2, 3, 4)$ [44]. In this way, we change the type-1 trapezoidal fuzzy sets into IT2TrFSs for this particular case. The detailed steps of implementing the proposed approach are explained as follows:

Step 1. Regarding Refs. [25], [26] and [27], the set of experts and the set of alternatives are defined as $E = \{e_1, e_2, e_3\}$, and $Z = \{z_1, z_2, z_3\}$, respectively. The set of criteria is denoted by $C_b = \{c_1, c_2, \dots, c_5\}$ and $C_c = \emptyset$. In this case, the estimations of three experts are equally important. Given this, the weight vector of experts is $\lambda = (\lambda_1, \lambda_2, \lambda_3) = (1/3, 1/3, 1/3)$.

Step 2. The linguistic scales used for this case include the following terms for determining the performance ratings: Very Poor (VP), Poor (P), Medium Poor (MP), Fair (F), Medium Good (MG), Good (G), and Very Good (VG). The importance weights of criteria are also characterized using the following seven-point linguistic scale: Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H), and Very High (VH). The three experts describe the performance ratings and importance weights of criteria using the IT2TrF linguistic scale mentioned above, which are shown in Table 19 of "Appendix A" [27].

Step 3. The weighted performance ratings $\hat{\tilde{\tilde{\Phi}}}^k_{ij}$ and importance weights $\hat{\tilde{\tilde{W}}}^k_j$ are computed considering the importance

Table 1 Aggregated performance ratings and importance weights of the criteria for the supplier selection problem

z_1	
$\tilde{\tilde{\hat{\Phi}}}_{11}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{12}$	[(0.707, 0.794, 0.794, 0.864;0.9, 0.9), (0.619, 0.794, 0.794, 0.933;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{13}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{14}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{15}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
z_2	
$\tilde{\tilde{\hat{\Phi}}}_{21}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{22}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{23}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{24}$	[(0.899, 0.966, 0.966, 0.983;0.9, 0.9), (0.831, 0.966, 0.966, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{25}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
z_3	
$\tilde{\tilde{\hat{\Phi}}}_{31}$	[(0.899, 0.966, 0.966, 0.983;0.9, 0.9), (0.831, 0.966, 0.966, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{32}$	[(0.849, 0.933, 0.933, 0.967;0.9, 0.9), (0.764, 0.933, 0.933, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{33}$	[(0.899, 0.966, 0.966, 0.983;0.9, 0.9), (0.831, 0.966, 0.966, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{34}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{35}$	[(0.849, 0.933, 0.933, 0.967;0.9, 0.9), (0.764, 0.933, 0.933, 1;1, 1)]
z_4	
$\tilde{\tilde{\hat{\Phi}}}_{41}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{42}$	[(0.73, 0.831, 0.831, 0.899;0.9, 0.9), (0.63, 0.831, 0.831, 0.966;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{43}$	[(0.663, 0.764, 0.764, 0.849;0.9, 0.9), (0.563, 0.764, 0.764, 0.933;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{44}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{45}$	[(0.849, 0.933, 0.933, 0.967;0.9, 0.9), (0.764, 0.933, 0.933, 1;1, 1)]
z_5	
$\tilde{\tilde{\hat{\Phi}}}_{51}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{52}$	[(0.73, 0.831, 0.831, 0.899;0.9, 0.9), (0.63, 0.831, 0.831, 0.966;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{53}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{54}$	[(0.663, 0.764, 0.764, 0.849;0.9, 0.9), (0.563, 0.764, 0.764, 0.933;1, 1)]
$\tilde{\tilde{\hat{\Phi}}}_{55}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\tilde{\tilde{\hat{W}}}_1$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{W}}}_2$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\tilde{\tilde{\hat{W}}}_3$	[(0.899, 0.966, 0.966, 0.983;0.9, 0.9), (0.831, 0.966, 0.966, 1;1, 1)]
$\tilde{\tilde{\hat{W}}}_4$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\tilde{\tilde{\hat{W}}}_5$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]

of each expert. Because the weight vector of experts $\lambda = (1/3, 1/3, 1/3)$, we have $\tilde{\tilde{\hat{\Phi}}}_{ij}^k = 3(1/3)\tilde{\tilde{\Phi}}_{ij}^k$ where $\tilde{\tilde{\hat{\Phi}}}_{ij}^k = \tilde{\tilde{\Phi}}_{ij}^k$ for every $z_i \in Z$, $c_j \in C$ and $e_k \in E$ by Eq. (15). Similarly, the importance weights are $\tilde{\tilde{\hat{W}}}_j^k = 3(1/3)\tilde{\tilde{W}}_j^k$ where $\tilde{\tilde{\hat{W}}}_j^k = \tilde{\tilde{W}}_j^k$ for each $c_j \in C$ and $e_k \in E$.

Step 4. The Bonferroni mean operator is employed for each $z_i \in \{z_1, z_2, z_3\}$, $c_j \in \{c_1, c_2, \ldots, c_5\}$ and

Table 2 Results of $L\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$ for the supplier selection problem

	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{4j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{5j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$
c_1	0.13	0.05	0.13	0.5	0.87	0.25
c_2	0.06	0.16	0.4	0.4	0.94	0.79
c_3	0.15	0.25	0.78	0.87	0.85	0.7
c_4	0.25	0.15	0.5	0.78	0.75	0.3
c_5	0.15	0.37	0.37	0.87	0.85	0.79
	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{4j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{5j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{4j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{5j}\right)$
c_1	0.5	0.87	0.95	0.75	0.75	0.95
c_2	0.91	0.91	0.84	0.21	0.78	0.78
c_3	0.94	0.54	0.75	0.3	0.91	0.95
c_4	0.75	0.91	0.85	0.7	0.85	0.94
c_5	0.79	0.54	0.63	0.21	0.5	0.9
	$L\left(\tilde{\tilde{\Phi}}_{4j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{4j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{4j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{4j} \geq \tilde{\tilde{\Phi}}_{5j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{5j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{5j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$
c_1	0.87	0.5	0.25	0.87	0.5	0.13
c_2	0.6	0.09	0.22	0.5	0.6	0.09
c_3	0.22	0.06	0.09	0.78	0.13	0.46
c_4	0.5	0.25	0.15	0.78	0.22	0.09
c_5	0.63	0.21	0.5	0.9	0.13	0.46
	$L\left(\tilde{\tilde{\Phi}}_{5j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{5j} \geq \tilde{\tilde{\Phi}}_{4j}\right)$				
c_1	0.05	0.13				
c_2	0.22	0.5				
c_3	0.05	0.22				
c_4	0.06	0.22				
c_5	0.1	0.1				

$e_k \in \{e_1, e_2, e_3\}$ to aggregate the performance ratings and importance weights of the criteria by Eqs. (17) and (18). By considering $p, q = 1$, The results of this step are shown in Table 5.

