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ORIGINAL ARTICLE

Exponential operational laws and new aggregation operators of intuitionistic Fuzzy information based on Archimedean T-conorm and T-norm

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Abstract Atanassov extended the fuzzy set to intuitionistic fuzzy set (IFS) whose basic components are intuitionistic fuzzy numbers (IFNs). IFSs and IFNs can depict the fuzzy characteristics of the objects comprehensively, and lots of operational laws have been introduced to facilitate the use of IFSs and IFNs for solving the practical problems under intuitionistic fuzzy environments. As a supplement of the existing operational laws, we define the exponential operational laws of IFSs and IFNs based on Archimedean t-conorm and t-norm (EOL-IFS-A and EOL-IFN-A), which can be considered as the more general forms of the original exponential operational law. After that, we study the properties of the EOL-IFS-A and EOL-IFN-A. Then, we develop an approach for multiple criteria decision making with intuitionistic fuzzy information. Finally, we give an example to illustrate the application of the developed approach, and make a detailed comparison with the existing method so as to show the advantages of our approach.

Keywords Intuitionistic fuzzy set · Intuitionistic fuzzy number · Exponential operational law · Archimedean t-conorm and t-norm · Multiple criteria decision making

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1 Introduction

Most of the existing mathematical tools for formal modeling, reasoning and computing are crisp, deterministic and precise in nature, which are not capable of dealing with the problems involving uncertainty, imprecision or fuzziness. Fuzzy set (FS) [1], characterized by the membership function, is suitable to deal with those uncertain or fuzzy problems. Later, Atanassov extended the FS to intuitionistic fuzzy set (IFS) [2, 3]. The IFS is constructed by three functions, i.e., the membership function, the non-membership function, and the indeterminacy function, and thus, the IFS can describe uncertainty and fuzziness more comprehensively than the FS. Atanassov [4] and De et al. [5] introduced some basic operational laws of IFSs. For simplicity, Xu and Yager [6–8] defined the concept of intuitionistic fuzzy number (IFN) and gave some operational laws of IFNs, such as "intersection", "union", "supplement", "power" and so on. Besides, Gou et al. [9] presented the exponential operational law of IFNs, which is an effective supplement for the calculations of IFNs.

Based on these operational laws of IFNs, lots of intuitionistic fuzzy aggregation operators have been developed, such as the intuitionistic fuzzy weighted averaging (IFWA) operator [7], the intuitionistic fuzzy weighted geometric (IFWG) operator [6], the intuitionistic fuzzy ordered weighted averaging (IFOWA) [8] operator, and the intuitionistic fuzzy weighted exponential aggregation (IFWEA) operator [9], etc.

With the advantage in depicting uncertain and fuzzy information, IFSs and IFNs have been widely applied in many practical areas of modern life, including aggregation techniques [6–16], distance measures [17–19], correlation measures [20–23], intuitionistic preference relations [24–26], dynamic decision making [27–29], intuitionistic fuzzy calculus [30–32], system fault analysis [33–35], and multiple criteria decision making (MCDM) [12, 13, 18, 21, 35–45]. For the decision making problem with a normal decision matrix and intuitionistic fuzzy weights, Gou et al. [9] introduced the exponential operational law and the IFWEA operator, but they are far from meeting the actual needs. In this paper, we define the exponential operational laws of IFSs and IFNs based on Archimedean t-conorm and t-norm (EOL-IFS-A and EOL-IFN-A), where the bases are positive real numbers and the exponents are IFSs or IFNs. With Archimedean t-conorm and t-norm [46–48], and the aggregation functions for the classical IFSs [6–16, 49], we can assign different functions based on t-conorm and t-norm to get different forms of exponential operational laws and aggregation operators for intuitionistic fuzzy information, which can help the decision makers to deal with different relationships of the aggregated intuitionistic fuzzy arguments and give them more choices.

The remainder of this paper is organized as follows: some basic knowledge related to IFSs, IFNs and t-conorm and t-norm are introduced in Sect. 2. Section 3 gives the definitions, properties of EOL-IFS-A and EOL-IFN-A, and develops an intuitionistic fuzzy exponential aggregation operator based on t-conorm and t-norm. Section 4 uses the operator to develop a MCDM approach for solving the problems with intuitionistic fuzzy information, and employs an example to illustrate the application of the developed approach. The paper ends with some conclusions in Sect. 5.

2 Preliminaries

In this section, we recall some basic concepts and operational laws of IFSs, IFNs and t-conorm and t-norm.

Definition 2.1 [2]. Let X be a fixed set, the IFS A can be defined as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$ with $\mu_A(x) \ge 0$, $\nu_A(x) \ge 0$ and $0 \le \mu_A(x) + \nu_A(x) \le 1$. $\mu_\alpha(x)$ and $\nu_\alpha(x)$ represent the membership degree and the non-membership degree, respectively. Moreover, $\pi_\alpha(x) = 1 - \mu_\alpha(x) - \nu_\alpha(x)$ is called the indeterminacy degree or hesitancy degree.

For convenience, the pair $\alpha = (\mu_\alpha, \nu_\alpha)$ is called an intuitionistic fuzzy number (IFN) [2], where $\mu_\alpha, \nu_\alpha \ge 0$ and $\mu_\alpha + \nu_\alpha \le 1$.

Definition 2.2 [50]. A function $T: [0, 1] \times [0, 1] \to [0, 1]$ is called a t-norm if it satisfies the following four conditions:

1. $T(1, x) = x$, for all x;
2. $T(x, y) = T(y, x)$, for all x and y;
3. $T(x, T(y, z)) = T(x, T(y, z))$, for all for all x, y and z;
4. If $x \le x'$ and $y \le y'$, then$T(x, y) \le T(x', y')$.

