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ORIGINAL ARTICLE

A novel decision making approach based on intuitionistic fuzzy soft sets

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Abstract Molodtsov's soft set was initiated as a general emerging mathematical tool to deal with uncertain problems, which is free from the limitations of other traditional mathematical tool. It has been proven that decision making based on soft sets boom in recent years in many different fields. In this paper, a novel multi-criteria ranking approach is generalized based on intuitionistic fuzzy soft sets. There will be only one optimal decision among all the selections, instead of several or all by this method. Firstly, we present several notations named degree-hesitation function, score function and accuracy function to intuitionistic fuzzy soft set, and then give several principles based on these concepts. Some different decision making algorithms can be got for different preference, and a concrete algorithm is proposed in a certain condition. Moreover, we introduced the weighted ranking approach to the weighted intuitionistic fuzzy soft set. At the same time, both of these situations are proved to be effective with the help of examples. Finally, we conclude the research and further research directions.

Keywords Intuitionistic fuzzy soft set · Choice-value · Degree-hesitation function · Score function

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1 Introduction

In recent years, we are meeting more and more concepts that are uncertain, imprecise and vague rather than precise in our daily life, such as some complicated problems in economics, engineering, environment, social science, medical science, etc. All such problems involve data which are not always crisp. However, most of our traditional tools for modeling and computing are crisp, deterministic and precise. These classical mathematical tools cannot be used successfully. To solve this conflict, a wide range of theories, such as probability theory, fuzzy set theory [1], intuitionistic fuzzy set theory [2], rough set theory [3], rough set over dual-universes [4], vague set theory [5], the interval mathematics [6], rough fuzzy set theory [7, 8] and intelligent algorithm [9–11]. are well known as mathematical approaches to manage vagueness.The uncertainty processing plays a key role in relation-based learning system, and the representation, measure, and handling of uncertainty have a significant impact on the performance of learning algorithms [12–15]. However, each of these theories has its inherent difficulties of inadequacy of the parametrization tool, which is pointed by Molodtsov [16].

Molodtsov [16] initiated soft set theory as a newly emerging mathematical tool to deal with uncertain problems, which is free from the above limitations. Research on soft sets has been very active and more and more important results have been achieved in the domain of the theoretical aspect. Maji et al. [17] introduced several algebraic operations in soft set theory and also extended crisp soft sets to fuzzy soft sets [18]. Akta and Cagman [19] initiated soft groups and showed that fuzzy groups can be viewed as a special case of the soft groups. Jun [20] applied soft sets to the theory of BCK/BCI-algebras. Jun and Park [21] reported applications of soft sets in ideal theory of BCK/

BCI-algebras. Feng et al. [22] defined soft semi-rings and several related notions to establish a connection between soft sets and semi-rings. Soft set theory is also usually associated with other mathematical theories by many researchers for its character, such as fuzzy soft set theory [23], vague soft set theory [24], soft theory of interval mathematics [25], intuitionistic fuzzy soft sets [26], rough soft set theory [27], algebras soft theory [21, 28]. Especially, Sun and Ma [29] presented a new concept of soft fuzzy rough set by combining the fuzzy soft set with the traditional fuzzy rough set and gave an approach to decision making problem based on soft fuzzy rough set model. Jiang et al. [30] research the entropy on intuitionistic fuzzy soft sets and on interval-valued fuzzy soft sets.

With the establishment and development of soft set theory, it has been proven that its applications boom in recent years in many different fields, such as data analysis [31, 32], combined forecasting [33], decision-making [34, 35], evaluation [36, 37], medical diagnosis [21], classification [36] and so on. Cagman and Enginoglu [38, 39] presented a soft matrix based decision making approach and a unt- int based decision making approach. Feng et al. [40] further presented a method on generalized uni-int decision making schemes based on choice value soft sets. To cope with fuzzy soft sets based decision making problems, Roy and Maji [35] presented a novel method of object recognition from an imprecise multi-observer data. Maji et al. [41] first applied soft sets to solve the decision making problem with the help of rough approach. To improve the soft sets based decision making, Chen et al. [42] presented a new definition on of soft set parameterization reduction. Kong et al. [43] argued that the Roy-Maji method [35] was incorrect and they presented a revised algorithm. Feng et al. [44] gave deeper insights into decision making based on fuzzy soft sets. They discussed the validity of the Roy-Maji method [26] and showed its limitations. By means of level soft sets, Feng et al. presented an adjustable approach to fuzzy soft sets based decision making. Recently, Jiang et al. [26] presented an adjustable approach to intuitionistic fuzzy soft sets [45, 46] based decision making by using level soft sets of intuitionistic fuzzy soft sets. They mainly extended the decision making approach presented by Feng et al. [44] to the intuitionistic fuzzy case. Li [48] discussed decision making based on intuitionistic fuzzy soft sets by means of grey relational analysis and D-S theory of evidence. Kong et al. [49] present the simplified probability to directly instead of the incomplete information, and demonstrate the equivalence between the weighted-average of all possible choice values approach and the simplified probability approach.

As concluded in these reviewed paper on applications of soft set, some of these techniques cover multi-criterion decision making (MCDM is the abbreviation for multi-criterion decision making) problem, such as in paper [26, 33–37, 43, 44]. But it seems that there is little investigation on multi-criteria decision making using intuitionistic fuzzy soft sets with multiple criteria being explicitly taken into account. On the other hand, most of them mainly solve the problem of how to select an optimal object from the entire candidate rather than completely rank or sort the objects. However, many practical problems in economics, engineering, environment, social science, medical science, etc., that involve completely ranking, such as it need to rank the supplier in supply chain according to their service level. To overcome this shortcoming, in this paper, we extend the decision making approach presented by Jiang et al. [26]. We can get completely ranking or sorting alternatives problem accurately. In this method, there will be only one optimal decision among all the selections, instead of several or all. we try to investigate the intuitionistic fuzzy soft set based decision making deeply.

The remainder of this paper is organized as follows: Sect. 2 briefly introduces the basic concepts which are needed in this paper such as (fuzzy) soft set and the level soft set of (intuitionistic) fuzzy soft set. In Sect. 3, we firstly present the concepts of the hesitation function and the accuracy function on the intuitionistic fuzzy soft set, and then proposed a novel multi-criteria decision making approach based on intuitionistic fuzzy soft sets. In Sect. 4, we further present the concept of the weighted hesitation function and the weighted accuracy function of the intuitionistic fuzzy soft set, and the mainly steps and the algorithm of the proposed weighted MCDM decision making approach based on intuitionistic fuzzy soft sets are also proposed in this paper. Meantime, an example in reality is applied to illustrate the principle of the method and validity successfully throughout the Sects. 3 and 4. At last we conclude our research and further research directions in Sect. 5.

