Int. J. Mach. Learn. & Cyber. (2016) 7:819–831
DOI 10.1007/s13042-015-0400-3

ORIGINAL ARTICLE

Dual hesitant fuzzy group decision making method and its application to supplier selection

Dejian Yu[1,2] · Deng-Feng Li[1] · José M. Merigó[3,4]

Received: 7 February 2015 / Accepted: 7 July 2015 / Published online: 8 August 2015

Abstract The concept of dual hesitant fuzzy set arising from hesitant fuzzy set is generalized by including a function reflecting the decision maker's fuzziness about the non-membership degree of the information provided. This paper studies some dual hesitant fuzzy information aggregation operators for aggregating dual hesitant fuzzy elements, such as dual hesitant fuzzy Heronian mean operator and dual hesitant fuzzy geometric Heronian mean operator. The research resulting dual hesitant fuzzy information aggregation operators finds an important role in group decision making (GDM) applications. It can fusion the experts' opinion to the comprehensive ones and based on which an optimal decision making scheme can be determined. The properties of the proposed operators are studied and the application on GDM are investigated. The effectiveness of the GDM method is demonstrated on the case study about supplier selection.

Keywords DHFS · Hesitant fuzzy set · GDM · Heronian mean · Supplier selection

✉ Deng-Feng Li
lidengfeng@fzu.edu.cn

Dejian Yu
yudejian62@126.com

1 School of Economics and Management, Fuzhou University, Fuzhou 350108, Fujian, China

2 School of Information, Zhejiang University of Finance and Economics, Hangzhou 310018, Zhejiang, China

3 Manchester Business School, University of Manchester, Booth Street West, Manchester M15 6PB, UK

4 Department of Management Control and Information Systems, University of Chile, Av. Diagonal Paraguay 257, 8330015 Santiago, Chile

1 Introduction

Intuitionistic fuzzy set (IFS) theory was first proposed by Atanassov [1] and it is a generalization of Fuzzy set (FS) [2]. Compared with the FS, the IFS have the additional parameters to control the non-membership degree and hesitant degree besides the membership degree function. The non-membership and hesitant parameters make IFS very useful in depicting the fuzziness in the real world and have successfully applied to pattern recognition [3–5], decision making [6–10] and so on.

Torra and Narukawa [11] has pointed out that both FS theory and IFS theory undergo the inherent disadvantages when define the membership degree of a given element to a predefined set. Besides, some other extended FS theory, such as type-2 FS [12, 13] and fuzzy multiset [14, 15] cannot handle this problem [11, 16]. The theory of hesitant fuzzy set (HFS) addresses these limitations [11, 16].

The HFS is based on strong ability on the description of the membership degree. For the membership functions, the HFS is more powerful in descripting the object than the IFS and some other extended FS. One can have some determined number in the description of non-membership degree in the HFS theory. This makes it convenient to describe a group of experts' opinion, especially when the experts are relatively independent. In this regard, the HFS theory has found many applications such as group decision making (GDM) [17–22].

Based on the extensive research of IFS and HFS, Zhu et al. [23] have combined the idea of IFS theory with HFS theory and introduced the concept of dual hesitant fuzzy set (DHFS). Similar to the IFS theory, DHFS also have membership degree function and non-membership degree function. However, these two functions are expressed by several determined numbers rather a single number and make the descriptions of the fuzziness of the real world more accurately than the other extended FS theory. Specifically, the DHFS is very helpful in GDM problem when the membership degree and non-membership degree functions are hard to be determined. Chen et al. [24] investigated the GDM method under the dual hesitant fuzzy environment. In their contribution, optimization models are adopted to determine the weight vector. Zhu and Xu [25] further extended the DHFS and developed the typical DHFS, some new operations and properties are also investigated. Farhadinia [26] studied the correlation coefficient for DHFS and applied it to medical diagnosis. Wang et al. [27] defined the correlation measures for DHFS and studied the clustering algorithm for DHFS. Ye [28] proposed the concept of weighted correlation coefficient and the predefined idea alternative and applied it investment management. Wang et al. [29] focused on the dual hesitant fuzzy information aggregation methods and proposed a series of aggregation operators, such as dual hesitant fuzzy weighted average (DHFWA) DHFWG, DHFOWA, DHFOWG, DHFHA and DHFHG operators.

It should be noted that the above aggregation operators are based on the assumption that the variables are independent of each other. In this paper, we further research the information fusion methods for dual hesitant fuzzy information which can descript the interrelations between the aggregated arguments quantitatively. To do this, the traditional Heronian mean (HM) [30] is applied to assist in this study. HM is a kind of average mean which can be used to describe the correlations of the aggregated arguments and it have been further researched in our previous works [31, 32]. The structure of HM is very similar to BM [33–37], however, the main difference and the advantage are studied in detail in our previous research results [32]. In this study, we first extended the HM to dual hesitant fuzzy environment and proposed some operators for aggregating dual hesitant fuzzy information, such as dual hesitant fuzzy HM (DHFHM) operator, dual hesitant fuzzy geometric HM (DHFGHM) operator and their weighted forms. Then, we investigate the GDM method based on the above operators. Finally, a real example about supplier selection is shown to illustrate the GDM method.

This paper will be organized as follows: In Sect. 2, we give an overview of the DHFS. We make effort on the extension of HM to DHFS in Sect. 3. In particular, we provide the definition of the DHFHM and DHFGHM aggregation operators and study their desirable properties, a GDM method and its application are presented in Sect. 4. Section 5 concludes this paper.

2 Some basic concepts

FS theory has become an indispensable part in decision making and some other human activities because of the need to express the associated ambiguity. A lot of extended FS theories have been forwarded to meet the growing demand for reality, such as multi-sets [15], vague sets [38], type-2 FSs [39], IFS [1], interval-valued IFS [40], HFS [11, 19] and DHFS [23]. In the past two years, DHFS has attracted the attention of many scholars. In this section, we will introduce the basic concept of DHFS to facilitate the follow-up in following sections.

Definition 1 [23] Let X be a given set. A DHFS D on the set X is expressed by the following equation.

$$D = \{\langle x, h(x), g(x) \rangle x \in X\} \tag{1}$$

In Eq. (1), $h(x)$ and $g(x)$ are two HFS, therefore, DHFS is the combination of two HFS. In the research contribution of Zhu et al. [23], the comparison laws of any two dual hesitant fuzzy elements (DHFEs) are also provided.

Definition 2 [23] Suppose $d = (h, g)$ be any two DHFEs. The scores function and accuracy function of d is defined as follows:

$$S(d) = \frac{1}{\#h}\sum\nolimits_{\gamma \in h} \gamma - \frac{1}{\#g}\sum\nolimits_{\eta \in g} \eta \tag{2}$$

$$p(d) = \frac{1}{\#h}\sum\nolimits_{\gamma \in h} \gamma + \frac{1}{\#g}\sum\nolimits_{\eta \in g} \eta \tag{3}$$

In many cases, the comparison can be processed based on Eq. (2). In other words, the larger the score of the DHFE, the larger DHFE.

Example 1 Let $d_1 = \{\{0.6, 0.7\}, \{0.1, 0.3\}\}, d_2 = \{\{0.2, 0.3, 0.4\}, \{0.6\}\}, d_3 = \{\{0.6, 0.7, 0.8\}, \{0.1\}\}$, be three DHFEs. According to the Definition 2, the following score functions can be obtained.

$$S(d_1) = \frac{1}{2}(0.6 + 0.7) - \frac{1}{2}(0.1 + 0.3) = 0.45$$

$$S(d_2) = \frac{1}{3}(0.2 + 0.3 + 0.4) - \frac{1}{1}(0.6) = -0.3$$

$$S(d_3) = \frac{1}{3}(0.6 + 0.7 + 0.8) - \frac{1}{1}(0.1) = 0.6$$

Since

$$S(d_3) > S(d_1) > S(d_2)$$

We have

$$d_3 \succ d_1 \succ d_2$$

Definition 3 Let d, d_1 and d_2 be three DHFEs, Then

(1) $d_1 \oplus d_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \{\{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}, \{\eta_1\eta_2\}\}$

(2) $d_1 \otimes d_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \eta_1 \in g_1, \eta_2 \in g_2} \{\{\gamma_1\gamma_2\}, \{\eta_1 + \eta_2 - \eta_1\eta_2\}\}$

(3) $nd = \cup_{\gamma \in h, \eta \in g}\{\{1-(1-\gamma)^n\}, \{\eta^n\}\}$

(4) $d^n = \cup_{\gamma \in h, \eta \in g}\{\{\gamma^n\}, \{1-(1-\eta)^n\}\}$

3 Dual hesitant fuzzy information aggregation with HM techniques

Based on the above operations given in Definition 3 [23], some aggregation operators for DHFE have been proposed, such as DHFWA operator, DHFWG operator, DHFOWA operator, DHFOWG operator, DHFHA operator and DHFHG operator [29]. However, the above aggregation operators can only deal with the situation that the aggregated arguments are independent. As everything in the real world is related to and interacted with each other. Investigate the correlated dual hesitant fuzzy information fusion method has important meaning to the development of DHFS theory and information fusion theory. HM is an important technique to deal with this situation and the definition is given as follows [30–32].