Definition 2.3 [50]. A function $S: [0, 1] \times [0, 1] \to [0, 1]$ is called a t-conorm if it satisfies the following four conditions:

1. $S(0, x) = x$, for all x;
2. $S(x, y) = S(y, x)$, for all x and y;
3. $S(x, S(y, z)) = S(x, S(y, z))$, for all for all x, y and z;
4. If $x \le x'$ and $y \le y'$, then $S(x, y) \le S(x', y')$.

Definition 2.4 [50]. If the t-norm function $T(x, y)$ is continuous and $T(x, x) < x$ for all $x \in (0, 1)$, then it is called an Archimedean t-norm. If an Archimedean t-norm is strictly increasing with respect to each variable for $x, y \in (0, 1)$, then it is called a strict Archimedean t-norm.

Definition 2.5 [50]. If a t-conorm function $S(x,y)$ is continuous and $S(x,x) > x$ for all $x \in (0, 1)$, then it is called an Archimedean t-conorm. If an Archimedean t-conorm is strictly increasing with respect to each variable for $x, y \in (0, 1)$, then it is called a strict Archimedean t-conorm.

Theorem 2.2 [50]. *Let* $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ $(i = 1, 2)$ *and* $\alpha = (\mu_\alpha, \nu_\alpha)$ *be three IFNs. Some basic operational laws for IFNs based on strict Archimedean t-conorm and t-norm can be expressed as follows*:

1. $\alpha_1 \oplus \alpha_2 = (S(\mu_{\alpha_1}, \mu_{\alpha_2}), T(\nu_{\alpha_1}, \nu_{\alpha_2})) = (h^{-1}(h(\mu_{\alpha_1}) + h(\mu_{\alpha_2})), g^{-1}(g(\nu_{\alpha_1}) + g(\nu_{\alpha_2})))$;
2. $\alpha_1 \otimes \alpha_2 = (T(\mu_{\alpha_1}, \mu_{\alpha_2}), S(\nu_{\alpha_1}, \nu_{\alpha_2})) = (g^{-1}(g(\mu_{\alpha_1}) + g(\mu_{\alpha_2})), h^{-1}(h(\nu_{\alpha_1}) + h(\nu_{\alpha_2})))$;
3. $\lambda\alpha = (h^{-1}(\lambda h(\mu_\alpha)), g^{-1}(\lambda g(\nu_\alpha)))$, $\lambda > 0$;
4. $\alpha^\lambda = (g^{-1}(\lambda g(\mu_\alpha)), h^{-1}(\lambda h(\nu_\alpha)))$, $\lambda > 0$.

3 Exponential operational law of IFNs based on Archimedean t-conorm and t-norm

3.1 Novel exponential operational laws

In Sect. 2, we have recalled some operational laws of IFSs and IFNs. However, we are still lack of one kind of exponential operational law, where the bases are positive real numbers and the exponents are IFSs or IFNs. Gou et al. [9] introduced a specific form of exponential operational law of IFSs and IFNs. In this section, we extend it to a much more general form, and propose the exponential operational laws of IFSs and IFNs based on Archimedean t-conorm and t-norm.

Definition 3.1 Let X be a fixed set, and $A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in X \right\}$ be an IFS on X, then

$$\lambda^A = \begin{cases} \left\{ \left\langle x, g^{-1}\left(\left(1 - \mu_A(x)\right) g(\lambda) \right), h^{-1}\left(\nu_A(x) g(\lambda) \right) \right\rangle | x \in X \right\}, & \lambda \in (0,1) \\ \left\{ \left\langle x, g^{-1}\left(\left(1 - \mu_A(x)\right) g\left(\frac{1}{\lambda}\right) \right), h^{-1}\left(\nu_A(x) g\left(\frac{1}{\lambda}\right) \right) \right\rangle | x \in X \right\} & \lambda \geq 1 \end{cases} \tag{1}$$

which is called the exponential operational law of IFSs based on Archimedean t-conorm and t-norm (EOL-IFS-A).

Definition 3.2 Let $\alpha = \left(\mu_\alpha, \nu_\alpha\right)$ be an IFN, then the exponential operational law of α is

$$\lambda^\alpha = \begin{cases} \left(g^{-1}\left(\left(1 - \mu_\alpha\right) g(\lambda) \right), h^{-1}\left(\nu_\alpha g(\lambda) \right) \right), & \lambda \in (0,1) \\ \left(g^{-1}\left(\left(1 - \mu_\alpha\right) g\left(\frac{1}{\lambda}\right) \right), h^{-1}\left(\nu_\alpha g\left(\frac{1}{\lambda}\right) \right) \right), & \lambda \geq 1 \end{cases} \tag{2}$$

which is called an exponential operational law of IFNs based on Archimedean t-conorm and t-norm (EOL-IFN-A), where λ^α is also an IFN.

The process of proving λ^α being an IFN is similar to the process of proving λ^A being an IFS, so we omit it here.

Compared to the exponential operational law given in Ref. [9], our exponential operational law (2) is more general and comprehensive. If we assign some specific forms to the function g, then we can get different forms of λ^α based on the well-known t-conorms and t-norms:

1. If $g(t) = -\log(t)$, then Eq. (2) reduces to

$$\lambda^\alpha = \begin{cases} \left(\lambda^{1 - \mu_\alpha}, \ 1 - \lambda^{\nu_\alpha} \right), & \lambda \in (0,1) \\ \left(\left(\frac{1}{\lambda}\right)^{1 - \mu_\alpha}, \ 1 - \left(\frac{1}{\lambda}\right)^{\nu_\alpha} \right), & \lambda \geq 1 \end{cases} \tag{3}$$

which is the exponential operational law of IFNs based on Algebraic t-conorm and t-norm.

