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ORIGINAL ARTICLE

New approach to MCDM under interval-valued intuitionistic fuzzy environment

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Abstract It is well-known that how to determine the weights of criteria is an important problem of multicriteria decision making. To make further description of the aforementioned, in this paper we introduce an extended TOPSIS method for multicriteria decision making with interval-valued intuitionistic fuzzy information, where the weighted vector of each alternative is determined by ranking corresponding evaluation information. Meanwhile, we construct a new method to measure the distance between alternatives and positive ideal solution as well as negative ideal solution, which is score distance. Finally, the detailed decision making procedure is proposed and an illustrative example is applied to demonstrate its validity. It is worth while to point out that the weights determination for criteria will be helpful to future research on decision making analysis.

Keywords Interval-valued intuitionistic fuzzy sets · Multicriteria decision making · Ranking of interval-valued intuitionistic fuzzy numbers · Score distance · TOPSIS

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1 Introduction

Multicriteria decision making (MCDM, for short) method, as an important part of modern decision science, has found its successful application in various areas, such as conflict analysis [1], urban planning and layout [13, 37, 43], guidance system [20], management [21, 38], pattern recognition [12], material selection [5, 6, 23], and so on. Because there exist many uncertainty or imprecision in practical problems, decision makers often have to face complicated situations during the process of decision making. In order to obtain a reasonable decision making result, some researchers have made a combination of MCDM with fuzzy sets [4], gray system [7, 32], rough sets [9, 18, 19, 34], neural network [8] and others [22, 26–29, 40]. On this bases, an increasing amount of literature has been engaged in this field.

Roughly speaking, the procedure of MCDM for certain problem can be summarized in following steps: (1) state the nature of the problem and propose the evaluation information of each alternative under all criteria; (2) determine the weights of criteria by means of corresponding techniques; (3) evaluate the whole performance of each alternative; (4) rank alternatives with respect to their whole performance scores and select the most desirable one. Because of the uncertainty or imprecision, the evaluation information may take different expressions with respect to practical problem, such as fuzzy data [10], intuitionistic fuzzy sets [25, 33], interval-valued intuitionistic fuzzy sets [35] and linguistic setting [36]. In fact, the key step is to obtain the whole performance score of each possible alternative. Certainly, how to determine weights of criteria also plays an important role in the process of decision making. Therefore, many techniques have been applied to deal with the determination of weights in recent years, such as optimization model method [16, 31, 36], correlation coefficient method [30], standard or maximizing deviation method [10, 33], and so on.

Meanwhile, it deserves to be pointed out that as one of the MCDM methods, the technique for order preference by similarity to an ideal solution(TOPSIS, for short) method proposed by Hwang and Yoon in [11] has aroused great interesting of researchers. The basic principle of which is that the chosen alternative should have the nearest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Because of its practicality, the TOPSIS method has been investigated widely by many researchers. Especially, the TOPSIS method and its extensions are studied under interval-valued intuitionistic fuzzy environment by many researchers. Such as Park et al. [17] and Ye [39] researched the extended TOPSIS method with interval-valued intuitionistic fuzzy numbers. In [42] Zhang and Yu investigated multi-attribute decision making problems under interval-valued intuitionistic fuzzy environment.

As is known to all, different criterion usually plays different role in practical decision making problem. But, the classical TOPSIS method gives equal treatment to all criteria which runs counter to cognitive laws of knowledge acquisition [15]. Hence, it is necessary to readjust the criteria weights when applying TOPSIS method. For this, in this study we propose a ranking-based weights determination method where the precondition is that all criteria can be ranked according to so-named "importance" of criteria. The importance of criteria can be reflected by the evaluation information provided by experts. At the same time, the similarity measure the classical TOPSIS method adopted is Euclidean distance. Here, by considering the score of evaluation information and criteria weights, a weighted score distance between any two alternatives is constructed. With above mentioned, this paper introduces a new procedure of MCDM using extended TOPSIS method under interval-valued intuitionistic fuzzy environment. To reflect the decision maker's subjective initiative, the overall score calculation method of classical TOPSIS is also modified.

