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ORIGINAL ARTICLE

An approach to interval-valued intuitionistic stochastic multi-criteria decision-making using set pair analysis

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Abstract This paper proposes an approach to interval-valued intuitionistic stochastic multi-criteria decision-making (MCDM) problems using set pair analysis. This approach is applicable to MCDM problems in which the criterion weights are incomplete or the weights are certain, and evaluation values of alternatives take the form of interval-valued intuitionistic stochastic variables. To begin with, we briefly introduce the concepts of interval-valued intuitionistic fuzzy set, interval-valued intuitionistic stochastic variable, and set pair analysis. Then, we define a new similarity measure between interval-valued intuitionistic fuzzy numbers, after which we establish a mathematical programming model based on the technique for order preference by similarity to an ideal solution method and the maximizing deviation method in order to determine criterion weights. We then use connection degree to represent interval-valued intuitionistic fuzzy information and transform the interval-valued intuitionistic stochastic decision-making matrixes into corresponding connection degree matrixes. Finally, we rank the alternatives according to the value of set pair potential after calculating the connection degree of each alternative. After defining the method, we apply it to a practical decision-making problem and provide a comparison analysis with existing methods to illustrate the feasibility and validity of the proposed approach.

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1 Introduction

Multi-criteria decision-making (MCDM) refers to evaluating, ranking, or selecting alternatives on the basis of several conflicting criteria or preferences. Due to the complexity of life, the real world contains many MCDM problems in which the preference information and criterion weights are inaccurate, uncertain, or incomplete. To address these circumstances, Zadeh introduced the concept of fuzzy set (FS) [1], wherein an element's membership degree with regard to a set is represented by a real number between zero and one. FS was identified as an excellent theory with which to describe inexact, uncertain, and imprecise information, and has been extended to many application domains such as e-commerce, link prediction, machine learning, fuzzy classification, intrusion detection [2–8]. Considering that the single membership function is insufficient to express the information completely, building upon FS, Atanassov introduced intuitionistic fuzzy set (IFS) [9], which takes the membership degree and non-membership degree into consideration simultaneously. IFS is more useful than FS in dealing with the presence of hesitancy and vagueness originating from imprecise information, and it has attracted great interest from scholars [10–14]. However, considering the complexity and uncertainty inherent in practical problems, interval numbers are more suitable than exact numbers for expressing membership and non-membership degrees. Therefore, Atanssov and Gargov [15] introduced interval-valued intuitionistic fuzzy set (IVIFS) as an important extension of IFS that can better characterize membership and non-membership degrees. Compared with FS and IFS, IVIFS may be more flexible in describing abundant information when uncertain information is involved [16, 17]. Interval-valued intuitionistic fuzzy number (IVIFN) [18] is a special case of IVIFS, and using IVIFN to model human judgment may be a more appropriate way to handle real-life problems. For example, when a paper is sent to a reviewer, he may use IVIFN ([0.3, 0.6], [0.1, 0.3], [0.1, 0.6]) to express his attitude toward the paper. The notation [0.3, 0.6] represents that the paper is 30–60 percent acceptable, [0.1, 0.3]means that the paper is 10–30 percent unacceptable, and [0.1, 0.6] expresses his hesitation. In this example, IVIFNs describe the real-life problem better than crisp numbers. Both IVIFSs and IVIFNs have been widely applied in MCDM problems, and research on the subject can be roughly classified into four main topics: aggregation operators [19–21], similarity (or distance) measures [22–25], extension of classic decision-making methods [26–28] and preference relation [29, 30]. Xu [19], Liu [20], and He et al. [21] proposed some operators for IVIFNs and applied them to MCDM problems. Chen [26] extended the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method to multi-criteria group decision-making problems with interval-valued intuitionistic fuzzy information.

In daily life, it is common for decision-makers to encounter MCDM problem in which the consequences of each alternative are presented in the form of stochastic variables with probability distributions. These problems are called stochastic MCDM problems, and they have extensive applications [31–35]. Zhou et al. [31] considered the bounded rationality of decision-makers while integrating regret theory and TOPSIS in order to handle grey stochastic MCDM problems with extended grey numbers. Wang et al. [32] extended the Elimination Et Choice Translating Reality (ELECTRE) to rough stochastic MCDM problems. Tan et al. [33] integrated prospect stochastic dominance degrees and Preference Ranking Organization Methods for Enrichment Evaluations II (PROMETHEE II) to handle stochastic MCDM problems. Okul et al. [34] combined stochastic multi-criteria acceptability analysis (SMAA) with TOPSIS, applying the new method to drug risk–benefit analysis and machine gun selection. Hu et al. [35] combined prospect theory with conjoint analysis in addressing dynamic stochastic MCDM problems with unknown weight information.

As decision-making environments and problems have become increasingly complex and uncertain, stochastic MCDM problems with intuitionistic fuzzy information have gradually received more attention. For example, Gao and Liu [36] developed a new approach to interval-valued intuitionistic stochastic MCDM problems by utilizing prospect theory and P-score function. Li et al. [37–39] introduced methods for solving stochastic MCDM problems with IFS and incomplete information, while Hu et al. [40] defined a new score function and distance measure for IFS and proposed an approach to intuitionistic stochastic MCDM problems based on prospect theory. Wang and Li [41] defined a new score function and applied it to intuitionistic stochastic MCDM problems.

Despite the volume of research, almost all of the methods for dealing with intuitionistic stochastic MCDM problems are based on prospect theory. In prospect theory, the complexity of the decision-making problems makes it difficult for decision-makers to provide an accurate reference point in the real decision-making process. Furthermore, the five parameters in the calculation functions of prospect theory, α, β, θ, γ and δ, are hard to determine because they depend on the psychological behavior of the decision-makers. In contrast, set pair analysis [42], introduced by Zhao, has no requirement to determine a reference point or parameters. Set pair analysis is a modified uncertainty theory that considers both certainties and uncertainties as an integrated certain-uncertain system. Set pair analysis is considered to be an excellent theory for analyzing and dealing with imprecise, inconsistent, and incomplete information, and it has been applied in such fields as risk assessment, agriculture, and decision-making [43–45]. In set pair analysis, connection degree functions as a useful tool to systematically describe certainty and uncertainty from the three aspects of identity degree, discrepancy degree, and contrary degree. Because set pair analysis uses a simple mathematic depiction to express ambiguous and imprecise information with clarity, it is utilized in research into MCDM problems [46–48]. Zhang [46] analyzed the compatibility between set pair analysis and IFS, providing a method for transforming intuitionistic fuzzy numbers into connection numbers. Yue et al. [47] developed a multi-criteria group decision-making approach based on set pair analysis, and Hu and Liu [48] combined cumulative prospect theory with set pair analysis for use in dynamic stochastic MCDM problems.

Randomness and intuitionistic logic commonly exist simultaneously in decision-making problems, but little systematic research has explored these problems. Previously published works recognize the advantages of set pair analysis theory, but most existing methods of handling intuitionistic stochastic MCDM problems are based on prospect theory, no relevant research has applied set pair analysis to deal with intuitionistic or interval-valued stochastic MCDM problems. This gap in the research motivates us to apply set pair analysis to interval-valued intuitionistic stochastic MCDM problems. The main contributions of this work are summarized below.

1. In order to derive criterion weights, this paper defines a new similarity measure of IVIFNs that considers hesitation information. This paper also constructs a mathematical programming model based on TOPSIS and the maximizing deviation method in order to determine optimal weight values when a decision-maker may provide incomplete or inconsistent opinions about weight information.
2. The proposed method introduces a new representation using connection degrees to express interval-valued intuitionistic fuzzy information on the basis of set pair analysis. The proposed method deals directly with interval-valued intuitionistic fuzzy information, avoiding any loss of fuzzy information. Furthermore, the set pair potential is employed to rank alternatives.
3. In contrast to existing interval-valued intuitionistic stochastic MCDM methods based on prospect theory, the proposed method based on set pair analysis does not need to consider parameter values or set a reference point. Besides, the calculations of the proposed method are relatively simple.

