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ORIGINAL ARTICLE

An emergency decision making method based on the multiplicative consistency of probabilistic linguistic preference relations

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Abstract

As the evolution of emergencies is often uncertain, it may lead to multiple emergency scenarios. According to the characteristics of emergency management, this paper proposes an emergency decision support method by using the probabilistic linguistic preference relations (PLPRs) whose elements are the pairwise comparisons of alternatives given by the decision-makers (DMs) in the form of probabilistic linguistic term sets (PLTSs). As the decision data are limited, it is difficult for the DMs to provide exact occurrence probabilities of all possible emergency scenarios. Thus, we propose a probability correction method by using the computer-aided tool named the case-based reasoning (CBR) to obtain more accurate and reasonable occurrence probabilities of the probabilistic linguistic elements (PLEs). Then, we introduce a multiplicative consistency index to judge whether a PLPR is consistent or not. Afterwards, an acceptable multiplicative consistency-based emergency decision support method is proposed to get more reliable results. Furthermore, a case study about the emergency decision making in a petrochemical plant fire accident is conducted to illustrate the proposed method. Finally, some comparative analyses are performed to demonstrate the feasibility and effectiveness of the proposed method.

Keywords Emergency decision making · Probabilistic linguistic preference relations (PLPRs) · Multiplicative consistency · Probability correction

1 Introduction

After the 9/11 incident happened in the United States, the international community has paid more attention to public safety and emergency management. Thus, how to choose an effective emergency response plan and organize it quickly to reduce casualties and property losses has been reconsidered by governments, public and scholars all over the world. In reality, a variety of emergencies occur frequently and the evolution of emergencies is often uncertain. This may lead to multiple emergency scenarios. Through the establishment and improvement of emergency response support technique, we can improve the efficiency of emergency decision making. Considering the urgency of time and the complicated situation when an emergency happens, emergency commanders often organize numerous experts from relevant departments. The experts may prefer to provide their opinions by linguistic terms [1]. For example, the linguistic term "*very important*", "*important*" or "*unimportant*" can be used to express the weight of a criterion. For the performance of an alternative, the linguistic terms "*high*" or "*low*" could be used. Due to the ambiguity and complexity of human

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cognition, one issue of emergency decision making is how to express the experts' evaluations or preference information accurately [2].

To represent uncertainty more accurately, the linguistic term set (LTS) has been extended to the uncertain linguistic term sets [3], the 2-tuple LTS [4], the virtual LTSs [5], the hesitant fuzzy LTSs (HFLTSs) [6] and the probabilistic LTS (PLTS) [7]. In the previous study of the emergency management, the LTSs have been introduced into various emergency decision-making processes. For example, Zhang et al. [8] constructed a fuzzy multi-attribute emergency decision support system, which is based on the traditional LTSs. Zhou et al. [9] proposed a fuzzy decision-making trial and evaluation laboratory (DEMATEL) method to figure out the critical success factors (CSFs) of emergency management by using LTSs. Afterwards, Li et al. [10] introduced an evidential DEMATEL method and then Zhou et al. [11] researched another DEMATEL method to identify CSFs in emergency management. Beyond that, Ju et al. [12] utilized the 2-tulpe LTSs to assess the emergency response capability. Xu et al. [13] investigated the group decision making (GDM) problems in emergency management with incomplete fuzzy LTSs.

To our knowledge, there is no work on emergency management with PLTSs. In addition, the PLTS is more flexible than other LTS extensions as it not only allows the experts to provide their judgments using more than one linguistic terms but also can reflect the different occurrence probabilities of all possible linguistic terms. In this study, we select the PLTS to represent the hesitancy of the DMs among multiple linguistic terms and the uncertain probabilities of the possible emergency scenarios. Take the evacuation plan selection problem as an example. In Wenchuan earthquake, to evacuate the people who live in the downstream of Tangjiashan barrier lake, the barrier lake may face three kinds of scenarios, "no dam break (1)", "small-scale dam break (2)", "large-scale dam break (3)", which are affected by some uncertain factors such as aftershock, precipitation, dam geological structure. In such a case, the barrier lake will lead to no dam break with the probability 20%, small-scale dam break with the probability 25%, and large-scale dam break with the probability 55%. The evaluation values of the schemes would vary in different emergency scenarios. Therefore, probability interpretations play a significant role in understanding the real situations and the DMs' behaviors. Compared with the other extended LTSs, the PLTS is an ideal decision-making tool as it can model the real emergency cases and avoid the loss of information.

Due to the uncertain evolution of the emergency scenarios, it is difficult for the DMs to provide accurate evaluations, especially when they face various options. To select the desirable alternatives, they usually provide their judgements by pairwise comparisons over the alternatives. It is found that the linguistic preference relation (LPR) [14, 15] is suitable to describe the DMs' subjective preferences since the linguistic variables are natural expressions of the DMs. For this good character, the LPR has wide applications in different types of decision-making problems [16, 17]. However, it still has limitations to capture the practical events due to that it ignores the occurring probabilities of the DMs' judgements. The probabilistic linguistic preference relation (PLPR) [18], whose elements are PLTSs, has been proposed to improve the practicability of the linguistic judgements. After introducing the definition of PLPR, Zhang et al. [18] discussed the consistency of the PLPR from the perspective of additive transitivity and then investigated the consensus reaching process for group decision making with PLPRs [19]. Nevertheless, the additive consistency of preference relation is sometimes beyond reasonable limits and thus has to be transformed by various formulas [20, 21], which may distort the original preference information. Some researchers have pointed out that the multiplicative consistency property does not have the limitation that occurs in the additive consistency [22–24]. Therefore, in this paper, we investigate another consistency property of PLPR, namely, the multiplicative consistency.

The multiplicative consistency has been investigated for various preference relations, such as the interval fuzzy preference relation [25], the intuitionistic fuzzy preference relation [26], the intuitionistic multiplicative preference relation [27], the linguistic preference relation [22], the hesitant fuzzy preference relation [23] and the hesitant fuzzy linguistic preference relation [24]. Even though the multiplicative consistency has been studied for the above preference relations, as far as we know, there is no research focused on the multiplicative consistency of the PLPR. Moreover, due to the difficulties of information collection in distinct emergency scenarios, it is hard to give specific numerical values to describe the occurrence probabilities of the elements in the PLPR. Besides, it seems difficult to obtain the accurate probabilities of the elements through subjective evaluations of the DMs. In this case, how to correct the probabilities of the PLPR becomes another problem that will be discussed in this paper.

To achieve the above purposes, we must address four basic issues: (1) the probability correction of the PLPR; (2) the multiplicative consistency identification of the PLPR; (3) the consistency improving of the PLPR; and (4) the emergency decision support method based on the acceptable multiplicative consistent PLPR. To do so, we define the similar degree between the historical event and the current event. Based on the introduced similarity threshold, a probability correction method is proposed to adjust the probabilities of the PLPR. By exploiting this method, we can obtain more credible occurrence probabilities of the elements in the PLPR. Next, we identify the multiplicative consistency of the PLPR and provide an iterative algorithm to improve the multiplicative consistency of the inconsistent PLPR. After that, an emergency decision support method based on the acceptable multiplicative consistency of the PLPR is proposed, which can improve the efficiency of emergency management. The main contributions of this paper can be summarized as follows:

1. We introduce the concept of multiplicative consistency of the PLPR. We then justify that the multiplicative consistency of the PLPR is more reasonable than the previously defined additive consistency.
2. We propose a probability correction method for the PLPR based on the computer-aided tool, the case-based reasoning (CBR). With this method, the historical information of the same kind of emergencies and the subjective judgements of the current emergency can be considered simultaneously.
3. We define a multiplicative consistency measure of the PLPR. An algorithm based on the multiplicative consistency of PLPR is developed for emergency decision support. Taking advantage of this method and the computer-aided tool, we can make a decision more effectively and quickly when a disaster occur.
4. We implement our proposed method to handle an emergency decision-making problem in a fire accident to illustrate the proposed method. Some comparative analyses and discussions are performed to demonstrate the feasibility and effectiveness of the proposed method.

The remainder of this paper is organized as follows: The concept of PLTS and multiplicative consistency of the PLPR are introduced in Sect. 2. In Sect. 3, we propose a probability correction method for the PLPR based on the CBR. In Sect. 4, the multiplicative consistency measure is developed, and the emergency decision support method based on the acceptable multiplicative consistent PLPR is proposed. Finally, a practical case of a fire accident in the petrochemical plant is given to illustrate our method, and then some further analyses are provided in Sect. 5. Section 6 ends the paper with some concluding comments.

2 PLTS and PLPR

2.1 PLTS

Given an additive LTS $S=\{s_\alpha|\alpha=0,1,\dots,2\tau\}$ (τ is a positive integer), then the PLTS is defined as [7]:

$$L_p=\left\{L_l(p_l)\middle|L_l\in S,p_l\geq 0,l=1,2,\dots,m,\sum_{l=1}^{m}p_l\leq 1\right\},\tag{1}$$

where $L_l(p_l)$ represents the linguistic term L_l associated with the probability p_l, and m is the number of all different linguistic terms in L_p.