Definition 4 Suppose there are two parameters $p,\ q > 0$, and $a_i (i = 1, 2, \ldots, n)$ be a group of non-negative real number. If

$$\mathrm{HM}^{p,q}(a_1, a_2, \ldots, a_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} a_i^p a_j^q\right)^{1/p+q} \quad (4)$$

then HM is called the Heronian mean.

Example 2 Let $a_1 = 2, a_2 = 3, a_3 = 4$ be three non-negative real number, based on the HM operator given above, we can have the following aggregated result (take $p = 3, q = 5$ for example).

$$\mathrm{HM}(2,3,4) = \left(\frac{2}{3(3+1)}(2^3 \times 2^5 + 2^3 \times 3^5 + 2^3 \times 4^5 + 3^3 \times 3^5 + 3^3 \times 4^5 + 4^3 \times 4^5)\right)^{\frac{1}{8}} = 3.41$$

Existing HM operator can be used to aggregate the real number [30], IFS [31] and Interval-valued IFS [32]. In this paper, we mainly focus on the HM operator under DHFS environment.

Definition 5 Let $d_j = (h_j, g_j)(j = 1, 2, \ldots, n)$ be a group of DHFEs. If

$$\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) = \left(\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} d_i^p \otimes d_j^q\right)^{1/p+q} \quad (5)$$

then DHFHM is called the dual hesitant fuzzy Heronian mean operator (DHFHM).

Theorem 1 *Let* $p, q > 0$, *and* $d_j = (h_j, g_j)(j = 1, 2, \ldots, n)$ *be a group of DHFEs, based on the* DHFHM *operator, the aggregated DHFE was given as follows.*

$$\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\left\{\left\{\left(1 - \prod_{i=1, j=i}^{n}\left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}, \left\{1 - \left(1 - \prod_{i=1, j=i}^{n}\left(1-(1-\eta_i)^p(1-\eta_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}\right\} \quad (6)$$

Proof According to the operations defined in Definition 2, we know,

$$d_i^p = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\xi_i^p\}, \{1-(1-\eta_i)^p\}\}, \quad d_j^q = \cup_{\xi_j \in h_j, \eta_j \in g_j}\left\{\left\{\xi_j^q\right\}, \left\{1-(1-\eta_j)^q\right\}\right\}, \quad (7)$$

and

$$d_i^p \otimes d_j^q = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\left\{\left\{\xi_i^p \xi_j^p\right\}, \left\{1-(1-\eta_i)^p(1-\eta_j)^q\right\}\right\} \quad (8)$$

Then

$$\sum_{i=1}^{n}\sum_{j=i}^{n} d_i^p \otimes d_j^q = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\left\{\left\{1 - \prod_{i=1, j=i}^{n}\left(1 - \xi_i^p \xi_j^q\right)\right\}, \left\{\prod_{i=1, j=i}^{n}\left(1-(1-\eta_i)^p(1-\eta_j)^q\right)^{\frac{2}{n(n+1)}}\right\}\right\} \quad (9)$$

and

$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n} h_i^p \otimes h_j^q$$

$$= \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ 1 - \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}} \right\}, \left\{ \prod_{i=1, j=i}^{n} \left(1 - (1-\eta_i)^p (1-\eta_j)^q\right)^{\frac{2}{n(n+1)}} \right\} \right\} \quad (10)$$

Therefore, we have

$$\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\}, \left\{ 1 - \left(1 - \prod_{i=1, j=i}^{n} \left(1 - (1-\eta_i)^p (1-\eta_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \right\} \quad (11)$$

which completes the proof of Theorem 1.The DHFHM operator has excellent properties such as idempotency, monotonicity, permutation and boundary. □

Property 1 (Idempotency) *If all DHFEs $d_i (i = 1, 2, \ldots, n)$ are equal, i.e., $d_i = d = \cup_{\xi \in h, \eta \in g} \{\{\xi\}, \{\eta\}\}$ for all i, $p > 0$, $q > 0$. Then*

$$\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) = \mathrm{DHFHM}(d, d, \ldots, d) = d \quad (12)$$

Proof Based on known information, we have

$$\begin{aligned} &\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) \\ &= \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{ \left\{ \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\}, \left\{ 1 - \left(1 - \prod_{i=1, j=i}^{n} \left(1 - (1-\eta_i)^p (1-\eta_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \right\} \\ &= \cup_{\xi \in h, \eta \in g} \left\{ \left\{ \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \xi^p \xi^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\}, \left\{ 1 - \left(1 - \prod_{i=1, j=i}^{n} \left(1 - (1-\eta)^p (1-\eta)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \right\} \\ &= \cup_{\xi \in h, \eta \in g} \left\{ \left\{ \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \xi^{p+q}\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\}, \left\{ 1 - \left((1-\eta)^{p+q}\right)^{\frac{1}{p+q}} \right\} \right\} = \cup_{\xi \in h, \eta \in g} \{\{\xi\}, \{\eta\}\} = d \end{aligned}$$

which completes the proof. □

Property 2 (Monotonicity) *If $d_i = \cup_{\xi_i \in h_i, \eta_i \in g_i} \{\{\xi_i\}, \{\eta_i\}\} (i = 1, 2, \ldots, n)$ and $k_i = \cup_{\alpha_i \in s_i, \beta_i \in t_i} \{\{\alpha_i\}, \{\beta_i\}\} (i = 1, 2, \ldots, n)$ be two sets of DHFEs, if $\xi_i \le \alpha_i$ and $\eta_i \ge \beta_i$ for all i, then*

$$\mathrm{DHFHM}(h_1, h_2, \ldots, h_n) \le \mathrm{DHFHM}(k_1, k_2, \ldots, k_n) \quad (13)$$

Proof On the one hand, Since $\xi_i \le \alpha_i$ for all i, then $\xi_i^p \xi_j^q \le \alpha_i^p \alpha_j^q$

$$\begin{aligned} &\Rightarrow \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}} \ge \prod_{i=1, j=i}^{n} \left(1 - \alpha_i^p \alpha_j^q\right)^{\frac{2}{n(n+1)}} \\ &\Rightarrow 1 - \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}} \le 1 - \prod_{i=1, j=i}^{n} \left(1 - \alpha_i^p \alpha_j^q\right)^{\frac{2}{n(n+1)}} \\ &\Rightarrow \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \le \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \alpha_i^p \alpha_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \\ &\Rightarrow \cup_{\xi_i \in d_i, \xi_j \in d_j} \left\{ \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \xi_i^p \xi_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \le \cup_{\alpha_i \in k_i, \alpha_j \in k_j} \left\{ \left(1 - \prod_{i=1, j=i}^{n} \left(1 - \alpha_i^p \alpha_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \end{aligned} \quad (14)$$

On the other hand, put the same thought into the proof of non-membership degree, we can easily get.

$$\begin{aligned} &\cup_{\eta_i \in g_i, \eta_j \in g_j} \left\{ 1 - \left(1 - \prod_{i=1, j=i}^{n} \left(1 - (1-\eta_i)^p (1-\eta_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \\ &\ge \cup_{\beta_i \in t_i, \beta_j \in t_j} \left\{ 1 - \left(1 - \prod_{i=1, j=i}^{n} \left(1 - (1-\beta_i)^p (1-\beta_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}} \right\} \end{aligned} \quad (15)$$

United Eqs. (14) with (15), we have

$$\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) \le \mathrm{DHFHM}(k_1, k_2, \ldots, k_n) \quad (16)$$

which completes the proof. □

Property 3 (Permutation) *If $d_i = \cup_{\xi_i \in h_i, \eta_i \in g_i} \{\{\xi_i\}, \{\eta_i\}\} (i = 1, 2, \ldots, n)$ be a set of DHFEs. Then*

$$\mathrm{DHFHM}(d_1, d_2, \ldots, d_n) = \mathrm{DHFHM}(\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_n) \quad (17)$$

where $(\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_n)$ is any permutation of $(d_1, d_2, \ldots, d_n)$.

