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Exponential stability for the neutral-type singular neural network with time-varying delays

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Abstract
This paper deals with the problem of global exponential stability for neutral type singular networks with time-varying delays. Delay-dependent criterion is proposed to guarantee the exponential stability of neutral type singular networks via linear matrix inequality (LMI) approach. Improved global exponential stability condition is derived by employing a new Lyapunov–Krasovskii functional and a rarely integral inequality. The developed result is less conservative than previous published ones in the literature, which is illustrated by representative numerical examples.

Keywords Neutral type descriptor system · Neural network · Exponential stability · Linear matrix inequality

1 Introduction

Over the recent decades there have been extensive studies of neural networks (NNs) including time-varying delays. The existence of time delays may result in instability or oscillation of a neural network system. Therefore, the stability of neural networks with time delay has been attracting the interest of a great number of researchers and has obtained some achievements [1–10], and the references cited therein. In general, neural networks model is similar to the general dynamical system with time-delay, and the condition of the stability of time-delay neural network can be divided into two parts: delay-independent conditions and delay-dependent conditions [11–13]. At present, there are a lot of studies, which focus on the delay-dependent conditions, because the delay-dependent case is less conservative than delay-independent case. Chen [14] obtained the delay-dependent exponential stability for a class of linear neural network condition with the technique of model transformation. A less conservative delay-dependent condition is proposed,

through establishing a new Lyapunov-functional and applying S-procedure in [15]. What's more, in order to reduce the conservatism of the stability criterion, the authors utilized the method of free weighting matrices in [16]. In some of earlier articles, a lot of important parts are ignored, such as the process in the derivative of Lyapunov function and the calculation of the upper limit for the derivative. However, in [17–19], the authors considered these items and obtained a less conservative for network systems.

Obviously, the neutral type descriptor system is more general than neutral system and descriptor system. Many practical processes can be modeled as general neutral type descriptor system, such as circuit analysis, computer aided design, real-time simulation of mechanical systems and optimal control [20, 21]. The neutral type descriptor systems contain delays in both its states and the derivatives of states. At the same time, it has the nature of the regular, impulse free, which lead to the search of the neutral-type singular neural network system is more hard to find the criterion of the stability. Recently, the neutral type descriptor system with time-delay has been studied in [22–25], and some valuable results are obtained. In [23, 24], the authors studied the stability of neutral type descriptor system with mixed delays and multiple time-varying delays, respectively. Han et al. [25] investigated the delay-dependent robust stability for uncertain singular neutral differential system with time-varying and distributed delays. In [26–28], Synchronization of singular complex dynamical networks with time-varying delays has been studied. However, the research for

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exponential stability of neutral type singular networks system is still very rare.

Motivated by the above discussion, in this paper, we consider the problem of the exponential stability of neutral type singular networks system. The main contributions of this paper are summarized as follows: (1) this paper is based on predecessors' research about the neural network system, a typical network model is considered. (2) A new construction method of Lyapunov functional is adopted. (3) A new stability theorem about singular neutral system is obtained. (4) The simulation results demonstrate the feasibility of the method.

In addition, the liberty of matrix and integral inequality methods are applied, which ensure that the neutral type singular neural network system is globally exponential stability with a unique equilibrium point. These conditions are expressed in terms of linear matrix inequality, which can be solved by using developed techniques in Matlab toolbox. Finally, numerical examples are provided to show the less conservatism and valid of the proposed LMI conditions.

2 System description and assumptions

Consider the neutral type descriptor neural networks with varying delays described by:

$$\begin{cases} E\dot{x}(t) = -Ax(t) + B\dot{x}(t - \tau(t)) + Cf(x(t)) + Df(x(t - \tau(t))) + J, \\ x(t) = \phi(t), \quad t \in [-\tau, 0]. \end{cases} \tag{1}$$

