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ORIGINAL ARTICLE

Three-way fuzzy concept lattice representation using neutrosophic set

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Abstract Recently, three-way concept lattice is studied to handle the uncertainty and incompleteness in the given attribute set based on acceptation, rejection, and uncertain regions. This paper aimed at analyzing the uncertainty and incompleteness in the given fuzzy attribute set characterized by truth-membership, indeterminacy-membership, and falsity membership functions of a defined single-valued neutrosophic set. For this purpose a method is proposed to generate the component wise three-way formal fuzzy concept and their hierarchical order visualization in the fuzzy concept lattice using the properties of neutrosophic graph, neutrosophic lattice, and Gödel residuated lattice with an illustrative example.

Keywords Formal concept analysis · Fuzzy concept lattice · Formal fuzzy concept · Three-way concept lattice · Neutrosophic set

1 Introduction

In the early eighties a mathematical model called as Formal Concept Analysis (FCA) was introduced by wille [1] based on applied lattice theory for knowledge processing tasks. In the last decade the properties of FCA has been applied in various research fields [2]. FCA provides some set of patterns called as formal concepts from a data set in form of binary matrix—(X, Y, R) where row represents set of objects (X), column represents set of attributes (Y), and each entries in the matrix represents binary relations among them ($R \subseteq X \times Y$). Further its provides hierarchical order visualization of generated formal concepts through a defined concept lattice structure [3]. For precise representation of uncertainty and incompleteness the mathematics of FCA is augmented with fuzzy context [4], heterogeneous context [5], interval-valued fuzzy context [6], bipolar fuzzy context [7], linked fuzzy context [8], possibility-theoretic [9], and rough set [10] based formal context. Recently these extensions of FCA and their future trends are analysed based on their suitable applicability in a defined universe [2]. Recently the research trends to analysis of three-way formal concept analysis [11], its connection with classical concept lattice [12], multi-scaled concept lattice [13], triadic-decision context [14], three-way incomplete context [15, 16], three-way cognitive concept learning [17] at different granulation [18]. Further three-way decision space [19], with fuzzy sets [20], hesitant fuzzy sets [21] and their knowledge reduction [22] is also studied. Motivated from these recent studied current paper focused on rigorous analysis of three-way fuzzy concept lattice based on truth-membership function, indeterminacy-membership function, and falsity-membership function of a defined neutrosophic set.

Fuzzy concept lattice represents the uncertainty and incompleteness in the given attribute set through a defined single fuzzy membership-value between 0 to 1. This single fuzzy membership value includes both acceptation and rejection part of the attributes with respect to the given context. Similarly, the interval-valued [23] (or bipolar [7]) fuzzy set represents the acceptation and rejection part of the attributes in the defined interval [0,1] ([−1,1]). These extensions does not represent the indeterminacy part of the attributes. Of course these extension represents the non-membership degree of attributes via 1 minus the computed

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degree of membership-value for their acceptation (or rejection). However the non-membership degree of an attribute cannot be computed using this traditional method for all the cases of given fuzzy attributes. In those cases fuzzy attributes may contain some hesitation or indeterminacy part which can be characterized by three independent regions i.e. truth-membership function, indeterminacy-membership function and falsity membership function independently. Table 1 represents the inevitable conditions for the existence of three-way fuzzy attributes in the formal fuzzy context. This paper focused on analysis of data with three-way fuzzy attributes with an illustrative example.

There are many example of data with three way fuzzy attributes as for example symptoms of a disease (or Algae [24]) may characterized by truth, indeterminacy, and falsity membership value independently. To represent these three membership value precisely neutrosophic set is introduced [25]. Let us suppose X be a space of points (objects) then a neutrosophic set N can be characterized by truth-membership function ($T_X(N)$), an indeterminacy function ($I_X(N)$), and a false membership function ($F_X(N)$) [26]. Further some related properties like complex neutrosophic set [27], shadow set [28], neutrosophic logic [29], neutrosophic graph [30], its partial ordering based on hesitant fuzzy sets [21] as well as its applications [31], in multi-criteria decision making process [32] are explored. Some other methods like fuzziness based classifier [33], fuzzy entropy [34], fuzzy decision tree [35], fuzzy based ensemble learning [36] and granular computing [18] is also studied. Motivated from these recent studies current paper focused on hierarchical order visualization of three-way fuzzy concept lattice using the properties of neutrosophic logic [29], neutrosophic graph [30] and its partial ordering [21]. The motivation is to analyze the interested pattern in the data with three-way fuzzy attributes using the generated formal concepts via neutrosphic set and its super-sub concept hierarchy. For this purpose componentwise Gödel residuated lattice and its properties is used in this paper. To objective is to accelerate the decision making process and characterization by truth-membership function, indeterminacy-membership function, and falsity-membership function independently. To achieve the goal current paper addresses following problems as marked $*$ in Table 2:

(1) How to represent the data with three-way decision space attributes in the formal fuzzy context?
(2) How to generate formal fuzzy concepts three-way decision space characterized by truth-membership function, indeterminacy-membership function and falsity-membership function, independently?
(3) How to visualize the generated three-way formal fuzzy concepts super and sub concept order in a defined concept lattice structure?

Current paper aimed at following proposals to solve the mentioned problems:

(1) To represent the three-way decision space attributes in the formal context using neutrosophic set,
(2) To propose an algorithm for generating three–way formal fuzzy concepts based properties of neutrosophic set, and
(3) One application of the proposed method is discussed with an illustrative example.

