

Multiobjective optimal power flow using a fuzzy based grenade explosion method

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Received: 26 January 2016 / Accepted: 2 May 2016 / Published online: 23 May 2016
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Abstract The aim of this paper is to solve the multiobjective optimal power flow (MOPF) problem using a new metaheuristic that is the grenade explosion method. The MOPF problem is formulated by assuming that the decision maker may have a fuzzy goal for each of the objective functions. Six objectives are considered which are: the minimization of generation fuel cost, the improvement of voltage profile, the enhancement of voltage stability, the reduction of emission and the minimization of active and reactive transmission losses. The proposed approach has been tested on the IEEE 30-bus test system. The obtained results show the effectiveness of the proposed method.

Keywords Multiobjective optimal power flow · Power system optimization · Grenade explosion method · Fuzzy logic

1 Introduction

Over the past half-century, optimal power flow (OPF) has been one of the foremost research works for the power system community. It has become the heart of economically efficient and reliable independent system operator power markets [1].

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The conventional power flow problem can be stated by specifying loads at PQ buses, generated powers and voltage magnitudes at PV buses in addition to a complete topological description of the network. The objective of such problem is to determine voltages and then to compute all the other quantities like currents, line flows and losses [2]. However, in the OPF problem, the generated powers have to be adjusted according to some criteria, for instance the minimum generating cost and under some constraints like power flow constraints. Therefore, the main purpose of the OPF problem is to give the optimal settings of a power system (generated powers, voltages at PV buses,...) by optimizing an objective function (minimum cost, minimum losses,...) while satisfying some equality and inequality constraints (power flow, operating limits of system components, security constraints...) [3].

Even 50 years after the problem was first formulated, researchers worldwide still have thirst for solving the OPF problem. Because it is complex economically, electrically and computationally [1]. The first methods that have been used to solve the OPF problem are deterministic or traditional ones. A detailed survey of such methods is given in [4]. It is reported in that paper that at present, the most powerful deterministic methods for OPF are: active-set sequential linear programming (SLP), sequential quadratic programming (SQP) methods and variants of primal-dual interior point methods (PDIPMs) [4]. Furthermore, some other approaches are tested. Some researchers try to reformulate the classical OPF to a semidefinite programming (SDP) model, in order to take advantage of the SDP technique as in [5]. Other researchers have proposed Branched and Bound method combining with SDP as in [6]. However, traditional methods have shown some limitations as reported in the literature [4, 7, 8].

Even tough, many works are still conducted in the field of traditional methods, progressively metaheuristics are becoming a serious and a reliable alternative for solving the OPF problem. Some examples of metaheuristics used to solve the OPF problem are: evolutionary programming (EP) [9], genetic algorithm (GA) [10], particle swarm optimization (PSO) [11], tabu search (TS) [12], simulated annealing (SA) [13], differential evolution (DE) [14], biogeography-based optimization (BBO) [15], imperialist competitive algorithm (ICA) [16], league championship algorithm (LCA) [17], black-hole-based optimization (BHBO) [18], teaching-learning-based optimization (TLBO) [19], differential search algorithm (DSA) [20], colliding bodies optimization (CBO) [21], electromagnetism-like mechanism (EM) [22] and backtracking search optimization (BSA) [23]. A review of many metaheuristics applied to solve the OPF problem is given in [7, 8].

However, due to the varied formulations and objectives of the OPF problem, no algorithm is the best in solving all OPF problems. Therefore, there is always a need for a new algorithm which can solve some of the OPF problems efficiently. Furthermore, in recent years, the field of metaheuristics has witnessed a true tsunami of novel methods [24]. These new metaheuristics are very efficient since they are tested on hard benchmarks and compared with many other methods. Therefore, this paper aims to apply a new metaheuristic, which has not received yet much attention in the power systems community that is the grenade explosion method (GEM) in order to solve the OPF problem.

The GEM is new metaheuristic based on the mechanism of grenade explosion [25,26]. Ahrari et al., demonstrate in [25,26] the superiority of the GEM method compared with other popular metaheuristics.

In this paper, the OPF problem is formulated as a multiobjective optimization problem. Six objectives are investigated that are: the minimization of generation fuel cost, the improvement of voltage profile, the enhancement of voltage stability, the reduction of emission and the minimization of active and reactive transmission losses. The objective functions are replaced with membership functions using fuzzification.

The remainder of this paper is organized as follows. First, the problem formulation is presented in Sect. 2. Then, the GEM method is presented in Sect. 3. Next, the results after solving different cases of OPF (single objective and multiobjective cases) using GEM are discussed in Sect. 4. Finally, the conclusions are drawn in the last section of this paper.

2 Problem formulation

The OPF is a power flow problem which gives the optimal settings of the control variables for a given settings of load by minimizing a predefined objective function such as the cost of active power generation. The OPF considers the operating limits of the system and it can be formulated as a nonlinear constrained optimization problem as follows:

$$\begin{aligned} & \text{Minimize } f(x,u) \\ & \text{Subject to } g(x, u) = 0 \\ & \text{and } h(x, u) \leq 0 \end{aligned} \quad (1)$$

where u is the vector of independent variables or control variables, x is the vector of dependent variables or state variables, $f(x, u)$ is the objective function, $g(x, u)$ is the set of equality constraints, and $h(x, u)$ is the set of inequality constraints.

In the multiobjective optimal power flow (MOPF) problem, instead of only one objective function a vector of objective functions is optimized. Therefore, the MOPF can be formulated as follows:

$$\begin{aligned} & \text{Minimize } F(x,u) = [f_1(x,u), f_2(x,u), \dots, f_k(x,u)]^T \\ & \text{Subject to } g(x,u) = 0 \\ & \text{and } h(x,u) \leq 0 \end{aligned} \quad (2)$$

where $F(x, u)$ is the vector of objective functions and k is the number of objective functions.

2.1 Design variables

2.1.1 Control variables

These are the set of variables which can be modified to satisfy the load flow equations. The set of control variables in the OPF problem formulation are:

P_G : active power generation at PV buses except the slack bus.

