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ORIGINAL ARTICLE

Image enhancement based on fractional directional derivative

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Abstract This paper introduces the directional derivative to fractional derivative and proposes a new mathematical method, fractional directional derivative (FDD), and gives the corresponding numerical calculation. Compared with the traditional fractional derivative, the coefficients of FDD along the eight directions in the image plane are not the same, which can reflect different fractional change rates along different directions and is benefit to enlarge the differences among the image textures. Experiments show that the capability of nonlinearly enhancing texture details by FDD is more obvious than those by the traditional fractional derivative and integer-order differentiation operators Laplacian, Butterworth high-pass filter.

Keywords FDD · Image enhancement · DV/BV · Average gradient

1 Introduction

Fractional derivative, also called non-integer derivative, is not a new concept: it dates back to Cauchy, Riemann, Liouville and Letnikov in the 19th century [1]. In comparison with integer-order differential, the fractional differential of direct current or low-frequency signal is often nonzero. In the last two decades, fractional differentiation has played a very important role in various physical sciences fields, such as mechanics, electricity, chemistry, biology, economics, time and frequency domains system identification, notably control theory, mechatronics and robotics [2–4]. Recently, fractal theory is already used in fractal image processing [5–12]. In general, the corresponding coefficients of traditional fractional differentiation (TFD) operator along the eight directions are $1, -v, \frac{-v(-v+1)}{2}, \ldots, \frac{\Gamma(-v+1)}{(n-1)!(\Gamma(-v+n))}$ respectively in image processing [5, 13, 14]. The coefficients along the eight directions in the image plane are the same, which is not conducive to reflect the different change rates of the image along different directions and is not benefit to enlarge the differences among textures. Based on this, we give the concept of fractional directional derivative (FDD) and its coefficients along the eight directions are not the same, which can reflect different fractional change rates along different directions and is benefit to enlarge the differences among the image textures.

2 Theory and numerical calculation of FDD

Assume that $z = f(x, y)$ has definition in some neighborhood D. Find the line which passes through $M_0(x_0, y_0)$ and parallels to l as follows:

$$\frac{x - x_0}{\cos\alpha} = \frac{y - y_0}{\sin\alpha} = t, \; l = (\cos\alpha, \sin\alpha), \tag{1}$$

where α is the directional angle. Let $M(x_0 + \Delta x, y_0 + \Delta y)$ satisfy (1) and $h = sgn(t) \cdot \overline{M_0 M}$. If the direction of vector M_0M is the same as l, then $t > 0, h > 0$, otherwise,

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$t < 0, h < 0$, that is to say , h changes along the positive or negative direction of l. If the limit

$$\lim_{h \to 0} \frac{f(M) - f(M_0)}{h} \tag{2}$$

exists, then (2) is called the directional derivative of $f(x, y)$ at M_0 along the direction l, write $\frac{\partial f(M_0)}{\partial l}$, which denotes the slope of tangent line at M_0 for the plane curve

$$\begin{cases} z = f(x, y) \\ \frac{x - x_0}{\cos\alpha} = \frac{y - y_0}{\sin\alpha} \end{cases}$$

and its geometric meaning is the same as general derivative.

It is well-known, the sufficient condition which (2) exists is that $f(x, y)$ has continuous partial derivatives of first order in some neighborhood D at the point M_0 and

$$\frac{\partial f(M_0)}{\partial l} = \frac{\partial f(M_0)}{\partial x}\cos\alpha + \frac{\partial f(M_0)}{\partial y}\sin\alpha. \tag{3}$$

Clearly, $\frac{\partial f}{\partial l}$ is the function with respect to x, y in D:

$$\frac{\partial f(x, y)}{\partial l} = \frac{\partial f(x, y)}{\partial x}\cos\alpha + \frac{\partial f(x, y)}{\partial y}\sin\alpha. \tag{4}$$

If $f(x, y)$ exists continuous partial derivatives of second order in some neighborhood D, making the direction l fixed and by (4), we obtain:

$$\begin{aligned} \frac{\partial^2 f}{\partial l^2} &= \frac{\partial^2 f}{\partial x^2}\cos^2\alpha + 2\frac{\partial^2 f}{\partial x \partial y}\sin\alpha\cos\alpha \\ &\quad + \frac{\partial^2 f}{\partial y^2}\sin^2\alpha \triangleq \left(\cos\alpha\frac{\partial}{\partial x} + \sin\alpha\frac{\partial}{\partial y}\right)^2 f(x, y), \end{aligned} \tag{5}$$

thus (5) is called the second order directional derivative of $f(x, y)$ at M_0 along the direction l.