Table 3 Results of $P\left(\tilde{\tilde{\Phi}}_{ij}\right)$ for the supplier selection problem

	c_1	c_2	c_3	c_4	c_5
$P\left(\tilde{\tilde{\Phi}}_{1j}\right)$	0.81	1.02	2.05	1.68	1.76
$P\left(\tilde{\tilde{\Phi}}_{2j}\right)$	2.49	3.55	3.03	2.71	2.97
$P\left(\tilde{\tilde{\Phi}}_{3j}\right)$	3.4	2.61	2.91	3.34	2.24
$P\left(\tilde{\tilde{\Phi}}_{4j}\right)$	2.49	1.41	1.15	1.68	2.24
$P\left(\tilde{\tilde{\Phi}}_{5j}\right)$	0.81	1.41	0.86	0.59	0.79

Step 5. The lower, upper and mean likelihoods $L^-\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$, $L^+\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$, and $L\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$, are calculated respectively by Eqs. (7) to (9). The obtained results of likelihoods are shown in Table 6.

Step 6. In this step, we obtain the likelihood-based performance index $P\left(\tilde{\tilde{\Phi}}_{ij}\right)$ for each aggregated performance rating $\tilde{\tilde{\Phi}}_{ij}$ for $z_i \in \{z_1, z_2, z_3\}$, $c_j \in \{c_1, c_2, \ldots, c_5\}$ by Eq. (23), The results are shown in Table 7.

The likelihood-based comprehensive evaluation values Y_i are computed in this step based on Eq. (25) for each alternative $z_i \in \{z_1, z_2, z_3\}$. The obtained results of Y_i are shown in Table 8.

Step 7. We obtain the signed distance-based evaluation value ε_i for each alternative by Eq. (27) and rank the alternatives in descending order of ε_i. The best alternative is determined by Eq. (28). The results are shown in Table 8. The results indicate $\varepsilon_2 > \varepsilon_3 > \varepsilon_1$, which makes the ranking order of the three alternatives as $z_2 \succ z_3 \succ z_1$. In this case, the best alternative is $z^* = z_2$.

Table 4 Results of Y_i for the supplier selection problem

	Likelihood-based comprehensive evaluation value Y_i	ε_i	Final ranking
z_1	$[(6.209, 6.825, 6.825, 7.072; 0.9, 0.9), (5.594, 6.825, 6.825, 7.32; 1, 1)]$	13.48	4
z_2	$[(12.629, 13.829, 13.829, 13.289; 0.9, 0.9), (11.428, 13.829, 13.829, 14.75; 1, 1)]$	27.32	1
z_3	$[(12.276, 13.503, 13.503, 14.001; 0.9, 0.9), (11.05, 13.829, 13.829, 14.5; 1, 1)]$	26.73	2
z_4	$[(7.5, 8.289, 8.289, 8.629; 0.9, 0.9), (6.71, 8.289, 8.289, 8.97; 1, 1)]$	16.37	3
z_5	$[(3.863, 4.211, 4.211, 4.335; 0.9, 0.9), (3.515, 4.211, 4.211, 4.46; 1, 1)]$	8.33	5

Table 5 Aggregated performance ratings $\tilde{\tilde{\Phi}}_{ij}$ and importance weights of criteria $\tilde{\tilde{W}}_j$ for the food production company

z_1	
$\tilde{\tilde{\Phi}}_{11}$	$[(5.627, 7.638, 7.638, 9.327; 1, 1), (5.627, 7.638, 7.638, 9.327; 1, 1)]$
$\tilde{\tilde{\Phi}}_{12}$	$[(4.865, 6.904, 6.904, 8.662; 1, 1), (4.865, 6.904, 6.904, 8.662; 1, 1)]$
$\tilde{\tilde{\Phi}}_{13}$	$[(5.508, 7.550, 7.550, 8.944; 1, 1), (5.508, 7.550, 7.550, 8.944; 1, 1)]$
$\tilde{\tilde{\Phi}}_{14}$	$[(8.307, 9.661, 9.661, 10; 1, 1), (8.307, 9.661, 9.661, 10; 1, 1)]$
$\tilde{\tilde{\Phi}}_{15}$	$[(3, 5, 5, 7; 1, 1), (3, 5, 5, 7; 1, 1)]$
z_2	
$\tilde{\tilde{\Phi}}_{21}$	$[(6.298, 8.307, 8.307, 9.661; 1, 1), (6.298, 8.307, 8.307, 9.661; 1, 1)]$
$\tilde{\tilde{\Phi}}_{22}$	$[(9, 10, 10, 10; 1, 1), (9, 10, 10, 10; 1, 1)]$
$\tilde{\tilde{\Phi}}_{23}$	$[(8.307, 9.661, 9.661, 10; 1, 1), (8.307, 9.661, 9.661, 10; 1, 1)]$
$\tilde{\tilde{\Phi}}_{24}$	$[(9, 10, 10, 10; 1, 1), (9, 10, 10, 10; 1, 1)]$
$\tilde{\tilde{\Phi}}_{25}$	$[(6.904, 8.662, 8.662, 9.661; 1, 1), (6.904, 8.662, 8.662, 9.661; 1, 1)]$
z_3	
$\tilde{\tilde{\Phi}}_{31}$	$[(6.083, 7.853, 7.853, 8.944; 1, 1), (6.083, 7.853, 7.853, 8.944; 1, 1)]$
$\tilde{\tilde{\Phi}}_{32}$	$[(6.904, 8.662, 8.662, 9.661; 1, 1), (6.904, 8.662, 8.662, 9.661; 1, 1)]$
$\tilde{\tilde{\Phi}}_{33}$	$[(6.904, 8.662, 8.662, 9.661; 1, 1), (6.904, 8.662, 8.662, 9.661; 1, 1)]$
$\tilde{\tilde{\Phi}}_{34}$	$[(6.904, 8.662, 8.662, 9.661; 1, 1), (6.904, 8.662, 8.662, 9.661; 1, 1)]$
$\tilde{\tilde{\Phi}}_{35}$	$[(6.298, 8.307, 8.307, 9.661; 1, 1), (6.298, 8.307, 8.307, 9.661; 1, 1)]$
$\tilde{\tilde{W}}_1$	$[(0.69, 0.862, 0.862, 0.966; 1, 1), (0.69, 0.862, 0.862, 0.966; 1, 1)]$
$\tilde{\tilde{W}}_2$	$[(0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1)]$
$\tilde{\tilde{W}}_3$	$[(0.764, 0.933, 0.933, 1; 1, 1), (0.764, 0.933, 0.933, 1; 1, 1)]$
$\tilde{\tilde{W}}_4$	$[(0.9, 1, 1, 1; 1, 1), (0.9, 1, 1, 1; 1, 1)]$
$\tilde{\tilde{W}}_5$	$[(0.428, 0.63, 0.63, 0.831; 1, 1), (0.428, 0.63, 0.63, 0.831; 1, 1)]$

Table 6 Results of $L\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$ for the food production company