V_G : voltage magnitudes at PV buses.

T: tap settings of transformers.

Q_C : shunt VAR compensation.

Hence, u can be expressed as:

$$u^T = [P_{G_2} \dots P_{G_{NG}}, V_{G_1} \dots V_{G_{NG}}, Q_{C_1} \dots Q_{C_{NC}}, T_1 \dots T_{NT}] \quad (3)$$

where NG, NT and NC are the number of generators, the number of regulating transformers and the number of VAR compensators, respectively.

2.1.2 State variables

These are the set of variables which describe any unique state of the system. The set of state variables for the OPF problem formulation are:

P_{G_1} : active power generation at slack bus.

V_L : voltage magnitudes at PQ buses or load buses.

Q_G : reactive power output of all generator units.

S_l : transmission line loadings (or line flow).

Hence, x can be expressed as:

$$x^T = [P_{G_1}, V_{L_1} \dots V_{L_{NL}}, Q_{G_1} \dots Q_{G_{NG}}, S_{l_1} \dots S_{l_{nl}}] \quad (4)$$

where NL, and nl are the number of load buses, and the number of transmission lines, respectively.

2.2 Objective functions

Many objective functions can be found in the literature for the OPF problem. In this paper, six objective functions are studied. These objectives are discussed in the following sections.

2.2.1 Objective function # 1: generation fuel cost

The generation fuel cost is usually expressed by a quadratic function as follows:

$$C = \sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \quad (5)$$

It is worth to mention here that the cost function given by (3) is convex. However, in some cases the cost function can be non-convex for instance when multi-fuels options are used as in [19,21].

2.2.2 Objective function # 2: voltage deviation

Bus voltage is one of the most important and significant indication of safety and service quality. The voltage profile can be improved by minimizing the load bus voltage deviations (VD) from 1.0 p.u, which is given by:

$$VD = \sum_{i=1}^{NL} |V_{L_i} - 1.0| \quad (6)$$

2.2.3 Objective function # 3: voltage stability index

Prediction of voltage instability is an issue of paramount importance. In [27] Kessel and Glavitch have developed a voltage stability index referred to as Lmax which is defined based on local indicators L_j and it is given by:

$$L_{max} = \max(L_j) \quad j = 1, 2, \dots, NL \quad (7)$$

where L_j is the local indicator of bus j and it is represented as follows:

$$L_j = \left| 1 - \sum_{i=1}^{NG} H_{LG_{ji}} \frac{V_i}{V_j} \right| \quad j = 1, 2, \dots, NL \quad (8)$$

where H matrix is generated by the partial inversion of Y_{bus} . More details can be found in [27].

The Lmax varies between 0 and 1 and the lower this indicator is the more the system is stable. Hence, in order to enhance the voltage stability, Lmax has to be minimized.

2.2.4 Objective function # 4: emission

There is an increasing worldwide emphasizes on environmental pollution reduction and control [28]. Therefore, it is desirable to adjust the OPF to account for emission which will drastically affects the whole system operation [28,29].

The total ton/h emission (E) of the atmospheric pollutants such as sulfur oxides SO_x and nitrogen oxides NO_x caused by fossil-fueled thermal units can be expressed by [29,30]:

$$E = \sum_{i=1}^{NG} 10^{-2} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) + (\omega_i e^{\mu_i P_{G_i}}) \quad (9)$$

where α_i , β_i , γ_i , ω_i and μ_i are coefficients of the i th generator emission characteristics.

2.2.5 Objective function # 5: active power transmission losses

The active power losses can be expressed by:

$$PL = \sum_{i=1}^{NB} P_i = \sum_{i=1}^{NB} P_{Gi} - \sum_{i=1}^{NB} P_{Di} \quad (10)$$

where NB is the number of busses and P_D is the active load demand.

2.2.6 Objective function # 6: reactive power transmission losses

The reactive power losses can be expressed by:

$$QL = \sum_{i=1}^{NB} Q_i = \sum_{i=1}^{NB} Q_{Gi} - \sum_{i=1}^{NB} Q_{Di} \quad (11)$$

where Q_D is the reactive load demand.

2.3 Constraints

OPF constraints can be classified into equality and inequality constraints, as detailed in the following sections.

2.3.1 Equality constraints

(a) Real power constraints:

$$P_{Gi} - P_{Di} - V_i \sum_{j=i}^{NB} V_j [G_{ij} \cos(\theta_{ij}) + B_{ij} \sin(\theta_{ij})] = 0 \quad (12)$$

(b) Reactive power constraints:

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=i}^{NB} V_j [G_{ij} \sin(\theta_{ij}) - B_{ij} \cos(\theta_{ij})] = 0 \quad (13)$$

where $\theta_{ij} = \theta_i - \theta_j$, G_{ij} and B_{ij} are the elements of the admittance matrix ($Y_{ij} = G_{ij} + j B_{ij}$) representing the conductance and susceptance between bus i and bus j , respectively.

2.3.2 Inequality constraints

(a) Generator constraints

$$V_{G_i}^{\min} \leq V_{G_i} \leq V_{G_i}^{\max}, \quad i = 1, \dots, NG \quad (14)$$

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}, \quad i = 1, \dots, NG \quad (15)$$

$$Q_{G_i}^{\min} \leq Q_{G_i} \leq Q_{G_i}^{\max}, \quad i = 1, \dots, NG \quad (16)$$

(b) Transformer constraints

$$T_i^{\min} \leq T_i \leq T_i^{\max}, \quad i = 1, \dots, NT \quad (17)$$

(c) Shunt VAR compensator constraints

$$Q_{C_i}^{\min} \leq Q_{C_i} \leq Q_{C_i}^{\max}, \quad i = 1, \dots, NC \quad (18)$$

(d) Security constraints

$$V_{L_i}^{\min} \leq V_{L_i} \leq V_{L_i}^{\max}, \quad i = 1, \dots, NL \quad (19)$$

$$S_{l_i} \leq S_{l_i}^{\max}, \quad i = 1, \dots, nl \quad (20)$$

2.3.3 Constraints handling

It is worth mentioning that control variables are self-constrained. The inequality constraints of dependent variables which contain load bus voltage magnitude; real power generation output at slack bus, reactive power generation output and line loading can be included into the objective function as quadratic penalty terms. In these terms, a penalty factor multiplied with the square of the disregard value of dependent variable is added to the objective function [19,20].

