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ORIGINAL ARTICLE

Interval-valued neutrosophic soft sets and its decision making

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Abstract In this paper, the notion of the interval valued neutrosophic soft set (*ivn*-soft sets) is defined which is a combination of an interval valued neutrosophic set [35] and a soft set [29]. Our *ivn*-soft sets generalizes the concept of the soft set, fuzzy soft set, interval valued fuzzy soft set, intuitionistic fuzzy soft set, interval valued intuitionistic fuzzy soft set and neutrosophic soft set. Then, we introduce some definitions and operations on *ivn*-soft sets sets. Some properties of *ivn*-soft sets which are connected to operations have been established. Also, the aim of this paper is to investigate the decision making based on *ivn*-soft sets by level soft sets. Therefore, we develop a decision making methods and then give a example to illustrate the developed approach.

Keywords Neutrosophic sets · Interval valued neutrosophic sets · Soft sets · Fuzzy sets · Intuitionistic fuzzy sets · Level soft set · Decision making

1 Introduction

Many fields deal with the uncertain data may not be successfully modeled by the classical mathematics, since concept of uncertainty is too complicate and not clearly defined object. But they can be modeled a number of different approaches including the probability theory, fuzzy set theory [39], intuitionistic fuzzy set [3], rough set theory [32], neutrosophic set theory [33] and some other mathematical tools. These theories have been applied in many real applications to handle uncertainty. In 1999, Molodtsov [29] succesfully proposed a completely new theory so-called *soft set theory* by using classical sets because its been pointed out that soft sets are not appropriate to deal with uncertain and fuzzy parameters. The theory is a relatively new mathematical model for dealing with uncertainty from a parametrization point of view.

After Molodtsov, there has been a rapid growth of interest in soft sets and their various applications such as; algebraic structures (e.g. [1, 2, 5, 37, 41]), optimization (e.g. [21]), lattice (e.g. [19, 31, 33]), topology (e.g. [8,11, 28, 34]), perron integration, data analysis and operations research (e.g. [29, 30]), game theory (e.g. [14, 29]), clustering (e.g. [4, 27]), medical diagnosis (e.g. [38]), and decision making under uncertainty (e.g. [9, 10, 11, 13, 16, 20, 26]). In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [9, 12, 16, 22]), rough sets (e.g. [15]) , intuitionistic fuzzy sets (e.g. [18, 23]), interval valued intuitionistic fuzzy sets (e.g. [17, 40]), neutrosophic sets (e.g. [24, 25]).

Intuitionistic fuzzy sets can only handle incomplete information because the sum of degree true, indeterminacy and false is one in intuitionistic fuzzy sets. But neutrosophic sets can handle the indeterminate information and inconsistent information which exists commonly in belief systems in neutrosophic set since indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. It is mentioned in [33, 35]. Therefore, Maji firstly proposed neutrosophic soft sets with operations, which is free of the difficulties mentioned above, in [25]. He also, applied to decision making problems in [24]. After Maji,

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the studies on the neutrosophic soft set theory have been studied increasingly (e.g. [6, 7]).

From academic point of view, the neutrosophic set and operators need to be specified because is hard to be applied to the real applications. So the concept of interval valued neutrosophic sets [35] which can represent uncertain, imprecise, incomplete and inconsistent information was proposed. In this paper, we first define interval valued neutrosophic soft sets (*ivn*-soft sets) which is generalizes the concept of the soft set, fuzzy soft set, interval valued fuzzy soft set, intuitionistic fuzzy soft set, interval valued intuitionistic fuzzy soft sets. Then, we introduce some definitions and operations of *ivn*-soft sets. Some properties of *ivn*-soft sets which are connected to operations have been established. Also, the aim of this paper is to investigate the decision making based on *ivn*-soft sets. By means of level soft sets, we develop an adjustable approach to *ivn*-soft sets based decision making and a examples are provided to illustrate the developed approach.

The relationship among *ivn*-soft set and other soft sets is illustrated as;

Soft set $\subseteq$ Fuzzy soft set
$\subseteq$ Intuitionistic fuzzy soft set (Interval valued fuzzy soft set)
$\subseteq$ Interval valued intuitionistic fuzzy soft set
$\subseteq$ Interval valued neutrosophic soft set

Therefore, interval valued neutrosophic soft set is a generalization other each the soft sets.

2 Preliminary

In this section, we present the basic definitions of neutrosophic set theory [33], interval valued neutrosophic set theory [35] and soft set theory [29] that are useful for subsequent discussions. More detailed explanations related to this subsection may be found in [6, 7, 12, 17, 24, 25, 33, 35, 36].

Definition 2.1 [33] A neutrosophic set A on the universe of discourse U is defined as

$$A = \left\{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\right\}$$

where $T_A, I_A, F_A : U \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or nonstandard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of [0, 1].

Here, $1^+ = 1 + \varepsilon$, where 1 is its standard part and ε its non-standard part. Similarly, $^-0 = 0 - \varepsilon$, where 0 is its standard part and ε its non-standard part.

Definition 2.2 [36] Let U be a space of points (objects), with a generic element in U denoted by u. A single valued neutrosophic sets A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(u)$; $I_A(u)$ and $F_A(u)$ are real standard or nonstandard subsets of [0, 1]. It can be written as

$$A = \{\langle u, (T_A(u), I_A(u), F_A(u))\rangle : u \in U, \; T_A(u), I_A(u), F_A(u) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $0 \leq supT_A(u) + supI_A(u) + supF_A(u) \leq 3$.

Definition 2.3 [35] Let U be a space of points (objects), with a generic element in U denoted by u. An interval value neutrosophic set (*IVN*-sets) A in U is characterized by truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $u \in U$; T_A, I_A and $F_A \subseteq [0, 1]$.

Thus, a *IVN*-sets over U can be represented by the set of

$$A = \{\langle T_A(u), I_A(u), F_A(u)\rangle / u : u \in U\}$$

Here, $(T_A(u), I_A(u), F_A(u))$ is called interval value neutrosophic number for all $u \in U$ and all interval value neutrosophic numbers over U will be denoted by *IVN*(U).

Example 2.4 Assume that the universe of discourse $U = \{u_1, u_2\}$ where u_1 and characterises the quality, u_2 indicates the prices of the objects. It may be further assumed that the values of u_1 and u_2 are subset of [0, 1] and they are obtained from a expert person. The expert construct an interval value neutrosophic set the characteristics of the objects according to by truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A as follows;

$$A = \{\langle [0.1, 1.0], [0.1, 0.4], [0.4, 0.7]\rangle / u_1, \langle [0.6, 0.9], [0.8, 1.0], [0.4, 0.6]\rangle / u_2\}$$

Definition 2.5 [35] Let A a interval valued neutrosophic sets. Then, for all $u \in U$,

1. A is empty, denoted $A = \widetilde{\emptyset}$, is defined by
$$\widetilde{\emptyset} = \{\langle [0, 0], [1, 1], [1, 1]\rangle / u : u \in U\}$$
2. A is universal, denoted $A = \widetilde{E}$, is defined by
$$\widetilde{E} = \{\langle [1, 1], [0, 0], [0, 0]\rangle / u : u \in U\}$$
3. The complement of A is denoted by $\overline{A}$ and is defined by
$$\bar{A} = \{\langle [infF_A(u), supF_A(u)], [1 - supI_A(u), 1 - \text{inf I\_A(u)}], [infT_A(u), supT_A(u)]\rangle / u : u \in U\}$$

Definition 2.6 [35] An interval valued neutrosophic set A is contained in the other interval valued neutrosophic set B, $A\tilde{\subseteq}B$, if and only if

$$\begin{array}{ll} infT_A(u) \leq infT_B(u) & supI_A(u) \geq supI_B(u) \\ supI_A(u) \geq supI_B(u) & infF_A(u) \geq infF_B(u) \\ infI_A(u) \geq infI_B(u) & supF_A(u) \geq supF_B(u) \end{array}$$

for all $u \in U$.