	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$
c_1	0.322	0.463	0.678	0.671	0.537	0.329
c_2	0	0.134	1	0.925	0.866	0.075
c_3	0.048	0.193	0.952	0.834	0.807	0.166
c_4	0.246	0.834	0.754	0.925	0.166	0.075
c_5	0.004	0.035	0.996	0.629	0.965	0.371

Table 7 Results of $P\left(\tilde{\tilde{\Phi}}_{ij}\right)$ for the food production company

	c_1	c_2	c_3	c_4	c_5
$P\left(\tilde{\tilde{\Phi}}_{1j}\right)$	0.785	0.134	0.241	1.08	0.039
$P\left(\tilde{\tilde{\Phi}}_{2j}\right)$	1.349	1.925	1.786	1.679	1.625
$P\left(\tilde{\tilde{\Phi}}_{3j}\right)$	0.866	0.941	0.973	0.241	1.336

4.3 Application to the facility location selection

In this subsection, we adopt the proposed approach to resolve the facility location selection problem of a Turkish textile company presented by Ertuğrul and Karakaşoğlu [28]. They resolved this problem with both fuzzy AHP and fuzzy TOPSIS, and compared the results. Here, we resolve the case to compare the results with both approaches in Ref [28] and validate the likelihood-based MCGDM approach.

The proposed application is related to the facility location problem of a Turkish Textile Company which is deduced in home-textile. This company encountered a huge growth in the demand for its products while there is no enough location space to respond to all needs [28]. Hence, the company tends to find a new location among three potential alternatives $Z = \{z_1, z_2, z_3\}$. A committee of three experts $E = \{e_1, e_2, e_3\}$ are also formed to evaluate five criteria for those alternatives. These five criteria are chosen as: favorable labor climate (c_1), proximity to markets (c_2), community considerations (c_3), quality of life (c_4), and proximity to suppliers and resources (c_5).

Ertuğrul and Karakaşoğlu [28] considered the seven-point linguistic rating scale with triangular fuzzy membership functions to determine the performance ratings and weight of criteria. We change these triangular fuzzy numbers to IT2TrFNs by adopting the approach used in Ref. [44]. A triangular fuzzy number $\tilde{A} = \left[a_1, a_2, a_3; h_A\right]$ can be characterized as an interval type-2 trapezoidal fuzzy set $\tilde{\tilde{A}} = \left[\left(a_1^-, a_2^-, a_2^-, a_3^-; h_A^-, h_A^+\right), \left(a_1^+, a_2^+, a_2^+, a_3^+; h_A^-, h_A^+\right)\right]$ in which $a_i^- = a_i^+ = a_i$ $(i = 1, 2, 3)$, and $h_A^- = h_A^+ = h_A$ [44]. We applied the T2TrFS to the performance ratings and the weights of criteria presented in Ref. [28]. The steps of implementing the proposed approach are summarized as follows:

Step 1. According to Ref. [28], the set of experts and the set of alternatives are defined as $E = \{e_1, e_2, e_3\}$, and $Z = \{z_1, z_2, z_3\}$, respectively. The set of benefit criteria is also denoted by $C_b = \{c_1, c_2, \ldots, c_5\}$ and $C_c = \emptyset$. The approximations of three experts are equally important following their equal weights. Given this, the weight vector of the experts is $\lambda = \left(\lambda_1, \lambda_2, \lambda_3\right) = (1/3, 1/3, 1/3)$.

Step 2. The linguistic terms utilized for this case include the following axioms for determining the performance ratings: Very Poor (VP), Poor (P), Medium Poor (MP), Fair (F), Medium Good (MG), Good (G), and Very Good (VG). The importance weights of criteria are also characterized using following seven-point linguistic scale: Very Low (VL), Low (L), Medium Low (ML), Medium (M), Medium High (MH), High (H), and Very High (VH). Three experts determine the performance ratings and the weight of criteria using the IT2TrF linguistic scales mentioned above as shown in Table 20 of "Appendix A" [28].

Step 3. The weighted performance ratings $\hat{\tilde{\tilde{\Phi}}}_{ij}^k$ and importance weights $\hat{\tilde{\tilde{W}}}_j^k$ are computed considering the importance of each expert. Because of the weight vector of experts $\lambda = (1/3, 1/3, 1/3)$, we have $\hat{\tilde{\tilde{\Phi}}}_{ij}^k = 3(1/3)\tilde{\tilde{\Phi}}_{ij}^k$ where $\hat{\tilde{\tilde{\Phi}}}_{ij}^k = \tilde{\tilde{\Phi}}_{ij}^k$ for each $z_i \in Z$, $c_j \in C$ and $e_k \in E$ by Eq. (15). Similarly, the importance weights are $\hat{\tilde{\tilde{W}}}_j^k = 3(1/3)\tilde{\tilde{W}}_j^k$ where $\hat{\tilde{\tilde{W}}}_j^k = \tilde{\tilde{W}}_j^k$ for each $c_j \in C$ and $e_k \in E$.

Step 4. The Bonferroni mean operator is utilized for obtaining the aggregated performance ratings and the weights of criteria for each $z_i \in \{z_1, z_2, z_3\}$, $c_j \in \{c_1, c_2, \ldots, c_5\}$ and $e_k \in \{e_1, e_2, e_3\}$ by Eqs. (17) and (18). By considering $p, q = 1$, the results are shown in Table 9.

Step 5. The lower, upper and mean likelihoods $L^-\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$, $L^+\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$, and $L\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$, are obtained by Eqs. (7) to (9). The obtained results are shown in Table 10.

Step 6. The likelihood-based performance index $P\left(\tilde{\tilde{\Phi}}_{ij}\right)$ for each aggregated performance rating $\tilde{\tilde{\Phi}}_{ij}$ for $z_i \in \{z_1, z_2, z_3\}$, $c_j \in \{c_1, c_2, \ldots, c_5\}$ by Eq. (23). The results are shown in Table 11.

The likelihood-based comprehensive evaluation values are computed by Eq. (25) for each alternative

Table 8 Results of Y_i and ε_i for food production company

	Likelihood-based comprehensive evaluation value Y_i	ε_i	Final rank
z_1	$[(1.835, 2.14, 2.14, 2.24; 1, 1), (1.835, 2.14, 2.14, 2.24; 1, 1)]$	4.23	3
z_2	$[(6.234, 7.456, 7.456, 8.043 : 1, 1), (6.234, 7.456, 7.456, 8.043 : 1, 1)]$	14.75	1
z_3	$[(2.976, 3.677, 3.677, 4.101; 1, 1), (2.976, 3.677, 3.677, 4.101; 1, 1)]$	7.28	2

