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Stochastic method for prediction of subsidence due to the underground coal mining integrated with GIS, a case study in Serbia

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Abstract

The problem of subsidence and damages on buildings above mining works is an inevitable consequence of underground coal mining in large mining basins. Significant development of various methods for prediction of the displacement parameters of the undermined terrain took place in the period from the middle of the last to the beginning of this century. In this paper, a new mathematical model of the stochastic Pataric-Stojanovic method for the predictive subsidence calculation is developed. A case study, as an example for the practical application of the proposed stochastic method, is given for the underground coal mine "Rembas"- Resavica, Serbia. An original MITSOUKO program package, supported by spatial analyses in the Geographic Information System (GIS), was developed for the predictive subsidence calculation based on the stochastic method proposed by the authors Pataric and Stojanovic. The comparative analysis of predicted and measured subsidence values confirms the possibility of using this stochastic method for predictive subsidence calculation in practice, which is a necessity of modern mining.

Keywords Underground coal mining · Mine subsidence · Stochastic prediction method · GIS · Spatial analysis

Introduction

Since the mid-nineteenth century, when the problem of ground subsidence and building damage caused by underground coal mining occurred in the Ruhr Basin, long-term measurements in worldwide coal mines with different geological and technological exploitation conditions have been carried out. The obtained extensive results were afterwards analyzed by the scientists from Germany, Belgium, Poland,

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Czech Republic, Russia, United Kingdom, Serbia, United States, Canada, Australia, China, India, Vietnam, Iran, Turkey, and other countries. Based on their work, original methods for predicting subsidence and deformation parameters were created.

The developed subsidence prediction methods with different starting assumptions for deriving equations that describe the subsidence curve are:

Profile function methods are based on the application of the influence function of the elementary excavated volume on the ground surface subsidence. Each elementary excavated volume causes individual subsidence in the rock massif and on the terrain surface, while their sum gives the total subsidence at the observed point of the land surface. Various scientific hypotheses are used, such as potential theories, attractive forces between masses, elements of the elasticity theory, which are far from the real composition of the rock massif. As a rule, the shape of the function is not related to the physico-mechanical properties of the rock massif and does not show a different character of deformations in different zones of the massif. Therefore, these methods offer the transition from an empirical to a theoretical approach. They originate and were mostly applied in Germany, and their

authors are Schmitz (1923); Keinhorst (1928), Bals (1931); Schleier (1937); Flaschenträger (1938); Sann (1949); Perz (1957); King and Whetton (1957); Hoffmann (1964); Kumar (1973); Peng and Chyan (1981); Daemen and Hood (1982); Karmis and Haycocks (1983), Alejano et al. (1999); Díez and Álvarez (2000); Asadi et al. (2004); Díaz-Fernández et al. (2010).

Influence function methods. The Influence Function Method was first proposed by Bals in 1931. In Poland, since 1950 two influence function methods have been developed based on applying distribution of impacts using the Gaussian normal distribution curve (Budryk 1953; Knothe 1953, 1957) as well as the method by Kochmanski's (1959) based on the distribution of impacts on a specified derived curve. Using these methods, a subsidence calculation can be performed for horizontal and slightly inclined coal seams: Zenc (1969), Brauner (1973a, b), Marr (1975), Ren et al. (1987, 1989), Karmis et al. (1990), Lin et al. (1992), Sheorey et al. (2000), Álvarez-Fernández et al. (2005), Luo and Cheng (2009), Saeidi et al. (2013), Luo (2015), Polanin (2015), Nie et al. (2017) and Malinowska et al. (2020).

Specialized software programs allow the calculation of subsidence and deformations on the basis of the profile function method and influence function method. Various computer program packages that have been used over the past decades are (Agioutantis et al. 2014):

- Subsidence Deformation Prediction System—SDPS (Carlson Software, Inc.), based on the profile function method and influence function method, developed by Karmis and Agioutantis, can predict the vertical subsidence, horizontal displacement, tensile and compressive strains, slope and curvature;
- Comprehensive and Integrated Subsidence Prediction Model (CISPM), based on the influence function method, for predicting surface subsidence induced by underground mining operations.

Stochastic Influence Function Methods start from the assumption that the massif is layered, divided by a series of cracks into a large number of rock blocks whose movements have a random character. The displacement of a rock massif, as the sum of the movements of a large number of clastic elements, obeys the laws of mathematical statistics (Litwiniszyn 1957, 1974, 2014; Liu and Liao 1965; Pataric and Stojanovic 1994; Vulkov 2001; Meng et al. 2014; Borela 2016; Cai et al. 2016; Malinowska et al. 2020).