It is worth mentioning that, Eqs. (3)–(20) present the complete formulation of the OPF problem adopted in this work. It is worth mentioning that other formulations exist which involve only quadratic and bilinear terms [31].

3 Grenade explosion method (GEM)

3.1 Overview

The GEM is a novel metaheuristic for optimizing real-valued bounded black-box optimization problems inspired by the mechanism of grenade exposition [25,26]. In the GEM, once the grenades explode, the resulting shrapnel hit objects that are located within a neighborhood radius called L_e . The damages caused by shrapnel on objects are calculated. The damage-per-shrapnel value indicates the value of objects in that area. In order to cause more damage, the next grenade is thrown in the location of

the greatest damage that has been caused. The overall damage caused by the hit is considered as the fitness of the solution at the object's location.

Furthermore, the GEM has a unique feature, which is the concept of agent's territory radius (R_t) [25,26]. Each agent (a grenade here), does not allow other agents to come closer than a certain distance that is R_t . Therefore, when several grenades expose in the search space, a high value of R_t guarantees that grenades are spared quit uniformly in the search space while a small value of R_t allows the grenades to search local regions together [25,26].

3.2 Algorithm

The pseudo code of the GEM is given in Algorithm 1. The GEM starts by scaling all independent variables within the interval $[-1,1]$. Then, the problem parameters like the number of grenades (N_g) and the maximum number of iterations ($Max_iterations$) are selected and L_e and R_t are initialized. After that, N_g grenades, distant by R_t from each other, are randomly generated in the n -dimensional scaled space. These grenades are ranked in a descending order based on their fitness. For each grenade N_q pieces of shrapnel are generated using the following expression:

$$X'_j = \{X_m + sign(r_m) \times |r_m|^p \times L_e\}, \quad j = 1, 2, \dots, N_q \quad (21)$$

where X_m is the location of the grenade, r_m is a uniformly distributed random number in $[-1,1]$ and p is a constant used to tune the intensity of the exploration. The value of p is updated using the probability of territory search (T_w) as follows:

$$p = \max \left\{ 1, n \times \frac{\log(\frac{R_t}{L_e})}{\log(T_w)} \right\} \quad (22)$$

While generating the N_q pieces of shrapnel, some produced shrapnel may collide to objects outside the scaled feasible space and they have to be transported to the $[-1,1]^n$ interval.

Then, the damage caused by every piece of shrapnel around a grenade is computed. If the fitness of the best generated point is better than the fitness of the current location of the grenade, the position of the grenade is updated and the grenade moves to the location of the best point.

To increase the global search ability, L_e and R_t are adjusted during the iterations. High values of these parameters are necessary to cover the whole search space in initial iterations, however, they have to be reduced over iterations with taking fitness value into account. The territory radius is updated as follows:

$$R_t = \frac{R_{t-initial}}{(R_{rd})^{(iteration\ Number/Max_iterations)}} \quad (23)$$

where R_{rd} represents the ratio of the value of R_t in the first iteration to its value in the last iteration and it has to be set before the algorithm starts.

Likewise, L_e is decreased over iterations as follows:

$$L_e = (L_{e-initial})^m (R_t)^{1-m}, \quad 0 \leq m \leq 1 \quad (24)$$

where m is calculated using the following expression:

$$m = m_{\max} - \left(\frac{\text{Iteration_number}}{\text{Max_iterations}} \right) (m_{\max} - m_{\min}) \quad (25)$$

In order to save global search ability, the rate of decrease of L_e during the iterations is slower than the one of R_t [25,26]. Once L_e and R_t are adjusted the value of p has to be updated accordingly. Finally, the termination criterion used in the GEM is the number of iterations i.e., if the number of iterations exceeds the maximum value the algorithm stops.

Algorithm 1: the pseudo code of the grenade explosion method [25].

- 1 Scale the independent variables' ranges to $[-1,1]$.
- 2 Select problem parameters: (Ng, Nq, $L_{e-initial}$, $R_{t-initial}$, Rrd, Max_iterations, m_{\max} , m_{\min} , T_w)
- 3 Let ($L_e=L_{e-initial}$, $R_t=R_{t-initial}$, Iteration_number=1)
- 4 Generate Ng grenades in random locations in an n-dimension space $\bar{X}_i \in [-1,1]^n$, ($i = 1, \dots, Ng$) which are at a distance of at least R_t apart from each other.
- 5 **While** Iteration number \leq Max iterations.
 - 6 Arrange the grenades based on their fitness (the best grenade is the first).
 - 7 Let: $i=1$.
 - 8 **While** N_q accepted points are generated around i th grenade.
 - 9 Generate a point (X') around i th grenade according to Eq. (21).
 - 10 If (X') is outside $[-1,1]^n$, transport it into $[-1,1]^n$.
 - 11 If (X') is a distance of at least R_t apart from the location of grenades 1, 2, . . . , $i-1$, then (X') is accepted.
 - 12 **End while** % N_q pieces of shrapnel are thrown.
 - 13 Calculate the fitness of the new generated points around the i th grenade. If the fitness of the best point is better than current location of the i th grenade, move the grenade to the location of the best point
 - 14 Let: $i = i+1$.
 - 15 If $Ng \geq i$ then go to 8.
 - 16 Reduce R_t according to Eq.(23).
 - 17 Calculate exponent m according to Eq. (25)
 - 18 Update L_e according to Eq. (24).
 - 19 Update exponent p according to Eq. (22).
- 20 **End While** % Number of iterations

4 The proposed multiobjective optimization approach

A complete optimal solution that simultaneously optimizes all the objective functions of a Multiobjective Optimization (MOO) problem usually does not exist especially when the objective functions present a conflicting behavior like in MOPF. Therefore, the task is to find a Pareto optimal solution. A point $x_0 \in X$ is said to be a Pareto optimal solution to an optimization problem P if there is no $x \in X$ such that $F(x) \leq F(x_0)$ [32].