Table 9 Aggregated performance ratings and the weights of the criteria for the textile facility location problem

z_1	
$\tilde{\tilde{\Phi}}_{11}$	[(7.659, 9.309, 9.309, 9.661;1, 1), (7.659, 9.309, 9.309, 9.661;1, 1)]
$\tilde{\tilde{\Phi}}_{12}$	[(7.659, 9.309, 9.309, 9.661;1, 1), (7.659, 9.309, 9.309, 9.661;1, 1)]
$\tilde{\tilde{\Phi}}_{13}$	[(4.655, 5.979, 5.979, 7.303;1, 1), (4.655, 5.979, 5.979, 7.303;1, 1)]
$\tilde{\tilde{\Phi}}_{14}$	[(7.326, 8.641, 8.641, 9.327;1, 1), (7.326, 8.641, 8.641, 9.327;1, 1)]
$\tilde{\tilde{\Phi}}_{15}$	[(7.326, 8.641, 8.641, 9.327;1, 1), (7.326, 8.641, 8.641, 9.327;1, 1)]
z_2	
$\tilde{\tilde{\Phi}}_{21}$	[(7.326, 8.641, 8.641, 9.327;1, 1), (7.326, 8.641, 8.641, 9.327;1, 1)]
$\tilde{\tilde{\Phi}}_{22}$	[(4.320, 5.477, 5.477, 6.633;1, 1), (4.320, 5.477, 5.477, 6.633;1, 1)]
$\tilde{\tilde{\Phi}}_{23}$	[(7.659, 9.309, 9.309, 9.661;1, 1), (7.659, 9.309, 9.309, 9.661;1, 1)]
$\tilde{\tilde{\Phi}}_{24}$	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]
$\tilde{\tilde{\Phi}}_{25}$	[(6.298, 7.483, 7.483, 8.660;1, 1), (6.298, 7.483, 7.483, 8.660;1, 1)]
z_3	
$\tilde{\tilde{\Phi}}_{31}$	[(5.627, 6.982, 6.982, 8.327;1, 1), (5.627, 6.982, 6.982, 8.327;1, 1)]
$\tilde{\tilde{\Phi}}_{32}$	[(4.655, 5.979, 5.979, 7.303;1, 1), (4.655, 5.979, 5.979, 7.303;1, 1)]
$\tilde{\tilde{\Phi}}_{33}$	[(7.659, 9.309, 9.309, 9.661;1, 1), (7.659, 9.309, 9.309, 9.661;1, 1)]
$\tilde{\tilde{\Phi}}_{34}$	[(4.320, 5.477, 5.477, 6.633;1, 1), (4.320, 5.477, 5.477, 6.633;1, 1)]
$\tilde{\tilde{\Phi}}_{35}$	[(5.627, 6.982, 6.982, 8.327;1, 1), (5.627, 6.982, 6.982, 8.327;1, 1)]
$\tilde{\tilde{W}}_1$	[(6.298, 8.30, 8.30, 9.66;1, 1), (6.298, 8.30, 8.30, 9.66;1, 1)]
$\tilde{\tilde{W}}_2$	[(4.898, 6.838, 6.838, 8.64;1, 1), (4.898, 6.838, 6.838, 8.64;1, 1)]
$\tilde{\tilde{W}}_3$	[(5.916, 8.144, 8.144, 9.309;1, 1), (5.916, 8.144, 8.144, 9.309;1, 1)]
$\tilde{\tilde{W}}_4$	[(5.259, 7.016, 7.016, 8.64;1, 1), (5.259, 7.016, 7.016, 8.64;1, 1)]
$\tilde{\tilde{W}}_5$	[(5.627, 7.707, 7.707, 9.327;1, 1), (5.627, 7.707, 7.707, 9.327;1, 1)]

Table 10 Results of $L\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$ for the facility location problem

	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$
c_1	0.722	0.947	0.278	0.906	0.053	0.094
c_2	1	1	0	0.283	0	0.717
c_3	0	0	1	0.5	1	0.5
c_4	0.947	1	0.053	0.826	0	0.174
c_5	0.84	0.906	0.16	0.712	0.094	0.288

Table 11 Results of $P\left(\tilde{\tilde{\Phi}}_{ij}\right)$ for the facility location problem

	c_1	c_2	c_3	c_4	c_5
$P\left(\tilde{\tilde{\Phi}}_{1j}\right)$	1.669	2	0	1.947	1.746
$P\left(\tilde{\tilde{\Phi}}_{2j}\right)$	1.184	0.283	1.5	0.879	0.872
$P\left(\tilde{\tilde{\Phi}}_{3j}\right)$	0.147	0.717	1.5	0.174	0.382

$z_i \in \{z_1, z_2, z_3\}$. Given this, the obtained Y_i are shown in Table 12.

Step 7. The signed distance-based evaluation value ε_i is obtained for each alternative $z_i \in \{z_1, z_2, z_3\}$ by Eq. (27). To determine the best alternative, the alternatives are ranked in descending order of ε_i by Eq. (28). The results are also shown in Table 12, which indicate $\varepsilon_1 > \varepsilon_2 > \varepsilon_3$ and $z_1 \succ z_2 \succ z_3$. In this case, the best alternative is $z^* = z_1$.

4.4 Application to the facility site selection

In this subsection, we apply the proposed approach to resolve a hypothetical facility site selection problem presented by Chu and Lin [29]. They proposed an interval arithmetic based fuzzy TOPSIS model and validated their approach by solving the hypothetical facility site selection problem. The results obtained by the proposed likelihood-based MCGDM approach are then compared with the results obtained by interval arithmetic based fuzzy TOPSIS approach proposed in [29].

According to Ref. [29], assume that a high technology company must select a site to build a new plant. After a preliminary screening, three potential sites z_1, z_2 and z_3 remain for further evaluations. A committee of three experts $E = \{e_1, e_2, e_3\}$ are formed to assess these three alternatives with respect to three benefit criteria including climate (c_1), labor force quality (c_2) and transportation availability (c_3). The steps of implementing the proposed approach are summarized as follows:

Step 1. Regarding to Chu and Lin [29], the set of experts and the set of alternatives are defined as $E = \{e_1, e_2, e_3\}$, and $Z = \{z_1, z_2, z_3\}$, respectively. The set of criteria is denoted by $C_b = \{c_1, c_2, c_3\}$ and $C_c = \emptyset$. In this case, the estimations of three experts are equally important, namely, the weight vector of experts is $\lambda = (\lambda_1, \lambda_2, \lambda_3) = (1/3, 1/3, 1/3)$.