Proof According to the operations of DHFE, we can get DHFHM

$$\begin{aligned}(d_1, d_2, \ldots, d_n) &= \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n} d_i^p \otimes d_j^q\right)^{1/p+q} \\ &= \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n} \dot{d}_i^p \otimes \dot{d}_j^q\right)^{1/p+q} \\ &= \mathrm{DHFHM}(\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_n)\end{aligned}$$

which completes the proof. □

Property 4 (Boundary) *If* $d_i = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\xi_i\}, \{\eta_i\}\}(i = 1, 2, \ldots, n)$ *be a set of DHFEs, and*

$$d^- = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\min(\xi_i)\}, \{\max(\eta_i)\}\}$$

$$d^+ = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\max(\xi_i)\}, \{\min(\eta_i)\}\} \quad (18)$$

then

$$d^- \leq \mathrm{DHFHM}(h_1, h_2, \ldots, h_n) \leq d^+ \quad (19)$$

Proof According to the proof of properties 1–3, the property 4 can be proved easily. We do not repeat here.

Example 1 Let $d_1 = \{\{0.3, 0.4, 0.5\}, \{0.2, 0.3\}\}, h_2 = \{\{0.7, 0.8\}, \{0.2\}\}, h_3 = \{\{0.6, 0.7\}, \{0.1, 0.2, 0.3\}\}$ be three DHFEs, then by Definitions of DHFHM operator and Theorem 1 described above, when the values of the parameters be set the determined numbers, the final aggregated DHFEs can be obtained, for example,

When $p = 0.5, q = 0.5$

$$\begin{aligned}\mathrm{DHFHM}(d_1, d_2, d_3) = \{&\{0.5421, 0.5793, 0.5858, 0.6219, \\ &0.5727, 0.6087, 0.6148, 0.6495, 0.6033, 0.6380, 0.6438, 0.6771\}, \\ &\{0.1624, 0.2000, 0.2310, 0.1901, 0.2310, 0.2644\}\}\end{aligned}$$

the score of the aggregated DHFE is 0.3982.

When $p = 1, q = 10$,

$$\begin{aligned}\mathrm{DHFHM}(d_1, d_2, d_3) = \{&\{0.6313, 0.6726, 0.7065, 0.7238, \\ &0.6363, 0.6770, 0.7120, 0.7289, 0.6415, 0.6814, 0.7172, 0.7338\}, \\ &\{0.1382, 0.2000, 0.2326, 0.1455, 0.2131, 0.2521\}\}\end{aligned}$$

the score of the aggregated DHFE is 0.4916.

When $p = 10, q = 10$,

$$\begin{aligned}\mathrm{DHFHM}(d_1, d_2, d_3) = \{&\{0.6475, 0.6762, 0.7337, 0.7421, \\ &0.6476, 0.6762, 0.7338, 0.7422, 0.6486, 0.6769, 0.7341, 0.7425\}, \\ &\{0.1494, 0.2000, 0.2202, 0.1585, 0.2202, 0.2481\}\}\end{aligned}$$

the score of the aggregated DHFE is 0.5007. □

From the definition of the DHFHM operator, we can find out that there are two parameters (p, q) to control the aggregated DHFE simultaneously. When one of these two parameters is set, the scores trends can be shown.

Case 1 Take $p = 1(q \in (0, 10])$ or $q = 1(p \in (0, 10])$ for example, scores trends were shown in Fig. 1.

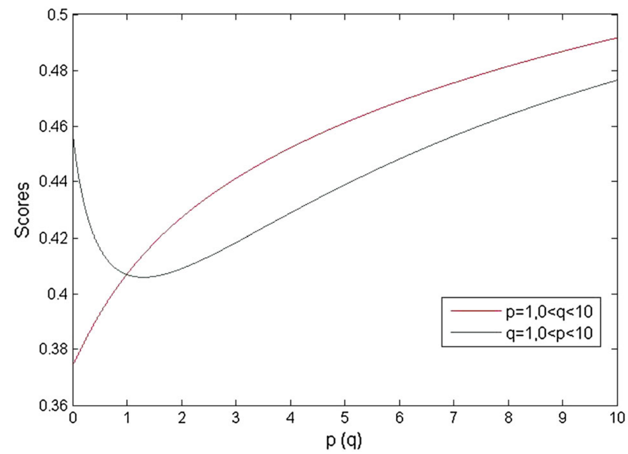

Fig. 1 Scores trends $p = 1$ and $q = 1$

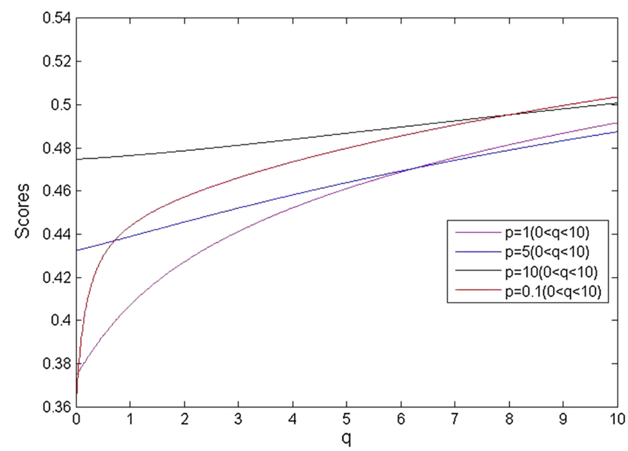

Fig. 2 Scores trends $p = 0.01, 1, 5, 10$

Case 2 Take $p = 0.1, 1, 5, 10(q \in (0, 10])$ for example, scores trends was shown in Fig. 2.

Case 3 Take $q = 0.1, 1, 5, 10(p \in (0, 10])$ for example, scores trends was shown in Fig. 3.

The effects of simultaneous varieties of p and q on score functions were analyzed on Fig. 4.

The DHFHM operator is one kind of average mean operator and it is the extension of some existing operators such as HFWA operator, HFOWA operator [18], DHFWA operator and DHFOWA operator [29]. Another kind of mean operator is geometric mean and it has been extended to fuzzy environment [41], IFS environment [42–45], interval-valued IFS environment [46, 47]. In the following, we study the geometric mean under dual hesitant fuzzy environment and introduce the dual hesitant fuzzy geometric Heronian mean (DHFGHM) operator.

Definition 6 Let $d_j = (h_j, g_j)(j = 1, 2, \ldots, n)$ be a collection of DHFEs. If

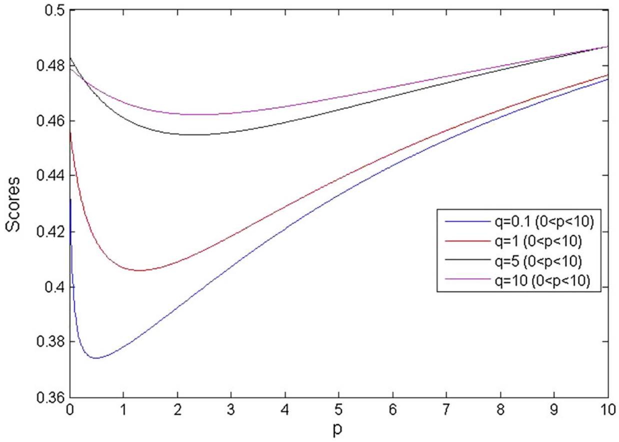

Fig. 3 Scores trends $q = 0.01, 1, 5, 10$

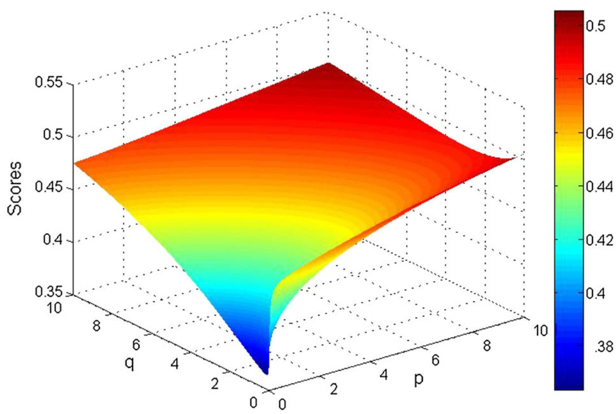

Fig. 4 Scores for DHFEs obtained by the DHFHM operator ($p \in (0, 10]$, $q \in (0, 10]$)

$$\mathrm{DHFGHM}(d_1, d_2, \ldots, d_n) = \frac{1}{p+q}\left(\underset{i=1,j=i}{\overset{n}{\otimes}}(pd_i \oplus qd_j)^{2/n(n+1)}\right) \tag{20}$$

then DHFGHM is called the dual hesitant fuzzy geometric HM operator (DHFGHM).