The remainder of this paper is organized as follows. Some basic concepts required are recalled briefly in Sect. 2 In Sect. 3, the technique for computing weights of criteria with respect to alternatives is constructed and the definition of score distance is proposed carefully. On this bases, a new method of MCDM is constructed in detail. In Sect. 4, an illustrative example is applied to demonstrate the validity of the new constructed method. And finally, conclusions are stated in Sect. 5.

2 Preliminaries

In this section we make a brief review of some preliminaries, which are ranking of interval-valued intuitionistic fuzzy sets and the TOPSIS method. Throughout this paper, let U be the universe of discourse, and $D([0,1])$ the set of all closed subintervals of unit interval [0, 1].

2.1 Ranking of interval-valued intuitionistic fuzzy sets

By taking the positive and negative aspects of the evaluation information into consideration, the intuitionistic fuzzy sets proposed by Atanassov [2] can be seen as the natural generalization of fuzzy sets proposed by Zadeh [41]. Later Atanassov and Gargov extend intuitionistic fuzzy sets to interval-valued intuitionistic fuzzy sets as follows:

Definition 1 (See [3]) An interval-valued intuitionistic fuzzy set on U can be expressed as

$$A = \{(u, [\mu_A^-(u), \mu_A^+(u)], [\nu_A^-(u), \nu_A^+(u)]) \mid u \in U\}, \quad (1)$$

where $[\mu_A^-(u), \mu_A^+(u)] \in D([0,1])$ and $[\nu_A^-(u), \nu_A^+(u)] \in D([0,1])$ with the condition $\mu_A^+(u) + \nu_A^+(u) \leq 1$ for all $u \in U$.

We call $[\mu_A^-(u), \mu_A^+(u)]$ the membership degree interval, $[\nu_A^-(u), \nu_A^+(u)]$ the nonmembership degree interval and $[\pi_A^-(u), \pi_A^+(u)]$ the hesitation degree interval of u to A, where $\pi_A^-(u) = 1 - \mu_A^+(u) - \nu_A^+(u)$ and $\pi_A^+(u) = 1 - \mu_A^-(u) - \nu_A^-(u)$.

For an interval-valued intuitionistic fuzzy set A based on U, the pair $([\mu_A^-(u), \mu_A^+(u)], [\nu_A^-(u), \nu_A^+(u)])$ is called an interval-valued intuitionistic fuzzy number [35] and denoted by $\widetilde{\alpha} = ([a,b],[c,d])$ for convenience.

To measure the information interval-valued intuitionistic fuzzy numbers contained, score function, accuracy function, membership uncertainty index function and hesitation uncertainty index function are defined as follows.

Definition 2 (See [35]) Let $\widetilde{\alpha} = ([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, then the score function is defined by

$$S(\widetilde{\alpha}) = \frac{1}{2}(a+b-c-d) \quad (2)$$

and the accuracy function is defined by

$$H(\widetilde{\alpha}) = \frac{1}{2}(a+b+c+d). \quad (3)$$

Definition 3: (See [31]) Let $\widetilde{\alpha} = ([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, then the membership uncertainty index function is defined by

$$T(\widetilde{\alpha}) = b+c-a-d \quad (4)$$

and hesitation uncertainty index function is defined by

$$G(\widetilde{\alpha}) = b+d-a-c. \quad (5)$$

Based on above measuring functions, in [31] Wang et al. proposed a procedure to compare any two interval-valued intuitionistic fuzzy numbers, taking $\widetilde{\alpha} = ([a_1, b_1], [c_1, d_1])$ and $\widetilde{\beta} = ([a_2, b_2], [c_2, d_2])$ for example, as:

(1) If $S(\widetilde{\alpha}) < S(\widetilde{\beta})$, then $\widetilde{\alpha}$ is smaller than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \prec \widetilde{\beta}$; if $S(\widetilde{\alpha}) > S(\widetilde{\beta})$, then $\widetilde{\alpha}$ is bigger than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \succ \widetilde{\beta}$; if $S(\widetilde{\alpha}) = S(\widetilde{\beta})$, then accuracy function $H(*)$ is used to compare $\widetilde{\alpha}$ and $\widetilde{\beta}$ with the same format as score function $S(*)$: (2) if $H(\widetilde{\alpha}) < H(\widetilde{\beta})$, then $\widetilde{\alpha}$ is smaller than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \prec \widetilde{\beta}$; if $H(\widetilde{\alpha}) > H(\widetilde{\beta})$, then $\widetilde{\alpha}$ is bigger than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \succ \widetilde{\beta}$; if $H(\widetilde{\alpha}) = H(\widetilde{\beta})$, then the membership uncertainty index function $T(*)$ is used to determine the order of $\widetilde{\alpha}$ and $\widetilde{\beta}$: (3) if $T(\widetilde{\alpha}) < T(\widetilde{\beta})$, then $\widetilde{\alpha}$ is bigger than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \succ \widetilde{\beta}$; if $T(\widetilde{\alpha}) > T(\widetilde{\beta})$, then $\widetilde{\alpha}$ is smaller than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \prec \widetilde{\beta}$; moreover,if $T(\widetilde{\alpha}) = T(\widetilde{\beta})$, then: (4) if $G(\widetilde{\alpha}) < G(\widetilde{\beta})$, then $\widetilde{\alpha}$ is bigger than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \succ \widetilde{\beta}$; if $G(\widetilde{\alpha}) > G(\widetilde{\beta})$, then $\widetilde{\alpha}$ is smaller than $\widetilde{\beta}$ and denoted by $\widetilde{\alpha} \prec \widetilde{\beta}$; if $G(\widetilde{\alpha}) = G(\widetilde{\beta})$, then $\widetilde{\alpha}$ and $\widetilde{\beta}$ contain the same information and denoted by $\widetilde{\alpha} = \widetilde{\beta}$.

Definition 4 (See [31]) Let $\widetilde{\alpha}$ and $\widetilde{\beta}$ be two interval-valued intuitionistic fuzzy numbers, then $\widetilde{\alpha} \preccurlyeq \widetilde{\beta}$ if and only if $\widetilde{\alpha} \prec \widetilde{\beta}$ or $\widetilde{\alpha} = \widetilde{\beta}$.

To perfect the method of ranking interval-valued intuitionistic fuzzy numbers, Nayagam et al. introduced another technique as follows.

Definition 5 (See [14]) Let $\widetilde{\alpha} = ([a, b], [c, d])$ be an interval-valued intuitionistic fuzzy number, then the general accuracy function of $\widetilde{\alpha}$ is defined as

$$LG(\widetilde{\alpha}) = \frac{a + b + \delta(2 - a - b - c - d)}{2}, \qquad (6)$$

where $\delta \in [0, 1]$ is a parameter depending on the individual's intention.

In particular, it has been proved that for any two interval-valued intuitionistic fuzzy numbers $\widetilde{\alpha}$ and $\widetilde{\beta}, \widetilde{\alpha} \preccurlyeq \widetilde{\beta}$ implies $LG(\widetilde{\alpha}) \leqslant LG(\widetilde{\beta})$.

2.2 The TOPSIS method

Mathematically speaking, the MCDM problem about m alternatives with n criteria can be expressed as

$$\begin{array}{c} \\ u_1 \\ u_2 \\ \vdots \\ u_m \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{pmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix} \end{array}, \qquad (7)$$

where $U = \{u_1, u_2, \ldots, u_m\}$ is the set of alternatives; $C_r = \{c_1, c_2, \ldots, c_n\}$ is the set of criteria; r_{ij} for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ is the evaluation information of alternative u_i under criterion c_j provided by experts, and what follows is to assign an overall evaluation value to each alternative by trading off techniques, i.e., the TOPSIS analysis procedure as:

1. Choose positive ideal solution(PIS) and negative ideal solution(NIS) as
$$PIS = \{r_1^+, r_2^+, \ldots, r_n^+\} \text{ and } NIS = \{r_1^-, r_2^-, \ldots, r_n^-\}, \qquad (8)$$
where r_j^+ represents the obtainable maximum value under c_j if c_j is beneficial criterion(larger is better), otherwise r_j^+ is the minimum value; r_j^- represents the obtainable minimum value under c_j if c_j is costly criterion(smaller is better), otherwise r_j^- is the maximum value.
2. Calculate the separation from the PIS and NIS between alternatives by
$$D_i^+ = \sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^+)^2} \text{ and } D_i^- = \sqrt{\sum_{j=1}^{n} (r_{ij} - r_j^-)^2} \qquad (9)$$
for $i = 1, 2, \ldots, m$.
3. Calculate the overall score of each alternative by
$$D_i = \frac{D_i^-}{D_i^+ + D_i^-}. \qquad (10)$$
4. Finally, the preferred orders can be obtained according to descending order to choose the best alternatives and the decision makers can make a choice, where $i = 1, 2, \ldots, m$.