Step 2. The linguistic scales used for this case include the following terms for determining the performance ratings: Very Poor (VP), Poor (P), Medium Poor (MP), Fair (F), Medium Good (MG), Good (G), and Very Good (VG). The importance weights of criteria are also characterized using following seven-point linguistic scale: Very Low (VL), Low

Table 12 Results of Y_i and ε_i for facility location problem

	Likelihood-based comprehensive evaluation value Y_i	ε_i	Final rank
z_1	[(40.375, 54.656, 54.656, 66.515;1, 1), (40.375, 54.656, 54.656, 66.515;1, 1)]	108.7	1
z_2	[(27.248, 36.873, 36.873, 43.577;1, 1), (27.248, 36.873, 36.873, 43.577;1, 1)]	73.01	2
z_3	[(16.377, 22.506, 22.506, 26.646;1, 1), (16.377, 22.506, 22.506, 26.646;1, 1)]	44.51	3

Table 13 Aggregated performance ratings and the weights of criteria for the textile facility site selection

z_1	
$\tilde{\tilde{\Phi}}_{11}$	[(5.627, 7.638, 7.638, 9.327;1, 1), (5.627, 7.638, 7.638, 9.327;1, 1)]
$\tilde{\tilde{\Phi}}_{12}$	[(5.508, 7.550, 7.550, 8.944;1, 1), (5.508, 7.550, 7.550, 8.944;1, 1)]
$\tilde{\tilde{\Phi}}_{13}$	[(8.307, 9.661, 9.661, 10;1, 1), (8.307, 9.661, 9.661, 10;1, 1)]
z_2	
$\tilde{\tilde{\Phi}}_{21}$	[(8.307, 9.661, 9.661, 10;1, 1), (8.307, 9.661, 9.661, 10;1, 1)]
$\tilde{\tilde{\Phi}}_{22}$	[(8.307, 9.661, 9.661, 10;1, 1), (8.307, 9.661, 9.661, 101, 1)]
$\tilde{\tilde{\Phi}}_{23}$	[(7.550, 8.944, 8.944, 9.661;1, 1), (7.550, 8.944, 8.944, 9.661;1, 1)]
z_3	
$\tilde{\tilde{\Phi}}_{31}$	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
$\tilde{\tilde{\Phi}}_{32}$	[(5.508, 7.550, 7.550, 8.944;1, 1), (5.508, 7.550, 7.550, 8.944;1, 1)]
$\tilde{\tilde{\Phi}}_{33}$	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
$\tilde{\tilde{W}}_1$	[(0.833, 0.966, 0.966, 1;1, 1), (0.833, 0.966, 0.966, 1;1, 1)]
$\tilde{\tilde{W}}_2$	[(0.766, 0.933, 0.933, 1;1, 1), (0.766, 0.933, 0.933, 1;1, 1)]
$\tilde{\tilde{W}}_3$	[(0.7, 0.833, 0.833, 0.9;1, 1), (0.7, 0.833, 0.833, 0.9;1, 1)]

Table 14 Results of $L\left(\tilde{\tilde{\Phi}}_{ij} \geq \tilde{\tilde{\Phi}}_{i'j}\right)$ for the facility site selection problem

	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{1j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{2j} \geq \tilde{\tilde{\Phi}}_{3j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{1j}\right)$	$L\left(\tilde{\tilde{\Phi}}_{3j} \geq \tilde{\tilde{\Phi}}_{2j}\right)$
c_1	0.079	0.202	0.921	0.768	0.798	0.232
c_2	0.048	0.5	0.952	0.952	0.5	0.048
c_3	0.787	0.768	0.213	0.525	0.232	0.475

Table 15 Results of $P\left(\tilde{\tilde{\hat{\Phi}}}_{ij}\right)$ for the facility site selection problem

	c_1	c_2	c_3
$P\left(\tilde{\tilde{\hat{\Phi}}}_{1j}\right)$	0.281	0.548	1.555
$P\left(\tilde{\tilde{\hat{\Phi}}}_{2j}\right)$	1.689	1.904	0.738
$P\left(\tilde{\tilde{\hat{\Phi}}}_{3j}\right)$	0.03	0.548	0.707

(L), Medium Low (ML), Medium (M), Medium High (MH), High (H), and Very High (VH). Three experts described the performance ratings and the weights of criteria using the IT2TrF linguistic scale mentioned above as shown in Table 21 of "Appendix A", or Table 1 of Chu and Lin [29].

Step 3. The weighted performance ratings $\tilde{\tilde{\hat{\Phi}}}_{ij}^k$ and importance weights $\tilde{\tilde{\hat{W}}}_j^k$ are computed considering the importance of each expert. Then, we have $\tilde{\tilde{\hat{\Phi}}}_{ij}^k = 3(1/3)\tilde{\tilde{\Phi}}_{ij}^k$ where $\tilde{\tilde{\hat{\Phi}}}_{ij}^k = \tilde{\tilde{\Phi}}_{ij}^k$ for every $z_i \in Z$, $c_j \in C$ and $e_k \in E$ by Eq. (15). Similarly, the importance weights are $\tilde{\tilde{\hat{W}}}_j^k = 3(1/3)\tilde{\tilde{W}}_j^k$ where $\tilde{\tilde{\hat{W}}}_j^k = \tilde{\tilde{W}}_j^k$ for each $c_j \in C$ and $e_k \in E$.

Step 4. The Bonferroni mean operator is now going to be employed for each $z_i \in \{z_1, z_2, z_3\}$, $c_j \in \{c_1, c_2, c_3\}$ and $e_k \in \{e_1, e_2, e_3\}$ to aggregate the performance ratings and the weights of criteria by Eqs. (17) and (18).By considering $p, q = 1$, the results of this step are shown in Table 13.

Step 5. The lower, upper and mean likelihoods $L^-\left(\tilde{\tilde{\hat{\Phi}}}_{ij} \ge \tilde{\tilde{\hat{\Phi}}}_{i'j}\right)$, $L^+\left(\tilde{\tilde{\hat{\Phi}}}_{ij} \ge \tilde{\tilde{\hat{\Phi}}}_{i'j}\right)$, and $L\left(\tilde{\tilde{\hat{\Phi}}}_{ij} \ge \tilde{\tilde{\hat{\Phi}}}_{i'j}\right)$, are obtained by Eqs. (7) to (9). The obtained results of the likelihoods for the facility site selection problem are shown in Table 14.

Step 6. The likelihood-based performance index $P\left(\tilde{\tilde{\hat{\Phi}}}_{ij}\right)$ for each aggregated performance rating for $z_i \in \{z_1, z_2, z_3\}$, $c_j \in \{c_1, c_2, c_3\}$ by Eq. (23). The results are shown in Table 15.

The likelihood-based comprehensive evaluation values Y_i are computed byg Eq. (25) for each alternative $z_i \in \{z_1, z_2, z_3\}$. The obtained Y_i are shown in Table 16.

Step 7. The signed distance-based evaluation value ε_i is obtained for each alternative $z_i \in \{z_1, z_2, z_3\}$ by Eq. (27). The alternatives are ranked in descending order of ε_i to determine the best alternative by Eq. (28). The results are also shown in Table 16. The results indicate $\varepsilon_2 > \varepsilon_3 > \varepsilon_1$ and $z_2 \succ z_3 \succ z_1$. In this case, the best alternative is $z^* = z_2$.