Similarly, if $f(x, y)$ exists $n+1$ order continuous partial derivatives in D, we can define the k-order directional derivative of $f(x, y)$ at M_0 along the direction l as follows.

$$\frac{\partial^k f}{\partial l^k} = \left(\cos\alpha\frac{\partial}{\partial x} + \sin\alpha\frac{\partial}{\partial y}\right)^k f(x, y),\ k = 1, 2, \ldots, n, n+1. \tag{6}$$

Thus, we can define the FDD of $f(x, y)$ at M_0 along the direction l as follows:

$$\frac{\partial^v f}{\partial l^v} = \left(\cos\alpha\frac{\partial}{\partial x} + \sin\alpha\frac{\partial}{\partial y}\right)^v f(x, y), \tag{7}$$

where v is any real number. From (7), in [15], the authors derived the directional derivative expression of Taylor formula for two-variable function and proposed fractional directional differentiation (also called FDD). Further, the authors discussed the construction of fractional directional differentiation mask in the four quadrants, respectively, for digital image. The differential coefficients of every direction are not the same along the eight directions in the four quadrants. While, in this paper, note that we will develop FDD of $f(x, y)$ at M_0 along the direction l instead of fractional directional differentiation (also called FDD) in [15]. Thus, we will obtain a new fractional operator whose coefficients are different from fractional directional differentiation operator in [15]. By (7), we have

$$\begin{aligned} \frac{\partial^v f}{\partial l^v} &= \sum_{k=0}^{\infty} (-1)^k \binom{v}{k} \frac{\partial^{v-k} f}{\partial x^{v-k}} \frac{\partial^k f}{\partial y^k} \cos^{v-k}\alpha \sin^k\alpha \\ &= \sum_{k=0}^{\infty} (-1)^k \left(\frac{v}{1} \cdot \frac{v-1}{2} \cdots \frac{v-k+1}{k}\right) \frac{\partial^{v-k} f}{\partial x^{v-k}} \frac{\partial^k f}{\partial y^k} \cos^{v-k}\alpha \sin^k\alpha \\ &= \frac{\partial^v f}{\partial x^v}\cos^v\alpha - v \cdot \frac{\partial^{v-1} f}{\partial x^{v-1}} \frac{\partial f}{\partial y}\cos^{v-1}\alpha\sin\alpha \\ &\quad + \frac{v(v-1)}{2} \frac{\partial^{v-2} f}{\partial x^{v-2}} \frac{\partial^2 f}{\partial y^2}\cos^{v-2}\alpha\sin^2\alpha + \cdots \end{aligned} \tag{8}$$

In the same way,

$$\begin{aligned} \frac{\partial^v f}{\partial l^v} &= \left(\sin\alpha\frac{\partial}{\partial y} + \cos\alpha\frac{\partial}{\partial x}\right)^v f(x, y) \\ &= \sum_{k=0}^{\infty} (-1)^k \binom{v}{k} \frac{\partial^{v-k} f}{\partial y^{v-k}} \frac{\partial^k f}{\partial x^k} \sin^{v-k}\alpha \cos^k\alpha \\ &= \sum_{k=0}^{\infty} (-1)^k \left(\frac{v}{1} \cdot \frac{v-1}{2} \cdots \frac{v-k+1}{k}\right) \frac{\partial^{v-k} f}{\partial y^{v-k}} \frac{\partial^k f}{\partial x^k} \sin^{v-k}\alpha \cos^k\alpha \\ &= \frac{\partial^v f}{\partial y^v}\sin^v\alpha - v\frac{\partial^{v-1} f}{\partial y^{v-1}} \frac{\partial f}{\partial x}\sin^{v-1}\alpha\cos\alpha \\ &\quad + \frac{v(v-1)}{2} \frac{\partial^{v-2} f}{\partial y^{v-2}} \frac{\partial^2 f}{\partial x^2}\sin^{v-2}\alpha\cos^2\alpha + \cdots. \end{aligned} \tag{9}$$

Taking the average of (8) and (9), we obtain

$$\begin{aligned} \frac{\partial^v f}{\partial l^v} &= \frac{1}{2}\left(\frac{\partial^v f}{\partial x^v}\cos^v\alpha + \frac{\partial^v f}{\partial y^v}\sin^v\alpha\right) \\ &\quad - \frac{v}{2}\left(\frac{\partial^{v-1} f}{\partial x^{v-1}} \frac{\partial f}{\partial y}\cos^{v-1}\alpha\sin\alpha + \frac{\partial^{v-1} f}{\partial y^{v-1}} \frac{\partial f}{\partial x}\sin^{v-1}\alpha\cos\alpha\right) \\ &\quad + \frac{v(v-1)}{4}\left(\frac{\partial^{v-2} f}{\partial x^{v-2}} \frac{\partial^2 f}{\partial y^2}\cos^{v-2}\alpha\sin^2\alpha \right. \\ &\quad \left. + \frac{\partial^{v-2} f}{\partial y^{v-2}} \frac{\partial^2 f}{\partial x^2}\sin^{v-2}\alpha\cos^2\alpha\right) + \cdots. \end{aligned} \tag{10}$$

