Modeling and Measurements of Intermixing Time in a Water Model of a Four Strand Steelmaking Tundish System

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Abstract Intermixing of two different grades of steel in a 17 tonne, 4 strand bloom casting has been investigated in a ∼0.3 scale, geometrically and dynamically similar, isothermal water model tundish system. In this, intermixing phenomena were studied by simulating mixing of flowing water in the tundish, having different dissolved tracer (electrolyte) concentrations. Electrical conductivity measurement technique was employed to monitor concentration of dissolved electrolyte near the strands as a function of time and thereby determine intermixing time. For each experimental condition, three measurements were made and based on such an average intermixing time was estimated. Reproducibility was found to be always within ±15 %. Influence of key operating variables such as, inflow rate from ladle, net outflow rate from tundish, residual liquid volume left over from the previous grade etc. on the duration over which intermixing occurs (referred to as in the text as the intermixing time) was investigated. It was found that any increase in residual volume as well as inflow rate tends to prolong intermixing time. In contrast, influence of outflow rate was quite the opposite. Furthermore, while variation of intermixing time among strands was only marginal, tundish interior design (viz., presence of flow modifiers, pouring box etc.) was found to have considerable influence on intermixing time. Flow phenomena (observed visually through the dispersion of $KMnO_4$ solution) in the given tundish was found to be practically symmetrical about the transverse centre-line and so was the associated intermixing time. Embodying a large number of experimental data, explicit correlations for intermixing time were derived in terms of principal operating variables through dimensional analysis and regression. Two different versions of correlations, applicable respectively to a flat bottom as well as a wedge fitted tundish systems, were developed. In SI unit, these are respectively represented as:

(a) $\tau_{int.mix} = 6.38 \times V_{res}^{0.86} Q_{in}^{0.48} Q_{out,T}^{-1.32}$

(b) $\tau_{int.mix} = 1.43 \times 10^{-2} V_{res}^{1.7} Q_{in}^{-0.02} Q_{out,T}^{-1.82}$

in which, Q_{in} is the input flow rate, $Q_{out,T}$ is the net outflow rate, V_{res} is the residual liquid volume and $\tau_{int.mix}$ is the intermixing time corresponding to a degree of 95 % homogeneity. Finally, adequacy and appropriateness of the proposed correlations are assessed both from theoretical and experimental stand points.

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1 Introduction

A tundish is an intermediate vessel, placed between a ladle and a mold, designed to supply and distribute molten steel to different continuous casting machines at a near constant rate. As the teeming ladle above the tundish is emptied, a new ladle containing molten steel from a subsequent heat is brought into replace the old ladle so that casting can

continue un-interruptedly. During ladle changeover operation, if the melt contained in the new ladle is of a different grade i.e., composition, mixing of two grades ensues in the tundish as soon as the new ladle is opened. This as a consequence results in the production of bloom/slab having a composition that is intermediate between the two successive grades. Mixed grade bloom/slab is often downgraded and re-circulated in the plant as scrap. High quality steel thus lost adversely influences plant performance. Efforts are therefore routinely made in the industry to minimize the production of intermixed bloom/slab. This necessitates quantification of intermixing time or intermixed cast length in terms of key operating variables.

During casting of two dissimilar grades of steel successively through the same tundish, composition, bath depth and weight of metal in the tundish vary with the time. To illustrate these time dependant phenomena better, Fig. 1 [1] has been included. There as seen, inter-mixing starts (corresponding to a time, $t = 0$) as the new ladle is opened and the new grade of steel, with a different composition, starts to flow into the tundish. At time $t < 0$, prior to opening of the new ladle, composition of melt is essentially that of the preceding heat/grade.

With reference to Fig. 1, in which, variation of composition/bath depth (weight) with time is shown schematically during ladle change over, five characteristic time periods can be identified. These are:

(i) $t \leq 0$; old ladle is continuously teemed into the tundish and steady bath height and composition are maintained,

(ii) $0 < t \leq t_1$; old ladle is removed and as a result, bath depth and tundish weight decrease continuously,

(iii) $t_1 < t \leq t_2$; new ladle is opened and mixing of two different grades in the tundish ensues. During the period, bath height as well as tundish weight increase continuously,

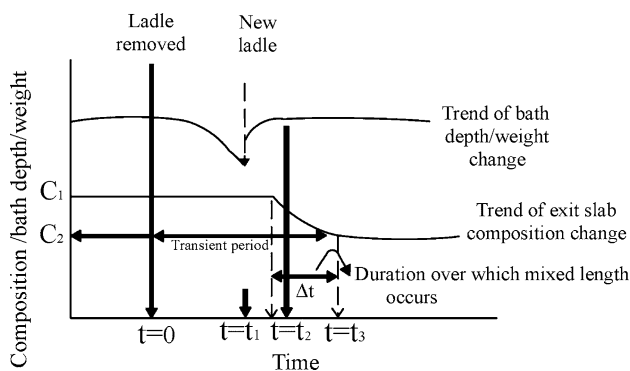(iv) $t_2 < t \leq t_3$; conditions within the tundish progressively move towards the steady state regime resulting

ultimately in constant bath depth and weight and finally,

(v) $t > t_3$ fully steady state conditions (in terms of composition, depth and weight) prevails since inflow rate is balanced against the net out flow rate resulting in the production of new grade bloom/slab.

On the basis of the preceding figure and foregone discussion, it is therefore evident that during the time period, $t_1 < t < t_3$, intermixing of two different grades takes place in the tundish. In the present work, the duration has been referred to as the intermixing or simply mixing time. Knowing the casting speed, physical dimensions of the caster and time taken by molten steel to travel physically from shroud to strand, corresponding intermixed bloom/slab length (and hence their weight) can be readily estimated.

Transient, three dimensional, multiphase (i.e., involving slag, bulk liquid steel and the ambient) and non isothermal (e.g., the residual liquid and new grade temperatures can vary by as much as 30–40 °C) turbulent flow simulation is a pre-requisite to the numerical modeling of grade intermixing phenomena. This is necessary as phase volume fractions continuously change during the unsteady period and individual phases tend to occupy different regions of tundish at different instants of time. Multiphase, turbulent flow calculations are invariably complex, time intensive and entail significant computational efforts. Furthermore, currently available computational fluid dynamics software yet do not provide a physically based and sound framework to carry out turbulent, multiphase, non isothermal flow simulation in large steel processing reactors with a great deal of accuracy. As a consequence, considerably idealized mathematical models have been applied to study grade intermixing phenomena [2–9]. Parallel to such, relatively simple, tank in series models have also been applied to mathematically model grade intermixing phenomena. The latter class of model often relies on such empirical inputs as the plug flow volume fraction etc. which are either deduced from experimentally determined "C" curve or largely assumed. Needless to mention that C curve based estimation and the associated residence time distribution theory cannot be applied to multi strand tundish (where there are several outlets as opposed to a single inlet) under un-steady state condition in a straight forward manner. This tends to limit the applicability of the so called "tank in series" approach to tundish flow modeling. In many mathematical studies, model predictions have been directly evaluated against plant trials and/or water modeling and reasonable agreement demonstrated [10–12]. Studies reported in the literature in general appear to indicate that intermixing time depends on residual volume of liquid and specific types of tundish furniture. In contrast, tundish filling rate has only marginal influence on intermixing time.

Fig. 1 Slab composition, tundish bath depth and tundish weight during ladle changeover operation [1]

Despite many studies [2–12] reported in the literature, the role of key operating variables on intermixing time remains yet to be quantified. Similarly, the influence tundish geometry and furniture exert on intermixing time is required to be investigated on a case to case basis, since these vary widely from one practice to another. Consequently, the purpose of the present work has been to carry out and present a systematic experimental study of grade intermixing phenomena in a multi strand steelmaking tundish system. In this, scaled water model of a four strand, delta shaped, bloom casting tundish has been employed to investigate and quantify intermixing time in terms of the key operating variables namely, residual volume, inflow and outflow rates. Two different designs of the tundish, flat bottom and wedge fitted were employed. In the following, experimental technique and results, dimensional analysis, regression of experimental data and derivation of specific correlations for intermixing time are presented.