Table 16 Results of Y_i and ε_i for the facility location problem

	Likelihood-based comprehensive evaluation value Y_i	ε_i	Final rank
z_1	$[(1.695, 2.05, 2.05, 2.29;1, 1), (1.695, 2.05, 2.05, 2.29;1, 1)]$	4.07	3
z_2	$[(3.35, 4.01, 4.01, 4.28;1, 1), (3.35, 4.01, 4.01, 4.28;1, 1)]$	7.92	1
z_3	$[(1.747, 2.08, 2.08, 2.24;1, 1), (1.747, 2.08, 2.08, 2.24;1, 1)]$	4.12	2

Table 17 Comparison analysis of the obtained results

Research source	Comparative method	Ranking results
The supplier selection problem		
Hatami-Marbini and Tavana [23]	Non-fuzzy extended ELECTRE I	$z_2 \sim z_3 \succ z_4 \sim z_1 \succ z_5$
Chen et al. [24]	An approval status approach	$z_2 \sim z_3 \succ z_4 \sim z_1 \sim z_5$
Chen [15]	IT2F ELECTRE using a net concordance approach	$z_2 \succ z_3 \succ z_4 \succ z_1 \succ z_5$
Current study	Likelihood-based MCGDM approach using Bonferroni mean operator	$z_2 \succ z_3 \succ z_4 \succ z_1 \succ z_5$
The food production problem		
Chen [25]	A Vertex distance-based TOPSIS with triangular fuzzy sets	$z_2 \succ z_3 \succ z_1$
Li [26]	Compromise ratio-based TOPSIS method	$z_2 \succ z_3 \succ z_1$
Chen and Niou [27]	Type-1 fuzzy preference relations	$z_2 \succ z_3 \succ z_1$
Current study	Likelihood-based MCGDM approach using Bonferroni mean operator	$z_2 \succ z_3 \succ z_1$
The facility location selection problem		
Ertuğrul and Karakaşoğlu [28]	Type-1 fuzzy TOPSIS	$z_1 \succ z_2 \succ z_3$
Ertuğrul and Karakaşoğlu [28]	Type-1 fuzzy AHP	$z_1 \succ z_2 \succ z_3$
Current study	Likelihood-based MCGDM approach using Bonferroni mean operator	$z_1 \succ z_2 \succ z_3$
The facility site selection problem		
Chu and Lin [29]	Interval arithmetic-based TOPSIS	$z_2 \succ z_3 \succ z_1$
Current study	Likelihood-based MCGDM approach using Bonferroni mean operator	$z_2 \succ z_3 \succ z_1$

4.5 Comparisons of the results

This subsection discusses the results to elucidate the advantages of the proposed MCGDM approach over pre-existent models. The practicability of the proposed approach is demonstrated by the applications to four cases. The results of the comparison analysis are indicated in Table 17.

We compare the final ranking orders of the supplier selection problem obtained by the likelihood-based MCGDM approach using Bonferroni mean operator with the results obtained in Refs. [14, 24, 37]. With respect to the results obtained by the proposed method, there is a high degree of similarity between the final ranking orders of five suppliers with the results shown in Table 17. The difference is that in the ranking order provided by Hatami-Marbini and Tavana [23], $z_2 \sim z_3 \succ z_4 \sim z_1 \succ z_5$, alternatives z_2 and z_3 are not comparable, which makes it difficult for experts to characterize the best alternatives. Similarly, in Ref. [24], the ranking order $z_2 \sim z_3 \succ z_4 \sim z_1 \sim z_5$ has the incomparable pair z_2 and z_3. The IT2F ELECTRE-based approach proposed by Chen [15] overcame the mentioned gap by deducing a precise ranking order in which the alternatives were differentiated using a net concordance approach. However, the procedure of IT2F ELECTRE approach proposed by Chen [15] contained complex computations which consumes time for obtaining the signed distance-based hybrid averaging operations, the weighted collective evaluation values and the concordance indices. Nevertheless, we elaborate the likelihood-based approach for MCGDM problems with the ability of determining the exact decisions by a soft computing procedure. The proposed likelihood-based MCGDM approach concludes the same ranking order by obtaining the likelihoods of IT2TrF preference relations for five suppliers $z_i \in \{z_1, z_2, \dots, z_5\}$ with regards to five criteria $c_j \in \{c_1, c_2, \dots, c_5\}$ based on the considerations of three experts $e_i \in \{e_1, e_2, e_3\}$ aggregated by a Bonferroni aggregation operator with $p, q = 1$.

Similarly, we illustrate the validation and effectiveness of the proposed approach by comparing the results of the proposed approach and the results obtained by these three approaches [13, 19, 40]. Chen [25] introduced a new fuzzy TOPSIS considering a vertex distance between triangular fuzzy sets. Le [26] also developed a TOPSIS method with introducing a compromise ratio under the fuzzy environment. Chen and Niou [27] utilized the concept of type-1 fuzzy preference relations to resolve the same case. From Table 17, three approaches obtained the same ranking order for three alternatives with respect to five criteria. Both Chen [25] and Li [26] utilized the TOPSIS method with different distance measures for ranking these alternatives. They determined the decision matrix based on experts' linguistic judgements and prepared the normalized decision matrix based on the TOPSIS principles. They both obtained the distances to positive and negative ideal solutions based on the performance of each alternative. Compared with the TOPSIS method, the proposed likelihood-based approach minimizes the computation steps by removing the calculation of distances and replacing them with lower, upper and likelihoods of IT2TrF preference relations. In other words, the proposed likelihood-based approach removes the hardships of conventional fuzzy TOPSIS by proposing a soft computing calculation and a new method for ranking the potential alternatives. With Eq. (24), there is no need to normalize the decision matrixes for changing the scales of cost criteria. Using the same equation can automatically alter the effects of cost criteria for each alternative. Although Chen and Niou [27] utilized the concept of preference relations of type-1 fuzzy sets to propose a far effective approach than conventional TOPSIS, the approach was limited with type-1 fuzzy sets. Additionally, Chen and Niou [27] computed the fuzzy preference relations for performance rating of each alternative based on the judgements of each expert, which increases the complexity of computations and time of the process. This is why the Bonferroni mean operator is employed in this study to aggregate the judgements of each expert based on their importance. Using the IT2TrFBAO, we can exploit a balanced value which embeds the impact of each expert.

The facility location selection problem presented by Ertuğrul and Karakaşoğlu [28] is resolved using the proposed approach to verify the accuracy of the obtained results. Ertuğrul and Karakaşoğlu [28] solved this particular case with both fuzzy TOPSIS and fuzzy AHP. The results indicated that both approaches are leading to the same conclusions. Although the fuzzy TOPSIS, fuzzy AHP and the proposed approach concluded the same ranking order $z_1 \succ z_2 \succ z_3$ for three alternatives, we compare the structures of these three approaches to clarify the advantages and disadvantages of each approach. In Ref. [28], they deduced that the fuzzy AHP requires more complex computations than fuzzy TOPSIS. As discussed early, the proposed likelihood-based MCGDM approach reduces the computational complexity of TOPSIS method by removing the distances to both positive and negative ideal solutions. Given this, the proposed approach is more facilitated than the fuzzy AHP while it removes the pairwise comparison of alternatives with respect to each criterion. They also mentioned the fact that TOPSIS is known as the best approach for addressing the rank reversal issue which is the consequence of changes in the number of alternatives when a non-optimal alternative is added or removed from the list of potential alternatives [28]. The proposed likelihood-based MCGDM approach is also very performable in cases where the number of alternatives is changing time to time. In the event, the fuzzy AHP is planned to handle the cases with a fixed number of alternatives. Regarding the nature of fuzzy AHP, each expert is asked to provide judgements about either the importance of one criterion against another criteria or its preference of one alterative on one criterion against another alternatives [28]. With respect to this fact, it is reasonable to assume in cases with huge number of criteria, the pairwise comparison process becomes unwieldy [28, 45]. In contrast, the proposed likelihood-based approach has few limitations in the number of criteria which is one of the advantages of this approach.