In order to simplify the calculation, we only take the top three terms in (10), and the differences of fractional partial derivative are expressed as respectively in (10) [5–7]:

$$\begin{aligned} \frac{\partial^v f(x, y)}{\partial x^v} &\approx f(x, y) + (-v)f(x-1, y) + \cdots \\ &\quad + \frac{\Gamma(n - v + 1)}{(n-1)!\Gamma(-v)} f(x - n + 1, y), \end{aligned} \tag{11}$$

$$\frac{\partial^v f(x,y)}{\partial y^v} \approx f(x,y) + (-v)f(x,y-1) + \cdots + \frac{\Gamma(n-v+1)}{(n-1)!\Gamma(-v)} f(x,y-n+1). \tag{12}$$

And the differences of $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$ in (10) can be expressed as

$$\frac{\partial f(x,y)}{\partial x} \approx f(x,y) - f(x-1,y) \tag{13}$$

$$\frac{\partial f(x,y)}{\partial y} \approx f(x,y) - f(x,y-1), \tag{14}$$

$$\frac{\partial^2 f(x,y)}{\partial x^2} \approx f(x,y) - 2f(x-1,y) + f(x-2,y) \tag{15}$$

$$\frac{\partial^2 f(x,y)}{\partial y^2} \approx f(x,y) - 2f(x,y-1) + f(x,y-2), \tag{16}$$

thus, by the top three terms of (11)–(12), and (13)–(16), (10) becomes

$$\begin{aligned}
\frac{\partial^v f}{\partial l^v} \approx{} & \frac{1}{2}\left(\frac{\partial^v f}{\partial x^v}\cos^v\alpha + \frac{\partial^v f}{\partial y^v}\sin^v\alpha\right) \\
& - \frac{v}{2}\left(\frac{\partial^{v-1} f}{\partial x^{v-1}}\frac{\partial f}{\partial y}\cos^{v-1}\alpha\sin\alpha + \frac{\partial^{v-1} f}{\partial y^{v-1}}\frac{\partial f}{\partial x}\sin^{v-1}\alpha\cos\alpha\right) \\
& + \frac{v(v-1)}{4}\left(\frac{\partial^{v-2} f}{\partial x^{v-2}}\frac{\partial^2 f}{\partial y^2}\cos^{v-2}\alpha\sin^2\alpha + \frac{\partial^{v-2} f}{\partial y^{v-2}}\frac{\partial^2 f}{\partial x^2}\sin^{v-2}\alpha\cos^2\alpha\right) \\
={} & \frac{1}{2}\left(\left(f(x,y) - vf(x-1,y) + \frac{v(v-1)}{2}f(x-2,y)\right)\cos^v\alpha\right. \\
& \left.+\left(f(x,y) - vf(x,y-1) + \frac{v(v-1)}{2}f(x,y-2)\right)\sin^v\alpha\right) \\
& - \frac{v}{2}\left(\left(f(x,y) - (v-1)f(x-1,y) + \frac{(v-1)(v-2)}{2}f(x-2,y)\right)\right. \\
& \left.\times(f(x,y) - f(x,y-1))\cos^{v-1}\alpha\sin\alpha\right) \\
& - \frac{v}{2}\left(\left(f(x,y) - (v-1)f(x,y-1) + \frac{(v-1)(v-2)}{2}f(x,y-2)\right)\right. \\
& \left.\times(f(x,y) - f(x-1,y))\sin^{v-1}\alpha\cos\alpha\right) \\
& + \frac{v(v-1)}{4}\left(\left(f(x,y) - (v-2)f(x-1,y) + \frac{(v-2)(v-3)}{2}f(x-2,y)\right)\right. \\
& \left.\times(f(x,y) - 2f(x,y-1) + f(x,y-2))\cos^{v-2}\alpha\sin^2\alpha\right) \\
& + \frac{v(v-1)}{4}\left(\left(f(x,y) - (v-2)f(x,y-1) + \frac{(v-2)(v-3)}{2}f(x,y-2)\right)\right. \\
& \left.\times(f(x,y) - 2f(x-1,y) + f(x-2,y))\sin^{v-2}\alpha\cos^2\alpha\right).
\end{aligned} \tag{17}$$

For the digital image, the shortest neighborhood distance between pixels is only one pixel. Therefore, the measurement for duration on x-coordinate or y-coordinate must take pixel as unit, and the minimum division must be $\Delta x = 1, \Delta y = 1$. To obtain the new FDD filter along eight directions, take the directional angles $\alpha = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$, see Fig. 1.