Other parts of the paper are constituted as follows: Sect. 2 contains preliminaries about FCA in the fuzzy setting and neutrosophic set. Section 3 contains the proposed method to generate the three-way formal fuzzy concepts using the

Table 1 Some inevitable conditions existence in a given fuzzy context

Conditions	Objects	Attributes	Relation
a	Binary	Binary	Bipolar or three-way space
b	Binary	Bipolar	Bipolar or three-way space
c	Binary	Three-way space	Three-way space
d	Three-way space	Three-way space	Three-way space

Table 2 Some benchmark papers on FCA with interval, bipolar and three-way decision space

	Fuzzy	Interval	Bipolar	Three-way space
Formal context	Burusco and Gonzales [4]	Burusco and Gonzales [6]	Singh and Aswani Kumar [37]	Antoni et al. [5], Smarandache [26] $*$
Formal concept	Bělohlávek [38]	Djouadi and Prade [9]	Singh and Aswani Kumar[7]	Qi et al. [11] $*$
Lattice	Ganter and and Wille [3]	Pollandt [39]	Djouadi [40]	Hu [21], Ashbacher [25]
Concept lattice	Berry and Sigayret [41]	Singh et al. [23]	Singh and Aswani Kumar [7]	$*$

properties of neutrosophic logic and componentwise Gödel logic. Section 4 demonstrates an illustrative example for the proposed method. Section 5 includes discussion followed by conclusions and references.

2 Preliminaries

2.1 Formal concept analysis in the fuzzy setting

FCA in the fuzzy setting is now a well established mathematical model to analyze the data with fuzzy attributes. It takes input in form of a fuzzy matrix and process it through a complete lattice structure defined in [0,1]. Let us suppose L is a scale of truth degrees of a given complete residuated lattice ($\mathbf{L}$). Then a formal fuzzy context is a triplet $\mathbf{K} = (X, Y, \tilde{R})$. In which X represents set of objects, Y represents set of attributes and $\tilde{R}$ represents an L-relation among X and Y, $\tilde{R}$: $X \times Y \rightarrow L$ [4]. Each relation $\tilde{R}(x, y) \in L$ represents the membership value at which the object $x \in X$ has the attribute $y \in Y$ in scale of truth degrees L (L is a support set of some complete residuated lattice $\mathbf{L}$).

A residuated lattice $\mathbf{L} = (L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$ is the basic structure of truth degrees, where 0 and 1 represent least and greatest elements respectively. $\mathbf{L}$ is a complete residuated lattice iff [39]:

(1) $(L, \wedge, \vee, 0, 1)$ is a bounded complete lattice with bound 0 and 1.
(2) $(L, \otimes, 1)$ is commutative monoid.
(3) $\otimes$ and $\rightarrow$ are adjoint operators (called multiplication and residuum, respectively), that is $a \otimes b \leq c$ iff $a \leq b \rightarrow c, \forall a, b, c \in L$.

The operators $\otimes$ and $\rightarrow$ are defined distinctly by Lukasiewicz, Gödel, and Goguen t-norms and their residua as given below [38];

Lukasiewicz:

- $a \otimes b = \max\,(a + b - 1, 0)$,
- $a \rightarrow b = \min\,(1 - a + b, 1)$.

Gödel:

- $a \otimes b = \min\,(a, b)$,
- $a \rightarrow b = 1$ if $a \leq b$, otherwise b.

Goguen:

- $a \otimes b = a \cdot b$,
- $a \rightarrow b = 1$ if $a \leq b$, otherwise b/a. Classical logic of FCA is an example of a complete residuated lattice which is represented as $(\{0, 1\}, \wedge, \vee, \otimes, \rightarrow, 0, 1)$.

For any $\mathbf{L}$-set $A \in L^X$ of objects an $\mathbf{L}$-set $A^{\uparrow} \in L^Y$ of attributes can be defined using UP operator of Galois connection as follows:

$$A^{\uparrow}(y) = \wedge_{x \in X}(A(x) \rightarrow \tilde{R}(x, y)).$$

Similarly, for any $\mathbf{L}$-set of $B \in L^Y$ of attributes an $\mathbf{L}$-set $B^{\downarrow} \in L^X$ of objects set can be defined using down operator of Galois connection as given below:

$$B^{\downarrow}(x) = \wedge_{y \in Y}(B(y) \rightarrow \tilde{R}(x, y)).$$

$A^{\uparrow}(y)$ is interpreted as the $\mathbf{L}$-set of attribute $y \in Y$ shared by all objects from A. Similarly, $B^{\downarrow}(x)$ is interpreted as the $\mathbf{L}$-set of all objects $x \in X$ having the same attributes from B in common. The formal fuzzy concept is a pair of $(A, B) \in L^X \times L^Y$ satisfying $A^{\uparrow} = B$ and $B^{\downarrow} = A$, where fuzzy set of objects A is called an extent and fuzzy set of attributes B is called an intent. The operators $(\uparrow, \downarrow)$ are known as Galois connection [3] and extended for interval-valued also [40].

A formal fuzzy concept is an ordered pair (A, B), where A is called as (fuzzy) extent, and B is called as (fuzzy) intent. The set of all formal fuzzy concepts generated from a given fuzzy context $\mathbf{K}$ equipped with partial order i.e. $(A_1, B_1) \leq (A_2, B_2) \Longleftrightarrow A_1 \subseteq A_2 (\Longleftrightarrow B_2 \subseteq B_1)$ for every formal fuzzy concept. Together with this ordering the complete lattice contains an infimum and a supremum for some of the formal concepts which can be find as follows [3]:

- $\wedge_{j \in J}(A_j, B_j) = (\bigcap_{j \in J} A_j, (\bigcup_{j \in J} B_j)^{\downarrow\uparrow})$,
- $\vee_{j \in J}(A_j, B_j) = ((\bigcup_{j \in J} A_j)^{\uparrow\downarrow}, \bigcap_{j \in J} B_j)$.

The properties of fuzzy concept lattice is applied in various fields for knowledge processing task [2] with augmentation in interval-valued [23] and bipolar [37] fuzzy concept lattice. Recently, three-way formal concept analysis [11], its connection with classical concept lattice [12] is studied. Further three-way decision space with fuzzy sets [20], their partial ordering [21], and approximation [19] is defined based on acceptation, rejection and uncertain regions. Current paper focused on analyzing the uncertainty and incompleteness in attributes using truth-membership, indeterminacy-membership, and falsity-membership value of a defined neutrosophic set [25]. Such that the formal concept can be generated using the properties of neutrosophic graph [30], neutrosophic logic [29], and its partial ordering [21]. To bring them on a common platform some of the common properties among the neutrosophic set, neutrosophic graph and concept lattice is discussed in the next section with an example.