Theorem 2 *Let* $p, q > 0$, *and* $d_j = (h_j, g_j)$ $(j = 1, 2, \ldots, n)$ *be a group of DHFEs, based on the* DHFGHM *operator, the aggregated DHFE was given as follows.*

$$\mathrm{DHFGHM}(d_1, d_2, \ldots, d_n) = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \left\{\left\{1 - \left(1 - \prod_{i=1,j=i}^{n}\left(1 - (1-\xi_i)^p(1-\xi_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}, \left\{\left(1 - \prod_{i=1,j=i}^{n}\left(1 - \eta_i^p \eta_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}\right\} \tag{21}$$

Proof According to the operations defined in Definition 2, we know,

$$pd_i = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{1 - (1-\xi_i)^p\}, \{\eta_i^p\}\} \tag{22}$$

$$qd_j = \cup_{\xi_j \in h_j, \eta_j \in g_j}\left\{\{1 - (1-\xi_j)^q\}, \left\{\eta_j^q\right\}\right\} \tag{23}$$

and

$$pd_i \oplus qd_j = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\left\{\{1 - (1-\xi_i)^p(1-\xi_j)^q\}, \left\{\eta_i^p \eta_j^p\right\}\right\} \tag{24}$$

Then

$$\underset{i=1,j=i}{\overset{n}{\otimes}}(pd_i \oplus qd_j) = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\left\{\left\{\prod_{i=1,j=i}^{n}\left(1 - (1-\xi_i)^p(1-\xi_j)^q\right)^{\frac{2}{n(n+1)}}\right\}, \left\{1 - \prod_{i=1,j=i}^{n}\left(1 - \eta_i^p \eta_j^q\right)\right\}\right\} \tag{25}$$

and

$$\underset{i=1,j=i}{\overset{n}{\otimes}}(pd_i \oplus qd_j)^{2/n(n+1)} = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\left\{\left\{\prod_{i=1,j=i}^{n}\left(1 - (1-\xi_i)^p(1-\xi_j)^q\right)^{\frac{2}{n(n+1)}}\right\}, \left\{1 - \prod_{i=1,j=i}^{n}\left(1 - \eta_i^p \eta_j^q\right)^{\frac{2}{n(n+1)}}\right\}\right\} \tag{26}$$

Therefore, we have

$$\mathrm{DHFGHM}(d_1, d_2, \ldots, d_n) = \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j} \times \left\{\begin{array}{l}\left\{1 - \left(1 - \prod_{i=1,j=i}^{n}\left(1 - (1-\xi_i)^p(1-\xi_j)^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}, \\ \left\{\left(1 - \prod_{i=1,j=i}^{n}\left(1 - \eta_i^p \eta_j^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}\end{array}\right\} \tag{27}$$

which completes the proof of Theorem 2.

Similar to the DHFHM operator, the DHFGHM operator also has some excellent properties such as idempotency, monotonicity, permutation and boundary. □

Property 5 (Idempotency) *If all DHFEs* $d_i(i = 1, 2, \ldots, n)$ *are equal, i.e.,* $d_i = d = \cup_{\xi \in h, \eta \in g}\{\{\xi\}, \{\eta\}\}$, *for all* i, $p > 0$, $q > 0$. *Then*

$$\mathrm{DHFGHM}(d_1, d_2, \ldots, d_n) = \mathrm{DHFGHM}(d, d, \ldots, d) = d \tag{28}$$

Property 6 (Monotonicity) *If* $d_i = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\xi_i\}, \{\eta_i\}\}(i = 1, 2, \ldots, n)$ *and* $k_i = \cup_{\alpha_i \in s_i, \beta_i \in t_i}\{\{\alpha_i\}, \{\beta_i\}\}(i = 1, 2, \ldots, n)$ *be two sets of DHFEs, if* $\xi_i \le \alpha_i$ *and* $\eta_i \le \beta_i$ *for all* i*, then*

$$\mathrm{DHFGHM}(h_1, h_2, \ldots, h_n) \le \mathrm{DHFGHM}(k_1, k_2, \ldots, k_n) \tag{29}$$

Property 7 (Permutation) *If* $d_i = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\xi_i\}, \{\eta_i\}\}(i = 1, 2, \ldots, n)$ *be a set of DHFEs. Then*

$$\mathrm{DHFGHM}(d_1, d_2, \ldots, d_n) = \mathrm{DHFGHM}(\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_n) \tag{30}$$

where $(\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_n)$ *is any permutation of* $(d_1, d_2, \ldots, d_n)$.

Property 8 (Boundary) *If* $d_i = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\xi_i\}, \{\eta_i\}\}(i = 1, 2, \ldots, n)$ *be a set of DHFEs, and*

$$d^- = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\min(\xi_i)\}, \{\max(\eta_i)\}\}, d^+ = \cup_{\xi_i \in h_i, \eta_i \in g_i}\{\{\max(\xi_i)\}, \{\min(\eta_i)\}\}$$

then

$$d^- \le \mathrm{DHFGHM}(h_1, h_2, \ldots, h_n) \le d^+ \tag{31}$$

Example 2 Let $d_1 = \{\{0.12, 0.34\}, \{0.51, 0.65\}\}, h_2 = \{\{0.56, 0.61, 0.74\}, \{0.21\}\}, h_3 = \{\{0.37\}, \{0.46, 0.57, 0.62\}\}$ be three DHFEs, then by Definitions of DHFGHM operator and Theorem 2 described above, when the parameters be set the determined numbers, the final aggregated DHFEs can be obtained, for example,When $p = 0.5, q = 0.5\mathrm{DHFGHM}(d_1, d_2, d_3)\mathrm{DHFHM}(d_1, d_2, d_3)$ the score of the aggregated DHFE is -0.0571.When $p = 1, q = 10$, $\mathrm{DHFHM}(d_1, d_2, d_3) = \{\{0.2400, 0.2401, 0.2404, 0.3867, 0.3881, 0.3910\}, \{0.4577, 0.5275, 0.5678, 0.5556, 0.5762, 0.5986\}\}$ the score of the aggregated DHFE is -0.2329.

When $p = 10, q = 10, \mathrm{DHFHM}(d_1, d_2, d_3) = \{\{0.1926, 0.1927, 0.1927, 0.3745, 0.3748, 0.3749\}, \{0.4756, 0.5307, 0.5711, 0.5953, 0.6031, 0.6154\}\}$ the score of the aggregated DHFE is -0.2815.

The definition of DHFGHM operator shows that there are two parameters (p, q) control the aggregated DHFE simultaneously. When one of these two parameters is set, the scores trends can be described in detail.

Case 1 Take $p = 1(q \in (0, 10])$ or $q = 1(p \in (0, 10])$ for example, scores trends were shown in Fig. 5.

Case 2 Take $p = 0.01, 1, 5, 10(q \in (0, 10])$ for example, scores trends was shown in Fig. 6.

Case 3 Take $q = 0.01, 1, 5, 10(p \in (0, 10])$ for example, scores trends was shown in Fig. 7.