2 Preliminaries

This section gives a briefly review of some basic notations and theories of the intuitionistic fuzzy sets [2], soft sets [16], fuzzy soft sets [18], and intuitionistic fuzzy soft sets [45, 46]. Throughout this paper U refers to the initial universe of objects, cases, selections and so on. E is a set of parameters, which are often attributes, characteristics or properties of the objects. The intuitionistic fuzzy set (IFS) was introduced by Atanassov [2] as a generalization of fuzzy set firstly developed by Zadeh [1]. In the following, the theory of the intuitionistic fuzzy set is introduced briefly.

Definition 2.1 [2] An intuitionistic fuzzy set A (*IFSA* for short) over the universe U is defined as the following form: $IFS = \{x, u_A(x), v_A(x), \pi_A(x) | x \in U\}$.

In this formula, the maps $u_A(x), v_A(x)$ and $\pi_A(x)$ represent the degree of membership, the degree of non-membership and the degree hesitance of the element$x$ to set A, respectively. They satisfy the following inequalities: $0 \le \pi_A(x) = 1 - \mu_A(x) - v_A(x) \le 1$ and $0 \le \mu_A(x), v_A(x) \le 1$.

A lot of researches have accomplished in the fuzziness related areas, covering applications and theoretical demoralizations. The main difference between the fuzzy sets and the intuitionistic fuzzy sets are summarized as follows: the theory of fuzzy set, described the vague concepts by only its membership function. In other word, the degrees of the membership and the nonmembership of an element to a set is equal. However, the degrees of the membership and the nonmembership of an element to a set in the theory of intuitionistic fuzzy sets are quite different. It is easy to see that every fuzzy set may be regarded as intuitionistic fuzzy set. Then, we make an introduction to the concepts of the score function and the accurancy function to the intuitionistic fuzzy numbers put forward by Chen and Tans [47], with a view to serve the novel approach.

Definition 2.2 [47] Let $IFS = \{x, u_A(x), v_A(x) | x \in U\}$ denote an intuitionistic fuzzy set over the universe U about the attribute a, the score function is defined as $s_a(x) = \mu_a(x) - v_a(x)$.

Accordingly, the degree-accuracy function presented by Hong and Choi [50] can be obtained by the formula $H_a(x) = \mu_a(x) + v_a(x)$.

In the above definition, $s_a(x)$ denotes to what degree an alternative meets the decision maker's expectations. $H_a(x)$ is the degree that how much information the decision maker knows. Before introducing the notion of the intuitionistic fuzzy soft sets, it is necessary to give the concept of the soft sets and fuzzy soft sets. Molodtsov [16] defined the initial soft set in the following way.

Definition 2.3 [16] Let $P(U)$ be the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

Clearly, a soft set (F, A) over the universe U can be regarded as a parameterized family of subsets of the universe U in [22–24], which gives an approximate (soft) description of the objects in U.

Example 2.1 Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ is a set of houses and $A = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of parameters, which stand for cheap, beautiful, size, location and in the surroundings respectively. Consider the mapping form parameter set A to the set of all subsets of U. Then, soft set (F, A) can describe an "attractive" house, and $(F, A) = \{$cheap houses $= \{h_2, h_4\}$, beautiful houses $= \{h_1, h_3\}$, big houses $= \{h_3, h_4, h_5\}$,good location houses $= \{h_1, h_3, h_5\}$,in the green surroundings $= \{h_1\}\}$.

A soft set also can be represented in the form of Table 1.

Maji et al. [17, 18] initiated the study on combination with fuzzy sets and soft sets, in which the fuzzy soft sets can be seen as a fuzzy generalization of (crisp) soft sets. The definition of the fuzzy soft sets is presented as follows.

Definition 2.4 [17] Let $\Re(\mathfrak{U})$ denotes the set of all fuzzy sets of $U$and $A \subseteq E$. A pair $(\tilde{F}, A)$ is called a fuzzy soft set over$U$, where $\tilde{F}$ is a mapping given by $\tilde{F}: A \rightarrow \Re(\mathfrak{U})$.

Example 2.2 Consider the Example 2.1. In real life , the value of the parameter is not only 0 or 1. We can characterize it by a membership degree. Then a fuzzy soft set $(\tilde{F}, A)$ can decribe the "attractiveness of the houses" under the fuzzy information.

$$(\tilde{F}, A) = \left\{ \tilde{F}(e_1) = \left\{ \frac{h_1}{0.5}, \frac{h_2}{0.2}, \frac{h_3}{0.8}, \frac{h_4}{0.4}, \frac{h_5}{0.7}, \frac{h_6}{0.2} \right\}, \right.$$
$$\tilde{F}(e_2) = \left\{ \frac{h_1}{0.9}, \frac{h_2}{0.8}, \frac{h_3}{0.2}, \frac{h_4}{0.7}, \frac{h_5}{0.4}, \frac{h_6}{0.9} \right\},$$
$$\tilde{F}(e_3) = \left\{ \frac{h_1}{0.2}, \frac{h_2}{0.8}, \frac{h_3}{0.5}, \frac{h_4}{0.4}, \frac{h_5}{0.2}, \frac{h_6}{0.7} \right\},$$
$$\tilde{F}(e_4) = \left\{ \frac{h_1}{0.2}, \frac{h_2}{0.5}, \frac{h_3}{0.3}, \frac{h_4}{0.5}, \frac{h_5}{0.8}, \frac{h_6}{0.6} \right\},$$
$$\left. \tilde{F}(e_5) = \left\{ \frac{h_1}{0.7}, \frac{h_2}{0.4}, \frac{h_3}{0.2}, \frac{h_4}{0.8}, \frac{h_5}{0.2}, \frac{h_6}{0.8} \right\} \right\}.$$

Similarly, we could also represent a fuzzy soft set in the form of Table 2. It is easy to see that every crisp soft set may be regarded as a fuzzy soft set [17, 18].