2.2 Neutrosophic set, its lattice and graph

In this section some basic properties of single-valued neutrosophic set, its graph and lattice structure is discussed to bridge it with concept lattice as given below:

Definition 1 (*Single-valued neutrosophic set*) [25]: Let $x \in X$ and X is a space of points (objects) then a neutrosophic set N in X can be characterized by a truth-membership function $T_N(x)$, a indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$. The $T_N(x)$, $I_N(x)$ and $F_N(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$as given below:

$$T_N : X \rightarrow]0^-, 1^+[,$$
$$I_N : X \rightarrow]0^-, 1^+[,$$
$$F_N : X \rightarrow]0^-, 1^+[.$$

The neutrosophic set can be represented as follows: $N = \{(x, T_N(x), I_N(x), F_N(x)) : x \in X\}$ where $0^- \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+$.

It is noted that $0^- = 0 - \epsilon$ where 0 is its standard part and ϵ is its non-standard part. Similarly, $1^+ = 1 + \epsilon$ ($3^+ = 3 + \epsilon$) where 1 (or 3) is standard part and ϵ is its non-standard part. The real standard (0,1) or [0,1] can be also used to represent the neutrosophic set.

Example 1 The symptoms of a disease can be characterized by truth-membership, indeterminacy-membership and falsity membership function, independently. Let us suppose doctor writes 0.8 as truth-membership function, 0.6 as indeterminacy-membership function and as 0.2 falsity-membership function for the symptoms of a given disease. It can be represented using a neutrosophic set as (0.8, 0.6, 0.2).

Definition 2 (*Single-valued neutrosophic set*) [27]: A single-valued neutrosophic set can be characterized by a truth-membership function $T_N(x)$, a indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$ independently, where $\{(T_N(x), I_N(x), F_N(x)) \in [0,1]^3\}$ for all $x \in X$. If X is continuous then:

$$N = \int \frac{(T_N(x), I_N(x), F_N(x))}{x} \quad \text{forall} \quad \text{x} \in \text{X}.$$

If X is discrete, then single-valued neutrosophic set can be defined as follows:

$$N = \sum \frac{(T_N(x), I_N(x), F_N(x))}{x} \quad \text{for all} \quad \text{x} \in \text{X}.$$

The single-valued neutrosophic set is used in this paper for decision making problem through a three-way $[0,1]^3$ concept lattice representation.

Definition 3 (*Union and Intersection of neutrosophic set*) [29]: Let N_1 and N_2 be two neutrosophic set in the universe of discourse X. Then union of N_1 and N_2 can be defined as follows:

- $N_1 \bigcup N_2 = \{(x, T_{N_1}(x) \vee T_{N_2}(x), I_{N_1}(x) \wedge I_{N_2}(x), F_{N_1}(x) \wedge F_{N_2}(x)) : x \in X\}$

The intersection of N_1 and N_2 can be defined as follows:

- $N_1 \bigcap N_2 = \{(x, T_{N_1}(x) \wedge T_{N_2}(x), I_{N_1}(x) \vee I_{N_2}(x), F_{N_1}(x) \vee F_{N_2}(x)) : x \in X\}$

This helps in finding a supremum and an infimum of any formal concepts for the three-way fuzzy concept lattice.

Definition 4 (*Lattice structure of neutrosophic set*) [21]: Let N_1 and N_2 be two neutrosophic set in the universe of discourse X. Then $N_1 \subseteq N_2$ iff $T_{N_1}(x) \leq T_{N_2}(x)$, $I_{N_1}(x) \geq I_{N_2}(x)$, $F_{N_1}(x) \geq F_{N_2}(x)$ for any $x \in X$. $(N, \wedge, \vee)$ is bounded lattice. Also the structure $(N, \wedge, \vee, (1,0,0), (0,1,1), \neg)$ follow the De Morgan's law. Hence this lattice structure can be used to represent the three-way fuzzy concept lattice and their concept using Gödel logic.

Definition 5 (*Single-valued neutrosophic graph*) [30]: Let $G = (V, E)$ is a neutrosophic graph in which the vertices (V) can be characterized by a truth-membership function $T_V(v_i)$, a indeterminacy-membership function $I_V(v_i)$ and a falsity-membership function $F_V(v_i)$ where $\left\{(T_V(v_i), I_V(v_i), F_V(v_i)) \in [0,1]^3\right\}$ for all $v_i \in V$. Similarly the edges (E) can be defined as a neutrosophic set $\left\{(T_E(V \times V), I_E(V \times V), F_E(V \times V)) \in [0,1]^3\right\}$ for all $V \times V \in E$ such that:

$$T_E(v_iv_j) \leq min[T_E(v_i, T_E(v_j)],$$
$$I_E(v_iv_j) \geq max[I_E(v_i, I_E(v_j)],$$
$$F_E(v_iv_j) \geq max[F_E(v_i, F_E(v_j)].$$

The single-valued neutrosophic graph is complete iff:

$$T_E(v_iv_j) = min[T_E(v_i, T_E(v_j)],$$
$$I_E(v_iv_j) = max[I_E(v_i, I_E(v_j)],$$
$$F_E(v_iv_j) = max[F_E(v_i, F_E(v_j)].$$

It is noted that $\left\{(T_E(v_iv_jV), I_E(v_iv_j), F_E(v_iv_j))\right\}$ = (0, 0, 0) $\forall (v_i, v_i) \in (V \times V \setminus E)$.