3 MCDM with interval-valued intuitionistic fuzzy sets

With the ranking procedure for interval-valued intuitionistic fuzzy numbers, we are able to describe a TOPSIS-based new version of MCDM with interval-valued intuitionistic fuzzy evaluation information from three aspects: the determination of weights of criteria for each possible alternative, the construction of score distance which is applied to measure the distance between alternative and positive/negative ideal solution, the detailed procedure of the new decision making method.

3.1 Criteria weights determination

During the process of MCDM, an interesting issue is the determination of the weighted vectors associated with each criterion. The more important the attribute plays in decision making, the greater the weight of it is, and vice versa. But how to obtain the ideal weights is still on the air. In what follows we propose a new method, from the view point of ranking interval-valued intuitionistic fuzzy numbers, to determine the weights of criteria. And the detailed process of the proposed method can be introduced as follows:

1. State problem: given that there is a MCDM problem and the evaluation information is expressed as Eq. (7), where $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ is the evaluation information of alternative i under criterion j. Moreover, $\mathbb{B}$ is the beneficial criteria set and $\mathbb{C}$ is the cost criteria set, such that $\mathbb{B} \bigcap \mathbb{C} = \emptyset$ and $\mathbb{B} \bigcup \mathbb{C} = C_r$.
2. Rank evaluation information: rank r_{ij} for $c_j \in \mathbb{B}$ and $c_j \in \mathbb{C}$ separately, according to score function $S(*)$, accuracy function $H(*)$, membership uncertainty index function $T(*)$ and hesitation uncertainty index function $G(*)$.
3. Calculate score: compute score of each alternative $LG(r_{ij})$ by Eq. (6) for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, where δ can be determined by the order of r_{ij}.
4. Determine criteria weights:
   - If $\mathbb{B} = C_r$, i.e., all criteria are beneficial criteria, then

$$\omega_{ij} = \frac{LG(r_{ij})}{LG(r_{i1}) + LG(r_{i2}) + \cdots + LG(r_{in})}, \qquad (11)$$

   in which case $\omega_i = (\omega_{i1}, \omega_{i2}, \ldots, \omega_{in})$ constitutes the weighted vector of alternative u_i under all criteria.
   - If $\mathbb{C} = C_r$, i.e., all criteria are cost criteria, then

$$\omega_{ij} = \frac{1 - LG(r_{ij})}{n - LG(r_{i1}) - LG(r_{i2}) - \cdots - LG(r_{in})}. \qquad (12)$$

- If $\mathbb{B} \neq \emptyset$ and $\mathbb{C} \neq \emptyset$, then compute the weights of criteria $c_j \in \mathbb{B}$ by

$$\omega_{ij} = \frac{LG(r_{ij})}{LG(r_{ij_1}) + \cdots + LG(r_{ij_n}) + (n - j_n) - LG(r_{ij^1}) - \cdots - LG(r_{ij^n})} \qquad (13)$$

and the weights of $c_j \in \mathbb{C}$ by

$$\omega_{ij} = \frac{1 - LG(r_{ij})}{LG(r_{ij_1}) + \cdots + LG(r_{ij_n}) + (n - j_n) - LG(r_{ij^1}) - \cdots - LG(r_{ij^n})}, \qquad (14)$$

where $c_{j_1}, c_{j_2}, \ldots, c_{j_n} \in \mathbb{B}$ and $c_{j^1}, c_{j^2}, \ldots, c_{j^n} \in \mathbb{C}$ such that $j_n + j^n = n$.