For convenience, we refer to $L(p)$ as a probabilistic linguistic elements (PLEs) and L_p as the set of all PLEs. Thus, the PLE $\{s_{\alpha 1}(0.2),s_{\alpha 2}(0.25),s_{\alpha 3}(0.55)\}$ is more suitable for the description of the example in Sect. 1.

Then the normalized PLTSs (NPLTSs) can be denoted as [7]:

$$L_p^N=\left\{L_l^N(p_l^N)\middle|L_l^N=L_l,p_l^N\geq 0,l=1,2,\dots,m,\sum_{l=1}^{m}p_l^N=1\right\},$$

where $p_l^N=p_l\Big/\sum_{l=1}^{m}p_l$.

Let $L(p)=\{L_l(p_l)|l=1,2,...,m\}$ be a PLE and r_l be the subscript of the linguistic term L_l, $E(L(p))=s_{\bar{\alpha}}$ is called the score of $L(p)$, where $\bar{\alpha}=\sum_{l=1}^{m}r_lp_l\Big/\sum_{l=1}^{m}p_l$. The comparison laws between $L(p)_1$ and $L(p)_2$ can be presented as: (1) if $E(L(p)_1)>E(L(p)_2)$, then $L(p)_1>L(p)_2$; (2) if$E(L(p)_1)<E(L(p)_2)$, then $L(p)_1<L(p)_2$. (3) If $E(L(p)_1)=E(L(p)_2)$, then $L(p)_1$ and $L(p)_2$ are said to be approximately equivalent, denoted as $L(p)_1\cong L(p)_2$.

Remark 1 Notice that the PLE is reduced to the (hesitant fuzzy linguistic element) HFLE when all probabilities are equal, i.e., $p_1=p_2=\cdots=p_m$, which shows that the PLE is a generalized HFLE. By including the probabilities of occurrence, the PLE is more readily utilized to present the uncertainty than the HFLE. In addition, probability information plays an important role in understanding the behavior of the DMs and the real situations of emergencies, making it an ideal decision-making tool for emergency decision support.

2.2 PLPR

Zhang et al. [18] gave the definition of the additive consistent PLPR as follows:

Definition 1 [18]. Let $P=(L(p)_{ij})_{n\times n}$ be a PLPR and $P^N=(L(p)_{ij}^N)_{n\times n}$ be its corresponding normalized PLPR. Then $P=(L(p)_{ij})_{n\times n}$ is an additively consistent PLPR if

$$L(p)_{ij}^N\cong L(p)_{ie}^N\oplus L(p)_{ej}^N.\tag{2}$$

However, it is obvious that the additive consistency property has some shortcomings. For example, let $\tau=4$, for a NPLPR: if $L_{12,1}^N(p_{12,1})=s_2(0.9)$ and $L_{23,1}^N(p_{23,1})=s_3(0.8)$, where $L_{ij,l}^N(p_{ij,l})$ is the lth element in $L(p)_{ij}^N$. Then by Eq. (2), we have $L_{13,1}^N(p_{13})\cong L_{12,1}^N(p_{12,1})\oplus L_{23,1}^N(p_{23,1})=s_{1.8}+s_{2.4}$

$= s_{4.2}$,which is outside of $\left[s_{-4}, s_4\right]$ and is unreasonable. Although it can be transformed into the value in $\left[s_{-4}, s_4\right]$ by using Wang and Xu's method [21], some preference information will be lost. In what follows, we introduce a new concept of the multiplicative consistency for the PLPR to solve this issue.

Definition 2 Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed set. The PLPR P on the set X can be represented by a matrix $P = \left(L(p)_{ij}\right)_{n\times n} \subset X \times X$ for all $i,j = 1,2,\ldots,n$. $L(p)_{ij} = \{L_{ij,l}(p_{ij,l}) | \, l = 1, 2, \ldots, \#L(p)_{ij}\}$ is a PLE on the LTS $S = \{s_\alpha | \alpha = 0, 1, \ldots, 2\tau\}$, where $p_{ij,l} > 0$ and $\sum_{l=1}^{\#L(p)_{ij}} p_{ij,l} \le 1$, $\#L(p)_{ij}$ is the numbers of possible elements in $L(p)_{ij}$. $L(p)_{ij}$ indicates the preference degree of the alternative x_i over x_j and must satisfy the following characteristics:

$$p_{ij,l} = p_{ji,l}, \quad L_{ij,l} \oplus L_{ji,l} = 2\tau, \quad L(p)_{ii} = \left\{s_\tau(1)\right\} = \left\{s_\tau\right\},$$

$\#L(p)_{ij} = \#L(p)_{ji}=m$ ($\#L(p)_{ij}$ and $\#L(p)_{ji}$ are the numbers of possible elements in $L(p)_{ij}$ and $L(p)_{ji}$ respectively)

and

$$L_{ij,l} \le L_{ij,l+1} \quad \text{for } i \le j, \quad L_{ji,l} \ge L_{ji,l+1} \quad \text{for } i \ge j,$$

where $L_{ij,l}$ is the lth linguistic term in $L(p)_{ij}$ $(i,j = 1, 2, \ldots, n$ and $l = 1, 2, \ldots, m-1)$.

Based on Definition 2, we can derive three conclusions: (1) the basic components of the PLPR are the PLEs; (2) the elements of the PLE in the upper triangular matrix of the PLPR are increasing; and (3) the elements of the PLE in the lower triangular matrix of the PLPR are decreasing. It is pointed out that (2) and (3) are identified for the convenience of calculations.

The PLEs and their corresponding probabilities should be specified to construct the first PLPR. In real emergency decision making, the PLEs are provided by the DMs depending on their experience and knowledge. However, it is sometimes difficult to obtain the exact occurrence probabilities of all elements in the PLEs. For example, in the second PLPR, the DMs are required to provide the precise probability information for all the values in the PLEs. Intuitively, this requirement is illogical and difficult. As the PLEs are more flexible than other fuzzy numbers or real numbers in considering the uncertain probabilities of the multiple scenarios with the dynamic evolvement process of emergency, the next section will focus on how to obtain more accurate and reasonable probabilities of the scenarios. Below we introduce the multiplicative consistency of the PLPR:

Definition 3 Let $B = \left(L(p)_{ij}\right)_{n\times n}$ be a PLPR and $B^N = \left(L(p)_{ij}^N\right)_{n\times n}$ be the corresponding NPLPR, for $i,j,e = 1,2,\ldots,n$, $i \ne j \ne e$. Then $B = \left(L(p)_{ij}\right)_{n\times n}$ is a multiplicative consistent PLPR if

$$E\left(L(p)_{ie,l}^N\right)E\left(L(p)_{ej,l}^N\right)E\left(L(p)_{ji,l}^N\right) = E\left(L(p)_{ei,l}^N\right)E\left(L(p)_{je,l}^N\right)E\left(L(p)_{ij,l}^N\right), \tag{3}$$

where $L(p)_{ij,l}^N$ is the lth element in $L(p)_{ij}^N$.

Theorem 1 Given a PLPR $P = \left(L(p)_{ij}\right)_{n\times n}$ and its NPLPR $P^N = \left(L(p)_{ij}^N\right)_{n\times n}$, $P = \left(L(p)_{ij}\right)_{n\times n}$ is a multiplicative consistent PLPR if

$$E\left(L(p)_{ij}\right) = \begin{cases} \dfrac{2\tau E\left(L(p)_{ie,l}^N\right)E\left(L(p)_{ej,l}^N\right)}{E\left(L(p)_{ik,l}^N\right)E\left(L(p)_{ej,l}^N\right) + \left(2\tau - E\left(L(p)_{ie,l}^N\right)\right)\left(2\tau - E\left(L(p)_{ej,l}^N\right)\right)} & i,j,e = 1,2,\ldots,n; \quad i \ne j \ne e \\ \tau, & \text{otherwise} \end{cases}. \tag{4}$$

Suppose that S is defined as: $S = \left\{ \begin{array}{l} s_0 : \text{very poor}, \; s_1 : \text{poor}, s_2 : \text{slightly poor}, \; s_3 : \text{fair}, \\ s_4 : \text{slightly good}, s_5 : \text{good}, \; s_6 : \text{very good}, \end{array} \right\}$.

