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ORIGINAL ARTICLE

A linguistic intuitionistic multi-criteria decision-making method based on the Frank Heronian mean operator and its application in evaluating coal mine safety

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Abstract Coal mine safety has been a pressing issue for many years, and it is a constant and non-negligible problem that must be addressed during any coal mining process. This paper focuses on developing an innovative multi-criteria decision-making (MCDM) method to address coal mine safety evaluation problems. Because lots of uncertain and fuzzy information exists in the process of evaluating coal mine safety, linguistic intuitionistic fuzzy numbers (LIFNs) are introduced to depict the evaluation information necessary to the process. Furthermore, the handling of qualitative information requires the effective support of quantitative tools, and the linguistic scale function (LSF) is therefore employed to deal with linguistic intuitionistic information. First, the distance, a valid ranking method, and Frank operations are proposed for LIFNs. Subsequently, the linguistic intuitionistic fuzzy Frank improved weighted Heronian mean (LIFFIWHM) operator is developed. Then, a linguistic intuitionistic MCDM method for coal mine safety evaluation is constructed based on the developed operator. Finally, an illustrative example is provided to demonstrate the proposed method, and its feasibility and validity are further verified by a sensitivity analysis and comparison with other existing methods.

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1 Introduction

In recent years, frequent major accidents during coal mining have attracted public attention and aroused extensive social concern. The scientific implementation of safety evaluations for coal mines represents a vital opportunity for effectively reducing coal mine accidents and achieving stable improvements in the working conditions in mines. A safety evaluation of coal mines involves comprehensively evaluating risks and ranking the extant projects. Because each coal mine can be characterized under multiple evaluation criteria, such as geological conditions, environmental security, technological advancement, and management quality, it is difficult for evaluators to make appropriate judgments about safety without applying scientific methods. Multi-criteria decision-making (MCDM) methods are therefore useful tools to assist in the safety evaluation of coal mines, and some MCDM methods have been specifically developed to address these problems [1, 2].

As society has evolved, decision-making problems have become more complex, such that it is now difficult for decision-makers to express specific preferences or opinions under uncertain and fuzzy environments, like in coal mine safety evaluation problems. To deal with fuzzy information, Zadeh [3] proposed fuzzy sets (FSs), which are now considered to be a useful tool in solving decision-making problems [4, 10, 25]. However, in some cases, the membership degree alone cannot describe information with sufficient precision. In order to address this issue, Atanassov

[5] introduced intuitionistic fuzzy sets (IFSs), which measure both membership degree and non-membership degree. Since their introduction, IFSs have been researched in great detail, and some extensions have been developed and applied to MCDM problems [6, 7, 23, 54]. Torra and Narukawa [8] first introduced the hesitant fuzzy sets (HFSs), an extension of traditional FSs that permits the membership degree of an element to be a set of several possible values in [0, 1]; the main purpose of HFSs is to model the uncertainty produced by human doubt when eliciting information [9].

Although fuzzy set theory has been developed and generalized, the need to define an element's membership or non-membership degrees as specific values in [0, 1] greatly confines its applications in many practical problems. Therefore, the above classic fuzzy sets are limited to use in quantitative environments, and they are incapable of handling qualitative information. However, in most decision-making environments, the best expression of decision-makers' preferences or opinions naturally takes a linguistic form because of the complexity of problems and the inherent vagueness of human preferences. Linguistic variables [10] are valid tools because the use of linguistic information reinforces the flexibility and reliability of classical decision models [11].

In recent years, linguistic variables that take the values of words or sentences from natural or artificial languages have been studied extensively and applied in various fields [12–14]. Xu [15] defined operations and developed some aggregation operators for linguistic variables represented by a single linguistic term. Subsequently, Xu [16] proposed uncertain linguistic variables (ULVs), which employ linguistic intervals rather than single linguistic values to depict fuzzy information; the use of a ULV suggests that the probabilities of all linguistic values in the interval are equal or obey a specific distribution [16–18]. Accompanying the promotion of preliminary linguistic models, some extended linguistic concepts have been developed. Linguistic sets consisting of several discrete linguistic terms have been proposed based on linguistic variables and HFSs, such as hesitant fuzzy linguistic term sets (HFLTSs) [19], hesitant fuzzy linguistic sets (HFLSs) [20], linguistic hesitant fuzzy sets (LHFSs) [21], and multi-hesitant fuzzy linguistic term sets (MHFLTSs) [22]. To improve the applicability of IFSs and accommodate them to complex and qualitative environments, Chen et al. [23] proposed linguistic intuitionistic fuzzy numbers (LIFNs) by integrating linguistic models and IFSs. LIFNs simultaneously consider the linguistic membership degree and linguistic non-membership degree, and they can effectively handle ambiguous and uncertain information.

Addressing decision-making problems with linguistic information implies the need for computing with words (CWW) [24, 25]. Several computational models have been developed to deal with linguistic information, and the primary ones are briefly described as follows. (1) One method involves the direct use of linguistic labels [15–18, 23, 26]; this method is easy to employ, but it cannot make the maximum use of the original information. (2) Another method represents linguistic information through fuzzy membership functions that convert linguistic information into fuzzy sets, such as triangular fuzzy numbers, trapezoidal fuzzy numbers, and type-2 fuzzy sets [27, 28]. However, this method inevitably causes information loss and distortion during the transformation process. (3) Another method, developed based on cloud model, can precisely depict the fuzziness and randomness of qualitative concepts, and their application in decision-making problems can be found in lots of researches [29, 30, 36]. (4) One method introduced the 2-tuple linguistic representation model [31, 32], which reinforces the accuracy of linguistic computations while improving the fuzzy linguistic method by utilizing a symbolic translation parameter [33, 34, 46, 47]. This method includes both a conversion process and an inverse conversion process. Based on the elicitation of this idea and the numerical scale model of linguistic term sets [33, 46], and considering the shortcomings of previous linguistic methods, Wang et al. [35] proposed linguistic scale functions (LSFs) to accommodate different semantic circumstances. LSFs are greatly flexible in addressing linguistic information and can effectively avoid the loss and distortion of original information, and has been successfully applied in all kinds of linguistic models [7, 13, 21, 22, 36, 37].

Aggregation operators are important tools for facilitating information fusion in decision-making problems, and they represent a consistently active topic for research. Most aggregation operators suppose that the arguments are mutually independent; however, in many practical problems, aggregated values may be correlative. For example, in coal mine safety evaluation problems, environmental security may be affected by geological condition, and human diathesis may be affected by management level. To deal with these kinds of problems, interrelationships among criteria values must be considered, creating an opportunity to utilize the Heronian mean (HM) operator [38]. Introduced by Beliakov et al. [38], the HM operator can capture the interrelationships among different aggregated arguments, making it one of the most important information fusion tools applied in decision-making problems [39, 40].

Most existing aggregation operators are defined based on the algebraic product and algebraic sum; however, algebraic operations lack flexibility and robustness. Frank operations [41], which are the generalization of algebraic operations, can overcome this defect. Compared to algebraic operations, Frank operations are made to be more flexible through the introducing of a parameter. In recent years, many scholars have studied Frank operations under fuzzy environments. Ji et al. [42] defined the single-valued neutrosophic Frank normalized prioritized Bonferroni mean aggregation operator. Zhang et al. [43] proposed some intuitionistic fuzzy Frank power aggregation operators. Subsequently, Zhang [44] developed the interval-valued intuitionistic fuzzy Frank aggregation operators, and Qin et al. [45] further proposed some hesitant fuzzy Frank aggregation operators.

Based on the analysis above, the primary motivations for this paper are summarized as follows:

1. As established previously, LIFNs are helpful tools for decision-makers to present assessments under uncertain or fuzzy environments. Thus, this paper introduces LIFNs as a mean to depict as fully as possible the complex information involved in the process of evaluating safe coal mines.
2. Frank operations are more robust and flexible than other algebraic operations, but they have not been studied in the context of LIFNs. Therefore, this paper proposes Frank operations for LIFNs.
3. Extant methods for evaluating coal mine safety cannot take into account the interrelationships among criteria values. In order to effectively evaluate coal mine safety, this paper proposes a linguistic intuitionistic aggregation operator based on the HM operator, thereby developing a comprehensive MCDM approach.

