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ORIGINAL ARTICLE

Pseudo almost periodic high-order cellular neural networks with complex deviating arguments

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Abstract In this paper, we propose and study the pseudo almost periodic high-order cellular neural networks with oscillating leakage coefficients and complex deviating arguments, which has not been studied in the existing literature. Applying the contraction mapping fixed point theorem and inequality analysis techniques, we establish a set of criteria for the existence and uniqueness of pseudo almost periodic solutions for this model, which can be easily tested in practice by simple algebra computations. The obtained results play an important role in designing high-order cellular neural networks with state-dependent delays. Moreover, some illustrative examples are given to demonstrate our theoretical results.

Keywords High-order cellular neural networks · Pseudo almost periodic solution · Existence · Oscillating leakage coefficient · Complex deviating argument

Mathematics Subject Classification 34C25 · 34K13 · 34K25

1 Introduction

Over the past 30 years, compared with the ordinary neural networks, the high-order neural networks (HNNs) have stronger approximation property, faster convergence rate, great stronger capacity and higher fault tolerance, and high-order cellular neural networks (HCNNs) has become an active research topic in different application areas, such as associtative memory, pattern recognition and signal processing (see [1–7] and the references cited therein). Recently, as pointed out in [8], it is significantly important in theory and application to study cellular neural networks (CNNs) with complex deviating arguments, which has a wider meaning than CNNs with time-varying delays or distributed delays since the models describing the complexity of real problems should reflect the effects of fluctuation factors (see [9–12]). Furthermore, a typical time delay called Leakage (or "forgetting") delay may exist in the negative feedback terms of the neural network system, and these terms are variously known as forgetting or leakage terms (see [1, 13, 14]). Usually, HCNNs with complex deviating arguments and involving time-varying delays, and continuously distributed delays in the leakage terms can be described as follows:

$$x_i'(t) = -c_i(t)x_i(t-\eta_i(t)) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)g_j(x_j(x_j(t))) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}(t)\bar{g}_j(x_j(x_j(t)))\bar{g}_l(x_l(x_l(t))) + I_i(t),\ i = 1, 2, \dots, n, \tag{1.1}$$

and

$$x_i'(t) = -c_i(t)\int_0^{+\infty} \delta_i(s)x_i(t-s)ds + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)g_j(x_j(x_j(t))) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}(t)\bar{g}_j(x_j(x_j(t)))\bar{g}_l(x_l(x_l(t))) + I_i(t),\ i = 1, 2, \dots, n, \tag{1.2}$$

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respectively. Here n corresponds to the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the ith unit at the time t, $c_i(t)$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at the time t, $a_{ij}(t)$, $b_{ij}(t)$ and $e_{ijl}(t)$ are the connection weights at the time t, $\eta_i(t) \ge 0$ denotes the leakage delay, $\delta_i(s) \ge 0$ denotes the leakage delay kernel, and $I_i(t)$ denotes the external inputs at time t, f_j, g_j and $\bar{g}_j$ are activation functions of signal transmission, $i, j \in J = \{1, 2, \dots, n\}$. From the basic theory on state-dependent delay-differential equations in [15], HCNNs (1.1) and (1.2) are special types of state-dependent delay-differential equations.

It should be mentioned that, compared with almost periodic phenomenon, pseudo almost periodic phenomenon which can be represented as an almost periodic process plus a ergodic component is more common [10, 16]. Therefore, it is more realistic to study the pseudo almost periodic phenomenon in neural networks models [17–22]. However, to the best of our knowledge, there is no result on the existence of pseudo almost periodic solutions for HCNNs with oscillating leakage coefficients and complex deviating arguments. This motivates us to derive some sufficient criteria on the existence of pseudo almost periodic solutions for Eqs. (1.1) and (1.2).

The remaining of this paper is organized as follows. In Sect. 2, we recall some basic definitions and lemmas, which play an important role in Sect. 3 to establish the existence and uniqueness of pseudo almost periodic solutions of Eqs. (1.1) and (1.2). The paper concludes with two examples to illustrate the effectiveness of the obtained results.

2 Preliminaries

In this section, we shall first recall some basic definitions, lemmas which are used in what follows. We designate by $AP(\mathbb{R}, \mathbb{R}^n)$ the set of the almost periodic functions from $\mathbb{R}$ to $\mathbb{R}^n$. Hereafter, we denote by $\mathbb{R}^n(\mathbb{R} = \mathbb{R}^1)$ the set of all n–dimensional real vectors (real numbers). For any $\{x_i\} = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$, we let $|x|$ denote the absolute-value vector given by $|x| = \{|x_i|\}$, and define $\|x\| = \max_{i\in J} |x_i|$. A matrix or vector $A \ge 0$ means that all entries of A are greater than or equal to zero. $A > 0$ can be defined similarly. For matrices or vectors A_1 and A_2, $A_1 \ge A_2$ (resp. $A_1 > A_2$) means that $A_1 - A_2 \ge 0$ (resp. $A_1 - A_2 > 0$). $BC(\mathbb{R}, \mathbb{R}^n)$ denotes the set of bounded and continuous functions from $\mathbb{R}$ to $\mathbb{R}^n$, and $BUC(\mathbb{R}, \mathbb{R}^n)$ denotes the set of bounded and uniformly continuous functions from $\mathbb{R}$ to $\mathbb{R}^n$. Note that $(BC(\mathbb{R}, \mathbb{R}^n), \|\cdot\|_\infty)$ is a Banach space, where $\|\cdot\|_\infty$ denotes the supremum norm $\|f\|_\infty := \sup_{t\in\mathbb{R}} \|f(t)\|$. For $h \in BC(\mathbb{R}, \mathbb{R})$, let h^+ and h^- be defined as

$$h^+ = \sup_{t\in\mathbb{R}} |h(t)|, \quad h^- = \inf_{t\in\mathbb{R}} |h(t)|.$$