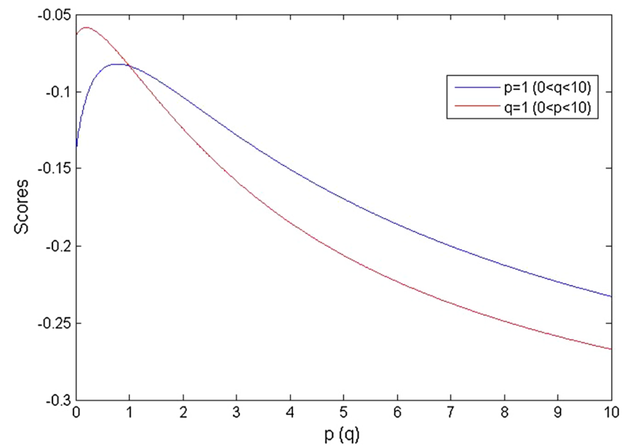

Fig. 5 Scores trends $p = 1$ and $q = 1$

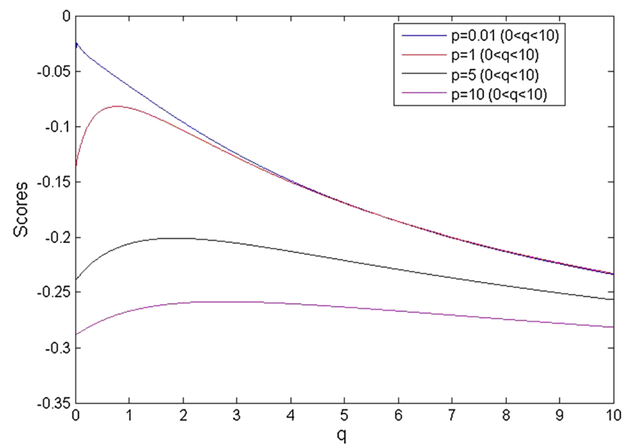

Fig. 6 Scores trends $p = 0.01, 1, 5, 10$

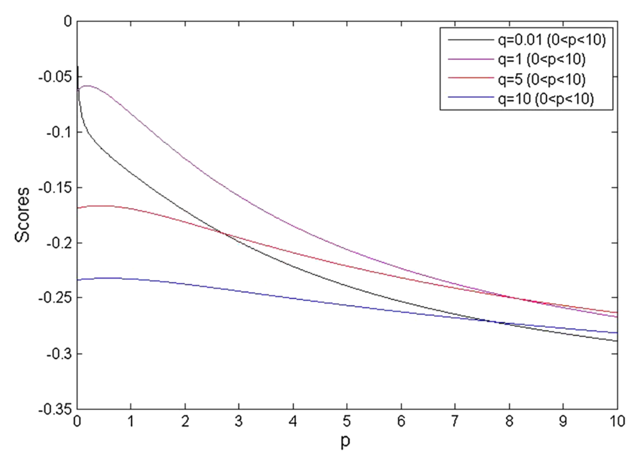

Fig. 7 Scores trends $q = 0.01, 1, 5, 10$

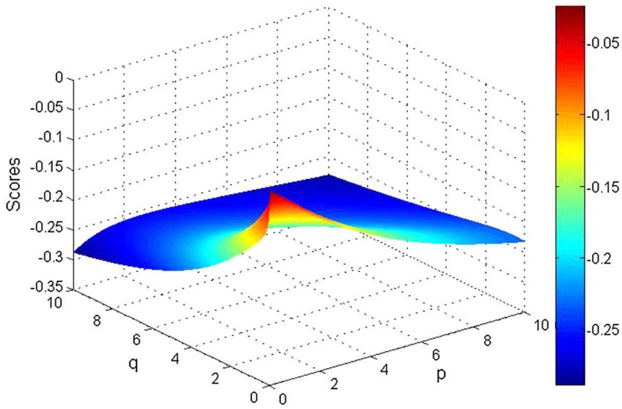

Fig. 8 Scores for DHFEs obtained by the DHFGHM operator ($p \in (0, 10]$, $q \in (0, 10]$)

Influences of simultaneous varieties of p and q on score functions were analyzed on Fig. 8.

4 GDM under dual hesitant fuzzy environment

Fuzziness, uncertainty and insufficient information are almost inevitable in all GDM and classification problems [48–50]. The GDM problem under dual hesitant fuzzy environment is chiefly researched in this section. In Sect. 4.1, we propose the weighted DHFHM and weighted DHFGHM operators. Algorithm for GDM using dual hesitant fuzzy information is investigated in Sect. 4.2. In Sect. 4.3, a real example of supplier selection presented to illustrate the proposed method.

4.1 Weighted DHFHM and weighted DHFGHM operators

The effects of the importance of aggregated arguments are not taken into account by DHFHM and DHFGHM operators. In the following, we introduced the weighted DHFHM and weighted DHFGHM operators.

Definition 7 Let $d_j = (h_j, g_j)(j = 1, 2, \ldots, n)$ be a collection of DHFEs whose weight vector is $w = (w_1, w_2, \ldots, w_n)^T$, which satisfies $w_i > 0$, $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i = 1$. If

$$\mathrm{DHFWHM}(h_1, h_2, \ldots, h_n) = \left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}(w_i d_i)^p \otimes (w_j d_j)^p\right)^{1/p+q} \quad (32)$$

$$\mathrm{DHFWGHM}(h_1, h_2, \ldots, h_n) = \frac{1}{p+q}\left(\bigotimes_{i=1,j=i}^{n}\left((pd_i)^{w_i} \oplus (qd_j)^{w_j}\right)^{2/n(n+1)}\right) \quad (33)$$

Then DHFWHM is called the dual hesitant fuzzy weighted Heronian mean operator (DHFWHM) and DHFWGHM is called the dual hesitant fuzzy weighted geometric Heronian mean operator (DHFWGHM).

Theorem 3 *Let* $p, q > 0$, *and* $d_j = (h_j, g_j)(j = 1, 2, \ldots, n)$ *be a collection of DHFEs whose weight vector is* $w = (w_1, w_2, \ldots, w_n)^T$, *which satisfies* $w_i > 0$, $i = 1, 2, \ldots, n$ *and* $\sum_{i=1}^{n} w_i = 1$. *then the aggregated value based on the DHFWHM or DHFWGHM is*

$$\begin{aligned}&\mathrm{DHFWHM}(d_1, d_2, \ldots, d_n)\\&= \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\\&\left\{\left\{\left(1 - \prod_{i=1,j=i}^{n}\left(1-(1-(1-\xi_i)^{w_i})^p(1-(1-\xi_j)^{w_j})^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\},\right.\\&\left.\left\{1-\left(1-\prod_{i=1,j=i}^{n}\left(1-(1-\eta_i^{w_i})^p(1-\eta_j^{w_j})^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}\right\}\end{aligned} \quad (34)$$

$$\begin{aligned}&\mathrm{DHFWHM}(d_1, d_2, \ldots, d_n)\\&= \cup_{\xi_i \in h_i, \xi_j \in h_j, \eta_i \in g_i, \eta_j \in g_j}\\&\left\{\left\{1-\left(1-\prod_{i=1,j=i}^{n}\left(1-(1-\xi_i^{w_i})^p(1-\xi_j^{w_j})^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\},\right.\\&\left.\left\{\left(1-\prod_{i=1,j=i}^{n}\left(1-(1-(1-\eta_i)^{w_i})^p(1-(1-\eta_j)^{w_j})^q\right)^{\frac{2}{n(n+1)}}\right)^{\frac{1}{p+q}}\right\}\right\}\end{aligned} \quad (35)$$

The proof of Theorem 3 is very similar to Theorems 1 and 2 and not to repeat them.

4.2 Dual hesitant fuzzy GDM method

Consider a multi-criteria GDM problem with a group of experts $(e_1, e_2, \ldots, e_p)$, a set of alternatives $(a_1, a_2, \ldots, a_m)$ and a collection of criteria $(c_1, c_2, \ldots, c_m)$. The experts $(e_1, e_2, \ldots, e_p)$ are responsible for the evaluation of each alternative against each criterion. The evaluation results of alternative a_i with the criteria c_j is presented by a DHFE $d_{ij} = \{\{h_{ij}\}, \{g_{ij}\}\}$. The decision matrix is illustrated as follows.

The decision making method consists of the following steps.

Step 1. Access to decision-making information based on the experts' evaluation and expressed by DHFE.