Example 2 The modern medical diagnosis system contains lots of incomplete, uncertain, and inconsistent information due to their large volume of information. To characterize the diagnoses of a patient doctor uses its truth-membership, indeterminacy-membership, and falsity-membership function. Hence this diagnoses can be represented more precisely through properties of neutrosophic set as shown in Table 3. Table 4 represents the corresponding relationship (E) among the diagnoses of a patients as shown in Table 4. This information can be visualized using a complete neutrosophic graph as shown in Fig. 1. Further to analyze this type of data set using the

Table 3 A neutrosophic set representation for diagnoses V of a patient for Example 2

	v_1	v_2	v_3
T_V	0.8	0.6	0.2
I_V	0.2	0.3	0.1
F_V	0.1	0.1	0.8

Table 4 A neutrosophic relationship among the diagnoses E of a patient for Example 2

	v_1v_2	v_2v_3	v_3v_1
T_E	0.6	0.2	0.2
I_E	0.3	0.3	0.2
F_E	0.1	0.8	0.8

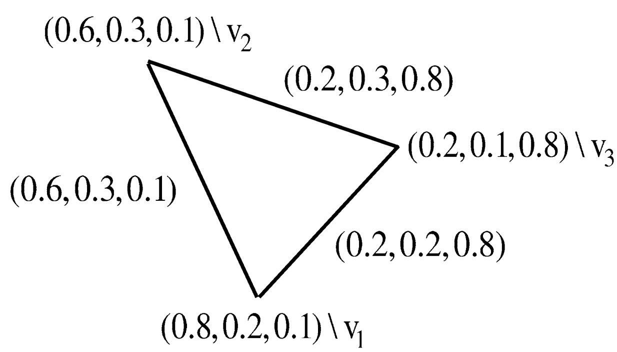

Fig. 1 A neutrosophic graph representation of Example 2

properties of neutrosophic graph based fuzzy concept lattice a method is proposed in the next section.

3 Proposed method for generating three-way fuzzy concept using neutrosophic set

In this section an algorithm is proposed for three-way fuzzy concepts generation using the properties of neutrosophic set and Gödel residuated lattice. Let us suppose a three-way fuzzy context $\mathbf{K} = (X, Y, \tilde{R})$ where, $|X| = n$, $|Y| = m$ and, $\tilde{R}$ represents corresponding three-way relationship among them using neutrosophic set. Then three-way formal fuzzy concepts can be generated as follows:

Definition 6 (*Three-way formal fuzzy concepts*) Let us suppose a neutrosophic set of attributes i.e. $(B) = \left\{y_j, (T_B(y_j), I_B(y_j), F_B(y_j)) \in [0,1]^3 : \forall y_j \in Y\right\}$ T_B characterized by a truth-membership function $T_B(y_j)$, a indeterminacy-membership function $I_B(y_j)$ and a falsity-membership function $F_B(y_j)$ independently. For the selected neutrosophic set of attributes find their covering neutrosophic set of objects i.e. $(A) = \{x_i, (T_A(x_i), I_A(x_i), F_A(x_i)) \in [0,1]^3 : \forall x_i \in X\}$ characterized by a truth-membership function $T_A(x_i)$, a indeterminacy-membership function $I_A(x_i)$ and a falsity-membership function $F_A(x_i)$ independently.

The obtain pair (*A*, *B*) is called as a formal fuzzy concept iff: $A^{\uparrow} = B$ and $B^{\downarrow} = A$. It can be interpreted as neutrosophic set of objects having maximal truth membership value, minimum indeterminacy and minimum falsity membership value with respect to integrating the information from the common set of fuzzy attributes in a defined three-way space $[0,1]^3$ using component-wise Gödel residuated lattice. After that, we cannot find any fuzzy set of objects (or attributes) which can make the membership value of the obtained fuzzy set of attributes (or objects) bigger. Then pair of neutrosophic set (*A*, *B*) is called as a formal concepts, where *A* is called as extent, and *B* is called as intent. This formal concepts can visualized as a node of three-way complete lattice.

Step (1) Let us suppose a three-way fuzzy context $\mathbf{K} = (X, Y, \tilde{R})$ where, $|X| = n$, $|Y| = m$ and, $\tilde{R}$ represents corresponding three-way relationship among them using neutrosophic set.
Step (2) Compute all the subset of attributes 2^m and represent them as s_j where $j \leq 2^m$.
Step (3) Set the maximum truth-membership value i.e. 1.0, minimum indeterminacy-membership value i.e. 0, and minimum falsity-membership value i.e. $B_{s_j} = (1,0,0)$. Assign this membership value for each subset of attributes as given below: $\{y_j, (T_B(y_j), I_B(y_j), F_B(y_j)) \in [0,1]^3 : \forall y_j \in Y\}$ where $= T_B(y_j) = 1$, $I_B(y_j) = 0$, and $F_B(y_j) = 0$).
Step (4) For the chosen subset of attributes (s_j) find its corresponding covering objects using Down operator(↓) i.e. $B_{s_j}^{\downarrow} = A_{s_i}$.
Step (5) The computed membership value for the accomplishment of necessary constraint for the obtained object set must be maximal membership degree for the accomplishment of a minimum desired property i.e.

$$T_{A_{s_i}}(x_i) = min_{j \in T_{B_{s_j}}} \mu_T^{\tilde{R}}(x_i, y_j),$$

$$I_{A_{s_i}}(x_i) = max_{j \in I_{B_{s_j}}} \mu_I^{\tilde{R}}(x_i, y_j),$$

$$F_{A_{s_i}}(x_i) = max_{j \in F_{B_{s_j}}} \mu_F^{\tilde{R}}(x_i, y_j).$$

where $\tilde{R}$ is the corresponding neutrosophic relationship among the object and attribute set.
Step (6) Now apply the UP operator (↑) of Galois connection on the constituted objects sets i.e $A_{s_i}^{\uparrow}$ to find the corresponding covering set of attributes i.e. B_{s_j} as follows:

$$T_{B_{s_j}}(y_j) = min_{j \in T_{A_{s_i}}} \mu_T^{\tilde{R}}(x_i, y_j),$$
$$I_{B_{s_j}}(y_j) = max_{j \in I_{A_{s_i}}} \mu_I^{\tilde{R}}(x_i, y_j),$$
$$F_{B_{s_j}}(y_j) = max_{j \in F_{A_{s_i}}} \mu_F^{\tilde{R}}(x_i, y_j).$$

Step (7) If the obtained truth-membership values, indeterminacy-membership value, and falsity-membership value for set of attributes is equal to initially considered subset of attributes i.e. B_{s_j} then the pair (A_{s_i}, B_{s_j}) forms a formal fuzzy concepts.