Actually, in most decision making problems, fuzzy soft set based decision making problems are essentially humanistic or subjective in nature because of different human understanding and vision systems. Considering this

Table 1 Tabular of soft set (F, A)

U	e_1	e_2	e_3	e_4	e_5
h_1	0	1	0	1	1
h_2	1	0	0	0	0
h_3	0	1	1	1	0
h_4	1	0	1	0	0
h_5	0	0	1	1	0
h_6	0	0	0	0	0

Table 2 Tabular of soft set $(\tilde{F}, A)$

U	e_1	e_2	e_3	e_4	e_5
h_1	0.5	0.9	0.2	0.2	0.7
h_2	0.2	0.8	0.8	0.5	0.4
h_3	0.8	0.2	0.5	0.3	0.2
h_4	0.4	0.7	0.8	0.5	0.8
h_5	0.7	0.4	0.2	0.8	0.2
h_6	0.2	0.9	0.7	0.6	0.8

factor, Feng et al. [44] presented an approach to fuzzy soft sets based decision making problems by using the novel concept called level soft sets, which is defined as follows.

Definition 2.5 [44] Let $\Upsilon = \left(\tilde{F}, A\right)$ denotes the fuzzy soft set over U and $A \subseteq E$, let $t - level$ soft set of the fuzzy soft set Υ is a crisp soft set defined by $L(\Upsilon; t) = (F_t, A) = \left\{x \in U : \tilde{F}(x)(a) \geq t\right\}$for all $a \in A$ where $t \in [0, 1]$ called threshold values.

In fact, these threshold values are chosen by decision makers in advance and represent their requirements on " membership levels". It is easy to see in a certain sense that the level soft set are soft generalizations of classical level fuzzy sets, if the fuzzy soft sets are regarded as extensions of fuzzy sets from the soft set theoretical viewpoint.

By combined the concepts both of the fuzzy soft set and the intuitionistic fuzzy set, Maji et al. [45, 46] proposed the concept of the intuitionistic fuzzy soft sets as follows.

Definition 2.6 [45] Let $I\Re(\mathfrak{U})$ denotes the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A pair $\langle F, A\rangle$ is called an intuitionistic fuzzy soft set over$U$ where F is a mapping given by $F : A \to I\Re(\mathfrak{U})$.

As a generalization of fuzzy soft set theory, intuitionistic fuzzy soft set theory makes descriptions of the objective world more realistic, practical and accurate in some cases.

In the following, the concept on level soft set to intuitionistic fuzzy soft set is given by Jiang et al. [32].

Definition 2.7 [32] Let $\varpi = \langle F, A\rangle$ be an intuitionistic fuzzy soft set over U and $A \subseteq E$. A $(s, t) - level$ intuitionistic fuzzy soft set $L(\varpi; (s, t))$ of the intuitionistic fuzzy soft set ϖ is a crisp soft set, where $L(\varpi; (s, t)) = \left(F_{(s,t)}, A\right) = \{x \in U : \mu_a(x) \geq s, v_a(x) \leq t\}$ for all $a \in A$.

In this definition, $s, t \in [0, 1]$ are also called threshold values, in which $s \in [0, 1]$ can be regarded as a given least threshold on membership values and $t \in [0, 1]$ can be regarded as a given greatest threshold on non-membership values. Analogously, the threshold values are in advance chosen by decision makers and represent their requirements on membership levels and nonmembership levels, respectively, in the process of real-life applications of intuitionistic fuzzy based decision making.

3 A novel approach to intuitionistic fuzzy soft set based decision making

3.1 Problem statement

At present, many practical problems in economics, engineering, environment, social science, medical science, etc., that involves completely ranking rather than select an optimal one. For example, evaluation of supply chain partners is an important decision involving multiple criteria and risk factors, and it needs to rank the suppliers completely in the supply chain of engineering project according to their service level. Because in practical business of the economics and management, ranking the cooperative supplier according to their service level is more significant in most cases to the decision maker for using different cooperation mode.

Suppose that there are six suppliers for an engineering project, which can be expressed by $U = \{s_1, s_2, s_1, s_3, s_4, s_5, s_6\}$. The six suppliers need to be completely ranked according to their service level by the decision maker. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ is a set of decision attributes or decision parameters, such as the right quality, the right price, the right quantity, the right delivery time and the right delivery place, which is called principle 5R in real life. Here, we apply the intuitionistic fuzzy number to denote the criterion, which are more close to the reality. Hence, to make our research easier to understand, in this paper, we assume the intuitionistic fuzzy soft set$\langle F, A\rangle$ represent the problem that the decision maker wants to rank

$$\langle F, A\rangle = \left\{F(a_1) = \left\{\frac{s_1}{(0.9, 0.1)}, \frac{s_2}{(0.7, 0.2)}, \frac{s_3}{(0.8, 0.2)}, \frac{s_4}{(0.5, 0.4)}, \frac{s_5}{(0.6, 0.3)}, \frac{s_6}{(0.7, 0.3)}\right\},\right.$$
$$F(a_2) = \left\{\frac{s_1}{(0.8, 0.1)}, \frac{s_2}{(0.7, 0.1)}, \frac{s_3}{(0.6, 0.3)}, \frac{s_4}{(0.4, 0.6)}, \frac{s_5}{(0.8, 0.2)}, \frac{s_6}{(0.6, 0.2)}\right\},$$
$$F(a_3) = \left\{\frac{s_1}{(0.6, 0.2)}, \frac{s_2}{(0.5, 0.2)}, \frac{s_3}{(0.4, 0.5)}, \frac{s_4}{(0.7, 0.3)}, \frac{s_5}{(0.8, 0.2)}, \frac{s_6}{(0.6, 0.2)}\right\},$$
$$F(a_4) = \left\{\frac{s_1}{(0.4, 0.5)}, \frac{s_2}{(0.9, 0.1)}, \frac{s_3}{(0.7, 0.1)}, \frac{s_4}{(0.7, 0.2)}, \frac{s_5}{(0.8, 0.1)}, \frac{s_6}{(0.8, 0.1)}\right\},$$
$$\left.F(a_5) = \left\{\frac{s_1}{(0.9, 0.1)}, \frac{s_2}{(0.4, 0.5)}, \frac{s_3}{(0.8, 0.1)}, \frac{s_4}{(0.7, 0.1)}, \frac{s_5}{(0.4, 0.5)}, \frac{s_6}{(0.7, 0.2)}\right\}\right\}.$$