The facility site selection problem proposed by Chu and Lin [29] is an illustrative case which validated the interval arithmetic-based TOPSIS approach. We utilize this case to ensure the effectiveness and dominancy of the proposed likelihood-based MCGDM approach over the state-of-the-art extensions of TOPSIS. Ref. [30] proposed interval arithmetic-based fuzzy principles to normalize the performance rating of each alternative and the weight of each criteria into a comparable scale. They utilized the α-cuts of a triangular fuzzy number to define some principles for arithmetic of fuzzy sets. With respect to these principles, they remodeled the fuzzy TOPSIS method in an interval-based fuzzy environment which enhanced the accuracy of results. Furthermore, the concept of α-cuts depends on choosing a right and accurate α to perform the cuts. it is a challenging task to select the most appropriate α while there are so many variables affecting the performance ratings of alternatives. Hence, it is more valuable to deal with fuzzy numbers directly than converting them into crisp sets. On the other hand, triangular fuzzy sets cannot contain enough information compared with trapezoidal fuzzy sets [46]. Besides, in Ref. [29], they utilized the average operator to determine the aggregated performance ratings and importance weights of criteria. The averaging operator is weak because it ignores the importance of each expert. In contrast, the Bonferroni mean operator is a powerful aggregating operator in the sense that it performs based on the importance of each criteria. Even if there is no difference between the importances of experts, the results obtained by the Bonferroni mean operator differs from the results obtained by the averaging operator.

5 Conclusions

With respect to the fact that decisions are made by analyzing the related data which always carry uncertainty, IT2TrFSs were proposed as a generalization for conventional fuzzy sets in which the degree of membership falls into a fuzzy set on the interval [0,1] [2]. The IT2TrFSs were established because the conventional type-1 fuzzy sets were incompetent to cover all the uncertainties that currently exist in real-world problems [12]. This study elaborated the single expert decision-making model based on the likelihoods of IT2TrF preference relations proposed by Chen [2] to a multi-expert MCGDM approach by employing the Bonferroni mean operator to aggregate the subjective preferences of experts in terms of IT2TrFSs. The proposed methodology was proved to be more facilitated, accurate, flexible, and effective than conventional group decision making approaches such as the fuzzy TOPSIS, fuzzy AHP, and fuzzy ELECTRE. This likelihood-based MCGDM approach was established based on the comparison of IT2TrFSs. The likelihoods of IT2TrF preference relations compressed the valuable information of the sophisticated IT2TrF into regular crisps to reduce the complexity of making decisions for experts.

By the likelihoods and the IT2TrFBAO, this study presented a multi-expert MCGDM approach which was validated by the applications to four pre-existent cases. Comparisons of the results also authenticated the effectiveness and applicability of the proposed group decision making approach.

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Appendix A

This appendix provides the IT2TrF data that are utilized to express the importance weights and performance rates for the presented cases. These provided data includes Table 18 which indicates the linguistic and IT2TrFs data expressed by three decision makers $e_j \in \{e_1, e_2, e_3\}$ for the supplier selection case including importance weights of each criteria $c_j \in \{c_1, c_2, \dots, c_5\}$ and the performance rate of each supplier $z_i \in \{z_1, z_2, z_3\}$ with respect to each criteria $c_j \in \{c_1, c_2, \dots, c_5\}$. Table 19 which indicates the linguistic and IT2TrFs of decision makers $e_j \in \{e_1, e_2, e_3\}$ for the food production company including importance weights assigned for each criteria $c_j \in \{c_1, c_2, \dots, c_5\}$ and the performance rate of each alternative $z_i \in \{z_1, z_2, z_3\}$ with respect to each criteria $c_j \in \{c_1, c_2, \dots, c_5\}$. Table 20 indicates the linguistic and IT2TrFs of decision makers $e_j \in \{e_1, e_2, e_3\}$ for the facility location selection problem including importance weights assigned for each criteria $c_j \in \{c_1, c_2, \dots, c_5\}$ and the performance rate of each supplier $z_i \in \{z_1, z_2, z_3\}$ with respect to

Table 18 The IT2TrFN rating $\boldsymbol{\Phi}_{ij}^{k}$. and the importance weights W_j^k for the supplier selection problem

IT2TrFN ratings	
$\boldsymbol{\Phi}_{11}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{11}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{11}^{3}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{12}^{3}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{12}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{12}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{13}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{13}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{13}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{14}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{14}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)].
$\boldsymbol{\Phi}_{14}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{15}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{15}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{15}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{21}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{21}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{21}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{22}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{22}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{22}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{23}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{23}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{23}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{24}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{24}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{24}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{25}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{25}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{25}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{31}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{31}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{31}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{32}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{32}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{32}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{33}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{33}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{33}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{34}^{1}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{34}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{34}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{35}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{35}^{2}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{35}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{41}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{41}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{41}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{42}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{42}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{42}^{3}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{43}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{43}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{43}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{44}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{44}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{44}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{45}^{1}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{45}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{45}^{3}$	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
$\boldsymbol{\Phi}_{51}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{51}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{51}^{3}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{52}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{52}^{2}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{52}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{53}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{53}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{53}^{3}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{54}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{54}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{54}^{3}$	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
$\boldsymbol{\Phi}_{55}^{1}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{55}^{2}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
$\boldsymbol{\Phi}_{55}^{3}$	[(0.6, 0.7, 0.7, 0.8;0.9, 0.9), (0.5, 0.7, 0.7, 0.9;1, 1)]
W_1^1	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_1^2	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_1^3	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_2^1	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
W_2^2	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
W_2^3	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
W_3^1	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
W_3^2	[(0.95, 1, 1, 1;0.9, 0.9), (0.9, 1, 1, 1;1, 1)]
W_3^3	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_4^1	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_4^2	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_4^3	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_5^1	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_5^2	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]
W_5^3	[(0.8, 0.9, 0.9, 0.95;0.9, 0.9), (0.7, 0.9, 0.9, 1;1, 1)]

Table 19 The IT2TrFN rating Φ_{ij}^k and the importance weight W_j^k for the food production company