Step (8) In the case when UP operator provides a new attribute which covers the constituted object set. In this case add the newly obtained attribute with their computed membership-value for truth, indeterminacy and falsity-membership value as per step 6. Now if the obtain attributes set-B_{s_j} and its covering objects set-A_{s_i} closed with the Galois connection then it forms a formal fuzzy concept (A_{s_i}, B_{s_j}).

Step (9) In last set the minimum truth-membership value i.e. 0, maximum-indeterminacy membership value i.e. 1, maximum-falsity membership value 1 for $\{y_j, (T_B(y_j), I_B(y_j), F_B(y_j)) \in [0,1]^3 : \forall y_j \in Y\}$. Find their covering objects using the Galois connection and vice versa. It will provide last concept.

Step (10) The proposed method uses the subset of attributes for generating the fuzzy concepts. Due to that the proposed method helps in constructing the hierarchical ordering among the generated concepts to visualize them in the concept lattice structure. It is one of the advantages of the proposed method.

Table 5 summarizes the proposed algorithm established above. The proposed algorithm starts generating the three-way fuzzy concepts based on the chosen subset of attributes (2^m) as shown in Steps 1 and 2. The proposed algorithm considered maximum acceptation membership-

Table 5 Proposed algorithm for three-way fuzzy concept generation using the neutrosophic set

Input: A three–way fuzzy context $\mathbf{K}=(X, Y, \tilde{R})$
where $|X|=n$, $|Y|=m$.
Output: The set of three–way fuzzy concepts
$\{\{x_i, (T_A(x_i), I_A(x_i), F_A(x_i))\}, \{y_j, (T_B(y_j), I_B(y_j), F_B(y_j))\}\}$
where $i \leq n$ and $j \leq m$

1. Find the subset of three–way attributes and represent them as s_j
2. **for** j=1 to 2^k
3. Set the membership value for the chosen subset as follows:
   $\left\{y_j, (T_{B_{s_j}}(y_j), I_{B_{s_j}}(y_j), F_{B_{s_j}}(y_j))\right\}$=(1, 0, 0)
   //Maximum acceptation of the given attribute set
4. Apply down operator on the chosen attribute set :
   $\left\{y_j, (T_{B_{s_j}}(y_j), I_{B_{s_j}}(y_j), F_{B_{s_j}}(y_j))\right\}^{\downarrow} = \left\{x_i, (T_{A_{s_i}}(x_i), I_{A_{s_i}}(x_i), F_{A_{s_i}}(x_i))\right\}$
5. Compute the membership value for the obtained objects set:
   $T_{A_{s_i}}(x_i) = min_{j \in T_{B_{s_j}}} \mu_T^{\tilde{R}}(x_i, y_j),$
   $I_{A_{s_i}}(x_i) = max_{j \in I_{B_{s_j}}} \mu_I^{\tilde{R}}(x_i, y_j),$
   $F_{A_{s_i}}(x_i) = max_{j \in F_{B_{s_j}}} \mu_F^{\tilde{R}}(x_i, y_j).$
6. Apply the Galois connection on the constituted set of objects:
   $\{x_i, (T_A(x_i), I_A(x_i), F_A(x_i))\}^{\uparrow}$
7. **if** $\left\{x_i, (T_{A_{s_i}}(x_i), I_{A_{s_i}}(x_i), F_{A_{s_i}}(x_i))\right\}^{\uparrow} = \left\{y_j, (T_{B_{s_j}}(y_j), I_{B_{s_j}}(y_j), F_{B_{s_j}}(y_j))\right\}$
   //Provides initially considered neutrosophic set of attributes (i.e. intent)
8. Then it is a concept.
9. **else**
10. Any other attribute $z \in Y$ covers the constituted objects.
11. Then compute its membership–value as follows:
    $T_{B_{s_j}}(y_j) = min_{j \in T_{A_{s_i}}} \mu_T^{\tilde{R}}(x_i, y_j),$
    $I_{B_{s_j}}(y_j) = max_{j \in I_{A_{s_i}}} \mu_I^{\tilde{R}}(x_i, y_j),$
    $F_{B_{s_j}}(y_j) = max_{j \in F_{A_{s_i}}} \mu_F^{\tilde{R}}(x_i, y_j).$
12. Now add this newly attribute as a intent (B_{s_j})
13. The concept is (A_{s_i}, B_{s_j}) is generated for chosen attribute.
14. **end if**
15. **end for**
16. For last concept set–(0.0,1.0,1.0) for $\{y_j, (T_B(y_j), I_B(y_j), F_B(y_j))\}$
17. Investigate the covering objects and their corresponding attributes using Galois connection and vice versa.
18. Write the last concept.
19. Construct the concept lattice
20. Derive the knowledge.

value (1.0, 0.0, 0.0) for the chosen subset of attributes using neutrosophic set as shown in the Step 3. The operator (↓) is applied on chosen subset of attributes as shown in Step 4. It provides maximal neutrosophic set of objects for covering the constituted subset of attributes using the component-wise Gödel residuated lattice as shown in Step 5. Consequently, the operator (↑) is applied on the obtained set of objects to find the covering attributes set with respect to integrating the information from the constituted objects set as shown in the Step 6. If the membership-valued of obtained neutrosophic set of attributes is equal to initially considered neutrosophic set of attributes, then the pair can be considered as formal concept (as shown in Steps 7 and 8). If any other extra attributes covers the constituted objects set then add it with as shown in Steps 10 and 11. Now this form a formal concepts for the chosen subset of attribute as shown in Steps 12 and 13. The last concept can be generated by using (0.0,1.0,1.0) membership value for the neutrosophic set of attributes y_j. Find the covering objects set for these attributes using the Galois connection and vice versa. In this way the proposed method provides all the concepts and their hierarchical ordering in the concept lattice for knowledge processing tasks.