Empirical methods are the most numerous group based on the results of systematic measurements in the local conditions of a coal basin. The values of subsidence in the main cross sections by the seam strike and dip were obtained on the basis of type curves. They are the result of approximation and averaging of curves obtained by measurements or theoretical assumptions. The analytical form of type curves, depending on the author, can vary (Avershyn 1947; Miller and Pierson 1958; Kolbenkov 1961, National Coal Board N.C.B. 1963; Anon 1975; Peng 1978; Hood et al. 1981; Kratzsch 1983; Kapp 1986; Reddish and Whittaker 1989; Kay and Carter 1992; Holla 1997; Holla and Barclay 2000; Seedsman 2004; Gale 2011). By applying empirical methods, high accuracy of subsidence prognosis can be achieved, but their disadvantage is that the results are based on direct measurements and are valid only in local conditions.

Numerical models. Theories of rock mechanics and mathematics are the base of numerical models for subsidence prediction. In mathematical modeling, mechanisms of rock deformations are approximated to components that can be defined and quantified since this view means understanding the physical behavior of attacked rocks. A reliable model is formed on the basis of defined mutual interactions of components. Many theoretical studies and mathematical modeling of underground mining-induced surface deformations are performed by applying methods that rely on numerical models. The most important methods are the *elastic methods* (e.g. Berry 1977; Jones and Bellamy 1973; Salamon 1977, 1991) and *the visco-elastic method* (e.g. Astin 1968; Zeng and Kou 1992). Numerical models include usage of the *finite-element methods-FEM* (e.g. Reddish 1984; Jones and Kohli 1985; Fitzpatrick 1987; Siriwardane and Amanat 1988; Najjar and Zaman 1993; Yang et al. 1993; Lewis and Sukirman 1994; Migliazza et al. 2009; Liu et al. 2017; Pariseau and McCarterr 2017; Yin et al. 2018), *boundary element method* (e.g. Crouch 1983; Wardle and Enever 1983; Gurtunca and Schumann 1986; McNabb 1987), and *distinct element method* (Coulthard and Dutton 1989) in predicting the ground subsidence and deformations. In addition, the numerical methods were used for the subsidence modeling and calculating the movement of rock strata (e.g. Yao et al. 1991; Alejano et al. 1999; Keilich et al. 2006; Zhao and Jinan 2011; Shabanimashcool and Li 2012; Unlua et al. 2013; Zhu et al. 2016; Ghabraie et al. 2015).

Values of in situ rock mass parameters are necessary for all numerical models. These models can simulate nearly every conceivable material property, inhomogeneity, bedding planes, anistropy, and various boundary conditions. All subsidence phenomena such as the decay of rock, detachment, sliding, and rotation of seam cannot be quantified in laboratory tests or incorporated into numerical models. The reliability of the prognosis depends on the number and degree of approximations of the real conditions that are always involved in subsidence modeling.

An intensive development of numerical modeling has started from the last decade of the twentieth century. The recent models have much wider use for many aspects related to subsidence because they provide a good simulation of stress and strain distributions within subsiding overburden,

block translation, crack propagation and groundwater movement.

Subsidence modeling faces and should overcome different problems. First of all, subsidence is a three-dimensional phenomenon. It takes place in different heterogenuous environments with characteristics that are not completely known; the testing is performed on small laboratory samples but some characteristics are not testable and therefore have to be assessed. The seams can behave as soils, weak rocks, elastically deformed rocks, plastically deformed shales, discontinuous blocky masses or their combinations. Unknown or partly known geological structures have influence on further deformation of the seams.

Programs based on finite element methods are universal software tool for modeling and they have been adopted for subsidence prediction.

Using 2D and 3D geotechnical software programs it is possible to get an image of the displacement distribution of the roof and the footwall of the excavated area. At the same time, limiting the influence area by draw angles, like in empirical, profile function, and stochastic methods, is not reliable, because deformations and displacements of points in the massif, from the lowest element of the excavated seam to the element of the upper surface of the model, behave approximately according to the laws of mechanics. Manipulating certain parameters is usually necessary for obtaining a model which approaches the measured subsidence profile results very closely. Compared with the numerical analysis method, the stochastic medium theory does not need a constitutive model of rock or soil and its mechanical and hydrogeological parameters (Djordjevic and Vusovic 2014).