The rest of this paper is organized as follows. Section 2 briefly reviews some basic concepts of IVIFS, interval-valued intuitionistic stochastic variable, and set pair analysis. Section 3 introduces a new similarity measure between IVIFNs and gives some examples to illustrate its validity. Section 4 presents an approach to interval-valued intuitionistic stochastic MCDM problems using set pair analysis. Section 5 uses an example to illustrate the efficiency and feasibility of the proposed approach and describe a comparison analysis with existing methods. Section 6 presents conclusions.

2 Preliminaries

This section provides a brief review of some concepts of IVIFS, interval-valued intuitionistic stochastic variable and set pair analysis.

2.1 IVIFS

Definition 1 [15, 18]: Let X be a universal set, and let int([0, 1]) be a set of all the closed subintervals of [0, 1]. Then an IVIFS $\tilde{A}$ in X is described by the form

$$\tilde{A} = \left\{ \left\langle x, \tilde{\mu}_{\tilde{A}}(x), \tilde{\nu}_{\tilde{A}}(x), \tilde{\pi}_{\tilde{A}}(x) \right\rangle | x \in X \right\},$$

where the functions $\tilde{\mu}_{\tilde{A}} : X \to \mathrm{int}([0,1])$ and $\tilde{\nu}_{\tilde{A}} : X \to \mathrm{int}([0,1])$ denote the interval membership and interval non-membership degrees, respectively, of element x to IVIFS $\tilde{A}$. For any $x \in X$ to IVIFS $\tilde{A}$, $\sup\left(\tilde{\mu}_{\tilde{A}}(x)\right) + \sup\left(\tilde{\nu}_{\tilde{A}}(x)\right) + \inf\left(\tilde{\pi}_{\tilde{A}}(x)\right) = 1$, and $\inf\left(\tilde{\mu}_{\tilde{A}}(x)\right) + \inf\left(\tilde{\nu}_{\tilde{A}}(x)\right) + \sup\left(\tilde{\pi}_{\tilde{A}} left(x)\right)=1$.. Call $\tilde{\pi}_{\tilde{A}}(x) = 1 - \tilde{\mu}_{\tilde{A}}(x) - \tilde{\nu}_{\tilde{A}}(x)$ the interval hesitation degree of element x to IVIFS $\tilde{A}$, for any $x \in X$ to the IVIFS $\tilde{A}$, $\tilde{\pi}_{\tilde{A}} : X \to \mathrm{int}([0,1])$. For convenience, the IVIFS can be denoted as follows:

$$\tilde{A} = \left\{ \left\langle x, \left[\tilde{\mu}^L_{\tilde{A}}(x), \tilde{\mu}^U_{\tilde{A}}(x)\right], \left[\tilde{\nu}^L_{\tilde{A}}(x), \tilde{\nu}^U_{\tilde{A}}(x)\right], \left[\tilde{\pi}^L_{\tilde{A}}(x), \tilde{\pi}^U_{\tilde{A}}(x)\right] \right\rangle | x \in X \right\}.$$

In practice, IVIFN $\widetilde{\alpha}$ can be denoted as follows: $\widetilde{\alpha} = ([a^L, a^U], [b^L, b^U], [c^L, c^U])$, where $[a^L, a^U] \subseteq [0,1]$, $[b^L, b^U] \subseteq [0,1]$ and $a^U + b^U < 1$.

Definition 2 [15]: Let $\widetilde{\alpha}_1 = \left([a^L_1, a^U_1], [b^L_1, b^U_1], [c^L_1, c^U_1]\right)$ and $\widetilde{\alpha}_2 = \left([a^L_2, a^U_2], [b^L_2, b^U_2], [c^L_2, c^U_2]\right)$ be two IVIFNs, then

1. $\widetilde{\alpha}_1 \leq \widetilde{\alpha}_2$ if and only if $a^L_1 \leq a^L_2$, $a^U_1 \leq a^U_2$, $b^L_1 \geq b^L_2$, and $b^U_1 \geq b^U_2$.
2. $\widetilde{\alpha}_1 = \widetilde{\alpha}_2$ if and only if $a^L_1 = a^L_2$, $a^U_1 = a^U_2$, $b^L_1 = b^L_2$, and $b^U_1 = b^U_2$.

Definition 3 [52]: Let $\widetilde{\alpha}_1 = \left([a^L_1, a^U_1], [b^L_1, b^U_1], [c^L_1, c^U_1]\right)$ and $\widetilde{\alpha}_2 = \left([a^L_2, a^U_2], [b^L_2, b^U_2], [c^L_2, c^U_2]\right)$ be two IVIFNs, then the normalized Hamming distance between two IVIFNs is defined as follows:

$$D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = \frac{1}{4}\left(\left|a^L_1 - a^L_2\right| + \left|a^U_1 - a^U_2\right| + \left|b^L_1 - b^L_2\right| + \left|b^U_1 - b^U_2\right| + \left|c^L_1 - c^L_2\right| + \left|c^U_1 - c^U_2\right|\right). \quad (1)$$

Definition 4 [18, 49]: Let $\widetilde{\alpha} = ([a^L, a^U], [b^L, b^U], [c^L, c^U])$ be an IVIFN, then, the score function of $\widetilde{\alpha}$ can be denoted as follows:

$$S(\widetilde{\alpha}) = \left(a^L + a^U - b^L - b^U\right)/2. \quad (2)$$

2.2 Interval-valued intuitionistic stochastic variable

An interval-valued intuitionistic stochastic variable [41] is a group of countable stochastic variables made up of a finite set of IVIFNs. The interval-valued intuitionistic stochastic variable is denoted as $\xi(\omega)$. Table 1 shows the probability distribution of $\xi(\omega)$.

In Table 1, $\xi(\omega)$ is an interval-valued intuitionistic stochastic variable, ω_i is the i-th possible value that would be taken by $\xi(\omega)$, m is the number of values that an interval-

Table 1 Probability distribution of $\xi(\omega)$

$\xi(\omega)$	ω_1	ω_2	…	ω_i	…	ω_m
P	P_1	P_2	…	P_i	…	P_m

valued intuitionistic stochastic variable can have. P_i is the probability with respect to ω_i, and the probability density function $f(\xi(\omega))$ can be denoted as $f(\xi(\omega) = \omega_i) = P_i$.

Exampel 1 Assume that the evaluation information about a company's profitability can be expressed as ([0.6, 0.7], [0.2, 0.3], [0, 0.2]) with a probability of 0.4 when the company is well-run, or ([0.4, 0.5], [0.2, 0.4], [0.1, 0.4]) with a probability of 0.6 when the company is not well-run. This information can be presented by the interval-valued intuitionistic stochastic variable $\xi(\omega)$ described in Table 2. Meanwhile, the probability density function can be denoted as follows:

$$f(\xi(\omega) = ([0.6, 0.7], [0.2, 0.3], [0, 0.2])) = 0.4,\ f(\xi(\omega) = ([0.4, 0.5], [0.2, 0.4], [0.1, 0.4])) = 0.6.$$

2.3 Set pair analysis

Set pair analysis, proposed by Zhao [42], is a modified uncertainty theory that considers both certainties and uncertainties as an integrated certain-uncertain system. Set pair analysis involves a useful tool called connection degree that describes the certainty and uncertainty systematically from three aspects: identity degree, discrepancy degree, and contrary degree. This subsection introduces some basic concepts of set pair analysis.