A simple PLPR based on S can be expressed as:

$$\begin{bmatrix} L_{11,l}(p_{11,l}) & L_{12,l}(p_{12,l}) & L_{13,l}(p_{13,l}) \\ L_{21,l}(p_{21,l}) & L_{22,l}(p_{22,l}) & L_{23,l}(p_{23,l}) \\ L_{31,l}(p_{31,l}) & L_{32,l}(p_{32,l}) & L_{33,l}(p_{33,l}) \end{bmatrix} = \begin{bmatrix} \{s_3(1)\} & \{s_2(p_{12,1}), s_3(p_{12,2})\} & \{s_3(p_{13,1}), s_4(p_{13,2})\} \\ \{s_4(p_{21,1}), s_3(p_{22,2})\} & \{s_3(1)\} & \{s_1(p_{23,1}), s_2(p_{23,2})\} \\ \{s_3(p_{31,1}), s_2(p_{32,2})\} & \{s_5(p_{32,1}), s_4(p_{32,2})\} & \{s_3(1)\} \end{bmatrix}.$$

Proof According Eq. (3), for any $i,j,e = 1,2,\cdots,n$; $i \ne j \ne e$, we have

$$
\begin{aligned}
&E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)E\left(L(p)^N_{ji,l}\right)=\mathrm{E}\left(L(p)^N_{ei,l}\right)E\left(L(p)^N_{je,l}\right)E\left(L(p)^N_{ij,l}\right)\\
&\Leftrightarrow E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)\left(2\tau-E\left(L(p)^N_{ij,l}\right)\right)=\left(2\tau-E\left(L(p)^N_{ie,l}\right)\right)\left(2\tau-E\left(L(p)^N_{ej,l}\right)\right)E\left(L(p)^N_{ij,l}\right)\\
&\Leftrightarrow 2\tau E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)-E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)E\left(L(p)^N_{ij,l}\right)=\left(2\tau-E\left(L(p)^N_{ie,l}\right)\right)\left(2\tau-E\left(L(p)^N_{ej,l}\right)\right)E\left(L(p)^N_{ij,l}\right)\\
&\Leftrightarrow 2\tau E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)=\left(2\tau-E\left(L(p)^N_{ie,l}\right)\right)\left(2\tau-E\left(L(p)^N_{ej,l}\right)\right)+E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)E\left(L(p)^N_{ij,l}\right)\\
&\Leftrightarrow E\left(L(p)^N_{ij,l}\right)=\frac{2\tau E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)}{E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)+\left(2\tau-E\left(L(p)^N_{ie,l}\right)\right)\left(2\tau-E\left(L(p)^N_{ej,l}\right)\right)}\quad i,j,e=1,2,\dots,n;\quad i\neq j\neq e,
\end{aligned}
$$

which completes the proof of Theorem 1.

Based on Eq. (4), it can be easily proven that:

$$
\begin{aligned}
E\left(L(p)^N_{ij,l}\right)&=\frac{2\tau E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)}{E\left(L(p)^N_{ie,l}\right)E\left(L(p)^N_{ej,l}\right)+\left(2\tau-E\left(L(p)^N_{ie,l}\right)\right)\left(2\tau-E\left(L(p)^N_{ej,l}\right)\right)}\\
&=\frac{2\tau}{1+\left(\frac{2\tau}{I\left(L(p)^N_{ie,l}\right)}-1\right)\left(\frac{2\tau}{I\left(L(p)^N_{ej,l}\right)}-1\right)},
\end{aligned}
\tag{5}
$$

which shows that $E\left(L(p)^N_{ij,l}\right)$ is an increasing function regarding to $E\left(L(p)^N_{ie,l}\right)$ and $E\left(L(p)^N_{ej,l}\right)$. Therefore, we have

$$
0\le E\left(L(p)^N_{ij,l}\right)=\frac{2\tau}{1+\left(\frac{2\tau}{E\left(L(p)^N_{ie,l}\right)}-1\right)\left(\frac{2\tau}{E\left(L(p)^N_{ej,l}\right)}-1\right)}\le 2\tau. \tag{6}
$$

Equation (6) implies that Eq. (4) overcomes the weakness that may happen in Eq. (2). Thus, the definition of the multiplicative consistency of PLPR is much more reasonable than that of the additive consistency.

3 Probability correction method of the PLPR

As mentioned above, the occurrence probabilities of the PLE is a key component of the PLPR, but yet, it is difficult to identify the probabilities of emergency scenarios as the limited decision data and possible evolvement of emergency scenarios. Therefore, in this section, a probability correction method is proposed to combine the subjective and objective data. By this method, we can simultaneously consider the historical information of the same kind of emergencies and the subjective judgment information of the DMs for the current emergency. Firstly, the objective probability estimation method is proposed.

3.1 Probability calculation by CBR

Rapid decision making and rapid disposal of the incident are very important for emergency management. Using the procedure of CBR, which is a method to combine problem-solving and learning and is one of the most successful applied subfields of artificial intelligence (AI) in recent years [28], we can find similar historical events or plans of the incident quickly to provide reference for the DMs. The CBR imitates the process of human reasoning and thinking, reflecting the use of remembered problems and solutions of people as a starting point for solving the new problem. Aamodt and Plaza's [29] described that the genetic CBR system is composed of 4 consecutive processes (i.e., retrieve, reuse, revise, and retain), which are known as the "4 REs". Due to the critical role of retrieval in the CBR cycle, a large number of studies have focused on the retrieval and similarity assessments [30, 31]. In the following, motivated by the similarity measures in Ref. [32], we define the similarity degree under the HFL environment as follows:

Definition 4 Let $\mathrm{Z}=\left\{\mathrm{Z}_q\middle|q=1,2,\dots,Q\right\}$ be the case set of the group like historical events, $C^q=\left\{C^q_k\middle|k=1,2,\dots,K\right\}$

be the attribute set of Z_q, $C^* = \left\{ C_k^* \middle| k = 1,2,\ldots,K \right\}$ be the attribute set of the current event, $H_S^{Z^*} = \cup_{L_{\delta_l^{Z^*}} \in H_S^{Z^*}} \left\{ L_{\delta_l^{Z^*}} \middle| l = 1,\ldots,\#H_S \right\}$ and $H_S^{Z_q} = \cup_{L_{\delta_l^{Z_q}} \in H_S^{Z_q}} \left\{ L_{\delta_l^{Z_q}} \middle| l = 1,\cdots,\#H_S \right\}$ be two HFLTSs on the LTS $S = \{s_\alpha | \alpha = 0,1,\ldots,2\tau\}$, which denotes the evaluation values with respect to C^* and C^q respectively. Suppose that the linguistic terms are arranged in ascending order, and $\#H_S$ is the number of possible linguistic terms in $H_S^{Z^*}$ and $H_S^{Z_q}$. Then the attribute similarity degree between the current event Z^* and the group like historical event Z_q stored in a case database can be defined as:

$$Sim\left(C_i^*, C_i^q\right) = 1 - \frac{1}{\#H_S} \sum_{l=1}^{\#H_S} \frac{\left|\delta_l^{Z^*} - \delta_l^{Z_q}\right|}{2\tau + 1}. \tag{7}$$

Remark 2 It should be noted that for some historical event Z_q, the corresponding attribute C_k^q may be unrecorded or missing, i.e., $C_k^q = \phi$. Then $Sim\left(C_k^*, C_k^q\right) = 0$. Thus, we can define $\zeta^q = \left\{ \zeta_k^q | k = 1,2,\cdots,K \right\}$ as the attribute indicator vector of C^q.

- For the attribute C_k^q, if Z_q lacks history information, then we denote its attribute indicator as $\zeta_k^q = 0$ for $k = 1,2,\ldots,K$ and $q = 1,2,\ldots,Q$. Thus, we have $Sim\left(C_k^*, C_k^q\right) = 0$.
- For the attribute C_k^q, if the history information of Z_q is expressed as the HFLE, then we denote the attribute indicator as $\zeta_k^q = 1$ for $k = 1,2,\ldots,K$ and $q = 1,2,\ldots,Q$. Thus, we have $Sim\left(C_k^*, C_k^q\right) = 1 - \frac{1}{\#H_S} \sum_{l=1}^{\#H_S} \frac{\left|\delta_l^{Z^*} - \delta_l^{Z_q}\right|}{2\tau+1}$ $\sum_{l=1}^{\#H_S} \frac{\left|\delta_l^{Z^*} - \delta_l^{Z_q}\right|}{2\tau+1}$.

Based on the attribute indicator vector $\zeta^q = \left\{ \zeta_1^q, \zeta_2^q, \ldots, \zeta_K^q \right\}$, Eq. (7) can be transferred into Eq. (8):

$$Sim\left(C_k^*, C_k^q\right) = \begin{cases} 1 - \frac{1}{\#H_S} \sum\limits_{l=1}^{\#H_S} \frac{\left|\delta_l^{Z^*} - \delta_l^{Z_q}\right|}{2\tau+1}, & \zeta_k^q = 1 \\ 0, & \zeta_k^q = 0 \end{cases}. \tag{8}$$

To calculate the global similarity between the current events and the historical events stored in the case database, the global similarity degree is defined as follows:

Definition 5 Let $\omega = \left\{ \omega_k | k = 1,2,\ldots,K \right\}$ be the priority vector of C_k^q satisfying $\omega_k \in [0,1]$, and $\sum_{k=1}^K \omega_k = 1$. Then the global similarity degree between the current event Z^* and the group like historical event Z_q stored in the case database according to different characteristics C_k^q can be defined as:

$$Sim\left(Z^*, Z_q\right) = \sum_{k=1}^{K} \omega_k \cdot Sim\left(C_k^*, C_k^q\right), \quad q = 1,2,\ldots,Q, \tag{9}$$

where $Sim(Z^*, Z_q) \in [0,1]$. The larger the value of $Sim(Z^*, Z_q)$ is, the higher the similarity degree between the current event and the historical event should be.