where $x(t) = \left[x_1(t)\, x_2(t) \cdots x_n(t)\right]^T \in R^n$ is the neuron state vector; the integer $n \geq 2$ denotes the number of units in neural network; $J = \left[J_1\, J_2 \cdots J_n\right]^T \in R^n$ is a constant input vector; $A = \text{diag}\{a_1,\, a_2, \cdots a_n\} \in R^n$ denotes the diagonal matrix of passive decay rates, and satisfying $a_i > 0$ for $i = 1, 2, \cdots, n$; $\dot{x}(t - \tau(t)) = \left[\dot{x}_1(t - \tau(t))\, \dot{x}_2(t - \tau(t)) \cdots \dot{x}_n(t - \tau(t))\right]^T \in R^n$ is the neutral item; E is a singular matrix and satisfy $rank(E) = r \leq n$; A, B, C and $D \in R^{n \times n}$ are the interconnection matrices representing the weight coefficient of the neurons. $\tau(t)$ is time-varying continuously function satisfies $0 \leq \tau(t) \leq \tau$, $0 < \dot{\tau}(t) \leq \bar{\tau}$, τ and $\bar{\tau}$ are constants. The initial vector $\phi(\cdot)$ is bounded and continuously differential on $[-\tau, 0]$.

Now letting $\tilde{x} = \left[\tilde{x}_1\, \tilde{x}_2 \cdots \tilde{x}_n\right]^T \in R^n$ is an equilibrium point of system (1), which is

$$\tilde{x}^*(t) \equiv 0, \ \tilde{x}^*(t - \tau(t)) \equiv 0.$$

Implying from (1) that

$$0 = -Ax^*(t) + Cf(x^*(t)) + Df(x^*(t - \tau(t))) + J.$$

Introducing the state deviation from equilibrium

$$z(t) = x(t) - \tilde{x} = \left[z_1(t)\, z_2(t) \cdots z_n(t)\right]^T.$$

So the system (1) is equivalent to the following:

$$\begin{cases} E\dot{z}(t) = -Az(t) + B\dot{z}(t - \tau(t)) + Cg(z(t)) + Dg(z(t - \tau(t))) \\ z(t) = \phi(t) - \tilde{x}, t \in [-\tau, 0], \end{cases} \tag{2}$$

where

$$z(t) = [z_1(t)\, z_2(t) \cdots z_n(t)]^T,$$
$$z(t) = [g_1(z_1(t)) \quad g_2(z_2(t)) \cdots g_n(z_n(t))]^T,$$
$$\dot{z}(t - \tau(t)) = [\dot{z}_1(t - \tau(t))\, \dot{z}_2(t - \tau(t)) \cdots \dot{z}_n(t - \tau(t))]^T$$
$$= \dot{x}(t - \tau(t)) - \tilde{x},$$
$$g(z(t - \tau(t))) = [g_1(z_1(t - \tau(t))) \cdots g_n(z_n(t - \tau(t)))]^T,$$
$$g_i(z_i(t)) = f_i(x_i(t)) - f_i(\tilde{x}_i) = f_i(z_i(t) - \tilde{x}_i)$$
$$- f_i(\tilde{x}_i), g_i(0) = 0,$$
$$g_i(z_i(t - \tau(t))) = f_i(x(t - \tau(t))) - f_i(\tilde{x}_i)$$
$$= f_i(z_i(t - \tau(t)) - \tilde{x}_i) - f_i(\tilde{x}_i).$$

Assumption 2.1 The nonlinear function $f : R^n \to R^n$ satisfies

$$\left[f(x) - f(y) - U(x - y)\right]^T \left[f(x) - f(y) - V(x - y)\right] \leq 0, \forall x, y \in R^n, \tag{3}$$

where U and V are constant matrices with appropriate dimensions.

Definition 2.1 Consider the linear system (2), if there exist two scalars $\varepsilon > 0$ and $\gamma \geq 1$ such that

$$\|z(t)\| \leq \gamma e^{-\varepsilon t} \|z\|_s, \quad t \geq 0,$$

Let $\|z\|_s = \sup_{-d \leq s \leq 0} \sqrt{\|z(s)\|^2 + \|\dot{z}(s)\|^2}$, then the system (2) is exponentially stable, where ε is called the exponential convergence rate, in which $\|\cdot\|$ denotes the Euclidean norm.

Definition 2.2 ([29]) The systems (2) is said to be regular and impulse-free, if and only if the pair (E, A) is said to be regular, impulse free.

Before proceeding with the main results, several lemmas are necessary.