Besides, we define the class of functions $PAP_0(\mathbb{R}, \mathbb{R}^n)$ as follows:

$$\left\{ f \in BC(\mathbb{R}, \mathbb{R}^n) | \lim_{T\to+\infty} \frac{1}{2T} \int_{-T}^{T} |f(t)| dt = \mathbf{0} \right\}.$$

A function $f \in BC(\mathbb{R}, \mathbb{R}^n)$ is called pseudo almost periodic if it can be expressed as

$$f = h + \varphi,$$

where $h \in AP(\mathbb{R}, \mathbb{R}^n)$ and $\varphi \in PAP_0(\mathbb{R}, \mathbb{R}^n)$. The collection of such functions will be denoted by $PAP(\mathbb{R}, \mathbb{R}^n)$. Then, $(PAP(\mathbb{R}, \mathbb{R}^n), \|.\|_\infty)$ is a Banach space and $AP(\mathbb{R}, \mathbb{R}^n)$ is a proper subspace of $PAP(\mathbb{R}, \mathbb{R}^n)$ [23].

For $i,\ j \in J$, it will be assumed that $c_i : \mathbb{R} \to \mathbb{R}$ is an almost periodic function, $\eta_i : \mathbb{R} \to [0,\ +\infty)$, $I_i,\ a_{ij},\ b_{ij}, e_{ijl} : \mathbb{R} \to \mathbb{R}$ are pseudo almost periodic on $\mathbb{R}$.

Let $L > 0,\ l > 0$ and $0 < \theta < 1$ be three constants such that

$$B^L = \{\varphi | \varphi \in PAP(\mathbb{R}, \mathbb{R}^n),\ \{|\varphi_i(t_1) - \varphi_i(t_2)|\} \le \{L|t_1 - t_2|\}, \text{ for all } t_1, t_2 \in \mathbb{R}\},$$

and

$$B^* = \left\{ \varphi \middle| ||\varphi - \varphi_0||_{\mathbf{Z}} \le \frac{\theta l}{1-\theta},\ \varphi \in \mathbf{Z} \right\},$$

where

$$\varphi_0 = \left\{ \int_{-\infty}^{t} e^{-\int_s^t c_i(w)dw} I_i(s) ds \right\},$$
$$\left\{ \max \left\{ K_i \frac{I_i^+}{\tilde{c}_i^-}, \left(1 + K_i \frac{c_i^+}{\tilde{c}_i^-}\right) I_i^+ \right\} \right\} \le \{l\}.$$

We also make the following assumptions which will be used later.

(S_0) For each $i \in J$,

$$M[c_i] = \lim_{T\to+\infty} \frac{1}{T} \int_{t}^{t+T} c_i(s) ds > 0,$$

and there exist a bounded and continuous function $\tilde{c}_i : \mathbb{R} \to (0,\ +\infty)$ and a positive constant K_i such that

$$e^{-\int_s^t c_i(u)du} \le K_i e^{-\int_s^t \tilde{c}_i(u)du} \text{ for all } t, s \in \mathbb{R} \text{ and } t - s \ge 0.$$

(S_1) For each $j \in J$, there exist nonnegative constants L_j^f, L_j^g, $L_j^{\bar{g}}$ and $M_j^{\bar{g}}$ such that

$$|f_j(u)-f_j(v)|\le L_j^f|u-v|,\ |g_j(u)-g_j(v)|\le L_j^g|u-v|,$$
$$|\bar{g}_j(u)-\bar{g}_j(v)|\le L_j^{\bar{g}}|u-v|,\quad \text{for all}\quad u,\ v\in\mathbb{R},$$

$$g_j(0)=f_j(0)=\bar{g}_j(0)=0,\ \sup_{u\in\mathbb{R}}|\bar{g}_j(u)|:=M_j^{\bar{g}}<+\infty.$$

Lemma 2.1 (see [19, Lemma 4 and Remark 5]). *Let* $\mathbf{Z}=\{f|f,f'\in PAP(\mathbb{R},\mathbb{R}^n)\}$ *be equipped with the induced norm defined by*

$$\|f\|_{\mathbf{Z}}=\max\{\|f\|_\infty,\|f'\|_\infty\}=\max\left\{\sup_{t\in\mathbb{R}}\|f(t)\|,\sup_{t\in\mathbb{R}}\|f'(t)\|\right\}.$$

Then, $\mathbf{Z}$ *is a Banach space.*

Lemma 2.2 (see [5, Lemma 2.2]) B^L *is a closed subset of* $PAP(\mathbb{R},\mathbb{R}^n)$.

3 Main results

In this section, we establish some sufficient conditions on the existence and uniqueness of pseudo almost periodic solutions of Eqs. (1.1) and (1.2).