During the last decades, the following numerical modeling software packages have been developed (Agioutantis et al. 2014):

LaModel, established by Heasley, is usually used in more complex mining situations, such as multiple seam mining, complex mine geometries, yielding pillars and variable topography.

1. ADONIS, based on finite element method, for improving the modeling and computational simulation in geotechnical engineering.
2. RS3 (Rocscience Inc.) is a finite element analysis program for underground and surface excavations, tunnels, foundation design etc.

In addition, commercial software programs such as Analysis of Longwall Pillar Stability (ALPS), Analysis of Retreat Mining Pillar Stability (ARMPS), Coal Mine Roof Rating (CMRR), Analysis of Roof Bolt Systems (ARBS), Analysis of Horizontal Stress Effects in Mining (AHSM), Analysis of Multiple Seam Stability (AMSS), Analysis of Retreat Mining Pillar Stability—Highwall Mining (ARMPS-HWM), all developed by NIOSH, Elfen horizon, and GEO5 Ground Loss are also available, but it remains unknown on which prediction models they are based.

Theoretical models based on physical concepts and understanding of displacement mechanism in a rock massive are needed to anticipate the continuous and discontinuous subsidence profiles. The shape and position of subsidence trough in relation to the excavated area depend on the geological and technological conditions of underground exploitation. For the horizontal coal seams, a subsidence trough is symmetrical in relation to the excavated area, while for sloped coal seams it becomes asymmetrical. Also, they provide deriving other factors of movement, such as tilt, curvature, and horizontal strain (Malinowska and Hejmanowski 2010).

Surface subsidence due to underground mining and evaluation of subsidence-induced damages of objects are nowadays growing problems in Serbian coal underground mines. When comparing the experience in studying the process of mining subsidence with other countries, Serbia's 40-years experience can be considered modest. The sloped seams, with a thickness of 10–20 m at small and medium depths and an extremely steep relief as well as not large population of the mine area are specific for Serbia. In some underground coal mines, the protective pillars were excavated under the facilities of minor importance and with higher values of permitted deformations such are local roads and railways, smaller watercourses and rural settlements (Pataric and Stojanovic 1994). Material costs and psychological responses have forced the underground coal mining companies in Serbia to start resolving these problems by subsidence prediction methods that minimize damages instead of previous repairing the buildings or compensating their owners.

The stochastic method proposed by the authors Pataric and Stojanovic has been applied for almost four decades in predictive calculations of subsidence at all brown coal mines with underground exploitation in Serbia, such are: "Aleksinac" coal mine in Aleksinac, "Soko" coal mine in Soko Banja, "Rembas" coal mine in Resavica and "Štavalj" coal mine in Sjenica. The results of long-term measurements at these mines with different naturally geological and mining-technological conditions of exploitation indicated a correlation between the elements related to the cause- excavation of the coal seam and the elements related to the consequence-subsidence on the ground surface.

By comparing calculated and measured subsidence values, it was revealed that using this stochastic method the extent of change on the ground surface can be successfully predicted, thus indicating the expected damage level of objects due to subsidence above mining works (Diyab 2011). This confirmed the possibility of applying the stochastic Pataric-Stojanovic method for a reliable subsidence prediction. Although tested and verified for decades, this

method has been offered and promoted at international conferences only until the 1990s.

Sophisticated technology is required to find, extract, and manage the resources in the earth's crust. Since mining is inherently spatial, requiring accurate knowledge of areas of the earth's surface and subsurface, geospatial technology is best suited to intelligently oversee all phases of the mine operation. Modern geoinformation technologies provided the use of progressive systems for the management of spatial data with their integration in the GIS (Fortheringham and Wegener 2000; Burrough and McDonnel 2006). GIS gives mining companies the tools they need to operate mines responsibly and at optimum efficiency (Banerjee et al. 2014).

GIS is a comprehensive technology that has been specifically designed to compile, process, analyze, display, and archive extensive volumes of data. A well designed, implemented, and supported GIS management structure is critical in the success of any mining operation. GIS replaced old map-analysis processes, traditional drawing tools, and drafting and database technologies. GIS uses spatially referenced data that carries out various management and analysis tasks. GIS map can combine many layers of information comes from a database.

One of the key areas of research are using geoinformation technologies (GIS) and remote sensing technologies (Trofymchuk et al. 2020; Korchenko et al. 2019; Anpilova et al. 2020), methods of mathematical modeling (Okhariev 2020) to address pressing environmental safety issues. Among them, it is important to note the study of hazardous processes in geological medium and soils (Trofymchuk et al. 2020), in particular, the problems of landslides and subsidence, and building damages in the zone of mining operations impact (Malinowska and Hejmanowski 2010).