The rest part of this paper is organized as follows. In Sect. 2, some concepts are reviewed briefly. In Sect. 3, LIFNs is introduced, and the distance, comparison method and Frank operations of LIFNs are presented. In Sect. 4, a linguistic intuitionistic aggregation operator is developed, and some desirable properties and special cases are discussed, and then, a method for addressing coal mine safety evaluation problems is described. In Sect. 5, a practical example of coal mine safety evaluation is used to verify the validity of the proposed method. A sensitivity analysis and a comparison analysis are then conducted. Finally, the conclusion is drawn in Sect. 6.

2 Preliminaries

This section introduces some concepts, including linguistic term sets, linguistic numerical scale models, intuitionistic fuzzy sets, and Frank operations, which are useful in the subsequent analysis.

2.1 Linguistic term sets

Let $S = \{s_i | i = 0, 1, 2, \cdots, 2t\}$ be a finite and completely ordered discrete term set with odd cardinality, where t is a nonnegative integer, and s_i represents a possible value for a linguistic variable. It is required that s_i and s_j satisfy the following properties [10]: (1) The set is ordered $s_i \le s_j$ if and only if $i \le j$, and (2) The set obeys the negation operation $neg(s_i) = s_j$ if $i + j = 2t$.

The linguistic term set S is a discrete set, but a continuous set is required to solve practical problems, especially in the process of aggregation operation. Xu [15] extended the discrete linguistic term set to its continuous form, which can be expressed as $\tilde{S} = \{s_i | i \in k\}$, where k is a large positive real number, and $s_i > s_j$ if $i > j$. If $s_i \in S$, then s_i is the original linguistic term; otherwise, s_i is the virtual linguistic term.

2.2 Linguistic numerical scale models

The transformation from linguistic terms to numerical values requires the effective support of quantitative tools. Dong et al. [33] first proposed the concept of numerical scale for a linguistic term set in order to accomplish this transformation. Subsequently, the numerical scale model was extended and applied widely in many fields [34, 46–48]. Motivated by this idea, Wang et al. [35] further proposed linguistic scale functions (LSFs) to convert linguistic terms into real numbers, which have been improved based on psychological theory and prospect theory. In fact, each LSF can be regarded as a specific linguistic numerical scale model. In linguistic evaluation scales with increasing linguistic subscripts, the absolute deviation between any two adjacent linguistic subscripts may increase or decrease.

Definition 1 [33, 35] Let $s_i \in S$ be a linguistic term, in which $S = \{s_i | i = 0, 1, 2, \cdots, 2t\}$. If $\theta_i \in [0, 1]$ is a numerical value, then the LSF is mapped from s_i to θ_i $(i = 0, 1, \cdots, 2t)$, which is defined as follows:

$$f : s_i \to \theta_i (i = 0, 1, \cdots, 2t), \tag{1}$$

where $0 \le \theta_0 < \theta_1 < \cdots < \theta_{2t} \le 1$. In this situation, θ_i reflects the preference of decision-makers when they choose the linguistic term s_i, such that the function f illustrates the semantics of s_i. A LSF is a strictly monotonously increasing function with respect to the subscript i. In the following, three kinds of LSFs are shown.

1. The following LSF is defined based on the subscript function $sub(s_i) = i$:

$$\text{LSF1:} f_1(s_i) = \theta_i = \frac{i}{2t}(i = 0, 1, \cdots, 2t).$$

The evaluation scale for the linguistic information presented above is divided evenly.

2. The following LSF is defined based on the exponential scale, and it is improved by comprehensively considering the real behavioral preference of decision-makers according to psychological theory:

$$\text{LSF2:} f_2(s_i) = \theta_i = \begin{cases} \frac{a^t - a^{t-i}}{2a^t - 2}(i = 0, 1, 2, ..., t) \\ \frac{a^t + a^{i-t} - 2}{2a^t - 2}(i = t+1, t+2, ..., 2t) \end{cases}.$$

Several researches have manifested the parameter a, which generally lies in the interval [1.36, 1.4] [49]. With the extension from the middle of the given linguistic term to both ends, the absolute deviation between adjacent linguistic terms also increases.

3. The following LSF is defined based on prospect theory's value function, as well as the decision-makers' different sensitivities with respect to the absolute deviation between adjacent linguistic subscripts:

$$\text{LSF3:} f_3(s_i) = \theta_i = \begin{cases} \frac{t^\alpha - (t-i)^\alpha}{2t^\alpha}(i = 0, 1, 2, ..., t) \\ \frac{t^\beta + (i-t)^\beta}{2t^\beta}(i = t+1, t+2, ..., 2t) \end{cases}.$$

Several experiments have been done [50] to determine that $\alpha = \beta = 0.88$. If $\alpha = \beta = 1$, then LSF3 is reduced to LSF1. With the extension from the middle of the given linguistic term to both ends, the absolute deviation between adjacent linguistic terms decreases.

To preserve all of the given information and facilitate calculations, the above function can be extended to $f^*:\tilde{S} \to R^+$, where $f^*(s_i) = \theta_i$ is a strictly monotonously increasing and continuous function. Therefore, the inverse function of f^* also exists, and it can be indicated as f^{*-1}.

Recently, the linguistic numerical scale models have made additional progress. On the one hand, Dong et al. [51, 52] proved that they can provide a unified framework to connect the traditional linguistic 2-tuples, virtual linguistic model, proportional 2-tuples, and the model based on linguistic hierarchy. On the other hand, Li et al. [53] showed that the linguistic numerical scale models can support linguistic group decision making by providing a better semantic representation model to personalize individual semantics in CWW.

2.3 Intuitionistic fuzzy sets

Definition 2 [5] Let X be a universe of discourse; then, an IFS A in X can be defined as follows:

$$A = \left\{ \langle x, u_A(x), v_A(x) \rangle | x \in X \right\}, \tag{2}$$

where $u_A:X \to [0,1]$ and $v_A:X \to [0,1]$ with $0 \le u_A(x) + v_A(x) \le 1$ for all $x \in X$. The functions u_A and v_A represent the membership degree and non-membership degree of x to A, respectively. Usually, $\pi_A(x)$ can be called the hesitation degree of x to A for $x \in X$ if $\pi_A(x) = 1 - u_A(x) - v_A(x)$. When $\pi_A(x) = 0$ for each $x \in X$, the IFS is reduced to a FS.

2.4 Frank operations

Definition 3 [41] Let a and b be two real numbers, which satisfy $a, b \in [0, 1]$, and let $\lambda \in (1, +\infty)$. Then, the Frank product $\otimes_F$ and Frank sum $\oplus_F$ between a and b can be defined as follows:

$$a \oplus_F b = 1 - \log_\lambda \left(1 + \frac{(\lambda^{1-a} - 1)(\lambda^{1-b} - 1)}{\lambda - 1} \right),$$

$$a \otimes_F b = \log_\lambda \left(1 + \frac{(\lambda^a - 1)(\lambda^b - 1)}{\lambda - 1} \right).$$

In addition, it can be easily proved that when $\lambda \to 1$, $a \oplus_F b \to a + b - ab$ and $a \otimes_F b \to ab$. The Frank product and Frank sum reduce to the algebraic triangular norm and conorm, respectively.

3 LIFNs and relevant concepts

In this section, the concept of LIFNs is introduced, and the distance for LIFNs is defined. Moreover, after discussing the drawbacks of existing method for comparing LIFNs, a valid ranking method for LIFNs is proposed. Subsequently, the novel linguistic intuitionistic operations is developed based on Frank operations and LSFs.