Lemma 2.3 ([30]) For any scalars $\tau_2 > \tau_1 > 0$ and any constant matrix $Z > 0$, if there are a vector function $\xi(t) : \left[\tau_1, \tau_2\right] \to R^m$ such that the integrations in the following are well-defined, then

$$-\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \xi^T(t) Z \xi(t) \mathrm{d}s \mathrm{d}\theta \leq -\frac{1}{\tau_s} \int_{-\tau_1}^{-\tau_2} \int_{t+\theta}^{t} \xi^T(t) \mathrm{d}s \mathrm{d}\theta$$
$$\times Z \times \int_{-\tau_1}^{-\tau_2} \int_{t+\theta}^{t} \xi(t) \mathrm{d}s \mathrm{d}\theta,$$

where $\tau_s = \left(\tau_2^2 - \tau_1^2\right)/2$.

Lemma 2.4 ([31]) For a given symmetric positive definite matrix $R > 0$, any functions any differential signal ω in $[a,b] \to R^n$ and any twice differential function $[a,b] \to R^n$, then the following inequality holds:

$$\int_a^b \dot{\omega}(u)R(\dot{\omega})(u)\mathrm{d}u \geq \frac{\mu^T(a,b)R\eta(a,b)}{b-a} + \frac{v_0^T(a,b)Rv(a,b)}{b-a},$$

w h e r e $\eta(a,b) = \omega(b) - \omega(a), v_0(a,b) = \frac{\omega(b)+\omega(a)}{2} - \frac{1}{b-a}\int_a^b \omega(u)\mathrm{d}u$.

3 The main results

Theorem 3.1 *The neutral type descriptor neural networks system* (2) *is global exponential stability with convergence rate $\alpha > 0$, if there exist some positive definite matrices P, Q_1, Q_2, Q_3, Q_4, R and H, a matrix two nonnegative constants μ and v, such that the following LMI condition is satisfied:*

$$E^T P = P^T E \geq 0. \tag{4}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & P^T B & \Pi_{16} & P^T D & \Pi_{18} \\ * & \Pi_{22} & 0 & 0 & \Pi_{25} & \Pi_{26} & E^T M^T D & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 & \Pi_{38} \\ * & * & * & \Pi_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & 0 \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix} \leq 0. \tag{5}$$

where $\Pi_{11} = 2\alpha E^T P - P^T A - A^T P + Q_1 + Q_2 - \tau^2 H - \lambda \frac{U^T V + V^T U}{2} - \frac{5}{4}e^{-2\alpha\tau}R$, $\Pi_{12} = A^T M E$, $\Pi_{13} = \frac{5}{4}e^{-2\alpha\tau}R$, $\Pi_{16} = \Pi_{16} = P^T C - \lambda\frac{U^T + V^T}{2}$, $\Pi_{18} = \tau H + \frac{1}{2\tau}e^{2\alpha\tau}R$, $\Pi_{22} = Q_3 + \tau^2 R - \frac{\tau^4}{4}H - E^T(M^T + M)E$, $\Pi_{25} = E^T M^T B$, $\Pi_{26} = E^T M^T C$, $\Pi_{33} = -e^{-2\alpha\tau}Q_2 - \frac{5}{4}e^{-2\alpha\tau}R$, $\Pi_{38} = -\frac{1}{2\tau}e^{-2\alpha\tau}R$, $\Pi_{44} = -(1-\bar{\tau})e^{-2\alpha\tau}Q_1$, $\Pi_{55} = -(1-\bar{\tau})e^{-2\alpha\tau}Q_3$, $\Pi_{66} = Q_4 - \lambda I$, $\Pi_{44} = -(1-\bar{\tau})e^{-2\alpha\tau}Q_4$, $\Pi_{88} = -H - \frac{4}{\tau^2}e^{-2\alpha\tau}R$.

Proof Firstly, we prove the system (2) is regular and impulse free. From Eq. (5), we can obtain the inequality $\Pi_{11} < 0$, that is to say $-P^T A - A^T P < 0$. Then, combining with Eq. (4), the pair (E, A) is regular and impulse free. Thus, the system (2) is regular and impulse free.

Next, we shall show the exponential stability of system (2) is global exponential stability.