Definition 6 To extract the historical events with high similarity degree as the reference cases, the similarity threshold of the current event and the historical event can be defined as:

$$\varsigma = \tau \cdot \max\left\{ Sim\left(Z^*, Z_q\right) \middle| q = 1,2,\ldots,Q \right\}, \tag{10}$$

where τ is the adjustment parameter and $\tau \in (0,1]$. The larger value τ indicates the higher similarity between the current event and the extracted historical event.

Remark 3 Once the global similarity degree $Sim\left(Z^*, Z_q\right)$ is obtained, we then conduct the comparison between $Sim\left(Z^*, Z_q\right)$ and the similarity threshold $\varsigma (0 \le \varsigma \le 1)$. It should be noted that the parameter ς is given by the experts based on the historical data, and can be adjusted according to the actual situations:

- If the global similarity degree $Sim(Z^*, Z_q) < \varsigma$, then the corresponding historical event Z_q will be removed.
- If the global similarity degree $Sim(Z^*, Z_q) \ge \varsigma$, then the corresponding historical event Z_q will be extracted and put into the set of the reference cases, which are denoted as $Z^{Sim} = \left\{ Z_f^{Sim} \middle| f = 1,2,\ldots,F \right\}$.

To calculate the objective scenario probability P^*, we let $\mathbb{S} = \left\{ \mathbb{S}_l \middle| l = 1,2,\ldots,m \right\}$ be the set of all the possible scenarios, where $\mathbb{S}_l$ is the lth possible scenario in $\mathbb{S}$, and $\mathbb{S}^{Z_f^{Sim}} = \left\{ \mathbb{S}_l^{Z_f^{Sim}} \middle| l = 1,2,\ldots,m; f = 1,2,\ldots,F \right\}$ is the indicator set of the possible scenarios. If the similar historical case Z_f^{Sim} appears in the scenario $\mathbb{S}_l$, then $\mathbb{S}_l^{Z_f^{Sim}} = 1$; otherwise, $\mathbb{S}_l^{Z_f^{Sim}} = 0$. Because the final scenario of the historical event

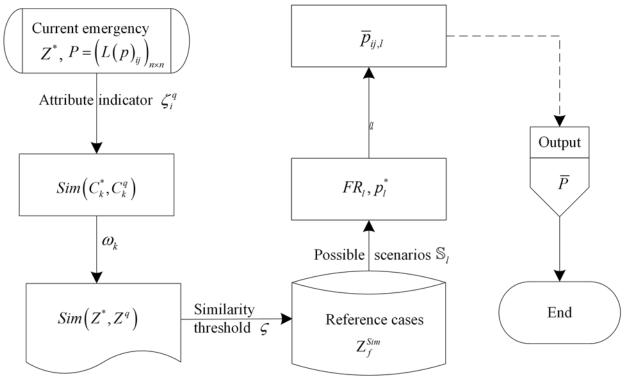

Fig. 1 Calculation process of the probability correction algorithm

is unique and has been determined, we have $\sum_{l=1}^{m} \mathbb{S}_l^{Z_f^{Sim}} = 1$. Let FR_l be the frequency of the scenario $\mathbb{S}_l$ finally happen in the reference cases Z_f^{Sim}, then it can be calculated according to the following formula:

$$FR_l = \sum_{f=1}^{F} \mathbb{S}_l^{Z_f^{Sim}} \quad l = 1, 2, \ldots, m. \tag{11}$$

Let $P^* = \left\{p_l^* | l = 1, 2, \ldots, m\right\}$ be the objective probability vector of the current emergency scenario, where p_l^* is the occurrence probability of the scenario $\mathbb{S}_l$ derived from the reference case statistics. Then p_l^* can be obtained by:

$$p_l^* = \frac{FR_l}{F}, \quad l = 1, 2, \ldots, m, \tag{12}$$

where F is the number of reference cases in Z^{Sim}, $p_l^* \in [0, 1]$ and $\sum_{l=1}^{m} p_l^* = 1$.

3.2 Probability correction method for the PLPR

Using the CBR-based probability estimation method, we can calculate the scenario probability of the current emergency based on the historical events. However, the external environment and interventions between the current emergency and the historical events are usually not the same. In addition, the historical events may also have some scene characteristics or influencing factors that are difficult to obtain or missing. Therefore, it is also necessary to estimate the probability through the subjective judgments of the DMs. Let $\alpha (0 \le \alpha \le 1)$ be the important degree of the subjective probability information. It is desirable that the modified PLPR should not only have more accurate probability information but also take into account the original preference information of the DM. Hence, it is proper to fuse the initial PLPR P and its corresponding objective probability information P^* to get a new PLPR $\bar{P} = \left(\bar{L}(p)_{ij}\right)_{n\times n}$, where each element's probability $\bar{p}_{ij,l}$ is defined as:

$$\bar{p}_{ij,l} = \alpha p_{ij,l} + (1-\alpha) p_l^*, \quad i,j = 1, 2, \ldots, n;\ l = 1, 2, \ldots, m, \tag{13}$$

where $\bar{p}_{ij,l} \in [0, 1]$ and $\sum_{l=1}^{m} \bar{p}_{ij,l} = 1$.

Remark 4 Notice that α also can be seen as a controlling parameter that is determined by the DM, the smaller the value of α, the closer $\bar{P}$ is to P. Obviously, $\bar{P}$ is also a PLPR. Based on the afore-mentioned analysis, we propose a probability correction algorithm of PLPR as follows:

A graph is utilized to illustrate the process of our algorithm as shown in Fig. 1.

Algorithm 1

Input: A PLPR $P=\left(L(p)_{ij}\right)_{n\times n}$;

Output: The modified PLPR $\bar{P}=\left(\bar{L}(p)_{ij}\right)_{n\times n}$.

Step 1. According the attribute indicator vector ζ_k^q, we determine the attribute similarity degree $Sim\left(C_k^*, C_k^q\right)$ between the current event Z^* and the group like historical events Z_q stored in the case database by using Eq. (8).

Step 2. Calculate the global similarity $Sim(Z^*, Z_q)$ according to Eq. (9).

Step 3. Determine the similarity threshold ς by Eq. (10), if the global similarity degree $Sim(Z^*, Z_q) < \varsigma$, then the corresponding historical case Z_q should be removed; Otherwise, the corresponding historical case Z_q should be extracted and put into the set of the reference cases, which is denoted as $Z^{Sim}=\left\{Z_f^{Sim} \middle| f=1,2,\cdots,F\right\}$.

Step 4. For the possible scenario $\mathbb{S}_l$, we calculate RF_l according to Eq. (11) and obtain the corresponding p_l^* by Eq. (12).

Step 5. Let $P=\left(L_{ij,l}\left(p_{ij,l}\right)\right)_{n\times n}$ be a PLPR, which contains the DM's initial judgements, then based on Eq. (13), we can obtain the corrected probability $\bar{p}_{ij,l}$

Step 6. Let $\bar{P}=\left(L_{ij,l}\left(\bar{p}_{ij,l}\right)\right)_{n\times n}$. Then output the probability modified PLPR $\bar{P}=\left(\bar{L}(p)_{ij}\right)_{n\times n}$.

Note The accuracy of the correction can be controlled by the adjusted parameter τ. The probability modified PLPR $\bar{P}$ contains not only the probability information of the initial PLPR P but also the objective probability information P^* derived from the CBR.

4 An emergency decision support method based on the acceptable multiplicative consistent PLPR

Similar to the fuzzy preference relation proposed by Orlovsky [33], the PLPR is based on the assumption that the DMs express their preferences in a logical way, that is, if A is better than B and B is better than C, then A is better than C. However, due to the complexity of decision-making issues, the DMs may provide inconsistent preference information and build inconsistent preference relations. If the consistency is unacceptable, the priority vector derived from the preference relation should be incorrect. Thus, some consistency tests were designed to validate the 'logical' assumption [34–36].

Consistency means that the DMs' preference information cannot be contrary. Many scholars have studied the consistency of linguistic preference relations [37–39]. However, there is no research on the multiplicative consistency of the PLPR. In order to obtain a more reasonable solution, we need to make sure that the PLPR is consistent. As for the inconsistent PLPR, a consistency improving process is required. In this section, we focus on establishing a consistency index for PLPR and provide a consistency improving model to repair the inconsistent PLPR. We then illustrate the use of multiplicative consistency in emergency decision making.

4.1 Multiplicative consistency index of the PLPR

Firstly, we introduce the multiplicative consistency of fuzzy preference relation:

Definition 7 [34]. Let $R=(r_{ij})_{n\times n}$ be a fuzzy preference relation with r_{ij} being the preference degree of the objective x_i over x_j, $r_{ii}=0.5$, and $r_{ij}+r_{ji}=1$. R is said to be consistent or multiplicative consistent if $r_{ie}\cdot r_{ej}\cdot r_{ji}=r_{ei}\cdot r_{je}\cdot r_{ij}$ $(i,j,e=1,2,\dots,n)$, which can be represented as:

$$r_{ij}=\frac{w_i}{w_i+w_j},\forall i,j=1,2,\dots,n, \tag{14}$$

where $w=(w_1,w_2,\dots,w_n)$ is the priority vector of R, satisfying $w_i\in[0,1]$ and $\sum_{i=1}^{n}w_i=1$.