Example 1 Given that there is a MCDM problem with three alternatives and five beneficial criteria under interval-valued intuitionistic fuzzy environment. And the concrete evaluation information is proposed by experts as

	u_1	u_2	u_3
c_1	$([0.25, 0.25], [0.75, 0.75])$	$([0.75, 0.75], [0.25, 0.25])$	$([0.50, 0.50], [0.50, 0.50])$
c_2	$([0.80, 0.80], [0.10, 0.10])$	$([0.70, 0.70], [0.10, 0.10])$	$([0.30, 0.30], [0.40, 0.40])$
c_3	$([0.50, 0.50], [0.50, 0.50])$	$([1.00, 1.00], [0.00, 0.00])$	$([0.75, 0.75], [0.25, 0.25])$
c_4	$([0.30, 0.30], [0.60, 0.60])$	$([0.20, 0.20], [0.10, 0.10])$	$([0.50, 0.50], [0.00, 0.00])$
c_5	$([0.50, 0.70], [0.10, 0.20])$	$([0.20, 0.40], [0.30, 0.50])$	$([0.00, 0.10], [0.60, 0.70])$.

At first, we compute the score of u_1 about $c_1, c_2, \ldots, c_5$ by Eq. (2) as

$$S(r_{11}) = -0.5, S(r_{12}) = 0.7, S(r_{13}) = 0, \quad S(r_{14}) = -0.25, S(r_{15}) = 0.45,$$

in which case we have $r_{11} \prec r_{14} \prec r_{13} \prec r_{15} \prec r_{12}$. Then calculate $LG(r_{1j})$ for $j = 1, 2, \ldots, 5$ as

$$LG(r_{11}) = 0.25 \quad LG(r_{12}) = 0.84, \quad LG(r_{13}) = 0.5, \quad LG(r_{14}) = 0.37, \quad LG(r_{15}) = 0.7,$$

where $\delta = 0.4$. Hence,

$$\omega_1 = (\omega_{11}, \omega_{12}, \omega_{13}, \omega_{14}, \omega_{15}) = (0.094, 0.3158, 0.188, 0.139, 0.2632).$$

Analogous, the weighted vectors of alternatives u_2 and u_3 can be expressed as

$$\omega_2 = (\omega_{21}, \omega_{22}, \omega_{23}, \omega_{24}, \omega_{25}) = (0.2187, 0.2274, 0.2915, 0.1399, 0.1225)$$

and

$$\omega_3 = (\omega_{31}, \omega_{32}, \omega_{33}, \omega_{34}, \omega_{35}) = (0.1968, 0.1654, 0.2953, 0.2756, 0.0669).$$

Therefore, the whole weights of criteria about this decision making problem is

$$\Omega = (\omega_1, \omega_2, \omega_3)^T = \begin{pmatrix} 0.0940 & 0.3158 & 0.1880 & 0.1390 & 0.2632 \\ 0.2187 & 0.2274 & 0.2915 & 0.1399 & 0.1225 \\ 0.1968 & 0.1654 & 0.2953 & 0.2756 & 0.0669 \end{pmatrix}.$$

3.2 Score distances

One of the key process of TOPSIS is to calculate the distance between the alternative and the maximum ideal solution and the distance between the alternative and the minimum ideal solution. Inspired by literature [25], we introduce score distance between two interval-valued intuitionistic fuzzy numbers as follows.

Definition 6 Let $\widetilde{\alpha}$ and $\widetilde{\beta}$ be two interval-valued intuitionistic fuzzy numbers, then we call

$$d(\widetilde{\alpha}, \widetilde{\beta}) = \frac{1}{2}|S(\widetilde{\alpha}) - S(\widetilde{\beta})| \tag{15}$$

the score distance between the score $S(\widetilde{\alpha})$ of $\widetilde{\alpha}$ and score $S(\widetilde{\beta})$ of $\widetilde{\beta}$.

With Eq. (2) one get that if $\widetilde{\alpha} = ([a_1, b_1], [c_1, d_1])$ and $\widetilde{\beta} = ([a_2, b_2], [c_2, d_2])$, then the score distance between $\widetilde{\alpha}$ and $\widetilde{\beta}$ is

$$\begin{aligned} d(\widetilde{\alpha}, \widetilde{\beta}) &= \frac{1}{2}|S(\widetilde{\alpha}) - S(\widetilde{\beta})| \\ &= \frac{1}{4}|(a_1 + b_1 - a_2 - b_2) - (c_1 + d_1 - c_2 - d_2)|. \end{aligned} \tag{16}$$

Because the maximum interval-valued intuitionistic fuzzy ideal solution and the minimum interval-valued intuitionistic fuzzy ideal solution can be distinguished only with score function $S(*)$ from arbitrary interval-valued intuitionistic fuzzy number. Therefore, the above constructed score distance is fully capable of measuring the distance between each possible alternative and the ideal solution.