Table 3 Tabular representation of intuitionistic fuzzy soft set

U	a_1	a_2	a_3	a_4	a_5
s_1	$(0.9, 0.1)$	$(0.8, 0.1)$	$(0.6, 0.2)$	$(0.4, 0.5)$	$(0.9, 0.1)$
s_2	$(0.7, 0.2)$	$(0.7, 0.1)$	$(0.5, 0.2)$	$(0.9, 0.1)$	$(0.4, 0.5)$
s_3	$(0.8, 0.2)$	$(0.6, 0.3)$	$(0.4, 0.5)$	$(0.7, 0.1)$	$(0.8, 0.1)$
s_4	$(0.5, 0.4)$	$(0.4, 0.6)$	$(0.7, 0.3)$	$(0.7, 0.2)$	$(0.7, 0.1)$
s_5	$(0.6, 0.3)$	$(0.8, 0.2)$	$(0.8, 0.2)$	$(0.8, 0.1)$	$(0.4, 0.5)$
s_6	$(0.7, 0.3)$	$(0.5, 0.3)$	$(0.6, 0.2)$	$(0.8, 0.1)$	$(0.7, 0.2)$

six suppliers according the five attributes.
Table 3 shows the tabular representation of the intuitionistic fuzzy soft set $\langle F, A\rangle$ to this problem. To illustrate our idea from a different perspective, the vwe make the value of the parameters is the same as [26], which does not affect the model establishment and application process. The problem is that it needs to rank these six suppliers completely according to their the services level expressed by principle 5R, which is actuallya problem on intuitionistic fuzzy soft set based decision making problem.

3.2 A novel approach based on intuitionistic fuzzy soft set

To solve the above problem, in this section, we present a novel approach about the sorting problems based on the synthesized degree- hesitation function and the synthesized degree-accuracy function to intuitionistic fuzzy soft sets based decision making. Firstly, we need to present several concepts on the synthesized degree-hesitation and the synthesized degree-accuracy as follows. These synthesizedconcepts based on intuitionistic fuzzy soft sets are developed from the related concepts based on intuitionistic fuzzy sets presented by Atanassov [2].

Definition 3.1 Let $I\Re(\mathfrak{U})$ denote the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A pair $\langle F, A\rangle = \left\{\left(x_i, \mu_{a_j}(x_i), \nu_{a_j}(x_i)\right)\right\}$, where $x_i \in U, a_j \in A, i = 1, \ldots, n, j = 1, \ldots, m$, is denoted by an intuitionistic fuzzy soft set over U. $\pi_A(x_i)$ is called the synthetical degree-hesitation function of the object $x_i \in U$ in all attributes A of $\langle F, A\rangle$, which is defined by the following formula: $\pi_A(x_i) = [j = 1]m\sum\pi_{aj}(x_i) = [j = 1]m\sum\left(1 - \mu_{aj}(x_i) - \nu_{aj}(x_i)\right)$.

Next, we give the score function and the accuracy function of the intuitionistic fuzzy soft set $\langle F, A\rangle$.

Definition 3.2 Let $I\Re(\mathfrak{U})$ denote the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A pair $\langle F, A\rangle = \left\{\left(x_i, \mu_{aj}(x_i), \nu_{aj}(x_i)\right)\right\}$, where $x_i \in U, a_j \in A, i = 1, \ldots, n$, $j = 1, \ldots, m$, is denoted by an intuitionistic fuzzy soft set over$U$. $s_A(x_i)$ is called the synthetical score function of the object $x_i \in U$ in all attributes A of $\langle F, A\rangle$,which is defined by the following formula: $s_A(x_i) = [j = 1]m\sum s_{aj}(x_i) = [j = 1]m\sum\left(\mu_{aj}(x_i) - \nu_{aj}(x_i)\right)$.

Definition 3.3 Let $I\Re(\mathfrak{U})$ denote the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A pair $\langle F, A\rangle = \left\{\left(x_i, \mu_{aj}(x_i), \nu_{aj}(x_i)\right)\right\}$, where $x_i \in U, a_j \in A, i = 1, \ldots, n, j = 1, \ldots, m$, is denoted by an intuitionistic fuzzy soft set over U. $H_A(x_i)$ is called the synthetical accuracy function of the object $x_i \in U$ in all attributes A which is defined by the following formula: $s_A(x_i) = [j = 1]m\sum s_{aj}(x_i) = [j = 1]\ m\sum\left(\mu_{aj}(x_i) - \nu_{aj}(x_i)\right)$.

Obviously, Definition 3.1 is an intuitionistic fuzzy soft extension of hesitation function of intuitionistic fuzzy numbers (See Definition 2.1). Definitions 3.2 and 3.3 are intuitionistic fuzzy soft extension of score function and accuracy function of intuitionistic fuzzy numbers (See Definition 2.2), respectively.

Example 3.1 (continued by the problem in Table 3) To the above example, the degree-hesitation of the object $x_i \in U$ in all attributes A of intuitionistic fuzzy soft set $\langle F, A\rangle$, the accuracy of the object $x_i \in U$ in all attributes A of intuitionistic fuzzy soft set $\langle F, A\rangle$ and the score of the object $x_i \in U$ in all attributes A of intuitionistic fuzzy soft set $\langle F, A\rangle$ can be seen from Tables 4, 5 and 6.
To solve the ranking problem of the six suppliers presented in Sect. 3.1, we need further present some principles based on the above notations as follows.

Principle 1 In MCDM ranking problem we rank the object according to the degree-hesitation of the object from the smallest to the largest, which is called the minimum degree-hesitation principle.

Principle 2 In MCDM ranking problem we rank the object according to the score of the object from the largest to the smallest, which is called the maximal score principle.

Principle 3 In MCDM ranking problem we rank the object according to the accuracy of the object from the largest to the smallest, which is called the maximal accuracy principle. At the same time, we also give another principle according to the concept of the choice-value of the intuitionistic fuzzy soft set in the paper [26].