Step 2. Aggregate all the dual hesitant fuzzy values $d_{ij}(j = 1, 2, \ldots, n)$ with respect of alternative $a_i(i = 1, 2, \ldots, m)$, and obtain the comprehensive evaluation result DHFE d_i based on the DHFWHM or DHFWGHM operators.

Step 3. Rank the DHFEs $d_i (i = 1, 2, \ldots, m)$ and choice the most appropriate one.

4.3 Supplier selection

Supplier selection is an indispensable part of supply chain and it is the activity to identify the most suitable supplier. In addition, supplier selection problem is difficult due to the following two reasons, (1) the evaluation criteria are correlated and conflicted; (2) the performance of the evaluation criteria is hard to be characterized quantitatively. However, the GDM method proposed in Sect. 4.2 can solve the above two difficulties effectively. The main reasons are (1) the aggregation operators such as DHFWHM and DHFWGHM are characterized by the ability to cope with the decision making problem which the attribute are correlated; (2) DHFS is a powerful technique to describe the vagueness of the performance of the evaluation criteria (Table 1).

Based on the existing research contributions [51–53], the following four criteria are selected to assess the performance of the alternatives and their weight are supposed as $(0.17, 0.32, 0.38, 0.13)^T$. (1) Relationship closeness c_1; (2) Product quality c_2; (3) Price competitiveness c_3; (4) Delivery performance c_4. Suppose there are four alternatives (a_1, a_2, a_3, a_4) need to be evaluated and three experts (e_1, e_2, e_3) responsible for this evaluations. In order to access to decision-making information, the review table (Table 2) is designed and sent to three experts. There experts are requested to fill in Table 2 with the numbers between 0 and 1. The decision making information can be obtained after the three experts expressed their preferences.

Take the performance of alternative a_1 against criterion c_1 for example, The DHFE {{0.3, 0.4},{0.6}} means (1) two of the three experts believe that the degree of alternative a_1 meet criteria c_1 is 0.3, and the remaining one expert think the degree is 0.4; (2) three experts all account the degree of alternative a_1 does not meet criteria c_1 is 0.6. According to the above analysis, the final evaluation result of the alternative a_1 against criterion c_1 can be expressed as a DHFE {{0.3, 0.4}, {0.6}}. Similarly, other DHFEs in Table 3 indicated the corresponding evaluation information.

Based on the decision making information described in Table 3, we first use the DHFWHM operator to obtain the comprehensive evaluation dual hesitant fuzzy information for every alternatives. Due to the relatively large numbers, we just list the score functions of the DHFEs.

From the Table 4, we find that different values of the parameters could lead to different ranking of the alternatives. For example, when $p = 0.01, q = 10$, the best alternative is a_3 and this result would be changed when parameters p and q be set the different values. When the value of parameter p fixed, the trends of the scores of four alternatives with different values of parameter q are presented in Fig. 9 $(p = 0.01)$ and Fig. 10 $(p = 10)$ respectively.

From Table 4 and Figs. 9 and 10, we find that different score functions can be obtained when p and q are set

Table 1 The decision matrix

	C_1	C_2	…	C_n
a_1	d_{11}	d_{12}	…	d_{1n}
a_2	d_{21}	h_{22}	…	d_{2n}
…	…	…	…	…
a_m	d_{m1}	h_{m2}	…	d_{mn}

Table 2 Review table for experts

No.	Criteria	Weights	Evaluations	Evaluation results
1	Relationship closeness (c_1)	0.17	The degree that the alternative a_i satisfies the criteria c_1	a_1 a_2 a_3 a_4
			The degree that the alternative a_i does not satisfy the criteria c_1	a_1 a_2 a_3 a_4
2	Product quality (c_2)	0.32	The degree that the alternative a_i satisfies the criteria c_2	a_1 a_2 a_3 a_4
			The degree that the alternative a_i does not satisfy the criteria c_2	a_1 a_2 a_3 a_4
3	Price competitiveness (c_3)	0.38	The degree that the alternative a_i satisfies the criteria c_3	a_1 a_2 a_3 a_4
			The degree that the alternative a_i does not satisfy the criteria c_3	a_1 a_2 a_3 a_4
4	Delivery performance (c_4)	0.13	The degree that the alternative a_i satisfies the criteria c_4	a_1 a_2 a_3 a_4
			The degree that the alternative a_i does not satisfy the criteria c_4	a_1 a_2 a_3 a_4

Table 3 Dual hesitant fuzzy decision matrix

	C_1	C_2	C_3	C_4
a_1	{{0.3,0.4},{0.6}}	{{0.7,0.9},{0.1}}	{{0.4},{0.2,0.3}}	{{0.5,0.6},{0.2}}
a_2	{{0.2,0.3},{0.5}}	{{0.6, 0.7},{0.2}}	{{0.7,0.8},{0.2}}	{{0.6},{0.1,0.2,0.3}}
a_3	{{0.4},{0.2,0.3}}	{{0.2,0.3,0.4},{0.6}}	{{0.7,0.8},{0.1}}	{{0.7},{0.2,0.3}}
a_4	{{0.6,0.7},{0.3}}	{{0.5},{0.4}}	{{0.3,0.4},{0.5}}	{{0.4, 0.6},{0.1,0.2}}

Table 4 Scores obtained by the DHFWHM operator and the rankings of alternatives

	Operators	a_1	a_2	a_3	a_4	*Ranking*
$p = 0.01, q = 10$	DHFWHM	−0.1899	−0.2207	−0.1180	−0.5871	$a_3 \succ a_1 \succ a_2 \succ a_4$
$p = q = 2$	DHFWHM	−0.3813	−0.3916	−0.3960	−0.6147	$a_1 \succ a_2 \succ a_3 \succ a_4$
$p = q = 5$	DHFWHM	−0.2344	−0.2785	−0.2084	−0.5963	$a_3 \succ a_1 \succ a_2 \succ a_4$
$p = 10, q = 0.01$	DHFWHM	−0.1609	−0.2427	−0.1528	−0.5825	$a_3 \succ a_1 \succ a_2 \succ a_4$
$p = 10, q = 10$	DHFWHM	−0.1567	−0.2171	−0.1128	−0.5797	$a_3 \succ a_1 \succ a_2 \succ a_4$

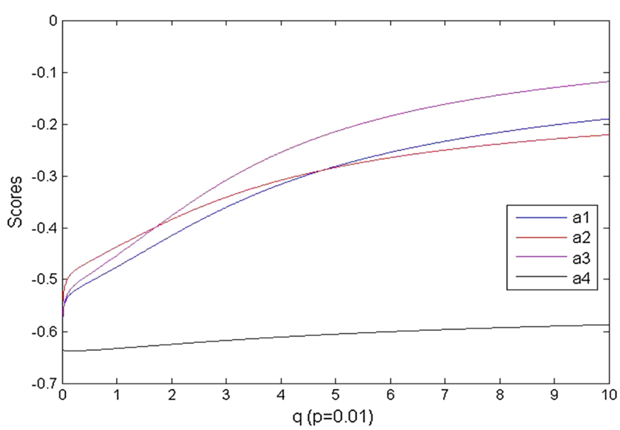

Fig. 9 Change trends of the scores of four alternatives ($q \in (0, 10],\ p = 0.01$)

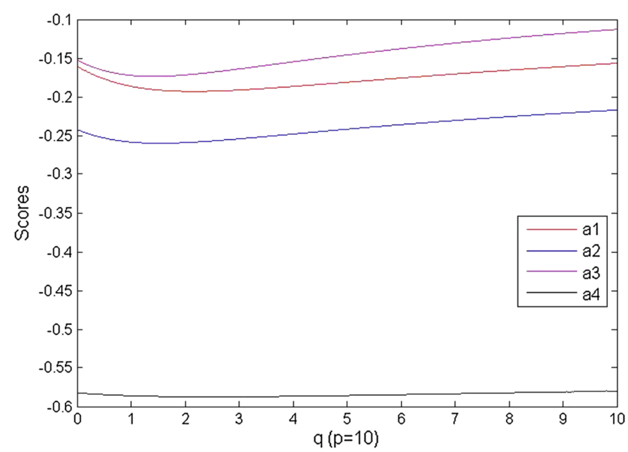

Fig. 10 Change trends of the scores of four alternatives ($q \in (0, 10],\ p = 10$)

different values. Figures 11, 12, 13, 14 show the scores for the four alternatives when the values of parameters p and q change simultaneously.