Note that an interval valued neutrosophic number $X = (T_X, I_X, F_X)$ is larger than the other interval valued neutrosophic number $Y = (T_Y, I_Y, F_Y)$, denoted $X\widehat{\leq}Y$, if and only if

$$\begin{array}{ll} infT_X \leq infT_Y & supI_X \geq supI_Y \\ supT_X \leq supT_Y & supF_X \geq supF_Y \\ infI_X \geq infI_Y & supF_X \geq supF_Y \end{array}$$

Definition 2.7 [35] Let A and B be two interval valued neutrosophic sets. Then, for all $u \in U$, $a \in R^+$,

1. Intersection of A and B, denoted by $A\tilde{\cap}B$, is defined by

$$A\tilde{\cap}B = \{\langle[min(infT_A(u), infT_B(u)), min(supT_A(u), supT_B(u))], [max(infI_A(u), infI_B(u)), max(supIA(x), supI_B(u))], [max(infF_A(u), infF_B(u)), max(supF_A(u), supF_B(u))]\rangle / u : u \in U\}$$

2. Union of A and B, denoted by $A\tilde{\cup}B$, is defined by

$$A\tilde{\cup}B = \{\langle[max(infT_A(u), infT_B(u)), max(supT_A(u), supT_B(u))], [min(infI_A(u), infI_B(u)), min(supI_A(u), supI_B(u))], [min(infF_A(u), infF_B(u)), min(supF_A(u), supF_B(u))]\rangle / u : u \in U\}$$

3. Difference of A and B, denoted by $A\tilde{\setminus}B$, is defined by

$$A\tilde{\setminus}B = \{\langle[min(infT_A(u), infF_B(u)), min(supT_A(u), supF_B(x))], [max(infI_A(u), 1 - supI_B(u)), max(supI_A(u), 1 - infI_B(u))], [max(infF_A(u), infT_B(u)), max(supF_A(u), supT_B(u))]\rangle / u : u \in U\}$$

4. Addition of A and B, denoted by $A\tilde{+}B$, is defined by

$$A\tilde{+}B = \{\langle[min(infT_A(u) + infT_B(u), 1), min(supT_A(u) + supT_B(u), 1)], [min(infI_A(u) + infI_B(u), 1), min(supI_A(u) + supI_B(u), 1)], [min(infF_A(u) + infF_B(u), 1), min(supF_A(u) + supF_B(u), 1)]\rangle / u : u \in U\}$$

5. Scalar multiplication of A, denoted by $A\tilde{.}a$, is defined by

$$A\tilde{.}a = \{\langle[min(infT_A(u).a, 1), min(supT_A(u).a, 1)], [min(infI_A(u).a, 1), min(supI_A(u).a, 1)], [min(infF_A(u).a, 1), min(supF_A(u).a, 1)]\rangle / u : u \in U\}$$

6. Scalar division of A, denoted by $A\tilde{/}a$, is defined by

$$A\tilde{/}a = \{\langle[min(infT_A(u)/a, 1), min(supT_A(u)/a, 1)], [min(infI_A(u)/a, 1), min(supI_A(u)/a, 1)], [min(infF_A(u)/a, 1), min(supF_A(u)/a, 1)]\rangle / u : u \in U\}$$

7. Truth-Favorite of A, denoted by $\tilde{\triangle}A$, is defined by

$$\tilde{\triangle}A = \{\langle[min(infT_A(u) + infI_A(u), 1), min(supT_A(u) + supI_A(u), 1)], [0, 0], [infF_A(u), supF_A(u)]\rangle / u : u \in U\}$$

8. False-Favorite of A, denoted by $\tilde{\nabla}A$, is defined by

$$\tilde{\nabla}A = \{\langle[infT_A(u), supT_A(u)], [0, 0], [min(infF_A(u) + infI_A(u), 1), min(supF_A(u) + supI_A(u), 1)]\rangle / u : u \in U\}$$

Definition 2.8 [29] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and $X \subseteq E$. Then a soft set F_X over U is a set defined by a function representing a mapping

$$f_X : E \rightarrow P(U) \; such \; that \; f_X(x) = \emptyset \; if \; x \notin X$$

Here, f_X is called approximate function of the soft set F_X, and the value $f_X(x)$ is a set called *x-element* of the soft set for all $x \in E$. It is worth noting that the set is worth noting that the sets $f_X(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set over U can be represented by the set of ordered pairs

$$F_X = \{(x, f_X(x)) : x \in E, f_X(x) \in P(U)\}$$

Example 2.9 Suppose that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the universe contains six house under consideration in a real agent and $E = \{x_1 = cheap, x_2 = beatiful, x_3 = greensurroundings, x_4 = costly, x_5 = large\}$.

If a customer to select a house from the real agent then, he/she can construct a soft set F_X that describes the characteristic of houses according to own requests. Assume that $f_X(x_1) = \{u_1, u_2\}$, $f_X(x_2) = \{u_1\}$, $f_X(x_3) = \emptyset$, $f_X(x_4) = U$, $\{u_1, u_2, u_3, u_4, u_5\}$ then the soft-set F_X is written by

$$F_X = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_4, u_5, u_6\}), (x_4, U), (x_5, \{u_1, u_2, u_3, u_4, u_5\})\}$$

The tabular representation of the soft set F_X is as follow (Table 1):

Definition 2.10 [26] Let $U = \{u_1, u_2, \ldots, u_k\}$ be an initial universe of objects, $E = \{x_1, x_2, \ldots, x_m\}$ be a set of parameters and F_X be a soft set over U. For any $x_j \in E$,

Table 1 The tabular representation of the soft set F_X

U	u_1	u_2	u_3	u_4	u_5	u_6
x_1	1	1	0	0	0	0
x_2	1	0	0	1	1	1
x_3	0	0	0	0	0	0
x_4	1	1	1	1	1	1
x_5	1	1	1	1	1	0

$f_X(x_j)$ is a subset of U. Then, the choice value of an object $u_i \in U$ is c_i, given by $c_i = \sum_j u_{ij}$, where u_{ij} are the entries in the table of the reduct-soft-set. That is,

$$u_{ij} = \begin{cases} 1, & u_i \in f_X(x_j) \\ 0, & u_i \notin f_X(x_j) \end{cases}$$

Example 2.11 Consider the above Example 2.9. Clearly,

$$c_1 = \sum_{j=1}^{5} u_{1j} = 4,$$

$$c_3 = c_6 = \sum_{j=1}^{5} u_{3j} = \sum_{j=1}^{5} u_{6j} = 2,$$

$$c_2 = c_4 = c_5 = \sum_{j=1}^{5} u_{2j} = \sum_{j=1}^{5} u_{4j} = \sum_{j=1}^{5} u_{5j} = 3$$

Definition 2.12 [13] Let F_X and F_Y be two soft sets. Then,

1. Complement of F_X is denoted by $F_X^{\tilde{c}}$. Its approximate function $f_{X^c}(x) = U \setminus f_X(x) \quad \text{for all } x \in E$
2. Union of F_X and F_Y is denoted by $F_X \tilde{\cup} F_Y$. Its approximate function $f_{X\tilde{\cup}F_Y}$ is defined by

   $$f_{X\tilde{\cup}Y}(x) = f_X(x) \cup f_Y(x) \quad \text{for all } x \in E.$$

3. Intersection of F_X and F_Y is denoted by $F_X \tilde{\cap} F_Y$. Its approximate function $f_{X\tilde{\cap}Y}$ is defined by

   $$f_{X\tilde{\cap}Y}(x) = f_X(x) \cap f_Y(x) \quad \text{for all } x \in E.$$

3 Interval-valued neutrosophic soft sets

In this section, we give interval valued neutrosophic soft sets (*ivn*-soft sets) which is a combination of an interval valued neutrosophic sets [35] and a soft sets [29]. Then, we introduce some definitions and operations of *ivn*-soft sets sets. Some properties of *ivn*-soft sets which are connected to operations have been established. Some of it is quoted from [6, 7, 12, 13, 17, 18, 24, 25, 35].

Definition 3.1 Let U be an initial universe set, $IVN(U)$ denotes the set of all interval valued neutrosophic sets of U and E be a set of parameters that are describe the elements of U. An interval valued neutrosophic soft sets(*ivn*-soft sets) over U is a set defined by a set valued function Υ_K representing a mapping

$$\upsilon_K : E \to IVN(U)$$

It can be written a set of ordered pairs

$$\Upsilon_K = \{(x, \upsilon_K(x)) : x \in E\}$$

Here, υ_K, which is interval valued neutrosophic sets, is called approximate function of the *ivn*-soft sets Υ_K and $\upsilon_K(x)$ is called x-approximate value of $x \in E$. The subscript K in the υ_K indicates that υ_K is the approximate function of Υ_K.

Generally, υ_K, υ_L, υ_M,... will be used as an approximate functions of Υ_K, Υ_L, Υ_M,..., respectively.

Note that the sets of all *ivn*-soft sets over U will be denoted by $IVNS(U)$.

Now let us give the following example for *ivn*-soft sets.

Example 3.2 Let $U = \{u_1, u_2\}$ be set of houses under consideration and E is a set of parameters which is a neutrosophic word. Consider $E = \{x_1 = cheap, x_2 = beatiful, x_3 = greensurroundings, x_4 = costly, x_5 = large\}$. In this case, we give an (*ivn*-soft sets) Υ_K over U as;

$$\begin{aligned}\Upsilon_K = \{&(x_1, \{\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle/u_1, \langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle/u_2\}),\\ &(x_2, \{\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle/u_1, \langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle/u_2\}),\\ &(x_3, \{\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle/u_1, \langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle/u_2\}),\\ &(x_4, \{\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle/u_1, \langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle/u_2\}),\\ &(x_5, \{\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.1]\rangle/u_1, \langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle/u_2\})\}\end{aligned}$$

The tabular representation of the *ivn*-soft set Υ_K is as follow (Table 2):

Definition 3.3 Let $\Upsilon_K \in IVNS(U)$. If $\upsilon_K(x) = \widetilde{\emptyset}$ for all $x \in E$, then Υ_K is called an empty *ivn*-soft set, denoted by $\Upsilon_{\widehat{\emptyset}}$.