Computer-based analytical methods that realistically simulate spatially distributed, time-dependent subsidence processes are desirable for the reliable design of mining layout to minimize the impact of underground excavation on the land surface. GIS is designed to support integrative modeling, perform interactive spatial analysis and comprehend different processes. Furthermore, based on the simulation of complex subsidence processes that are spatially distributed and progressive in time, it is possible to create innovative thematic maps containing land-surface properties (Zahiri et al. 2006; McDonald 2010; Cao et al. 2019; Cai et al. 2016; Trofymchuk et al. 2020; Suh 2020). The stochastic calculation model for mine subsidence without a GIS would be time-consuming and very complicated in cases of a large number of excavation areas (Esaki et al. 2003, 2008; Miao et al. 2008; Unlua et al. 2013).

The aim of this paper is to present a new approach for subsidence prediction based on the stochastic Pataric-Stojanovic

method with integration into GIS applied to the exploitation of the "Strmosten" deposit in the underground coal mine ''Rembas''-Resavica (Serbia) for the period from 2018 to 2038. The stochastic Pataric-Stojanovic method for the prognosis of displacement of undermined terrain is original since its derived equations contain coefficients that do not have a specific physical meaning but can be related to the geometric characteristics of the displacement process in the massif. Among them are the draw angles as the basic parameters of the displacement process. It gives correct results of the prognostic calculation of subsidence and deformations at any point of the undermined terrain for the seam slope from 10° to 60° when the footwall is also affected by the displacement process. Basic formulas can be applied both for inclined and horizontal seams. Furthermore, it provides calculating the draw angles for any conditional subsidence value at the boundary points based on the angle of full subsidence (θ) obtained from the empirical equations. The subsidence calculations are implemented by the original program package MITSOUKO, created by Vusovic, based on Pataric- Stojanovic stochastic method. Spatial analysis in GIS is based on the integration of the subsidence prediction results using MITSOUKO software and data processing in the ArcGIS computer program. The predicted subsidence values should be checked by geodetic surveying during the entire mining period. The obtained results of benchmarks leveling along the profile lines during the first year, as well as their numerical and graphical interpretation, provided comparing measured and calculated subsidence values and proved the reliability of the applied stochastic method in practice, which is a necessity in modern mining.

Stochastic method for mine subsidence prediction

Land movements, caused by underground coal mining, are very complex and therefore difficult for modeling because of the complicated behavior of the overlying rock mass and land profiles. The subsidence is seriously affected by geological conditions and characteristics such are tectonic strains, groundwater, and faults.

The surface topography does not influence the subsidence magnitude, but it changes the shape and position of the subsidence trough (Reddish and Whittaker 1989). In the mid-twentieth century, for the mining-subsidence prediction, several idealized media were used. Since the behavior of overlying strata is complicated due to numerous known and unknown factors that affect and conduct moving of the rock mass, the stochastic theory of ground moving was used in these models for the subsidence prediction (Kratzsch 1983;

Liu and Liao 1965). Generally, it is easier to predict the definite subsidence than the movements caused by sequential mining processes. Therefore, the stochastic theory can be a universal method for mine-subsidence prediction (Pataric and Stojanovic 1979, 1982, 1985, 1988, 1991; Vulkov 2001; Esaki et al. 2008; Djamaluddin et al. 2011; Cai et al. 2016; Blachowski 2016; Borela 2016; Malinowska et al. 2020).

The stochastic theory model

The idea of the stochastic theory model was introduced by Pokrovsky in 1929 (Pataric and Stojanovic 1994) and further was applied and developed by other authors of stochastic methods. The stochastic model was originally applied in soil mechanics. During studying the pressure variation with depth due to the concentrated force on the surface, Pokrovsky concluded that the pressure change at the horizontal plane can be represented by the standard Gaussian distribution curve. This consideration was related to sand, gravel and severely cracked soils. The massif movement, which is a sum of the movements of a large number of clastic elements, obeys the laws of mathematical statistics.

Litwiniszyn (1957) proposed stochastic subsidence model presuming the ground mass as a discontinuous medium where element movement towards a collapsing cavity is considered as a Markovian process (Borela 2016). According to the stochastic theory, moving of the rock mass above the excavated element might happen randomly with a certain likeliness. It assumes that the rock mass can be moved from one location to another and its shape can vary under unit element mining, however, its total volume remains the same. The procedure is based on the concept of stohastic process (Fig. 1). Since its beginning, this theory has undergone numerous and constant improvements. Numerous solutions of rock movement computing in various geological and mining conditions have been realized based on this the stochastic method. This method proved to be effective allowing to find theoretical solutions to many problems (Djamaluddin et al. 2012).