Theorem 3.1 *Let* (S_0) *and* (S_1) *hold. Suppose that there exist constants* $\theta>0,\ \alpha_i>0$ *and* $L>0$, *such that*

$$\begin{aligned}\theta=\max_{i\in J}\Bigg\{&\left(1+c_i^+\frac{K_i}{\tilde{c}_i^-}\right)\Bigg[c_i^+n_i^+ +\sum_{j=1}^n\left(a_{ij}^+L_j^f+b_{ij}^+L_j^g\right)\\&+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\Bigg],\\&\frac{K_i}{\tilde{c}_i^-}\left[c_i^+\eta_i^+ +\sum_{j=1}^n(a_{ij}^+L_j^f+b_{ij}^+L_j^g)+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right]\Bigg\}\end{aligned}\tag{3.1}$$

$$\begin{aligned}\Bigg\{&\Bigg[c_i^+ + c_i^+\eta_i^+ +\sum_{j=1}^n\left(a_{ij}^+L_j^f+b_{ij}^+L_j^g\right)\\&+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\Bigg]\frac{l}{1-\theta}+I_i^+\}\le\{L\Bigg\},\end{aligned}\tag{3.2}$$

and

$$\begin{aligned}-\tilde{c}_i^-+K_i\Bigg[&c_i^+\eta_i^+ +\sum_{j=1}^n a_{ij}^+L_j^f+\sum_{j=1}^n b_{ij}^+L_j^g(1+L)\\&+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+(1+L)\left(M_l^{\bar{g}}L_j^{\bar{g}}+M_j^{\bar{g}}L_l^{\bar{g}}\right)\Bigg]\le-\alpha_i,\end{aligned}$$

$$\frac{\tilde{c}_i^- -\alpha_i}{K_i}+c_i^+\left(1-\frac{\alpha_i}{\tilde{c}_i^+}\right)<1,\ i\in J.\tag{3.3}$$

Then (1.1) has a unique pseudo almost periodic solution in the region $\mathbf{B}=B^L\bigcap B^*$.

Proof By Lemmas 2.1 and 2.2, one can show that $\mathbf{B}$ is a closed subset of $\mathbf{Z}$. For any $\varphi\in\mathbf{B}$, notice that $M[c_i]>0,\ i=1,\ 2,\dots,n$, using a similar argument as that in the proof of Theorem 3.1 in [11], we obtain that the nonlinear pseudo almost periodic differential equations,

$$\begin{aligned}x_i'(t)=&-c_i(t)x_i(t)+c_i(t)\int_{t-\eta_i(t)}^t\varphi_i'(s)ds+\sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t))\\&+\sum_{j=1}^n b_{ij}(t)g_j(\varphi_j(\varphi_j(t)))\\&+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}(t)\bar{g}_j(\varphi_j(\varphi_j(t)))\bar{g}_l(\varphi_l(\varphi_l(t)))+I_i(t),\ i\in J,\end{aligned}\tag{3.4}$$

has exactly one pseudo almost periodic solution:

$$\begin{aligned}x^\varphi(t)=&\left\{x_i^\varphi(t)\right\}\\=&\Bigg\{\int_{-\infty}^t e^{-\int_s^t c_i(u)du}\Bigg[c_i(s)\int_{s-\eta_i(s)}^s\varphi_i'(u)du+\sum_{j=1}^n a_{ij}(s)f_j(\varphi_j(s))\\&+\sum_{j=1}^n b_{ij}(s)g_j(\varphi_j(\varphi_j(s)))\\&+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}(s)\bar{g}_j(\varphi_j(\varphi_j(s)))\bar{g}_l(\varphi_l(\varphi_l(s)))+I_i(s)\Bigg]ds\Bigg\},\end{aligned}\tag{3.5}$$

and

$$\begin{aligned}(x^\varphi(t))'=\Bigg\{&\Bigg[c_i(t)\int_{t-\eta_i(t)}^t\varphi_i'(u)du+\sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t))\\&+\sum_{j=1}^n b_{ij}(t)g_j(\varphi_j(\varphi_j(t)))\\&+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}(t)\bar{g}_j(\varphi_j(\varphi_j(t)))\bar{g}_l(\varphi_l(\varphi_l(t)))+I_i(t)\Bigg]\\&-c_i(t)x_i^\varphi(t)\Bigg\}\in PAP(\mathbb{R},\mathbb{R}^n).\end{aligned}\tag{3.6}$$

Now, we define a mapping $T{:}\mathbf{B}\to PAP(\mathbb{R},\mathbb{R}^n)$ by setting

$$(T\varphi)(t) = x^{\varphi}(t), \quad \forall \varphi \in \mathbf{B}.$$

First we show that for any $\varphi \in \mathbf{B}$, $T\varphi = x^{\varphi} \in \mathbf{B}$.

Note that

$$\begin{aligned}\|\varphi_0\| &= \max_{i\in J}\left\{\sup_{t\in\mathbb{R}}\left|\int_{-\infty}^{t} e^{-\int_s^t c_i(w)dw} I_i(s)ds\right|\right\}\\ &\le \max_{i\in J}\left\{\sup_{t\in\mathbb{R}} K_i\left|\int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i(w)dw} I_i(s)ds\right|\right\}\\ &\le \max_{i\in J}\left\{K_i\frac{I_i^+}{\tilde{c}_i^-}\right\} \le l, \varphi_{0'}\\ \|\varphi_0'\| &= \max_{i\in J}\left\{\sup_{t\in\mathbb{R}} |-c_i(t)\varphi_0(t) + I_i(t)|\right\}\\ &\le \max_{i\in J}\left\{\left(1 + K_i\frac{c_i^+}{\tilde{c}_i^-}\right)I_i^+\right\} \le l.\end{aligned}$$