Definition 3.4 Let $\Upsilon_K \in IVNS(U)$. If $\upsilon_K(x) = \widetilde{E}$ for all $x \in E$, then Υ_K is called a universal *ivn*-soft set, denoted by $\Upsilon_{\hat{E}}$.

Table 2 The tabular representation of the *ivn*-soft set Υ_K

U	u_1	u_2
x_1	$\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle$	$\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle$
x_2	$\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle$	$\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle$
x_3	$\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle$	$\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle$
x_4	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle$	$\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle$
x_5	$\langle[0.0,0.9],[1.0,1.0],[1.0,1.1]\rangle$	$\langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle$

Example 3.5 Assume that $U=\{u_1,u_2\}$ is a universal set and $E=\{x_1,x_2,x_3,\ x_4,x_5\}$ is a set of all parameters. Consider the tabular representation of the $\Upsilon_{\hat{\emptyset}}$ is as follows (Table 3);

The tabular representation of the $\Upsilon_{\hat{E}}$ is as follows (Table 4);

Definition 3.6 Let $\Upsilon_K,\Upsilon_L\in IVNS(U)$. Then, Υ_K is an *ivn*-soft subset of Υ_L, denoted by $\Upsilon_K\widehat{\subseteq}\Upsilon_L$, if $\upsilon_K(x)\tilde{\subseteq}\upsilon_L(x)$ for all $x\in E$.

Example 3.7 Assume that $U=\{u_1,u_2\}$ is a universal set and $E=\{x_1,x_2,x_3,\ x_4,x_5\}$ is a set of all parameters. Consider the tabular representation of the Υ_K is as follows (Table 5);

The tabular representation of the Υ_L is as follows (Table 6);

Clearly, by Definition 3.6, we have $\Upsilon_K\widehat{\subseteq}\Upsilon_L$.

Remark 3.8 $\Upsilon_K\widehat{\subseteq}\Upsilon_L$ does not imply that every element of Υ_K is an element of Υ_L as in the definition of the classical subset.

Proposition 3.9 Let $\Upsilon_K,\Upsilon_L,\Upsilon_M\in IVNS(U)$. Then,

Table 3 The tabular representation of the *ivn*-soft set $\Upsilon_{\hat{\emptyset}}$

U	u_1	u_2
x_1	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$
x_2	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$
x_3	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$
x_4	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$
x_5	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.0],[1.0,1.0],[1.0,1.0]\rangle$

Table 4 The tabular representation of the *ivn*-soft set $\Upsilon_{\hat{E}}$

U	u_1	u_2
x_1	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$
x_2	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$
x_3	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$
x_4	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$
x_5	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$	$\langle[1.0,1.0],[0.0,0.0],[0.0,0.0]\rangle$

Table 5 The tabular representation of the *ivn*-soft set Υ_K

U	u_1	u_2
x_1	$\langle[0.5,0.7],[0.8,0.9],[0.2,0.5]\rangle$	$\langle[0.3,0.6],[0.3,0.9],[0.2,0.8]\rangle$
x_2	$\langle[0.0,0.3],[0.6,0.8],[0.3,0.9]\rangle$	$\langle[0.1,0.8],[0.8,0.9],[0.3,0.5]\rangle$
x_3	$\langle[0.1,0.7],[0.4,0.5],[0.8,0.9]\rangle$	$\langle[0.2,0.5],[0.5,0.7],[0.6,0.8]\rangle$
x_4	$\langle[0.2,0.4],[0.7,0.9],[0.6,0.9]\rangle$	$\langle[0.3,0.9],[0.6,0.9],[0.3,0.9]\rangle$
x_5	$\langle[0.0,0.2],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.1],[0.9,1.0],[0.2,0.9]\rangle$

Table 6 The tabular representation of the *ivn*-soft set Υ_L

U	u_1	u_2
x_1	$\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle$	$\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle$
x_2	$\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle$	$\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle$
x_3	$\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle$	$\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle$
x_4	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle$	$\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle$
x_5	$\langle[0.0,0.9],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle$

1. $\Upsilon_K\widehat{\subseteq}\Upsilon_{\hat{E}}$
2. $\Upsilon_{\hat{\emptyset}}\widehat{\subseteq}\Upsilon_K$
3. $\Upsilon_K\widehat{\subseteq}\Upsilon_K$
4. $\Upsilon_K\widehat{\subseteq}\Upsilon_L$ and $\Upsilon_L\widehat{\subseteq}\Upsilon_M\Rightarrow\Upsilon_K\widehat{\subseteq}\Upsilon_M$

Proof They can be proved easily by using the approximate function of the *ivn*-soft sets. □

Definition 3.10 Let $\Upsilon_K,\Upsilon_L\in IVNS(U)$. Then, Υ_K and Υ_L are *ivn*-soft equal, written as $\Upsilon_K=\Upsilon_L$, if and only if $\upsilon_K(x)=\upsilon_L(x)$ for all $x\in E$.

Proposition 3.11 Let $\Upsilon_K,\Upsilon_L,\Upsilon_M\in IVNS(U)$. Then,

1. $\Upsilon_K=\Upsilon_L$ and $\Upsilon_L=\Upsilon_M\Leftrightarrow\Upsilon_K=\Upsilon_M$
2. $\Upsilon_K\widehat{\subseteq}\Upsilon_L$ and $\Upsilon_L\widehat{\subseteq}\Upsilon_K\Leftrightarrow\Upsilon_K=\Upsilon_L$

Proof The proofs are trivial. □

Definition 3.12 Let $\Upsilon_K\in IVNS(U)$. Then, the complement $\Upsilon_K^{\hat{c}}$ of Υ_K is an *ivn*-soft set such that
$\upsilon_K^{\hat{c}}(x)=\overline{\upsilon_K}(x)$, for all $x\in E$.

Example 3.13 Consider the above Example 3.7, the complement $\Upsilon_L^{\hat{c}}$ of Υ_L can be represented into the following table (Table 7);

Proposition 3.14 Let $\Upsilon_K\in IVNS(U)$. Then,

1. $(\Upsilon_K^{\hat{c}})^{\hat{c}}=\Upsilon_K$
2. $\Upsilon_{\hat{\emptyset}}^{\hat{c}}=\Upsilon_{\hat{E}}$
3. $\Upsilon_{\hat{E}}^{\hat{c}}=\Upsilon_{\hat{\emptyset}}$

Proof By using the fuzzy approximate functions of the *ivn*-soft set, the proofs can be straightforward. □

Table 7 The tabular representation of the *ivn*-soft set $\Upsilon_L^{\hat{c}}$

U	u_1	u_2
x_1	$\langle[0.1, 0.5], [0.1, 0.2], [0.6, 0.8]\rangle$	$\langle[0.1, 0.7], [0.1, 0.8], [0.5, 0.8]\rangle$
x_2	$\langle[0.3, 0.7], [0.2, 0.5], [0.1, 0.4]\rangle$	$\langle[0.2, 0.3], [0.1, 0.4], [0.1, 0.9]\rangle$
x_3	$\langle[0.7, 0.8], [0.5, 0.9], [0.2, 0.9]\rangle$	$\langle[0.5, 0.7], [0.4, 0.9], [0.4, 0.9]\rangle$
x_4	$\langle[0.6, 0.9], [0.1, 0.4], [0.6, 0.9]\rangle$	$\langle[0.1, 0.8], [0.2, 0.4], [0.5, 0.9]\rangle$
x_5	$\langle[1.0, 1.0], [0.0, 0.0], [0.0, 0.9]\rangle$	$\langle[0.2, 0.5], [0.0, 0.2], [0.0, 0.9]\rangle$

Theorem 3.15 Let $\Upsilon_K \in IVNS(U)$. Then, $\Upsilon_K \widehat{\subseteq} \Upsilon_L \Leftrightarrow \Upsilon_L^{\hat{c}} \widehat{\subseteq} \Upsilon_K^{\hat{c}}$

Proof By using the fuzzy approximate functions of the *ivn*-soft set, the proofs can be straightforward. □

Definition 3.16 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, union of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cup} \Upsilon_L$, is defined by

$$\upsilon_{K\widehat{\cup}L}(x) = \upsilon_K(x)\widetilde{\cup}\upsilon_L(x) \quad \text{for all } x \in E.$$

Example 3.17 Consider the above Example 3.7, the union of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cup} \Upsilon_L$, can be represented into the following table (Table 8);

Theorem 3.18 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, $\Upsilon_K \widehat{\cup} \Upsilon_L$ is the smallest *ivn*-soft set containing both Υ_K and Υ_L.