Similar to the above definition, the multiplicative consistency of PLPR can be defined as follows:

Definition 8 For a PLPR $P=\left(L(p)_{ij}\right)_{n\times n}\subset X\times X$, where $L(p)_{ij}=\left\{L_{ij,l}(p_{ij,l})\middle|l=1,2,\dots,\#L(p)_{ij}\right\}$ is a PLE on the LTS $S=\left\{s_\alpha|\alpha=0,1,\dots,2\tau\right\}$ with $\#L(p)_{ij}$ being the number of possible elements in $L(p)_{ij}$. $L_{ij,l}$ is the lth linguistic term in $L(p)_{ij}$ and $p_{ij,l}\in[0,1]$ is the corresponding probability. Then P is multiplicative consistent if

$$E\left(L(p)_{ie}\right)E\left(L(p)_{ej}\right)E\left(L(p)_{ji}\right)=E\left(L(p)_{ei}\right)E\left(L(p)_{je}\right)E\left(L(p)_{ij}\right),\quad i,j,e=1,2,\dots,n, \tag{15}$$

which also can be represented by the following formula:

$$\frac{1}{2\tau}E\left(L(p)_{ij}\right)=\frac{w_i}{w_i+w_j}=\frac{1}{2\tau}\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l},\quad i,j=1,2,\dots,n, \tag{16}$$

where $E\left(L(p)_{ij}\right)$ is the expected value of the PLE $L(p)_{ij}$, $L(p)_{ij}=\left\{L_{ij,l}(p_{ij,l})\middle|l=1,2,\dots,\#L(p)_{ij}\right\}$ is the element of the PLPR P, $r_{ij,l}$ is the subscript of $L_{ij,l}$, and $w=(w_1,w_2,\dots,w_n)$ is the priority vector of P satisfying $w_i\in[0,1]$, and $\sum_{i=1}^{n}w_i=1$.

Theorem 2 If $E\left(L(p)_{ij}\right)=\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}=\frac{2\tau w_i}{w_i+w_j}$ holds, then

$$E\left(L(p)_{ji}\right)=\sum_{l=1}^{\#L(p)_{ji}}\gamma_{ji,l}\cdot p_{ji,l}=\frac{2\tau w_j}{w_i+w_j}. \tag{17}$$

Proof For $\gamma_{ji,l}=2\tau-\gamma_{ij,l}$, $p_{ji,l}=p_{ij,l}$, and $\#L_{ij}=\#L_{ji}=m$, then

$$E\left(L(p)_{ji}\right)=\sum_{l=1}^{\#L(p)_{ji}}\gamma_{ji,l}\cdot p_{ji,l}=\sum_{l=1}^{\#L(p)_{ji}}\left(2\tau-\gamma_{ij,l}\right)\cdot p_{ij,l}=2\tau\sum_{l=1}^{\#L(p)_{ji}}p_{ji,l}-\sum_{l=1}^{\#L(p)_{ji}}\gamma_{ij,l}\cdot p_{ij,l}$$
$$=2\tau-\frac{2\tau w_i}{w_i+w_j}=\frac{2\tau w_j}{w_i+w_j},$$

which completes the proof of Theorem 2.

Theorem 2 shows that if the preference relations in the upper triangular matrix are consistent, the preference relations in the lower triangular matrix of the PLPR are consistent. Therefore, the multiplicative consistency test can be performed on the preference relations in the upper triangular matrix of the PLPR.

Based on Eq. (16), we define a function as:

$$F_{ij}(w)=(w_i+w_j)\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}-2\tau w_i. \tag{18}$$

- If $F_{ij}(w)=0$ for all $i,j=1,2,\dots,n$, then the HFLPR is perfectly multiplicative consistent. we say that the DM satisfies the priority vector with a satisfaction degree of 1.
- If the HFLPR is acceptable consistent, then the DM does not fully satisfy the priority vector. The DM's satisfaction is reduced to some deviation limits.

Actually, the perfectly consistent PLPRs are difficult to construct, so we focus on how to deal with the acceptable consistent PLPR in the decision-making process. For a PLPR, it is called to be acceptable consistent if

$$F_{ij}(w)=(w_i+w_j)\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}-2\tau w_i\approx 0. \tag{19}$$

Thus, to obtain the multiplicative consistent preferences as much as possible, we minimize $F_{ij}(w)$ for all $i<j$. Then, the priorities of the alternatives and the optimal probability distributions of the PLEs can be obtained by solving the following probabilistic linguistic multi-objective programming model:

$$\min F_{ij}(w)=(w_i+w_j)\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}-2\tau w_i$$
$$s.t.\begin{cases}\sum_{i=1}^{n}w_i=1,w_i\geq 0\\ \sum_{l=1}^{\#L(p)_{ij}}p_{ij,l}=1,p_{ij,l}\geq 0\\ i,j=1,2,\ldots,n;i<j\end{cases}\quad (20)$$

The solution of the above problem can be found by solving the following model:

$$\min \mathfrak{I}=\sum_{i=1}^{n-1}\sum_{j=2,j>i}^{n}\left(s_{ij}d_{ij}^{+}+t_{ij}d_{ij}^{-}\right)$$
$$s.t.\begin{cases}(w_i+w_j)\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}-2\tau w_i-s_{ij}d_{ij}^{+}+t_{ij}d_{ij}^{-}=0\\ \sum_{i=1}^{n}w_i=1,w_i\geq 0\\ \sum_{l=1}^{\#L(p)_{ij}}p_{ij,l}=1,p_{ij,l}\geq 0\\ d_{ij}^{+},d_{ij}^{-}\geq 0\\ i,j=1,2,\ldots,n;i<j\end{cases},\quad (21)$$

where d_{ij}^{+} and d_{ij}^{-} are respectively the positive deviation and the negative deviation relative to the goal $F_{ij}(w)$, s_{ij} and t_{ij} are respectively the weights corresponding to d_{ij}^{+} and d_{ij}^{-}. Without loss of generality, we assume that all goals $F_{ij}(w)$ $(i,j=1,2,\ldots,n;\quad i<j)$ are fair, namely, $s_{ij}=t_{ij}=1$ $(i,j=1,2,\ldots,n;\quad i<j)$. Thus, Eq. (21) is equal to the following model:

$$\min \mathfrak{I}=\sum_{i=1}^{n-1}\sum_{j=2,j>i}^{n}\left(d_{ij}^{+}+d_{ij}^{-}\right)$$
$$s.t.\begin{cases}(w_i+w_j)\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}-2\tau w_i-d_{ij}^{+}+d_{ij}^{-}=0\\ \sum_{i=1}^{n}w_i=1,w_i\geq 0\\ \sum_{l=1}^{\#L(p)_{ij}}p_{ij,l}=1,p_{ij,l}\geq 0\\ d_{ij}^{+},d_{ij}^{-}\geq 0\\ i,j=1,2,\cdots,n;i<j\end{cases}.\quad (22)$$

Solving Eq. (22), we can obtain the positive deviations $d_{ij}^{+}(i,j=1,2,\ldots,n)$ and the negative deviations $d_{ij}^{-}(i,j=1,2,\ldots,n)$ of the PLPRs $P=\{L_{ij,l}(p_{ij,l})|l=1,2,\ldots,\#L(p)_{ij}\}$. Then, the PLPR is perfectly multiplicative consistent if $\mathfrak{I}=0$ in Eq. (22). In practice, the decision making environment is usually of great complexity and uncertainty. The perfectly consistent PLPR is hard to achieve, and the priority vector derived from the PLPR is possibly incorrect if the consistency is unacceptable. Therefore, in this following, the multiplicative consistency index (CI_M) is defined to measure the multiplicative consistent degree of the PLPR, and then an iterative optimization algorithm is provided to improve the multiplicative consistency of the PLPR.

Definition 9 If $P=\left\{L_{ij,l}(p_{ij,l})\middle|l=1,2,\ldots,\#L(p)_{ij}\right\}$ $(i,j=1,2,\cdots,n)$ is a PLPR, where $\#L(p)_{ij}$ is the number of possible elements in $L(p)_{ij}$, d_{ij}^{+} and d_{ij}^{-} are respectively the positive deviation and the negative deviation obtained by Eq. (22), then the multiplicative consistency index of P can be defined as:

$$CI_M(P)=\frac{2\sum_{i=1}^{n-1}\sum_{j=2,j>i}^{n}\left(d_{ij}^{+}+d_{ij}^{-}\right)}{n(n-1)}.\quad (23)$$

Theorem 3 For the multiplicative consistency index $CI_M(P)$, if $CI_M(P)=0$, then the PLPR P is perfectly multiplicative consistent.

Proof If $CI_M(P)=0$, then $\sum_{i=1}^{n-1}\sum_{j=2,j>i}^{n}(d_{ij}^{+}+d_{ij}^{-})=0$. Since $d_{ij}^{+}\geq 0$ and $d_{ij}^{-}\geq 0$, we can get $d_{ij}^{+}=d_{ij}^{-}=0$, where $i,j=1,2,\ldots,n$ and $i<j$.