Definition 5 [42, 48]: If E and F are two interrelated sets, the set pair can be denoted as $H(E, F)$. With respect to the set pair $H(E, F)$, the connection degree between sets E and F can be denoted as $\eta_{(H)} = \frac{I}{N} + \frac{D}{N}i + \frac{C}{N}j$, where N is the total number of characteristics of the set pair $H(E, F)$, while I, C, and $D = N - I - C$ represent, respectively, the numbers of identity characteristics, contrary characteristics, and discrepancy characteristics. The identity degree, discrepancy degree and contrary degree of the discussed set pair are represented by I/N, D/N and C/N, respectively. i denotes the coefficient of discrepancy degree and $i \in [0, 1]$, j denotes the coefficient of contrary degree and $j = -1$. For convenience, let $a = I/N$, $b = D/N$ and $c = C/N$, and then $\eta = \frac{I}{N} + \frac{D}{N}i + \frac{C}{N}j$ can be denoted as follows:

$$\eta = a + bi + cj. \tag{3}$$

Table 2 Probability distribution of $\xi(\omega)$

$\xi(\omega)$	([0.6, 0.7], [0.2, 0.3], [0, 0.2])	([0.4, 0.5], [0.2, 0.4], [0.1, 0.4])
P	0.4	0.6

Definition 6 [42, 48]: If the connection degree $\eta = a + bi + cj$ satisfies $c \neq 0$, the set pair potential, which characterizes the approximate degree between the two interrelated sets, can be denoted as follows:

$$Shi(\eta) = a/c. \tag{4}$$

Zhang et al. [46] point out the compatibility between set pair analysis and IFS. A given intuitionistic fuzzy value (μ_{ij}, v_{ij}, π_{ij}) can be represented in the form of a connection degree as

$$\eta_{ij} = a_{ij} + b_{ij}\breve{k} + c_{ij}\breve{l}, \tag{5}$$

where $a_{ij} = \mu_{ij}$, $b_{ij} = \pi_{ij}$, and $c_{ij} = v_{ij}$. The coefficient of discrepancy degree is represented by $\breve{k}$, while $\breve{l}$ is the coefficient of contrary degree and $\breve{l} = -1$.

3 A new similarity measure between IVIFNs

Scholars have studied the topic of similarity measures between IFSs and IVIFSs from various points of view. Some of similarity measures are extensions of well-known distance measures, such as similarity measures between IFSs or IVISs based on Hamming Distance [22, 50], Euclidian Distance [22] and Hausdorff distance [23, 51]. However, some similarity measures for IFSs and IVIFSs are entirely new, instead of extending well-known distance measures, these includes similarity measures based on entropy measure [25], preference relations [52], and cosine similarity measure [53, 54].

Definition 7 [22]: Let $\widetilde{\alpha}_1 = \left(\left[a_1^L, a_1^U\right], \left[b_1^L, b_1^U\right], \left[c_1^L, c_1^U\right]\right)$ and $\widetilde{\alpha}_2 = \left(\left[a_2^L, a_2^U\right], \left[b_2^L, b_2^U\right], \left[c_2^L, c_2^U\right]\right)$ be two IVIFNs, Xu [22] defined the similarity measures between $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ as listed below.

$$S_{X1}(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 - \frac{1}{4}\left(\left|a_1^L - a_2^L\right| + \left|b_1^L - b_2^L\right| + \left|a_1^U - a_2^U\right| + \left|b_1^U - b_2^U\right|\right), \tag{6}$$

$$S_{X2}(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 - \max\left(\left|a_1^L - a_2^L\right|, \left|b_1^L - b_2^L\right|, \left|a_1^U - a_2^U\right|, \left|b_1^U - b_2^U\right|\right), \tag{7}$$

$$S_{X3}(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 - \sqrt{\frac{1}{4}\left[\left(a_1^L - a_2^L\right)^2 + \left(b_1^L - b_2^L\right)^2 + \left(a_1^U - a_2^U\right)^2 + \left(b_1^U - b_2^U\right)^2\right]}, \tag{8}$$

$$S_{X4}(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 - \sqrt{\max\left\{\left(a_1^L - a_2^L\right)^2, \left(b_1^L - b_2^L\right)^2, \left(a_1^U - a_2^U\right)^2, \left(b_1^U - b_2^U\right)^2\right\}}. \tag{9}$$

The similarity measures between IVIFNs listed above have a common drawback in which they only compared the differences in membership and non-membership degrees. Wei et al. [25] and Singh [53] also defined the similarity measures between IVIFNs based on entropy measure and cosine similarity measure respectively. Similarly, these two similarity measures also ignored the differences in hesitation degrees. The similarity measure without considering hesitation information is inaccurate and produces some counterintuitive results in some special cases. To address this, we define a new similarity measure between IVIFNs incorporating the hesitation degree and the score function in the following.

Definition 8: Let $\widetilde{\alpha}_1 = \left(\left[a_1^L, a_1^U\right], \left[b_1^L, b_1^U\right], \left[c_1^L, c_1^U\right]\right)$ and $\widetilde{\alpha}_2 = \left(\left[a_2^L, a_2^U\right], \left[b_2^L, b_2^U\right], \left[c_2^L, c_2^U\right]\right)$ be two IVIFNs, the similarity measure between $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ can be denoted as follows:

$$M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 - \frac{|S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_2)|}{4} - \frac{D(\widetilde{\alpha}_1, \widetilde{\alpha}_2)}{2}, \tag{10}$$

where $S(\widetilde{\alpha}_1) = \left(a_1^L + a_1^U - b_1^L - b_1^U\right)/2$, $S(\widetilde{\alpha}_2) = \left(a_2^L + a_2^U - b_2^L - b_2^U\right)/2$, and $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = \frac{1}{4}\left(\left|a_1^L - a_2^L\right| + \left|a_1^U - a_2^U\right| + \left|b_1^L - b_2^L\right| + \left|b_1^U - b_2^U\right| + \left|c_1^L - c_2^L\right| + \left|c_1^U - c_2^U\right|\right)$.

Theorem 1 Let $\widetilde{\alpha}_1$ and $\widetilde{\alpha}_2$ be two IVIFNs, the similarity measure $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2)$ satisfies the following properties:

1. $0 \le M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \le 1$.
2. $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = M(\widetilde{\alpha}_2, \widetilde{\alpha}_1)$.
3. $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1$ if and only if $\widetilde{\alpha}_1 = \widetilde{\alpha}_2$.
4. If $\widetilde{\alpha}_1 \le \widetilde{\alpha}_2 \le \widetilde{\alpha}_3$, then $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \ge M(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$ and $M(\widetilde{\alpha}_2, \widetilde{\alpha}_3) \ge M(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$.