Table 4 degree-hesitation of $\langle F, A\rangle$

U	π_{a_1}	π_{a_2}	π_{a_3}	π_{a_4}	π_{a_5}	π_A
s_1	0	0.1	0.2	0.1	0	0.4
s_2	0.1	0.2	0.3	0	0.1	0.7
s_3	0	0.1	0.1	0.2	0.1	0.5
s_4	0.1	0	0	0.1	0.2	0.4
s_5	0.1	0	0	0.1	0.1	0.3
s_6	0	0.2	0.2	0.1	0.1	0.6

Table 5 accurancy of $\langle F, A\rangle$

U	H_{a_1}	H_{a_2}	H_{a_3}	H_{a_4}	H_{a_5}	H_A
s_1	1	0.9	0.8	0.9	1	4.6
s_2	0.9	0.8	0.7	1	0.9	4.3
s_3	1	0.9	0.9	0.8	0.9	4.5
s_4	0.9	1	1	0.9	0.8	4.6
s_5	0.9	1	1	0.9	0.9	4.7
s_6	1	0.8	0.8	0.9	0.9	4.4

Table 6 score of $\langle F, A\rangle$

U	S_{a_1}	S_{a_2}	S_{a_3}	S_{a_4}	S_{a_5}	S_A
s_1	0.8	0.7	0.4	−0.1	0.8	2.6
s_2	0.5	0.6	0.3	0.8	−0.1	2.1
s_3	0.6	0.3	−0.1	0.6	0.7	2.1
s_4	0.1	−0.2	0.4	0.5	0.6	1.4
s_5	0.3	0.6	0.6	0.7	−0.1	2.1
s_6	0.4	0.2	0.4	0.7	0.5	2.2

Principle 4 In MCDM ranking problem we rank the object according to the choice-value of the object from the largest to the smallest, which is called the maximal choice-value principle.

In general, there actually exists no one unique or uniform criterion for the evaluation (ranking or selection) of different decision alternatives. Actually, according to the preference of different decision maker, if we choose the above principles with different order, it will emerge 24 algorithms about the problem. Of course, the minimum degree-hesitation principle and the maximal accuracy principle seem too similar. However, if we just choose the above three principles, there are still six algorithms to the decision maker. Because the aim of this paper is to present a novel effective approach in order to completely rank the candidate in the MCDM problem, we will not compare the different version algorithms. Here, we just choose one of them. We call this novel MCDM ranking approach is $Max\{choice-value\}-Min\{hesitation\}-Max\{score\}$. That is to say, the first principle is the maximal choice-value principle, the second principle is the minimum degree-hesitation principle, and the last principle is the maximal score principle.

At present, the above novel MCDM ranking approach to intuitionistic fuzzy soft sets based decision making is showed by the following algorithm.

Algorithm 1

Step 1: Input the intuitionistic fuzzy soft set $\varpi = \langle F, A\rangle$;

Step 2: Choose the level value $s, t \in [0, 1]$ according the preference of the decision maker;

Step 3: Compute the level soft set $L(\varpi; (s, t)) = \left(F_{(s,t)}, A\right)$.

Step 4: Present the level soft set $L(\varpi; (s, t))$ in tabular form and compute the choice value of object s_i for $\forall i$;

Step 5: Rank the objects according to the choice value C_i from the largest to the smallest. If we can sort all the alternatives by the strict order, we get the result. Otherwise, continue to do the next step;

Step 6: Compute the degree of hesitation$\pi_a(x)$ with the membership of element $x \in U$ to $a \in A$ in intuitionistic fuzzy soft set $\varpi = \langle F, A\rangle$and furthermore compute the sum degree-hesitation $\pi_A(x)$ with the element $x \in U$;

Step 7: Ranking the objects which cannot be sorted by the choice- value C_i , according to $\pi_A(x)$ from the smallest to the largest. If we can sort all the alternatives by the strict order, we get the result. Otherwise, continue to do the next step;

Step 8: Compute the degree of accuracy $H_a(x)$ with the membership of element $x \in U$ to $a \in A$ in intuitionistic fuzzy soft set $\varpi = \langle F, A\rangle$ and furthermore compute the sum degree-hesitation $H_A(x)$ with the element $x \in U$;

Step 9: Ranking the objects which cannot be sorted by the degree of hesitation $\pi_A(x)$ and the choice value C_i, according to $H_A(x)$ from the largest to the smallestt.

At this moment, we should sort all the alternatives by the strict order. The bubble sort method has provided a reference for the implementation of our algorithm 1. The average time complexity of Algorithm 1 is $O(n^2)$.

There are some remarks here.

Remark 1 Different pairs of level value can be get according the preference of the decision maker. For example, the dicision maker can choose the $(s, t)-level$ decision rule, or choose the $(mid, mid)-level$ decision rule, or choose the $(top, bot)-level$ decision rule, or choose the $(top, top)-level$ decision rule, or choose the $(bot, bot)-level$ decision rule;

Remark 2 Different level soft set can be get from different pairs of level value. We give six version level soft set of the intuitionistic fuzzy soft sets seen in [32], which can be seen in detail. For example, the $(s, t)-level$ soft set $L(\varpi; (s, t))$ or $(mid, mid)-level$ soft set $L(\varpi; (mid, mid))$ or $(top, bot)-level$ soft set $L(\varpi; (top, bot))$or $(top, top)-$

Table 7 $L(0.7, 0.3)$

U	a_1	a_2	a_3	a_4	a_5
s_1	1	1	0	0	1
s_2	1	1	0	1	0
s_3	1	0	0	1	1
s_4	0	0	1	1	1
s_5	1	0	0	1	1
s_6	1	0	0	1	1

Table 8 $L(mid, mid)$

U	a_1	a_2	a_3	a_4	a_5
s_1	1	1	1	0	1
s_2	1	1	0	1	0
s_3	1	0	0	0	1
s_4	0	0	0	0	1
s_5	0	1	1	1	0
s_6	0	0	1	1	1

Table 9 $L(top, bot)$

U	a_1	a_2	a_3	a_4	a_5
s_1	1	1	0	0	1
s_2	0	0	0	1	0
s_3	0	0	0	0	0
s_4	0	0	0	0	0
s_5	0	0	0	1	1
s_6	0	0	0	0	0

level soft set $L(\varpi; (top, top))$or $(bot, bot) - level$ soft set $L(\varpi; (bot, bot))$;

Remark 3 In terms of the computing the choice-value of different version level soft set of the intuitionistic fuzzy soft sets, the related theory can be seen in [26, 41, 43, 44].

Remark 4 In this paper, we give the results on every version of level soft set. However, we just can get only one ranking result in every version of level soft set in practice. Because in practice, the decision maker just can choose one pair of level value according their preference. It needs to emphasiz that only one ranking result can be get in a certain level value.