In order to make a detailed discussion on score distance, we first introduce the notion of interval-valued intuitionistic fuzzy vector here.

Definition 7 Let $\widetilde{A} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n)$ be a n-dimensional vector, if each component of it is an interval-valued intuitionistic fuzzy number, then we call $\widetilde{A}$ an interval-valued intuitionistic fuzzy vector.

From above definition, the score distance between any two interval-valued intuitionistic fuzzy vectors can be defined as

Definition 8 Let $\widetilde{A} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n)$ and $\widetilde{B} = (\widetilde{\beta}_1, \widetilde{\beta}_2, \ldots, \widetilde{\beta}_n)$ be two interval-valued intuitionistic fuzzy vectors, then we call

$$d(\widetilde{A}, \widetilde{B}) = \left[\sum_{j=1}^{n} (\omega_j d(\widetilde{\alpha}_j, \widetilde{\beta}_j))^{\lambda}\right]^{1/\lambda} \tag{17}$$

the weighted score distance between $\widetilde{A}$ and $\widetilde{B}$ with respect to parameter λ, where ω_j is the weight of component $\widetilde{\alpha}_j$ with $\sum_{j=1}^{n} \omega_j = 1$, and $\lambda \in [1, +\infty)$.

Based on above definition, we can drive the following conclusions:

Theorem 1 Let $\widetilde{A} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n)$ be an interval-valued intuitionistic fuzzy vector, then $d(\widetilde{A}, \widetilde{A}) = 0$.

Proof It can be proved easily by Definition 6 and Definition 8.

This completes the proof.

Theorem 2 Let $\widetilde{A} = (\widetilde{\alpha}_1, \widetilde{\alpha}_2, \ldots, \widetilde{\alpha}_n)$ and $\widetilde{B} = (\widetilde{\beta}_1, \widetilde{\beta}_2, \ldots, \widetilde{\beta}_n)$ be two interval-valued intuitionistic fuzzy vectors, then $d(\widetilde{A}, \widetilde{B}) = d(\widetilde{B}, \widetilde{A})$.

Proof With Eq. (15) we have that $d(\widetilde{\alpha}_j, \widetilde{\beta}_j) = d(\widetilde{\beta}_j, \widetilde{\alpha}_j)$ for any $\widetilde{\alpha}_j \in \widetilde{A}$ and $\widetilde{\beta}_j \in \widetilde{B}$. Thus, $d(\widetilde{A}, \widetilde{B}) = d(\widetilde{B}, \widetilde{A})$ can be derived from Eq. (17).

This completes the proof.

It is trivial but interesting that we can use the above defined score distance to measure any two interval-valued intuitionistic fuzzy vectors with precondition that the two interval-valued intuitionistic fuzzy vectors can be distinguished from each other by score function. If it can not be differentiated from score function, the accuracy function based accuracy distance will be constructed to measure it. In fact, it can be found that the score distance is adequate for the problem of TOPSIS.

3.3 TOPSIS-based MCDM with interval-valued intuitionistic fuzzy numbers

Since Hwang and Yoon proposed TOPSIS method, it has found its successful applications in many fields, especially for the problem of MCDM. The main idea of this technique is to compare every alternative with the positive and negative ideal solution. One alternative will be selected as the best choice if it is the nearest to the positive ideal solution and the farthest to the negative ideal solution [24]. Hereinafter, the detailed procedure for TOPSIS-based MCDM with interval-valued intuitionistic fuzzy information will be constructed as follows.

Problem statement: Given that there is a MCDM problem with m possible alternatives and n criteria. The evaluation information of it is described as Eq. (7). Parameters λ and θ (the function of it will be explained in the following) are predefined by decision maker's intension, and δ can be determined according to the ranking results of evaluation information by score function, accuracy function, etc.