2 Experimental Work

2.1 Scaling Down of Full Scale Tundish and Description of the Physical Model

Experimental work on grade intermixing was carried out in a 0.316 scale water model of a 17 tonne, four strand bloom casting tundish. Model tundish, as shown in Fig. 2 was built from 20 mm thick Perspex^{TM} sheet maintaining complete geometrical similarity. The model tundish, as one would note, was fitted with stopper rods which were operated manually to lower down or raise, so that tundish nozzle opening can be controlled and thereby, outflow rate regulated to maintain a desired bath depth in the tundish. Principal dimensions and operating parameters of the full scale and model tundish are summarized in Table 1. There, the volumetric in-flow rate in the model tundish was calculated on the basis of the well accepted Froude criterion viz., $Q_{mod} = \lambda Q_{f.s}^{5/2}$ [13], in which λ is the geometrical scale
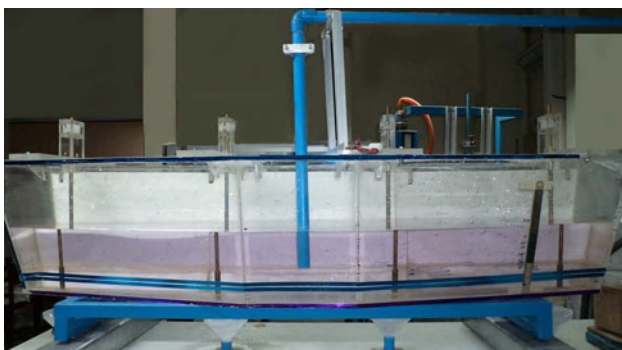

Fig. 2 A photograph of the 0.316 scale, partially filled, four strand, water model tundish system

factor and is equal to 0.316. Rate of water inflow into the tundish was regulated by a pre-calibrated rotameter while out flow rate was controlled through the movement of the stopper rods. Two specific designs of the bloom casting tundish system were investigated and these included:

(i)   a planner basal design with a pouring box and
(ii) a wedge fitted basal design with an embedded pouring box.

The schematics of model tundish together with the design of the basal wedge are shown in Figs. 3 and 4 respectively.

Dimensional analysis of industrial tundish flows involving different phases under non isothermal condition indicates that similarity of different dimensionless groups such as Re, Fr, Ri and We etc. between model and full scale system are important. These numbers influence flows and hence are expected to influence the associated mixing phenomena. However, for the quasi steady-state, isothermal, and single-phase flow of water in a scale-down tundish model, only the Re and Fr number similitude suffice. It is rater well known that in reduced scale modeling studies employing fluid of identical kinematic viscosity it is impossible to respect both Reynolds and Froude similarity simultaneously (see Table 1). In many earlier studies therefore, it has been assumed that flow phenomena in tundish are largely dominated by the inertial and gravitational forces (i.e., Froude number) rather than the viscous forces (Reynolds number).

The scaling equation adapted in the study i.e., $Q_{mod} = \lambda Q_{f.s}^{5/2}$ is the consequence of Froude similitude and obtained by equating Froude number between model and full scale systems, e.g., $N_{Fr,Model} = N_{Fr,Full\ scale}$. It is instructive to note here that numerous studies on tundish flow modeling has relied on the above mentioned Froude scaling criterion developed originally by Mazumdar and coworkers more than a decade ago [13]. To substantiate the point further, Froude based modeling results obtained through numerical solution of turbulent Navier–Stokes equations [14] and their direct comparison with water model as well as plant scale trial results during grade intermixing is illustrated in Fig. 5. There, close agreement between prediction and measurements is at once apparent. Given such, characterization of tundish flows solely in terms of Froude number can hardly be an issue of debate.

2.2 Experimental Results

2.2.1 Measurements of Intermixing Time

Intermixing time has been experimentally measured by monitoring the variation of conductance of flowing water (∞ concentration of an added electrolyte) with time at the tundish exit nozzle. During each experiment, a given amount of salt water of homogeneous concentration, representing a particular (i.e., the previous) steel grade, was

Table 1 Physical dimensions and operating parameters in full scale and experimental model in the present investigation

Parameters	Full scale system	0.316 scale model
Maximum length at the base (m)	5.442	1.72
Maximum length at the top (m)	5.87	1.85
Maximum width at the top (m)	1.67	0.527
Depth of liquid at normal operating level (mm)	560–705 (steady state bath depth is 678 mm)	168–222 (steady state bath depth is 214 mm)
Tundish nozzle diameter (mm)	45	14.22
Shroud diameter (mm)	84	26.54
Average throughput rate (m^3/s)	0.00398	2.2×10^{-4}
Minimum flow rate (m^3/s)	0.0014	0.8×10^{-4}
Maximum flow rate (m^3/s)	0.0085	4×10^{-4}
Estimated Froude number	0.092	0.092
Estimated Reynolds number	29820	10553

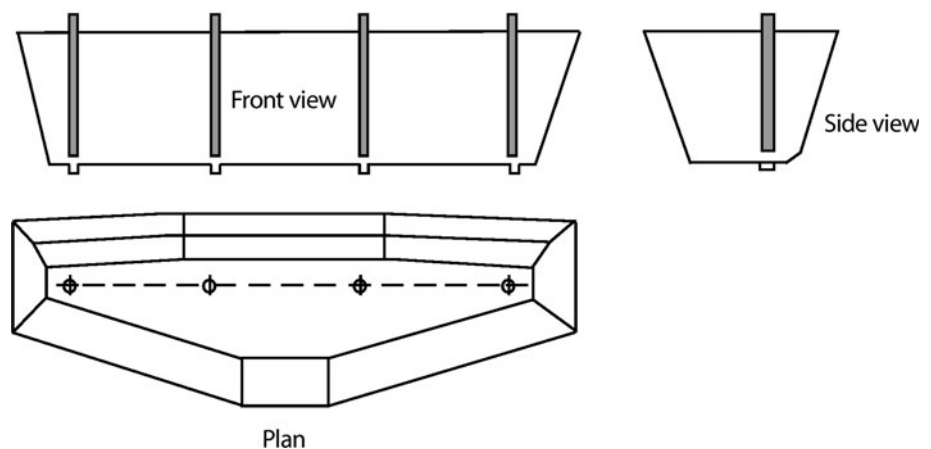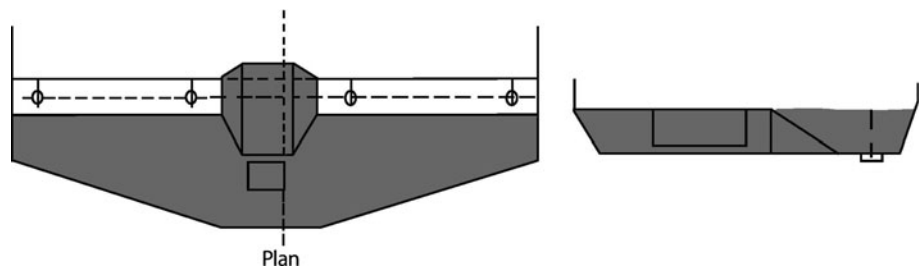**Fig. 3** Sectional views of the of the four strand bloom casting model tundish

Fig. 4 Schematic of the wedge placed on the tundish bottom illustrating the embedded pouring box and the contour of the basal wedge

intermixed with continuously flowing, pure water, representing steel of different grade/composition and the conductivity of mixed water measured dynamically by a CyberScan™ conductivity probe placed in the vicinity of the tundish exit nozzle. Because of the symmetry of the given geometry, conductivity was measured simultaneously at two adjacent, outlet nozzles, located on one half of the tundish.