3.1 Linguistic intuitionistic fuzzy numbers

Definition 4 [23] Let $s_u, s_v \in \tilde{S}_{[0,2t]}$, where $\tilde{S}_{[0,2t]} = \tilde{S} = \{s_i | 0 \le i \le 2t\}$ is a continuous linguistic term set, $2t$ is the maximal subscript of the linguistic term s_i in $\tilde{S}$, and $\phi = (s_u, s_v)$. If $u + v \le 2t$, then we call ϕ the linguistic intuitionistic fuzzy number (LIFN), and s_u and s_v represent the linguistic membership degree and linguistic non-membership degree, respectively. Moreover, if $s_u, s_v \in S$, then ϕ is an original LIFN; otherwise, ϕ is a virtual LIFN.

3.2 Distance for LIFNs

Definition 5 Let $\phi_1 = (s_{u_1}, s_{v_1})$ and $\phi_2 = (s_{u_2}, s_{v_2})$ be two arbitrary LIFNs, and $s_{u_1}, s_{v_1}, s_{u_2}, s_{v_2} \in \tilde{S}_{[0,2t]}$. When $f^*(s_i)$ is a LSF, d is a mapping, and d: $\phi \times \phi \to R^+$, the Hamming distance between ϕ_1 and ϕ_2 can be defined as

$$d_H(\phi_1, \phi_2) = \frac{1}{2}\Big(\left| f^*(s_{u_1}) - f^*(s_{u_2}) \right| + \left| f^*(s_{v_1}) - f^*(s_{v_2}) \right| + \left| f^*(s_{2t-u_1-v_1}) - f^*(s_{2t-u_2-v_2}) \right| \Big) \tag{3}$$

Property 1 *The following properties of distance measurement described in* Definition 5 *can be easily proved*:

1. $0 \le d_H(\phi_1, \phi_2) \le 1$,
2. $d_H(\phi_1, \phi_2) = d_H(\phi_2, \phi_1)$,
3. *If* ϕ_3 *is an arbitrary LIFN, then* $d_H(\phi_1, \phi_3) \le d_H(\phi_1, \phi_2) + d_H(\phi_2, \phi_3)$.

3.3 Comparison method for LIFNs

Definition 6 [23] Let $\phi = (s_u, s_v)$ be an arbitrary LIFN. Then, the linguistic score index and accuracy index of ϕ can be defined as $S(\phi) = u - v$,and $H(\phi) = u + v$.

Definition 7 [23] Let $\phi_1 = \{s_{u_1}, s_{v_1}\}$and $\phi_2 = \{s_{u_2}, s_{v_2}\}$be two arbitrary LIFNs. Based on Definition 6, the method for comparing ϕ_1 and ϕ_2 can be given as follows:

1. if $s(\phi_1) < s(\phi_2)$, then $\phi_1 < \phi_2$.
2. if $s(\phi_1) = s(\phi_2)$, then the following hold:

1. if $H(\phi_1) < H(\phi_2)$, then $\phi_1 < \phi_2$.
2. if $H(\phi_1) = H(\phi_2)$, then $\phi_1 = \phi_2$.

However, the above comparison method for LIFNs has obvious disadvantages that cannot be ignored. First, the score index and accuracy index are calculated directly based on the subscripts of linguistic terms; this not only leads to the loss and distortion of original information, but also cannot reveal critical distinctions in the final results under various semantic situations. Second, the comparison method outlined above cannot separately make use of the score index or accuracy index to obtain final rankings in some situations. Instead, it needs to combine both of these indices to compare different LIFNs, resulting in a loss of coherence to some extent. Third, for some particular LIFNs, the above comparison method cannot account for the objective influence of the indeterminacy degree, and it cannot produce reasonable rankings. This is demonstrated in Example 1.

Example 1 Let $S = \{s_0$ =extremely poor, s_1=very poor, s_2=poor, s_3=slightly poor, s_4=fair, s_5=slightly good, s_6=good, s_7=very good, s_8=extremely good$\}$ be a linguistic term set. Further, let $\phi_1 = (s_{3.2}, s_{0.8})$,$\phi_2 = (s_{4.8}, s_{2.8})$, $\phi_3 = (s_4, s_{3.6})$, and $\phi_4 = (s_2, s_{0.4})$ be four LIFNs. Then, their linguistic score index and linguistic accuracy index can be calculated as follows:

$$S(\phi_1) = 2.4, S(\phi_2) = 2, S(\phi_3) = 0.4, \text{ and } S(\phi_4) = 1.6;$$

$$H(\phi_1) = 4, H(\phi_2) = 7.6, H(\phi_3) = 7.6, \text{ and } H(\phi_4) = 2.4.$$

Based on Definition 7, there is $\phi_1 \succ \phi_2 \succ \phi_4 \succ \phi_3$.

Obviously, the comparison method described in Definition 7 suggests that ϕ_1 is bigger than ϕ_2, and ϕ_4 is bigger than ϕ_3. Although the linguistic score index of ϕ_1 is bigger than that of ϕ_2, and the linguistic score index of ϕ_4 is bigger than that of ϕ_3, the complete information contained within ϕ_2(*i.e.* $s_{4.8} + s_{2.8}$) is greater than that contained within ϕ_1(i.e. $s_{3.2} + s_{0.4}$), and the complete information contained within ϕ_3(i.e. $s_4 + s_{3.6}$) is greater than that contained within ϕ_4(i.e. $s_2 + s_{0.4}$). Therefore, it is unreasonable to conclude that ϕ_1 is bigger than ϕ_2, or that ϕ_4 is bigger than ϕ_3.

For this reason, it is necessary to explore a more appropriate comparison method for LIFNs. Szmidt and Kacprzyk [54] comprehensively accounted for the indeterminacy degree of IFNs and the distance in pursuing the positive ideal solution; ultimately, they proposed an effective ranking method for IFNs. In the same way, a valid ranking method for LIFNs can be developed based on the distance measurement of LIFNs, which appropriately considers the indeterminacy degree of LIFNs and can overcome the drawbacks of the comparison method described in Definition 7.

Definition 8 Let $\phi = (s_u, s_v)$ be an arbitrary LIFN, where $s_u, s_v \in \tilde{S}_{[0,2t]}$. Further, let $f^*(s_i)$ be a LSF, $P = (s_{2t}, s_0)$ be the positive ideal point, and $d_H(P, \phi)$ be the Hamming distance between P and ϕ. Then, a measurement function for ϕ can be defined as

$$R(\phi) = 0.5(1 + f^*(s_{2t-u-v}))d_H(P, \phi) \tag{4}$$

Based on the above definition, it is obvious that the smaller $R(\phi)$ is, the better ϕ is.

Example 2 Let ϕ_1, ϕ_2, ϕ_3, and $\phi_4$be the same as in Example 1. If LSF be $f_1^*(s_i)$, then the measurement function value can be calculated as follows:

$R(\phi_1) = 0.4500$, $R(\phi_2) = 0.2100$, $R(\phi_3) = 0.2625$, a n d $R(\phi_4) = 0.6375$.

Therefore, $\phi_2 \succ \phi_3 \succ \phi_1 \succ \phi_4$ can be obtained, which is more reasonable than the ranking result in Example 1.

3.4 Frank operations for LIFNs

In the following, some essential algorithms for LIFNs are established based on LSFs and Frank operations.