We get

$$||\varphi||_{\mathbf{Z}} \le ||\varphi - \varphi_0||_{\mathbf{Z}} + ||\varphi_0||_{\mathbf{Z}} \le \frac{\theta l}{1-\theta} + l = \frac{l}{1-\theta}. \quad (3.7)$$

It follows from Eqs. (3.1), (3.2) and (3.3) that

$$\begin{aligned}|(T\varphi)(t) - \varphi_0(t)| \le &\left\{K_i \int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i(u)du} \Big| c_i(s)\int_{s-\eta_i(s)}^{s} \varphi_i'(u)du + \sum_{j=1}^{n} a_{ij}(s)f_j(\varphi_j(s))\right.\\ &\left. + \sum_{j=1}^{n} b_{ij}(s)g_j(\varphi_j(\varphi_j(s))) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}(s)\bar{g}_j(\varphi_j(\varphi_j(s)))\bar{g}_l(\varphi_l(\varphi_l(s)))\Big]|ds\right\}\\ \le &\left\{K_i \int_{-\infty}^{t} e^{-\int_s^t \tilde{c}_i^- du}[c_i^+\eta_i^+ + \sum_{j=1}^{n}\left(a_{ij}^+L_j^f + b_{ij}^+L_j^g\right) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\Big]ds\|\varphi\|_{\mathbf{Z}}\right\}\\ \le &\left\{\frac{K_i}{\tilde{c}_i^-}\left[c_i^+\eta_i^+ + \sum_{j=1}^{n}\left(a_{ij}^+L_j^f + b_{ij}^+L_j^g\right) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right]\frac{l}{1-\theta}\right\} \le \left\{\frac{\theta l}{1-\theta}\right\}, \ \forall t \in \mathbb{R},\end{aligned}$$

and

$$\begin{aligned}&|((T\varphi)(t) - \varphi_0(t))'|\\ &\le \left\{\left(1 + c_i^+\frac{K_i}{\tilde{c}_i^-}\right)\left[c_i^+\eta_i^+ + \sum_{j=1}^{n}\left(a_{ij}^+L_j^f + b_{ij}^+L_j^g\right) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right]\frac{l}{1-\theta}\right\}\\ &\le \left\{\frac{\theta l}{1-\theta}\right\}, \ \forall t \in \mathbb{R}.\end{aligned}$$

Consequently,

$$||T\varphi - \varphi_0||_{\infty} \le \frac{\theta l}{1-\theta}, \quad ||(T\varphi - \varphi_0)'||_{\infty} \le \frac{\theta l}{1-\theta}, \quad (3.8)$$

$$||T\varphi||_{\infty} \le ||T\varphi - \varphi_0||_{\infty} + ||\varphi_0||_{\infty} \le \frac{\theta l}{1-\theta} + l = \frac{l}{1-\theta},$$

$$\begin{aligned}|((T\varphi)(t))'| = &\left\{|-c_i(t)((T\varphi)(t))_i + c_i(t)\int_{t-\eta_i(t)}^{t}\varphi_i'(s)ds\right.\\ &+ \sum_{j=1}^{n} a_{ij}(t)f_j(\varphi_j(t)) + \sum_{j=1}^{n} b_{ij}(t)g_j(\varphi_j(\varphi_j(t)))\\ &\left.+ \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}(t)\bar{g}_j(\varphi_j(\varphi_j(t)))\bar{g}_l(\varphi_l(\varphi_l(t))) + I_i(t)|\right\}\\ \le &\left\{\left[c_i^+ + c_i^+\eta_i^+ + \sum_{j=1}^{n}\left(a_{ij}^+L_j^f + b_{ij}^+L_j^g\right)\right.\right.\\ &\left.\left.+ \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right]\frac{l}{1-\theta} + I_i^+\right\} \le \{L\}, \ \forall t \in \mathbb{R},\end{aligned}$$

and

$$\begin{aligned}&|(T\varphi)(t_1) - (T\varphi)(t_2)|\\ &= \left\{|(((T\varphi)(t))_i)'\Big|_{t=t_1+\Delta(t_2-t_1)}(t_1 - t_2)|\right\} \le \left\{L|t_1 - t_2|\right\},\end{aligned} \quad (3.9)$$

where for all $t_1, t_2 \in \mathbb{R}$, $\Delta \in (0, 1)$, and $t_1 + \Delta(t_2 - t_1)$ is the mean point in Lagrange's mean value theorem. Thus, Eqs. (3.8) and (3.9) yield $T\varphi \in \mathbf{B}$. So, the mapping T is a self-mapping from $\mathbf{B}$ to $\mathbf{B}$.