Before starting an experiment, the tundish was filled up to the steady state operating level with salty water (complete homogeneity was ensured) keeping the outlet nozzles closed. Subsequently, the inlet flow was stopped and salt water drained out till the bath depth corresponded to a pre

determined level so as to mimic the desired residual volume from the previous grade. Once the desired residual volume was established, inlet was opened and pure, salt free, water was pumped into the tundish through the shroud at the desired rate. This corresponded to time, $t = 0$ in all grade intermixing experiments. Simultaneously, as the bath level started to rise, the stopper rod heights were lowered to reduce tundish nozzle opening so as to maintain outflow at a near constant rate, as in practice. Finally, inlet and out flow rates were made equal such that a steady operating bath level is established in the system. From the moment salt free water was pumped into the tundish, conductivity measurement was initiated and continued till the local
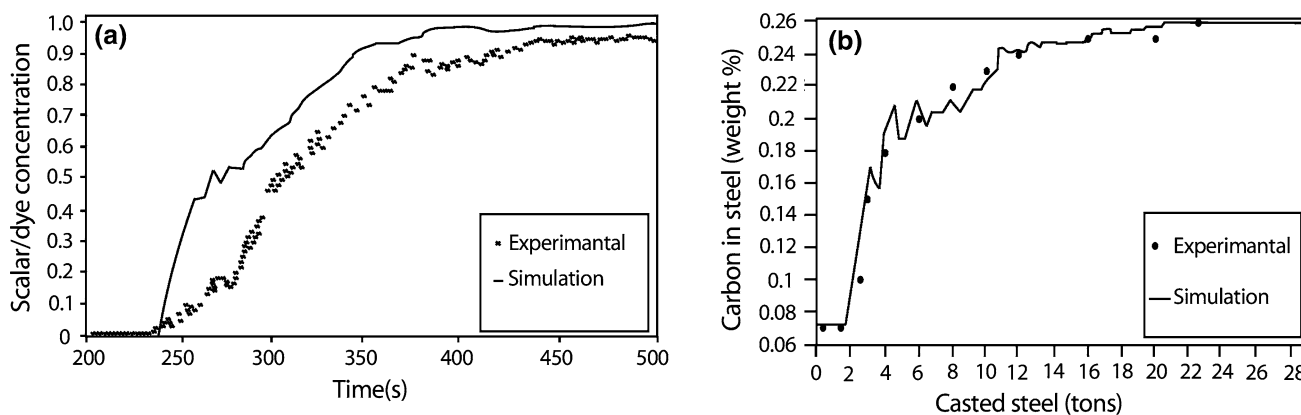
Fig. 5 Experimentally determined tracer concentration and its comparison with numerical computation **a** water model and **b** full scale tundish systems [14]

conductivity (in the vicinity of the exit nozzle) attained a practically constant value, equivalent to that of pure, salt free, water. Data on conductance thus obtained was saved in a computer as a text file to generate a typical conductivity versus time plot (viz., the characteristic intermixing curve which is essentially the so called F curve) as shown in Fig. 6. By definition, the duration over which intermediate conductivity prevails is the intermixing time.

Referring to Fig. 6, it is seen that concentration (i.e., a measure of conductance) changes asymptotically with time particularly during the later stages of mixing in the tundish. This tends to make determination of perfect intermixing time difficult. Therefore, for the sake of convenience, 95 % mixing criterion (with respect to the current/new grade) was employed to estimate intermixing time. Thus, from the time, $t = 0$ to the time dimensionless concentration/conductance at the strand e.g., $|(C(t) - C_{new})/(C_{old} - C_{new})|$

reaches a value of 0.05 has been taken as the representative intermixing time in the present case.

The procedure for estimation of 95 % mixing time from the conductivity versus time plot is also illustrated in Fig. 6. There, $t = 0$ indicates the instant at which salt free water was introduced (simulating flow from a new ladle) into the tundish. As seen, the probe starts detecting a change in water's conductivity after a little while since the incoming water takes time to physically travel up to the strand. Following such, the conductivity of flowing water continuously decreases attaining a final value, representative of pure, salt free water. Assuming that at large time, final uniform concentration (that of pure salt free water) prevails, the dimensionless conductivity/concentration defined above can be readily estimated. Evidently, the time at which the ±5 % deviation line intersects the F curve (truly an inverse F curve since the chosen initial

Fig. 6 A typical grade intermixing curve obtained from Perspex model tundish
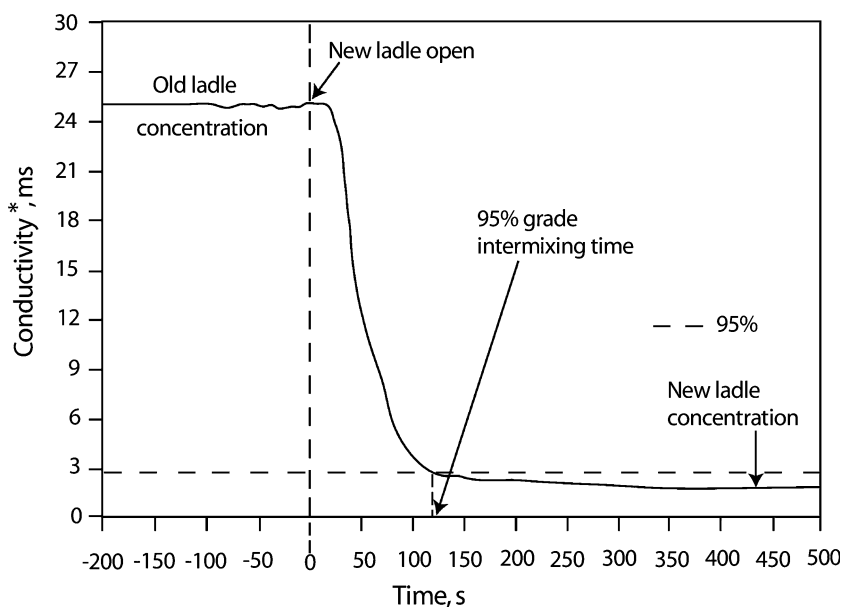
concentration is higher than the final concentration), corresponds to the 95 % intermixing time.

2.2.2 Stopper Rod Movement and Regulation of the Outflow Rate

In industrial practice, as the old ladle is exhausted of liquid steel, melt level in tundish continuously recedes till a new ladle is opened. During such time, the stopper rod height is adjusted to control strand opening such that casting rate is not significantly affected. To simulate such a behavior as accurately as possible, appropriately designed stopper rods were fabricated and placed above each strand. Each stopper rod was mounted on a threaded rotational unit, capable of rotating clockwise or anti-clockwise in order to provide vertical movement (up or down) to stopper rods. Thus, one full rotation of the unit in clockwise direction resulted in 1 mm upward movement of a stopper rod. This allowed us to control strand opening and thereby, regulate outflow at the desired rate during filling or emptying of the tundish.

Calibration curves between stopper rod movement and outflow rates were established a priori. Three different calibration charts were prepared corresponding to three distinct outflow rates shown in Table 1. The stopper rods which are operated manually currently are being integrated with computerized stepper motors so that their movement and the resultant outflow rate can be automated.

3 Results and Discussion

3.1 The Influence of Operating Variables and Tundish Designs on Grade Intermixing Times

Grade intermixing time was measured experimentally as a function of three principal operating variables namely, residual volume (i.e., previous grade volume), inflow rate and net outflow rate. Furthermore, experiments were carried out in two different tundish geometries and these included a traditional flat bottom tundish as well as a raised bottom, wedge fitted modified design tundish. A large number of experimental conditions were investigated and for each condition, at least three measurements of intermixing time were made. On the basis of such, an average, representative intermixing time was estimated for each specific experimental condition. With reference to Fig. 3, it is seen that the given tundish geometry is physically symmetrical about the central y–z plane. As a consequence of such, tracer ($KMnO_4$ solution) dispersion and intermixing phenomena in the tundish were found to be largely symmetrical. This allowed us to carry out mixing time experiments at two of the four strands, located on one half of the given tundish system. Intermixing time in as a function of operating parameters and tundish designs are summarized in Tables 2, 3 and 4. On the basis of such, several inferences can be made:

(i) Experiments and hence intermixing times estimated there from, are reasonably reproducible. Experimental uncertainty in measurements, as seen, is no more than ±15 %.