Proof

1. Solve the nonlinear programming problem: $\max(D(\widetilde{\alpha}_1, \widetilde{\alpha}_2)) = \max\left(\frac{1}{4}\left(\left|a_1^L - a_2^L\right| + \left|a_1^U - a_2^U\right| + \left|b_1^L - b_2^L\right| + \left|b_1^U - b_2^U\right| + \left|c_1^L - c_2^L\right| + \left|c_1^U - c_2^U\right|\right)\right)$, subject to $0 \le a_1^L \le a_1^U \le 1$, $0 \le b_1^L \le b_1^U \le 1$, $0 \le c_1^L \le c_1^U \le 1$, $0 \le a_2^L \le a_2^U \le 1$, $0 \le b_2^L \le b_2^U \le 1$, $0 \le c_2^L \le c_2^U \le 1$, $a_1^L + b_1^L \le 1$, $a_1^U + b_1^U \le 1$, $a_2^L + b_2^L \le 1$, $a_2^U + b_2^U \le 1$, $a_1^U + b_1^U + c_1^L = 1$, $a_1^L + b_1^L + c_1^U = 1$, $a_2^U + b_2^U + c_2^L = 1$ and $a_2^L + b_2^L + c_2^U = 1$. By using WinQSB, we can get $\max(D(\widetilde{\alpha}_1, \widetilde{\alpha}_2)) = 1$ if $\widetilde{\alpha}_1 = ([0.0001, 0.0005], [0.5426, 0.9995], [0, 0.4573])$ and $\widetilde{\alpha}_2 = ([0.9997, 0.9998], [0.0001, 0.0002], [0, 0.0002])$. In other words, $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \le 1$. Clearly, $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \ge 0$, and thus $0 \le D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \le 1$ can be obtained.
   Xu [18] proved that $-1 \le S(\widetilde{\alpha}_1) \le 1$ and $-1 \le S(\widetilde{\alpha}_2) \le 1$. Since $0 \le \frac{\left|S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_2)\right|}{4} \le \frac{1}{2}$ and $0 \le \frac{D(\widetilde{\alpha}_1, \widetilde{\alpha}_2)}{2} \le \frac{1}{2}$, we can get $0 \le M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \le 1$.
2. It is apparent that $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = D(\widetilde{\alpha}_2, \widetilde{\alpha}_1)$, so $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 - \frac{\left|S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_2)\right|}{4} - \frac{D(\widetilde{\alpha}_1, \widetilde{\alpha}_2)}{2} = M(\widetilde{\alpha}_2, \widetilde{\alpha}_1) = 1 - \frac{\left|S(\widetilde{\alpha}_2) - S(\widetilde{\alpha}_1)\right|}{4} - \frac{D(\widetilde{\alpha}_2, \widetilde{\alpha}_1)}{2}$, or in simpler terms, $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = M(\widetilde{\alpha}_2, \widetilde{\alpha}_1)$.
3. The definition of $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2)$ provides that we have $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 1 \Leftrightarrow S(\widetilde{\alpha}_1) = S(\widetilde{\alpha}_2)$, $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) = 0 \Leftrightarrow \left|a_1^L - a_2^L\right| = \left|a_1^U - a_2^U\right| = \left|b_1^L - b_2^L\right| = \left|b_1^U - b_2^U\right| = \left|c_1^L - c_2^L\right| = \left|c_1^U - c_2^U\right| = 0 \Leftrightarrow \widetilde{\alpha}_1 = \widetilde{\alpha}_2$.
4. If $\widetilde{\alpha}_1 \le \widetilde{\alpha}_2 \le \widetilde{\alpha}_3$, then $a_1^L + a_1^U \le a_2^L + a_2^U$ and $b_1^L + b_2^U \ge b_2^L + b_2^U$; therefore, $S(\widetilde{\alpha}_1) \le S(\widetilde{\alpha}_2)$. Similarly, $S(\widetilde{\alpha}_2) \le S(\widetilde{\alpha}_3)$, such that $S(\widetilde{\alpha}_1) \le S(\widetilde{\alpha}_2) \le S(\widetilde{\alpha}_3)$. It is apparent that $|S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_2)| \le |S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_3)|$ and $|S(\widetilde{\alpha}_3) - S(\widetilde{\alpha}_2)| \le |S(\widetilde{\alpha}_3) - S(\widetilde{\alpha}_1)|$.

If $\widetilde{\alpha}_1 \le \widetilde{\alpha}_2 \le \widetilde{\alpha}_3$, then $|a_1^L - a_2^L| \le |a_1^L - a_3^L|$, $|a_1^U - a_2^U| \le |a_1^U - a_3^U|$, $|b_1^L - b_2^L| \le |b_1^L - b_3^L|$, $|b_1^U - b_2^U| \le |b_1^U - b_3^U|$, $|c_1^L - c_2^L| \le |c_1^L - c_3^L|$ and $|c_1^U - c_2^U| \le |c_1^U - c_3^U|$; therefore, $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \le D(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$. A similar process yields $D(\widetilde{\alpha}_2, \widetilde{\alpha}_3) \le D(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$. Since $|S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_2)| \le |S(\widetilde{\alpha}_1) - S(\widetilde{\alpha}_3)|$, $|S(\widetilde{\alpha}_3) - S(\widetilde{\alpha}_2)| \le |S(\widetilde{\alpha}_3) - S(\widetilde{\alpha}_1)|$, $D(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \le D(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$, and $D(\widetilde{\alpha}_2, \widetilde{\alpha}_3) \le D(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$, we can obtain $M(\widetilde{\alpha}_1, \widetilde{\alpha}_2) \ge M(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$ and $M(\widetilde{\alpha}_2, \widetilde{\alpha}_3) \ge M(\widetilde{\alpha}_1, \widetilde{\alpha}_3)$.

The following examples compare the proposed similarity measure between IVIFNs with the existing similarity measures, and validate the applicability and flexibility of the proposed similarity measure.

Example 2 Let $\widetilde{\alpha}_3 = ([0.3, 0.4], [0, 0.5], [0.1, 0.7])$, $\widetilde{\alpha}_4 = ([0.1, 0.5], [0, 0.3], [0.2, 0.9])$, and $\widetilde{\alpha}_5 = ([0.3, 0.6], [0.1, 0.4], [0, 0.6]$ be three IVIFNs, by applying Eqs. (7) and (9) we obtain $S_{X2}(\widetilde{\alpha}_3, \widetilde{\alpha}_4) = S_{X2}(\widetilde{\alpha}_3, \widetilde{\alpha}_5) = 0.8$ and $S_{X4}(\widetilde{\alpha}_3, \widetilde{\alpha}_4) = S_{X4}(\widetilde{\alpha}_3, \widetilde{\alpha}_5) = 0.8$.

By applying Eqs. (6), (8) and (10), we obtain

$$S_{X1}(\widetilde{\alpha}_3, \widetilde{\alpha}_4) = 0.875 \text{ and } S_{X1}(\widetilde{\alpha}_3, \widetilde{\alpha}_5) = 0.9, \text{ so } S_{X1}(\widetilde{\alpha}_3, \widetilde{\alpha}_4) < S_{X1}(\widetilde{\alpha}_3, \widetilde{\alpha}_5);$$

$$S_{X3}(\widetilde{\alpha}_3, \widetilde{\alpha}_4) = 0.85 \text{ and } S_{X3}(\widetilde{\alpha}_3, \widetilde{\alpha}_5) = 0.8775, \text{ so } S_{X3}(\widetilde{\alpha}_3, \widetilde{\alpha}_4) < S_{X3}(\widetilde{\alpha}_3, \widetilde{\alpha}_5);$$

$$M(\widetilde{\alpha}_3, \widetilde{\alpha}_4) = 0.8875 \text{ and } M(\widetilde{\alpha}_3, \widetilde{\alpha}_5) = 0.9, \text{ so } M(\widetilde{\alpha}_3, \widetilde{\alpha}_4) < M(\widetilde{\alpha}_3, \widetilde{\alpha}_5).$$

Example 3 Let $\widetilde{\alpha}_6 = ([0, 0.5], [0.1, 0.4], [0.1, 0.9])$, $\widetilde{\alpha}_7 = ([0.3, 0.4], [0.2, 0.5], [0.1, 0.5])$ and $\widetilde{\alpha}_8 = ([0.1, 0.4], [0.4, 0.5], [0.1, 0.5])$ be three IVIFNs, by applying Eqs. (6), (7), (8) and (9) we obtain $S_{X1}(\widetilde{\alpha}_6, \widetilde{\alpha}_7) = S_{X1}(\widetilde{\alpha}_6, \widetilde{\alpha}_8) = 0.85$, $S_{X2}(\widetilde{\alpha}_6, \widetilde{\alpha}_7) = S_{X2}(\widetilde{\alpha}_6, \widetilde{\alpha}_8) = 0.7$, $S_{X3}(\widetilde{\alpha}_6, \widetilde{\alpha}_7) = S_{X3}(\widetilde{\alpha}_6, \widetilde{\alpha}_8) = 0.8268$, and $S_{X4}(\widetilde{\alpha}_6, \widetilde{\alpha}_7) = S_{X4}(\widetilde{\alpha}_6, \widetilde{\alpha}_8) = 0.7$.

By applying Eq. (10), we obtain

$$M(\widetilde{\alpha}_6, \widetilde{\alpha}_7) = 0.875 \text{ and } M(\widetilde{\alpha}_6, \widetilde{\alpha}_8) = 0.825\ , \text{ so } M(\widetilde{\alpha}_6, \widetilde{\alpha}_7) > M(\widetilde{\alpha}_6, \widetilde{\alpha}_8).$$

The above results indicate that using different similarity measures can produce different results. In Example 2, the similarity measures S_{X2} and S_{X4} defined by Xu [22] have lower discrimination, such that they cannot be used to classify this example. The result obtained from the proposed similarity measure agrees with the results from S_{X1} and S_{X3} defined by Xu [22]. The consistent results illustrate the feasibility and validity of the proposed similarity measure. In Example 3, all similarity measures defined by Xu [22] lose their effectiveness. And the reason might be the ignorance of hesitant degrees. Compared with existing similarity measures between IVIFNs, the proposed similarity measure takes all of the information into account, it can characterize the degrees of similarity between IVIFNs more accurate and effective. Furthermore, the examples also demonstrate the feasibility and applicability of the proposed similarity measure.