Liu and Liao (1965) established stochastic method named Stochastic Medium Theory Model (SMTM), a profile function based on statistic medium algorithm method, for the prognosis of underground mining-induced surface movement, which is the most commonly used method in China. To minimize damages on surface objects in the zone of underground mining impact, serious research on predicting ground surface movement by this method has been carried out in China. Nowadays, SMTM is also used for calculating ground movement caused by tunnel construction (Li et al. 2019; Chen et al. 2020). The Stochastic Medium Theory Model arose into Probability Integral
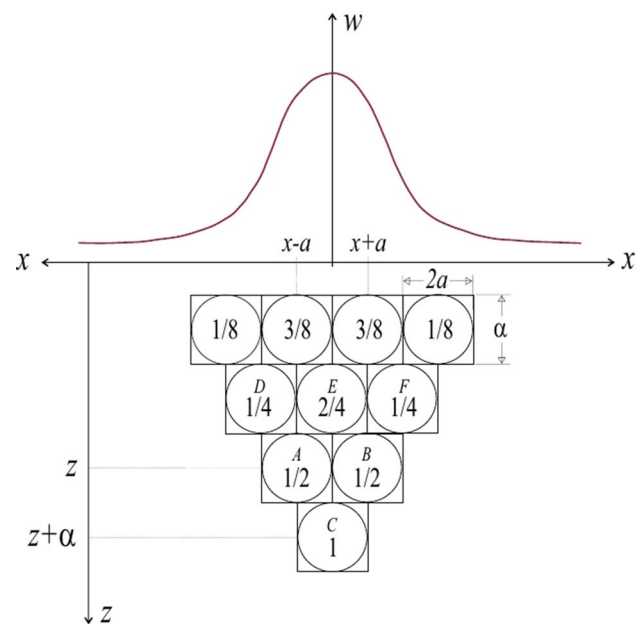
Fig. 1 Development of movement in a stochastic medium (Litwiniszyn 1957)

Method (PIM), based on the statistical theory which is more reliable and easy to use for subsidence and deformations prediction in the entire excavation field. To calculate the surface deformation caused by coal mining, the rock is approximated as statistic medium and formed with small statistic movement elements. When one element in the first layer was excavated, the two elements from the second layer have equal chances to occupy it and in that case the surface subsidence is similar to the Gaussian curve.

Stochastic method by Pataric and Stojanovic

The Pataric–Stojanovic stochastic method presents a model based on original mathematical formulas. This method applies the mathematical statistics and starts from the assumption of Litwiniszyn that the massif is multi-layered, divided by a series of cracks into a large number of elements whose movements have stochastic character. This environment can be presented by symmetrically arranged elements with approximately similar dimensions. Such an area does not exist in nature, but this assumption is statistically correct because the pressure change curve in a homogeneous medium is symmetrical and therefore elements in the profile must be symmetrical. Owing to this
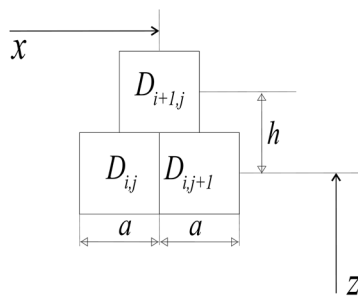
symmetry, the force got by one element is transmitted and equally divided into two parts on which it relies.

Movements in a massif with horizontal seams

The mathematical model by Litwiniszyn (Fig. 1) can be used to study the plane subsidence of a layered massif. The subsidence will be plane if a long area of constant width ($2a$) and height (d) has been excavated. The assumption is that the rock massif consists of horizontal layers divided by a series of cracks into equal, symmetrically arranged elements (Fig. 2). When the element from the lowest layer is removed, the elements in the higher layers start moving. Their movement is assumed to be stochastic. A place that remains vacant in the lowest layer will be occupied by one of the elements that lie directly above it. Owing to symmetry, the probabilities of these events are the same, so their value equals 1/2. With further following the probabilities for moving elements from higher layers, the same regularity as in the example in Fig. 1.

But, in this case, the boundary transition from a discrete distribution to a continuous massif is performed differently: the positions of the individual elements are determined by coordinates. Since the subsidence is plane, two coordinates are sufficient: an abscissa x parallel to the layers (horizontally) and an elevation z vertically with the upward direction-to determine the position of the elements in the massif.