Furthermore, for $\varphi, \psi \in \mathbf{B}$, according to (S_0), (S_1), Eqs. (3.1), (3.2), (3.5) and (3.6), we get

$$
\begin{aligned}
|(T\varphi)(t)-(T\psi)(t)| \le \Bigg\{ \int_{-\infty}^{t} e^{-\int_s^t c_i(u)du}\Bigg[|c_i(s)|\int_{s-\eta_i(s)}^{s}|\varphi_i'(u)-\psi_i'(u)|du \\
+\sum_{j=1}^{n} a_{ij}^{+}L_j^{f}|\varphi_j(s)-\psi_j(s)| + \sum_{j=1}^{n} b_{ij}^{+}L_j^{g}|\varphi_j(\varphi_j(s))-\psi_j(\psi_j(s))| \\
+\sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(M_l^{\bar g}L_j^{\bar g}|\varphi_j(\varphi_j(s))-\psi_j(\psi_j(s))| + M_j^{\bar g}L_l^{\bar g}|\varphi_l(\varphi_l(s))-\psi_l(\psi_l(s))|)\Bigg]ds\Bigg\} \\
\le \Bigg\{ \int_{-\infty}^{t} e^{-\int_s^t c_i(u)du}\Bigg[|c_i(s)|\int_{s-\eta_i(s)}^{s}|\varphi_i'(u)-\psi_i'(u)|du + \sum_{j=1}^{n} a_{ij}^{+}L_j^{f}|\varphi_j(s)-\psi_j(s)| \\
+\sum_{j=1}^{n} b_{ij}^{+}L_j^{g}(|\varphi_j(\varphi_j(s))-\psi_j(\varphi_j(s))| + |\psi_j(\varphi_j(s))-\psi_j(\psi_j(s))|) \\
+\sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(M_l^{\bar g}L_j^{\bar g}(|\varphi_j(\varphi_j(s))-\psi_j(\varphi_j(s))| + |\psi_j(\varphi_j(s))-\psi_j(\psi_j(s))|) \\
+M_j^{\bar g}L_l^{\bar g}(|\varphi_l(\varphi_l(s))-\psi_l(\varphi_l(s))| + |\psi_l(\varphi_l(s))-\psi_l(\psi_l(s))|))\Bigg]ds\Bigg\} \\
\le \Bigg\{ \int_{-\infty}^{t} e^{-\int_s^t \tilde c_i(u)du}K_i\Bigg[c_i^{+}\eta_i^{+} + \sum_{j=1}^{n} a_{ij}^{+}L_j^{f} + \sum_{j=1}^{n} b_{ij}^{+}L_j^{g}(1+L) \\
+\sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(1+L)(M_l^{\bar g}L_j^{\bar g} + M_j^{\bar g}L_l^{\bar g})\Bigg]ds\|\varphi(t)-\psi(t)\|_{\mathbf{Z}}\Bigg\} \\
\le \Bigg\{ \int_{-\infty}^{t} e^{-\int_s^t \tilde c_i(u)du}d\Bigg(-\int_s^t \tilde c_i(u)du\Bigg) - \alpha_i\int_{-\infty}^{t} e^{-\int_s^t \tilde c_i(u)du}ds\Bigg\}\|\varphi(t)-\psi(t)\|_{\mathbf{Z}} \\
\le \Bigg\{1-\alpha_i\int_{-\infty}^{t} e^{-\int_s^t \tilde c_i^{+}du}ds\Bigg\}\|\varphi(t)-\psi(t)\|_{\mathbf{Z}} \le \Bigg\{1-\frac{\alpha_i}{\tilde c_i^{+}}\Bigg\}\|\varphi(t)-\psi(t)\|_{\mathbf{Z}},
\end{aligned} \tag{3.10}
$$

and

$$
\begin{aligned}
|((T\varphi)(t))'-((T\psi)(t))'| le\Bigg\{\Bigg[c_i^{+}\eta_i^{+} + \sum_{j=1}^{n} a_{ij}^{+}L_j^{f} + \sum_{j=1}^{n} b_{ij}^{+}L_j^{g}(1+L) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(1+L)(M_l^{\bar g}L_j^{\bar g} + M_j^{\bar g}L_l^{\bar g})\Bigg]\|\varphi(t)-\psi(t)\|_{\mathbf{Z}} \\
+c_i^{+}\int_{-\infty}^{t} e^{-\int_s^t c_i(u)du}\Bigg[c_i^{+}\eta_i^{+} + \sum_{j=1}^{n} a_{ij}^{+}L_j^{f} + \sum_{j=1}^{n} b_{ij}^{+}L_j^{g}(1+L) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(1+L)(M_l^{\bar g}L_j^{\bar g} + M_j^{\bar g}L_l^{\bar g})\Bigg]ds\|\varphi(t)-\psi(t)\|_{\mathbf{Z}}\Bigg\} \\
\le \Bigg\{\Bigg[c_i^{+}\eta_i^{+} + \sum_{j=1}^{n} a_{ij}^{+}L_j^{f} + \sum_{j=1}^{n} b_{ij}^{+}L_j^{g}(1+L) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(1+L)(M_l^{\bar g}L_j^{\bar g} + M_j^{\bar g}L_l^{\bar g})\Bigg]\|\varphi(t)-\psi(t)\|_{\mathbf{Z}} \\
+c_i^{+}\int_{-\infty}^{t} e^{-\int_s^t \tilde c_i(u)du}K_i\Bigg[c_i^{+}\eta_i^{+} + \sum_{j=1}^{n} a_{ij}^{+}L_j^{f} + \sum_{j=1}^{n} b_{ij}^{+}L_j^{g}(1+L) + \sum_{j=1}^{n}\sum_{l=1}^{n} e_{ijl}^{+}(1+L)(M_l^{\bar g}L_j^{\bar g} + M_j^{\bar g}L_l^{\bar g})]ds\|\varphi(t)-\psi(t)\|_{\mathbf{Z}}\Bigg\} \\
\le \Bigg\{\frac{\tilde c_i^{-}-\alpha_i}{K_i} + c_i^{+}(1-\frac{\alpha_i}{\tilde c_i^{+}})\Bigg\}\|\varphi(t)-\psi(t)\|_{\mathbf{Z}}.
\end{aligned} \tag{3.11}
$$