2. If $g(t) = \log\left(\frac{2-t}{t}\right)$, then Eq. (2) reduces to

$$\lambda^\alpha = \begin{cases} \left(\frac{2\lambda^{1 - \mu_\alpha}}{(2-\lambda)^{1 - \mu_\alpha} + \lambda^{1 - \mu_\alpha}}, \ \frac{(2-(1-\mu_\alpha))^\lambda - (1-\mu_\alpha)^\lambda}{(2-(1-\mu_\alpha))^\lambda + (1-\mu_\alpha)^\lambda} \right), & \lambda \in (0,1) \\ \left(\frac{2\left(\frac{1}{\lambda}\right)^{1 - \mu_\alpha}}{\left(2-\left(\frac{1}{\lambda}\right)\right)^{1 - \mu_\alpha} + \left(\frac{1}{\lambda}\right)^{1 - \mu_\alpha}}, \ \frac{(2-(1-\mu_\alpha))^{\left(\frac{1}{\lambda}\right)} - (1-\mu_\alpha)^{\left(\frac{1}{\lambda}\right)}}{(2-(1-\mu_\alpha))^{\left(\frac{1}{\lambda}\right)} + (1-\mu_\alpha)^{\left(\frac{1}{\lambda}\right)}} \right), & \lambda \geq 1 \end{cases} \tag{4}$$

which is the exponential operational law of IFNs based on Einstein t-conorm and t-norm.

3. If $g(t) = \log\left(\frac{\gamma+(1-\gamma)t}{t}\right)$, then Eq. (2) reduces to

$$\lambda^\alpha = \begin{cases} \left(\frac{\gamma\lambda^{1 - \mu_\alpha}}{1+(\gamma-1)(1-\lambda)^{1 - \mu_\alpha} + (\gamma-1)\lambda^{1 - \mu_\alpha}}, \ \frac{(1+(\gamma-1)(1-\lambda))^{\nu_\alpha} - \lambda^{\nu_\alpha}}{(1+(\gamma-1)(1-\lambda))^{\nu_\alpha} + (\gamma-1)\lambda^{\nu_\alpha}} \right), & \lambda \in (0,1) \\ \left(\frac{\gamma\left(\frac{1}{\lambda}\right)^{1 - \mu_\alpha}}{1+(\gamma-1)\left(1-\left(\frac{1}{\lambda}\right)\right)^{1 - \mu_\alpha} + (\gamma-1)\left(\frac{1}{\lambda}\right)^{1 - \mu_\alpha}}, \ \frac{\left(1+(\gamma-1)\left(1-\left(\frac{1}{\lambda}\right)\right)\right)^{\nu_\alpha} - \left(\frac{1}{\lambda}\right)^{\nu_\alpha}}{\left(1+(\gamma-1)(1-\lambda)\left(\frac{1}{\lambda}\right)\right)^{\nu_\alpha} + (\gamma-1)\left(\frac{1}{\lambda}\right)^{\nu_\alpha}} \right), & \lambda \geq 1 \end{cases} \tag{5}$$

which is the exponential operational law of IFNs based on Hamacher t-conorm and t-norm.

4. If $g(t) = \log\left(\frac{\gamma-1}{\lambda^t-1}\right)$, then Eq. (2) reduces to

$$\lambda^\alpha = \begin{cases} \left(\log_\gamma\left(1 + \frac{(\gamma^\lambda-1)^{1-\mu_\alpha}}{(\gamma-1)^{-\mu_\alpha}}\right), \ 1 - \left(\log_\gamma\left(1 + \frac{(\gamma^\lambda-1)^{\nu_\alpha}}{(\gamma-1)^{\nu_\alpha-1}}\right)\right) \right), & \lambda \in (0,1) \\ \left(\log_\gamma\left(1 + \frac{\left(\gamma^{\left(\frac{1}{\lambda}\right)}-1\right)^{1-\mu_\alpha}}{(\gamma-1)^{-\mu_\alpha}}\right), \ 1 - \left(\log_\gamma\left(1 + \frac{\left(\gamma^{\left(\frac{1}{\lambda}\right)}-1\right)^{\nu_\alpha}}{(\gamma-1)^{\nu_\alpha-1}}\right)\right) \right), & \lambda \geq 1 \end{cases} \tag{6}$$

which is the exponential operational law of IFNs based on Frank t-conorm and t-norm.

As we can see, the EOL-IFN-A has a lot to do with the original exponential operational law defined by Gou et al. [9]. That is to say, the latter is the specific form of the former when Algebraic t-conorm and t-norm are assigned. In fact, some frequently-used forms of operational laws of IFNs can be obtained by those operational laws expressed by Archimedean t-conorm and t-norm.

Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ $(i = 1, 2)$ be two IFNs, λ and k be two real numbers, then we can get some operational formulas based on Definition 3.2 and Theorem 2.2. In what follows, we only consider the situation where $\lambda \in (0, 1)$, and the expressions can be similarly deduced when $\lambda \geq 1$.