In Eq. (22), for $(w_i+w_j)\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}-2\tau w_i=0$ $(i,j=1,2,\ldots,n;\ i\leq j)$, we have $\sum_{l=1}^{\#L(p)_{ij}}\gamma_{ij,l}\cdot p_{ij,l}=\frac{2\tau w_i}{(w_i+w_j)}$, i.e., $E\left(L(p)_{ij}\right)=\frac{2\tau w_i}{(w_i+w_j)}$.

According to Definition 12, we can conclude that the PLPR is of the perfectly multiplicative consistency, which completes the proof of Theorem 3.

Theorem 3 demonstrates that the smaller the value of $CI_M(P)$ is, the better the multiplicative consistency level of P would be. In general, if $CI_M(P)\leq 0.01$, then the multiplicative consistency of P is acceptable; Otherwise, the multiplicative consistency of P is unacceptable.

4.2 Emergency decision support method with the acceptable consistent PLPR

The acceptable consistent PLPR indicates that the preference information provided by the DM is basically consistent. However, it is possible that $CI_M(P)\geq 0.01$. In such a case, the multiplicative consistency level of P should be improved. In this section, we establish an emergency decision support method with acceptable consistent PLPR. The probability correction is first considered to obtain more accurate occurrence probabilities of elements in PLEs. After that, to be more logical and get rid of confusion, we put forward a consistency improving algorithm to repair the unacceptable consistent PLPR into acceptable consistent one. This method is suitable for emergency decision making as it provides support for the vagueness and uncertain characteristics of emergency information. Meanwhile, the method takes into consideration the possible evolutions of disaster scenarios. The steps of the algorithm are summarized as follows:

Figure 2 is provided to illustrate the above calculation process.

Algorithm 2

Input: A PLPR $P=\left(L(p)_{ij}\right)_{n\times n}$ of alternatives;

Output: The ranking of the alternatives.

Step 1. Construct the PLPR decision matrix $P=\left(L(p)_{ij}\right)_{n\times n}\subset X\times X$ with respect to the emergency evolution and the occurring possibilities of different emergency scenarios.

Step 2. Use Algorithm 1 to get the probability modified PLPR $\bar{P}=\left(\bar{L}(p)_{ij}\right)_{n\times n}$, where $\bar{L}(p)_{ij}=\left\{L_{ij,l}\left(\bar{p}_{ij,l}\right)\middle| l=1,2,\cdots,m\right\}$ $(i,j\in 1,2,\cdots,n)$ is a PLE of $\bar{P}$.

Step 3. Use Eq. (22) to calculate the positive deviations $d_{ij}^{+}\ (i,j=1,2,\cdots,n)$ and the negative deviations $d_{ij}^{-}\ (i,j=1,2,\cdots,n)$ of the modified PLPR $\bar{P}=\left(\bar{L}(p)_{ij}\right)_{n\times n}$.

Step 4. Use Eq. (23) to calculate the multiplicative consistency index $CI_M\left(\bar{P}\right)$ for $\bar{P}=\left(\bar{L}(p)_{ij}\right)_{n\times n}$ $(i,j=1,2,\cdots,n)$.

Step 5. If $CI_M(P)\le 0.01$, which indicates that the multiplicative consistency of P is acceptable, then go to step 7; otherwise, continue to the next step.

Step 6. Calculate the adjusted value d^* according to

$$d^*=\left\{d_{ij}^{+}>0,d_{ij}^{-}>0\middle| i=1,2,\cdots,n-1;\ j=2,3,\cdots,n; i<j\right\} \tag{24}$$

For a given adjusted parameter $\sigma\ (0\le\sigma\le 1)$:

- If $d^*=d_{ij}^{-}$, then the modified elements can be calculated by $\gamma_{ij,l}^{*}=\gamma_{ij,l}^{\sigma}\cdot\left(\gamma_{ij,l}+d_{ij}^{-}\right)^{1-\sigma}$, where $\gamma_{ij,l}$ is the subscript of the lth element in $\bar{L}(p)_{ij}$;
- If $d^*=d_{ij}^{+}$, then the modified elements can be calculated by $\gamma_{ij,l}^{*}=\gamma_{ij,l}^{\sigma}\cdot\left(\gamma_{ij,l}-d_{ij}^{-}\right)^{1-\sigma}$, where $\gamma_{ij,l}$ is the subscript of the lth element in $\bar{L}(p)_{ij}$.

By this step, we can obtain the consistency improved PLPR $\bar{P}^{m*(\theta)}=\left(\bar{L}^{*}(p)_{ij}^{m(\theta)}\right)_{n\times n}$, where $\bar{L}^{*}(p)_{ij}=\left\{L_{ij,l}^{*}\left(\bar{p}_{ij,l}\right)\middle| l=1,2,\cdots,m\right\}$ is a PLE of $\bar{P}^{m*(\theta)}$. θ indicates the number of iterations. Let $\theta=\theta+1$, and return to Step 3.

Step 7. Output w_i and the ranking of the alternatives.

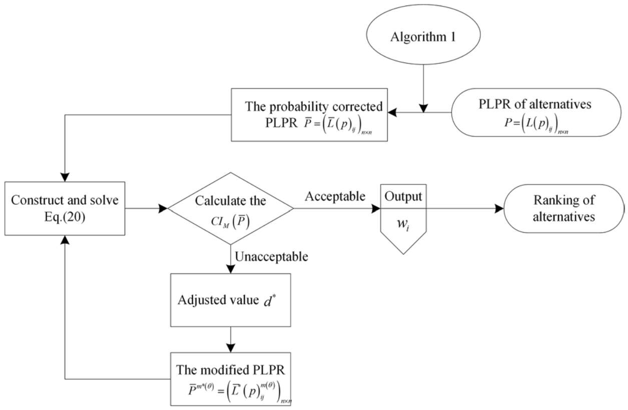

Fig. 2 The calculation process of the multiplicative consistency-based method

It is worth pointing out that through this algorithm, we can improve the multiplicative consistency of any PLPR automatically without losing much original information. The procedure can improve the inconsistent PLPR without the participation of the DM. Thus, it can reach a quick decision and show some advantages in emergency decision making. This algorithm is convergent, and the derived $\bar{P}^{m^*(\theta)}$ has weak transitivity.

5 Illustrative example: fire decision support system

5.1 The application of the proposed method

In this section, we apply the above method to an application in emergency decision support for a fire accident in the petrochemical plant under the probabilistic linguistic environment. A petrochemical plant irregularity does the "hydrogen desulfurization agent" operations on the crude oil pipeline and continued filling when the tanker stopped unloading. It causes "hydrogen desulfurization agent" in the oil pipeline local enrichment. Strong oxidation reaction, resulting in an oil pipeline explosion, causes a fire. Since the characteristics of the petrochemical plant are prone to chain reaction, the commander of the fire department holds an emergency meeting and the decision must be "one-time equipped with adequate fire power, rapid response, timely rescue, cooling and fire extinguishing" principle, which aims to quickly and effectively control the situation to prevent further expand deteriorate. Suppose that the emergency decision support system intends to select the best one from four emergency response alternatives, denoted as A_1, A_2, A_3 and A_4. These four alternatives are:

A_1 Set out five foam fire engines, three low pressure water tank fire engines for cooling the surrounding tank and an ambulance.

A_2 Set out five foam fire engines, three low pressure water tank fire engines, three high-power foam fire engines, three high pressure water tanker fire engines, and an ambulance.

A_3 Set out five foam fire engines, three low pressure water tank fire engines, three high-power foam fire engines, three high pressure water tanker fire engines, two lift-up fire engines, and an ambulance.

A_4 Set out five foam fire engines, three low pressure water tank fire engines, three high-power foam fire engines, three high pressure water tanker fire engines, two lift-up fire engines, a fire helicopter, and two ambulances.

By consulting the relevant experts in the field of fire, and with the help of fire emergency support system, three possible scenarios for the fire were identified as follows:

$\mathbb{S}_1$ This is a fire in the independent area.

$\mathbb{S}_2$ Under the influence of the fire in the independent area, resulting in the adjacent tank was detonated, making the adjacent areas appeared fire.

$\mathbb{S}_3$ A fire broke out in the entire chemical plant.