Definition 9 Let $\phi_1 = (s_{u_1}, s_{v_1})$ and $\phi_2 = (s_{u_2}, s_{v_2})$ be two arbitrary LIFNs, f^* be a LSF, f^{*-1} be the inverse function of f^*, and $\lambda > 0$. Then, the operations for LIFNs can be defined as follows:

1. $$\phi_1 \oplus_F \phi_2 = \left(f^{*-1}\left(1 - \log_\lambda\left(1 + \frac{(\lambda^{1-f^*(s_{u_1})}-1)(\lambda^{1-f^*(s_{u_2})}-1)}{\lambda-1}\right)\right), f^{*-1}\left(\log_\lambda\left(1 + \frac{(\lambda^{f^*(s_{v_1})}-1)(\lambda^{f^*(s_{v_2})}-1)}{\lambda-1}\right)\right)\right);$$
2. $$\phi_1 \otimes_F \phi_2 = \left(f^{*-1}\left(\log_\lambda\left(1 + \frac{(\lambda^{f^*(s_{u_1})}-1)(\lambda^{f^*(s_{u_2})}-1)}{\lambda-1}\right)\right), f^{*-1}\left(1 - \log_\lambda\left(1 + \frac{(\lambda^{1-f^*(s_{v_1})}-1)(\lambda^{1-f^*(s_{v_2})}-1)}{\lambda-1}\right)\right)\right);$$
3. $$k \cdot_F \phi_1 = \left(f^{*-1}\left(1 - \log_\lambda\left(1 + \frac{(\lambda^{1-f^*(s_{u_1})}-1)^k}{(\lambda-1)^{k-1}}\right)\right), f^{*-1}\left(\log_\lambda\left(1 + \frac{(\lambda^{f^*(s_{v_1})}-1)^k}{(\lambda-1)^{k-1}}\right)\right)\right), k > 0;$$
4. $$\phi_1^{\hat{}_F k} = \left(f^{*-1}\left(\log_\lambda\left(1 + \frac{(\lambda^{f^*(s_{u_1})}-1)^k}{(\lambda-1)^{k-1}}\right)\right), f^{*-1}\left(1 - \log_\lambda\left(1 + \frac{(\lambda^{1-f^*(s_{v_1})}-1)^k}{(\lambda-1)^{k-1}}\right)\right)\right), k > 0.$$

Property 2 *Let $\phi = (s_u, s_v)$, $\phi_1 = (s_{u_1}, s_{v_1})$, $\phi_2 = (s_{u_2}, s_{v_2})$, and $\phi_3 = (s_{u_3}, s_{v_3})$ be four arbitrary LIFNs, and k, k_1, $k_2 > 0$. Then, the following properties can be easily proved*:

1. $\phi_1 \oplus_F \phi_2 = \phi_2 \oplus_F \phi_1$,
2. $\phi_1 \otimes_F \phi_2 = \phi_2 \otimes_F \phi_1$,
3. $(\phi_1 \oplus_F \phi_2) \oplus_F \phi_3 = \phi_1 \oplus_F (\phi_2 \oplus_F \phi_3)$,
4. $(\phi_1 \otimes_F \phi_2) \otimes_F \phi_3 = \phi_1 \otimes_F (\phi_2 \otimes_F \phi_3)$,
5. $k \cdot_F (\phi_1 \oplus_F \phi_2) = k \cdot_F \phi_1 \oplus_F k \cdot_F \phi_2$,
6. $(\phi_1 \otimes_F \phi_2)^{\hat{}_F k} = \phi_1^{\hat{}_F k} \otimes_F \phi_2^{\hat{}_F k}$,
7. $(k_1 + k_2) \cdot_F \phi = k_1 \cdot_F \phi \oplus_F k_2 \cdot_F \phi$,
8. $\phi^{\hat{}_F (k_1+k_2)} = \phi^{\hat{}_F k_1} \otimes_F \phi^{\hat{}_F k_2}$.

4 Aggregation operator for LIFNs

Based on Frank operations and the HM operator, this section proposes a linguistic intuitionistic fuzzy aggregation operator. Further, some desirable properties and special cases are discussed with regard to the parameters λ,p, and q. Finally, a method for addressing MCDM problems with LIFNs is developed.

4.1 Heronian mean (HM) operator

Definition 10 [38] Let $a_i (i = 1, 2, ..., n)$ be a collection of nonnegative crisp data, and $p, q \geq 0$; then

$$H^{p,q}(a_1, a_2, ..., a_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} a_i^p a_j^q \right)^{\frac{1}{p+q}}, \quad (5)$$

is called the Heronian mean (HM) operator with parameter.

4.2 The linguistic intuitionistic fuzzy Frank improved weighted Heronian mean operator

In this subsection, the HM operator is extended to the situations in which the input arguments consist of linguistic intuitionistic fuzzy information. The linguistic intuitionistic fuzzy Frank improved weighted Heronian mean (LIFFIWHM) operator, and its corresponding theorems are described below.

Definition 11 Let $\phi_i = (s_{u_i}, s_{v_i}) (i = 1, 2, ..., n)$ be a collection of LIFNs. For any $p, q \geq 0$, the linguistic intuitionistic fuzzy Frank improved weighted Heronian mean (LIFFIWHM) operator can be defined as

$$LIFFIWHM^{p,q}(\phi_1, \phi_2, ..., \phi_n) = \left(\frac{1}{\underset{i=1,j=i}{\overset{n}{\oplus_F}} w_i w_j} \cdot_F \underset{i=1,j=i}{\overset{n}{\oplus_F}} w_i w_j \cdot_F ((\phi_i)^{\hat{}_F p} \otimes_F (\phi_j)^{\hat{}_F q}) \right)^{\hat{}_F \frac{1}{p+q}}, \quad (6)$$

where $w=(w_1,w_2,...,w_n)$ is the weight vector of $\phi_i(i=1,2,...,n)$, $w_i \geq 0$, and $\sum_{i=1}^{n} w_i = 1$.

According to the Frank operations of LIFNs, the following results can be obtained.

Theorem 1 *Let $\phi_i=(s_{u_i},s_{v_i})(i=1,2,...,n)$ be a collection of LIFNs, and $p,q \geq 0$. Then, the aggregated value obtained by the LIFFIWHM operator is*

$$LIFFIWHM^{p,q}(\phi_1,\phi_2,...,\phi_n)=\left(\frac{1}{\bigoplus_{F\,i=1,j=i}^{n} w_iw_j}\cdot_F \bigoplus_{F\,i=1,j=i}^{n} w_iw_j \cdot_F \left((\phi_i)^{\wedge_F p}\otimes_F (\phi_j)^{\wedge_F q}\right)\right)^{\wedge_F \frac{1}{p+q}} =$$

$$\left(f^{*-1}\left(\log_\lambda\left(1+(\lambda-1)\left(\frac{1-\left(\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}\right)^{w_iw_j}\right)^k}{1+(\lambda-1)\left(\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}\right)^{w_iw_j}\right)^k}\right)^{\frac{1}{p+q}}\right)\right),\right.$$

$$\left. f^{*-1}\left(1-\log_\lambda\left(1+(\lambda-1)\left(\frac{1-\left(\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}\right)^{w_iw_j}\right)^k}{1+(\lambda-1)\left(\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}\right)^{w_iw_j}\right)^k}\right)^{\frac{1}{p+q}}\right)\right)\right). \quad (7)$$

where $k=\frac{1}{\bigoplus_{F\,i=1,j=i}^{n} w_iw_j}$.

In the following, some desirable properties of the LIFFIWHM operator will be discussed.

Theorem 2 (idempotency) *Let $\phi=\phi_i$ for all $i=1,2,...,n$; then, $LIFFIWHM^{p,q}(\phi_1,\phi_2,...,\phi_n)=\phi$.*

Theorem 3 (monotonicity) *Let $a_i=(s_{u(a_i)},s_{v(a_i)})$ $(i=1,2,...,n)$ and $b_i=(s_{u(b_i)},s_{v(b_i)})(i=1,2,...,n)$ be two collection of LIFNs. For any $p,q \geq 0$, if $s_{u(a_i)} \geq s_{u(b_i)}$ and $s_{v(a_i)} \leq s_{v(b_i)}$ for all $i=1,2,...,n$, then $LIFFIWHM^{p,q}(a_1,a_2,...,a_n) \geq LIFFIWHM^{p,q}(b_1,b_2,...,b_n)$.*

Theorem 4 (boundedness) *Let $\phi_i=(s_{u_i},s_{v_i})$ $(i=1,2,...,n)$ be a collection of LIFNs, $a=(s_{u(a)},s_{v(a)})=(\max_i\{s_{u_i}\},\min_i\{s_{v_i}\})$, and $b=(s_{u(b)},s_{v(b)})=(\min_i\{s_{u_i}\},\max_i\{s_{v_i}\})$. Then, $b \leq LIFFIWHM^{p,q}(\phi_1,\phi_2,...,\phi_n) \leq a$.*

Theorem 2 can be easily proved according to the properties of LIFNs' operations as described in Property 2. The proofs of Theorem 1 and Theorems 3–4 are shown in the Appendix.