Now, let us take a look at the example of the problem stated in Table 3 according to algorithm 1. Firstly, Tables 7, 8, 9, 10, 11 and 12 are the results of the step1 to step 3, which are the computing results of different level soft set of the intuitionistic fuzzy soft sets. Secondly, we compute the choice-value of different version level soft set of intuitionistic fuzzy soft set according to 4, as is shown in Table 13.

Table 10 $L(top, top)$

U	a_1	a_2	a_3	a_4	a_5
s_1	1	1	0	0	1
s_2	0	0	0	1	0
s_3	0	0	0	0	0
s_4	0	0	0	0	0
s_5	0	1	1	0	0
s_6	0	0	0	0	0

Table 11 $L(bot, bot)$

U	a_1	a_2	a_3	a_4	a_5
s_1	1	1	1	0	1
s_2	0	1	1	1	0
s_3	0	0	0	1	1
s_4	0	0	0	0	1
s_5	0	0	1	1	0
s_6	0	0	1	1	0

Table 12 $L(bot, top)$

U	a_1	a_2	a_3	a_4	a_5
s_1	1	1	1	1	1
s_2	1	1	1	1	1
s_3	1	1	1	1	1
s_4	1	1	1	1	1
s_5	1	1	1	1	1
s_6	1	1	1	1	1

Then we rank the suppliers according to the choice-value from the largest to the smallest, namely, the principle of the Max{choice-value}. The ranking result we get from step 5 is as follows.

$L(0.7, 0.3) : s_1 = s_2 = s_3 = s_4 = s_5 = s_6;$
$L(mid, mid) : s_1 \succ s_2 = s_5 = s_6 \succ s_3 \succ s_4;$
$L(top, bot) : s_1 \succ s_2 = s_5 \succ s_3 = s_4 = s_6;$
$L(top, top) : s_1 \succ s_5 \succ s_2 \succ s_3 = s_4 = s_6;$
$L(bot, bot) : s_1 \succ s_2 \succ s_3 = s_5 = s_6 \succ s_4;$
$L(bot, top) : s_1 = s_2 = s_3 = s_4 = s_5 = s_6.$

According to step 5, we cannot sort all the alternatives by the strict order from the maximal choice-value principle. Hence, we need to compute the degree-hesitation of the intuitionistic fuzzy soft set, as is shown in Table 4.

Next, we further start using the minimum degree-hesitation principle on the basis of the result from the first principle, and we will get the ranking result as follows.

$L(0.7, 0.3) : s_5 \succ s_1 = s_4 \succ s_6 \succ s_2 \succ s_3;$
$L(mid, mid) : s_1 \succ s_5 \succ s_6 \succ s_2 \succ s_3 \succ s_4;$
$L(top, bot) : s_1 \succ s_5 \succ s_2 \succ s_4 \succ s_3 \succ s_6;$

Table 13 Choice value of different level soft set $L(\varpi;(s,t))$

U	$C(0.7,0.3)$	$C(mid,mid)$	$C(top,bot)$	$C(top,top)$	$C(bot,bot)$	$C(bot,top)$
s_1	3	4	3	3	4	5
s_2	3	3	1	1	3	5
s_3	3	2	0	0	2	5
s_4	3	1	0	0	1	5
s_5	3	3	1	2	2	5
s_6	3	3	0	0	2	5

$L(top,top): s_1 \succ s_5 \succ s_2 \succ s_4 \succ s_3 \succ s_6;$
$L(bot,bot): s_1 \succ s_2 \succ s_5 \succ s_3 \succ s_6 \succ s_4;$
$L(bot,top): s_5 \succ s_1 \succ s_4 \succ s_3 \succ s_6 \succ s_2.$

Apparently, we can still not get the complete ranking result. We need to apply the third principle, that is, the maximal score principle. Let's continue the algorithm from step 8 to step 9. The scores of different version level soft set are seen in Table 6. At the same time, we can also get the completely ranking order in the rule of which is $L(0.7,0.3): s_5 \succ s_1 \succ s_4 \succ s_6 \succ s_2 \succ s_3$.

Up to now, we get the completely ranking order in any version rule on level soft set of intuitionistic fuzzy soft set as follows:

$L(0.7,0.3): s_5 \succ s_1 \succ s_4 \succ s_6 \succ s_2 \succ s_3;$
$L(mid,mid): s_1 \succ s_5 \succ s_6 \succ s_2 \succ s_3 \succ s_4;$
$L(top,bot): s_1 \succ s_5 \succ s_2 \succ s_4 \succ s_3 \succ s_6;$
$L(top,top): s_1 \succ s_5 \succ s_2 \succ s_4 \succ s_3 \succ s_6;$
$L(bot,bot): s_1 \succ s_2 \succ s_5 \succ s_3 \succ s_6 \succ s_4;$
$L(bot,top): s_5 \succ s_1 \succ s_4 \succ s_3 \succ s_6 \succ s_2.$

Actually, it is easy to get the just only one optimal selection, if we sort all the alternatives by the strict order. As for the above example, s_1 is the optimal supplier in rule $L(mid,mid), L(top,bot), L(bot,bot)$, and $L(top,top)$, s_5 is the optimal supplier in rule $L(0.7,0.3)$and $L(bot,top)$.

Hence, the above algorithm is more general in terms of MCDM ranking problem and MCDM optimal selection problem.

3.3 Comparisons with other literatures

At present, there is few investigation on using intuitionistic fuzzy soft sets, though there are abundant researches on MCDM. Actually, there is another adjustable approach to intuitionistic fuzzy soft sets based decision making, which is presented by Jiang et al. [26]. Step 1 to step 4 in this paper is similar with some of the algorithm presented by Jiang et al. [32]. Another two steps of the algorithm in Jiang et al. [26] are just like:

Step 5a: The optimal decision is to selects_k if $C_k = max\{C_i\}$.

Step 6a: If there is more than one value, then any of one of $s_k$may be chosen.

The above algorithm presented by Jiang et al. [26] mainly solved the problem of selecting an optimal object from all candidates. It is easy to get the optimal selection by using the above-mentioned step 5a and step 6a, and in $L(\varpi;(0.7,0.3))$ the optimal selection may be any one of the set $\{s_1,s_2,s_3,s_4,s_5,s_6\}$, and the optimal selection in $L(\varpi,(mid;mid))$ is s_1, and the optimal selection in $L(\varpi,(top;bot))$ is s_1, and the optimal selection in $L(\varpi,(bot;bot))$is s_1, and the optimal selection in $L(\varpi,(top;bot))$is s_1, and the optimal selection in $L(\varpi,(bot;top))$ may be any one of the set $\{s_1,s_2,s_3,s_4,s_5,s_6\}$.