For the MOPF problem, considering the vague or fuzzy nature of human judgment, it is quite logic to assume that the decision maker (DM) may have a fuzzy goal for each of the objective functions. Therefore, objective functions can be replaced by fuzzy membership functions.

The following steps describe the proposed MOPF approach in this paper:

Step 1: Minimization and maximization of each objective function separately in order to calculate the individual minimum and maximum values of each objective function under constraints.

Step 2: Computation of the membership functions $\mu_1, \mu_2, \dots, \mu_k$ by taking into account the calculated individual maximum and maximum values of each objective function in step 1.

Step 3: Computation of the aggregation function $\mu_D(\mu_1, \mu_2, \dots, \mu_k)$.

Step 4: Maximization of μ_D using the GEM method.

Step 5: If the DM is satisfied with the current results i.e., the values of the membership functions, go to step 6. Otherwise, modify the aggregation function then go to step 4.

Step 6: Stop and print optimal results.

The first step of the of the proposed approach consists of minimizing then maximizing each objective function separately in order to compute the maximum and minimum values of each objective function. These two values correspond to the best and worst values that can each objective function reach, respectively. Then, using these values, fuzzy membership functions are calculated. The membership function of the i th objective is defined by [32]:

$$\mu_i = \begin{cases} 1 & f_i \leq f_i^{\min} \\ \frac{f_i^{\max} - f_i}{f_i^{\max} - f_i^{\min}} & f_i^{\min} < f_i < f_i^{\max} \\ 0 & f_i \geq f_i^{\max} \end{cases} \quad (26)$$

where: f_i^{\max} and f_i^{\min} are the maximum and minimum values of the i th objective function, respectively. Therefore, a membership function of a given objective is equal to 1 if this objective is fully satisfied (Fig. 1).

Once the membership functions are calculated, they are aggregated in one objective function as shown in Fig. 2. The aggregation function can be seen as the satisfaction degree of the objective defined by the user [33].

There are many aggregation methods that can be used. In this paper four methods are investigated.

Fig. 1 Linear fuzzy membership function for fuzzy goal

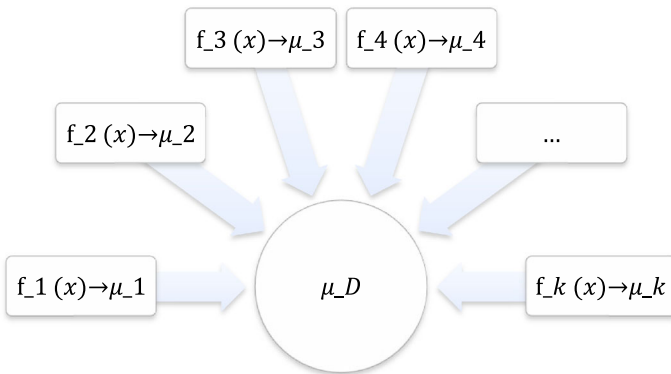
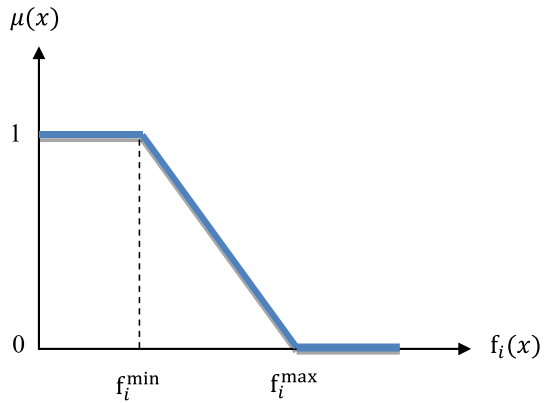


Fig. 2 Flow chart of fuzzy aggregation for the six objective functions

The weighted sum method: The first aggregation function investigated in this paper is the most evident one that is the weighted sum method where the membership functions are multiplied by weights then summed to give an aggregation function as follows:

$$\mu_d = \sum_{i=1}^k \omega_i \mu_i \tag{27}$$

where $\omega_1, \omega_1, \omega_2, \dots, \omega_k$ are weighting factors or weights. Usually, the weights are greater than or equal to 0 and their sum equals 1.

The exponential weighted method: The second aggregation function considered in this study is the exponential weighted method. This method has been proposed due to the inability of the weighted sum method to capture points on non-convex portions of the Pareto optimal surface [34]. Therefore, μ_D can be expressed by:

$$\mu_d = \sum_{i=1}^k (e^{p\omega_i} - 1) e^{p\mu_i} \tag{28}$$

where $p = 1$ in this study.

The minimum operator: For the third aggregation function, we consider the well-known fuzzy decision of Belmann and Zadeh or the minimum operator as follows [33]:

$$\mu_d = \min(\mu_1, \mu_2, \dots, \mu_k) \quad (29)$$

The ε -constraint method: In many cases, all objective functions do not have the same importance, for instance reducing the generation fuel cost is usually more important than reducing losses or emission. In fact, the fuel cost minimization is the most important objective to achieve. However, at the same time the remaining objectives have to be taken into consideration in a way or another. In this case using a method like the ε -constraint method is very interesting. This method (also called the e-constraint or trade-off method) optimizes one of the objective functions while imposing constraints on the remaining objective functions. In other words, it minimizes one objective function (the most important one) and at the same time it maintains acceptable levels for the remaining objective functions [35]. Thus, the aggregation function can be expressed by:

$$\mu_d = \begin{matrix} \mu_1 \\ \mu_2 \leq \varepsilon_2 \\ \vdots \\ \mu_k \leq \varepsilon_k \end{matrix} \quad (30)$$

where $\{\varepsilon_2, \varepsilon_3, \dots, \varepsilon_k\}$ are the vector of constraints to be chosen by the DM or sometimes it can be imposed by some regulations.