4 An interval-valued intuitionistic stochastic MCDM method using set pair analysis

This section describes an interval-valued intuitionistic stochastic MCDM method using set pair analysis.

For an interval-valued intuitionistic stochastic MCDM problem, let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria, and let $\theta = \{\theta_1, \theta_2, \ldots, \theta_S\}$ be a set of statuses. The probability of status θ_k can be denoted as p_k, where $0 \le p_k \le 1$ and $\sum_{k=1}^{s} p_k = 1$. The weights vector of the criteria can be denoted as $W = (w_1, w_2, \ldots, w_n)$, satisfying $\sum_{j=1}^{n} w_j = 1$, $0 \le w_j \le 1$, $j = 1, 2, \ldots, n$. Let $D_k = (\xi_{ijk})_{m \times n \times k}$ represent the interval-valued intuitionistic stochastic decision-making matrix, where $\xi_{ijk} = ([a_{ijk}^L, a_{ijk}^U], [b_{ijk}^L, b_{ijk}^U], [c_{ijk}^L, c_{ijk}^U])$ is an interval-valued intuitionistic stochastic variable that refers to the criterion value of A_i with respect to criterion C_j in the kth status. The problem to be addressed concerns ranking the alternatives or choosing the best alternative.

The interval-valued intuitionistic stochastic MCDM approach using set pair analysis proceeds as follows.

Step 1. Determine the criterion weights.

The TOPSIS method, first developed by Hwang and Yoon [55], is a well-known classical MCDM method. In TOPSIS, the alternatives are ranked by evaluating the shortest distance from the ideal solution among the maximum and minimum criteria values according to certain criteria. The maximizing deviation method [56], proposed by Wang, is a method for handling MCDM problems with numerical information and determining criterion weights. The basic idea of the maximizing deviation method is that a criterion with a larger deviation value among alternatives should be assigned a greater weight. Base on the TOPSIS and maximizing deviation methods, we can construct a new method to determine criterion weights.

(1) Determine the positive ideal alternative and the negative ideal alternative.

The positive ideal alternative A^+ is chosen according to the following formula:

$$A^+ = \left(\left[a^{L+}, a^{U+}\right], \left[b^{L+}, b^{U+}\right], \left[c^{L+}, c^{U+}\right]\right), \tag{11}$$

where, $a^{L+} = \max_{i=1}^{m}(a_{ijk}^L)$, $a^{U+} = \max_{i=1}^{m}(a_{ijk}^U)$, $b^{L+} = \min_{i=1}^{m}(b_{ijk}^L)$, $b^{U+} = \min_{i=1}^{m}(b_{ijk}^U)$, $c^{L+} = 1 - \max_{i=1}^{m}(a_{ijk}^U) - \min_{i=1}^{m}(b_{ijk}^U)$, $c^{U+} = 1 - \max_{i=1}^{m}(a_{ijk}^L) - \min_{i=1}^{m}(b_{ijk}^L)$, and $1 \le i \le m$, $1 \le j \le n$, $1 \le k \le s$.

The negative ideal alternative A^- is chosen according to the following formula:

$$A^- = \left(\left[a^{L-}, a^{U-}\right], \left[b^{L-}, b^{U-}\right], \left[c^{L-}, c^{U-}\right]\right), \tag{12}$$

where, $a^{L-} = \min_{i=1}^{m}(a_{ijk}^L)$, $a^{U-} = \min_{i=1}^{m}(a_{ijk}^U)$, $b^{L-} = \max_{i=1}^{m}(b_{ijk}^L)$, $b^{U-} = \max_{i=1}^{m}(b_{ijk}^U)$, $c^{L-} = 1 - \min_{i=1}^{m}(a_{ijk}^U) - \max_{i=1}^{m}(b_{ijk}^U)$, $c^{U-} = 1 - \min_{i=1}^{m}(a_{ijk}^L) - \max_{i=1}^{m}(b_{ijk}^L)$, and $1 \le i \le m$, $1 \le j \le n$, $1 \le k \le s$.

(2) Calculate the overall relative closeness between alternative A_i and the ideal alternative with respect to C_j.

Use Eq. (10) to calculate the similarity between each criterion value and both positive ideal alternative A^+ and the negative ideal alternative A^-, as follows:

$$M^+\left(\xi_{ijk}\right) = M\left(\xi_{ijk}, A^+\right), \tag{13}$$

$$M^-\left(\xi_{ijk}\right) = M\left(\xi_{ijk}, A^-\right). \tag{14}$$

Calculate the relative closeness between each criterion value and the ideal alternative with the following formula:

$$\overline{M}\left(\xi_{ijk}\right) = M^+\left(\xi_{ijk}\right) / \left(M^+\left(\xi_{ijk}\right) + M^-\left(\xi_{ijk}\right)\right). \tag{15}$$

The overall relative closeness between alternative A_i and the ideal alternative with respect to C_j can be determined as follows:

$$M_{ij} = \sum_{k=1}^{s} \bar{M}\left(\xi_{ijk}\right) P_k. \tag{16}$$

(3) Establish the mathematical programming model.

Calculate the deviation of the relative closeness of alternative A_i and alternative A_l with respect to C_j by applying the following formula:

$$d_{il}^j = \left|M_{ij} - M_{lj}\right|. \tag{17}$$

The deviation of all alternatives to other alternatives with respect to C_j can be determined according to the following formula:

$$d_j = \sum_{i=1}^{m}\sum_{l=1}^{m}\left|M_{ij} - M_{lj}\right|. \tag{18}$$

Establish the mathematical programming model on the basis of the maximizing deviation method as follows:

$$\max\ L(W) = \sum_{j=1}^{n} w_j d_j$$
$$s.t\ \begin{cases} \sum_{j=1}^{n} w_j = 1 \\ w_j \geq 0, \quad j = 1, 2, \ldots, n. \end{cases} \tag{19}$$

The weights $W^* = (w_1^*, w_2^*, \ldots, w_n^*)$ can be derived by solving the programming model (19).

Step 2. Calculate the integrated connection degree η_i of alternative A_i.

On the basis of compatibility between set pair analysis and IFS, transform the interval-valued intuitionistic fuzzy value $\xi_{ijk} = ([a_{ijk}^L, a_{ijk}^U], [b_{ijk}^L, b_{ijk}^U], [c_{ijk}^L, c_{ijk}^U])$ into a corresponding connection degree η_{ijk}, and then transform the interval-valued intuitionistic stochastic decision-making matrix $D_k = (\xi_{ijk})_{m\times n\times k}$ into a connection degree matrix $M_k = (\eta_{ijk})_{m\times n\times k}$ using the following formula:

$$\eta_{ijk} = a_{ijk} + b_{ijk}\breve{k} + c_{ijk}\breve{l}, \tag{20}$$

where $a_{ijk} = [a_{ijk}^L, a_{ijk}^U]$, $b_{ijk} = [c_{ijk}^L, c_{ijk}^U]$, and $c_{ijk} = [b_{ijk}^L, b_{ijk}^U]$. $\breve{k}$ is the coefficient of discrepancy degree, $\breve{l}$ is the coefficient of contrary degree and $\breve{l} = -1$.

Calculate the overall connection degree matrix $C_{ij} = (\eta_{ij})_{m\times n}$ of alternative A_i with respect to C_j using the following formula:

$$\eta_{ij} = \sum_{k=1}^{s} a_{ijk}p_k + \sum_{k=1}^{s} b_{ijk}p_k\breve{k} + \sum_{k=1}^{s} c_{ijk}p_k\breve{l}. \tag{21}$$

Calculate the integrated connection degree of alternative A_i by applying the following formula:

$$\eta_i = \sum_{j=1}^{m} w_j\eta_{ij} = \sum_{j=1}^{m} w_j \sum_{k=1}^{s} a_{ijk}p_k + \sum_{j=1}^{m} w_j \sum_{k=1}^{s} b_{ijk}p_k\breve{k} + \sum_{j=1}^{m} w_j \sum_{k=1}^{s} c_{ijk}p_k\breve{l}. \tag{22}$$

Step 3. Determine the set pair potential.