In the following, some special cases of the LIFFIWHM operator will be discussed.

1. If $q=0$, then the LIFFIWHM operator degenerates to the linguistic intuitionistic fuzzy Frank generalized weighted average (LIFFGWA) operator as follows:

$$
\begin{aligned}
LIFFGWA(\phi_1,\phi_2,\ldots,\phi_n) &= \left(\mathop{\oplus_F}\limits_{k=1}^{n} w'_k \cdot_F ((\phi_k)^\wedge{}_F p)\right)^{\wedge_F \frac{1}{p}} \\
&= \left(f^{*-1}\left(\log_\lambda\left(1+(\lambda-1)\left(\frac{1-\prod\limits_{k=1}^{n}\left(\frac{(\lambda-1)^p-(\lambda^{f^*(s_{u_k})}-1)^p}{(\lambda-1)^p+(\lambda-1)(\lambda^{f^*(s_{u_k})}-1)^p}\right)^{w'_k}}{1+(\lambda-1)\prod\limits_{k=1}^{n}\left(\frac{(\lambda-1)^p-(\lambda^{f^*(s_{u_k})}-1)^p}{(\lambda-1)^p+(\lambda-1)(\lambda^{f^*(s_{u_k})}-1)^p}\right)^{w'_k}}\right)^{\frac{1}{p}}\right)\right),\right. \\
&\left. f^{*-1}\left(1-\log_\lambda\left(1+(\lambda-1)\left(\frac{1-\prod\limits_{k=1}^{n}\left(\frac{(\lambda-1)^p-(\lambda^{1-f^*(s_{v_k})}-1)^p}{(\lambda-1)^p+(\lambda-1)(\lambda^{1-f^*(s_{v_k})}-1)^p}\right)^{w'_k}}{1+(\lambda-1)\prod\limits_{k=1}^{n}\left(\frac{(\lambda-1)^p-(\lambda^{1-f^*(s_{v_k})}-1)^p}{(\lambda-1)^p+(\lambda-1)(\lambda^{1-f^*(s_{v_k})}-1)^p}\right)^{w'_k}}\right)^{\frac{1}{p}}\right)\right)\right)
\end{aligned}
\tag{8}
$$

where $w'_k = \frac{\mathop{\oplus_F}\limits_{j=k}^{n} w_k w_j}{\mathop{\oplus_F}\limits_{i=1,j=i}^{n} w_i w_j}, (k = 1,2,\ldots,n)$, $w'_k \geq 0$, and $\sum\limits_{k=1}^{n} w'_k = 1$.

2. If $p = 1$ and $q = 0$, then the LIFFIWHM operator degenerates to the linguistic intuitionistic fuzzy Frank weighted average (LIFFWA) operator as follows:

$$
\begin{aligned}
LIFFWA(\phi_1,\phi_2,\ldots,\phi_n) &= \mathop{\oplus_F}\limits_{k=1}^{n} w'_k \cdot_F (\phi_k) \\
&= \left(f^{*-1}\left(1-\log_\lambda\left(1+\prod_{k=1}^{n}(\lambda^{1-f^*(s_{u_k})}-1)^{w'_k}\right)\right),\right. \\
&\left. f^{*-1}\left(\log_\lambda\left(1+\prod_{k=1}^{n}(\lambda^{f^*(s_{v_k})}-1)^{w'_k}\right)\right)\right).
\end{aligned}
\tag{9}
$$

3. If $p \to 0$ and $q = 0$, then the LIFFIWHM operator reduces to the linguistic intuitionistic fuzzy Frank weighted geometric (LIFFWG) operator as follows:

$$
\begin{aligned}
LIFFWG(\phi_1,\phi_2,\ldots,\phi_n) &= \prod_{k=1}^{n} \cdot_F \phi_k^{w'_k} \\
&= \left(f^{*-1}\left(\log_\lambda\left(1+\prod_{k=1}^{n}(\lambda^{f^*(s_{u_k})}-1)^{w'_k}\right)\right),\right. \\
&\left. f^{*-1}\left(1-\log_\lambda\left(1+\prod_{k=1}^{n}(\lambda^{1-f^*(s_{v_k})}-1)^{w'_k}\right)\right)\right).
\end{aligned}
\tag{10}
$$

4. If $\lambda \to 1$, then the LIFFIWHM operator reduces to the linguistic intuitionistic fuzzy improved weighted Heronian mean (LIFIWHM) operator as follows:

$$
\begin{aligned}
&\lim_{\lambda\to 1} LIFFIWHM^{p,q}(\phi_1,\phi_2,\ldots,\phi_n) \\
&= LIFIWHM^{p,q}(\phi_1,\phi_2,\ldots,\phi_n) \\
&= \left(\frac{1}{\mathop{\oplus}\limits_{i=1,j=i}^{n} w_i w_j} \mathop{\oplus}\limits_{i=1,j=i}^{n} w_i w_j((\phi_i)^p \otimes (\phi_j)^q)\right)^{\frac{1}{p+q}} = \\
&\left(f^{*-1}\left(\left(1-\prod_{i=1,j=i}^{n}\left(1-(f^*(s_{u_i}))^p(f^*(s_{u_j}))^q\right)^{\frac{w_i w_j}{\mathop{\oplus}\limits_{i=1,j=i}^{n} w_i w_j}}\right)^{\frac{1}{p+q}}\right),\right. \\
&\left. f^{*-1}\left(1-\left(1-\prod_{i=1,j=i}^{n}\left(1-(1-f^*(s_{v_i}))^p(1-f^*(s_{v_j}))^q\right)^{\frac{w_i w_j}{\mathop{\oplus}\limits_{i=1,j=i}^{n} w_i w_j}}\right)^{\frac{1}{p+q}}\right)\right)
\end{aligned}
\tag{11}
$$

4.3 A MCDM method for coal mine safety evaluation under linguistic intuitionistic fuzzy circumstance

Based on the previous discussions, this subsection employs the LIFFIWHM operator to develop a new MCDM method to handle coal mine safety evaluation problems.

Coal mine safety evaluation problems with linguistic intuitionistic fuzzy information involve a group of coal mines denoted by $A = \{a_1, a_2, \ldots, a_n\}$. Each coal mine is assessed by means of m criteria, denoted by $C = \{c_1, c_2, \ldots, c_m\}$, whose weight vector is $w = (w_1, w_2, \ldots, w_n)$, satisfying $w_j \in [0,1]$ and $\sum_{j=1}^{m} w_j = 1$. Let $R = (\phi_{ij})_{m\times n}$ be the linguistic intuitionistic fuzzy evaluation matrix, where $\phi_{ij} = (s_{u_{ij}}, s_{v_{ij}})$ is an evaluation value expressed by LIFNs, in which $s_{u_{ij}}$ indicates the linguistic membership to which coal mine a_i satisfies criterion c_j, while $s_{v_{ij}}$ indicates the linguistic non-membership to which coal mine a_i satisfies criterion c_j.

In the following, a MCDM method is developed to address coal mine safety evaluation problems under linguistic intuitionistic fuzzy environments. The main procedures are described as follows:

Step 1. Normalize the evaluation matrix.

First, it is necessary to normalize all evaluation values to the same magnitude grade in order to eliminate the influence of different dimensions on the operation process. The normalization of the evaluation values can be expressed as follows:

For benefit criteria, $\phi'_{ij} = \phi_{ij} = (s_{u_{ij}}, s_{v_{ij}})$; for cost criteria, $\phi'_{ij} = \phi^c_{ij} = (s_{v_{ij}}, s_{u_{ij}})$..

The normalized evaluation matrix is expressed as $R' = (\phi'_{ij})_{m\times n}$.

Step 2. Obtain comprehensive evaluations.

The comprehensive evaluation ϕ'_i of coal mine a_i can be obtained utilizing the LIFFIWHM operator.

Step 3. Calculate the measurement function value of each ϕ'_i.

The measurement function value $R(\phi'_i)(i = 1, 2, ..., n)$ of ϕ'_i can be calculated according to Definition 8.