Proof The proofs can be easily obtained from Definition 3.16. □

Proposition 3.19 Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,

1. $\Upsilon_K \widehat{\cup} \Upsilon_K = \Upsilon_K$
2. $\Upsilon_K \widehat{\cup} \Upsilon_{\widehat{\emptyset}} = \Upsilon_K$
3. $\Upsilon_K \widehat{\cup} \Upsilon_{\hat{E}} = \Upsilon_{\hat{E}}$
4. $\Upsilon_K \widehat{\cup} \Upsilon_L = \Upsilon_L \widehat{\cup} \Upsilon_K$
5. $(\Upsilon_K \widehat{\cup} \Upsilon_L) \widehat{\cup} \Upsilon_M = \Upsilon_K \widehat{\cup} (\Upsilon_L \widehat{\cup} \Upsilon_M)$

Proof The proofs can be easily obtained from Definition 3.16. □

Definition 3.20 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, intersection of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cap} \Upsilon_L$, is defined by

$$\upsilon_{K\widehat{\cap}L}(x) = \upsilon_K(x)\widetilde{\cap}\upsilon_L(x) \quad \text{for all } x \in E.$$

Table 8 The tabular representation of the *ivn*-soft set $\Upsilon_K \widehat{\cup} \Upsilon_L$

U	u_1	u_2
x_1	$\langle[0.6, 0.8], [0.8, 0.9], [0.1, 0.5]\rangle$	$\langle[0.5, 0.8], [0.2, 0.9], [0.1, 0.7]\rangle$
x_2	$\langle[0.1, 0.4], [0.5, 0.8], [0.3, 0.7]\rangle$	$\langle[0.1, 0.9], [0.6, 0.9], [0.2, 0.3]\rangle$
x_3	$\langle[0.2, 0.9], [0.1, 0.5], [0.7, 0.8]\rangle$	$\langle[0.4, 0.9], [0.1, 0.6], [0.5, 0.7]\rangle$
x_4	$\langle[0.6, 0.9], [0.6, 0.9], [0.6, 0.9]\rangle$	$\langle[0.5, 0.9], [0.6, 0.8], [0.1, 0.8]\rangle$
x_5	$\langle[0.0, 0.9], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.9], [0.8, 1.0], [0.2, 0.5]\rangle$

Table 9 The tabular representation of the *ivn*-soft set $\Upsilon_K \widehat{\cap} \Upsilon_L$

U	u_1	u_2
x_1	$\langle[0.5, 0.7], [0.8, 0.9], [0.2, 0.5]\rangle$	$\langle[0.3, 0.6], [0.3, 0.9], [0.2, 0.8]\rangle$
x_2	$\langle[0.0, 0.3], [0.6, 0.8], [0.3, 0.9]\rangle$	$\langle[0.1, 0.8], [0.8, 0.9], [0.3, 0.5]\rangle$
x_3	$\langle[0.1, 0.7], [0.4, 0.5], [0.8, 0.9]\rangle$	$\langle[0.2, 0.5], [0.5, 0.7], [0.6, 0.8]\rangle$
x_4	$\langle[0.2, 0.4], [0.7, 0.9], [0.6, 0.9]\rangle$	$\langle[0.3, 0.9], [0.6, 0.9], [0.3, 0.9]\rangle$
x_5	$\langle[0.0, 0.2], [1.0, 1.0], [1.0, 1.0]\rangle$	$\langle[0.0, 0.1], [0.9, 1.0], [0.2, 0.9]\rangle$

Example 3.21 Consider the above Example 3.7, the intersection of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\cap} \Upsilon_L$, can be represented into the following table (Table 9);

Proposition 3.22 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, $\Upsilon_K \widehat{\cap} \Upsilon_L$ is the largest *ivn*-soft set containing both Υ_K and Υ_L.

Proof The proofs can be easily obtained from Definition 3.20. □

Proposition 3.23 Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,

1. $\Upsilon_K \widehat{\cap} \Upsilon_K = \Upsilon_K$
2. $\Upsilon_K \widehat{\cap} \Upsilon_{\widehat{\emptyset}} = \Upsilon_{\widehat{\emptyset}}$
3. $\Upsilon_K \widehat{\cap} \Upsilon_{\hat{E}} = \Upsilon_K$
4. $\Upsilon_K \widehat{\cap} \Upsilon_L = \Upsilon_L \widehat{\cap} \Upsilon_K$
5. $(\Upsilon_K \widehat{\cap} \Upsilon_L) \widehat{\cap} \Upsilon_M = \Upsilon_K \widehat{\cap} (\Upsilon_L \widehat{\cap} \Upsilon_M)$

Proof The proof of the Propositions 1- 5 are obvious. □

Remark 3.24 Let $\Upsilon_K \in IVNS(U)$. If $\Upsilon_K \neq \Upsilon_{\widehat{\emptyset}}$ or $\Upsilon_K \neq \Upsilon_{\hat{E}}$, then $\Upsilon_K \widehat{\cup} \Upsilon_K^{\hat{c}} \neq \Upsilon_{\hat{E}}$ and $\Upsilon_K \widehat{\cap} \Upsilon_K^{\hat{c}} \neq \Upsilon_{\widehat{\emptyset}}$.

Proposition 3.25 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, De Morgan's laws are valid

1. $(\Upsilon_K \widehat{\cup} \Upsilon_L)^{\hat{c}} = \Upsilon_K^{\hat{c}} \widehat{\cap} \Upsilon_L^{\hat{c}}$
2. $(\Upsilon_K \widehat{\cap} \Upsilon_L)^{\hat{c}} = \Upsilon_K^{\hat{c}} \widehat{\cup} \Upsilon_L^{\hat{c}}$.

Proof The proofs can be easily obtained from Definition 3.12, Definition 3.16 and Definition 3.20. □

Proposition 3.26 Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,

1. $\Upsilon_K \widehat{\cup} (\Upsilon_L \widehat{\cap} \Upsilon_M) = (\Upsilon_K \widehat{\cup} \Upsilon_L) \widehat{\cap} (\Upsilon_K \widehat{\cup} \Upsilon_M)$
2. $\Upsilon_K \widehat{\cap} (\Upsilon_L \widehat{\cup} \Upsilon_M) = (\Upsilon_K \widehat{\cap} \Upsilon_L) \widehat{\cup} (\Upsilon_K \widehat{\cap} \Upsilon_M)$
3. $\Upsilon_K \widehat{\cup} (\Upsilon_K \widehat{\cap} \Upsilon_L) = \Upsilon_K$
4. $\Upsilon_K \widehat{\cap} (\Upsilon_K \widehat{\cup} \Upsilon_L) = \Upsilon_K$

Proof The proofs can be easily obtained from Definition 3.16 and Definition 3.20. □

Definition 3.27 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, OR operator of Υ_K and Υ_L, denoted $\Upsilon_K \widehat{\bigvee} \Upsilon_L$, is defined by a set valued function Υ_O representing a mapping

Table 10 The tabular representation of the *ivn*-soft set $\Upsilon_K\hat{/}5$

U	u_1	u_2
x_1	$\langle[0.1, 0.14], [0.16, 0.18], [0.04, 0.1]\rangle$	$\langle[0.06, 0.12], [0.15, 0.18], [0.04, 0.16]\rangle$
x_2	$\langle[0.0, 0.06], [0.12, 0.16], [0.16, 0.18]\rangle$	$\langle[0.02, 0.16], [0.16, 0.18], [0.15, 0.25]\rangle$
x_3	$\langle[0.02, 0.14], [0.08, 0.1], [0.16, 0.18]\rangle$	$\langle[0.04, 0.1], [0.1, 0.14], [0.12, 0.16]\rangle$
x_4	$\langle[0.04, 0.08], [0.14, 0.18], [0.12, 0.18]\rangle$	$\langle[0.15, 0.18], [0.12, 0.18], [0.06, 0.18]\rangle$
x_5	$\langle[0.0, 0.04], [0.2, 0.2], [0.2, 0.2]\rangle$	$\langle[0.0, 0.05], [0.18, 0.2], [0.04, 0.18]\rangle$

$$\upsilon_O : E \times E \to IVN(U)$$

where

$$\upsilon_O(x, y) = \upsilon_K(x)\tilde{\cup}\upsilon_L(y) \quad \text{for all } (x, y) \in E \times E.$$

Definition 3.28 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, AND operator of Υ_K and Υ_L, denoted $\Upsilon_K\widehat{\bigwedge}\Upsilon_L$, is defined by a set valued function Υ_A representing a mapping

$$\upsilon_A : E \times E \to IVN(U)$$

where

$$\upsilon_A(x, y) = \upsilon_K(x)\tilde{\cap}\upsilon_L(y) \quad \text{for all } (x, y) \in E \times E.$$

Proposition 3.29 Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,

1. $(\Upsilon_K\widehat{\bigvee}\Upsilon_L)^{\hat{c}} = \Upsilon_K^{\hat{c}}\widehat{\bigwedge}\Upsilon_L^{\hat{c}}$
2. $(\Upsilon_K\widehat{\bigwedge}\Upsilon_L)^{\hat{c}} = \Upsilon_K^{\hat{c}}\widehat{\bigvee}\Upsilon_L^{\hat{c}}$.
3. $(\Upsilon_K\widehat{\bigvee}\Upsilon_L)\widehat{\bigvee}\Upsilon_M = \Upsilon_K\widehat{\bigvee}(\Upsilon_L\widehat{\bigvee}\Upsilon_M)$
4. $(\Upsilon_K\widehat{\bigwedge}\Upsilon_L)\widehat{\bigwedge}\Upsilon_M = \Upsilon_K\widehat{\bigwedge}(\Upsilon_L\widehat{\bigwedge}\Upsilon_M)$