Calculate the set pair potential of alternative A_i by applying the following formula:

$$shi(\eta_i) = \frac{\sum_{j=1}^{m} w_j \sum_{k=1}^{s} a_{ijk}p_k}{\sum_{j=1}^{m} w_j \sum_{k=1}^{s} c_{ijk}p_k} = \left[\eta_i^-, \eta_i^+\right],\ i = 1, 2, \ldots, m. \tag{23}$$

Step 4. Rank the alternatives according to the values of set pair potential.

Normalize the interval number $shi(\eta_i) = [\eta_i^-, \eta_i^+]$. To rank an interval number on the basis of set pair analysis, the interval number must be normalized so as to transform the various interval number $shi(\eta_i) = [\eta_i^-, \eta_i^+]$ into the comparable interval number $shi(\ddot{\eta}_i) = \left[\ddot{\eta}_i^-, \ddot{\eta}_i^+\right]$, in which the lower limit $\ddot{\eta}^-$ and the upper limit $\ddot{\eta}^+$ satisfy $0 \leq \ddot{\eta}^- \leq 1$ and $0 \leq \ddot{\eta}^+ \leq 1$, respectively, The transformation formula is as follows:

$$\begin{cases} \ddot{\eta}_i^- = 1 - \eta_i^{\min}/\eta_i^- \\ \ddot{\eta}_i^+ = 1 - \eta_i^{\min}/\eta_i^+ \end{cases}, \tag{24}$$

where, $\eta_i^{\min} = \min\limits_{1\leq i\leq m} \eta_i^-$.

Transform the normalized interval number $shi(\ddot{\eta}_i) = \left[\ddot{\eta}_i^-, \ddot{\eta}_i^+\right]$ into a set pair connection degree. The transformation formula is as follows:

$$\mu_i\left(\left[\ddot{\eta}_i^-, \ddot{\eta}_i^+\right], 1\right) = a_i + b_i\breve{k} + c_i\breve{l}, \tag{25}$$

where $a_i = \frac{\ddot{\eta}_i^- - 0}{1-0} = \ddot{\eta}_i^-$, $b_i = \frac{\ddot{\eta}_i^+ - \ddot{\eta}_i^-}{1-0} = \ddot{\eta}_i^+ - \ddot{\eta}^-$, and $c_i = \frac{1-\ddot{\eta}_i^+}{1-0} = 1 - \ddot{\eta}_i^+$.

Calculate the set pair potential as follows:

$$shi(\mu_i) = \frac{a_i}{c_i}. \tag{26}$$

The larger the value of $shi(\mu_i)$, the better the alternative A_i will be.

5 Numerical example

This section applies the proposed method to solve a practical problem and compares the proposed approach with existing methods to illustrate its effectiveness and operability.

5.1 Illustration of the proposed method

Example 4 ABC MANUFACTURE is a manufacturing company that is primarily involved in the development and production of mobile phone parts. In order to enhance the company's market competitiveness, the company executives decide to select the best enterprise among four peer enterprises $\{A_1, A_2, A_3, A_4\}$ to form a corporate union. ABC MANUFACTURE engages an expert to help select the best enterprise. The expert assesses the four enterprises on the basis of three criteria, namely production capability C_1, research and development capability C_2 and the cash flow capacity C_3. The expert provides information on the four enterprises based on the evaluation criterion and the operational status of each enterprise. Three possible statuses θ_1, θ_2, and θ_3 may describe the operational status of an enterprise; θ_1 indicates that the enterprise is well-run, θ_2

indicates that the enterprise is operating at a satisfactory level, and θ_3 indicates the enterprise is not well-run. The corresponding probabilities of each status occurring are 0.3, 0.5 and 0.2, respectively.

The evaluations given by decision-maker are transformed into interval-valued intuitionistic stochastic variables in order to reflect any fuzzy or uncertain information in the evaluations. $D_1 = (\xi_{ij1})$, $D_2 = (\xi_{ij2})$, and $D_3 = (\xi_{ij3})$ contain the transformed evaluation information for statuses θ_1, θ_2, and θ_3, respectively. For example, in the fourth row and the first column of decision matrix D_2, the interval-valued intuitionistic stochastic variable $\xi_{412} = ([0.6, 0.7], [0.1, 0.2], [0.1, 0.3])$ indicates the expert's belif that when the enterprise is operating at a satisfactory level, enterprise A_4 is 60–70 percent productive and 10–20 percent unproductive; the range of 10–30 percent expresses his hesitation. The vector of criterion weights given by the expert is denoted as $\Omega = \{0.1 \le w_1 \le 0.3, 0.2 \le w_2 \le 0.4, 0.5 \le w_3 \le 0.7, w_1 + w_2 + w_3 = 1\}$.

(2) Calculate the overall relative closeness between alternative A_i and the ideal alternative with respect to C_j.

The overall relative closeness can be obtained by using Eqs. (10), (13), (14), (15) and (16). The results are shown in Table 3.

(3) Establish the mathematical program model.

The deviation of all alternatives to other alternatives with respect to C_j can be obtained using Eqs. (17) and (18), with the following results

$d_1 = 0.936, d_2 = 0.518, d_3 = 0.946.$

Establish the mathematical programming model as follows, according to Eq. (19):

$$\max L(W) = 0.936w_1 + 0.518w_2 + 0.946w_3$$
$$s.t. \begin{cases} w_1 + w_2 + w_3 = 1 \\ 0.1 \le w_1 \le 0.3 \\ 0.2 \le w_2 \le 0.4 \\ 0.5 \le w_3 \le 0.7 \end{cases}$$

$$D_1 = \begin{bmatrix} ([0.8,0.9],[0.0,0.1],[0.0,0.2]) & ([0.7,0.8],[0.1,0.2],[0.0,0.2]) & ([0.5,0.7],[0.1,0.3],[0.0,0.4]) \\ ([0.6,0.7],[0.2,0.3],[0.0,0.2]) & ([0.5,0.7],[0.2,0.3],[0.0,0.3]) & ([0.3,0.6],[0.2,0.3],[0.1,0.5]) \\ ([0.4,0.5],[0.2,0.4],[0.1,0.4]) & ([0.6,0.8],[0.0,0.1],[0.1,0.4]) & ([0.7,0.9],[0.0,0.1],[0.0,0.3]) \\ ([0.7,0.8],[0.1,0.2],[0.0,0.2]) & ([0.5,0.6],[0.2,0.3],[0.1,0.3]) & ([0.4,0.5],[0.2,0.4],[0.1,0.4]) \end{bmatrix}$$

$$D_2 = \begin{bmatrix} ([0.7,0.8],[0.1,0.2],[0.0,0.2]) & ([0.6,0.7],[0.1,0.3],[0.0,0.3]) & ([0.6,0.8],[0.0,0.2],[0.0,0.4]) \\ ([0.5,0.7],[0.1,0.2],[0.1,0.4]) & ([0.8,0.9],[0.0,0.1],[0.0,0.2]) & ([0.5,0.6],[0.2,0.3],[0.1,0.3]) \\ ([0.5,0.7],[0.2,0.3],[0.0,0.3]) & ([0.4,0.5],[0.1,0.3],[0.2,0.5]) & ([0.6,0.7],[0.1,0.2],[0.1,0.3]] \\ ([0.6,0.7],[0.1,0.2],[0.1,0.3]) & ([0.7,0.8],[0.1,0.2],[0.0,0.2]) & ([0.5,0.7],[0.1,0.3],[0.0,0.4]) \end{bmatrix}$$

$$D_3 = \begin{bmatrix} ([0.6,0.7],[0.1,0.3],[0.0,0.3]) & ([0.4,0.7],[0.1,0.2],[0.1,0.5]) & ([0.6,0.7],[0.1,0.3],[0.0,0.3]) \\ ([0.4,0.6],[0.1,0.2],[0.2,0.5]) & ([0.4,0.5],[0.3,0.4],[0.1,0.3]) & ([0.5,0.7],[0.2,0.3],[0.0,0.3]) \\ ([0.2,0.4],[0.0,0.1],[0.5,0.8]) & ([0.3,0.6],[0.0,0.1],[0.3,0.7]) & ([0.5,0.7],[0.1,0.2],[0.1,0.4]) \\ ([0.7,0.8],[0.0,0.1],[0.1,0.3]) & ([0.8,0.9],[0.0,0.1],[0.0,0.2]) & ([0.7,0.8],[0.1,0.2],[0.0,0.2]) \end{bmatrix}$$

Step 1. Determine the criterion weights.