1. $\lambda^{\alpha_1} \oplus \lambda^{\alpha_2} = \left(h^{-1}\left(h\left(g^{-1}\left(\left(1-\mu_{\alpha_1}\right)g(\lambda)\right)\right) + h\left(g^{-1}\left(\left(1-\mu_{\alpha_2}\right)g(\lambda)\right)\right)\right),\ g^{-1}\left(g\left(h^{-1}\left(\nu_{\alpha_1}g(\lambda)\right)\right) + g\left(h^{-1}\left(\nu_{\alpha_2}g(\lambda)\right)\right)\right)\right)$;
2. $\lambda^{\alpha_1} \otimes \lambda^{\alpha_2} = \left(g^{-1}\left(2-\mu_{\alpha_1}-\mu_{\alpha_2}\right)g(\lambda),\ h^{-1}\left(\left(\nu_{\alpha_1}+\nu_{\alpha_2}\right)g(\lambda)\right)\right)$;
3. $\lambda^{\alpha_1 \oplus \alpha_2} = \left(g^{-1}\left(\left(g^{-1}\left(h\left(\mu_{\alpha_1}\right)+h\left(\mu_{\alpha_2}\right)\right)\right)g(\lambda)\right),\ h^{-1}\left(g^{-1}\left(g\left(\nu_{\alpha_1}\right)+g\left(\nu_{\alpha_2}\right)\right)g(\lambda)\right)\right)$;
4. $\lambda^{\alpha_1 \otimes \alpha_2} = \left(g^{-1}\left(\left(h^{-1}\left(g\left(\mu_{\alpha_1}\right)+g\left(\mu_{\alpha_2}\right)\right)\right)g(\lambda)\right), h^{-1}\left(h^{-1}\left(h\left(\nu_{\alpha_1}\right)+h\left(\nu_{\alpha_2}\right)\right)g(\lambda)\right)\right)$;
5. $k\lambda^{\alpha} = \left(h^{-1}\left(kh\left(g^{-1}\left(\left(1-\mu_{\alpha}\right)g(\lambda)\right)\right)\right), g^{-1}\left(kg\left(h^{-1}\left(\nu_{\alpha}g(\lambda)\right)\right)\right)\right)$.

3.2 Some basic properties of EOL-IFN-As

Theorem 3.1 *Let $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ be two IFNs, $\lambda \in (0, 1)$, then*

(1) $\lambda^{\alpha_1} \oplus \lambda^{\alpha_2} = \lambda^{\alpha_2} \oplus \lambda^{\alpha_1}$; (2) $\lambda^{\alpha_1} \otimes \lambda^{\alpha_2} = \lambda^{\alpha_2} \otimes \lambda^{\alpha_1}$.

Theorem 3.2 *Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ $(i = 1, 2, 3)$ be three IFNs, $\lambda \in (0, 1)$, then*

(1) $(\lambda^{\alpha_1} \oplus \lambda^{\alpha_2}) \oplus \lambda^{\alpha_3} = \lambda^{\alpha_1} \oplus (\lambda^{\alpha_2} \oplus \lambda^{\alpha_3})$; (2) $(\lambda^{\alpha_1} \otimes \lambda^{\alpha_2}) \otimes \lambda^{\alpha_3} = \lambda^{\alpha_1} \otimes (\lambda^{\alpha_2} \otimes \lambda^{\alpha_3})$.

Theorem 3.3 *Let $\alpha_1 = (\mu_{\alpha_1}, \nu_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, \nu_{\alpha_2})$ be two IFNs, $\lambda \in (0, 1)$, $k > 0$, then*

(1) $k(\lambda^{\alpha_1} \oplus \lambda^{\alpha_2}) = k\lambda^{\alpha_1} \oplus k\lambda^{\alpha_2}$; (2) $(\lambda^{\alpha_1} \otimes \lambda^{\alpha_2})^k = (\lambda^{\alpha_1})^k \otimes (\lambda^{\alpha_2})^k$.

In Theorem 3.3, we only have two EOL-IFN-As λ^{α_1} and λ^{α_2}, we can extend (1) and (2) of Theorem 3.3 to the general forms with a collection of n EOL-IFN-As $\lambda^{\alpha_1}, \lambda^{\alpha_2}, \ldots, \lambda^{\alpha_n}$:

$k(\lambda^{\alpha_1} \oplus \lambda^{\alpha_2} \oplus \cdots \oplus \lambda^{\alpha_n}) = k\lambda^{\alpha_1} \oplus k\lambda^{\alpha_2} \oplus \cdots \oplus k\lambda^{\alpha_n}$
$(\lambda^{\alpha_1} \otimes \lambda^{\alpha_2} \otimes \cdots \otimes \lambda^{\alpha_n})^k = (\lambda^{\alpha_1})^k \otimes (\lambda^{\alpha_2})^k \otimes \cdots \otimes (\lambda^{\alpha_n})^k$.

Theorem 3.4 *Let $\alpha = (\mu_\alpha, \nu_\alpha)$ be an IFN, $\lambda \in (0, 1)$, and $k_1, k_2 > 0$, then*

(1) $k_1\lambda^\alpha \oplus k_2\lambda^\alpha = (k_1 + k_2)\lambda^\alpha$; (2) $(\lambda^\alpha)^{k_1} \otimes (\lambda^\alpha)^{k_2} = (\lambda^\alpha)^{k_1+k_2}$.

Similar to Theorem 3.3, we can further extend (1) and (2) to the following general forms:

$k_1\lambda^\alpha \oplus k_2\lambda^\alpha \oplus \cdots \oplus k_n\lambda^\alpha = (k_1 + k_2 + \cdots k_n)\lambda^\alpha$ and $(\lambda^\alpha)^{k_1} \otimes (\lambda^\alpha)^{k_2} \otimes \cdots \otimes (\lambda^\alpha)^{k_n} = (\lambda^\alpha)^{k_1+k_2+\cdots+k_n}$.

Theorem 3.5 *Let $\alpha = (\mu_\alpha, \nu_\alpha)$ be an IFN, if $\lambda_1 \geq \lambda_2$, then we can get*

(1) $(\lambda_1)^\alpha \geq (\lambda_2)^\alpha$, *if* $\lambda_1, \lambda_2 \in (0, 1)$; (2) $(\lambda_1)^\alpha \leq (\lambda_2)^\alpha$, *if* $\lambda_1, \lambda_2 \geq 1$.

Proof When $\lambda_1, \lambda_2 \in (0, 1)$, based on the EOL-IFN-As, we know.