In order to circumvent the numerical problem posed by the constraints they are augmented in into μ_d using the following expression:

$$\mu_d = \mu_{d\text{Augmented}} = \mu_1 - \sum_j \lambda_j (\mu_j - \varepsilon_j)^2 \quad (31)$$

where λ_j are penalty factors and j takes the value of all the violated constraints.

5 Application and results

The proposed GEM method has been applied to the IEEE 30-bus test system. This test system has a total generation capacity of 435 MW and its main characteristics are given in Table 1. Detailed data about these test systems can be derived from [36]. Cost and emission coefficients used in this paper are given in Table 2.

It is worth to mention that, the developed software program is written in the commercial MATLAB computing environment and applied on a 2.20 GHz i7 personal computer with 8.00 GB-RAM using parallel processing to run the 30 different runs.

This paper investigates 12 cases. The first six cases are single objective OPF cases whilst the remaining cases are MOPF cases.

The control parameters of the GEM method are given Table 3. These parameters have been selected after several tests.

Table 1 The main characteristics of the IEEE 30-bus test system

System characteristics	Value	Details
Buses	30	–
Branches	41	–
Generators	6	Buses: 1, 2, 5, 8, 11 and 13
Shunts	9	Buses: 10, 12, 15, 17, 20, 21, 23, 24 and 29
Transformers	4	Branches: 11, 12, 15 and 36
Control variables	24	–

Table 2 Cost and emission coefficients for the IEEE 30-bus test system

Bus	a	b	c	d	β	γ	ω	μ
1	0	2	0.00375	18	–5.554	6.49	2.00E–04	2.857
2	0	1.75	0.0175	16	–6.047	5.638	5.00E–04	3.333
5	0	1	0.0625	14	–5.094	4.586	1.00E–06	8
8	0	3.25	0.00834	12	–3.55	3.38	2.00E–03	2
11	0	3	0.025	13	–5.094	4.586	1.00E–06	8
13	0	3	0.025	13.5	–5.555	5.151	1.00E–05	6.667

Table 3 Control parameters of the GEM method

Name	Description	Value
L_e	Neighborhood radius	10
Max_iterations	Maximum no. of iterations	150
Mmin	Exploitation ratio	0.3
N_g	No. of grenades	4
N_q	No. of shrapnel	50
Psin	Weight of optimal search direction	0.1
Rrd	Reduction of territory radius	100
R_t	Territory radius	5

5.1 Single objective OPF

The single objective OPF cases are:

CASE 1: minimization of generation fuel cost.

CASE 2: improvement of voltage profile by the minimization of voltage deviation.

CASE 3: enhancement of the voltage stability by the minimization of voltage stability index.

CASE 4: reduction of emission.

CASE 5: minimization of active losses.

CASE 6: minimization of reactive losses.

Table 4 Optimal settings of control variables for single objective OPF cases using GEM.

Name	CASE 1	CASE 2	CASE 3	CASE 4	CASE 5	CASE 6
P_{G1}	177.1995	176.6211	139.7908	63.9687	51.4565	52.1519
P_{G2}	48.6683	23.0226	45.5509	67.4512	79.8979	79.9897
P_{G5}	21.3889	41.6352	23.3529	49.9982	49.9960	49.8542
P_{G8}	20.8814	24.4240	34.9950	34.9986	34.9991	34.9443
P_{G11}	11.8876	13.9965	29.9416	29.9997	29.9973	29.5857
P_{G13}	12.0001	12.0107	16.0002	39.9996	39.9395	39.8252
V_{G1}	1.1000	1.0464	1.0992	1.0919	1.0895	1.1000
V_{G2}	1.0803	1.0276	1.0810	1.0834	1.0828	1.0922
V_{G5}	1.0542	1.0162	1.0670	1.0661	1.0640	1.0824
V_{G8}	1.0636	1.0022	1.0717	1.0743	1.0718	1.0909
V_{G11}	1.1000	1.0140	1.0932	1.0993	1.0987	1.0986
V_{G13}	1.0998	1.0207	1.0924	1.0993	1.0996	1.0999
$T_{11(6-9)}$	1.0296	1.0172	0.9985	0.9699	1.0179	0.9943
$T_{12(6-10)}$	0.9006	0.9003	0.9011	0.9758	0.9095	0.9922
$T_{15(4-12)}$	0.9731	0.9867	1.0056	0.9766	0.9715	0.9903
$T_{36(28-27)}$	0.9557	0.9682	0.9583	0.9678	0.9619	0.9888
QC_{10}	4.9963	0.9632	3.1034	3.0313	3.4362	4.2438
QC_{12}	4.9931	0.0116	0.5164	4.0171	4.7518	1.1823
QC_{15}	4.9931	2.3100	4.9002	3.7516	3.5556	3.2978
QC_{17}	4.9977	0.0579	3.0580	4.9944	4.9976	4.2262
QC_{20}	4.3337	4.9897	0.0565	3.7166	3.7225	4.8981
QC_{21}	4.9997	4.9920	1.6048	4.9989	4.8917	0.6604
QC_{23}	2.6505	4.9956	3.4331	2.6582	3.2080	4.7053
QC_{24}	4.9865	4.9859	4.9470	4.9530	4.9583	4.8848
QC_{29}	2.2507	2.5850	4.7145	3.0087	2.1188	3.4300
C (\$/h)	799.0463	840.6457	816.9095	943.6358	966.7473	965.1707
VD	1.9312	0.1163	1.8320	1.9504	1.9755	1.9143
L_{max}	0.1264	0.1487	0.1257	0.1269	0.1265	0.1278
E (ton/h)	0.3665	0.3629	0.2802	0.2048	0.2072	0.2075
PL (MW)	8.6257	8.3101	6.2313	3.0160	2.8863	2.9511
QL (MVar)	-1.1038	1.3925	-10.061	-23.396	-21.798	-24.4826

The proposed GEM has been applied to solve CASE 1 through CASE 6 and the obtained optimal settings are given in Table 4.