From the above results, the optimal selection cannot be ensured just only one; even any one of all candidates may be the best. For example, in the rule of $L(\varpi,(0.7,0.3))$ and $L(\varpi,(bot;top))$, we almost select any one of the six supplier as the optimal one according to the algorithm. That is to say, the algorithm from Jiang et al. [26] is not effective enough .

At the same time, there is a question, whether is it feasible to the problem of ranking the six suppliers. We will further rank the supplier according to the choice-value from the largest to the smallest after step 5a, and the ranking result is as follows:

$L(0.7,0.3): s_1 = s_2 = s_3 = s_4 = s_5 = s_6;$
$L(mid,mid): s_1 \succ s_2 = s_5 = s_6 \succ s_3 \succ s_4;$
$L(top,bot): s_1 \succ s_2 = s_5 \succ s_3 = s_4 = s_6;$
$L(top,top): s_1 \succ s_5 \succ s_2 \succ s_3 = s_4 = s_6;$
$L(bot,bot): s_1 \succ s_2 \succ s_3 = s_5 = s_6 \succ s_4;$
$L(bot,top): s_2 = s_2 = s_3 = s_4 = s_5 = s_6.$

It is easy to see from the ranking results that it cannot completely rank the suppliers according to any different rule version of level soft set of intuitionistic fuzzy soft set. This result just has a small significance to practical MCDM ranking problem. However, it is proved that the approach in this paper can not only solve MCDM ranking problem, but also MCDM optimal selection problem. In other words, the research in this paper is more generalized, which includes but not limited to the MCDM optimal selection problem. Moreover, the algorithm presented in this paper is more effective in terms of MCDM optimal selection problem and MCDM ranking problem.

4 Weighted intuitionistic fuzzy soft set based decision making

In this section, we put foward an adjustable weighted MCDM ranking approach to weighted intuitionistic fuzzy soft sets . Several notations have to be presented here.

Definition 4.1 Let $I\Re(\mathfrak{U})$ denote the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A triple $\langle F, A, \omega\rangle = \left\{\left(x_i \mu_{a_j}(x_i), \upsilon_{a_j}(x_i), \omega\right)\right\}$ is a weighted intuitionistic fuzzy soft set over U, where $x_i \in U, a_j \in A, i = 1, \ldots, n, j = 1, \ldots, m$, and $\omega : A \to [0,1]$ is a weight function specifying the weight $w_j = \omega(a_j)$ for each attribute $a_j \in A$. $\pi_A(x_i)$ is called the weighted degree-hesitation function of the object $x_i \in U$ in all attributes A of $\langle F, A, \omega\rangle$, which is defined by the following formula: $\pi_A(x_i) = [j=1]m\sum\omega(a_j)\pi_{a_J}(x_i) = [j=1]m\sum\omega(a_j)\left(1 - \mu_{a_J}(x_i) - \nu_{a_J}(x_i)\right)$.

Next, we give the score function and the accuracy function of the intuitionistic fuzzy soft set $\langle F, A, \omega\rangle$.

Definition 4.2 Let $I\Re(\mathfrak{U})$ denote the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A triple $\langle F, A, \omega\rangle = \left\{\left(x_i \mu_{a_j}(x_i), \upsilon_{a_j}(x_i), \omega\right)\right\}$ is a weighted intuitionistic fuzzy soft set over U, where $x_i \in U, a_j \in A, i = 1, \ldots, n, j = 1, \ldots, m$, and $\omega : A \to [0,1]$ is a weight function specifying the weight $w_j = \omega(a_j)$ for each attribute $a_j \in A$. $s_A(x_i)$ is called the weighted degree-hesitation function of the object $x_i \in U$ in all attributes A of $\langle F, A, \omega\rangle$, which is defined by the following formula: $s_A(x_i) = [j=1]m\sum\omega(a_j)s_{a_J}(x_i) = [j=1]m\sum\omega(a_j)\left(\mu_{a_j}(x_i) - \nu_{a_J}(x_i)\right)$.

Definition 4.3 Let $I\Re(\mathfrak{U})$ denote the set of all the intuitionistic fuzzy sets of U and $A \subseteq E$. A triple $\langle F, A, \omega\rangle = \left\{\left(x_i \mu_{a_j}(x_i), \upsilon_{a_j}(x_i), \omega\right)\right\}$ is a weighted intuitionistic fuzzy soft set over U, where $x_i \in U, a_j \in A, i = 1, \ldots, n, j = 1, \ldots, m$, and $\omega : A \to [0,1]$ is a weight function specifying the weight $w_j = \omega(a_j)$for each attribute $a_j \in A$. $H_A(x_i)$is called the weighted degree-hesitation function of the object $x_i \in U$ in all attributes A of $\langle F, A, \omega\rangle$, which is defined by the following formula: $H_A(x_i) = [j=1]m\sum\omega(a_j)H_{a_J}(x_i) = [j=1]m\sum\omega(a_j)\left(\mu_{a_j}(x_i) + \nu_{a_J}(x_i)\right)$.

In the above definitions, every intuitionistic fuzzy soft set is a weighted intuitionistic fuzzy soft set [26]. Of course, every degree-hesitation function or score function or accuracy function of the intuitionistic fuzzy soft set can be regarded as a weighted degree-hesitation function or score function or accuracy function of the weighted intuitionistic fuzzy soft set.