(1) Determine the positive and negative ideal alternatives.

The positive ideal alternative and negative ideal alternative can be derived using Eqs. (11) and (12). The results are as follows:

$$A^+ = ([0.8, 0.9], [0, 0.1], [0, 0.2]),\ A^- = ([0.2, 0.4], [0.3, 0.4], [0.2, 0.5]).$$

The weights $W = (0.1, 0.2, 0.7)$ can easily be obtained using LINGO.

Step 2. Calculate the integrated connection degree η_i of alternative A_i.

The stochastic connection degree matrixes $M_1 = (\eta_{ij1})$, $M_2 = (\eta_{ij2})$, and $M_3 = (\eta_{ij3})$ are derived by applying Eq. (20), and the results are as follows:

Table 3 Overall relative closeness between each alternative and the ideal alternative

	C_1	C_2	C_3
A_1	0.6033	0.5463	0.5504
A_2	0.5132	0.5645	0.4694
A_3	0.4648	0.4924	0.5678
A_4	0.5736	0.5736	0.5017

$$M_1 = \begin{bmatrix} [0.8,0.9]+[0,0.2]\breve{k}+[0,0.1]\breve{l} & [0.7,0.8]+[0,0.2]\breve{k}+[0.1,0.2]\breve{l} & [0.5,0.7]+[0,0.4]\breve{k}+[0.1,0.3]\breve{l} \\ [0.6,0.7]+[0,0.2]\breve{k}+[0.2,0.3]\breve{l} & [0.5,0.7]+[0,0.3]\breve{k}+[0.2,0.3]\breve{l} & [0.3,0.6]+[0.1,0.5]\breve{k}+[0.2,0.3]\breve{l} \\ [0.4,0.5]+[0.1,0.4]\breve{k}+[0.2,0.4]\breve{l} & [0.6,0.8]+[0.1,0.4]\breve{k}+[0,0.1]\breve{l} & [0.7,0.9]+[0,0.3]\breve{k}+[0,0.1]\breve{l} \\ [0.7,0.8]+[0,0.2]\breve{k}+[0.1,0.2]\breve{l} & [0.5,0.6]+[0.1,0.3]\breve{k}+[0.2,0.3]\breve{l} & [0.4,0.5]+[0.1,0.4]\breve{k}+[0.2,0.4]\breve{l} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} [0.7,0.8]+[0,0.2]\breve{k}+[0.1,0.2]\breve{l} & [0.6,0.7]+[0,0.3]\breve{k}+[0.1,0.3]\breve{l} & [0.6,0.8]+[0,0.4]\breve{k}+[0,0.2]\breve{l} \\ [0.5,0.7]+[0.1,0.4]\breve{k}+[0.1,0.2]\breve{l} & [0.8,0.9]+[0,0.2]\breve{k}+[0,0.1]\breve{l} & [0.5,0.6]+[0.1,0.3]\breve{k}+[0.2,0.3]\breve{l} \\ [0.5,0.7]+[0,0.3]\breve{k}+[0.2,0.3]\breve{l} & [0.4,0.5]+[0.2,0.5]\breve{k}+[0.1,0.3]\breve{l} & [0.6,0.7]+[0.1,0.3]\breve{k}+[0.1,0.2]\breve{l} \\ [0.6,0.7]+[0.1,0.3]\breve{k}+[0.1,0.2]\breve{l} & [0.7,0.8]+[0,0.2]\breve{k}+[0.1,0.2]\breve{l} & [0.5,0.7]+[0,0.4]\breve{k}+[0.1,0.3]\breve{l} \end{bmatrix}$$

$$M_3 = \begin{bmatrix} [0.6,0.7]+[0,0.3]\breve{k}+[0.1,0.3]\breve{l} & [0.4,0.7]+[0.1,0.5]\breve{k}+[0.1,0.2]\breve{l} & [0.6,0.7]+[0,0.3]\breve{k}+[0.1,0.3]\breve{l} \\ [0.4,0.6]+[0.2,0.5]\breve{k}+[0.1,0.2]\breve{l} & [0.4,0.5]+[0.1,0.3]\breve{k}+[0.3,0.4]\breve{l} & [0.5,0.7]+[0,0.3]\breve{k}+[0.2,0.3]\breve{l} \\ [0.2,0.4]+[0.5,0.8]\breve{k}+[0,0.1]\breve{l} & [0.3,0.6]+[0.3,0.7]\breve{k}+[0,0.1]\breve{l} & [0.5,0.7]+[0.1,0.4]\breve{k}+[0.1,0.2]\breve{l} \\ [0.7,0.8]+[0.1,0.3]\breve{k}+[0.0,0.1]\breve{l} & [0.8,0.9]+[0,0.2]\breve{k}+[0,0.1]\breve{l} & [0.7,0.8]+[0,0.2]\breve{k}+[0.1,0.2]\breve{l} \end{bmatrix}$$

The overall connection degree matrix $C = (\eta_{ij})_{m\times n}$ can be obtained through Eq. (21), with the following results:

$$C = \begin{bmatrix} [0.71,0.81]+[0,0.22]\breve{k}+[0.07,0.19]\breve{l} & [0.59,0.73]+[0.02,0.31]\breve{k}+[0.1,0.25]\breve{l} & [0.57,0.75]+[0,0.38]\breve{k}+[0.05,0.25]\breve{l} \\ [0.51,0.68]+[0.09,0.36]\breve{k}+[0.13,0.23]\breve{l} & [0.63,0.76]+[0.02,0.25]\breve{k}+[0.12,0.22]\breve{l} & [0.44,0.62]+[0.08,0.36]\breve{k}+[0.2,0.3]\breve{l} \\ [0.41,0.58]+[0.13,0.43]\breve{k}+[0.16,0.29]\breve{l} & [0.44,0.61]+[0.19,0.51]\breve{k}+[0.05,0.2]\breve{l} & [0.61,0.76]+[0.07,0.32]\breve{k}+[0.07,0.17]\breve{l} \\ [0.65,0.75]+[0.07,0.27]\breve{k}+[0.08,0.18]\breve{l} & [0.66,0.76]+[0.03,0.23]\breve{k}+[0.11,0.21]\breve{l} & [0.51,0.66]+[0.03,0.36]\breve{k}+[0.13,0.31]\breve{l} \end{bmatrix}$$

The integrated connection degree of alternative A_i can be obtained using Eq. (22) as follows:

$$\eta_1 = [0.588,0.752]+[0.004,0.35]\breve{k}+[0.062,0.244]\breve{l},\ \eta_2 = [0.485,0.654]+[0.069,0.338]\breve{k}+[0.177,0.277]\breve{l},$$

$$\eta_3 = [0.556,0.712]+[0.1,0.369]\breve{k}+[0.075,0.188]\breve{l},\ \eta_4 = [0.554,0.689]+[0.034,0.325]\breve{k}+[0.121,0.277]\breve{l}.$$

Step 3. Determine the set pair potential.

Using Eq. (23), the set pair potential $shi(\eta_i)$ of alternative A_i can be determined as follows:

$shi(\eta_1) = [2.4098,\ 12.129]$, $shi(\eta_2) = [1.7509,\ 3.6949]$, $shi(\eta_3) = [2.9574,\ 9.4933]$, $shi(\eta_4) = [2,\ 5.6942]$.