Proof The proof of the Propositions 1–4 are obvious. □

Definition 3.30 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, difference of Υ_K and Υ_L, denoted $\Upsilon_K\widehat{\setminus}\Upsilon_L$, is defined by

$$\upsilon_{K\widehat{\setminus}L}(x) = \upsilon_K(x)\tilde{\setminus}\upsilon_L(x) \quad \text{for all } x \in E.$$

Definition 3.31 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then, addition of Υ_K and Υ_L, denoted $\Upsilon_K\hat{+}\Upsilon_L$, is defined by

$$\upsilon_{K\hat{+}L}(x) = \upsilon_K(x)\tilde{+}\upsilon_L(x) \quad \text{for all } x \in E.$$

Proposition 3.32 Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,

1. $\Upsilon_K(x)\hat{+}\Upsilon_L(x)\hat{=}\Upsilon_L(x)\hat{+}\Upsilon_K(x)$
2. $(\Upsilon_K(x)\hat{+}\Upsilon_L(x))\hat{+}\Upsilon_M(x) = \Upsilon_K(x)\hat{+}(\Upsilon_L(x)\hat{+}\Upsilon_M(x))$

Proof The proofs can be easily obtained from Definition 3.31. □

Definition 3.33 Let $\Upsilon_K \in IVNS(U)$. Then, scalar multiplication of Υ_K, denoted $a\hat{\times}\Upsilon_K$, is defined by

$$a\hat{\times}\Upsilon_K = a\tilde{.}\upsilon_K \quad \text{for all } x \in E.$$

Proposition 3.34 Let $\Upsilon_K, \Upsilon_L, \Upsilon_M \in IVNS(U)$. Then,

1. $\Upsilon_K(x)\hat{\times}\Upsilon_L(x) = \Upsilon_L(x)\hat{\times}\Upsilon_K(x)$
2. $(\Upsilon_K(x)\hat{\times}\Upsilon_L(x))\hat{\times}\Upsilon_M(x) = \Upsilon_K(x)\hat{\times}(\Upsilon_L(x)\hat{\times}\Upsilon_M(x))$

Proof The proofs can be easily obtained from Definition 3.33. □

Definition 3.35 Let $\Upsilon_K \in IVNS(U)$. Then, scalar division of Υ_K, denoted $\Upsilon_K\hat{/}a$, is defined by

$$\Upsilon_K\hat{/}a = \Upsilon_K\tilde{/}a \quad \text{for all } x \in E.$$

Example 3.36 Consider the above Example 3.7, for $a = 5$, the scalar division of Υ_K, denoted $\Upsilon_K\hat{/}5$, can be represented into the following table (Table 10);

Definition 3.37 Let $\Upsilon_K \in IVNS(U)$. Then, truth-Favorite of Υ_K, denoted $\hat{\triangle}\Upsilon_K$, is defined by

$$\hat{\triangle}\Upsilon_K = \tilde{\triangle}\upsilon_K \quad \text{for all } x \in E.$$

Example 3.38 Consider the above Example 3.7, the truth-Favorite of Υ_K, denoted $\hat{\triangle}\Upsilon_K$, can be represented into the following table (Table 11);

Proposition 3.39 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then,

1. $\hat{\triangle}\hat{\triangle}\Upsilon_K = \hat{\triangle}\Upsilon_K$
2. $\hat{\triangle}(\Upsilon_K\hat{\cup}\Upsilon_K)\hat{\subseteq}\hat{\triangle}\Upsilon_K\hat{\cup}\hat{\triangle}\Upsilon_K$
3. $\hat{\triangle}(\Upsilon_K\hat{\cap}\Upsilon_K)\hat{\subseteq}\hat{\triangle}\Upsilon_K\hat{\cap}\hat{\triangle}\Upsilon_K$
4. $\hat{\triangle}(\Upsilon_K\hat{+}\Upsilon_K) = \hat{\triangle}\Upsilon_K\hat{+}\hat{\triangle}\Upsilon_K$

Proof The proofs can be easily obtained from Definition 3.16, Definition 3.20 and Definition 3.37. □

Definition 3.40 Let $\Upsilon_K \in IVNS(U)$. Then, False-Favorite of Υ_K, denoted $\hat{\triangledown}\Upsilon_K$, is defined by

Table 11 The tabular representation of the *ivn*-soft set $\hat{\triangle}\Upsilon_K$

U	u_1	u_2
x_1	$\langle[1.0, 1.0], [0.0, 0.0], [0.2, 0.5]\rangle$	$\langle[0.6, 1.0], [0.0, 0.0], [0.2, 0.8]\rangle$
x_2	$\langle[0.6, 1.0], [0.0, 0.0], [0.3, 0.9]\rangle$	$\langle[0.9, 1.0], [0.0, 0.0], [0.3, 0.5]\rangle$
x_3	$\langle[0.5, 1.0], [0.0, 0.0], [0.8, 0.9]\rangle$	$\langle[0.7, 1.0], [0.0, 0.0], [0.6, 0.8]\rangle$
x_4	$\langle[0.9, 1.0], [0.0, 0.0], [0.6, 0.9]\rangle$	$\langle[0.9, 1.0], [0.0, 0.0], [0.3, 0.9]\rangle$
x_5	$\langle[1.0, 1.0], [0.0, 0.0], [1.0, 1.0]\rangle$	$\langle[0.9, 1.0], [0.0, 0.0], [0.2, 0.9]\rangle$

Table 12 The tabular representation of the *ivn*-soft set $\widehat{\bigtriangledown}\Upsilon_K$

U	u_1	u_2
x_1	$\langle[0.5,0.7],[0.0,0.0],[1.0,1.0]\rangle$	$\langle[0.3,0.6],[0.0,0.0],[0.5,1.0]\rangle$
x_2	$\langle[0.0,0.3],[0.0,0.0],[0.9,1.0]\rangle$	$\langle[0.1,0.8],[0.0,0.0],[1.0,1.0]\rangle$
x_3	$\langle[0.1,0.7],[0.0,0.0],[1.0,1.0]\rangle$	$\langle[0.2,0.5],[0.0,0.0],[1.0,1.0]\rangle$
x_4	$\langle[0.2,0.4],[0.0,0.0],[1.0,1.0]\rangle$	$\langle[0.3,0.9],[0.0,0.0],[0.9,1.0]\rangle$
x_5	$\langle[0.0,0.2],[0.0,0.0],[1.0,1.0]\rangle$	$\langle[0.0,0.1],[0.0,0.0],[1.0,1.0]\rangle$

$$\widehat{\bigtriangledown}\Upsilon_K = \widetilde{\bigtriangledown}\upsilon_K \quad \text{for all } x \in E.$$

Example 3.41 Consider the above Example 3.7, the False-Favorite of Υ_K, denoted $\widehat{\bigtriangledown}\Upsilon_K$, can be represented into the following table (Table 12);

Proposition 3.42 Let $\Upsilon_K, \Upsilon_L \in IVNS(U)$. Then,

1. $\widehat{\bigtriangledown}\widehat{\bigtriangledown}\Upsilon_K \mathbin{\widehat{=}} \widehat{\bigtriangledown}\Upsilon_K$
2. $\widehat{\bigtriangledown}(\Upsilon_K \mathbin{\widehat{\cup}} \Upsilon_K) \mathbin{\widehat{\subseteq}} \widehat{\bigtriangledown}\Upsilon_K \mathbin{\widehat{\cup}} \widehat{\bigtriangledown}\Upsilon_K$
3. $\widehat{\bigtriangledown}(\Upsilon_K \mathbin{\widehat{\cap}} \Upsilon_K) \mathbin{\widehat{\subseteq}} \widehat{\bigtriangledown}\Upsilon_K \mathbin{\widehat{\cap}} \widehat{\bigtriangledown}\Upsilon_K$
4. $\widehat{\bigtriangledown}(\Upsilon_K \mathbin{\widehat{+}} \Upsilon_K) = \widehat{\bigtriangledown}\Upsilon_K \mathbin{\widehat{+}} \widehat{\bigtriangledown}\Upsilon_K$

Proof The proof can be easily obtained from Definition 3.16, Definition 3.20 and Definition 3.40. □

Theorem 3.43 Let P be the power set of all *ivn*-soft sets defined in the universe U. Then $(P, \widehat{\cap}, \widehat{\cup})$ is a distributive lattice.

Proof The proof can be easily obtained by showing properties; idempotency, commutativity, associativity and distributivity. □

4 *ivn*-soft set based decision making

In this section, we present an adjustable approach to *ivn*-soft set based decision making problems by extending the approach to interval-valued intuitionistic fuzzy soft set based decision making [40]. Some of it is quoted from [18, 26, 35, 40].

Definition 4.1 Let $\Upsilon_K \in IVNS(U)$. Then a *relation form* of Υ_K is defined by

$$R_{\Upsilon_K} = \{(r_{\Upsilon_K}(x,u)/(x,u)) : r_{\Upsilon_K}(x,u) \in IVN(U), x \in E, u \in U\}$$

where $r_{\Upsilon_K} : E \times U \to IVN(U)$ *and* $r_{\Upsilon_K}(x,u) = \upsilon_{K(x)}(u)$ for all $x \in E$ and $u \in U$.