For positive definite matrices P, Q_1, Q_2, Q_3, Q_4, R and H. Let us consider the Lyapunov functional candidate to be

$$V(z_t) = V_1(z_t) + V_2(z_t) + V_3(z_t) + V_4(z_t) + V_5(z_t) + V_6(z_t) + V_7(z_t), \tag{6}$$

where

$$V_1(z_t) = e^{2\alpha t}z^T(t)E^T Pz(t);$$

$$V_2(z_t) = \int_{t-\tau(t)}^t e^{2\alpha s}z^T(s)Q_1 z(s)\mathrm{d}s;$$

$$V_3(z_t) = \int_{t-\tau}^t e^{2\alpha s}z^T(s)Q_2 z(s)\mathrm{d}s;$$

$$V_4(z_t) = \int_{t-\tau(t)}^t e^{2\alpha s}\dot{z}^T(s)Q_3 \dot{z}(s)\mathrm{d}s;$$

$$V_5(z_t) = \int_{t-\tau(t)}^t e^{2\alpha s}g^T(s)Q_4 g(z(s))\mathrm{d}s;$$

$$V_6(z_t) = \tau\int_{-\tau}^0 \int_{t+\beta}^t e^{2\alpha s}\dot{z}^T(s)R\dot{z}(s)\mathrm{d}s\mathrm{d}\beta;$$

$$V_7(z_t) = \frac{\tau^2}{2}\int_{-\tau}^0 \int_\theta^0 \int_{t+\beta}^t e^{2\alpha s}\dot{z}^T(s)H\dot{z}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta.$$

Taking the time derivative of $V(t)$ along the trajectories of system (2) yields

$$\dot{V}(z_t) = \dot{V}_1(z_t) + \dot{V}_2(z_t) + \dot{V}_3(z_t) + \dot{V}_4(z_t) + \dot{V}_5(z_t) + \dot{V}_6(z_t) + \dot{V}_7(z_t), \tag{7}$$

where

$$\dot{V}_1(z_t) = 2\alpha e^{2\alpha t}z^T(t)E^T Pz(t) + 2\alpha e^{2\alpha t}z^T(t)P^T E\dot{z}(t)$$
$$= e^{2\alpha t}(z^T(t)(2\alpha E^T P - P^T A - A^T P)z(t)) + 2z^T(t)P^T Cg(z(t))$$
$$+ 2z^T(t)P^T B\dot{z}(t-\tau(t)) + 2z^T(t)P^T Dg(z(t-\tau(t))));$$

$$\dot{V}_2(z_t) \leq e^{2\alpha t}\left[z^T(t)Q_1 z(t) - e^{-2\alpha\tau}z^T(t-\tau(t))Q_1 z(t-\tau(t))(1-\bar{\tau})\right];$$

$$\dot{V}_3(z_t) \leq e^{2\alpha t}\left[z^T(t)Q_2 z(t) - e^{-2\alpha\tau}z^T(t-\tau)Q_2 z(t-\tau)\right];$$

$$\dot{V}_4(z_t) \leq e^{2\alpha t}\left[\dot{z}^T(t)Q_3 \dot{z}(t) - e^{-2\alpha\tau}\dot{z}^T(t-\tau(t))Q_3 \dot{z}(t-\tau(t))(1-\bar{\tau})\right];$$

$$\dot{V}_5(z_t) \leq e^{2\alpha t}\left[g^T(z(t))Q_4 g(z(t))\right]$$
$$- e^{2\alpha t}\left[e^{-2\alpha\tau}g^T(z(t-\tau(t)))Q_4 g(z(t-\tau(t)))(1-\bar{\tau})\right];$$

$$\dot{V}_6(z_t) \leq e^{2\alpha t}\left[\tau^2\dot{z}^T(t)R\dot{z}(t) - e^{-2\alpha\tau}\tau\int_{t-\tau}^t \dot{z}^T(s)R\dot{z}(s)\mathrm{d}s\right];$$

$$\dot{V}_7(z_t) \leq e^{2\alpha t}\left[-\frac{\tau^4}{4}\dot{z}^T(t)H\dot{z}(t) - \frac{\tau^2}{2}\int_{-\tau}^0 \int_{t+\theta}^t \dot{z}^T(s)H\dot{z}(s)\mathrm{d}s\mathrm{d}\theta\right].$$