Step 4. Rank the alternatives according to the values of set pair potential.

The normalized interval number $shi(\ddot{\eta}_i)$ can be derived as follows using Eq. (24).

$$shi(\ddot{\eta}_1) = [0.2734, 0.8556],\ shi(\ddot{\eta}_2) = [0, 0.5261],\ shi(\ddot{\eta}_3) = [0.4080, 0.8156],\ shi(\ddot{\eta}_4) = [0.1245, 0.6925].$$

Transform the interval number into a set pair connection degree according to Eq. (25)

$$\mu_1(shi(\ddot{\eta}_1), 1) = 0.2734 + 0.5822\breve{k} + 0.1444\breve{l},$$

$$\mu_2(shi(\ddot{\eta}_2), 1) = 0 + 0.5261\breve{k} + 0.4739\breve{l},$$

$$\mu_3(shi(\ddot{\eta}_3), 1) = 0.408 + 0.4076\breve{k} + 0.1844\breve{l},$$

$$\mu_4(shi(\ddot{\eta}_4), 1) = 0.1245 + 0.568\breve{k} + 0.3075\breve{l}.$$

Rank the alternatives according to Eq. (26), as follows:

$shi(\mu_1) = 1.8941$, $shi(\mu_2) = 0$, $shi(\mu_3) = 2.2119$, $shi(\mu_4) = 0.405$,

$shi(\mu_2) < shi(\mu_4) < shi(\mu_1) < shi(\mu_3)$.

The order of the alternatives is $A_2 \prec A_4 \prec A_1 \prec A_3$, indicating that the best enterprise for ABC MANUFACTURE to choose is enterprise A_3.

5.2 Comparison analysis and discussion

In order to verify the feasibility of the proposed decision-making approach based on IVIFSs and set pair analysis, we conducted a comparison analysis based on the same illustrative example.

The comparison analysis includes two cases. One incorporates the methods outlined in Li et al. [37] and Li et al. [38], which are compared to the proposed method using intuitionistic fuzzy information. In the other case, the method introduced in Gao and Liu [36] is compared with the proposed approach using interval-valued intuitionistic fuzzy information.

Case 1: The proposed method is compared with other methods using intuitionistic fuzzy information.

In order to apply the methods in Li et al. [37] and Li et al. [38], all interval-valued intuitionistic fuzzy evaluation values are translated into intuitionistic fuzzy evaluation values using the mean values of membership, non-membership and hesitation.

In the method developed by Li et al. [37], the intuitionistic fuzzy distance is defined, and the prospect decision matrix is obtained based on prospect theory and intuitionistic fuzzy distance. Then, the comprehensive prospect values which are in the form of intuitionistic fuzzy numbers are calculated and the alternatives are ranked using the score function. According the Li et al. [37], the score function values are $S_1 = -0.3394$, $S_2 = -0.6060$, $S_3 = -0.3668$ and $S_4 = -0.4069$.

The method introduced in Li et al. [38] consists of two main steps. First the intuitionistic fuzzy information is transformed into crisp values through a new score function. Then, based on prospect theory, the prospect matrix is obtained, and the alternatives are ranked by comprehensive prospect values. According to Li et al. [38], the comprehensive prospect values are $W_1 = 0.7843$, $W_2 = 0.3610$, $W_3 = 0.9917$, $W_4 = 0.5533$.

Case 2: The proposed method is compare with the method using interval-valued intuitionistic fuzzy information.

The method developed in Gao and Liu [36] consists of two main steps. First, the IVIFNs are transformed into real numbers through the proposed P-score function, and the corresponding P-score function matrix is constructed. Then, the positive and negative prospect values are computed, and the integrated prospect values are derived. And the alternatives can be ranked according to the integrated prospect values.

Table 4 summarizes the ranking results according to different methods, based on the criterion weights obtained using the proposed programming model.

As Table 4 shows, different methods yield different ranking results, but all of the methods except that found in in Li et al. [37] identify A_3 as the optimal alternative. This inconsistency likely occurs because Li et al. [37] uses the intuitionistic fuzzy distances between the evaluation values and reference point to construct the prospect matrix. The validity and efficiency of the reference point [0.5, 0.5] in Li et al. [37] requires further study and the score function used to rank alternatives in Li et al. [37] needs to be improved.

Table 4 Ranking results using different methods

Methods	Ranking results
Li et al.'s [37] method	$A_2 \prec A_4 \prec A_3 \prec A_1$
Li et al. 's [38] method	$A_2 \prec A_4 \prec A_1 \prec A_3$
Gao and Liu's [36] method	$A_2 \prec A_4 \prec A_1 \prec A_3$
The proposed approach	$A_2 \prec A_4 \prec A_1 \prec A_3$

Although the ranking results obtained through the methods developed by Gao and Liu [36] and Li et al. [38] are consistent with the results of the proposed method, the decision-making processes are different. Both the method introduced by Gao and Liu [36] and that introduced by Li et al. [38] are based on prospect theory, with one of the methods applying to interval-valued intuitionistic fuzzy environment and the other dealing with intuitionistic fuzzy information. These two methods use different score functions to transform interval-valued intuitionistic information or intuitionistic fuzzy information into real numbers during the first step of the decision-making process, which may cause a loss of fuzzy information. Besides, all of the methods except the proposed method are based on prospect theory, it is difficult to determine the reference point and the parameters, because they depend on the psychological behavior of the decision-makers.

According to the above comparison analysis, the proposed method for addressing intuitionistic stochastic MCDM problems has the following advantages.

1. The proposed approach represents interval-valued intuitionistic fuzzy information using connection degrees, which can simultaneously characterize the membership degree, non-membership degree and hesitation degree with a simple mathematic depiction and we can directly compute with connection degrees without transformation. Compared with methods that involve transforming IVIFNs into real numbers, the proposed approach's used of connection degrees can effectively avoids the loss of information.
2. Considering the difficulty to determine the reference points and parameters, the proposed method is more applicable to hand MCDM problem in which the decision-makers do not need to consider reference point and parameters. The calculations of the proposed approach are relatively simple. Besides, the proposed method can also be extended to address stochastic MCDM problems under IFS environments.

3. The results obtained by the proposed similarity measure might be more accurate for it takes the hesitation degree into account. And the new method to determine criterion weights on the basis of TOPSIS and the deviation maximizing method can effectively deal with the MCDM problem with incomplete criterion weights information.

6 Conclusion

In this paper, we present an approach for MCDM problems based on set pair analysis and IVIFSs, in which the criteria values are interval-valued intuitionistic stochastic variables. In the proposed method, an optimization weight model based on TOPSIS and the maximizing deviation methods is constructed to determine the criterion weights. Evaluation values are expressed in corresponding connection degrees based on the compatibility between set pair analysis and IFS. Combining set pair analysis and IVIFSs, alternatives are ranked according to the values of set pair potential. The feasibility and applicability of the proposed method was illustrated by an example, and a comparison analysis verified the validity of the proposed method and demonstrated its advantages over existing methods.

The proposed method fully considers the advantages of set pair analysis by using the connection degree to represent interval-valued fuzzy information and ranking the alternatives according to set pair potential. The proposed method effectively overcomes the shortcomings of the existing methods as was discussed earlier. Nevertheless, interval-valued intuitionistic fuzzy information cannot be directly given by decision-makers. Normally, the evaluation values of multiple decision-makers are collected and transformed into interval-valued intuitionistic fuzzy information and this preparation needs to be completed by the experts who have certain understanding of IFS or IVIFS. Besides, the method of determining the criterion weights in the proposed method is applicable to handle the situation that the criteria information is incomplete and, on the basis of this method, it is worth studying how to calculate the criterion weights in the case that nobody is capable of giving them. In future study, we will also study the interval-valued intuitionistic stochastic MCDM problems, and extend them to other fields, such as selection of green suppliers or evaluation of environmental comprehensive service providers, etc.

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