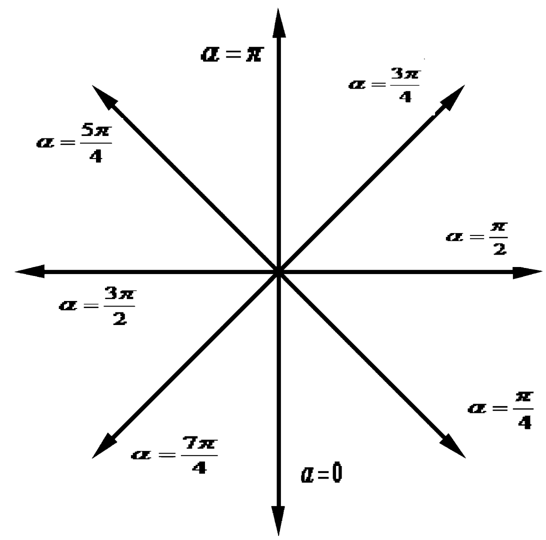

Fig. 1 The eight directions of FDD

(a) When $\alpha = 0$, $l = (\cos\alpha, \sin\alpha) = (1, 0)$, that is, $\cos\alpha = 1, \sin\alpha = 0$, we obtain

$$\frac{\partial^v f}{\partial l^v} = \frac{1}{2}\frac{\partial^v f}{\partial x^v} \approx \frac{1}{2}\left(f(x,y) - vf(x-1,y) + \frac{v(v-1)}{2}f(x-2,y)\right). \tag{18}$$

(b) When $\alpha = \frac{\pi}{4}, l = (\cos\alpha, \sin\alpha) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, that is, $\cos\alpha = \frac{\sqrt{2}}{2}, \sin\alpha = \frac{\sqrt{2}}{2}$, we obtain

$$\begin{aligned}
\frac{\partial^v f}{\partial l^v} \approx{} & \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^v f}{\partial x^v} + \frac{\partial^v f}{\partial y^v}\right) \\
& - \frac{v}{2}\left(\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^{v-1} f}{\partial x^{v-1}}\frac{\partial f}{\partial y} + \frac{\partial^{v-1} f}{\partial y^{v-1}}\frac{\partial f}{\partial x}\right) \\
& + \frac{v(v-1)}{4}\left(\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^{v-2} f}{\partial x^{v-2}}\frac{\partial^2 f}{\partial y^2} + \frac{\partial^{v-2} f}{\partial y^{v-2}}\frac{\partial^2 f}{\partial x^2}\right).
\end{aligned} \tag{19}$$

(c) When $\alpha = \frac{\pi}{2}$, $l = (\cos\alpha, \sin\alpha) = (0, 1)$, that is, $\cos\alpha = 0, \sin\alpha = 1$, we have

$$\frac{\partial^v f}{\partial l^v} = \frac{1}{2}\frac{\partial^v f}{\partial y^v} \approx \frac{1}{2}\left(f(x,y) - vf(x,y-1) + \frac{v(v-1)}{2}f(x,y-2)\right). \tag{20}$$

(d) When $\alpha = \frac{3\pi}{4}$, $l = (\cos\alpha, \sin\alpha) = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, that is, $\cos\alpha = -\frac{\sqrt{2}}{2}, \sin\alpha = \frac{\sqrt{2}}{2}$, we have