According to Lemma 2.3, we can obtain

$$-\frac{\tau^2}{2}\int_{t+\theta}^0 \int_{t+\theta}^t \dot{z}^T(s)H\dot{z}(s)\mathrm{d}s\mathrm{d}\theta \leq -z^T(t)\tau^2 Hz(t) + z^T(t)\tau H\int_{t-\tau}^t z(s)\mathrm{d}s$$
$$+ \int_{t-\tau}^t z^T(s)\mathrm{d}s(\tau H)z(t) - \int_{t-\tau}^t z^T(s)\mathrm{d}sH\int_{t-\tau}^t z(s)\mathrm{d}s.$$

Then, according to Lemma 2.4, it can get,

$$
\begin{aligned}
\dot{V}_6(z_t) \leq & e^{-2\alpha t}\tau^2 \dot{z}^T(t)R\dot{z}(t) - e^{2\alpha(t-\tau)}(z(t) \\
& - z(t-\tau))^T R(z(t) - z(t-\tau)) \\
& - \frac{1}{4}e^{2\alpha(t-\tau)}\left(z(t) - z(t-\tau) - \frac{2}{\tau}\int_{t-\tau}^{t} z(s)\mathrm{d}s\right)^T \\
& R\left(z(t) - z(t-\tau) - \frac{2}{\tau}\int_{t-\tau}^{t} z(s)\mathrm{d}s\right).
\end{aligned}
$$

On the other hand, based on Assumption 2.1, we have that any $\lambda > 0$,

$$
y(t) = \lambda \begin{bmatrix} z(t) \\ g(z(t)) \end{bmatrix}^T \begin{bmatrix} \bar{U} & \bar{V} \\ * & I \end{bmatrix} \begin{bmatrix} z(t) \\ g(z(t)) \end{bmatrix} \leq 0, \tag{8}
$$

where $\bar{U} = \frac{U^T V + V^T U}{2}, \bar{U} = \frac{U^T + V^T}{2}$.

Then, according to the system (2) can get the following equation:

$$
\begin{aligned}
0 = & \dot{z}^T(t)E^T M^T(-Az(t) + B\dot{z}(t-\tau(t)) + Cg(z(t)) \\
& + Dg(z(t-\tau(t)))) - Az(t) + B\dot{z}(t-\tau(t) \\
& + Cg(z(t)) + Dg(z(t-\tau(t))))^T ME\dot{z}(t) \\
& - \dot{z}^T(t)E^T(M^T + M)E\dot{z}(t). \tag{9}
\end{aligned}
$$

Substituting (8) and (9) into (7) yield that

$$
\dot{V}(z_t) \leq e^{2\alpha t}Z_1^T \Pi Z_1.
$$

Define the extended vector

$$
\begin{aligned}
Z_1^T = & \left[z^T(t)\dot{z}^T(t)z^T(t-\tau)z^T(t-\tau(t))\dot{z}^T(t-\tau(t)) \right. \\
& \left. g^T(z(t))g^T(z(t-\tau(t))) \int_{t-\tau}^{t} z^T(s)\mathrm{d}s \right],
\end{aligned}
$$

If $\Pi < 0$, that is to say $\dot{V}(z_t) < 0$.

Based on the Lyapunov method, then system (2) is asymptotically stable, that is to say, the system (1) is asymptotically stable.

Furthermore, we prove the exponential stability of the neural networks:

According to $\dot{V}(z_t) < 0$, we can get the inequality $V(z_t) < V(z_0)$, where

$$
\begin{aligned}
V(z_0) = & z^T(0)Pz(0) + \int_{t-\tau(0)}^{0} e^{2\alpha s}z^T(s)Q_1 z(s)\mathrm{d}s \\
& + \int_{t-\tau}^{0} e^{2\alpha s}z^T(s)Q_2 z(s)\mathrm{d}s + \int_{t-\tau(0)}^{0} e^{2\alpha s}\dot{z}^T(s)Q_3 \dot{z}(s)\mathrm{d}s \\
& + \int_{t-\tau(0)}^{0} e^{2\alpha s}g^T(z^T(s))Q_4 g(z(s))\mathrm{d}s + \tau\int_{\beta}^{0} e^{2\alpha s}\dot{z}^T(s)R\dot{z}(s)\mathrm{d}s\mathrm{d}\beta \\
& + \frac{\tau^2}{2}\int_{t-\tau}^{0}\int_{\beta}^{0} e^{2\alpha s}\dot{z}^T(s)H\dot{z}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta \\
\leq & \left(\lambda_M(P) + 4\tau\lambda_M(Q_i) + \tau^3\lambda_M(R) + \tau^5\lambda_M(H)\right) \cdot \|z\|_s^2.
\end{aligned}
$$