Step 4. Rank all the alternatives and select the safest one.

Finally, all of the alternatives can be ranked, and the safest one can be identified in accordance with $R(\phi'_i)$.

5 Illustrative example

This section provides a practical coal mine safety evaluation problem to highlight the applicability of the proposed method, confirm its suitability through a sensitivity analysis, and demonstrate its strengths through a comparative analysis with other existing approaches.

Safety is not only a consistent concern in coal mining but also a fundamental topic during the process of coal mine production. Safety evaluation is one of the key questions in coal mining that must be emphasized at the highest levels, and it is therefore important to establish a scientific and feasible method for evaluating coal mine safety. The underground working conditions inherent in coal mining involve many perils and uncertainties, and decision-makers often have an ambiguous understanding of evaluation information. This situation is highly suitable for employing qualitative tools rather than numerical values to conduct assessments [1]. In this context, the proposed MCDM method based on LIFNs, as described in Sect. 4, can be used to address the practical problem of coal mine safety evaluation.

A state mining bureau reviews the safety conditions of five coal mines in a given area according to correlative assessment methods and production regulations. The five coal mines are denoted by $A = \{a_1, a_2, a_3, a_4, a_5\}$. Many factors affect the safety environment, and the following four criteria are considered based on detailed investigation: c_1, technological equipment; c_2, geological conditions; c_3, human diathesis; c_4, management quality [1]. The weight vector for these criteria is $w = (0.25, 0.22, 0.35, 0.18)$. Furthermore, the linguistic terms provided in Example 1 are employed in the evaluation process; in order to obtain more original information and more accurately reflect reality, the evaluation values provided by experts are transformed into LIFNs and denoted by $\phi_{ij} = (s_{u_{ij}}, s_{v_{ij}})(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$. The transformed evaluation matrix is presented in Table 1.

5.1 Illustration of the proposed approach

The main procedures for identifying the safest coal mine are described in the following steps.

Step 1. Normalize the evaluation matrix.

There is no need to normalize the evaluations in this case, because all of the criteria are of the benefit type.

Step 2. Obtain comprehensive evaluations.

By using the LIFFIWHM operator (let $\lambda = 2$, $p = q = 1$, and LSF be $f^*_1(s_i)$), the comprehensive evaluations $\phi_i(i = 1, 2, 3, 4, 5)$ of each coal mine can be obtained as $\phi_1 = (s_{6.0797}, s_{1.3349})$, $\phi_2 = (s_{5.4870}, s_{1.5764})$, $\phi_3 = (s_{5.4734}, s_{1.8375})$, $\phi_4 = (s_{5.4870}, s_{1.9009})$, and $\phi_5 = (s_{5.4748}, s_{1.8598})$.

Step 3. Calculate the measurement function value of each ϕ_i.

The measurement function value $R(\phi_i)(i = 1, 2, ..., n)$ of the comprehensive evaluation ϕ_i can be calculated according to Definition 8, with the following results:

$R(\phi_1) = 0.1288, R(\phi_2) = 0.1754, R(\phi_3) = 0.1715, R(\phi_4) = 0.1691$, and $R(\phi_5) = 0.1710$.

Step 4. Rank all alternatives and select the safest one.

Table 1 Transformed evaluation matrix

	c_1	c_2	c_3	c_4
a_1	(s_5, s_1)	(s_6, s_2)	(s_7, s_1)	(s_5, s_2)
a_2	(s_6, s_2)	(s_6, s_1)	(s_5, s_2)	(s_5, s_1)
a_3	(s_7, s_1)	(s_5, s_2)	(s_4, s_3)	(s_6, s_1)
a_4	(s_6, s_2)	(s_6, s_1)	(s_5, s_2)	(s_5, s_3)
a_5	(s_6, s_1)	(s_4, s_2)	(s_6, s_2)	(s_5, s_3)

Table 2 Rankings of coal mines with different LSFs and λ

	LSF1	LSF2	LSF3
$\lambda = 1$	$a_1 \succ a_4 \succ a_3 \succ a_5 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
$\lambda = 2$	$a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
$\lambda = 5$	$a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
$\lambda = 8$	$a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
$\lambda = 10$	$a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
$\lambda = 20$	$a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$

Table 3 Rankings of coal mines with different LSFs and p, q

	LSF1	LSF2	LSF3
$p \to 0, q = 0$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$	$a_1 \succ a_3 \succ a_4 \succ a_5 \succ a_2$	$a_1 \succ a_4 \succ a_2 \succ a_5 \succ a_3$
$p = 0.1, q = 0$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$	$a_3 \succ a_1 \succ a_4 \succ a_2 \succ a_5$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$
$p = 1, q = 0$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$	$a_3 \succ a_1 \succ a_4 \succ a_2 \succ a_5$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$
$p = 2, q = 1$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$	$a_1 \succ a_3 \succ a_4 \succ a_5 \succ a_2$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$
$p = 5, q = 1$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$	$a_1 \succ a_3 \succ a_4 \succ a_5 \succ a_2$	$a_1 \succ a_3 \succ a_4 \succ a_2 \succ a_5$
$p = q = 0.1$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$	$a_1 \succ a_3 \succ a_4 \succ a_5 \succ a_2$	$a_1 \succ a_4 \succ a_2 \succ a_5 \succ a_3$
$p = q = 1$	$a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_4 \succ a_5 \succ a_2 \succ a_3$
$p = q = 2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_4 \succ a_5 \succ a_2$
$p = q = 5$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$
$p = 0.001, q = 1$	$a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2$	$a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2$	$a_1 \succ a_5 \succ a_4 \succ a_2 \succ a_3$
$p = 0.1, q = 1$	$a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2$	$a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2$	$a_1 \succ a_5 \succ a_4 \succ a_2 \succ a_3$
$p = 0.5, q = 1$	$a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2$	$a_1 \succ a_5 \succ a_3 \succ a_4 \succ a_2$	$a_1 \succ a_5 \succ a_4 \succ a_2 \succ a_3$
$p = 1, q = 2$	$a_1 \succ a_5 \succ a_3 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2$
$p = 1, q = 5$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$	$a_1 \succ a_3 \succ a_5 \succ a_4 \succ a_2$

Step 3 makes it clear that the final ranking of the five coal mines is $a_1 \succ a_4 \succ a_5 \succ a_3 \succ a_2$, with the safest coal mine of a_1.

5.2 Discussion of the influence of parameters λ, p, q

The above safety ranking of the coal mine options was obtained by the LIFFIWHM operator under the parameters $\lambda = 2$ and $p = q = 1$. It is necessary to discuss whether or not the final rankings change when different values of these parameters are used. Moreover, since LSFs are utilized to deal with linguistic information, three different LSFs are always tested in this situation.

First, the influence of different values of λ in the Frank operations is discussed, with the final rankings for the coal mines shown in Table 2.

The above data show that the rankings obtained based on LSF1 remain constant except where $\lambda = 1$, while the rankings obtained based on LSF2 and LSF3 are always constant. However, it is worth noting that the rankings are different under different LSFs. This indicates that the LIFFIWHM operator is not sensitive to changing values of λ under the semantic situation in the above coal mine safety evaluation problem. Moreover, the safest coal mine is always a_1 no matter what value of λ or LSF is used. The consistency of this result fully demonstrates the reliability and accuracy of the proposed method.

Second, the influence on the final ranking of different values for parameters p and q is also analyzed. In order to precisely reflect the effect of the values of p and q, and the interrelationships between these values, the values of p and q can be divided into three categories. In the first category, the value of p exceeds that of q; the value of p equals that of q in the second category, and the value of p is smaller than that of q in the third category. The final rankings of the coal mines are shown in Table 3.