In view of Eq. (3.3), we have

$$0 < 1 - \frac{\alpha_i}{\tilde{c}_i^+} < 1,\ \rho = \max\left\{ \max_{1\le i\le n}\left\{1 - \frac{\alpha_i}{\tilde{c}_i^+}\right\},\right.$$
$$\left.\max_{1\le i\le n}\left\{\frac{\tilde{c}_i^- - \alpha_i}{K_i} + c_i^+\left(1 - \frac{\alpha_i}{\tilde{c}_i^+}\right)\right\}\right\} < 1,$$

which, together with Eqs. (3.10) and (3.11), give us that $\|T\varphi - T\psi\|_{\mathbf{Z}} \le \rho\|\varphi - \psi\|_{\mathbf{Z}}$, and the mapping $T{:}\mathbf{B} \longrightarrow \mathbf{B}$ is a contraction mapping. Therefore, the mapping T possesses a unique fixed point

$$x^* = (x_1^*(t), x_2^*(t), \dots, x_n^*(t)) \in \mathbf{B},\ Tx^* = x^*.$$

By Eq. (3.6), x^* satisfies Eq. (1.1). So Eq. (1.1) has a unique continuously differentiable pseudo almost periodic solution x^*. The proof of Theorem 3.1 is now completed.

Remark 3.1 Obviously, Theorem 3.1 of [11] is only a special case of Theorem 3.1 without high-order term $\sum_{j=1}^n \sum_{l=1}^n e_{ijl}(t)\bar{g}_j(x_j(x_j(t)))\bar{g}_l(x_l(x_l(t)))$.

Remark 3.2 From (3.1), (3.2) and (3.3), one can check that the conditions in Theorem 3.1 can be easily satisfied under the sufficiently small leakage delays $\eta_i(t)$, and (3.3) is not satisfied when the leakage delays in (1.1) are sufficiently large. This implies that the delays (oscillating leakage coefficients) effect the stability of (1.1).

Next, we will show the existence of pseudo almost periodic solutions of (1.2). We assume that the following conditions are satisfied.

$(\hat{S}_0)$ For each $i \in J$,

$$M[c_i] = \lim_{T\to+\infty}\frac{1}{T}\int_t^{t+T} c_i(s)ds > 0,$$

and there exist a bounded continuous function $\bar{c}_i{:}\mathbb{R} \to (0, +\infty)$ and a positive constant K_i such that

$$e^{-\int_s^t c_i(u)\int_0^{+\infty}\delta_i(v)dvdu} \le K_i e^{-\int_s^t \bar{c}_i(u)\int_0^{+\infty}\delta_i(v)dvdu}$$
for all $t, s \in \mathbb{R}$ and $t - s \ge 0$.

$(\hat{S}_2)$ for $i \in J$, the delay kernel $\delta_i{:}\mathbb{R} \to [0, +\infty)$ is a continuous function with $0 < \int_0^\infty \delta_i(v)dv < +\infty$ and $\int_0^\infty v\delta_i(v)e^{\kappa v}dv < +\infty$ for a certain positive constant κ.

Let $L > 0,\ l > 0$ and $0 < \theta < 1$ be three constants such that

$$B^L = \left\{\varphi|\varphi \in PAP(\mathbb{R}, \mathbb{R}^n),\ \left\{|\varphi_i(t_1) - \varphi_i(t_2)|\right\} \le \left\{L|t_1 - t_2|\right\},\right.$$
$$\left.\text{for all } t_1, t_2 \in \mathbb{R}\right\},$$

and

$$\bar{B}^* = \left\{\varphi\middle|\,\|\varphi - \bar{\varphi}_0\|_{\mathbf{Z}} \le \frac{\theta l}{1-\theta},\ \varphi \in \mathbf{Z}\right\},$$

where

$$\bar{\varphi}_0 = \left\{\int_{-\infty}^t e^{-\int_s^t c_i(w)\int_0^{+\infty}\delta_i(v)dvdw} I_i(s)ds\right\},$$
$$\left\{\max\left\{K_i\frac{I_i^+}{\int_0^{+\infty}\delta_i(v)dv\bar{c}_i^{\,-}},\ (1 + K_i\frac{c_i^+}{\bar{c}_i^{\,-}})I_i^+\right\}\right\} \le \left\{l\right\}.$$

Then, we get

Theorem 3.2 *Let* $(\hat{S}_0)$, (S_1) *and* $(\hat{S}_2)$ *hold. Moreover, assume that there exist constants* $\theta > 0,\ \alpha_i > 0$ *and* $L > 0$, *such that*

$$\theta = \max_{i\in J}\left\{\left(1 + c_i^+\frac{K_i}{\bar{c}_i^{\,-}}\right)\left[c_i^+\int_0^{+\infty} v\delta_i(v)dv + \sum_{j=1}^n\left(a_{ij}^+L_j^f + b_{ij}^+L_j^g\right) + \sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right],\right.$$

$$\left.\frac{K_i}{\int_0^{+\infty}\delta_i(v)dv\bar{c}_i^{\,-}}\left[c_i^+\int_0^{+\infty} v\delta_i(v)dv + \sum_{j=1}^n\left(a_{ij}^+L_j^f + b_{ij}^+L_j^g\right) + \sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right]\right\} < 1,$$