IT2TrFN ratings	
A_{11}^1	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{11}^2	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{11}^3	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{12}^1	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{12}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{12}^3	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{13}^1	[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]
A_{13}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{13}^3	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{14}^1	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{14}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{14}^3	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{15}^1	[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]
A_{15}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{15}^3	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{21}^1	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{21}^2	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{21}^3	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{22}^1	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{22}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{22}^3	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{23}^1	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{23}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{23}^3	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{24}^1	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{24}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{24}^3	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{25}^1	[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]
A_{25}^2	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{25}^3	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{31}^1	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{31}^2	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{31}^3	[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]
A_{32}^1	[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]
A_{32}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{32}^3	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{33}^1	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{33}^2	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{33}^3	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{34}^1	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{34}^2	[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]
A_{34}^3	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
A_{35}^1	[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]
A_{35}^2	[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]
A_{35}^3	[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]
W_1^1	[(0.7, 0.9, 0.9, 1;1, 1), (0.7, 0.9, 0.9, 1;1, 1)]
W_1^2	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_1^3	[(0.5, 0.7, 0.7, 0.9;1, 1), (0.5, 0.7, 0.7, 0.9;1, 1)]
W_2^1	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_2^2	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_2^3	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_3^1	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_3^2	[(0.7, 0.9, 0.9, 1;1, 1), (0.7, 0.9, 0.9, 1;1, 1)]
W_3^3	[(0.7, 0.9, 0.9, 1;1, 1), (0.7, 0.9, 0.9, 1;1, 1)]
W_4^1	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_4^2	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_4^3	[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]
W_5^1	[(0.3, 0.5, 0.5, 0.7;1, 1), (0.3, 0.5, 0.5, 0.7;1, 1)]
W_5^2	[(0.5, 0.7, 0.7, 0.9;1, 1), (0.5, 0.7, 0.7, 0.9;1, 1)]
W_5^3	[(0.5, 0.7, 0.7, 0.9;1, 1), (0.5, 0.7, 0.7, 0.9;1, 1)]

Table 20 The IT2TrFN rating Φ_{ij}^k and the importance weight W_j^k for the facility location selection problem

IT2TrFN ratings	
A_{11}^1	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{11}^2	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{11}^3	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{12}^1	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{12}^2	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{12}^3	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{13}^1	[(4, 5, 5, 6;1, 1), (4, 5, 5, 6;1, 1)]
A_{13}^2	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]
A_{13}^3	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]
A_{14}^1	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{14}^2	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{14}^3	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{15}^1	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{15}^2	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{15}^3	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{21}^1	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{21}^2	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{21}^3	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{22}^1	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]
A_{22}^2	[(4, 5, 5, 6;1, 1), (4, 5, 5, 6;1, 1)]
A_{22}^3	[(4, 5, 5, 6;1, 1), (4, 5, 5, 6;1, 1)]
A_{23}^1	[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]
A_{23}^2	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{23}^3	[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]
A_{24}^1	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]
A_{24}^2	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]
A_{24}^3	[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]

Table 20 (continued)

IT2TrFN ratings	
A_{25}^1	$[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]$
A_{25}^2	$[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]$
A_{25}^3	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{31}^1	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{31}^2	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{31}^3	$[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]$
A_{32}^1	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{32}^2	$[(4, 5, 5, 6;1, 1), (4, 5, 5, 6;1, 1)]$
A_{32}^3	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{33}^1	$[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]$
A_{33}^2	$[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]$
A_{33}^3	$[(8, 10, 10, 10;1, 1), (8, 10, 10, 10;1, 1)]$
A_{34}^1	$[(4, 5, 5, 6;1, 1), (4, 5, 5, 6;1, 1)]$
A_{34}^2	$[(4, 5, 5, 6;1, 1), (4, 5, 5, 6;1, 1)]$
A_{34}^3	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{35}^1	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{35}^2	$[(5, 6.5, 6.5, 8;1, 1), (5, 6.5, 6.5, 8;1, 1)]$
A_{35}^3	$[(7, 8, 8, 9;1, 1), (7, 8, 8, 9;1, 1)]$
W_1	$[(0.8, 1, 1, 1;1, 1), (0.8, 1, 1, 1;1, 1)]$
W_2	$[(0.7, 0.93, 0.93, 1;1, 1), (0.7, 0.93, 0.93, 1;1, 1)]$
W_3	$[(0.7, 0.87, 0.87, 1;1, 1), (0.7, 0.87, 0.87, 1;1, 1)]$
W_4	$[(0.5, 0.7, 0.7, 0.9;1, 1), (0.5, 0.7, 0.7, 0.9;1, 1)]$
W_5	$[(0.7, 0.8, 0.8, 0.9;1, 1), (0.7, 0.8, 0.8, 0.9;1, 1)]$

Table 21 The IT2TrFN rating Φ_{ij}^k and the importance weight W_j^k for the facility site selection problem

IT2TrFN ratings	
A_{11}^1	$[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]$
A_{11}^2	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{11}^3	$[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]$
A_{12}^1	$[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]$
A_{12}^2	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{12}^3	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{13}^1	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{13}^2	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{13}^3	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{21}^1	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{21}^2	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{21}^3	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{22}^1	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{22}^2	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{22}^3	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{23}^1	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{23}^2	$[(9, 10, 10, 10;1, 1), (9, 10, 10, 10;1, 1)]$
A_{23}^3	$[(5, 7, 7, 9;1, 1), (5, 7, 7, 9;1, 1)]$

Table 21 (continued)

IT2TrFN ratings	
A_{31}^1	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{31}^2	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{31}^3	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{32}^1	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{32}^2	$[(3, 5, 5, 7;1, 1), (3, 5, 5, 7;1, 1)]$
A_{32}^3	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{33}^1	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{33}^2	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
A_{33}^3	$[(7, 9, 9, 10;1, 1), (7, 9, 9, 10;1, 1)]$
W_1^1	$[(0.7, 0.9, 0.9, 1;1, 1), (0.7, 0.9, 0.9, 1;1, 1)]$
W_1^2	$[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]$
W_1^3	$[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]$
W_2^1	$[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]$
W_2^2	$[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]$
W_2^3	$[(0.7, 0.9, 0.9, 1;1, 1), (0.7, 0.9, 0.9, 1;1, 1)]$
W_3^1	$[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]$
W_3^2	$[(0.9, 1, 1, 1;1, 1), (0.9, 1, 1, 1;1, 1)]$
W_3^3	$[(0.3, 0.5, 0.5, 0.7;1, 1), (0.3, 0.5, 0.5, 0.7;1, 1)]$

each criteria $c_j \in \{c_1, c_2, \ldots, c_5\}$. Table 21 also indicates the linguistic and IT2TrFs of decision makers $e_j \in \{e_1, e_2, e_3\}$ for the facility site selection problem including importance weights assigned for each criteria $c_j \in \{c_1, c_2, c_3\}$ and the performance rate of each alternative $z_i \in \{z_1, z_2, z_3\}$ with respect to each criteria $c_j \in \{c_1, c_2, c_3\}$.

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