Decision process:

1. Determine beneficial criteria set $\mathbb{B}$ and cost criteria set $\mathbb{C}$ such that $|\mathbb{B}| + |\mathbb{C}| = n$ and $\mathbb{B} \bigcap \mathbb{C} = \emptyset$.
2. Rank evaluation information of each alternative with Eqs. (2)–(5), from which the one can obtain the value of LG for each criteria by Eq. (6).
3. Determine the weights of criteria for each alternative by Eqs. (11)–(14)
4. Determine ideal solution: for $c_j \in \mathbb{B}$, the positive ideal solution is $r_j^+ = ([1, 1], [0, 0])$ and the negative ideal solution is $r_j^- = ([0, 0], [1, 1])$; for $c_j \in \mathbb{C}$, the positive ideal solution is $r_j^+ = ([0, 0], [1, 1])$ and the negative ideal solution is $r_j^- = ([1, 1], [0, 0])$.
5. According to Eq. (17), calculate the separation of each possible alternative to positive ideal solution and negative ideal solution by

$$D_i^+ = \left(\sum_{j=1}^{n} (\omega_{ij} d(r_{ij}, r_j^+))^\lambda \right)^{1/\lambda} \tag{18}$$

and

$$D_i^- = \left(\sum_{j=1}^{n} (\omega_{ij} d(r_{ij}, r_j^-))^\lambda \right)^{1/\lambda} \tag{19}$$

for $i = 1, 2, \ldots, m$.

6. Calculate the performance score of each alternative by equation

$$D_i = \theta D_i^+ + (1 - \theta) D_i^- \tag{20}$$

where θ is the decision maker's preference to the positive ideal solution. If the separation of each alternative to positive ideal solution is paid more attention, then θ is bigger than 1 − θ, that is, θ > 0.5; else, θ < 0.5.

7. Rank D_i for $i = 1, 2, \ldots, m$ and select the best or worst alternative.

Remark 1 In fact, the proposed procedure for selecting best or worst possible alternative endows certain flexibility to decision makers. Such as the selection of parameter θ, in which case it is more suitable for resolving of practical decision problems.

4 Illustrative example

In this section we consider a synthetic problem concerning the MCDM for interval-valued intuitionistic fuzzy numbers which is used to make a best option by a speculative enterpriser.

Given that there are four possible alternatives for an enterprise to select: a car company (u_1), a food company (u_2), a clothes company (u_3) and a training center (u_4). The factors that must be considered before gathering evaluation information conclude risk (c_1), benefit (c_2), social and political response (c_3) and environment (c_4). Once the possible alternatives and corresponding factors is determined, the concrete evaluation information can be estimated by experts who are commissioned by the investor. Because of the limitation of knowledge, the expert utilizes an interval-valued intuitionistic fuzzy number to express their judgment. Finally, the assessment report about this problem can be listed as

In what follows, we use our proposed decision making method to make an advisable decision making.

Step (1): Generally speaking, higher risk usually provides higher benefit, and vice versa. Here, we suppose that they are independent. Therefore, "risk (c_1)" can be regarded as a cost criterion, and "benefit (c_2)" can be regarded as a beneficial criterion. In addition, the investor wishes to get more support from society and government for their investment, thus the "social and political response (c_3)" can be regarded as a beneficial criterion. And needless to say, "environment (c_4)" is a beneficial criterion.

Step (2): After ranking the evaluation information for each alternative, the value of LG can be expressed as

$$LG = \begin{pmatrix} 0.2700 & 0.7200 & 0.6940 & 0.5080 \\ 0.4850 & 0.5175 & 0.5200 & 0.7613 \\ 0.4525 & 0.4750 & 0.6010 & 0.5450 \\ 0.4350 & 0.5850 & 0.7850 & 0.6363 \end{pmatrix},$$

where δ = 0.35.

Step (3): By corresponding calculation, we have that the weights of criteria is

Table 1 The performance value report of MCDM

	c_1	c_2
u_1	([0.10, 0.30], [0.50, 0.70])	([0.60, 0.70], [0.05, 0.25])
u_2	([0.40, 0.50], [0.40, 0.50])	([0.50, 0.50], [0.40, 0.50])
u_3	([0.30, 0.50], [0.40, 0.50])	([0.20, 0.40], [0.10, 0.30])
u_4	([0.40, 0.40], [0.50, 0.50])	([0.50, 0.60], [0.30, 0.40])
	c_3	c_4
u_1	([0.55, 0.75], [0.2, 0.25])	([0.20, 0.50], [0.15, 0.25])
u_2	([0.30, 0.60], [0.3, 0.40])	([0.60, 0.80], [0.10, 0.15])
u_3	([0.55, 0.60], [0.3, 0.40])	([0.40, 0.55], [0.30, 0.35])
u_4	([0.70, 0.80], [0.1, 0.20])	([0.50, 0.65], [0.20, 0.30])