${\lambda_1}^\alpha = \left(g^{-1}\left(\left(1-\mu_\alpha\right)g\left(\lambda_1\right)\right),\ h^{-1}\left(\nu_\alpha g\left(\lambda_1\right)\right)\right)$ and ${\lambda_2}^\alpha = \left(g^{-1}\left(\left(1-\mu_\alpha\right)g\left(\lambda_2\right)\right),\ h^{-1}\left(\nu_\alpha g\left(\lambda_2\right)\right)\right)$.

It is obvious that Archimedean t-norm is a strictly decreasing function g:$[0, 1] \rightarrow [0, \infty]$, and Archimedean t-conorm is a strictly increasing function h:$[0, 1] \rightarrow [0, \infty]$. It's easy to prove that the function $g^{-1}(x)$ is strictly decreasing and $h^{-1}(x)$ is strictly increasing. Therefore, if $\lambda_1, \lambda_2 \in (0, 1)$ and $\lambda_1 \geq \lambda_2$, then we have $g(\lambda_1) \leq g(\lambda_2)$. Thus

$$g^{-1}\left(\left(1-\mu_\alpha\right)g\left(\lambda_1\right)\right) \geq g^{-1}\left(\left(1-\mu_\alpha\right)g\left(\lambda_2\right)\right) \tag{7}$$

Similarly, we can also prove.

$$h^{-1}\left(\nu_\alpha g\left(\lambda_1\right)\right) \leq h^{-1}\left(\nu_\alpha g\left(\lambda_2\right)\right) \tag{8}$$

Combining Eqs. (7) and (8), we get $s\left(\left(\lambda_1\right)^\alpha\right) \geq s\left(\left(\lambda_2\right)^\alpha\right)$. Therefore, we conclude that if $\lambda_1, \lambda_2 \in (0, 1)$, then $(\lambda_1)^\alpha \geq (\lambda_2)^\alpha$.

However, when $\lambda_1, \lambda_2 \geq 1$ and $\lambda_1 \geq \lambda_2$, we can obtain $0 < 1/\lambda_1 \leq 1/\lambda_2 \leq 1$. Based on what we have discussed above, we can get $(\lambda_1)^\alpha \leq (\lambda_2)^\alpha$. This completes the proof of the theorem.

Next, let's take a look at some special values of λ^α:

1. If $\lambda = 1$, then $\lambda^\alpha = (1, 0)$;
2. If $\alpha = (1, 0)$, then $\lambda^\alpha = (1, 0)$;
3. If $\alpha = (0, 1)$, then $\lambda^\alpha = (\lambda, 1 - \lambda)$.

3.3 The aggregation technique of EOL-IFN-As

Based on the EOL-IFS-As and their basic properties, we can get a novel aggregation operator:

Definition 3.3 Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ $(i = 1, 2, \ldots, n)$ be a collection of IFNs, $\lambda_i \in (0, 1)$ $(i = 1, 2, \ldots, n)$, and let AIFWEA:$\Theta^n \rightarrow \Theta$. If

$$AIFWEA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \lambda_1^{\alpha_1} \otimes \lambda_2^{\alpha_2} \otimes \cdots \otimes \lambda_n^{\alpha_n} = \left(g^{-1}\left(\sum_{i=1}^{n} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right) \right) \tag{9}$$

then the function AIFWEA is called an Archimedean intuitionistic fuzzy weighted exponential aggregation (AIFWEA) operator, where $\alpha_i (i = 1, 2, \ldots, n)$ are the exponential weights of $\lambda_i (i = 1, 2, \ldots, n)$.

Now we prove the formula (9) by using the mathematical induction:

When $n = 2$, we have $\text{AIFWEA}(\alpha_1, \alpha_2) = \lambda_1^{\alpha_1} \otimes \lambda_2^{\alpha_2}$.

According to Definition 3.2 and Theorem 2.2, $\text{AIFWEA}(\alpha_1, \alpha_2)$ can be written as:

$$\text{AIFWEA}(\alpha_1, \alpha_2) = \lambda_1^{\alpha_1} \otimes \lambda_2^{\alpha_2} = \left(g^{-1}\left((1-\mu_{\alpha_1}) g(\lambda_1) + (1-\mu_{\alpha_2}) g(\lambda_2) \right), h^{-1}\left(\nu_{\alpha_1} g(\lambda_1) + \nu_{\alpha_2} g(\lambda_2) \right) \right)$$

Suppose that when $n = k$, the equation

$$AIFWEA(\alpha_1, \alpha_2, \ldots, \alpha_k) = \left(g^{-1}\left(\sum_{i=1}^{k} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{k} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right) \right)$$

holds and the aggregated value is also an IFN. Then when $k = n + 1$, by the operational laws in Sect. 3.1, we get

$$AIFWEA(\alpha_1, \alpha_2, \ldots, \alpha_k, \alpha_{k+1}) = AIFWEA(\alpha_1, \alpha_2, \ldots, \alpha_k) \otimes \lambda_{k+1}^{\alpha_{k+1}}$$

$$= \left(g^{-1}\left(\sum_{i=1}^{k+1} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{k+1} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right) \right)$$

whose result is also an IFN, by which we can draw a conclusion that when $k = n + 1$, Eq. (9) holds. Therefore, Eq. (9) holds for all n. The proof is completed.

Theorem 3.6 (Boundedness) *Let* $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ $(i = 1, 2, \cdots, n)$ *and*

$$\alpha_{\min} = \left(\min_i \{\mu_{\alpha_i}\}, \max_i \{\nu_{\alpha_i}\} \right), \quad \alpha_{\max} = \left(\max_i \{\mu_{\alpha_i}\}, \min_i \{\nu_{\alpha_i}\} \right)$$

$$\alpha^+ = \text{AIFWEA}(\alpha_{max}, \alpha_{max}, \cdots, \alpha_{max}) = \left(g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \max_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{n} \left(\min_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \right) \right)$$

$$\alpha^- = \text{AIFWEA}(\alpha_{\min}, \alpha_{\min}, \cdots, \alpha_{\min}) = \left(g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \min_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{n} \left(\max_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \right) \right)$$

then $\alpha^- \le \text{AIFWEA}(\alpha_1, \alpha_2, \cdots, \alpha_n) \le \alpha^+$.