Due to the difficulties of information collection and the uncertainties in situation evolution, it is hard to give the exact numerical values of each emergency alternative under different scenarios. Suppose that the experts provide the assessment values of the emergency response alternatives $A_i (i = 1, 2, 3, 4)$ with their subjective estimation of the possible probability for the scenarios. All the assessment values are represented by the PLEs, which are based on the given LTS $S = \{s_0 : \text{very poor},\ s_1 : \text{poor}, s_2 : \text{slightly poor},\ s_3 : \text{fair},\ s_4 : \text{slightly good}, s_5 : \text{good},\ s_6 : \text{very good}\}$. Then the PLPR $P = \left(L(p)_{ij}\right)_{4\times 4}$ can be constructed (see Table 1), which considers the occurrence possibilities of the different emergency scenarios:

Why we ask the experts to provide the PLPR? There are two reasons for this choice. Firstly, the PLPR is more flexible than other preference relations that the DMs can give their judgements by pairwise comparison of alternatives using linguistic variables and their corresponding probabilities. Secondly, the PLPR possesses the ability of comprehensively portray the uncertain probability of multiple emergency scenarios. The process to determine the emergency response alternatives in fire accident can be presented as:

Step 1 By retrieving the information in the database, there are about 36 kinds of similar petrochemical plant fire related information which were collected at home and abroad, i.e.,

Table 1 The PLPR between alternatives

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_1(0.4), s_3(0.4), s_4(0.2)\}$	$\{s_1(0.3), s_3(0.3), s_4(0.4)\}$	$\{s_2(0.3), s_4(0.4), s_5(0.3)\}$
A_2	$\{s_5(0.4), s_3(0.4), s_2(0.2)\}$	$\{s_3(1)\}$	$\{s_2(0.2), s_3(0.4), s_4(0.4)\}$	$\{s_2(0.3), s_3(0.3), s_5(0.4)\}$
A_3	$\{s_5(0.3), s_3(0.3), s_2(0.4)\}$	$\{s_4(0.2), s_3(0.4), s_2(0.4)\}$	$\{s_3(1)\}$	$\{s_2(0.2), s_3(0.4), s_5(0.4)\}$
A_4	$\{s_4(0.3), s_2(0.4), s_1(0.3)\}$	$\{s_4(0.3), s_3(0.3), s_1(0.4)\}$	$\{s_4(0.2), s_3(0.4), s_1(0.4)\}$	$\{s_3(1)\}$

$\mathrm{Z} = \left\{ \mathrm{Z}_q \middle| q = 1, 2, \ldots, 36 \right\}$. Taking the historical events Z_1, Z_2 and the current fire accident Z^* as examples. Consider the following criteria $C_k (k = 1, 2, \ldots, 7)$: the fire intensity size (C_1), meteorological conditions (C_2), intensity degree of the surrounding buildings (C_3), intensity degree of staff (C_4), visibility level (C_5), escape evacuation routes (C_6), and completeness of the fire-fighting facilities (C_7). The weight vector of these seven criteria is $\omega = (0.13, 0.17, 0.14, 0.16, 0.11, 0.15, 0.14)^T$. Since these criteria are all qualitative, it is convenient and feasible to contrast the attribute values by HFLTSs, which are based on the given LTS $S = \{s_0$: terrible, s_1 : very bad, s_2 : bad, s_3 : medium, s_4 : well, s_5 : very well, s_6 : perfect$\}$.

According the attribute indication vector ζ_k^1, we can calculate the attribute similarity degree $Sim(C_k^*, C_k^1)$ and $Sim(C_k^*, C_k^2)$ for $k = 1, 2, \ldots, 7$ of the current event Z^* and the historical event $\mathrm{Z}_1, \mathrm{Z}_2$ on each criterion C_k based on Eq. (8), as shown in Tables 2 and 3.

Step 2 Based on Tables 2 and 3, we utilize Eq. (9) to determine the global similarity between the current fire accident Z^* and the historical events $\mathrm{Z}_1, \mathrm{Z}_2$, respectively:

$Sim(\mathrm{Z}^*, \mathrm{Z}_1) = \sum_{k=1}^{7} \omega_k \cdot Sim(C_k^*, C_k^1) = 0.748$; $Sim(\mathrm{Z}^*, \mathrm{Z}_2) = \sum_{k=1}^{7} \omega_k \cdot Sim(C_k^*, C_k^2) = 0.876$.

Repeat Step 1 and Step 2 to get all $Sim(\mathrm{Z}^*, \mathrm{Z}_q)$. Then, we obtain $\max\{Sim(\mathrm{Z}^*, \mathrm{Z}_q)\} = 0.876$, where $q = 1, 2, \ldots, 36$.

Step 3 Observe the similarity between historical cases, the DMs determine the similarity threshold as $\varsigma = 0.8$. Then, 8 historical cases are removed, whose global similarity degrees are less than 0.8. The other 28 historical cases are extracted and put into the set of the reference cases, denoted as $\mathrm{Z}^{Sim} = \left\{ \mathrm{Z}_f^{Sim} \middle| f = 1, 2, \ldots, 28 \right\}$.

Step 4 For the three possible scenarios $\mathbb{S}_1$, $\mathbb{S}_2$ and $\mathbb{S}_3$, based on Eq. (11) we have $FR_1 = 4, FR_2 = 6, FR_2 = 18$ and the corresponding $p^* = (0.14, 0.22, 0.64)$ can be obtained by Eq. (12).

Step 5 Let $\alpha = 0.5$, which means that the subjective and objective probabilities are equally important. Thus, based on Eq. (13), we can get the modified PLPR $\bar{P} = \left(L_{ij,l}(\bar{p}_{ij,l})\right)_{4\times4}$ (presented in Table 4), where $\bar{p}_{ij,l} = 0.5p_l^* + 0.5p_{ij,l}$ $\quad (i,j = 1, 2, 3, 4;\ l = 1, 2, 3)$.

Step 6 For the modified $\bar{P} = \left(\bar{L}(p)_{ij}\right)_{4\times4} (i,j = 1, 2, \ldots, 4)$, we construct the optimization model based on Eq. (22), and thus we get $d_{12}^+ = d_{12}^- = d_{13}^- = d_{14}^+ = d_{14}^- = d_{23}^- = d_{24}^- = d_{34}^+ = d_{34}^- = 0$, $d_{13}^- = 0.0533$, $d_{23}^+ = 0.0302$ and $d_{24}^- = 0.1429$.

Step 7 Use Eq. (23) to calculate the consistency index $CI_M(\bar{P}) = 0.0377 > 0.01$, which implies that the multiplicative consistency of $\bar{P}$ is unacceptable. To get the more reasonable decision- making results, the multiplicative consistency of $\bar{P}$ should be improved.

Step 8 Calculate the adjusted value $d^* = \{d_{13}^-, d_{23}^+, d_{24}^-\} = \{0.0533, 0.0302, 0.1429\}$, the number of iterations and the accuracy of the modification can be controlled by the adjusted parameter σ. Without loss of generality, let $\sigma = 0.1$. Then we have $\gamma_{13,l}^* = \gamma_{13,l}^{0.1} \cdot \left(\gamma_{13,l} + d_{13}^-\right)^{0.9}$, $\gamma_{23,l}^* = \gamma_{23,l}^{0.1} \cdot \left(\gamma_{23,l} - d_{23}^+\right)^{0.9}$ and $\gamma_{24,l}^* = \gamma_{24,l}^{0.1} \cdot \left(\gamma_{24,l} + d_{24}^-\right)^{0.9}$

Table 2 The attribute similarity degree between Z^* and Z_1

Attribute	Z_1	Z^*	ω_k	ζ_k^1	$Sim(C_k^*, C_k^1)$
C_1	$\{s_2, s_3\}$	$\{s_1, s_2\}$	0.13	1	0.857
C_2	$\{s_3, s_4\}$	$\{s_4, s_5\}$	0.17	1	0.857
C_3	$\{s_3, s_4\}$	$\{s_1, s_2\}$	0.14	1	0.714
C_4	$\{s_4, s_5\}$	$\{s_1, s_2\}$	0.16	1	0.571
C_5	–	$\{s_2, s_3\}$	0.11	0	0
C_6	$\{s_4, s_5\}$	$\{s_4, s_5\}$	0.15	1	1
C_7	$\{s_5, s_6\}$	$\{s_5, s_6\}$	0.14	1	1

Table 3 The attribute similarity degree between Z^* and Z_2

Attribute	Z_2	Z^*	ω_k	ζ_k^1	$Sim(C_k^*, C_k^2)$
C_1	$\{s_1, s_2\}$	$\{s_1, s_2\}$	0.13	1	1
C_2	$\{s_3, s_4\}$	$\{s_4, s_5\}$	0.17	1	0.857
C_3	$\{s_3, s_4\}$	$\{s_1, s_2\}$	0.14	1	0.714
C_4	$\{s_2, s_3\}$	$\{s_1, s_2\}$	0.16	1	0.857
C_5	$\{s_3, s_4\}$	$\{s_2, s_3\}$	0.11	0	0.857
C_6	$\{s_5, s_6\}$	$\{s_4, s_5\}$	0.15	1	0.857
C_7	$\{s_5, s_6\}$	$\{s_5, s_6\}$	0.14	1	1

Table 4 The modified PLPR $\bar{P} = \left(L_{ij,l}(\bar{p}_{ij,l})\right)_{4\times4}$

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_1(0.27), s_3(0.31), s_4(0.42)\}$	$\{s_1(0.22), s_3(0.26), s_4(0.52)\}$	$\{s_2(0.22), s_4(0.31), s_5(0.47)\}$
A_2		$\{s_3(1)\}$	$\{s_2(0.17), s_3(0.31), s_4(0.52)\}$	$\{s_2(0.22), s_3(0.26), s_5(0.52)\}$
A_3			$\{s_3(1)\}$	$\{s_2(0.17), s_3(0.31), s_5(0.52)\}$
A_4				$\{s_3(1)\}$

Table 5 The consistency improved PLPR $\bar{P}^{m^*(1)} = \left(\bar{L}^*(p)_{ij}\right)^{m^*(1)}$

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_1(0.27), s_3(0.31), s_4(0.42)\}$	$\{s_{1.05}(0.22), s_{3.05}(0.26), s_{4.05}(0.52)\}$	$\{s_2(0.22), s_4(0.31), s_5(0.47)\}$
A_2		$\{s_3(1)\}$	$\{s_{1.97}(0.17), s_{2.97}(0.31), s_{3.97}(0.52)\}$	$\{s_{2.12}(0.22), s_{3.12}(0.26), s_{5.12}(0.52)\}$
A_3			$\{s_3(1)\}$	$\{s_2(0.17), s_3(0.31), s_5(0.52)\}$
A_4				$\{s_3(1)\}$

$(l = 1, 2, 3)$. The consistency improved PLPR $\bar{P}^{m^*(1)} = \left(\bar{L}^*(p)_{ij}\right)^{m^*(1)}$ $(i, j = 1, 2, 3, 4)$ can be obtained as Table 5.