(ii) Experimental data summarized in Tables 2 and 3 indicate that intermixing time measured at strands 1 and 2 are not significantly different. To illustrate this point further, in Fig. 7, percent deviations between intermixing times recorded at the two consecutive strands have been plotted for various experimental conditions in the wedge fitted tundish. There, it is readily apparent that strand to strand variation, by and large, is not appreciable (barring only four cases) and lies within the limit of experimental uncertainty (±15 %). Consequently, for all practical purpose, it has been assumed that intermixing time for two adjacent strands is essentially identical. Due to symmetry, this further implies that intermixing time registered at any of the four strand is a reasonable representative of the entire tundish system.

(iii) Average value of the parameter t_f is also shown in Tables 2 and 3 for the two successive strands, strand number 1 and 2 respectively. The parameter, as pointed out already, represents the first appearance of fresh salt free water at a strand and is a measure of the time taken by incoming water stream to physically travel up to the relevant strand. Thus between the time $t = 0$ and $t = t_f$, although intermixing occur in the tundish, the probe does not register the same, since mixed liquid takes time to travel from the shroud region up to strand, where the probe tip is located. Looked at from a such a standpoint, meaningful grade intermixing time and therefore, the duration over which mixed bloom production ensues is equivalent to: {grade intermixing time—t_f}.

To illustrate the general trend on the influence of operating variables on grade intermixing times, Fig. 8a–c has been included in the text. On the basis of such, following generalizations can be made viz.,

(i) Residual volume has pronounced effect on grade intermixing time. Within the range of experimental conditions studied, it is seen that as residual volume is increased, intermixing time increases.

(ii) Out-flow rate has strong and quite the opposite influence on intermixing time. It is evident from Fig. 8b that as out flow rate increases, intermixing time decreases.

(iii) Finally, of the three principal variables investigated, in-flow rate appears to have relatively weak and only marginal influence on intermixing time.

Table 2 Intermixing time at the outer strand (nozzle 1) for different experimental conditions in the wedge fitted tundish

Condition number	Nozzle	$Q_{out,T}$ (m³/s)	Initial residual volume V_{res} (m³)	Q_{in} (m³/s)	Average[a] t_f (s)	Grade transition time (s) for 95 % mixing			
						Trial 1	Trial 2	Trial 3	Avg
1,1	1	0.00022	0.036	0.0004	13.75	299	274	310	294.3
1,2	1	0.00022	0.036	0.00036		309	263	304	292
1,3	1	0.00022	0.036	0.00032		298	260	277	278.3
1,4	1	0.00022	0.036	0.00028		286	247	247.5	260
1,5	1	0.00022	0.0297	0.0004		227	247.5	176	237.2
1,6	1	0.00022	0.0297	0.00036		243	207	145	225
1,7	1	0.00022	0.0297	0.00032		261	208.5	190	234.7
1,8	1	0.00022	0.0297	0.00028		253	230	196	241.5
1,9	1	0.00022	0.0234	0.0004		121	120.5	93	120.7
1,10	1	0.00022	0.0234	0.00036		127	146.5	89	136.7
1,11	1	0.00022	0.0234	0.00032		99	148.5	131.5	123.7
1,12	1	0.00022	0.0234	0.00028		100.5	169	155	134.7
1,13	1	0.000242	0.036	0.0004		227	185.5	206	206.2
1,14	1	0.000242	0.036	0.00036		218	210	226	218
1,15	1	0.000242	0.036	0.00032		202	218	206	210
1,16	1	0.000242	0.036	0.00028		192.5	172	185	182.2
1,17	1	0.000242	0.0297	0.0004		129.5	157.5	142	143.5
1,18	1	0.000242	0.0297	0.00036		132	165	151	148.5
1,19	1	0.000242	0.0297	0.00032		151	164	151	157.5
1,20	1	0.000242	0.0297	0.00028		178	188	174	183
1,21	1	0.000242	0.0234	0.0004		107	100.5	106	103.7
1,22	1	0.000242	0.0234	0.00036		85	72	80	78.5
1,23	1	0.000242	0.0234	0.00032		114.5	104	111	109.2
1,24	1	0.000242	0.0234	0.00028		124	85	105	104.5
1,25	1	0.000198	0.036	0.0004		364	354	374	364
1,26	1	0.000198	0.036	0.00036		357.5	323	368	357.5
1,27	1	0.000198	0.036	0.00032		341	333	369	341
1,28	1	0.000198	0.036	0.00028		336	346	323	336
1,29	1	0.000198	0.0297	0.0004		297.5	313.5	304	305.5
1,30	1	0.000198	0.0297	0.00036		221	278	245	249.5
1,31	1	0.000198	0.0297	0.00032		234	272	249	253
1,32	1	0.000198	0.0297	0.00028		239	259	243	249
1,33	1	0.000198	0.0234	0.0004		163	154.5	148	158.7
1,34	1	0.000198	0.0234	0.00036		170.5	111	120	140.7
1,35	1	0.000198	0.0234	0.00032		186	160.5	149.5	173.2
1,36	1	0.000198	0.0234	0.00028		184	171.5	169.5	177.7
1,37	1	0.00022	0.036	0.00022		286	276	320	294
1,38	1	0.00022	0.0297	0.00022		176.5	199	219	198.5
1,39	1	0.00022	0.0234	0.00022		174	179	181	178
1,40	1	0.000242	0.036	0.000242		192	211	209	204
1,41	1	0.000242	0.0297	0.000242		195	192	175.5	187.5
1,42	1	0.000242	0.0234	0.000242		152	140	131	141
1,43	1	0.000198	0.036	0.000198		294	302	262	286
1,44	1	0.000198	0.0297	0.000198		275	264	220	253
1,45	1	0.000198	0.0234	0.000198		182	180	202	188

[a] This is the time taken by the tracer to physically travel from shroud to strand and is characteristically known as the minimum break through time

Table 3 Intermixing time at the inner strand (nozzle 2) for different experimental conditions in the wedge fitted tundish

Condition number	Nozzle	$Q_{out,T}$ (m³/s)	Initial residual volume V_{res} (m³)	Q_{in} (m³/s)	Average t_f (s)	Grade transition time (s) for 95 % mixing			
						Trial 1	Trial 2	Trial 3	Avg
2,1	2	0.00022	0.036	0.0004	8.25	324	266	292	279
2,2	2	0.00022	0.036	0.00036		339	246	279	262.5
2,3	2	0.00022	0.036	0.00032		317	243	332	287.5
2,4	2	0.00022	0.036	0.00028		324	230.5	298	264.2
2,5	2	0.00022	0.0297	0.0004		292	230.5	166	230.5
2,6	2	0.00022	0.0297	0.00036		263.5	219	193.5	219
2,7	2	0.00022	0.0297	0.00032		305	192.5	181.5	192.5
2,8	2	0.00022	0.0297	0.00028		303	234	197	234
2,9	2	0.00022	0.0234	0.0004		207	127	142.5	127
2,10	2	0.00022	0.0234	0.00036		192	142.5	136	142.5
2,11	2	0.00022	0.0234	0.00032		189.5	138	148.5	138
2,12	2	0.00022	0.0234	0.00028		200	148.5	164.5	148.5
2,13	2	0.000242	0.036	0.0004		228	192	211.5	210.5
2,14	2	0.000242	0.036	0.00036		205	211	199	205
2,15	2	0.000242	0.036	0.00032		210	226.5	215.2	217.2
2,16	2	0.000242	0.036	0.00028		226	210.5	210	215.5
2,17	2	0.000242	0.0297	0.0004		159.5	161.2	160	160.2
2,18	2	0.000242	0.0297	0.00036		145	152	150	149
2,19	2	0.000242	0.0297	0.00032		139	181.7	160	160.2
2,20	2	0.000242	0.0297	0.00028		150	134	149.5	144.5
2,21	2	0.000242	0.0234	0.0004		109	103	104.5	105.5
2,22	2	0.000242	0.0234	0.00036		94	88	90.2	90.7
2,23	2	0.000242	0.0234	0.00032		112	109.5	110	110.5
2,24	2	0.000242	0.0234	0.00028		118	91.5	109.2	106.2
2,25	2	0.000198	0.036	0.0004		332	341.5	322.5	332
2,26	2	0.000198	0.036	0.00036		340	343.7	336.3	340
2,27	2	0.000198	0.036	0.00032		357	359.2	354.8	357
2,28	2	0.000198	0.036	0.00028		354	361.5	346.5	354
2,29	2	0.000198	0.0297	0.0004		258	308	289	285
2,30	2	0.000198	0.0297	0.00036		226	257	240.5	242.5
2,31	2	0.000198	0.0297	0.00032		227	260	228.5	238.5
2,32	2	0.000198	0.0297	0.00028		225	277.5	248.2	250.2
2,33	2	0.000198	0.0234	0.0004		164	161	159.5	161.5
2,34	2	0.000198	0.0234	0.00036		146	159.3	149.2	151.5
2,35	2	0.000198	0.0234	0.00032		154.5	191	171.2	172.2
2,36	2	0.000198	0.0234	0.00028		159	192.2	181.3	177.5
2,37	2	0.00022	0.036	0.00022		310	291	240.5	280.5
2,38	2	0.00022	0.0297	0.00022		162	186	199.5	182.5
2,39	2	0.00022	0.0234	0.00022		161	172	174	169
2,40	2	0.000242	0.036	0.000242		199	189	182	190
2,41	2	0.000242	0.0297	0.000242		159	168	186	171
2,42	2	0.000242	0.0234	0.000242		145	148	133	142
2,43	2	0.000198	0.036	0.000198		294	301	305	300
2,44	2	0.000198	0.0297	0.000198		222	241	249.5	237.5
2,45	2	0.000198	0.0234	0.000198		191	184.5	214	196.5