If the width is marked with $2a$ and height with h, the coordinates of particular elements are: $D_{i,j}(x - a, z)$, $D_{i,j+1}(x + a, z)$, $D_{i+1,j+1}(x, z + h)$.

For the boundary transition from a discrete division to a continuous massif, the starting point is the position of the elements that are defined by the coordinates, ie:

$$F(x, z + h) = \frac{1}{2} \cdot [F(x - a, z) + F(x + a, z)], \tag{1}$$

where $F(x, z)$ is the *function of subsidence probability* (Pataric and Stojanovic 1994).

The assumption is that the excavated area Ω_2 has a rectangle shape with width $2a$ and thickness d, so if a coordinate origin is placed in the intersection of rectangle diagonals, it will be:

$$U(x) = \iint\limits_{\Omega_2} F(x, \overline{x}, H, \overline{z}) \, d\overline{x} \, d\overline{z}. \tag{2}$$

Since the thickness d is negligible with respect to the depth H of the seam, instead of the integral, its mean value can be taken without a significant error:

$$\int\limits_{-d/2}^{d/2} F(x, \overline{x}, H, \overline{z}) \, d\overline{z} = d \cdot F(x, \overline{x}, H, 0).$$

Using the function from the Eq. (1) the subsidence is:

$$\Phi(t) = \frac{2}{\sqrt{2\pi}} \cdot \int\limits_0^t e^{-\frac{1}{2} \cdot s^2} \mathrm{d}s, \tag{3}$$

the subsidence is definitely calculated by the formula (Pataric and Stojanovic 1994):

$$U(x) = U_0 \cdot X_0(x), \tag{4}$$

where:

$$X_0(x) = \frac{1}{2} \cdot \left[\Phi\left(\frac{a + x}{n}\right) + \Phi\left(\frac{a - x}{n}\right) \right].$$

Movements in a massif with inclined seams

In the case of inclined seams, the starting assumption that the massif is divided by a series of horizontal and vertical cracks into elements of the same dimensions. Equation (4) can only be applied to profiles along the strike. However, the subsidence curve is not symmetrical by the seam dip, so there is anisotropy unlike the process in a massif with horizontal seams. The same mathematical model can be used assuming that the elements, with still identical dimensions due to homogeneity, are no longer symmetrically arranged,

Fig. 2 Moving process in the massif under plane subsidence

Fig. 3 Moving process in the massif with inclined seams
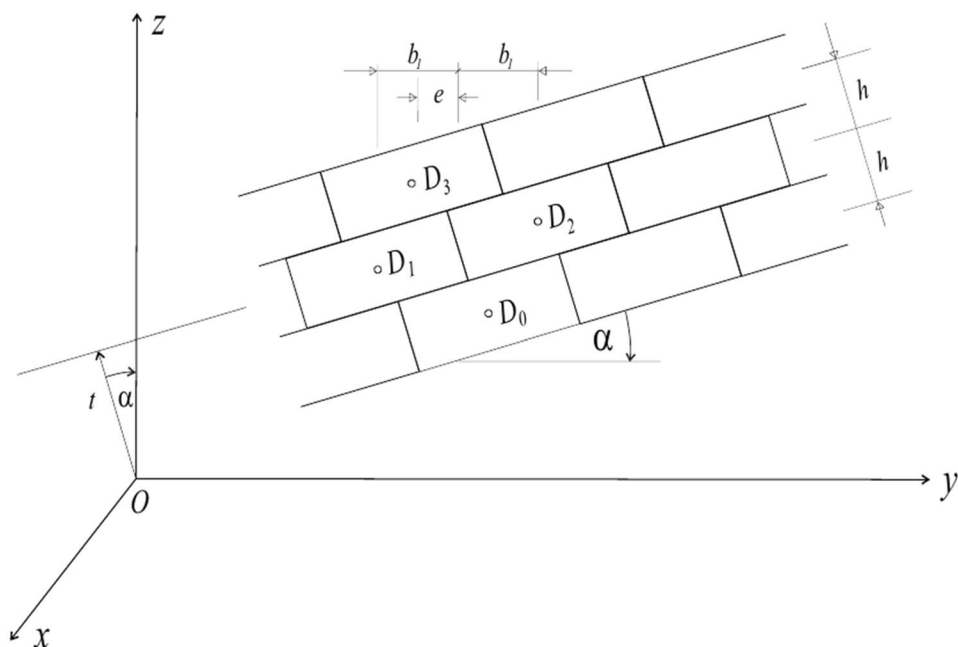
but the center of each element is shifted by the dip for a certain length e, whereby $0 < e < b_1$ (Fig. 3).