Letting $\delta_0 = \lambda_M(P) + \tau\lambda_M(Q_1) + \lambda_M(Q_2) + \tau\lambda_M(Q_3) + \tau^3\lambda_M(R)$, then $\lambda_M(P)$, $\lambda_M(Q_1)$, $\lambda_M(Q_2)$, $\lambda_M(Q_3)$ and $\lambda_M(R)$ stand for the largest characteristic value of matrix P, Q_1, Q_2, Q_3, Q_4 and R, respectively. For any variable t, form the chosen Lyapunov functional, we can get the following inequality:

$$
V(z_t) \geq e^{2\alpha t}z^T(t)Pz(t) \geq \lambda_m(P)e^{2\alpha t}\|z(t)\|^2,
$$

which leads us to

$$
\|z(t)\| \leq \sqrt{\frac{\delta_0}{\lambda_m(P)}}\|z\|_s e^{-\alpha t}, \quad t \geq 0.
$$

By Definition 2.1, the system (2) is global exponential stability with a convergence rate α. This implies that the equilibrium point $\tilde{x}$ of system (2) is globally exponentially stable with convergence rate α. The proof is completed.

Remark 1 When $E = I$, the system (1) reduces to the traditional neutral type neural networks system:

$$
\dot{x}(t) = -A(x) + B\dot{x}(t-\tau(t)) + Cf(x(t)) + Df(x(t-\tau(t))) + J.
$$

We can transform the neutral type neural networks system to the system in [28, 29]. However, the system in this paper is more widely used than the traditional neutral type system.

4 Numerical examples

Example 4.1 Considering the system (2):

$$
E\dot{z}(t) = -Az(t) + B\dot{z}(t-\tau(t)) + Cg(z(t)) + Dg(z(t-\tau(t))),
$$

with the following parameter:

$$
E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}, C = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix},
$$

$$
D = \begin{bmatrix} -2 & 0.1 \\ 0.1 & -0.4 \end{bmatrix}, \quad \dot{\tau}(t) = 0.1, \alpha = \mu = v = 0.1.
$$

Using MATLAB LMI Toolbox and Theorem 3.1, we obtain maximum allowable upper bounds $\tau(t) < 4.5$. we choice $f(z) = \tanh(z)$, by condition (3) in Assumption 2.1, which we can obtain

$$
U = \begin{bmatrix} -0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, V = \begin{bmatrix} -0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}.
$$

and have the following results:

$$P = \begin{bmatrix} 1.498 & 0.4403 \\ 0.4403 & 6.3053 \end{bmatrix}, Q_1 = \begin{bmatrix} 19.6132 & -0.2745 \\ -0.2745 & 23.0388 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 12.7215 & -0.1907 \\ -0.1907 & 15.8559 \end{bmatrix}, Q_3 = \begin{bmatrix} 48.2162 & -1.7476 \\ -1.7476 & 49.6480 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 3.2815 & 0.0013 \\ 0.0013 & 3.4988 \end{bmatrix}, R = \begin{bmatrix} 40.4538 & 0.4348 \\ 0.4348 & 32.0820 \end{bmatrix},$$

$$H = \begin{bmatrix} 8.8263 & 0.0088 \\ 0.0088 & 7.1792 \end{bmatrix}.$$

It is straightforward to check that the conditions in Theorem 3.1 hold. It follows from Theorem 3.1 that system (2) can be exponential stability.

We can simulate the results shown in Fig. 1. The Fig. 1 shows that the singular neutral type neural networks systems are stabilized to zero after a while, and we can say the systems are global exponential stability.