Now, we will give the related MCDM ranking algorithm to weighted intuitionistic fuzzy soft set, which still takes the approach of $Max\{choece - value\} - Min\{hesititation\} - Max\{score\}$. For it is similar when using the algorithm in different version level soft set. Here we just take $L(mid; mid)$for example

Algorithm 2

Step 1: Input a weighted intuitionistic fuzzy soft set $\langle F, A, \omega\rangle$;

Step 2: Choose the level value according the preference of the decision maker;

Step 3: Here choose the $(mid, mid) - level$ decision rule;

Step 4: Compute the weighted level soft set $L(\varpi; (s,t)) = \left(F_{(s,t)}, A\right)$ with $(mid, mid) - level$ soft set $L(\varpi; (mid, mid))$;

Step 5: Present the weighted level soft set of $L(\varpi, (mid, mid))$ in tabular form and compute the choice value C_i of object s_i for $\forall i$;

Step 6: Rank the objects according to the choice value C_i from the largest to the smallest. If we can sort all the alternatives by the strict order, we get the result. Otherwise, continue to do the next step;

Step 7: Compute the weighted degree of hesitation $\pi_A(x)$ with the membership of element $x \in U$ to $a \in A$ in the weighted intuitionistic fuzzy soft set $\varpi = \langle F, A, \omega\rangle$;

Step 8: Rank the objects which cannot be sorted by the choice value C_i according to $\pi_A(x)$ from the smallest to the largest. If we can sort all the alternatives by the strict order, we get the result. Otherwise, continue to do the next step;

Step 9: Compute the weighted degree of score function $s_A(x)$ with the membership of element $x \in U$ to $a \in A$ in the weighted intuitionistic fuzzy soft set $\varpi = \langle F, A, \omega\rangle$;

Step 10: Rank the objects which cannot be sorted by the degree of hesitation $\pi_A(x)$ and the choice value C_i , according to $s_A(x)$ from the largest to the smallest.

At this moment, we should sort all the alternatives by the strict order, and the computation ends. The average time complexity of Algorithm 2 is also $O(n^2)$.

Example 4.1 Let see an example expressed by Table 14. Suppose that the decision maker define the weight function $w_j = \omega(a_j)$ according their preference on the basis of the problem of Table 3 in Sect. 3.1. Here, $w_1 = 0.6, w_2 = 0.9, w_3 = 0.7, w_4 = 0.9, w_5 = 0.9$. Table 14 is the representation of the weighted intuitionistic fuzzy soft set $\varpi = \langle F, A, \omega \rangle$.

Table 15 is the tabular representation of the $(mid, mid) - level$ soft set of $\varpi = \langle F, A, \omega \rangle$, and the ranking result is $s_1 \succ s_5 = s_6 \succ s_2 \succ s_3 \succ s_4$ according to the principle of $Max\{choice - value\}$, which is obtained from step 1 to step 6.

It is easy to see that the result is not completely ranked. We need to further use the weighted degree-hesitation to finish the complete ranking. Table 16 is the representation of the weighted degree-hesitation $\pi_A(x)$ to the intuitionistic fuzzy soft set $\varpi = \langle F, A, \omega \rangle$.

Based on the principle (from the step 5 to the step 6), we can get the ranking result: $s_1 \succ s_5 = s_6 \succ s_2 \succ s_3 \succ s_4$. It is easy to find we cannot rank the objects s_5 and s_6. We need to rank the objects which cannot be sorted by the choice value according to the weighted degree of hesitation $\pi_A(x)$ from the smallest to the largest (from the step 7 to the step 8). We can get the ranking result: $s_5 \succ s_1 \succ s_4 \succ s_3 \succ s_6 \succ s_2$. For all the alternatives are with the strict order, so the ranking result $s_5 \succ s_1 \succ s_4 \succ s_3 \succ s_6 \succ s_2$ is the final ranking result.

Table 14 Tabular representation of $\varpi = \langle F, A, \omega \rangle$

U	$a_1(0.6)$	$a_2(0.9)$	$a_3(0.7)$	$a_4(0.9)$	$a_5(0.9)$
s_1	(0.9, 0.1)	(0.8, 0.1)	(0.6, 0.2)	(0.4, 0.5)	(0.9, 0.1)
s_2	(0.7, 0.2)	(0.7, 0.1)	(0.5, 0.2)	(0.9, 0.1)	(0.4, 0.5)
s_3	(0.8, 0.2)	(0.6, 0.3)	(0.4, 0.5)	(0.7, 0.1)	(0.8, 0.1)
s_4	(0.5, 0.4)	(0.4, 0.6)	(0.7, 0.3)	(0.7, 0.2)	(0.7, 0.1)
s_5	(0.6, 0.3)	(0.8, 0.2)	(0.8, 0.2)	(0.8, 0.1)	(0.4, 0.5)
s_6	(0.7, 0.3)	(0.5, 0.3)	(0.6, 0.2)	(0.8, 0.1)	(0.7, 0.2)

Table 15 The weighted choice-value of $L(mid, mid)$

U	$a_1(0.6)$	$a_2(0.9)$	$a_3(0.7)$	$a_4(0.9)$	$a_5(0.9)$	C_w
s_1	1	1	1	0	1	3.1
s_2	1	1	0	1	0	2.4
s_3	1	0	0	0	1	1.5
s_4	0	0	0	0	1	0.9
s_5	0	1	1	1	0	2.5
s_6	0	0	1	1	1	2.5

Table 16 The weighted degree-hesitation $\pi_A(x)$ to $\varpi = \langle F, A, \omega \rangle$

U	$\pi_{a_1}(0.6)$	$\pi_{a_2}(0.9)$	$\pi_{a_3}(0.7)$	$\pi_{a_4}(0.9)$	$\pi_{a_5}(0.9)$	$\pi_A(x)$
s_1	1	0.1	0.2	0.1	0	0.32
s_2	0.1	0.2	0.3	0	0.1	0.54
s_3	0	0.1	0.1	0.2	0.1	0.43
s_4	0.1	0	0	0.1	0.2	0.33
s_5	0.1	0	0	0.1	0.1	0.24
s_6	0	0.2	0.2	0.1	0.1	0.5

5 Conclusions

In this paper, a novel MCDM approach is generalized. Firstly, we present several notations on degree-hesitation function, score function, accuracy function of intuitionistic fuzzy soft set and further give several principles of the minimum degree-hesitation principle, the maximal score principle, the maximal accuracy principle and the maximal choice-value principle. We just choose three of them and generate one approach called the approach of principle. Moreover, a concrete algorithm on MCDM ranking problem to intuitionistic fuzzy soft set is presented, which is prpved to be more effective with an example. At the same time, we introduce the weighted MCDM ranking approach to the weighted intuitionistic fuzzy soft set based on the presented concepts on the weighted degree-hesitation, the score function and the accuracy function, respectively. It is necessary to emphasis that the approach presented in this paper can not only solve the MCDM completely ranking problem, but also can get the only one optimal-selection. Hence, it is a more generalized approach. As far as future research directions are concerned, for the extended score function and the extended accuracy function of intuitionistic fuzzy numbers were stated by Wang et al. [51]. It is desirable to further apply more appropriate extended score function and the extended accuracy function of intuitionistic fuzzy numbers to other practical applications based on intuitionistic fuzzy soft set.

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