As seen in Fig. 3, the coordinate origin is placed at an arbitrary point O of the seam that is being excavated, the z-axis is directed vertically upwards, the x-axis is directed horizontally along the seam strike, and the y-axis is in the direction of the seam dip. The direction of the x-axis is chosen in a way that the coordinate triedar is right.

The equation of the plane of the seam at the distance t from the coordinate origin is:

$$t = z \cos \alpha - y \sin \alpha. \tag{5}$$

In this case, the place of the lowest removed element D_0 will be occupied by the element D_1 or by the element D_2, but owing to the disturbed symmetry, the probabilities of these events are not equal. Their values can be calculated as the resistances of the simply supported beam of span $2b_1$ loaded with a unit eccentric force, so the probability is determined as follows:

$$\left(\frac{1}{2} + \frac{e}{2b_1}\right) F\left(x, y + b_1, t\right) + \left(\frac{1}{2} - \frac{e}{2b_1}\right) F\left(x, y - b_1, t\right) = = F(x, y - e, t + h). \tag{6}$$

The massif is assumed to be anisotropic, therefore the subsidence curves in the profiles by the seam strike are symmetrical, and asymmetrical in the profiles by the seam strike.

When deriving the equations, it is assumed that the centers of the elements remain in the same vertical plane during subsidence. In the general case, the center of the element during the subsidence will move both by the strike and by the dip, so the function of the subsidence probability F

depends on all coordinates. Due to the independence of the influence of individual coordinates, the equations:

$$F\left(x, \bar{x}, y, \bar{y}, z, \bar{z}\right) = F\left(x, \bar{x}, t, \bar{t}\right) F\left(y, \bar{y}, t, \bar{t}\right), \tag{7}$$

represents the total subsidence probability for inclined seams.

## Basic formulas for subsidence calculation of the horizontal seam

When excavating layered deposits, in modern mining practice the used excavation methods (the pillar excavation methods and the longwall mining methods) that remove the rectangular excavation panels of the layer, whereby the sides of the rectangle are parallel to the coordinate axes-O_ξ by the seam strike and O_η by the seam dip (Fig. 4). The usual assumptions are:

- the excavation area is prismatic, with a constant thickness (d);

- the base of the coal seam is rectangular with a width by strike ($2a$) and a width by dip ($2l$);
- the width by strike ($2a$) is large enough to meet the conditions of the plane subsidence.

Fig. 4 Coordinate system based on the stochastic model
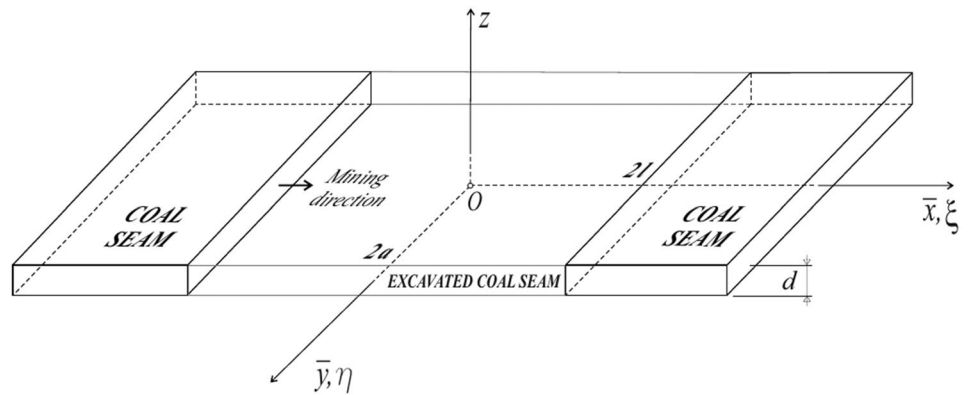
The subsidence in a point $M(x, y, H)$ of the terrain surface $z = H$ of the undermined massif based on the Eq. (7) will be a double integral:

$$U(x, y) = \iint_{\Omega_2} F\left(x, \bar{x}, y, \bar{y}, z, \bar{z}\right) d\bar{x} \, d\bar{y} \, d\bar{z}. \tag{8}$$

When the seam thickness d is negligible in relation to H, $d \leq H \leq \bar{z}$, and by introducing new variables: $\bar{x} = \xi$, $\bar{y} = \eta$, $\bar{z} = 0$, $z = H$, it is obtained:

$$U(x, y) = \iint_{\Omega_2} F(x, \xi, y, \eta, H, 0) \, d\bar{x} \, d\bar{y}. \tag{9}$$