$$\begin{aligned}\frac{\partial^v f}{\partial l^v} &\approx \frac{1}{2}\left(\frac{\partial^v f}{\partial x^v}\left(-\frac{\sqrt{2}}{2}\right)^v + \frac{\partial^v f}{\partial y^v}\left(\frac{\sqrt{2}}{2}\right)^v\right)\\ &- \frac{v}{2}\left(\frac{\partial^{v-1} f}{\partial x^{v-1}}\frac{\partial f}{\partial y}\left(-\frac{\sqrt{2}}{2}\right)^{v-1}\frac{\sqrt{2}}{2}\right.\\ &\left.+ \frac{\partial^{v-1} f}{\partial y^{v-1}}\frac{\partial f}{\partial x}\left(\frac{\sqrt{2}}{2}\right)^{v-1}\left(-\frac{\sqrt{2}}{2}\right)\right)\\ &+ \frac{v(v-1)}{4}\left(\frac{\partial^{v-2} f}{\partial x^{v-2}}\frac{\partial^2 f}{\partial y^2}\left(-\frac{\sqrt{2}}{2}\right)^{v-2}\left(\frac{\sqrt{2}}{2}\right)^2\right.\\ &\left.+ \frac{\partial^{v-2} f}{\partial y^{v-2}}\frac{\partial^2 f}{\partial x^2}\left(\frac{\sqrt{2}}{2}\right)^{v-2}\left(-\frac{\sqrt{2}}{2}\right)^2\right)\\ &= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)^v\left((-1)^v\frac{\partial^v f}{\partial x^v} + \frac{\partial^v f}{\partial y^v}\right)\\ &- \frac{v}{2}\left(\frac{\sqrt{2}}{2}\right)^v\left((-1)^{v-1}\frac{\partial^{v-1} f}{\partial x^{v-1}}\frac{\partial f}{\partial y} - \frac{\partial^{v-1} f}{\partial y^{v-1}}\frac{\partial f}{\partial x}\right)\\ &+ \frac{v(v-1)}{4}\left(\frac{\sqrt{2}}{2}\right)^v\left((-1)^{v-2}\frac{\partial^{v-2} f}{\partial x^{v-2}}\frac{\partial^2 f}{\partial y^2} + \frac{\partial^{v-2} f}{\partial y^{v-2}}\frac{\partial^2 f}{\partial x^2}\right).\end{aligned} \tag{21}$$

(e) When $\alpha = \pi, l = (\cos\alpha, \sin\alpha) = (-1, 0)$, that is, $\cos\alpha = -1, \sin\alpha = 0$, we have

$$\begin{aligned}\frac{\partial^v f}{\partial l^v} = \frac{(-1)^v}{2}\frac{\partial^v f}{\partial x^v} &\approx \frac{(-1)^v}{2}\left(f(x,y) - vf(x-1,y)\right.\\ &\left.+ \frac{v(v-1)}{2}f(x-2,y)\right).\end{aligned} \tag{22}$$

(f) When $\alpha = \frac{5\pi}{4}, l = (\cos\alpha, \sin\alpha) = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, that is, $\cos\alpha = -\frac{\sqrt{2}}{2}, \sin\alpha = -\frac{\sqrt{2}}{2}$, we have

$$\begin{aligned}\frac{\partial^v f}{\partial l^v} &\approx \frac{1}{2}\left(-\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^v f}{\partial x^v} + \frac{\partial^v f}{\partial y^v}\right)\\ &- \frac{v}{2}\left(-\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^{v-1} f}{\partial x^{v-1}}\frac{\partial f}{\partial y} + \frac{\partial^{v-1} f}{\partial y^{v-1}}\frac{\partial f}{\partial x}\right)\\ &+ \frac{v(v-1)}{4}\left(-\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^{v-2} f}{\partial x^{v-2}}\frac{\partial^2 f}{\partial y^2} + \frac{\partial^{v-2} f}{\partial y^{v-2}}\frac{\partial^2 f}{\partial x^2}\right).\end{aligned} \tag{23}$$

(g) When $\alpha = \frac{3\pi}{2}, l = (\cos\alpha, \sin\alpha) = (0, -1)$, that is, $\cos\alpha = 0, \sin\alpha = -1$, we have

$$\begin{aligned}\frac{\partial^v f}{\partial l^v} = \frac{(-1)^v}{2}\frac{\partial^v f}{\partial y^v} &\approx \frac{(-1)^v}{2}\left(f(x,y) - vf(x,y-1)\right.\\ &\left.+ \frac{v(v-1)}{2}f(x,y-2)\right).\end{aligned} \tag{24}$$

(h) When $\alpha = \frac{7\pi}{4}, l = \cos\alpha, \sin\alpha) = (\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, that is, $\cos\alpha = \frac{\sqrt{2}}{2}, \sin\alpha = -\frac{\sqrt{2}}{2}$, we have

$$\begin{aligned}\frac{\partial^v f}{\partial l^v} &\approx \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^v f}{\partial x^v} + (-1)^v\frac{\partial^v f}{\partial y^v}\right)\\ &- \frac{v}{2}\left(\frac{\sqrt{2}}{2}\right)^v\left(-\frac{\partial^{v-1} f}{\partial x^{v-1}}\frac{\partial f}{\partial y} + (-1)^{v-1}\frac{\partial^{v-1} f}{\partial y^{v-1}}\frac{\partial f}{\partial x}\right)\\ &+ \frac{v(v-1)}{4}\left(\frac{\sqrt{2}}{2}\right)^v\left(\frac{\partial^{v-2} f}{\partial x^{v-2}}\frac{\partial^2 f}{\partial y^2} + (-1)^{v-2}\frac{\partial^{v-2} f}{\partial y^{v-2}}\frac{\partial^2 f}{\partial x^2}\right).\end{aligned} \tag{25}$$