Table 3 shows that the rankings of the coal mines change with varying values of p and q under different LSFs. The safest coal mine is always a_1, except in situations where $p = 0.1, q = 0$ and $p = 1, q = 0$ under LSF2. In addition, it is worth noting that the rankings are basically same with different values of λ, but they apparently change with changes to p and q. This suggests that the LIFFIWHM operator is more sensitive to variations in p and q than to

Table 4 Ranking results acquired utilizing different methods

Methods	Ranking results
Chen et al.'s method with the LIFWA operator [23]	$a_1 \succ a_3 \succ a_2 \succ a_5 \succ a_4$
Chen et al.'s method with the LIFWG operator [23]	$a_1 \succ a_2 \succ a_4 \succ a_5 \succ a_3$
Xu's method with the ULWA operator [16]	$a_1 \succ a_2 \succ a_4 \succ a_5 \succ a_3$
Xu's method with the ULWGM operator [17]	$a_1 \succ a_2 \succ a_4 \succ a_5 \succ a_3$
Wei et al.'s method with the ULWBM operator [18]	$a_1 \succ a_2 \succ a_3 \succ a_4 \succ a_5$
Wei et al.'s method with the ULWGBM operator [18]	$a_1 \succ a_2 \succ a_4 \succ a_5 \succ a_3$

changes in λ under the semantic situation in this coal mine safety evaluation problem.

The preceding discussion demonstrates that the values of λ, p and q, as well as the LSF, can influence the final ranking of the coal mines. Broadly, λ, p and q are correlated with the thinking mode of the decision-makers. The bigger the values of λ, p and q, the more optimistic the decision-makers are; meanwhile, the smaller the values of λ, p and q, the more pessimistic the decision-makers are. Therefore, the proposed method for evaluating coal mine safety is very flexible, and decision-makers can choose appropriate values of λ, p and q and different LSFs according to their preferences and actual semantic situation in order to obtain the most precise result.

6 Comparative analysis and discussion

In order to further verify the validity of the method proposed in this paper, we conducted a comparative analysis by applying other existing methods to the example described above in the context of LIFNs or ULVs.

In their method, Chen et al. [23] proposed many aggregation operators based on LIFNs to aggregate evaluation information, such as the linguistic intuitionistic fuzzy weighted averaging (LIFWA) and linguistic intuitionistic fuzzy weighted geometric (LIFWG) operators. Then, the alternatives were ranked utilizing the linguistic score index and accuracy index.

Other scholars have proposed a series of aggregation operators for ULVs under uncertain linguistic environments, and some were selected for comparison here. Xu [16] proposed the uncertain linguistic weighted averaging (ULWA) operator, Xu [17] proposed the uncertain linguistic weighted geometric mean (ULWGM) operator, and Wei et al. [18] proposed both the uncertain linguistic weighted Bonferroni mean (ULWBM) operator and the uncertain linguistic weighted geometric Bonferroni mean (ULWGBM) operator.

The coal mine safety evaluation problem presented above can be addressed using these methods. First, the LIFNs must be transformed into ULVs. Let (s_u, s_v) be a LIFN; then, $[s_u, s_{2t-v}]$ is an ULV, and s_u and s_{2t-v} are the lower and upper limits, respectively [23]. For example, LIFN (s_5, s_1) can be transformed into ULV $[s_5, s_7]$. The ranking results acquired utilizing different methods are displayed in Table 4.

The data in Table 4 show that the same ranking is produced by Chen et al.'s method with the LIFWG operator [23], Xu's method with the ULWA operator [16], Xu's method with the ULWGM operator [17], and Wei et al.'s method with the ULWGBM operator [18]. In contrast, Chen et al.'s method with the LIFWA operator [23] and Wei et al.'s method with the ULWBM operator [18] produce two other rankings.

However, the rankings obtained by these extant methods are consistently different from the proposed method's results in all cases. There are some possible reasons for this difference in rankings. First, the extant methods and the proposed method use different operations. The operations in the extant methods are defined by simply dealing with the subscript of the linguistic term; this strategy has some non-negligible shortcomings and leads to the loss and distortion of the original information. However, the operations in the proposed method are constructed based on Frank operations and LSFs, which can effectively negate the drawbacks in the preexisting methods; additionally, different parameter values of λ and different LSFs are flexible and applicable under distinct semantic situations. Second, Chen et al.'s method [23], Xu's method [16], and Xu's method [17] do not consider the interrelationships among different input arguments, while the proposed method take these relationships into account with the HM operator. Although Wei et al.'s method using both ULWBM and ULWGBM [18] can reflect the correlations among different input arguments, there are two other drawbacks that cannot be ignored. On the one hand, the BM operator generates some unnecessary redundancy. For example, given a set of input variables $x_i (i = 1, 2, ..., n)$, the BM operator considers not only the correlation between x_i and $x_j (i \neq j)$, but also the interrelationships between x_j

and $x_i (i \neq j)$. On the other hand, although the BM operator can reflect the interrelationships between each pair of x_i and $x_j (i \neq j)$, it ignores the relationship between x_i and itself. However, the method proposed in this paper, which uses the HM operator, can counter these drawbacks of ULWBM and ULWGBM [18]. Finally, the extant methods and the proposed method have radically different ways to determine the final ranking for alternatives. Chen et al.'s method [23] uses the linguistic score index and linguistic accuracy index to compare comprehensive evaluation values, and other three extant methods in Refs. [16, 18] obtain final rankings by utilizing the possibility degree and complementary matrix; in contrast, the proposed method employs the new measurement function of LIFNs to compare the comprehensive evaluation values for each alternative, and then the final ranking is determined. Compared to the comparison method in Ref. [23], the proposed method in this paper is more reasonable and feasible, as discussed in Sect. 3.

Based on the above comparison analysis, we can conclude that the proposed method can be successfully utilized in evaluating the safety of coal mines, and it can identify the safest coal mine more precise than other methods. By using LSFs, the proposed method is more flexible and effective in dealing with MCDM problems involving linguistic information, and it is more robust than the other extant methods that are based on algebraic operations because of the introduction of Frank operations. Moreover, the proposed method takes into account the correlations among different criteria values by utilizing the HM operator.

7 Conclusion

To deal with situations where decision-makers employ qualitative values rather than real numbers to reflect the uncertainty and vagueness of coal mine safety evaluation problems, LIFSs were employed in this paper. In order to improve the applicability and validity of methods that use linguistic intuitionistic aggregation operators, the LIFFIWHM operator was proposed, and it was employed to establish an innovative MCDM approach for evaluating coal mine safety. First, a reliable comparison method and Frank operations were introduced for LIFNs. Then, with the aid of the LSFs, Frank operations, and HM operator, a novel linguistic intuitionistic aggregation operators was developed. Finally, the approach was tested using an example of coal mine safety evaluation problem, and it was further validated through a sensitivity analysis. The feasibility and reliability of the proposed approach were demonstrated through a comparison with other existing methods.

The main contributions of this research are the introduction of the Frank operations for LIFNs, which are more robust and flexible than the existing linguistic intuitionistic operations, as well as the integration of LIFSs with the HM operator, which allows the developed operators not only to capture correlations among criteria values but also to accommodate situations where the input arguments consist of linguistic intuitionistic fuzzy information. Moreover, the proposed approach is fairly flexible to use; the results may change when different LSFs are used, and the parameters λ,p, and q, all of which appear in the developed operators, may also affect the final results. Therefore, decision-makers can choose appropriate values for λ,p, and q according to their preferences, and they can choose a proper LSF based on practical semantic circumstances to obtain the most accurate result.

Future research will focus on introducing many meaningful models or methods to deal with linguistic intuitionistic fuzzy information in a broader CWW perspective, such as 2-tuple linguistic model [51], discrete fuzzy numbers linguistic computational model [25], and multigranular fuzzy linguistic method [14]. Moreover, we will consider developing some effective methods that use LIFNs to address problems involving incomplete information [6] and non-cooperative behaviors [55], which occur extensively in practice.

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Appendix

Proof of Theorem. In the following, Theorem 1 will be proved utilizing the mathematical induction on n.