In order to assess the effectiveness of the proposed method we have compared it to some other well-known methods. The results of this comparison are displayed in Table 5. For more comparison, and since CASE 1 has been widely investigated in the literature, the results found for this case are compared to those found in the literature in Table 6. It appears that the GEM outperforms many other optimization methods.

Table 5 Comparisons of GEM with other well-known methods using our codes

Method	GEM	GA	PSO	ABC	BBO
C (\$/h)	799.0463	800.3108	799.9409	799.0563	799.0860
VD	0.1163	0.1580	0.1451	0.1073	0.1085
Lmax	0.1257	0.1261	0.1262	0.1256	0.1252
E (ton/h)	0.2048	0.2052	0.2050	0.2047	0.2048
PL (MW)	2.8863	3.1493	3.1437	2.8496	2.8732
QL (MVar)	-24.4826	-21.6345	-23.6355	-24.7876	-24.7480

Table 6 Comparison of simulation results for CASE 1 with those found in the literature

Method	Cost	Method description	References
GEM	799.0463	Grenade explosion method	
BBO	799.1116	Biogeography-based optimization	[15]
LCA	799.1974	League championship algorithm	[17]
DE	799.2891	Differential evolution	[14]
SA	799.45	Simulated annealing	[13]
BHBO	799.9217	Black-hole-based optimization	[18]
EADHDE	800.1579	Genetic evolving ant direction HDE	[37]
EADDE	800.2041	Evolving ant direction differential evolution	[38]
PSO	800.41	Particle swarm optimization	[11]
FPSO	800.72	Fuzzy particle swarm optimization	[39]
IGA	800.805	Improved genetic algorithms	[40]
PSO	800.96	Particle swarm optimization	[39]
GAF	801.21	Fuzzy genetic algorithm	[39]
ICA	801.843	Imperialist competitive algorithm	[16]
EGA	802.06	Enhanced genetic algorithm	[41]
TS	802.2900	Tabu search	[12]
MDE	802.376	Modified differential evolution algorithm	[42]
IEP	802.465	Improved evolutionary programming	[43]
EP	802.62	Evolutionary programming	[9]
RGA	804.02	Refined genetic algorithm	[10]
GM	804.853	Gradient method	[44]
GA	805.94	Genetic algorithm	[10]

Figure 3 shows the evolution of cost and penalty term versus iterations for CASE 1. In Fig. 4 the evolution of the voltage deviation versus iterations for CASE 2 is shown. Furthermore, the voltage profile obtained for CASE 2 is sketched in Fig. 5. The evolution of voltage stability index versus iterations for CASE 3 is given in Fig. 6.

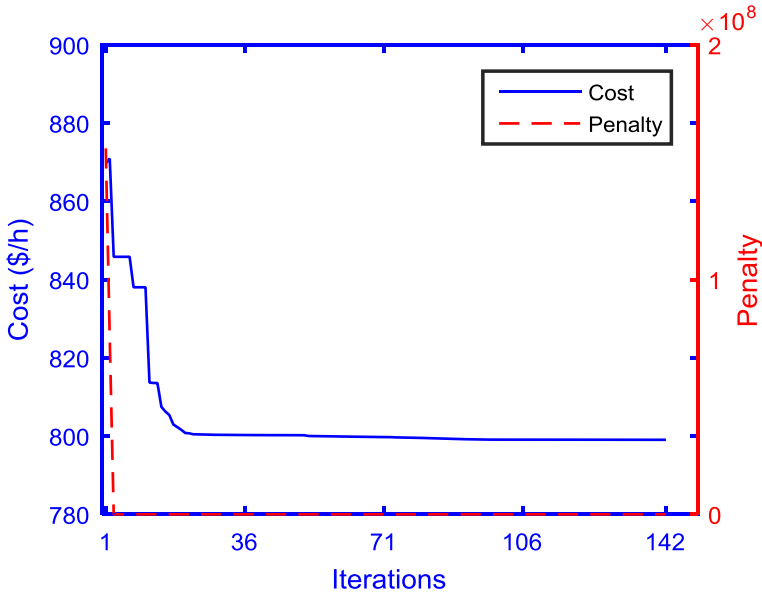


Fig. 3 Evolution of the objective function versus iterations for CASE 1

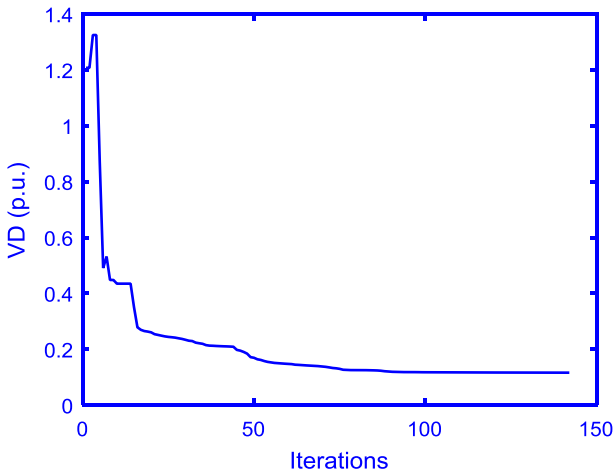


Fig. 4 Evolution of VD versus iterations for CASE 2

Before starting solving the MOPF cases, let us start by analyzing the results found when only one objective is considered at the time. In Table 7, the fuzzy membership functions of all objectives are calculated. We can see from this table that, when an objective function is optimized its membership function equals to 1 i.e., the objective is fully achieved. However, the remaining membership functions lie between 0 and 1. For instance, when the cost is minimized $\mu_E = 0.3061$ and $\mu_{QL} = 0.4605$ while