Example 4.2 Considering the system (2):

$$E\dot{z}(t) = -Az(t) + B\dot{z}(t - \tau(t)) + Cg(z(t)) + Dg(z(t - \tau(t))),$$

with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 2 & 0 \\ 0 & 3.5 \end{bmatrix}, C = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad B = 0, \alpha = 0.25. \tag{10}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, \quad B = 0, \alpha = 0.05. \tag{11}$$
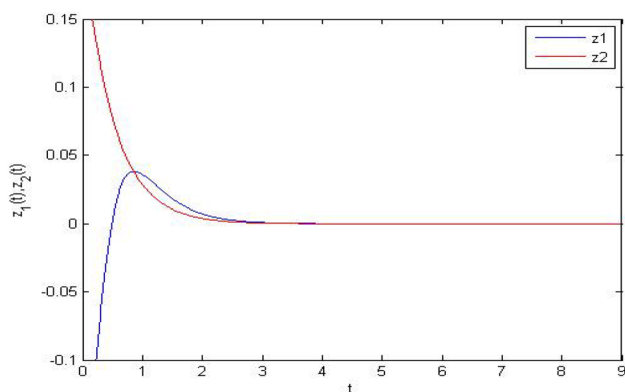

Fig. 1 State response of system with vary-time delays

By applying our criteria and using MATLAB LMI toolbox, we have the following comparative result listed in Tables 1 and 2. From Tables 1 and 2, we can see our results are much less conservative than those in [32, 33].

Remark 2 In this paper, through the comparison with the article [32, 33] in constant and time varying delays, the conclusion was obtained. The conclusion of this paper has better conservation.

Remark 3 A rare integral inequality and model transformation technique make the results have less conservative than some existing literatures.

5 Conclusions

This paper solve the global exponentially stable of neutral type singular networks with time-varying delays. In terms of LMI approach, model transformation technique and a new integral inequality, by using a new Lyapunov–Krasovskii functional, a less conservative criterion of global exponential stability for the networks system is given. The criterion is presented in terms of linear matrix inequalities, which can be easily solved by MATLAB Toolbox. Finally, two examples are presented to illustrate the effectiveness of the method.

Table 1 Some upper bounds for the time delays of system (2) with (10)

Results	Constant delay	Varied delay
[32]	$\tau(t) \leq 1$	$\tau(t) \leq 0.07$
[33]	$\tau(t) \leq 5.9$	$\tau(t) \leq 2.8$
Ours	$\tau(t) \leq 8.7$	$\tau(t) \leq 7.4$

Table 2 Some upper bounds for the time delays of system (2) with (11)

Result	Constant delay	Varied delay
[32]	$\tau(t) \leq 0.74$	$\tau(t) \leq 0.5$
[33]	$\tau(t) \leq 29.132$	$\tau(t) \leq 22.2$
Ours	$\tau(t) \leq 40.43$	$\tau(t) \leq 38.56$