Proof Since

$$1 - \min_i \{\mu_{\alpha_i}\} \ge 1 - \mu_{\alpha_i}, \ \max_i \{\nu_{\alpha_i}\} \ge \nu_{\alpha_i}$$
$$\text{and } 1 - \max_i \{\mu_{\alpha_i}\} \le 1 - \mu_{\alpha_i}, \ \min_i \{\nu_{\alpha_i}\} \le \nu_{\alpha_i}$$

and the values of the functions $g(x)$ and $h(x)$ should be no less than 0, then $g(\lambda_i) \ge 0$, and for any $i = 1, 2, \cdots, n$, we have

$$\left(1 - \min_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \ge (1-\mu_{\alpha_i}) g(\lambda_i), \ \max_i \{\nu_{\alpha_i}\} g(\lambda_i) \ge \nu_{\alpha_i} g(\lambda_i)$$

$$\left(1 - \max_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \le (1-\mu_{\alpha_i}) g(\lambda_i), \ \min_i \{\nu_{\alpha_i}\} g(\lambda_i) \le \nu_{\alpha_i} g(\lambda_i)$$

By adding n inequalities, where the value of i increases from 1 to n by 1 each time, we get

$$\sum_{i=1}^{n} \left(\left(1 - \min_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \right) \ge \sum_{i=1}^{n} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right),$$
$$\sum_{i=1}^{n} \left(\max_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \ge \sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right)$$

$$\sum_{i=1}^{n} \left(\left(1 - \max_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \right) \le \sum_{i=1}^{n} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right),$$
$$\sum_{i=1}^{n} \left(\min_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \le \sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right)$$

According to Definition 2.5, we know that $g^{-1}(x)$ is a strictly decreasing function and $h^{-1}(x)$ is a strictly increasing function, then the following four inequalities hold:

$$g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \min_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \right) \right) \le g^{-1}\left(\sum_{i=1}^{n} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right) \right),$$
$$h^{-1}\left(\sum_{i=1}^{n} \left(\max_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \right) \ge h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right)$$

$$g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \max_i \{\mu_{\alpha_i}\}\right) g(\lambda_i) \right) \right) \ge g^{-1}\left(\sum_{i=1}^{n} \left((1-\mu_{\alpha_i}) g(\lambda_i) \right) \right),$$
$$h^{-1}\left(\sum_{i=1}^{n} \left(\min_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \right) \le h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right)$$

Since

$$\alpha^- = \text{AIFWEA}(\alpha_{\min}, \alpha_{\min}, \cdots, \alpha_{\min}) = \left(g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \min_i \{\mu_{\alpha_i}\} \right) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{n} \left(\max_i \{\nu_{\alpha_i}\} g(\lambda_i) \right) \right) \right)$$

$$\text{AIFWEA}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \left(g^{-1}\left(\sum_{i=1}^{n} \left((1 - \mu_{\alpha_i}) g(\lambda_i) \right) \right), h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right) \right)$$

then based on the score function $s(\alpha) = \mu_\alpha - \nu_\alpha$, we can get the conclusion that $s(\alpha^-) \leq s(\text{AIFWEA}(\alpha_1, \alpha_2, \ldots, \alpha_n))$. In a similar way, we can easily prove $s(\text{AIFWEA}(\alpha_1, \alpha_2, \ldots, \alpha_n)) \leq s(\alpha^+)$. Thus, Theorem 3.6 holds, which completes the proof.

Theorem 3.7 (Monotonicity) *Let* $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ $(i = 1, 2, \ldots, n)$ *and* $\alpha_i^* = \left(\mu_{\alpha_i^*}, \nu_{\alpha_i^*}\right)$ $(i = 1, 2, \cdots, n)$ *be two collections of IFNs, if* $\mu_{\alpha_i} \leq \mu_{\alpha_i^*}$ *and* $\nu_{\alpha_i^*} \leq \nu_{\alpha_i}$, *for any* i. *Then*

$$\text{AIFWEA}(\alpha_1, \alpha_2, \ldots, \alpha_n) \leq \text{AIFWEA}(\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*) \tag{10}$$

Proof Since

$$1 - \mu_{\alpha_i} \geq 1 - \mu_{\alpha_i^*} \rightarrow \sum_{i=1}^{n} \left((1 - \mu_{\alpha_i}) g(\lambda_i) \right) \geq \left(\sum_{i=1}^{n} \left(\left(1 - \mu_{\alpha_i^*} \right) g(\lambda_i) \right) \right) \rightarrow g^{-1}\left(\sum_{i=1}^{n} \left((1 - \mu_{\alpha_i}) g(\lambda_i) \right) \right) \leq g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \mu_{\alpha_i^*} \right) g(\lambda_i) \right) \right)$$

$$\nu_{\alpha_i^*} \leq \nu_{\alpha_i} \rightarrow \nu_{\alpha_i^*} g(\lambda_i) \leq \nu_{\alpha_i} g(\lambda_i) \rightarrow \sum_{i=1}^{n} \left(\nu_{\alpha_i^*} g(\lambda_i) \right) \leq \sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \rightarrow h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i^*} g(\lambda_i) \right) \right) \leq h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right)$$

then the score functions of IFNs α and α^* are

$$s(\alpha) = g^{-1}\left(\sum_{i=1}^{n} \left((1 - \mu_{\alpha_i}) g(\lambda_i) \right) \right) - h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right),$$

$$s(\alpha^*) = g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \mu_{\alpha_i^*} \right) g(\lambda_i) \right) \right) - h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i^*} g(\lambda_i) \right) \right)$$