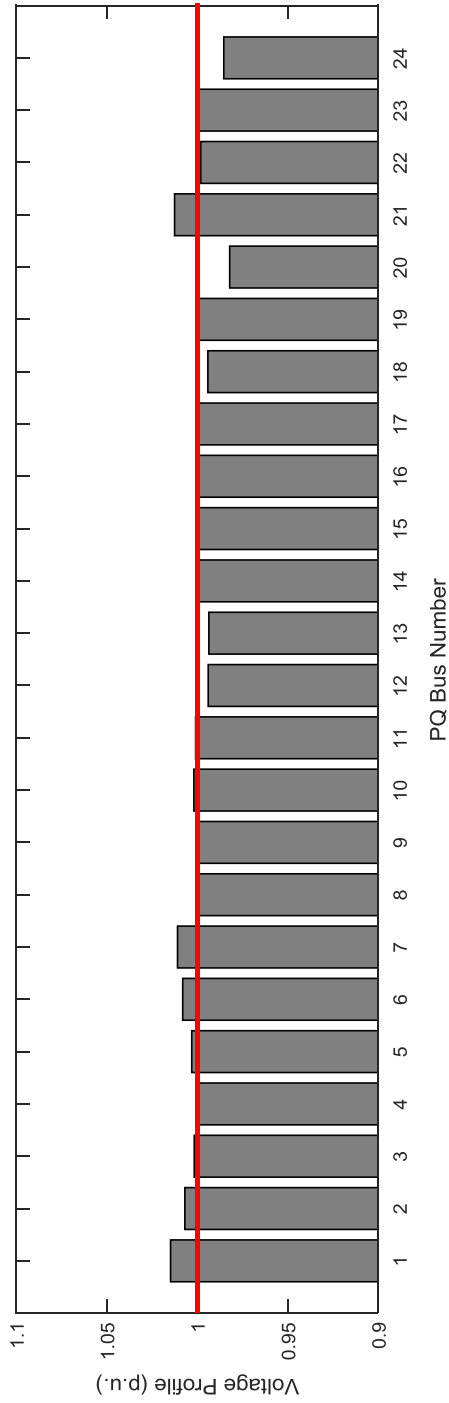


Fig. 5 Voltage profile for CASE 2

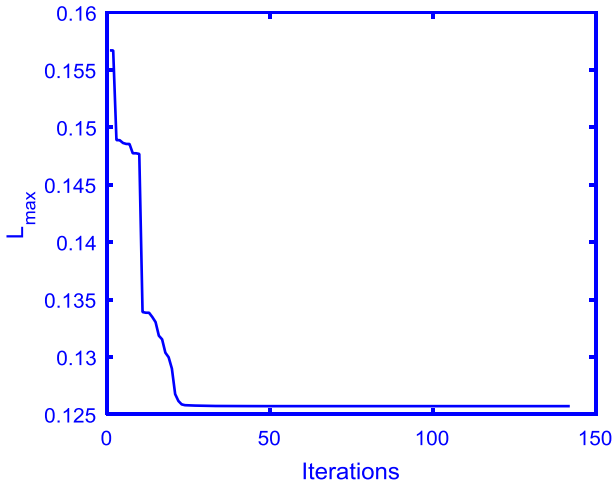


Fig. 6 Evolution of L_{\max} versus iterations for CASE 3

Table 7 Membership functions for the six objectives

	C	VD	L_{\max}	E	PL	QL
C	1	0.0376	0.9790	0.3061	0.4494	0.4451
VD	0.7617	1	0.3028	0.3215	0.4797	0.3859
L_{\max}	0.8977	0.0901	1	0.6762	0.6791	0.6577
E	0.1719	0.0273	0.9639	1	0.9876	0.9742
PL	0.0395	0.0141	0.9762	0.9895	1	0.9363
QL	0.0485	0.0465	0.9362	0.9883	0.9938	1.0000

$\mu_{L_{\max}} = 0.9790$. These values indicate the nature and intensity of the conflict between these objective functions. The same analysis can be made on the remaining objectives.

Furthermore, a statistical analysis has been made in order to compute the correlation coefficients between the six objectives (Table 8). Note that the correlation coefficient (r) is a dimensionless measure of the degree of linear association or dependence between two sets of data X and Y that varies between $[-1, 1]$ where: $r = 0$ indicates that there is no linear dependence between X and Y , $r = 1$ indicates that there is a total dependence with X and Y varying in the same direction and $r = -1$ indicates that there is a total dependence with X and Y varying in the opposite direction [45]. Therefore, the closer the coefficient is to either -1 or 1 , the stronger the correlation between the sets of data. The correlation coefficients can be arranged, into a symmetrical correlation matrix, where each element is the correlation coefficient of the respective column and row variables. Since the matrix is symmetrical, we show only the upper triangular part of this matrix. From Table 8, it appears for example that there is a strong correlation between cost and emission in opposite directions ($r = -0.8948$). the coefficient of correlation computed between emission and active losses ($r = 0.9938$) shows that

Table 8 The matrix of correlation coefficients

	C	VD	Lmax	E	PL	QL
C	1	0.3587	-0.2619	-0.8948	-0.9383	-0.9058
VD		1	-0.9914	-0.6056	-0.5681	-0.6359
Lmax			1	0.5517	0.5021	0.5772
E				1	0.9938	0.9906
PL					1	0.9908
L						1

there is a strong correlation in the same direction between these two objectives. The same analysis can be made on the remaining results.

5.2 Multiobjective OPF

The six MOPF cases investigated in this paper are summarized below:

CASE 7: the weighted sum method is used as an aggregation function with $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5 = \omega_6 = \omega = \frac{1}{6}$.

CASE 8: in this case, the minimum operator is used to generate the aggregation function.

CASE 9: exponential weighted method is used here to generate the aggregation function with $\omega_1 = \omega_2 = \omega_3 = \omega_4 = \omega_5 = \omega_6 = \omega = 1$.

CASE 10: the ϵ -constraint method is explored where the objective to be optimized is the cost and $\epsilon_2 = 0.75, \epsilon_3 = 0.75, \epsilon_4 = 0.75, \epsilon_5 = 0.75$ and $\epsilon_6 = 0.75$ are the constraints that are imposed on the remaining objectives.

CASE 11: the ϵ -constraint method is used where the objective to be optimized is the cost and $\epsilon_2 = 0.75, \epsilon_3 = 0.75, \epsilon_4 = 0.5, \epsilon_5 = 0.5$ and $\epsilon_6 = 0.5$ are the constraints that are imposed on the remaining objectives.