That is, $r_{\Upsilon_K}(x,u) = \upsilon_{K(x)}(u)$ is characterized by truth-membership function T_K, a indeterminacy-membership function I_K and a falsity-membership function F_K. For each point $x \in E$ and $u \in U$; T_K, I_K and $F_K \subseteq [0,1]$.

Example 4.2 Consider the above Example 3.7, then, $r_{\Upsilon_K}(x,u) = \upsilon_{K(x)}(u)$ can be given as follows

$$\upsilon_{K(x_1)}(u_1) = \langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle,$$
$$\upsilon_{K(x_1)}(u_2) = \langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle,$$
$$\upsilon_{K(x_2)}(u_1) = \langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle,$$
$$\upsilon_{K(x_2)}(u_1) = \langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle,$$
$$\upsilon_{K(x_3)}(u_1) = \langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle,$$
$$\upsilon_{K(x_3)}(u_2) = \langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle,$$
$$\upsilon_{K(x_4)}(u_1) = \langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle,$$
$$\upsilon_{K(x_4)}(u_2) = \langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle,$$
$$\upsilon_{K(x_5)}(u_1) = \langle[0.0,0.9],[1.0,1.0],[1.0,1.0]\rangle,$$
$$\upsilon_{K(x_5)}(u_2) = \langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle.$$

Zhang et al.[40] introduced level-soft set and different thresholds on different parameters in interval-valued intuitionistic fuzzy soft sets. Taking inspiration these definitions we give level-soft set and different thresholds on different parameters in *ivn*-soft sets.

Definition 4.3 Let $\Upsilon_K \in IVNS(U)$. For $\alpha, \beta, \gamma \subseteq [0,1]$, the (α, β, γ)-level soft set of Υ_K is a crisp soft set, denoted $(\Upsilon_K; \langle\alpha, \beta, \gamma\rangle)$, defined by

$$(\Upsilon_K; \langle\alpha, \beta, \gamma\rangle) = \{(x_i, \{u_{ij} : u_{ij} \in U, \mu(u_{ij}) = 1\}) : x_i \in E\}$$

where,

$$\mu(u_{ij}) = \begin{cases} 1, & (\alpha, \beta, \gamma) \mathbin{\hat{\leq}} \upsilon_{K(x_i)}(u_j) \\ 0, & \text{others} \end{cases}$$

for all $u_j \in U$.

Obviously, the definition is an extension of level soft sets of interval-valued intuitionistic fuzzy soft sets [40].

Remark 4.4 In Definition 4.3, $\alpha = (\alpha_1, \alpha_2) \subseteq [0,1]$ can be viewed as a given least threshold on degrees of truth-membership, $\beta = (\beta_1, \beta_2) \subseteq [0,1]$ can be viewed as a given greatest threshold on degrees of indeterminacy-membership and $\gamma = (\gamma_1, \gamma_2) \subseteq [0,1]$ can be viewed as a given greatest threshold on degrees of falsity-membership. If $(\alpha, \beta, \gamma) \mathbin{\widehat{\leq}} \upsilon_{K(x_i)}(u)$, it shows that the degree of the truth-membership of u with respect to the parameter x_i is not less than α, the degree of the indeterminacy-membership of u with respect to the parameter x_i is not more than γ and the degree of the falsity-membership of u with respect to the parameter x_i is not more than β. In practical applications of *inv*-soft sets, the thresholds α, β, γ are pre-established by decision makers and reflect decision makers' requirements on "truth-membership levels", "indeterminacy-membership levels" and "falsity-membership levels", respectively.

Example 4.5 Consider the above Example 3.7.

Clearly the ([0.3, 0.4], [0.3, 0.5], [0.1, 0.2])-level soft set of Υ_K as follows

Table 13 The tabular representation of the *ivn*-soft set $\widehat{\bigtriangledown}\Upsilon_K$

U	u_1	u_2
x_1	$\langle[0.5,0.7],[0.8,0.9],[0.2,0.5]\rangle$	$\langle[0.3,0.6],[0.3,0.9],[0.2,0.8]\rangle$
x_2	$\langle[0.0,0.3],[0.6,0.8],[0.3,0.9]\rangle$	$\langle[0.1,0.8],[0.8,0.9],[0.3,0.5]\rangle$
x_3	$\langle[0.1,0.7],[0.4,0.5],[0.8,0.9]\rangle$	$\langle[0.2,0.5],[0.5,0.7],[0.6,0.8]\rangle$
x_4	$\langle[0.2,0.4],[0.7,0.9],[0.6,0.9]\rangle$	$\langle[0.3,0.9],[0.6,0.9],[0.3,0.9]\rangle$
x_5	$\langle[0.0,0.2],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.1],[0.9,1.0],[0.2,0.9]\rangle$
U	u_3	u_4
x_1	$\langle[0.5,0.8],[0.8,0.9],[0.3,0.9]\rangle$	$\langle[0.1,0.9],[0.5,0.9],[0.2,0.4]\rangle$
x_2	$\langle[0.9,0.9],[0.2,0.3],[0.3,0.5]\rangle$	$\langle[0.7,0.9],[0.1,0.3],[0.5,0.6]\rangle$
x_3	$\langle[0.8,0.9],[0.1,0.7],[0.6,0.8]\rangle$	$\langle[0.8,0.9],[0.1,0.2],[0.5,0.6]\rangle$
x_4	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle$	$\langle[0.5,0.9],[0.6,0.8],[0.5,0.8]\rangle$
x_5	$\langle[0.8,0.9],[0.0,0.4],[0.7,0.7]\rangle$	$\langle[0.7,0.9],[0.5,1.0],[0.6,0.5]\rangle$
U	u_5	u_6
x_1	$\langle[0.6,0.8],[0.8,0.9],[0.1,0.5]\rangle$	$\langle[0.5,0.8],[0.2,0.9],[0.1,0.7]\rangle$
x_2	$\langle[0.1,0.4],[0.5,0.8],[0.3,0.7]\rangle$	$\langle[0.1,0.9],[0.6,0.9],[0.2,0.3]\rangle$
x_3	$\langle[0.2,0.9],[0.1,0.5],[0.7,0.8]\rangle$	$\langle[0.4,0.9],[0.1,0.6],[0.5,0.7]\rangle$
x_4	$\langle[0.6,0.9],[0.6,0.9],[0.6,0.9]\rangle$	$\langle[0.5,0.9],[0.6,0.8],[0.1,0.8]\rangle$
x_5	$\langle[0.0,0.9],[1.0,1.0],[1.0,1.0]\rangle$	$\langle[0.0,0.9],[0.8,1.0],[0.2,0.5]\rangle$

$$(\Upsilon_K;\langle[0.3,0.4],[0.3,0.5],[0.1,0.2]\rangle) = \{(x_1,\{u_1\}),(x_4,\{u_1,u_2\})\}$$

Note 4.6 In some practical applications the thresholds α,β,γ decision makers need to impose different thresholds on different parameters. To cope with such problems, we replace a constant value the thresholds by a function as the thresholds on truth-membership values, indeterminacy-membership values and falsity-membership values, respectively.

Theorem 4.7 Let $\Upsilon_K,\Upsilon_L\in IVNS(U)$. Then,

1. $(\Upsilon_K;\langle\alpha_1,\beta_1,\gamma_1\rangle)$ and $(\Upsilon_K;\langle\alpha_2,\beta_2,\gamma_2\rangle)$ are $\langle\alpha_1,\beta_1,\gamma_1\rangle$-level soft set and $\langle\alpha_2,\beta_2,\gamma_2\rangle$-level soft set of Υ_K, respectively. If $\langle\alpha_2,\beta_2,\gamma_2\rangle\widehat{\leq}\langle\alpha_1,\beta_1,\gamma_1\rangle$, then we have $(\Upsilon_K;\langle\alpha_1,\beta_1,\gamma_1\rangle)\tilde{\subseteq}(\Upsilon_K;\langle\alpha_2,\beta_2,\gamma_2\rangle)$.
2. $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle)$ and $(\Upsilon_L;\langle\alpha,\beta,\gamma\rangle)$ are $\langle\alpha,\beta,\gamma$-level soft set Υ_K and Υ_L, respectively. If $\Upsilon_K\widehat{\subseteq}\Upsilon_L$, then we have $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle)\tilde{\subseteq}(\Upsilon_L;\langle\alpha,\beta,\gamma\rangle)$.

Proof The proof of the theorems are obvious. □

Definition 4.8 Let $\Upsilon_K\in IVNS(U)$. Let an interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K}:E\to IVN(U)$ in U which is called a threshold interval-valued neutrosophic set. The level soft set of Υ_K with respect to $\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K}$ is a crisp soft set, denoted by $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K})$, defined by;

$$(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K}) = \{(x_i,\{u_{ij}:u_{ij}\in U,\mu(u_{ij})=1\}):x_i \in E\}$$

where,

$$\mu(u_{ij})=\begin{cases}1, & \langle\alpha,\beta,\gamma\rangle_{\Upsilon_K}(x_i)\widehat{\leq}\upsilon_{K(x_i)}(u_j)\\ 0, & others\end{cases}$$

for all $u_j\in U$.