Step 9 Repeat Steps 6–8 to improve the multiplicative consistency of $\bar{P}^{m^*(1)}$ until we get $CI_M\left(\bar{P}^{m^*(3)}\right) = 0.0066 < 0.01$, which shows that the consistency of $\bar{P}^{m^*(3)}$ is acceptable and the consistency improved PLPR $\bar{P}^{m^*(3)}$ can be shown in Table 6.

Step 10 Output the priority vector derived from the acceptable consistent PLPR $\bar{P}^{m^*(3)}$, i.e., $w = (0.29, 0.31, 0.25, 0.14)^T$. Then the ranking of the alternatives can be determined as $A_2 \succ A_1 \succ A_3 \succ A_4$. Thus, A_2 is selected as the most appropriate response action in this emergency event.

5.2 Comparative analyses and discussions

To show the rationality of our proposed method in handing the emergency decision making problems, below we perform some comparative analyses with the result based only on the subjective preference information. For the above example, if we directly use the multiplicative consistency-based method according to Table 1 without the probability correction process, then we can get the priority weight vector $w^* = (0.24, 0.31, 0.27, 0.18)^T$ according to Algorithm 2, i.e., $A_2 \succ A_3 \succ A_1 \succ A_4$. The changes of the priority values can be shown in Fig. 3.

From Fig. 3, we find that A_2 is still the most appropriate response action in the emergency, while A_4 is still the most inappropriate response. It illustrates the validity and applicability of the proposed method, that is, the actual calculation results are basically consistent with the subjective cognition of people. However, it can be seen that the orderss of A_1 and A_3 are changed. The reason for this may be that the evolution of emergencies is always of great complexity and uncertainty, and it is difficult for the DMs to provide exact occurrence probabilities of all possible emergency scenarios. By using the probability correction method based on the CBR, we can obtain more accurate and reasonable occurrence probabilities of the elements in the PLEs. According to the revised PLPR, we may get more reliable results.

In fact, the PLPR is an extended expression form of HFLPR. To justify that using the PLPRs to express the DMs' preferences is rational, we use the proposed multiplicative consistency-based method to cope with the same emergency decision making problem under the hesitant fuzzy linguistic environment that the occurrence probabilities of multiple

Table 6 The consistency improved PLPR $\bar{P}^{m^*(3)} = \left(\bar{L}^*(p)_{ij}\right)^{m^*(3)}$

	A_1	A_2	A_3	A_4
A_1	$\{s_3(1)\}$	$\{s_1(0.27), s_3(0.31), s_4(0.42)\}$	$\{s_{1.08}(0.22), s_{3.08}(0.26), s_{4.08}(0.52)\}$	$\{s_2(0.22), s_4(0.31), s_5(0.47)\}$
A_2		$\{s_3(1)\}$	$\{s_{1.95}(0.17), s_{2.95}(0.31), s_{3.95}(0.52)\}$	$\{s_{2.25}(0.22), s_{3.25}(0.26), s_{5.25}(0.52)\}$
A_3			$\{s_3(1)\}$	$\{s_2(0.17), s_3(0.31), s_5(0.52)\}$
A_4				$\{s_3(1)\}$

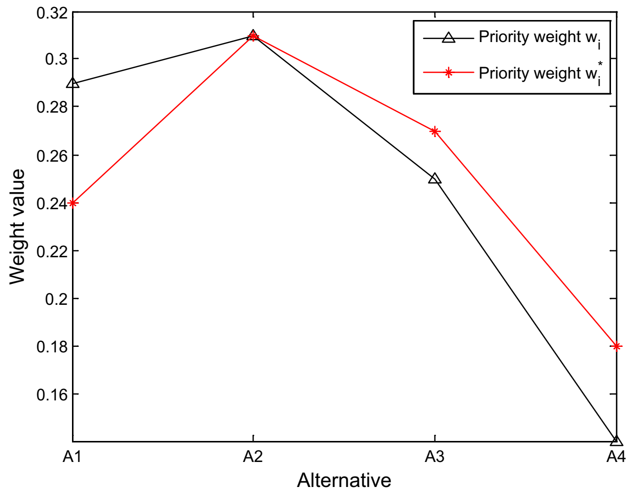

Fig. 3 Comparison of the results considering different preference information

scenarios are not considered. We remove the probabilities, and then the PLPR in Table 1 changes to the HFLPR as in Table 7.

According to Algorithm 2, we can get the ranking of the four emergency response alternatives from the HFLPR. The decision results derived from the PLPR and the HFLPR can be compared by Table 8.

As it can be seen from Table 8, the emergency response plans A_2 and A_3, A_1 and A_4 have the same priority orders derived from the HFLPR. Therefore, we do not know which one is the best decision. The reason for this is that it loses some information in the hesitant fuzzy linguistic environment. It takes the occurring probabilities of the possible emergency scenarios to be equal. However, the evolution of emergencies may lead to multiple emergency scenarios. The uncertainty of emergency scenarios determines that it is difficult for the experts to provide desirable evaluations especially when they face various options. The probability interpretation plays an important part in understanding the DMs' behaviors. Thus, the emergency decision making method under the probabilistic linguistic environment is more suitable for the evaluation of emergency response alternatives as it can simulate the emergency environment without loss of information.

Table 8 The decision results derived from the PLPR and the HFLPR

Plan method	A_1	A_2	A_3	A_4	Ranking
HFLPR	0.22	0.28	0.28	0.22	$A_2 = A_3 \succ A_1 = A_4$
PLPR	0.29	0.31	0.26	0.14	$A_2 \succ A_1 \succ A_3 \succ A_4$

Compared with the traditional emergency decision making methods [40–43], the decision making method with multiplicative consistent PLPR has the following advantages:

1. The PLPR can depict numerous linguistic preference relations and their corresponding occurring probabilities, which is reasonable in the situation where the uncertain evolution of emergency may lead to multiple scenarios.
2. The CBR-based probability correction method is a useful computer-aided tool. We can obtain the more accurate and more reasonable occurrence probabilities of the elements in the PLEs.
3. The inconsistent and paradoxical PLPRs can be improved with a consistent improving algorithm, which leads to the more reasonable results.

6 Concluding remarks

To address the evolution of emergencies, this paper has presented a decision support method with acceptable multiplicative consistent PLPRs. Actually, it is unpractical to provide the accurate probability values to describe the occurring possibilities of the different emergency scenarios. To get the more feasible and dependable results, this paper has focused on the probability correction and consistency improving for the PLPRs. Firstly, this paper has defined the multiplicative consistency of PLPR and proven that it is superior to the previously proposed additive consistency. Then, we have proposed the probability correction method based on the CBR. The more credible PLPRs have been obtained, which consider the subjective and objective probability information simultaneously. However, it is possible that the probability modified PLPRs can be inconsistent and some preference relations can be paradoxical. To address this issue, this paper has further constructed the multiplicative consistency

Table 7 The HFLPR between alternatives

	A_1	A_2	A_3	A_4
A_1	$\{s_3\}$	$\{s_1, s_3, s_4\}$	$\{s_1, s_3, s_4\}$	$\{s_2, s_4, s_5\}$
A_2	$\{s_5, s_3, s_2\}$	$\{s_3\}$	$\{s_2, s_3, s_4\}$	$\{s_2, s_3, s_5\}$
A_3	$\{s_5, s_3, s_2\}$	$\{s_4, s_3, s_2\}$	$\{s_3\}$	$\{s_2(0.2), s_3(0.4), s_5(0.4)\}$
A_4	$\{s_4(0.3), s_2(0.4), s_1(0.3)\}$	$\{s_4(0.3), s_3(0.3), s_1(0.4)\}$	$\{s_4(0.2), s_3(0.4), s_1(0.4)\}$	$\{s_3(1)\}$

index $CI_M(P)$ to evaluate the multiplicative consistency of the probability modified PLPRs, and then provided an iterative optimization algorithm to improve the multiplicative consistency of the unacceptable ones. Finally, an example based on the fire accident of the petrochemical plant has been addressed, which has demonstrated the feasibility and the effectiveness of the proposed method in emergency decision making.

In the future, we may investigate the cognitive and psychological differences of DMs in the emergency decision-making process. When the occurrence probabilities of the elements in the PLPR are unknown or incomplete, what we should do to deal with it is still a question to be studied.

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