Complexity Let, number of objects-$|X| = n$ and number of attributes $|Y| = m$ in the given three-way fuzzy context. The proposed algorithm computes subset of attributes-(Y) which takes 2^m complexity. Then finds covering objects set using the properties of neutrosophic set i.e. characterized by truth-membership value, indeterminacy-membership value and falsity-membership value. It takes three time to search the corresponding objects and compute their membership-value using Gödel logic. In this case total computational complexity for the proposed method is $O(2^m * 3n)$.

4 Three-way fuzzy concept lattice representation using neutrosophic set

Recently, three-way concept lattice is studied in various research fields for knowledge processing tasks. As for example three-way formal concept analysis [11], its connection with classical concept lattice [12], three way decision space [20], as well as its partial ordering using hesitant fuzzy sets [21]. Further extensive study is reported on defining complex neutrosophic set [27], shadow set [28], neutrosophic logic [29], neutrosophic graph [30], its partial ordering [21] and approximation [19] with its applications [32] in various fields [2]. Current paper focused on analysis of data with three-way decision space attribute based on acceptation, rejection and indeterminacy regions of a defined neutrosophic set using the fuzzy concept lattice. For this purpose an algorithm is proposed in Sect. 3. To illustrate the proposed method one example is given as below:

Example 3 Let us consider a company wants to invest the money on following [31]:

(1) x_1 as a car company,
(2) x_2 as a food company,
(3) x_3 as computer company, and
(4) x_4 as an arms company.

The decision is based on following parameters (Y):

(1) y_1 as risk analysis;
(2) y_2 as growth analysis, and
(3) y_3 as environment impact analysis.

It can be observed that the decision parameter can be characterized by truth-membership function, indeterminacy-membership function, and falsity membership function independently. The company can collect the data for this three-way decision attributes and represent them in the tabular matrix format. For this purpose listed company can be considered as set of objects $X = (x_1 =$ car company, $x_2 =$ food company, $x_3 =$ computer company) and the decision parameters can be considered as set of attributes $Y = (y_1 =$ risk analysis, $y_2 =$ growth analysis, $y_3 =$ environment impact analysis). Let us suppose the decision attribute risk analysis (y_1) effect the establishment of a car company by 0.4 truth-membership value, 0.2 indeterminacy-membership value, and 0.3 falsity-membership value. Then it can be represented as $\tilde{R}_{(}x_1, y_1) = (0.4,\ 0.2,\ 0.3)$ using neutrosophic set in the tabular matrix. Similarly other attributes and their corresponding relationship can be depicted as shown in Table 6.

Now the problem is to analyze the investment of company based on the given decision parameters. For this purpose company needs some patterns i.e. formal concepts generated from Table 16 to analyze its preferences for establishment of a firms. To achieve this goal an algorithm is proposed in Table 5 using that following three-way fuzzy concepts can be generated from Table 6:

Step (1) All the generated subsets are as follows:

1. $\{(1.0, 0.0, 0.0)/y_1\}$,
2. $\{(1.0, 0.0, 0.0)/y_2\}$,
3. $\{(1.0, 0.0, 0.0)/y_3\}$,
4. $\{(1.0, 0.0, 0.0)/y_1, (1.0, 0.0, 0.0)/y_2\}$,
5. $\{(1.0, 0.0, 0.0)/y_2, (1.0, 0.0, 0.0)/y_3\}$,
6. $\{(1.0, 0.0, 0.0)/y_1, (1.0, 0.0, 0.0)/y_3\}$,
7. $\{(1.0, 0.0, 0.0)/y_1, (1.0, 0.0, 0.0)/y_2,$ $(1.0, 0.0, 0.0)/y_3\}$,

Table 6 A three-way fuzzy context representation using neutrosophic set

	y_1	y_2	y_3
x_1	(0.4, 0.2, 0.3)	(0.4, 0.2, 0.3)	(0.2, 0.2, 0.5)
x_2	(0.6, 0.1, 0.2)	(0.6, 0.1, 0.2)	(0.5, 0.2, 0.2)
x_3	(0.3, 0.2, 0.3)	(0.5, 0.2, 0.3)	(0.5, 0.3, 0.2)
x_4	(0.7, 0.0, 0.1)	(0.6, 0.1, 0.2)	(0.4, 0.3, 0.2)

8. $\{(0.0, 1.0, 1.0)/y_1, (0.0, 1.0, 1.0)/y_2, (0.0, 1.0, 1.0)/y_3\}$.

Step (2) Let us choose the subset of attributes 1.$\{(1.0, 0.0, 0.0)/y_1\}$ and find its covering objects set using Down operator (↓) as follows:

$\{(1.0, 0.0, 0.0)/y_1\}^{\downarrow} = \{(0.4, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.3, 0.2, 0.3)/x_3 + (0.7, 0.0, 0.1)/x_4\}$.

Step (3) Now apply UP operator on these constituted object set to find the extra covering attribute set

$\{(0.4, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.3, 0.2, 0.3)/x_3 + (0.7, 0.0, 0.1)/x_4\}^{\uparrow} = \{(1.0, 0.0, 0.0)/y_1 + (0.4, 0.2, 0.3)/y_2 + (0.2, 0.3, 0.5)/y_3\}$. Hence the generate neutrosophic formal concept is as given below:

1. $\{(0.5, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.3, 0.2, 0.3)/x_3 + (0.7, 0.0, 0.1)/x_4, (1.0, 0.0, 0.0)/y_1 + (0.4, 0.2, 0.3)/y_2 + (0.2, 0.3, 0.5)/y_3\}$.