CASE 12: the ϵ -constraint method is tested where the objective to be optimized is the cost and $\epsilon_2 = 0.5, \epsilon_3 = 0.5, \epsilon_4 = 0.5, \epsilon_5 = 0.5$ and $\epsilon_6 = 0.5$ are the constraints that are imposed on the remaining objectives.

In order to compare the relative efficiencies of the different MOO techniques applied to solve MOPF problems, we use the Euclidean distance N , which is defined as follows:

$$N = \left\{ \sum_{i=1}^k [\mu_i - 1]^2 \right\}^{\frac{1}{2}} \tag{32}$$

N gives an indication on how much a solution is close to the utopia point. Therefore, the lower the value of N the better the MOO is [46].

The GEM has been applied to solve the above mentioned MOPF cases and the optimal results are given in Table 9. The N index is given in the last row of Table 9.

From Table 9, we can make the following comments:

Table 9 Optimal settings of control variables for MOPF cases using GEM.

Name	CASE 7	CASE 8	CASE 9	CASE 10	CASE 11	CASE 12
P_{G1}	96.7143	116.5585	193.6970	159.9338	155.5580	159.9338
P_{G2}	58.9434	61.2016	21.9052	51.2471	54.7509	51.2471
P_{G5}	37.5882	41.1760	32.7805	20.3517	24.6556	20.3517
P_{G8}	34.9061	28.8258	10.5019	23.2239	28.2992	23.2239
P_{G11}	29.9594	19.4952	10.0000	16.3598	10.0952	16.3598
P_{G13}	30.1076	21.7421	24.3501	20.0865	18.1389	20.0865
V_{G1}	1.0925	1.0807	1.0839	1.0984	1.0877	1.0984
V_{G2}	1.0817	1.0675	1.0605	1.0796	1.0691	1.0796
V_{G5}	1.0553	1.0382	1.0503	1.0456	1.0353	1.0456
V_{G8}	1.0573	1.0283	1.0461	1.0547	1.0309	1.0547
V_{G11}	1.0275	1.0240	1.0957	1.0463	1.0510	1.0463
V_{G13}	1.0118	1.0385	1.0997	1.0799	1.0256	1.0799
$T_{11(6-9)}$	1.0476	1.0266	0.9007	1.0220	1.0216	1.0220
$T_{12(6-10)}$	1.0241	0.9843	0.9991	0.9129	1.0229	0.9129
$T_{15(4-12)}$	1.0467	1.0106	1.0156	1.0231	0.9994	1.0231
$T_{36(28-27)}$	0.9199	0.9231	1.0994	0.9856	0.9236	0.9856
QC_{10}	4.2395	4.9840	4.9120	1.6421	2.4062	1.6421
QC_{12}	1.4495	2.1116	0.5579	1.6167	2.6837	1.6167
QC_{15}	3.2112	2.5207	4.9640	2.8157	0.5640	2.8157
QC_{17}	3.7457	4.9803	4.9847	1.5851	0.9694	1.5851
QC_{20}	4.3249	4.9990	4.9779	2.2872	2.3416	2.2872
QC_{21}	0.7353	3.1349	0.9259	4.2197	4.8982	4.2197
QC_{23}	3.4977	4.9984	3.6500	4.0903	4.4645	4.0903
QC_{24}	4.7835	4.4227	2.2163	2.7915	2.1013	2.7915
QC_{29}	2.9589	0.6670	3.3934	3.4645	4.4144	3.4645
C (\$/h)	867.2577	849.5033	830.1851	803.7616	806.9091	803.7616
VD	0.6913	0.6612	1.2445	1.0590	0.5877	1.0590
L_{max}	0.1312	0.1352	0.1529	0.1364	0.1340	0.1364
E (ton/h)	0.2237	0.2482	0.4154	0.3213	0.3140	0.3213
PL (MW)	4.8190	5.5993	9.8348	7.8029	8.0978	7.8029
QL (MVar)	-15.8415	-12.3067	3.0400	-4.0942	-3.6115	-4.0942
N	0.5966	0.6605	1.6597	1.0307	0.9177	1.0307

The weighted sum method gives excellent results as shown in CASE 7. Furthermore, it is probably the simplest way to solve the MOPF problem. However, the weights associated with each objective functions have to be carefully selected, which is a cumbersome task for the DM.

The minimum operator method applied in CASE 8 achieves good results ($N = 0.6605$). However, there is no control on the objectives. In other words, this method

treats all objectives as they have the same importance which not the case in MOPF problems.

The exponential weighted method used in CASE 9 gives bad results as shown by the higher value of the Euclidian distance ($N = 1.66$).

The ε -constraint method used in CASE 10, CASE 11 and CASE 12 is very interesting in solving MOPF problems for many reasons. As previously mentioned, in power system planning, minimizing the generating fuel cost is for sure the most important objective however; other issues and/or objectives have to be taken into consideration as additional constraints rather than to be included as objectives. This is more true in complex systems where the DM does not know exactly the values of objective functions and then he cannot tune or select weights like in weighted methods. However, he can impose constraints as percentages of the objective functions even without knowing their values.

6 Conclusion

In this paper, a recently developed metaheuristic, which is the grenade explosion method, is used to solve multiobjective optimal power flow problems. Six objective functions are considered namely: the minimization of generation fuel cost, the improvement of voltage profile, the enhancement of voltage stability, the reduction of emission and the minimization of active and reactive transmission losses. The fuzzy decision making approach is utilized to handle the multiobjective OPF problem. The correlation between different objectives is identified. Many multiobjective approaches are investigated which are the weighted sum method, the exponential weighted method, the minimum operator and the ε -constraint method. The obtained results show the efficiency of the GEM method when solving the OPF problem for single objective and multiobjective cases.

Acknowledgments Dr. M. A. Abido would like to acknowledge the support provided by King Abdulaziz City for Science and Technology (KACST) through the Science and Technology Unit at King Fahd University of Petroleum and Minerals (KFUPM) for funding this work through Project # 14-ENE265-04 as a part of the National Science, Technology and Innovation Plan (NSTIP).

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