With the inequalities

$$g^{-1}\left(\sum_{i=1}^{n} \left((1 - \mu_{\alpha_i}) g(\lambda_i) \right) \right) \leq g^{-1}\left(\sum_{i=1}^{n} \left(\left(1 - \mu_{\alpha_i^*} \right) g(\lambda_i) \right) \right),$$

$$h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i^*} g(\lambda_i) \right) \right) \leq h^{-1}\left(\sum_{i=1}^{n} \left(\nu_{\alpha_i} g(\lambda_i) \right) \right)$$

we know $s(\alpha) \leq s(\alpha^*)$, based on which we can see that Eq. (10) holds.

4 The application of the EOL-IFN-As and the AIFWEA operator

In this section, we apply the AIFWEA operator to develop a MCDM method, which involves the following steps:*Step 1* Consider a MCDM problem, there are m alternatives $Y_i (i = 1, 2, \ldots, m)$ and n criteria $G_j (j = 1, 2, \ldots, n)$. The decision maker constructs the decision matrix $R = (\gamma_{ij})_{m \times n}$, where γ_{ij} represents the degree that the decision maker prefers the alternative Y_i with respect to the criterion G_j. Moreover, the weights of the criteria are expressed as the IFNs $\alpha_j = \left(\mu_{\alpha_j}, \nu_{\alpha_j}\right)$ $(j = 1, 2, \ldots, n)$, where μ_{α_j} indicates the degree that the decision maker prefers the criterion G_j, and ν_{α_j} indicates the degree that the decision maker does not prefers the criterion G_j.

Step 2 Transform the decision matrix $R = (\gamma_{ij})_{m \times n}$ into the normalized decision matrix $D = (\lambda_{ij})_{m \times n}$, where

$$\lambda_{ij} = \begin{cases} \gamma_{ij}, & \text{for benifit attribute } G_j \\ 1 - \gamma_{ij}, & \text{for cost attribute } G_j \end{cases}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n$$

Step 3 Utilize the AIFWEA operator to aggregate the characteristics $\lambda_{ij} (i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n)$ and the IFNs $\alpha_j = \left(\mu_{\alpha_j}, \nu_{\alpha_j}\right)(j = 1, 2, \ldots, n)$ to get the overall value $d_i = \text{AIFWEA}(\alpha_1, \alpha_2, \ldots, \alpha_n)$ of each alternative Y_i.

Step 4 Utilize the score function to calculate the scores $s(d_i)$ $(i = 1, 2, \ldots, m)$ of the overall valuesd$_i (i = 1, 2, \ldots, m)$, which also represent the scores of the alternatives $Y_i (i = 1, 2, \ldots, m)$.

Step 5 Utilize the scores $s(d_i)(i = 1, 2, \ldots, m)$ to rank and select the alternatives $Y_i (i = 1, 2, \ldots, m)$. If two alternatives Y_{k_1} and Y_{k_2} have the same scores, we have to calculate the accuracy degrees $h(d_{k_1})$ and $h(d_{k_2})$ of these two alternatives. Then we rank the alternatives Y_{k_1} and Y_{k_2} by $h(d_{k_1})$ and $h(d_{k_2})$.

Example 4.1 Two past eight on April 20th, 2013, a violent earthquake occurred in Lushan County of Sichuan Province, Ya'an city. Most buildings in Longmen Town, the epicenter of the quake, collapsed and caused great damage. After the rescue operation, with the great leadership

Table 1 Intuitionistic fuzzy decision matrix

γ_{ij}	G1	G2	G3	G4
Y1	0.8	0.6	0.9	0.3
Y2	0.7	0.7	0.8	0.3
Y3	0.7	0.8	0.6	0.2
Y4	0.8	0.9	0.6	0.2

Table 2 Normalized intuitionistic fuzzy decision matrix

λ_{ij}	G1	G2	G3	G4
Y1	0.8	0.6	0.9	0.7
Y2	0.7	0.7	0.8	0.7
Y3	0.7	0.8	0.6	0.8
Y4	0.8	0.9	0.6	0.8

of China central government and the State Council and the help from all circles of the society, post-earthquake reconstruction goes well.

Now, the local government plans to build a new library for a middle school in Longmen town. Four construction companies present their designs and compete against each other for reaching the construction project. We have to choose the best design among those four designs.

A high level of earthquake resistance is strictly required and all these four designs perfectly meet the requirement. Besides, after discussion, four criteria are chosen to evaluate these designs of library.

G_1: Energy conservation and environmental protection, which mean that the library should be environmental friendly and save energy as much as possible. For example, the library should take the most advantage of the natural daylight instead of totally depending on the lamps.

G_2: Functional and humanized, which require that the library should have complete and specific function zones in good architectural compositions. Those function zones don't interact each other. Otherwise, the design would consider the rationality of landscape with humanization design.

G_3: Advanced building technique. It requires that the design should have a rational structure and construction scheme, which leads to a stronger and more sustainable building. With an advanced building technique, the building will be more accessible for erection and construction.

G_4: Cost, which requires that the budget of the whole program should be reasonable and acceptable.

After carefully evaluating all the alternatives, the experts give the degrees that they prefer the alternatives Y_i $(i = 1, 2, 3, 4)$ with respect to the criteria G_j $(j = 1, 2, 3, 4)$ in some real numbers ranging from 0 to 1 as shown in Table 1.

Meanwhile, the weight vector of all criteria is given as $\alpha = ((0.4, 0.4), (0.7, 0.2), (0.3, 0.5), (0.5, 0.4))$.