Only taking (11)–(16) into (18)–(25) correspondingly, we can achieve the numerical calculation of FDD. From (18)–(25), we see that the coefficients of FDD along the eight directions are not the same, which can reflect different fractional change rates along different directions and is benefit to enlarge the differences among textures. This is also the biggest difference with the traditional fractional derivative. Therefore, FDD not only can nonlinearly enhance the contour feature in the smooth area, but also can enhance evidently high-frequency edge feature in those areas where gray changes remarkably. In order to simplify the operation, by comparing the absolute value outputs of the eight directions, the biggest one is taken as FDD grayscale of image.

3 Experiments and result analysis

This section aims at demonstrating that FDD operator has better capability in image enhancement [6, 7, 16, 17]. We

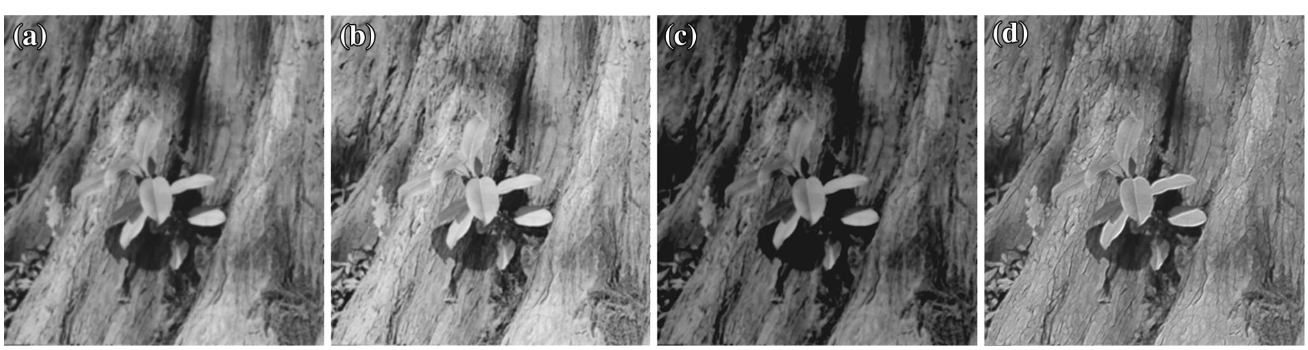

Fig. 2 The comparison of Laplacian operator, Butterworth high-pass filter and FDD. **a** Original image, **b** Laplacian operator, **c** Butterworth high-pass filter, **d** FDD $v = 0.03$.

first need to do FDD for an image $s(x, y)$ by (18)–(25) respectively. And then we select the maximum absolute value of the eight directions as grayscale of the point. In order to validate the performance of FDD, we compare with traditional integer order differential operator Laplacian, Butterworth high-pass filter and TFD operator. Compared with integer-order differential operator Laplacian and Butterworth high-pass filter, the results are shown in Fig. 2.

Figure 2 shows that the original image is more blurred and Laplacian operator enhances detail information. It is also obvious that enhanced image by Butterworth high-pass filter becomes dark and loses some texture detail information. While FDD operator achieves best visual effects and is better than Laplacian operator and Butterworth high-pass filter. To explain this phenomenon in Fig. 2, the grey level histogram is given in Fig. 3.

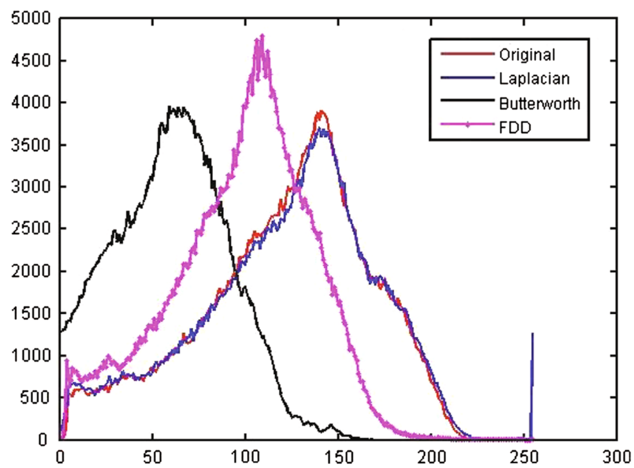

Fig. 3 The grey level histogram in Fig. 2

It is easy to see that the envelope curve of grey level histogram by Laplacian operator almost has no significant difference with that of the origin image, but when the grey value is 255, the number of pixel points increases suddenly, which leads to a large number of light spot and is respected as noise. Consequently, the enhance effect is not obvious. For Butterworth high-pass filter, the grey range of the grey level histogram becomes smaller than that of the original image, which implies the increase of low grey value, so the whole image becomes dark. The grey level histogram of FDD almost keeps the density function (envelope curve) with a normal distribution and the whole graph shifts, so FDD has best visual effects.