Step (4) Similarly following concepts can be generated using other given subset of attribute shown in step 1:

2. $\{(0.4, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.5, 0.2, 0.3)/x_3 + (0.6, 0.1, 0.2)/x_4, (1.0, 0.0, 0.0)/y_2 + (0.3, 1.0, 0.3)/y_1 + (0.2, 0.3, 0.5)/y_3\}$,
3. $\{(0.2, 0.2, 0.5)/x_1 + (0.5, 0.2, 0.2)/x_2 + (0.5, 0.3, 0.3)/x_3 + (0.4, 0.3, 0.2)/x_4, (1.0, 0.0, 0.0)/y_3 + (1.0, 0.0, 0.0)/y_2 + (0.3, 0.0, 0.0)/y_1\}$,
4. $\{(0.4, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.3, 0.2, 0.3)/x_3 + (0.6, 0.1, 0.2)/x_4, (1.0, 0.0, 0.0)/y_1 + (1.0, 0.0, 0.0)/y_2 + (0.2, 0.3, 0.5)/y_3\}$,
5. $\{(0.2, 0.2, 0.5)/x_1 + (0.5, 0.2, 0.2)/x_2 + (0.5, 0.3, 0.3)/x_3 + (0.4, 0.3, 0.2)/x_4, (1.0, 0.0, 0.0)/y_2 + (1.0, 0.0, 0.0)/y_3 + (0.3, 0.0, 0.0)/y_1\}$,
6. $\{(0.2, 0.2, 0.5)/x_1 + (0.5, 0.2, 0.2)/x_2 + (0.3, 0.3, 0.3)/x_3 + (0.4, 0.3, 0.2)/x_4, (1.0, 0.0, 0.0)/y_1 + (1.0, 0.0, 0.0)/y_3 + (1.0, 0.0, 0.0)/y_2\}$,
7. $\{(0.2, 0.2, 0.5)/x_1 + (0.5, 0.2, 0.2)/x_2 + (0.3, 0.3, 0.3)/x_3 + (0.4, 0.3, 0.2)/x_4, (1.0, 0.0, 0.0)/y_1 + (1.0, 0.0, 0.0)/y_2 + (1.0, 0.0, 0.0)/y_3\}$,
8. $\{(1.0, 0.0, 0.0)/x_1 + (1.0, 0.0, 0.0)/x_2 + (1.0, 0.0, 0.0)/x_3 + (1.0, 0.0, 0.0)/x_4, (0.3, 0.2, 0.3)/y_1 + (0.4, 0.2, 0.3)/y_2 + (0.2, 0.3, 0.5)/y_3\}$.

In the above generated concepts number 3 and 5 is similar. Subsequently concept number 6 and 7 is also same. Hence following are the distinct formal concepts generated from Table 6:

1. $\{(0.5, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.3, 0.2, 0.3)/x_3 + (0.7, 0.0, 0.1)/x_4, (1.0, 0.0, 0.0)/y_1 + (0.4, 0.2, 0.3)/y_2 + (0.2, 0.3, 0.5)/y_3\}$.
2. $\{(0.4, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.5, 0.2, 0.3)/x_3 + (0.6, 0.1, 0.2)/x_4, (1.0, 0.0, 0.0)/y_2 + (0.3, 1.0, 0.3)/y_1 + (0.2, 0.3, 0.5)/y_3\}$,
4. $\{(0.4, 0.2, 0.3)/x_1 + (0.6, 0.1, 0.2)/x_2 + (0.3, 0.2, 0.3)/x_3 + (0.6, 0.1, 0.2)/x_4, (1.0, 0.0, 0.0)/y_1 + (1.0, 0.0, 0.0)/y_2 + (0.2, 0.3, 0.5)/y_3\}$,
5. $\{(0.2, 0.2, 0.5)/x_1 + (0.5, 0.2, 0.2)/x_2 + (0.5, 0.3, 0.3)/x_3 + (0.4, 0.3, 0.2)/x_4, (1.0, 0.0, 0.0)/y_2 + (1.0, 0.0, 0.0)/y_3 + (0.3, 0.0, 0.0)/y_1\}$,
7. $\{(0.2, 0.2, 0.5)/x_1 + (0.5, 0.2, 0.2)/x_2 + (0.3, 0.3, 0.3)/x_3 + (0.4, 0.3, 0.2)/x_4, (1.0, 0.0, 0.0)/y_1 + (1.0, 0.0, 0.0)/y_2 + (1.0, 0.0, 0.0)/y_3\}$,
8. $\{(1.0, 0.0, 0.0)/x_1 + (1.0, 0.0, 0.0)/x_2 + (1.0, 0.0, 0.0)/x_3 + (1.0, 0.0, 0.0)/x_4, (0.3, 0.2, 0.3)/y_1 + (0.4, 0.2, 0.3)/y_2 + (0.2, 0.3, 0.5)/y_3\}$,

The above generated vague formal concepts and their hierarchical ordering is shown in Fig. 2. From that following information can be extracted:

- Concept number 1 represents the attribute y_1 has maximum acceptation membership value for the given

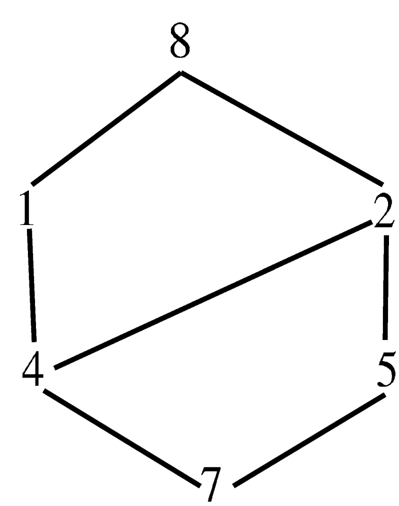

Fig. 2 A three-way fuzzy concept lattice representation using neutrosophic set for the context of Table 6

object set i.e. (1,0,0). Based on this attribute the object $(0.7, 0.0, 0.1)/x_4$ is more suitable due to its maximum truth-membership value i.e. 0.7, minimum indeterminacy-membership value i.e. 0.0, and minimum falsity-membership value i.e. 0.1.