In those four given criteria, the criterion G_4 (cost) is a cost criterion, so we have to transform the decision matrix $R = (\gamma_{ij})_{m\times n}$ into the normalized decision matrix as follows (Table 2):

If we choose the AIFWEA operator based on Algebraic t-conorm and t-norm, in which $g(t) = -\log t$, and $h(t) = -\log(1 - t)$, then the AIFWEA operator reduces to the IFWEA operator [9]:

$$IFWEA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \left(\prod_{i=1}^{n} \lambda_i^{1-\mu_{\alpha_i}},\ 1 - \prod_{i=1}^{n} \lambda_i^{\nu_{\alpha_i}} \right)$$

Thus, the aggregated value of the alternative Y_1 can be computed by

$$d_1 = IFWEA(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \left(\prod_{i=1}^{4} \lambda_{1i}^{1-\mu_{\alpha_i}}, 1 - \prod_{i=1}^{4} \lambda_{1i}^{\nu_{\alpha_i}} \right)$$
$$= (0.58320,\ 0.32076)$$

In the same way, we can easily get other aggregated values:

$d_2 = (0.51916,\ 0.37390)$, $d_3 = (0.54919,\ 0.35043)$,
$d_4 = (0.53012,\ 0.36555)$

Then, we utilize the score function to calculate the scores of the aggregated values of those four alternatives:

$$s(d_1) = 0.26244,\ s(d_2) = 0.14526,$$
$$s(d_3) = 0.19876,\ s(d_4) = 0.16457.$$

After that, we rank the scores of those four criteria, by which we get the ranking of the alternatives: $Y_1 > Y_3 > Y_4 > Y_2$. That is to say, the alternative Y_1 is the best one.

If we can choose other forms of t-conorm and t-norm, it may lead to a different aggregated values. For example, if we choose the AIFWEA operator based on Einstein t-conorm and t-norm, we have $g(t) = \log\left(\frac{2-t}{t}\right)$ and $h(t) = \log\left(\frac{2-(1-t)}{1-t}\right)$. The aggregated value of the alternative Y_1 can be computed by

$$d_1 = IFWEA(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$$
$$= \left(\frac{2}{\prod_{i=1}^{4} \left(\frac{2-\lambda_{1i}}{\lambda_{1i}}\right)^{1-\mu_{\alpha_i}} + 1}, 1 - \frac{2}{\prod_{i=1}^{4} \left(\frac{2-\lambda_{1i}}{\lambda_{1i}}\right)^{\upsilon_{\alpha_i}} + 1} \right)$$
$$= (0.55879,\ 0.32730)$$

In the same way, we can easily get the aggregated values of the other alternatives:

$d_2 = (0.48080,\ 0.38923)$; $d_3 = (0.52312,\ 0.35714)$; $d_4 = (0.49973,\ 0.37487)$

and then calculate the score values of the aggregated values of the criteria Y_i $(i = 1,\ 2,\ 3,\ 4)$:

$s(d_1) = 0.23149,\ s(d_2) = 0.09157,\ s(d_3) = 0.16598,$ $s(d_4) = 0.12486$

According to the scores of those four criteria, the ranking of the alternatives should be: $Y_1 > Y_3 > Y_4 > Y_2$. That is to say, the alternative Y_1 is the best one.

Therefore, the AIFWEA operator provides a new way to aggregate intuitionistic fuzzy information and make decisions. As we can see from the above example, the AIFWEA operator and the traditional IFWG operator can be applied in different situations. In the situation where the weights are given by real numbers, and the degrees that the decision maker prefers the alternatives with respect to the criteria are expressed by IFNs, we can use the traditional IFWG operators to help make decisions. In the situation where the weights of criteria are given by IFNs, the degrees that the decision maker prefers the alternatives with respect to the criteria are given by real numbers, where the traditional aggregation operators are not capable, we can use the AIFWEA operator to help make decisions. The IFWG operator can't replace the AIFWEA operator in the latter situation because they are the different aggregation functions. The AIFWEA operator is $AIFWEA(\alpha_1, \alpha_2, \ldots, \alpha_n) = {\lambda_1}^{\alpha_1} \otimes {\lambda_2}^{\alpha_2} \otimes \cdots \otimes {\lambda_n}^{\alpha_n}$ $(\lambda_i \in (0,1),\ i = 1,\ 2,\ \ldots,\ n)$, and the IFWG operator is $\mathrm{IFWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = {\alpha_1}^{\lambda_1} \otimes {\alpha_2}^{\lambda_2} \otimes \cdots \otimes {\alpha_n}^{\lambda_n}$ $(\lambda_i \in (0,1),\ i = 1,\ 2,\ \ldots,\ n)$. As we can see, the positions of the weights and the characteristics are different, thus the meanings of aggregations are different. This unique characteristic makes the AIFWEA operator play an irreplaceable role in fuzzy information aggregation system.

5 Conclusions

In this paper, we have given the novel exponential operational laws, i.e., EOL-IFN-As and EOL-IFN-As, which construct the basic operational systems of IFSs and IFNs. Also we have investigated their properties and correlations. Based on the EOL-IFN-As, we have developed the AIFWEA operator. Some specific cases of the AIFWEA operators have been developed, including Algebraic intuitionistic fuzzy weighted exponential aggregation operator and Einstein intuitionistic fuzzy weighted exponential aggregation operator by assigning specific functions to t-conorm and t-norm. Moreover, these two new proposed aggregation operators also satisfy all the properties that the AIFWEA operator owns. Finally, we have used the AIFWEA operator to propose a MCDM method for solving the practical problems with intuitionistic fuzzy information, and verified it by an illustrative example involving the section of designing scheme of the new library of a middle school.

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