$$\left\{\left[c_i^+\int_0^{+\infty}\delta_i(v)dv + c_i^+\int_0^{+\infty} v\delta_i(v)dv + \sum_{j=1}^n(a_{ij}^+L_j^f + b_{ij}^+L_j^g) + \sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+L_j^{\bar{g}}M_j^{\bar{g}}\right]\frac{l}{1-\theta} + I_i^+\right\} \le \left\{L\right\},$$

$$-\int_0^{+\infty}\delta_i(v)dv\bar{c}_i^{\,-} + K_i\left[c_i^+\int_0^{+\infty} v\delta_i(v)dv + \sum_{j=1}^n a_{ij}^+L_j^f + \sum_{j=1}^n b_{ij}^+L_j^g(1+L)\right.$$

$$\left.+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}^+(1+L)(M_l^{\bar{g}}L_j^{\bar{g}} + M_j^{\bar{g}}L_l^{\bar{g}})\right] \le -\alpha_i,$$

and

$$\frac{\int_0^{+\infty}\delta_i(v)dv\bar{c}_i^{\,-}-\alpha_i}{K_i} + c_i^+\int_0^{+\infty}\delta_i(v)dv\left(1-\frac{\alpha_i}{\int_0^{+\infty}\delta_i(v)dv\bar{c}_i^+}\right) < 1,\ i\in J.$$

Then Eq. (1.2) has a unique pseudo almost periodic solution in the region $\bar{\mathbf{B}} = B^L \bigcap \bar{B}^*$.

Proof For any $\varphi \in \bar{\mathbf{B}}$, in view of $(\hat{S}_2)$, an argument similar to the one used in Lemma 3.1 of [17] shows that

$$\int_0^\infty \delta_i(s)\int_{t-s}^t \varphi_i'(u)duds = \int_0^\infty \delta_i(s)ds\varphi_i(t) - \int_0^\infty \delta_i(s)\varphi_i(t-s)ds \in PAP(\mathbb{R},\mathbb{R}),$$

for all $i\in J$. Thus, the nonlinear pseudo almost periodic differential equations,

$$\begin{aligned} x_i'(t) = &-c_i(t)\int_0^{+\infty}\delta_i(v)dvx_i(t) + c_i(t)\int_0^{+\infty}\delta_i(v)\int_{t-v}^t\varphi_i'(u)dudv \\ &+\sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t)) + \sum_{j=1}^n b_{ij}(t)g_j(\varphi_j(\varphi_j(t))) \\ &+\sum_{j=1}^n\sum_{l=1}^n e_{ijl}(t)\bar{g}_j(\varphi_j(\varphi_j(t)))\bar{g}_l(\varphi_l(\varphi_l(t))) + I_i(t),\ i\in J, \end{aligned}$$

has exactly one pseudo almost periodic solution:

$$\begin{aligned} x^\varphi(t) = \Bigg\{ \int_{-\infty}^t e^{-\int_s^t c_i(u)\int_0^{+\infty}\delta_i(v)dvdu}\Bigg[c_i(s)\int_0^{+\infty}\delta_i(v)\int_{s-v}^s\varphi_i'(u)dudv \\ +\sum_{j=1}^n a_{ij}(s)f_j(\varphi_j(s)) + \sum_{j=1}^n b_{ij}(s)g_j(\varphi_j(\varphi_j(s))) \\ +\sum_{j=1}^n\sum_{l=1}^n e_{ijl}(t)\bar{g}_j(\varphi_j(\varphi_j(t)))\bar{g}_l(\varphi_l(\varphi_l(t))) + I_i(s)\Bigg]ds\Bigg\}, \end{aligned}$$

and

$$\begin{aligned} (x^\varphi(t))' = \Bigg\{\Bigg[c_i(t)\int_0^{+\infty}\delta_i(v)\int_{t-v}^t\varphi_i'(u)dudv \\ +\sum_{j=1}^n a_{ij}(t)f_j(\varphi_j(t)) + \sum_{j=1}^n b_{ij}(t)g_j(\varphi_j(\varphi_j(t))) \\ +\sum_{j=1}^n\sum_{l=1}^n e_{ijl}(t)\bar{g}_j(\varphi_j(\varphi_j(t)))\bar{g}_l(\varphi_l(\varphi_l(t))) + I_i(t)\Bigg] \\ - c_i(t)\int_0^{+\infty}\delta_i(v)dvx_i^\varphi(t)\Bigg\} \in PAP(\mathbb{R},\mathbb{R}^n). \end{aligned}$$

Then we complete the proof similarly to that of Theorem 3.1.

4 Examples and remarks

Consider the following HCNNs with oscillating leakage coefficients and complex deviating arguments:

$$\left\{\begin{array}{rcl} x_1'(t) &=& -\frac{1}{4}(2\sin 400t+1)x_1(t-\frac{|\cos t|}{100}) + \frac{|\cos t|}{100}\arctan(x_1(t)) + \frac{|\cos\sqrt{2}t|}{100}\arctan(x_2(t)) \\ && +\frac{|\cos\sqrt{3}t|}{100}\sin(x_1(x_1(t))) + \frac{|\cos\sqrt{5}t|}{100}\sin(x_2(x_2(t))) \\ && +\frac{|\cos\sqrt{2}t|}{1000}\cos(x_1(x_1(t)))\cos(x_1(x_1(t))) + \frac{|\cos\sqrt{3}t|}{1000}\cos(x_1(x_1(t)))\cos(x_2(x_2(t))) \\ && +\frac{|\cos\sqrt{3}t|}{1000}\cos(x_2(x_2(t)))\cos(x_1(x_1(t))) + \frac{|\cos 2t|}{1000}\cos(x_2(x_2(t)))\cos(x_2(x_2(t))) \\ && +\frac{1}{100}e^{-t^4\sin^2 t}, \\ x_2'(t) &=& -\frac{1}{4}(2\sin 400t+1)x_2(t-\frac{|\sin t|}{100}) + \frac{|\sin t|}{100}\arctan(x_1(t)) + \frac{|\sin\sqrt{2}t|}{100}\arctan(x_2(t)) \\ && +\frac{|\sin\sqrt{3}t|}{100}\sin(x_1(x_1(t))) + \frac{|\sin\sqrt{5}t|}{100}\sin(x_2(x_2(t))) \\ && +\frac{|\sin\sqrt{2}t|}{1000}\cos(x_1(x_1(t)))\cos(x_1(x_1(t))) + \frac{|\sin\sqrt{3}t|}{1000}\cos(x_1(x_1(t)))\cos(x_2(x_2(t))) \\ && +\frac{|\sin\sqrt{3}t|}{1000}\cos(x_2(x_2(t)))\cos(x_1(x_1(t))) + \frac{|\sin 2t|}{1000}\cos(x_2(x_2(t)))\cos(x_2(x_2(t))) \\ && +\frac{1}{100}e^{-t^4\cos^2 t}, \end{array}\right. \quad (4.1)$$

and

$$\begin{cases} x_1'(t) &= -\frac{1}{4}(2\sin 400t+1)\int_0^{+\infty} e^{-s}x_1(t-s)ds + \frac{|\cos t|}{100}\arctan(x_1(t)) + \frac{|\cos\sqrt{3}t|}{100}\arctan(x_2(t)) \\ &+ \frac{|\cos\sqrt{5}t|}{100}\sin(x_1(x_1(t))) + \frac{|\cos\sqrt{7}t|}{100}\sin(x_2(x_2(t))) \\ &+ \frac{|\cos\sqrt{2}t|}{1000}\cos(x_1(x_1(t)))\cos(x_1(x_1(t))) + \frac{|\cos\sqrt{3}t|}{1000}\cos(x_1(x_1(t)))\cos(x_2(x_2(t))) \\ &+ \frac{|\cos\sqrt{3}t|}{1000}\cos(x_2(x_2(t)))\cos(x_1(x_1(t))) + \frac{|\cos 2t|}{1000}\cos(x_2(x_2(t)))\cos(x_2(x_2(t))) \\ &+ \frac{1}{100}e^{-t^2\sin^4 t}, \\ x_2'(t) &= -\frac{1}{4}(2\sin 400t+1)\int_0^{+\infty} e^{-s}x_2(t-s)ds + \frac{|\sin t|}{100}\arctan(x_1(t)) + \frac{|\sin\sqrt{3}t|}{100}\arctan(x_2(t)) \\ &+ \frac{|\sin\sqrt{5}t|}{100}\sin(x_1(x_1(t))) + \frac{|\sin\sqrt{7}t|}{100}\sin(x_2(x_2(t))) \\ &+ \frac{|\sin\sqrt{2}t|}{1000}\cos(x_1(x_1(t)))\cos(x_1(x_1(t))) + \frac{|\sin\sqrt{3}t|}{1000}\cos(x_1(x_1(t)))\cos(x_2(x_2(t))) \\ &+ \frac{|\sin\sqrt{3}t|}{1000}\cos(x_2(x_2(t)))\cos(x_1(x_1(t))) + \frac{|\sin 2t|}{1000}\cos(x_2(x_2(t)))\cos(x_2(x_2(t))) \\ &+ \frac{1}{100}e^{-t^2\cos^4 t}. \end{cases} \tag{4.2}$$

Obviously, it is straightforward to show directly that system (4.1) and system (4.2) satisfy the conditions in Theorem 3.1 and Theorem 3.2, respectively. Therefore, either system (4.1) or system (4.2) has exactly one pseudo almost periodic solution in the region $\mathbf{B} = B^{\frac{1}{5}} \bigcap B^*$.

Remark 4.1 Systems (4.1) and (4.2) are a very simple HCNNs with oscillating leakage coefficients and complex deviating arguments in [1–5, 7], only HCNNs with nonoscillating leakage coefficients are studied. One can observe that all results there and the references cited therein can not be applicable for system (4.1) or (4.2). Moreover, the existence of pseudo almost periodic solutions on HCNNs with complex deviating arguments has not been touched in [1, 6, 8–33] and hence the results there cannot be applied to system (4.1) or (4.2).

5 Conclusions

In this paper, the existences and uniqueness of pseudo almost periodic solutions for high-order cellular neural networks models with complex deviating arguments and involving time-varying delays, and continuously distributed delays in the leakage terms have been discussed. By employing differential inequality techniques, several sufficient conditions have been obtained to ensure the existence and uniqueness of pseudo almost periodic for the considered neural networks. The proposed results extend and improve some known results. In order to demonstrate the usefulness of the presented results, some numerical examples are given. The established results are compared with those of recent methods existing in the literature. We expect to extend this work to the anti-periodic solution and weighted pseudo almost periodic solution problems on high-order cellular neural networks models with oscillating leakage coefficients and complex deviating arguments.

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