Proof Firstly, the following equation needs to be proved.

$$\bigoplus_{F\,i=1,j=i}^{n} w_i w_j \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_j)^{\wedge_F q}) = \left(f^{*-1}\left(1-\log_\lambda\left(1+(\lambda-1)\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}\right)^{w_i w_j}\right)\right), f^{*-1}\left(\log_\lambda\left(1+(\lambda-1)\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}\right)^{w_i w_j}\right)\right)\right). \tag{12}$$

1. For $n = 2$, the following equation can be calculated easily.

$$\bigoplus_{F\,i=1,j=i}^{2} w_i w_j \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_j)^{\wedge_F q}) = \left(f^{*-1}\left(1-\log_\lambda\left(1+(\lambda-1)\prod_{i=1,j=i}^{2}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}\right)^{w_i w_j}\right)\right), f^{*-1}\left(\log_\lambda\left(1+(\lambda-1)\prod_{i=1,j=i}^{2}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}\right)^{w_i w_j}\right)\right)\right).$$

2. If Eq. (11) holds for $n = k$, there is

$$\bigoplus_{F\,i=1,j=i}^{k} w_i w_j \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_j)^{\wedge_F q}) = \left(f^{*-1}\left(1-\log_\lambda\left(1+(\lambda-1)\prod_{i=1,j=i}^{k}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}\right)^{w_i w_j}\right)\right), f^{*-1}\left(\log_\lambda\left(1+(\lambda-1)\prod_{i=1,j=i}^{k}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}\right)^{w_i w_j}\right)\right)\right). \tag{13}$$

Then, when $n = k + 1$, the following equation can be obtained

$$\bigoplus_{F\,i=1,j=i}^{k+1} w_i w_j \cdot_F ((\phi_i)^{\hat{}_F p} \otimes_F (\phi_j)^{\hat{}_F q}) = \bigoplus_{F\,i=1,j=i}^{k} w_i w_j \cdot_F ((\phi_i)^{\hat{}_F p} \otimes_F (\phi_j)^{\hat{}_F q}) \oplus_F \bigoplus_{F\,i=1}^{k+1} w_i w_{k+1} \cdot_F ((\phi_i)^{\hat{}_F p} \otimes_F (\phi_{k+1})^{\hat{}_F q}). \tag{14}$$

According to the operations of LIFNs, the following result can be calculated.

$$\bigoplus_{F\,i=1}^{k+1} w_i w_{k+1} \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_{k+1})^{\wedge_F q}) = \left(f^{*-1}\left(1-\log_\lambda\left(1+(\lambda-1)\prod_{i=1}^{k+1}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_{k+1}})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_{k+1}})}-1)^q}\right)^{w_i w_{k+1}}\right)\right), f^{*-1}\left(\log_\lambda\left(1+(\lambda-1)\prod_{i=1}^{k+1}\left(\frac{(\lambda-1)^{p+q}-(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{f^*1-(s_{v_{k+1}})}-1)^q}{(\lambda-1)^{p+q}+(\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{f^*1-(s_{v_{k+1}})}-1)^q}\right)^{w_i w_{k+1}}\right)\right)\right). \tag{15}$$

Equation (15) can be easily proved by utilizing the mathematical induction on $k+1$, and the proof is omitted here.

Thus, by utilizing Eqs. (13) and (15), Eq. (14) can be converted into

$$\bigoplus_{F\, i=1,j=i}^{k+1} w_i w_j \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_j)^{\wedge_F q}) = \bigoplus_{F\, i=1,j=i}^{k} w_i w_j \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_j)^{\wedge_F q}) \oplus_F \bigoplus_{F\, i=1}^{k+1} w_i w_{k+1} \cdot_F ((\phi_i)^{\wedge_F p} \otimes_F (\phi_{k+1})^{\wedge_F q})$$
$$= \left(f^{*-1}\left(1 - \log_\lambda\left(1 + (\lambda-1)\prod_{i=1,j=i}^{k+1}\left(\frac{(\lambda-1)^{p+q} - (\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}{(\lambda-1)^{p+q} + (\lambda-1)(\lambda^{f^*(s_{u_i})}-1)^p(\lambda^{f^*(s_{u_j})}-1)^q}\right)^{w_i w_j}\right)\right),\right.$$
$$\left. f^{*-1}\left(\log_\lambda\left(1 + (\lambda-1)\prod_{i=1,j=i}^{k+1}\left(\frac{(\lambda-1)^{p+q} - (\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}{(\lambda-1)^{p+q} + (\lambda-1)(\lambda^{1-f^*(s_{v_i})}-1)^p(\lambda^{1-f^*(s_{v_j})}-1)^q}\right)^{w_i w_j}\right)\right)\right).$$

That is, Eq. (12) also holds for $n = k+1$. Thus, Eq. (12) is true for all n.

Then, by using Eq. (12), Eq. (6) can be calculated easily based on the operations of LIFNs, and Eq. (7) can be eventually acquired. Therefore, the proof Theorem 1 is completed.

Proof of Theorem 3.

Proof Let $LIFFIWHM^{p,q}(a_1, a_2, \dots, a_n) = (s_{u(a)}, s_{v(a)})$and $LIFFIWHM^{p,q}(b_1, b_2, \dots, b_n) = (s_{u(b)}, s_{v(b)})$. Since f^*, f^{*-1} and $\log_\lambda, (\lambda > 1)$ is a strictly monotonously increasing and continuous function, and $s_{u(a_i)} \ge s_{u(b_i)}$ and $s_{u(a_j)} \ge s_{u(b_j)}$ for all $i, j = 1, 2, \dots, n$, then the following inequalities can be obtained.

$$f^*(s_{u(a_i)}) \ge f^*(s_{u(b_i)}), f^*(s_{u(a_j)}) \ge f^*(s_{u(b_j)}) \Rightarrow (\lambda^{f^*(s_{u(a_i)})}-1)^p(\lambda^{f^*(s_{u(a_j)})}-1)^q \ge (\lambda^{f^*(s_{u(b_i)})}-1)^p(\lambda^{f^*(s_{u(b_j)})}-1)^q$$
$$\Rightarrow \left(\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q} - (\lambda^{f^*(s_{u(a_i)})}-1)^p(\lambda^{f^*(s_{u(a_j)})}-1)^q}{(\lambda-1)^{p+q} + (\lambda-1)(\lambda^{f^*(s_{u(a_i)})}-1)^p(\lambda^{f^*(s_{u(a_j)})}-1)^q}\right)^{w_i w_j}\right)^k$$
$$\le \left(\prod_{i=1,j=i}^{n}\left(\frac{(\lambda-1)^{p+q} - (\lambda^{f^*(s_{u(b_i)})}-1)^p(\lambda^{f^*(s_{u(b_j)})}-1)^q}{(\lambda-1)^{p+q} + (\lambda-1)(\lambda^{f^*(s_{u(b_i)})}-1)^p(\lambda^{f^*(s_{u(b_j)})}-1)^q}\right)^{w_i w_j}\right)^k$$
$$\Rightarrow s_{u(a)} \ge s_{u(b)}.$$

In the same way, the inequality $s_{v(a)} \le s_{v(b)}$ can also be obtained. Since $s_{u(a)} \ge s_{u(b)}$ and $s_{v(a)} \le s_{v(b)}$, then, $LIFFIWHM^{p,q}(a_1, a_2, \dots, a_n) \ge LIFFIWHM^{p,q}(b_1, b_2, \dots, b_n)$.

Proof of Theorem 4.

Proof Since $s_{u(a)} \ge s_{u_i}$ and $s_{v(a)} \le s_{v_i}$ for all $i = 1, 2, \dots, n$, then according to Theorems 2 and 3, we can obtain $LIFFIWHM^{p,q}(\phi_1, \phi_2, \dots, \phi_n) \le LIFFIWHM^{p,q}(a, a, \dots, a) = a$. Since $s_{u(b)} \le s_{u_i}$ and $s_{v(b)} \ge s_{v_i}$ for all $i = 1, 2, \dots, n$, then we can also obtain $b = LIFFIWHM^{p,q}(b, b, \dots, b) \le LIFFIWHM^{p,q}(\phi_1, \phi_2,$ $b = L$. Thus, $b \le LIFFIWHM^{p,q}(\phi_1, \phi_2, \dots, \phi_n) \le a$ is true.

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