Obviously, the definition is an extension of level soft sets of interval-valued intuitionistic fuzzy soft sets [40].

Remark 4.9 In Definition 4.8, $\alpha=(\alpha_1,\alpha_2)\subseteq[0,1]$ can be viewed as a given least threshold on degrees of truth-membership, $\beta=(\beta_1,\beta_2)\subseteq[0,1]$ can be viewed as a given greatest threshold on degrees of indeterminacy-membership and $\gamma=(\gamma_1,\gamma_2)\subseteq[0,1]$ can be viewed as a given greatest threshold on degrees of falsity-membership of u with respect to the parameter x.

If $\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K}(x_i)\widehat{\leq}\upsilon_{K(x_i)}(u)$ it shows that the degree of the truth-membership of u with respect to the parameter x_i is not less than α, the degree of the indeterminacy-membership of u with respect to the parameter x_i is not more than γ and the degree of the falsity-membership of u with respect to the parameter x_i is not more than β.

Definition 4.10 Let $\Upsilon_K\in IVNS(U)$. Based on Υ_K, we can define an interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}:E\to IVN(U)$ by

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}(x_i)=\sum_{u\in U}\upsilon_{K(x_i)}(u)/|U|$$

for all $x\in E$.

The interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}$ is called the avg-threshold of the *ivn*-soft set Υ_K. In the following discussions, the avg-level decision rule will mean using the avg-threshold and considering the avg-level soft set in *ivn*-soft sets based decision making.

Let us reconsider the *ivn*-soft set Υ_K in Example 3.7. The avg-threshold $\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}$ of Υ_K is an interval-valued neutrosophic set and can be calculated as follows:

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}(x_1)=\sum_{i=1}^{2}\upsilon_{K(x_1)}(u_i)/|U| =\langle[0.55,0.8],[0.5,0.9],[0.1,0.6]\rangle$$

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}(x_2)=\sum_{i=1}^{2}\upsilon_{K(x_2)}(u_i)/|U| =\langle[0.1,0.65],[0.55,0.85],[0.25,0.5]\rangle$$

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}(x_3)=\sum_{i=1}^{2}\upsilon_{K(x_3)}(u_i)/|U| =\langle[0.15,0.9],[0.1,0.55],[0.6,0.75]\rangle$$

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}(x_4)=\sum_{i=1}^{2}\upsilon_{K(x_4)}(u_i)/|U| =\langle[0.55,0.9],[0.6,0.85],[0.35,0.85]\rangle$$

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}(x_5)=\sum_{i=1}^{2}\upsilon_{K(x_5)}(u_i)/|U|$$
$$=\langle[0.0,0.9],[0.9,1.0],[0.6,0.75]\rangle$$

Therefore, we have

$$\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}=\{\langle[0.55,0.8],[0.5,0.9],[0.1,0.6]\rangle/x_1,$$
$$\langle[0.1,0.65],[0.55,0.85],[0.25,0.5]\rangle/x_2,$$
$$\langle[0.15,0.9],[0.1,0.55],[0.6,0.75]\rangle/x_3,$$
$$\langle[0.55,0.9],[0.6,0.85],[0.35,0.85]\rangle/x_4,$$
$$\langle[0.0,0.9],[0.9,1.0],[0.6,0.75]\rangle/x_5\}$$

Example 4.11 Consider the above Example 3.7. Clearly; $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})=\{(x_5,\{u_2\})\}$

Definition 4.12 Let $\Upsilon_K\in IVNS(U)$. Based on Υ_K, we can define an interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K}:A\to IVN(U)$ by

$$\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K}$$
$$=\Big\{\langle[max_{u\in U}\{infT_{\upsilon_{K(x_i)}(u)}\},max_{u\in U}\{supT_{\upsilon_{K(x_i)}(u)}\}],$$
$$[min_{u\in U}\{infI_{\upsilon_{K(x_i)}(u)}\},min_{u\in U}\{supI_{\upsilon_{K(x_i)}(u)}\}],$$
$$[min_{u\in U}\{infF_{\upsilon_{K(x_i)}(u)}\},min_{u\in U}\{supF_{\upsilon_{K(x_i)}(u)}\}]\rangle/x_i:x_i\in E\Big\}$$

The interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K}$ is called the max-min-min-threshold of the *ivn*-soft set Υ_K. In what follows the Mmm-level decision rule will mean using the max-min-min-threshold and considering the Mmm-level soft set in *ivn*-soft sets based decision making.

Definition 4.13 Let $\Upsilon_K\in IVNS(U)$. Based on Υ_K, we can define an interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K}:E\to IVN(U)$ by

$$\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K}$$
$$=\Big\{\langle[min_{u\in U}\{infT_{\upsilon_{K(x_i)}(u)}\},min_{u\in U}\{supT_{\upsilon_{K(x_i)}(u)}\}],$$
$$[min_{u\in U}\{infI_{\upsilon_{K(x_i)}(u)}\},min_{u\in U}\{supI_{\upsilon_{K(x_i)}(u)}\}],$$
$$[min_{u\in U}\{infF_{\upsilon_{K(x_i)}(u)}\},min_{u\in U}\{supF_{\upsilon_{K(x_i)}(u)}\}]\rangle/x_i:x_i\in E\Big\}$$

The interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K}$ is called the min-min-min-threshold of the *ivn*-soft set Υ_K. In what follows the mmm-level decision rule will mean using the min-min-min-threshold and considering the mmm-level soft set in *ivn*-soft sets based decision making.

Theorem 4.14 Let $\Upsilon_K\in IVNS(U)$. Then, $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})$, $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K})$, $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K})$ are the avg-level soft set, Mmm-level soft set, mmm-level soft set of $\Upsilon_K\in IVNS(U)$, respectively. Then,

1. $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})$
2. $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K})$

Proof The proof of the theorems are obvious. □

Theorem 4.15 Let $\Upsilon_K,\Upsilon_L\in IVNS(U)$. Then,

1. Let $\langle\alpha_1,\beta_1,\gamma_1\rangle^i_{\Upsilon_K}$ and $\langle\alpha_2,\beta_2,\gamma_2\rangle^i_{\Upsilon_K}$ for $i\in\{avg, Mmm, mmm\}$ be two threshold interval-valued neutrosophic sets. Then, $(\Upsilon_K;\langle\alpha_1,\beta_1,\gamma_1\rangle_{\Upsilon_K})$ and $(\Upsilon_K;\langle\alpha_2,\beta_2,\gamma_2\rangle_{\Upsilon_K})$ are $\langle\alpha_1,\beta_1,\gamma_1\rangle_{\Upsilon_K}$-level soft set and $\langle\alpha_2,\beta_2,\gamma_2\rangle_{\Upsilon_K}$-level soft set of Υ_K, respectively. If $\langle\alpha_2,\beta_2,\gamma_2\rangle_{\Upsilon_K}\widehat{\leq}\langle\alpha_1,\beta_1,\gamma_1\rangle_{\Upsilon_K}$, then we have $(\Upsilon_K;\langle\alpha_1,\beta_1,\gamma_1\rangle_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_K;\langle\alpha_2,\beta_2,\gamma_2\rangle_{\Upsilon_K})$.
2. Let $\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K}$ be a threshold interval-valued neutrosophic sets. Then, $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K})$ and $(\Upsilon_L;\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K})$ are $\langle\alpha,\beta,\gamma$-level soft set Υ_K and Υ_L, respectively. If $\Upsilon_K\widehat{\subseteq}\Upsilon_L$, then we have $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K})\tilde{\subseteq}(\Upsilon_L;\langle\alpha,\beta,\gamma\rangle_{\Upsilon_K})$.

Proof The proof of the theorems are obvious. □

Now, we construct an *ivn*-soft set decision making method by the following algorithm;

Algorithm:

1. Input the *ivn*-soft set Υ_K,
2. Input a threshold interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}$ (or $\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K},\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K}$) by using avg-level decision rule (or Mmm-level decision rule, mmm-level decision rule) for decision making.
3. Compute avg-level soft set $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})$ (or Mmm-level soft set $((\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K})$, mmm-level soft set $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K})))$
4. Present the level soft set $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})$ (or the level soft set$((\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{Mmm}_{\Upsilon_K}$, the level soft set $(\Upsilon_K;\langle\alpha,\beta,\gamma\rangle^{mmm}_{\Upsilon_K})))$ in tabular form.
5. Compute the choice value c_i of u_i for any $u_i\in U$,
6. The optimal decision is to select u_k if $c_k=max_{u_i\in U}c_i$.

Remark 4.16 If k has more than one value then any one of u_k may be chosen.

If there are too many optimal choices in Step 6, we may go back to the second step and change the threshold (or decision rule) such that only one optimal choice remains in the end.