Table 4 Intermixing time at the outer strand (nozzle 2) for different experimental conditions in the wedge fitted tundish

Condition number	Nozzle	$Q_{out,T}$ (m³/s)	Initial residual volume V_{res} (m³)	Q_{in} (m³/s)	Average t_f (s)	Grade transition time (s) for 95 % mixing			
						Trial 1	Trial 2	Trial 3	Avg
2,1	2	0.00022	0.036	0.0004	8.25	515	520	504	513
2,2	2	0.00022	0.036	0.00036		480	484	503	489
2,3	2	0.00022	0.036	0.00032		468	512	475	485
2,4	2	0.00022	0.036	0.00028		484	502	467	484.3
2,5	2	0.00022	0.0297	0.0004		414	440	453	435.6
2,6	2	0.00022	0.0297	0.00036		465	470	384	439.6
2,7	2	0.00022	0.0297	0.00032		402	425	406	411
2,8	2	0.00022	0.0297	0.00028		400	428	355	394.3
2,9	2	0.00022	0.0234	0.0004		352	401	385.5	379.5
2,10	2	0.00022	0.0234	0.00036		368	389.5	391	382.8
2,11	2	0.00022	0.0234	0.00032		322	377	383	360.6
2,12	2	0.00022	0.0234	0.00028		355	394	395	381.3
2,13	2	0.000242	0.036	0.0004		380	418	408	402
2,14	2	0.000242	0.036	0.00036		390.5	431	366	395.8
2,15	2	0.000242	0.036	0.00032		415	439	392	415.3
2,16	2	0.000242	0.036	0.00028		355	375	398	376
2,17	2	0.000242	0.0297	0.0004		316	373.5	352.5	347.3
2,18	2	0.000242	0.0297	0.00036		338	390.5	345	357.8
2,19	2	0.000242	0.0297	0.00032		309	358	317	328
2,20	2	0.000242	0.0297	0.00028		323	362.5	319	334.8
2,21	2	0.000242	0.0234	0.0004		289	346.5	275.5	303.6
2,25	2	0.000198	0.036	0.0004		659	642	720.5	673.8
2,26	2	0.000198	0.036	0.00036		666	649	651.5	655.5
2,27	2	0.000198	0.036	0.00032		535	550	668.5	584.5
2,28	2	0.000198	0.036	0.00028		512	566.5	534	537.5
2,29	2	0.000198	0.0297	0.0004		560	605	575	580
2,33	2	0.000198	0.0234	0.0004		407	431	413	417
2,34	2	0.000198	0.0234	0.00036		396	426.5	415.5	412.6
2,35	2	0.000198	0.0234	0.00032		423.5	434	466	441.1
2,36	2	0.000198	0.0234	0.00028		423.5	454	432	436.5

Such trends in experimental results, as one would note here, are consistent with previous work reported in the literature [4, 7].

Residual volume expectedly has pronounced influence on intermixing time. As the residual volume is increased, corresponding volume of intermixed liquid also tend to increase since inflow rate is typically somewhat greater than the out flow rate. Thus, the duration over which intermixed liquid flows out from the tundish also tends to increase. Similarly, as the outflow rate is increased, the mixed liquid leaves the tundish at a much faster rate reducing the duration of intermixing. In contrast, as in-flow rate is changed, intermixing time for any given residual volume and outflow rate is only affected marginally. This

is so as the rate of approach of equilibrium bath depth is dictated by a net difference between inflow and out flow rates.

Intermixing time in two different tundish designs i.e., wedge fitted versus and flat bottom tundish is shown under similar operating conditions in Fig. 9. There, it is readily apparent that intermixing times in flat bottom tundish are significantly higher than those in the wedge fitted tundish. This is consistent and expected since the wedge physically occupies a reasonable portion of the tundish and thereby, reduces the available volume for intermixing. It is to be noted here that for the same bath depth, effective residual volume in presence of a basal wedge is somewhat smaller than that without it. In addition to such, flow phenomena

Fig. 7 Percentage deviation in intermixing times for two adjacent strands for various experimental conditions in the wedge fitted tundish
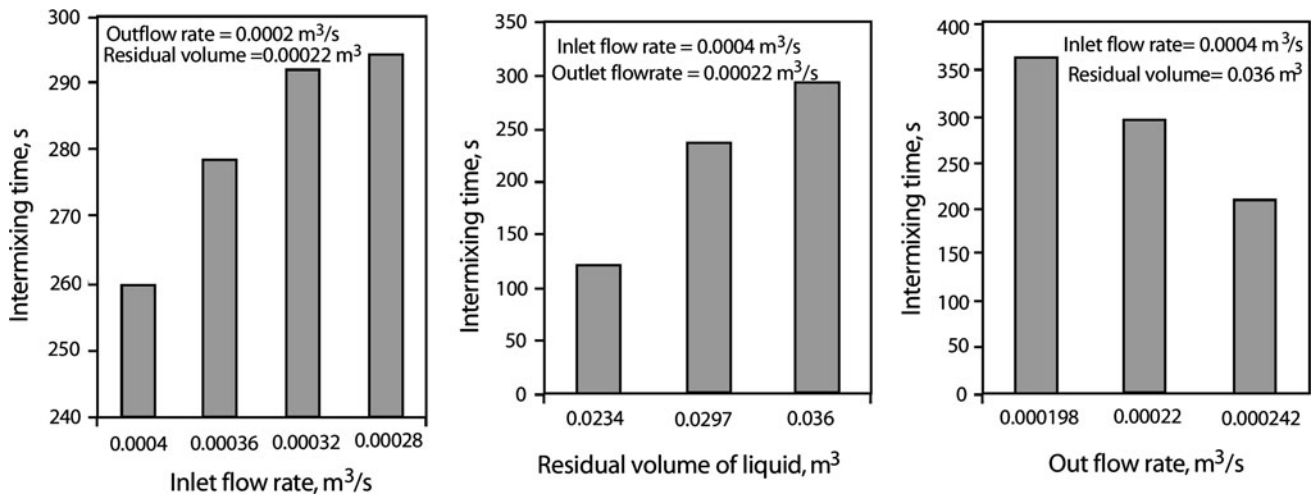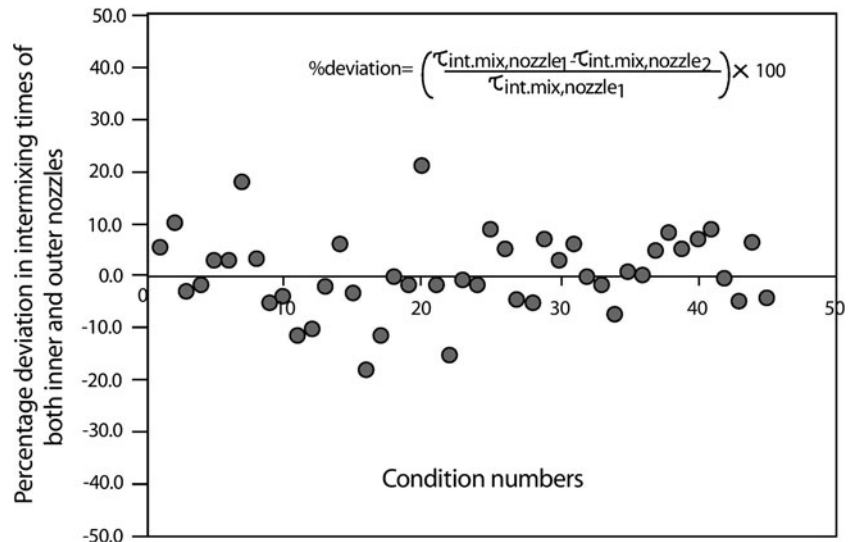

$$\%\text{deviation} = \left(\frac{\tau_{int.mix,nozzle_1} - \tau_{int.mix,nozzle_2}}{\tau_{int.mix,nozzle_1}} \right) \times 100$$