- Concept number 2 represents that based on attribute y_2 the object $(0.6, 0.1, 0.2)/x_2$ and $(0.6, 0.1, 0.2)/x_4$ are more suitable due to their maximum truth-membership value i.e. 0.6, minimum indeterminacy-membership value i.e. 0.1, and minimum falsity-membership value i.e. 0.2.
- Concept number 4 represents that based on attributes y_1y_2 the object $(0.6, 0.1, 0.2)/x_2$ and $(0.6, 0.1, 0.2)/x_4$ are more suitable due to their maximum truth-membership value i.e. 0.6, minimum indeterminacy-membership value i.e. 0.1, and minimum falsity-membership value i.e. 0.2.
- Concept number 7 represents that based on attributes $y_1y_2y_3$ the object $(0.5, 0.2, 0.2)/x_2$ and $(0.4, 0.3, 0.2)/x_4$ are more suitable. Due to its highest truth-membership, minimum indeterminacy-membership value and minimum falsity-membership value.
- Concept number 8 represents that all objects and their impact analysis is based on attribute $(0.4, 0.2, 0.3)/y_2$ characterized by its maximum truth-membership value i.e. 0.4, minimum indeterminacy-membership value i.e. 0.2, minimum falsity-membership values i.e. 0.3. Next most valuable decision parameter will be attribute $(0.3, 0.3, 0.3)/y_1$.

The information extracted from Fig. 2 shows that based on given parameters the object x_4 (i.e. an arms company) is most suitable option for the company to invest the money. The next choice will be x_2 i.e. a food company. This derived analysis from the proposed method is in good agreement with hybrid vector similarity method [31]. Moreover the proposed method provides rigorous analysis based on each parameter using the generated three-way fuzzy concepts. Further it provides hierarchical ordering among them within complexity $O(2^m * 3n)$. We can believe that the proposed method will be helpful in various research fields for multi-criteria decision making process [31], three-way decision making process [21] and incomplete context [15].

5 Discussions

FCA in the fuzzy setting is well established mathematical model for handling uncertainty and incompleteness in data with fuzzy attributes. Further it enhanced with interval-valued [6] and bipolar [37] fuzzy set for adequate representation of attributes based on given context. These available approaches in FCA with fuzzy setting represents acceptation and rejection of attributes in the seized scale [0,1] or [−1,1]. Same time these extensions represents the non-membership of an attributes to the given context via 1 minus the fuzzy membership degree of an element. This condition cannot be possible for each of the cases of fuzzy attributes set. Some time the attribute may contain some hesitant or indeterminacy part which may be independent of its acceptation and rejection part. To handle this type of uncertainty recently three-way concept lattice [12] and its partial ordering based on hesitant fuzzy sets [21] is studied to measure the acceptation, rejection and uncertain regions, independently [19]. Also the properties of neutrosophic set [26], neutrosophic logic [29], and neutrosophic graph [30] is studied to characterized this type of uncertainty based on truth-membership function, indeterminacy-membership function, and falsity-membership function independently. Further its application is also shown in complex neutrosophic set [27], three-way cognitive concept learning, three-way multi-attribute decision making [31] as well as three-way multi-criteria decision making process [32]. These recent work validated that neutrosophic set precisely represents the acceptation, rejection and uncertain region via a defined boundary when compare to interval-valued and bipolar fuzzy set (comparison is shown in Table 7). Influenced from these recent analysis current paper focused on three-way fuzzy concept lattice representation using the properties of neutrosophic set, its lattice, and neutrosophic graph theory. To achieve this goal following problems are addressed in this paper:

Table 7 Comparison of fuzzy set and its extensive set theory

	Fuzzy	Interval	Bipolar	Neutrosophic set
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Co-domain	Single-value in [0, 1]	Unipolar interva [0,1]	Bipolar interval [−1,1]	$[0,1]^3$
Uncertainty	Yes	Yes	Yes	Yes
True	Yes	Yes	Yes	Yes
Falsity	No	No	No	Yes
Negativity	No	No	[−1, 0)	Yes in [0,1]
Indeterminacy	No	No	No	Yes

(1) A three fuzzy context represented by truth-membership value, indeterminacy-membership value, and falsity-membership value as shown in Table 6,
(2) A method is proposed to generate the three-way formal fuzzy concept concepts in Sect. 3,
(3) One application of the proposed method is also discussed in Sect. 4 with an illustrative example.

It can be observed that Fig. 2 represents the hierarchical ordering visualization of three-way formal fuzzy concept using neutrosophic set. It means the proposed method gives a way to analyze the data with three-way decision attribute based on acceptation, rejection and uncertain regions through the properties of fuzzy concept lattice and neutrosophic graph. Such that we can find some interested pattern characterized by truth, indeterminacy and falsity membership value. For better understanding an application of the proposed method is demonstrated in Sect. 4 to analyze the three-way multi-decision attribute data set. It can be observed that the analysis derived from the proposed method is agreement with vector analysis similarity measure of neutrosophic set [31]. Moreover the proposed method provides many three-way formal fuzzy concepts to refine the multi-criteria decision process rigorous. In future our work will focus on interval-valued and bipolar neutrosophic graph representation of concept lattice. Simultaneously, work will be focused on finding some similar (three-way) fuzzy concepts at different granulation of their weight as shown in [42, 43].

6 Conclusions

This paper focused on adequate analysis of three-way fuzzy concept lattice using neutrosophic set. For this purpose a method is proposed to generate the three-way fuzzy concepts and their hierarchical order visualization in the concept lattice using neutrosophic graph and component-wise Gödel residuated lattice within $O(2^m * 3n)$ complexity. One application of the proposed method is also discussed to analyze the multi-criteria decision making process. Further work will be focused on interval-valued and bipolar neutrosophic graph representation of concept lattice.

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