Secondly, we compare FDD with TFD operator. The parts of enhanced effects are shown in Fig. 4.

Since both FDD and TFD can enhance image in detail. In order to quantify enhanced effect in detail, the paper adopts DV and BV [18] to evaluate the enhanced image. DV and BV can divide image into detail region and background region by a threshold. DV is the average local variance of the detail region, BV is the average local variance of the background region, and the threshold is the local variance that corresponds to the peak of local variance's grey level histogram for the original image. For

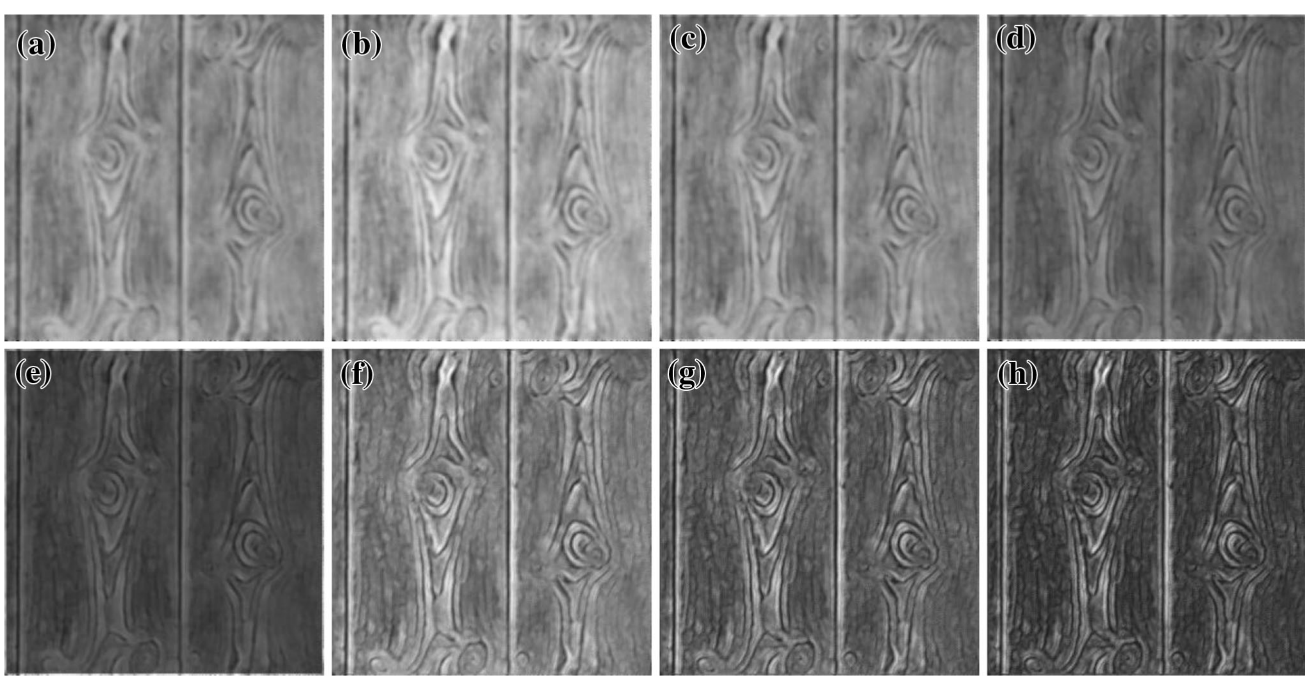

Fig. 4 The comparison of FDD and TFD with different order. **a** Original image, **b** TFD $v = 0.05$, **c** TFD $v = 0.2$, **d** TFD $v = 0.4$, **e** TFD $v = 0.6$, **f** FDD $v = 0.05$, **g** FDD $v = 0.1$, **h** FDD $v = 0.2$.