Fig. 8 Variation of intermixing time as a function of principal operating variables
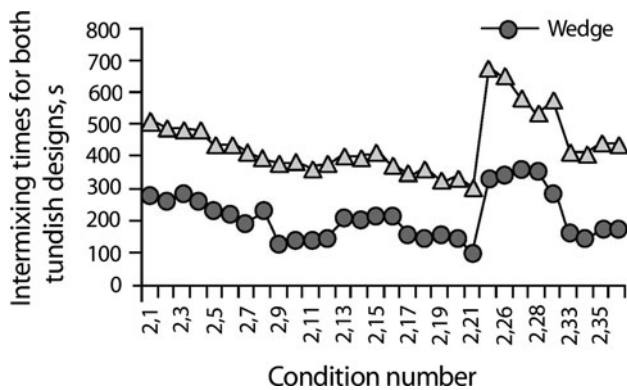
Fig. 9 Comparison of intermixing times for two different tundish designs under identical operating conditions

with and without wedge are also remarkably different. Since flow influences mixing, consequently different intermixing times are expected for different tundish designs. Given such, it is reasonable to assume that specific type of tundish furniture (or flow modifiers as these are customarily called) is likely to influence intermixing time.

To appreciate the relevance and significance of the experimental results presented above, it is important to note that model grade intermixing times are not as large as their full scale counterpart. Thus, intermixing time of say, 300 s in the model would translate to about 533 s (on the basis of $\tau_{mod} = \tau_{full\ scale}\ \lambda^{0.5}$) in the full scale under equivalent condition. Furthermore, considering a typical casting speed of 1 m/min, the estimated industrial intermixing time corresponds to a bloom length of about 10 m. Given that the tundish has four strands, about 40 m bloom length (equivalent to 15–20 tonnes of steel depending on bloom section size) is therefore lost/downgraded every sequence. This is evidently significant and suggests that

every effort must be made to limit the production of intermixed bloom/slab in the cast shop.

3.2 Development of Correlations for Grade Intermixing Time

To represent the influence of operating variables on grade intermixing times through appropriate mathematical correlations/expressions, dimensional analysis was carried out. Subsequently, non-linear regression method was applied to the experimental data, presented in Tables 2, 3 and 4, to deduce explicit forms of such correlations. Since tundish flows are largely dominated by Froude number, $(= u^2/gL)$ and that flows are intricately related to mixing phenomena [15], consequently for tundish system, the following functional relationship has been assumed i.e.,

$$\tau_{\text{int.mix}} = f\left(V_{\text{res}}, Q_{\text{in}}, Q_{\text{out,T}}, g\right) \tag{1}$$

in which, $\tau_{\text{int.mix}}$ is the intermixing time (s), V_{res} is the residual volume of the liquid present in the tundish (m^3), Q_{in} is the input flow rate (m^3/s), $Q_{\text{out,T}}$ is the total output flow rate i.e., through all four strands (m^3/s), and g is the acceleration due to gravity (m/s^2).

Intermixing time, as one would normally anticipate, is a function of mixing energy and the geometry of the system [15]. In the present scenario, mixing energy is directly proportional to the kinetic energy of the incoming fluid which in turn is a function of the volumetric inflow rate. This, as shown via Eq. 1, has been explicitly accommodated in the present study. While the influence of inflow rate on grade intermixing is expected to be substantial, diffusivity on the other hand is expected to have only marginal influence since we are concerned with macro mixing instead of micro mixing phenomena. Furthermore, steel and water are Newtonian fluids and have similar kinematic viscosity. Since flow phenomena is Froude dominated (Froude number is independent of density and viscosity), the influence of density and viscosity on grade intermixing is expected to be only marginal. It is to be emphasized here that the correlation developed in this work is valid for only steel/water system and should not be generalized to other fluid systems having substantially different thermo-physical properties.

On the basis of Raleigh's method of the indices, the above functional relationship can be expressed via:

$$\tau_{\text{int.mix}} = C Q_{\text{in}}^a Q_{\text{out,T}}^b V_{\text{res}}^c g^d. \tag{2}$$

On the basis of Buckingham π-theorem, one can expect three independent non-dimensional π groups to govern the above relationship. Non-dimensional equivalent of Eq. 2 can therefore be represented as:

$$f(\pi_1, \pi_2, \pi_3) = 0. \tag{3}$$

Embodying the principles of dimensional homogeneity, the three different π groups namely, dimensionless intermixing time, dimensionless flow rate and Froude number can be found out and represented via:

$$\pi_1 = \frac{\tau_{\text{int.mix}} Q_{\text{in}}}{V_{\text{res}}} \tag{4a}$$

$$\pi_2 = \frac{Q_{\text{out,T}}}{Q_{\text{in}}} \tag{4b}$$

and

$$\pi_3 = \frac{V_{\text{Res}}^{5/3} g}{Q_{\text{in}}^2}. \tag{4c}$$

It is instructive to note here that the dimensionless group viz., $V_{\text{Res}}^{5/3} g/Q_{\text{in}}^2$ is the ratio of inertial to gravitational forces and is therefore truly the inverse Froude number. It is readily confirmed that $V_{\text{Res}}^{5/3} g/Q_{\text{in}}^2$ is dimensionally equivalent to:

$$\frac{(L^3)^{5/3} g}{Q_{\text{in}}^2} = \frac{Lg}{\left(Q_{\text{in}}^2/L^4\right)} = \frac{Lg}{U^2} = \frac{\text{gravity force}}{\text{inertial force}} = N_{\text{Fr}}^{-1}. \tag{5}$$

In which U and L are respectively a velocity and length scale in the system.

On the basis of the three representative π groups, the functional relationship expressed through Eq. 2 can be represented in the following non dimensional form e.g.,

$$\left(\frac{\tau_{\text{int.mix}} Q_{\text{in}}}{V_{\text{res}}}\right) = K \left(\frac{Q_{\text{out,T}}}{Q_{\text{in}}}\right)^a \left(\frac{V_{\text{res}}^{1.66} g}{Q_{\text{in}}^2}\right)^b \tag{6}$$

To determine the explicit values of the pre-exponent, K, as well as the exponents "a" and "b" in Eq. 6, multiple, non linear regression [16] was employed. Towards this, experimental data presented in Tables 2, 3 and 4 were applied. For the wedge fitted tundish, regression yielded $K = 5.5 \times 10^{-3}$, $a = -1.82$ and $b = 0.42$. Accordingly, corresponding explicit form of the correlation is represented as:

$$\left(\frac{\tau_{\text{int.mix}} Q_{\text{in}}}{V_{\text{res}}}\right) = 5.5 \times 10^{-3} \left(\frac{Q_{\text{out,T}}}{Q_{\text{in}}}\right)^{-1.82} \left(\frac{V_{\text{res}}^{1.66} g}{Q_{\text{in}}^2}\right)^{0.42} \tag{7}$$