References

1. Li C, Feng G (2009) Delay-interval-dependent stability of recurrent neural networks with time-varying delay. Neurocomputing 72:1179–1183
2. Hua C, Long C, Guan X (2006) New results on stability analysis of neural networks with time-varying delays. Phys Lett A 352:335–340
3. Rakkiyappan R, Balasubramaniam P (2009) Delay-dependent robust stability analysis of uncertain stochastic neural networks with discrete interval and distributed time-varying delays. Phys Lett A 72:3231–3237
4. Tian J, Zhong S (2011) Improved delay-dependent stability criteria for neural networks with time-varying delay. Appl Math Comput 217:10278–10288
5. Tian J, Xie X (2010) New asymptotic stability criteria for neural networks with time-varying delay. Phys Lett A 374:938–943
6. He X, Li C, Huang T, Li C, Huang J (2014) A recurrent neural network for solving bilevel linear programming problem. IEEE Trans Neural Netw Learning Syst 25:824–830
7. He X, Huang T, Yu J, Li C, Li C (2016) An inertial projection neural network for solving variational inequalities. IEEE Trans Cybern 47:809–814
8. Zhou C, Zhang W, Yang X, Xu C, Feng J (2017) Finite-time synchronization of complex-valued neural networks with mixed delays and uncertain perturbations. Neural Process Lett 46:271–291
9. Feng J, Ma Q, Qin S (2017) Exponential stability of periodic solution for impulsive memristor-based Cohen–Grossberg neural networks with mixed delays. Int J Pattern Recognit Artif Intell 31:1–17
10. Yang X, Feng Z, Feng J, Cao J (2016) Synchronization of discrete-time neural networks with delays and Markov jump topologies based on tracker information. Neural Netw Off J Int Neural Netw Soc 85(C):157–164
11. Han Q (2004) On robust stability of neutral systems with time-varying discrete delay and norm-bounded uncertainty. Automatica 40:1087–1092
12. Wang Y, Wang Z, Liang J (2010) On robust stability of stochastic genetic regulatory net-works with time-delays: a delay fractioning approach. IEEE Trans Syst Man Cybern 40:729–740
13. Ma Y, Ma N (2016) Synchronization for complex dynamical networks with mixed mode-dependent time delays. Adv Differ Equ 242:1–18
14. Chen W, Lu X, Guan Z (2009) Delay-dependent exponential stability of neural networks with variable delay: an LMI approach. IEEE Trans Circuit Syst II 53:837–842
15. He Y, Wu M, She J (2006) An improved global asymptotic stability criterion for delayed cellular neural networks. IEEE Trans Neural Netw 17:250–252
16. He Y, Wu M, She J (2006) Delay-dependent exponential stability of delayed neural networks with time-varying delay. IEEE Trans Circuit Syst II 53:553–557
17. He Y, Liu G, Rees D (2007) New delay-dependent stability criteria for neural networks with time-varying delay. IEEE Trans Neural Netw 18:310–314
18. Liu Q, Wang J (2015) A projection neural network for constrained quadratic minimax optimization. IEEE Transactions Neural Netw Learning Syst 26:2891–2900
19. Li C, Yu X, Huang T, Chen G, He X (2015) A generalized hopfield network for nonsmooth constrained convex optimization: lie derivative approach. IEEE Trans Neural Netw Learning Syst 27:1–14
20. Ma Y, Zheng Y (2015) Projective lag synchronization of Markovian jumping neural networks with mode-dependent mixed time-delays based on an integral sliding mode controller. Neurocomputing 168:626–636
21. Ma Y, Zheng Y (2015) Synchronization of continuous-time Markovian jumping singular complex networks with mixed mode-dependent time delays. Neurocomputing 156:52–59
22. He Y, Liu G, Rees D (2007) Improved stability criteria for neural networks with time-varying interval delay. IEEE Trans Neural Netw 18:1850–1854
23. Li H, Li H, Zhong S (2007) Stability of neutral type descriptor system with mixed delays. Chaos Solitons Fractals 33:1796–1800
24. Zhao Y, Ma Y (2012) Stability of neutral-type descriptor systems with multiple time-varying delays. Adv Differ Equ 15:1–7
25. Han R, Jiang W (2009) Delay-dependent robust stability criteria for uncertain singular neutral differential systems with time-varying and distributed delays. Ann Differ Equ 25:150–160
26. Rakkiyappanan R, Kaviarasana B, Rihanb F, Lakshmananb S (2015) Synchronization of singular Markovian jumping complex networks with additive time-varying delays via pinning control. J Franklin Inst 352:3178–3195
27. Koo J, Ji D, Wona S (2010) Synchronization of singular complex dynamical networks with time-varying delays. Appl Math Comput 217:3916–3923
28. Li H, Ning Z, Yin Y, Tang Y (2013) Synchronization and state estimation for singular complex dynamical networks with time-varying delays. Commun Nonlinear Sci Numer Simul 18:194–208
29. Han Q (2009) A new delay-dependent absolute stability criterion for a class of nonlinear neutral systems. Automatica 44:272–277
30. Kwon O, Lee S, Park J, Cha E (2012) New approaches on stability criteria for neural networks with interval time-varying delays. Appl Math Comput 218:9953–9964
31. Kwon O, Park J (2006) Exponential stability of uncertain dynamic systems including state delay. Appl Math Lett 19:901–907
32. Zhang Q, Wei X, Xu J (2009) Delay-dependent exponential stability of cellular neural networks with time-varying delays. Chaos Solitons Fractals 23:1363–1369
33. Gau R, Lien C, Hsieh J (2007) Global exponential stability for uncertain cellular neural networks with multiple time-varying delays via LMI approach. Chaos Solitons Fractals 32:1258–1267