Remark 4.17 The aim of designing the Algorithm is to solve *ivn*-soft sets based decision making problem by using level soft sets. Level soft sets construct bridges between *ivn*-soft sets and crisp soft sets. By using level soft sets, we need not treat *ivn*-soft sets directly but only cope with crisp soft sets derived from them after choosing certain thresholds or decision strategies such as the mid-level or the top?bottom-level decision rules. By the Algorithm, the choice value of an object in a level soft set is in fact the number of fair attributes which belong to that object on the premise that the degree of the truth-membership of u with respect to the parameter x is not less than "truth-membership levels", the degree of the indeterminacy-membership of u with respect to the parameter x is not more than "indeterminacy-membership levels" and the degree of the falsity-membership of u with respect to the parameter x is not more than "falsity-membership levels".

Example 4.18 Suppose that a customer to select a house from the real agent. He can construct a *ivn*-soft set Υ_K that describes the characteristic of houses according to own requests. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the universe contains six house under consideration in an real agent and $E = \{x_1 = cheap, x_2 = beatiful, x_3 = greensurroundings, x_4 = costly, x_5 = large\}$.

Now, we can apply the method as follows:

1. Input the *ivn*-soft set Υ_K as,
2. Input a threshold interval-valued neutrosophic set $\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}$ by using avg-level decision rule for decision making as;

$$\begin{aligned}\langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K} = \{&\langle[0.41,0.76],[0.56,0.9],[0.18,0.63]\rangle/x_1,\\ &\langle[0.31,0.7],[0.46,0.66],[0.31,0.58]\rangle/x_2,\\ &\langle[0.41,0.8],[0.21,0.53],[0.61,0.76]\rangle/x_3,\\ &\langle[0.45,0.81],[0.61,0.86],[0.45,0.86]\rangle/x_4,\\ &\langle[0.25,0.65],[0.7,0.9],[0.61,0.76]\rangle/x_5\ \}\end{aligned}$$

3. Compute avg-level soft set $(\Upsilon_K; \langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})$ as;

$$\begin{aligned}(\Upsilon_K; \langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K}) = \{&(x_2,\{u_3\}),(x_3,\{u_4\}),(x_4,\{u_6\},\\ &(x_5,\{u_3\})\}\end{aligned}$$

4. Present the level soft set $(\Upsilon_K; \langle\alpha,\beta,\gamma\rangle^{avg}_{\Upsilon_K})$ in tabular form as (Table 14);
5. Compute the choice value c_i of u_i for any $u_i \in U$ as;

Table 14 The tabular representation of the soft set F_X

U	u_1	u_2	u_3	u_4	u_5	u_6
x_1	0	0	0	0	0	0
x_2	0	0	1	0	0	0
x_3	0	0	0	1	0	0
x_4	0	0	0	0	0	1
x_5	0	0	1	0	0	0

$$c_1 = c_2 = c_5 = \sum_{j=1}^{5} u_{1j} = \sum_{j=1}^{6} u_{2j} = \sum_{j=1}^{5} h_{5j} = 0,$$

$$c_4 = c_6 = \sum_{j=1}^{5} u_{4j} = \sum_{j=1}^{6} h_{6j} = 1$$

$$c_3 = \sum_{j=1}^{5} u_{3j} = 2$$

6. The optimal decision is to select u_3 since $c_3 = max_{u_i \in U} c_i$.

Note that this decision making method can be applied for group decision making easily with help of the Definition 3.27 and Definition 3.28.

5 Conclusion

In this paper, the notion of the interval valued neutrosophic soft sets (*ivn*-soft sets) is defined which is a combination of an interval valued neutrosophic sets[35] and a soft sets[29]. Then, we introduce some definitions and operations of *ivn*-soft sets sets. Some properties of *ivn*-soft sets which are connected to operations have been established. Finally, we propose an adjustable approach by using level soft sets and illustrate this method with some concrete examples. This novel proposal proves to be feasible for some decision making problems involving *ivn*-soft sets. It can be applied to problems of many fields that contain uncertainty such as computer science, game theory, and so on.

References

1. Acar U, Koyuncu F, Tanay B (2010) Soft sets and soft rings. Comput Math Appl 59:3458–3463
2. Aktaş H, Çağman N (2007) Soft sets and soft groups. Inf Sci 177:2726–2735
3. Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
4. Awang MI, Rose ANM, Herawan T, Deris MM (2010) Soft set approach for selecting decision attribute in data clustering. In: Advanced data mining and applications lecture notes in computer science, vol 6441, pp 87–98
5. Aygünoglu A, Aygün H (2009) Introduction to fuzzy soft groups. Comput Math Appl 58:1279–1286
6. Broumi S (2013) Generalized neutrosophic soft set. Int J Comput Sci Eng Inf Technol (IJCSEIT) 3/2. doi:10.5121/ijcseit.2013.3202
7. Broumi S, Smarandache F (2013) Intuitionistic neutrosophic soft set. J Inf Comput Sci 8(2):130–140
8. Çağman, Karataş S, Enginoğlu S (2011) Soft topology. Comput Math Appl 62:351–358
9. Çağman N, Erdoğan F, Enginoğlu S (2011) FP-soft set theory and its applications. Ann Fuzzy Math Inf 2(2):219–226

10. Çağman N, Enginoğlu S (2010) Soft set theory and uni–int decision making. Eur J Oper Res 207:848–855
11. Çağman N, Deli I (2012) Means of FP-soft sets and its applications. Hacet J Math Stat 41(5):615–625
12. Çağman N, Erdoğan F, Enginoğlu S (2011) FP-soft set theory and its applications. Ann Fuzzy Math Inf 2(2):219–226
13. Çağman N, Enginoğlu S (2010) Soft set theory and uni–int decision making. Eur J Oper Res 207:848–855
14. Çağman N, Deli II (2013) Soft games. http://arxiv.org/abs/1302.4568
15. Feng F, Li C, Davvaz B, Irfan Ali M (2010) Soft sets combined with fuzzy sets and rough sets: a tentative approach. Soft Comput 14:899–911
16. Feng F, Jun YB, Liu X, Li L (2010) An adjustable approach to fuzzy soft sets based decision making. J Comput Appl Math 234:10–20
17. Jiang Y, Tang Y, Chen Q, Liu H, Tang J (2010) Interval-valued intuitionistic fuzzy soft sets and their properties. Comput Math Appl 60:906–918
18. Jiang Y, Tang Y, Chen Q (2011) An adjustable approach to intuitionistic fuzzy soft sets based decision making. Appl Math Model 35:824–836
19. Karaaslan F, Çağman N, Enginoğlu S (2012) Soft lattices. J New Results Sci 1:5–17
20. Kharal A (2010) Distance and similarity measures for soft sets. New Math Nat Comput 06:321. doi:10.1142/S1793005710001724
21. Kovkov DV, Kolbanov VM, Molodtsov DA (2007) Soft sets theory-based optimization. J Comput Syst Sci Int 46(6):872–880
22. Maji PK, Biswas R, Roy AR (2001) Fuzzy soft sets. J Fuzzy Math 9(3):589–602
23. Maji PK, Roy AR, Biswas R (2004) On intuitionistic fuzzy soft sets. J Fuzzy Math 12(3):669–683
24. Maji PK (2012) A neutrosophic soft set approach to a decision making problem. Ann Fuzzy Math Inf 3(2):313–319
25. Maji PK (2013) Neutrosophic soft set. Comput Math Appl 45:555–562
26. Maji PK, Roy AR (2002) An application of soft sets in a decision making problem. Comput Math Appl 44:1077–1083
27. Mamat R, Herawan T, Deris MM (2013) MAR: maximum attribute relative of soft set for clustering attribute selection. Knowl Based Syst 52:11–20
28. Min WK (2011) A note on soft topological spaces. Comput Math Appl 62:3524–3528
29. Molodtsov DA (1999) Soft set theory-first results. Comput Math Appl 37:19–31
30. Molodtsov DA (2004) The theory of soft sets (in Russian). URSS Publishers, Moscow
31. Nagarajan EKR, Meenambigai G (2011) An application of soft sets to lattices. Kragujev J Math 35(1):75–87
32. Pawlak Z (1982) Rough sets. Int J Inf Comput Sci 11:341–356
33. Smarandache F (2005) Neutrosophic set, a generalisation of the intuitionistic fuzzy sets. Int J Pure Appl Math 24:287–297
34. Shabir M, Naz M (2011) On soft topological spaces. Comput Math Appl 61:1786–1799
35. Wang H, Smarandache F, Zhang YQ, Sunderraman R (2005) Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. In: Neutrosophic book series, vol 5. Hexis, Arizona
36. Wang H, Smarandache F, Zhang YQ, Sunderraman R (2010) Single valued neutrosophic sets. Multispace Multistructure 4:410–413
37. Şerife Yılmaz, Kazancı O (2013) Soft lattices(ideals, filters) related to fuzzy point. In: U.P.B. Scientific Bulletin, Series A, 75/3, pp 75–90
38. Yüksel S, Dizman T, Yildizdan G, Sert U (2013) Application of soft sets to diagnose the prostate cancer risk. J Inequal Appl. doi:10.1186/1029-242X-2013-229
39. Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
40. Zhang Z, Wang C, Tian D, Li K (2014) A novel approach to interval-valued intuitionistic fuzzy soft set based decision making. Appl Math Model 38:1255–1270
41. Zhan J, Jun YB (2010) Soft BL-algebras based on fuzzy sets. Comput Math Appl 59:2037–2046