In terms of the principal operating variables, the preceding equation in SI unit can be cast as:

$$\tau_{\text{int.mix}} = 1.43 \times 10^{-2} V_{\text{res}}^{1.7} Q_{\text{in}}^{-0.02} Q_{\text{out,T}}^{-1.82} \tag{8}$$

The adequacy of the proposed correlation is illustrated in Fig. 10. There, reasonably close fitting between experiment and prediction confirms the adequacy and effectiveness of Eq. 8.
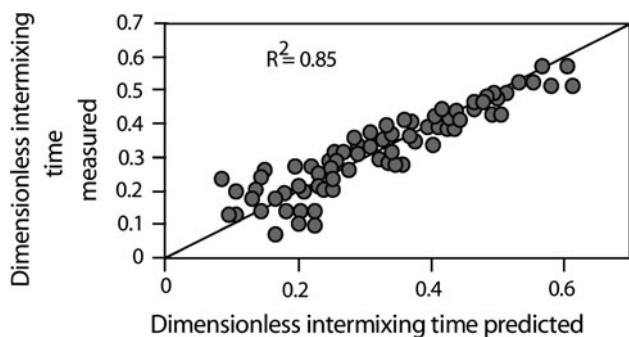
Fig. 10 Comparison of predicted and measured dimensionless intermixing times for the wedge fitted tundish

Similarly, intermixing time correlation for the flat bottom tundish system was derived in dimensionless form and represented as:

$$\left(\frac{\tau_{\text{int.mix}} Q_{\text{in}}}{V_{\text{res}}}\right) = 7.66 \times \left(\frac{Q_{\text{out,T}}}{Q_{\text{in}}}\right)^{-1.32} \left(\frac{V_{\text{res}}^{1.66} g}{Q_{\text{in}}^2}\right)^{-0.08} \quad (9)$$

In terms of principal operating variables, the preceding equation is represented in SI unit as:

$$\tau_{\text{int.mix}} = 6.38 \times V_{\text{res}}^{0.86} Q_{\text{in}}^{0.48} Q_{\text{out,T}}^{-1.32} \quad (10)$$

The fitness of the preceding relationship to the experimental data is illustrated in Fig. 11. This evidently demonstrates the adequacy and the appropriateness of Eq. 10.

Regression models presented above clearly indicate that operating variables influence intermixing times differently. This is essentially a reflection of experimental results and is consistent with those presented in Tables 2, 3 and 4 as well as Fig. 8. Remarkably however, internal geometry of the tundish has also substantial influence on intermixing time as is evident from the markedly different pre exponents in the two respective correlations.

The correlations developed in this study are specific to a given tundish geometry and a set of well defined operating conditions. The correlations are essentially empirical in nature and therefore, should not be extrapolated beyond their currently prescribed limit of validity. Similar models in principle can be derived from different stand points such as, artificial neural network etc. It is important to note here that in deriving such macroscopic models, the details of the flow structure in the system become inconsequential. Indeed, many macroscopic process models developed for ladle flows and mixing [15, 17] have clearly shown that detailed consideration of flow is not important and does not tend to limit model applicability and performance to any significant extent.

3.3 General Adequacy of the Proposed Correlations

A vast majority of the experimental results shown in Tables 2, 3 and 4, were obtained under unsteady state condition in which, the bath depth following the introduction of salt free water (new grade) increased due to a net imbalance between inflow and out flow rates. This is typical of the actual industrial practice. A limiting situation, though rare in practice, is the steady state operation in which, the inflow and out flow rates are balanced throughout, resulting in a constant, invariant bath depth/ volume (equivalent to the initial residual volume) at all times. Some results shown in Tables 2, 3 and 4 corresponded to this latter situation.

To assess the adequacy of the correlations presented in the preceding section, particularly under steady state condition, some new additional experiments on intermixing time were carried out in the wedge fitted tundish. A direct comparison between prediction and measurements is illustrated in Fig. 12. There, reasonable agreement between the two is readily apparent. Intermixing time correlations presented earlier, as one would note here, were derived on the basis of experimental data obtained from two adjacent strands located on one half of the tundish. To assess their adequacy further, a few validation data were generated by

Fig. 11 Comparison of predicted and measured dimensionless intermixing times for the flat bottom tundish
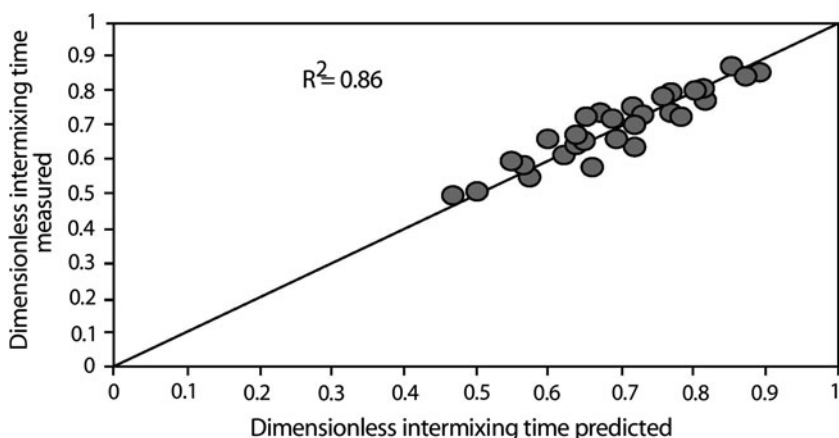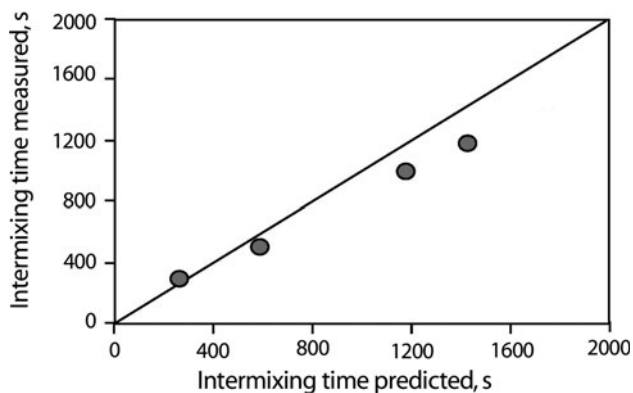
Fig. 12 Comparison of predicted and measured intermixing time under steady state condition for the wedge fitted tundish

carrying out additional experiments in which, measuring probes were placed at strands located on the other half of the tundish. Details of experimental measurements together with a comparison between measurement and prediction are shown in Table 5. Once again, excellent agreement between the two is readily apparent.

On the basis of Eq. 8, the ratio of intermixing times in model and full scale system can be expressed as:

$$\frac{\tau_{int.mix,mod}}{\tau_{int.mix,f.s}} = \left(\frac{V_{res,m}}{V_{res,f.s}}\right)^{1.7}\left(\frac{Q_{in,m}}{Q_{f.s,mod}}\right)^{-0.02}\left(\frac{Q_{out,T,mod}}{Q_{out,T,f.s}}\right)^{-1.82} \quad (11)$$

Since flow rates are scaled according to $Q_{mod} = \lambda^{5/2}Q_{f.s}$ (viz., the Froude criterion) and furthermore, as geometrically similar systems entails $V_{res,mod} = \lambda^3 V_{res,f.s}$, consequently, the preceding relationship can be readily simplified to:

$$\frac{\tau_{int.mix,mod}}{\tau_{int.mix,f.s}} = \left(\lambda^3\right)^{1.7}\left(\lambda^{2.5}\right)^{-0.02}\left(\lambda^{2.5}\right)^{-1.82} \quad (12)$$

providing the following, final explicit expression viz.,

$$\frac{\tau_{int.mix,mod}}{\tau_{int.mix,f.s}} = \lambda^{1/2} \quad (13)$$

Equation 13 is a reflection of kinematic similarity in Froude dominated geometrically similar flow systems and follows directly from the principles of physical modeling

[1]. On the basis of experimental results and analysis presented in this section, it is therefore reasonable to conclude that the intermixing time correlations developed in this work are reliable, predictive and fundamentally consistent.