4.3.1 Comparative study

The above ranking results and the analysis about the scores for every alternative are based on the DHFWHM operator. In order to show the advantages and rationality of the method proposed in the above Section, we compare it with DHFWA operator proposed by Wang et al. [29]. The DHFWA operator is defined by Wang et al. as follows. Let $d_j = \left(h_j, g_j\right)(j = 1, 2, \ldots, n)$ be a collection of DHFEs. A dual hesitant fuzzy weighted averaging (DHFWA) operator is a mapping $D^n \to D$ such that

$$\begin{aligned} \mathrm{DHFWA}(d_1, d_2, \ldots, d_n) &= \overset{n}{\underset{j=1}{\oplus}}\left(\omega_j d_j\right) \\ &= \omega_1 d_1 \oplus \omega_2 d_2 \oplus \cdots \oplus \omega_n d_n \end{aligned}$$

$$= \cup_{\gamma_j \in h_j, \eta_j \in g_j} \left\{ \left\{ 1 - \prod_{j=1}^{n} \left(1 - \gamma_j\right)^{\omega_j} \right\}, \left\{ \prod_{j=1}^{n} \eta_j^{\omega_j} \right\} \right\} \quad (36)$$

Based on DHFWA operator, we can get the overall preference values for each alternative (a_i). $d(a_1) = \{\{0.5182, 0.5319, 0.6610, 0.6707, 0.5306, 0.5440, 0.6697, 0.6792\}, \{0.1931, 0.2253\}\}$. The score of $d(a_1)$ is $s(d(a_1)) = 0.3915$. Similarly, $d(a_2) = \{\{0.5966, 0.6542, 0.6321, 0.6846, 0.6056, 0.6619, 0.6403, 0.6917\}, \{0.2136, 0.2337, 0.2464\}\}$. The score of $d(a_2)$ is $s(d(a_2)) = 0.4147$. $d(a_3) = \{\{0.5380, 0.6040, 0.5574, 0.6206, 0.5787, 0.6388\}, \{0.2184, 0.2303, 0.2340, 0.2467\}\}$. The score of $d(a_3)$ is $s(d(a_3)) = 0.3573$. $d(a_4) = \{\{0.4398, 0.4686, 0.4717, 0.4988, 0.4666, 0.4940, 0.4969, 0.5227\}, \{0.3462, 0.3789\}\}$. The score of $d(a_4)$ is $s(d(a_4)) = 0.1198$.

Since $s(d(a_2)) > s(d(a_1)) > s(d(a_3)) > s(d(a_4))$, then the second alternative should be selected. This result is quite different from the result based on DHFWHM

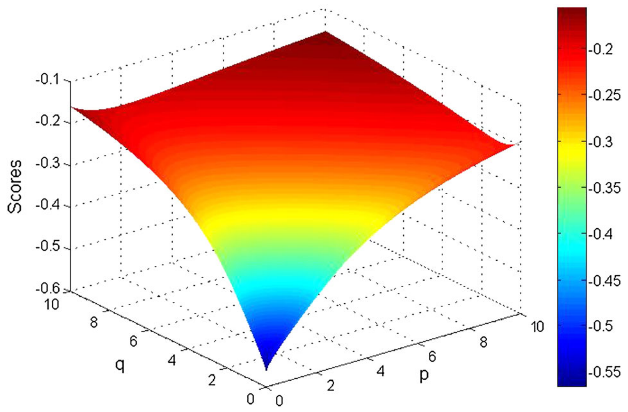

Fig. 11 Scores for alternative a_1 obtained by the DHFWHM operator

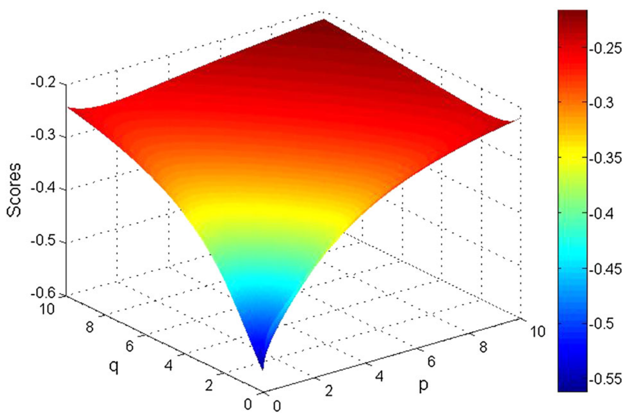

Fig. 12 Scores for alternative a_2 obtained by the DHFWHM operator

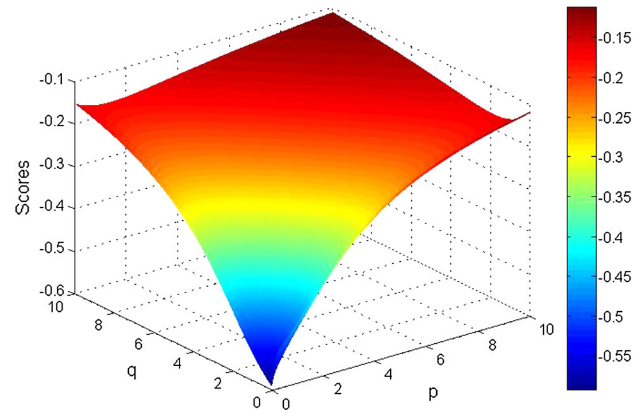

Fig. 13 Scores for alternative a_3 obtained by the DHFWHM operator

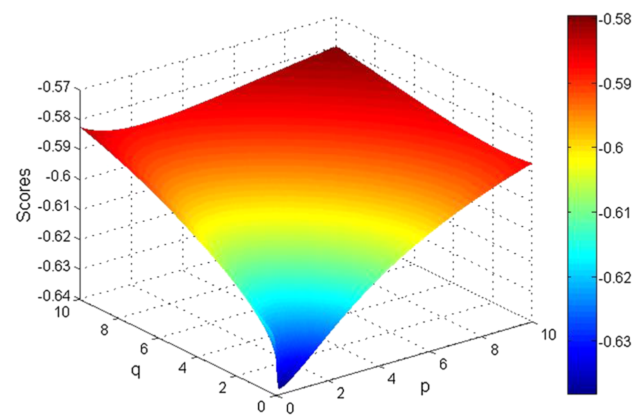

Fig. 14 Scores for alternative a_4 obtained by the DHFWHM operator

operator which was shown in Table 4. The main reason is that the DHFWA operator [29] does not take into account the relationship between various attributes but considered the aggregated arguments independent. Therefore, the dual hesitant fuzzy aggregation operators proposed in this paper are good complement to existing research.

5 Concluding remarks

This paper proposes some dual hesitant fuzzy information aggregation operators by introducing two parameters to reflect the decision makers' preference. Based on which, a multi-criteria GDM method under dual hesitant fuzzy environment is introduced and applied to evaluate suppliers, where criteria are described quantitatively with the DHFE. The final evaluation DHFE of each supplier can be achieved by DHFWHM operator. Decision can be made depends on the comprehensive DHFE for every alternative, the bigger the score function of the comprehensive DHFE the better the performance. A numerical example is presented to illustrate the effectiveness of the proposed method. The proposed multi-criteria GDM method can also be used to other evaluation problem under dual hesitant fuzzy environment. The following points should be taken into consideration when the method is adopted.

(1) Different values of the parameters represent different attitude of the decision makers which produce different comprehensive values and further cause different ranking results. Attention should be paid to the selection of appropriate values for parameter according the decision makers' preference.

(2) Access to decision-making information based on the experts' evaluation and expressed by DHFE correctly is a critical step of the proposed method. It should be noted that there are two kinds of information must be collected, namely membership degree and non-membership degree.

(3) The GDM method proposed in this paper should be compared with some other traditional methods to observe whether the final decision result changes or not. Further

study about real case in real situations worth studied which would enlarge the scope of the proposed GDM approach.

Acknowledgments The authors wish to thank the anonymous reviewers and Editor-in-Chief for their constructive comments on this study. This work has been supported by China National Natural Science Foundation (No. 71301142), Zhejiang Science & Technology Plan of China (2015C33024), Zhejiang Provincial Natural Science Foundation of China (No. LQ13G010004), Project Funded by China Postdoctoral Science Foundation (No. 2014M550353) and the National Education Information Technology Research (No. 146242069).