Table 1 Comparison of DV/BV in Fig. 4

Image	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
DV/BV	3.7058	4.1557	4.2848	4.6895	5.1663	6.5547	8.5827	9.0326

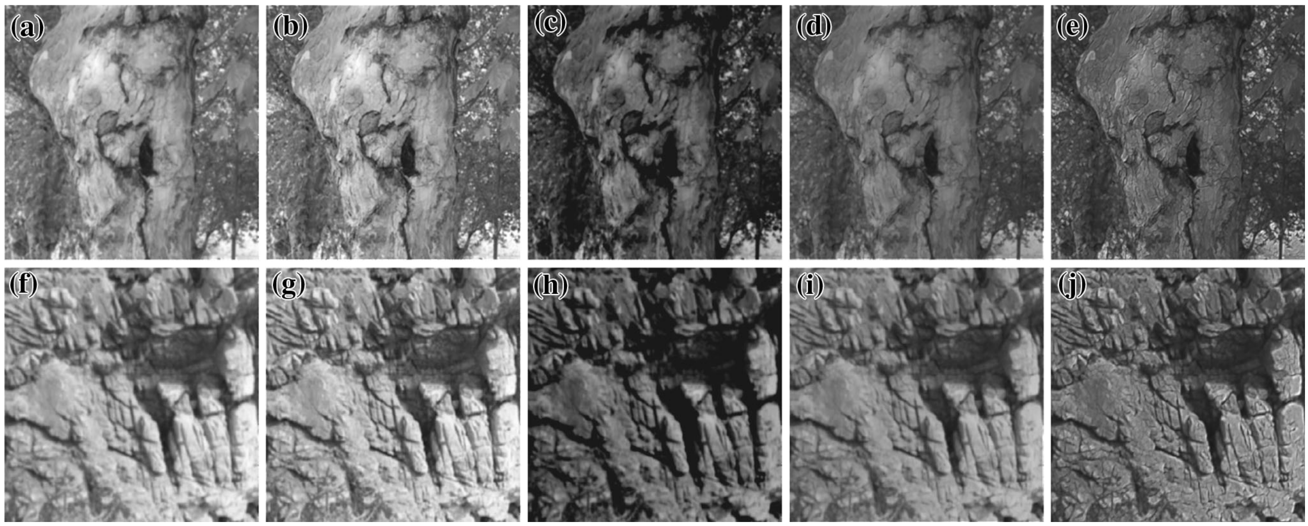

Fig. 5 The comparison of different methods. **a** Original image, **b** Laplacian operator, **c** Butterworth high-pass filter, **d** TFD $v = 0.3$, **e** FDD $v = 0.05$, **f** original image, **g** Laplacian operator, **h** Butterworth high-pass filter, **i** TFD $v = 0.2$, **j** FDD $v = 0.05$.

Table 2 DV/BV and average gradient in Fig. 5

Image	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
DV/BV	3.821	3.957	3.949	4.124	4.803	4.096	3.822	3.847	4.002	4.701
Ave.grad.	4.289	5.754	6.334	7.916	3.872	4.949	3.272	4.739	4.994	7.114

enhanced images, the larger DV is and the smaller BV is, the better enhanced effect is achieved in general. For the sake of convenience, DV/BV is used as the enhancement evaluation criterion. The results of images in Fig. 4 are shown in Table 1.

Table 1 gives the fact that the DV/BV values of FDD and TFD are both greater than that of the original image, and DV/BV of FDD changes obviously with the increase of the fractional order.

To further compare FDD with Laplacian operator, Butterworth high-pass filter and TFD, we do the following some experiments. The results are shown in Fig. 5.

In order to further illustrate the performance of FDD, the average gradient [19–22] of enhanced images in Fig. 5 are given. See Table 2.

As shown in Table 2, the values of DV/BV with Laplacian operator, Butterworth high-pass filter and FDD are greater than that of the origin image. Although Butterworth high-pass filter has the greatest DV/BV value, the enhanced image becomes dark, so extraneous noise is introduced. Although Laplacian operator has the greatest average gradient value, the enhanced image is over sharp. Comparing with TFD, both the average gradient and DV/BV by FDD are greater than that by TFD, which means FDD can enhance both the margins and the textures. Therefore, FDD could nonlinearly preserve the low-frequency contour feature in the smooth area to the furthest degree, and as well as, nonlinearly enhance high-frequency marginal information in those areas where gray scale changes frequently. FDD could nonlinearly enhance the comprehensive texture details.

4 Conclusion

The paper introduces the concept and theory of FDD. The coefficients of FDD along the eight directions in the image plane are not the same, which can reflect different fractional change rates along different directions and is benefit to enlarge the differences among textures. Compared with Laplacian operator, Butterworth high-pass filter and TFD, the experiments indicate that Laplacian operator makes enhanced image over sharp and introduces extraneous noise; the grey range of enhanced images by Butterworth high-pass filter becomes narrower, so some detail information lose. Although TFD can enhance image non-linearly, the amplitude is smaller than FDD. Therefore, FDD achieves better enhanced effect than the above three operators. Therefore, FDD is a new method and technology for image enhancement.

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