As a final point, throughout the text, intermixing time up to a degree of 95 % was considered. However, depending on the composition of the two successive grades and the prescribed tolerance limit, mixing criterion might change and ascribed to a value in the range of 80–99 %. Thus, if the desired criterion is intended to be more or less stringent than the one considered here, one has to refer back to the tracer response curves, estimate intermixing times up to the desired degree there from and follow the procedure described in the text (e.g., a fresh regression of experimental data and so on) to arrive at appropriate correlations. This is cumbersome and tends to limit the generality of the correlations developed in this work.

Intermixing times corresponding to different mixing criteria, say Y_1 and Y_2, can be readily correlated from the statistical theory of mixing [18] as:

$$\frac{\tau_{int.mix}(Y_2)}{\tau_{int.mix}(Y_1)} = \frac{\log(1-Y_2)}{\log(1-Y_1)} \quad (14)$$

Equation 14 further indicates that intermixing times, say, up to a degree of 85 % can be correlated with 95 % intermixing times via:

$$\tau_{int.mix}(85\,\%) = \tau_{int.mix}(95\,\%) \times 0.633 \quad (15)$$

On the basis of the above, Eq. 10 applicable to 95 % mixing mark has been translated to the 85 % mixing criterion and represented as:

$$\tau_{int.mix,85\%} = 4.038 V_{res}^{0.86} Q_{in}^{-0.02} Q_{out,T}^{-1.32} \quad (16)$$

To assess the adequacy and appropriateness of Eq. 16 and to develop thereby a frame work for estimating intermixing times for any arbitrarily chosen mixing criterion, 85 % intermixing times were estimated from experimental tracer response curves (Sect. 2.2.2) under a wide variety of conditions. Results thus obtained are summarized in Table 6. A direct comparison between experimental and predicted (via Eq. 16) 85 % intermixing time is illustrated in Fig. 13. There, agreement between the two is reasonable indicating essentially that correlations derived for a specific degree of

Table 5 A comparison between measured and predicted intermixing times for nozzle 3 (located close to shroud but in the different half) in the wedge fitted tundish

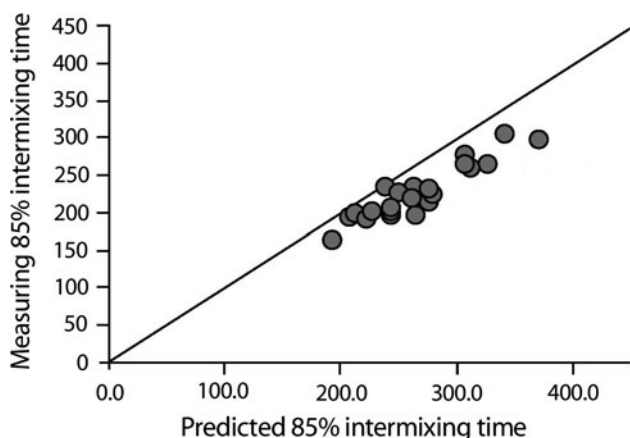| $Q_{out,T}$ (m³/s)	Q_{in} (m³/s)	Residual volume of liquid (m³)	Measured intermixing time (s)				Predicted intermixing time from correlation (s)
			Exp. 1	Exp. 2	Exp. 3	Average	
0.00022	0.00036	0.036	291	264	292.5	282.5	265.7
0.00022	0.00032	0.0297	181.5	191	184.5	185.5	189.8
0.00022	0.0004	0.0234	142.5	99	111.5	117.5	127.1

Fig. 13 Comparison between predicted and measured 85 % intermixing times for flat bottom tundish

mixing can be translated, as a first approximation, to a different degree of mixing on the basis of the statistical theory of turbulence.

4 Conclusions

From the present physical and mathematical modeling investigation of grade intermixing phenomena in a water model of a four strand, delta shaped, bloom casting tundish system, the following general conclusions can be drawn:

(i) Intermixing phenomena in the present geometrically symmetrical tundish system has been found to be largely symmetrical. Furthermore, in the four strand tundish system, intermixing times for both inner and outer nozzles are practically identical although time taken by the liquid to travel from inlet to two successive nozzles/strands is somewhat different. This has confirmed that monitoring intermixing time at one of the four nozzles is adequate for simulation of grade intermixing phenomena in the given tundish.

(ii) The residual volume of liquid has significant influence on the grade intermixing time and it is observed that the grade intermixing time increases with the increase in residual volume of liquid.

Table 6 Experimentally measured and predicted (via Eq. 16) 85 % intermixing times for nozzle 2 in the flat bottom tundish

$Q_{out,T}$ (m³/s)	Initial residual volume V_{res} (m³)	Q_{in} (m³/s)	t_f (s)	85 % grade intermixing time (s)				Predicted 85 % grade intermixing time (s)
				Exp. 1	Exp. 2	Exp. 3	Avg	
0.00022	0.036	0.0004	8.25	261	278	255	264.7	324.8
0.00022	0.036	0.00036	8.25	275	270	244	263.0	309.6
0.00022	0.036	0.00032	8.25	258	283	255	265.3	307.1
0.00022	0.036	0.00028	8.25	277	285	271	277.7	306.7
0.00022	0.0297	0.0004	8.25	179	233	232	214.7	275.8
0.00022	0.0297	0.00036	8.25	234	260	200	231.3	278.4
0.00022	0.0297	0.00032	8.25	233	220	235.5	229.5	260.2
0.00022	0.0297	0.00028	8.25	223	233.5	220	225.5	249.7
0.00022	0.0234	0.0004	8.25	175	213	203	197.0	240.3
0.00022	0.0234	0.00036	8.25	192	201	211	201.3	242.4
0.00022	0.0234	0.00032	8.25	194	205	208	202.3	228.3
0.00022	0.0234	0.00028	8.25	195	223	204	207.3	241.4
0.000242	0.036	0.0004	8.25	212	227.5	233.5	224.3	254.5
0.000242	0.036	0.00036	8.25	215	254	213	227.3	250.6
0.000242	0.036	0.00032	8.25	220	251	237	236.0	263.0
0.000242	0.036	0.00028	8.25	222	233	241	232.0	238.1
0.000242	0.0297	0.0004	8.25	173	211	186	190.0	219.9
0.000242	0.0297	0.00036	8.25	194	220.5	190	201.5	226.6
0.000242	0.0297	0.00032	8.25	187	222	180	196.3	207.7
0.000242	0.0297	0.00028	8.25	193	215	195	201.0	212.0
0.000242	0.0234	0.0004	8.25	150	180	158	162.7	192.2
0.000198	0.036	0.00028	8.25	309	319	290	306.0	340.3
0.000198	0.0234	0.0004	8.25	190	222	186	199.3	264.0
0.000198	0.0234	0.00036	8.25	210	237.5	210	219.2	261.3
0.000198	0.0234	0.00032	8.25	215	226	231.5	224.2	279.3
0.000198	0.0234	0.00028	8.25	232	237	234	234.3	276.4

(iii) In addition to residual volume, out flow rate from the tundish was found to have pronounced influence on grade intermixing time. It has been observed that intermixing time decreases with the increase of the total outlet flow rate. In contrast, the influence of ladle opening flow rate on intermixing time is remarkably less pronounced.

(iv) The internal geometry of the tundish influences intermixing time profoundly. Intermixing times for a flat bottom tundish are substantially higher than those in the wedge fitted tundish.

(v) The correlations developed in this study viz.,

(a) $\tau_{\text{int.mix}} = 6.38 \times V_{\text{res}}^{0.86} Q_{\text{in}}^{0.48} Q_{\text{out,T}}^{-1.32}$

(b) $\tau_{\text{int.mix}} = 1.43 \times 10^{-2} V_{\text{res}}^{1.7} Q_{\text{in}}^{-0.02} Q_{\text{out,T}}^{-1.82}$

for wedge fitted tundish and for flat bottom tundish which are valid for 95 % mixing mark are reasonably reliable, reasonably predictive and internally consistent and provide the requisite frame work for a prior estimation of intermixing time in the given tundish system.

(vi) Intermixing time corresponding to a given mixing criterion can be translated reasonably well to a different criterion on the basis of the statistical theory of turbulence.

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