$$\Omega = \begin{pmatrix} 0.2753 & 0.2715 & 0.2617 & 0.1915 \\ 0.2226 & 0.2237 & 0.2247 & 0.3290 \\ 0.2525 & 0.2190 & 0.2772 & 0.2513 \\ 0.2197 & 0.2275 & 0.3053 & 0.2475 \end{pmatrix}.$$

Step (4): Because criterion c_1 is cost and c_2, c_3 and c_4 are beneficial, then the positive ideal solution and negative ideal solution are

$$\begin{aligned} r_1^+ &= ([0,0],[1,1]), & r_1^- &= ([1,1],[0,0]); \\ r_2^+ &= ([1,1],[0,0]), & r_1^- &= ([0,0],[1,1]); \\ r_3^+ &= ([1,1],[0,0]), & r_1^- &= ([0,0],[1,1]); \\ r_4^+ &= ([1,1],[0,0]), & r_1^- &= ([0,0],[1,1]). \end{aligned}$$

Step (5): Compute the separation of each possible alternative to positive ideal solution and negative ideal solution by Eqs. (18) and (19):

$$D_1^+ = 0.0770 \quad D_2^+ = 0.0985$$
$$D_3^+ = 0.1084 \quad D_4^+ = 0.0848$$

and

$$D_1^- = 0.1771 \quad D_2^- = 0.1648$$
$$D_3^- = 0.1430 \quad D_4^- = 0.1730$$

where $\lambda = 2$.

Step (6): Compute the whole performance of each alternative with $\theta = 0.7$:

$$D_1 = 0.1070 \quad D_2 = 0.1184$$
$$D_3 = 0.1188 \quad D_4 = 0.1113$$

Step (7): With above steps, we have that $D_3 \succ D_2 \succ D_4 \succ D_1$, where the notation "$\succ$" means superior to. Therefore, the alternative u_3(clothes company) is the best choice for the enterpriser.

Remark 2 It is well-known that in literature [39] Ye also proposed an extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection. Next, we make a brief comparison between these two methods. The similarities and differences of these two methods are listed as follows:

- Both of their estimation information are interval-valued intuitionistic fuzzy numbers.
- For our proposed method, the weights of criteria are determined by estimation information provided by experts. For Ye's method, the weights of criteria are proposed by decision maker randomly.
- The finial order of alternatives in our method can be changed by θ, where the value of θ is the decision maker's preference to the positive ideal solution. But, the order of alternatives in Ye's method is computed by traditional TOPSIS method.
- In our method, one expert is commissioned by the investor, or all evaluation information from some experts is combined into one before delivering it to decision maker. In Ye's method, it is a group decision making problem.
- Generally speaking, during the process of decision making, in our method decision maker only need to provide the parameter θ. And in Ye's method, the weights of criteria($w = (w_1, w_2, \ldots, w_n)$) is predefined by decision makers.

With above comparison, one have that if we pay equal attention to four criteria in Ye's method, i.e., $w_1 = w_2 = w_3 = w_4 = 0.25$, then we have that the closeness coefficient of the four alternatives are

$$D_1 = 0.5356 \quad D_2 = 0.5000$$
$$D_3 = 0.5502 \quad D_4 = 0.6370$$

in which case $D_4 \succ D_3 \succ D_1 \succ D_2$.

5 Conclusion

In this paper we discussed the problem of multicriteria decision making from viewpoint of the determination of weights of criteria. Based on detailed discussion on the existing ranking methods for interval-valued intuitionistic fuzzy numbers the score-oriented weights determination method was proposed and in addition, we proposed a similarity measure method, so called score distance, to measure the difference between alternatives and ideal solution, in which case extended TOPSIS method for multicriteria decision making under interval-valued intuitionistic fuzzy numbers was introduced carefully. And an illustrative example was applied to show the validity of the proposed decision making method. These results will be helpful for the issue of how to determine the weights of criteria during the process of decision making analysis.

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