References

1. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96
2. Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353
3. Zhang Z, Wang M, Hu Y, Yang J, Ye Y, Li Y (2013) A dynamic interval-valued intuitionistic fuzzy sets applied to pattern recognition. Math Probl Eng 2013(408012):16
4. Papakostas GA, Hatzimichailidis AG, Kaburlasos VG (2013) Distance and similarity measures between intuitionistic fuzzy sets: a comparative analysis from a pattern recognition point of view. Pattern Recognit Lett 34(14):1609–1622
5. Boran FE, Akay D (2014) A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition. Inf Sci 255:45–57
6. Wei GW, Zhao XF (2012) Some induced correlated aggregating operators with intuitionistic fuzzy information and their application to multiple attribute group decision making. Expert Syst Appl 39(2):2026–2034
7. Tao F, Zhao D, Zhang L (2010) Resource service optimal-selection based on intuitionistic fuzzy set and non-functionality QoS in manufacturing grid system. Knowl Inf Syst 25(1):185–208
8. Wei GW, Wang HJ, Lin R (2011) Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information. Knowl Inf Syst 26(2):337–349
9. Tan CQ, Chen XH (2010) Intuitionistic fuzzy Choquet integral operator for multi-criteria decision making. Expert Syst Appl 37(1):149–157
10. He YD, Chen HY, Zhou LG, Han B, Zhao Q, Liu J (2014) Generalized intuitionistic fuzzy geometric interaction operators and their application to decision making. Expert Syst Appl 41(5):2484–2495
11. Torra V, Narukawa Y (2009). On hesitant fuzzy sets and decision. In: IEEE international conference on fuzzy systems, 2009. FUZZ-IEEE 2009, pp 1378–1382
12. Dubois DJ (1980) Fuzzy sets and systems: theory and applications (Vol. 144). Academic press, New York
13. Miyamoto S (2005) Remarks on basics of fuzzy sets and fuzzy multisets. Fuzzy Sets Syst 156(3):427–431
14. Yager RR (1986) On the theory of bags. Int J Gen Syst 13(1):23–37
15. Miyamoto S. (2000). Multisets and fuzzy multisets. In Soft computing and human-centered machines (pp. 9-33). Springer, Tokyo
16. Torra V (2010) Hesitant fuzzy sets. Int J Intell Syst 25(6):529–539
17. Xu ZS, Xia MM (2011) On distance and correlation measures of hesitant fuzzy information. Int J Intell Syst 26(5):410–425
18. Xia MM, Xu ZS (2011) Hesitant fuzzy information aggregation in decision making. Int J Approx Reason 52(3):395–407
19. Xia MM, Xu ZS, Chen N (2013) Some hesitant fuzzy aggregation operators with their application in group decision making. Group Decis Negot 22(2):259–279
20. Zhu B, Xu ZS, Xia MM (2012) Hesitant fuzzy geometric Bonferroni means. Inf Sci 205:72–85
21. Zhu B, Xu ZS (2014) Consistency measures for hesitant fuzzy linguistic preference relations. IEEE Trans Fuzzy Syst 22(1):35–45
22. Zhu B, Xu ZS (2013) Hesitant fuzzy Bonferroni means for multi-criteria decision making. J Oper Res Soc 64(12):1831–1840
23. Zhu B, Xu ZS, Xia MM (2012) Dual hesitant fuzzy sets. J Appl Math. 879629:13. doi:10.1155/2012/879629
24. Chen Y, Peng X, Guan G, Jiang H (2014) Approaches to multiple attribute decision making based on the correlation coefficient with dual hesitant fuzzy information. J Intell Fuzzy Syst 26(5):2547–2556
25. Zhu B, Xu ZS (2014) Some results for dual hesitant fuzzy sets. J Intell Fuzzy Syst 26:1657–1668
26. Farhadinia B (2014) Correlation for dual hesitant fuzzy sets and dual interval-valued hesitant fuzzy sets. Int J Intell Syst 29(2):184–205
27. Wang L, Ni M, Zhu L (2013) Correlation measures of dual hesitant fuzzy sets. J Appl Math 593739:12
28. Ye J (2014) Correlation coefficient of dual hesitant fuzzy sets and its application to multiple attribute decision making. Appl Math Model 38(2):659–666
29. Wang H, Zhao XF, Wei GW (2014) Dual hesitant fuzzy aggregation operators in multiple attribute decision making. J Intell Fuzzy Syst 26(5):2281–2290
30. Beliakov G, Pradera A, Calvo T (2007) Aggregation functions: a guide for practitioners. Springer, Berlin
31. Yu DJ (2013) Intuitionistic fuzzy geometric Heronian mean aggregation operators. Appl Soft Comput 13(2):1235–1246
32. Yu DJ, Wu YY (2012) Interval-valued intuitionistic fuzzy Heronian mean operators and their application in multi-criteria decision making. Afr J Bus Manag 6(11):4158–4168
33. Bonferroni C (1950) Sulle medie multiple di potenze. Bollettino dell'Unione Matematica Italiana 5(3–4):267–270
34. Yager RR (2009) On generalized Bonferroni mean operators for multi-criteria aggregation. Int J Approx Reaso 50(8):1279–1286
35. Xu ZS, Yager RR (2011) Intuitionistic fuzzy Bonferroni means. IEEE Trans Syst Man Cybern B Cybern 41(2):568–578
36. Xia MM, Xu ZS, Zhu B (2012) Generalized intuitionistic fuzzy Bonferroni means. Int J Intell Syst 27(1):23–47
37. Xia M, Xu Z, Zhu B (2013) Geometric Bonferroni means with their application in multi-criteria decision making. Knowl Based Syst 40:88–100
38. Gau WL, Buehrer DJ (1993) Vague sets. IEEE Trans Syst Man Cybern 23(2):610–614
39. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning—I. Inf Sci 8(3):199–249
40. Atanassov KT, Gargov G (1989) Interval valued intuitionistic fuzzy sets. Fuzzy Sets Syst 31(3):343–349
41. Chen SJ, Chen SM (2005) Fuzzy information retrieval based on geometric-mean averaging operators. Comput Math Appl 49(7):1213–1231
42. Xu ZS, Yager RR (2006) Some geometric aggregation operators based on intuitionistic fuzzy sets. Int J Gen Syst 35(4):417–433
43. Wang WZ, Liu XW (2011) Intuitionistic fuzzy geometric aggregation operators based on Einstein operations. Int J Intell Syst 26(11):1049–1075
44. Yu DJ (2012) Group decision making based on generalized intuitionistic fuzzy prioritized geometric operator. Int J Intell Syst 27(7):635–661

45. Zhang ZM (2013) Generalized Atanassov's intuitionistic fuzzy power geometric operators and their application to multiple attribute group decision making. Inf Fus 14(4):460–486
46. Wang WZ, Liu XW (2013) The multi-attribute decision making method based on interval-valued intuitionistic fuzzy Einstein hybrid weighted geometric operator. Comput Math Appl 66(10):1845–1856
47. Yu DJ (2013) Decision making based on generalized geometric operator under interval-valued intuitionistic fuzzy environment. J Intell Fuzzy Syst 25(2):471–480
48. Wang XZ, Dong CR (2009) Improving generalization of fuzzy IF–THEN rules by maximizing fuzzy entropy. IEEE Trans Fuzzy Syst 17(3):556–567
49. Wang XZ, Dong LC, Yan JH (2012) Maximum ambiguity-based sample selection in fuzzy decision tree induction. IEEE Trans Knowl Data Eng 24(8):1491–1505
50. Wang XZ, Xing HJ, Li Y, Hua Q, Dong CR, Pedrycz WA (2014) Study on relationship between generalization abilities and fuzziness of base classifiers in ensemble learning. IEEE Trans Fuzzy Syst. doi:10.1109/TFUZZ.2014.2371479
51. Chu TC, Varma R (2012) Evaluating suppliers via a multiple levels multiple criteria decision making method under fuzzy environment. Comput Ind Eng 62(2):653–660
52. Dalalah D, Hayajneh M, Batieha F (2011) A fuzzy multi-criteria decision making model for supplier selection. Expert Syst Appl 38(7):8384–8391
53. Boran FE, Genç S, Kurt M, Akay D (2009) A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Syst Appl 36(8):11363–11368