If the sides of the excavation panel Ω_2 are $2a$ by the seam strike and $2l$ by the seam dip, the coordinate origin is at the intersection point of the rectangle diagonals will be: $-a \leq \xi \leq a$; $-l \leq \eta \leq l$ and using the function:

$$\Phi(x) = \frac{2}{\sqrt{2\pi}} \cdot \int_0^x e^{-\frac{1}{2} \cdot t^2} dt, \tag{10}$$

a definite formula for calculating *subsidence during horizontal seam excavation* is obtained (Pataric and Stojanovic 1994):

$$U(x, y) = U_0 \cdot X_0(x) \cdot Y_0(y), \tag{11}$$

where:

$$X_0(x) = \frac{1}{2} \cdot \left[\Phi\left(\frac{a+x}{n}\right) + \Phi\left(\frac{a-x}{n}\right)\right],$$

$$Y_0(y) = \frac{1}{2} \cdot \left[\Phi\left(\frac{l+y}{n}\right) + \Phi\left(\frac{l-y}{n}\right)\right].$$

The function Φ from Eq. (10) represents the standard Gaussian distribution curve.

**Basic formulas for subsidence calculation of the inclined seam**

If during the excavation of the inclined layers, the exploitation area (Ω_3) is of arbitrary shape, the total subsidence $M(x, y, H)$ on the surface of the undermined massif based on (7) will be:

$$U(x, y) = \iiint_{\Omega_3} F\left(x, \bar{x}, y, \bar{y}, H, z\right) d\bar{x} \, d\bar{y} \, d\bar{z}. \tag{12}$$

If the seam dip angle (α) and the seam thickness (d) are constant, the area Ω_3 is prismatic with the base Ω_2 (Fig. 5).

By introducing new variables: $\bar{x} \equiv \xi$; $\bar{y} = \eta \cdot \cos\alpha - \xi \cdot \sin\alpha$; $\bar{z} = \eta \cdot \sin\alpha + \xi \cdot \cos\alpha$, the subsidence calculation can be simplified, because if $\xi = 0 \Rightarrow \bar{y} = \eta \cdot \cos\alpha$; $\bar{z} = \eta \cdot \sin\alpha$ the difference when $z = H$ is: $t - \bar{t} = (z - \bar{z}) \cdot \cos\alpha - (y - \bar{y}) \cdot \sin\alpha = H \cdot \cos\alpha - y \cdot \sin\alpha$ and no longer depends on the variable η.

If the sides of the excavation panel Ω_2 are $2a$ by the seam strike and $2l$ by the seam dip, with the coordinate origin at
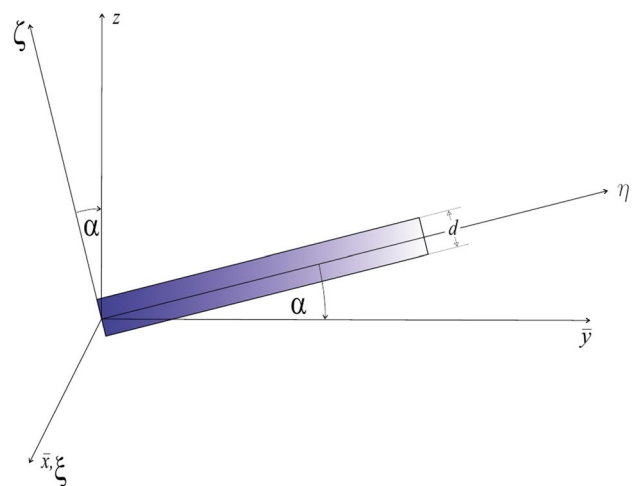
Fig. 5 Plane subsidence of inclined seam

the intersection point of the diagonals of the rectangle the expression for calculating *subsidence during inclined seams excavation* is obtained (Pataric and Stojanovic 1994):

$$U(x, y) = U_0 \cdot X(x, y) \cdot Y(y), \tag{13}$$

where by:

$$X(x, y) = \frac{1}{2} \cdot \left[\Phi \left(p \cdot \frac{a + x}{\sqrt{H \cdot ctg\alpha - y}} \right) + \Phi \left(p \cdot \frac{a - x}{\sqrt{H \cdot ctg\alpha - y}} \right) \right],$$

$$Y(y) = \frac{1}{2} \cdot \left[\Phi \left(q \cdot \frac{b + m + y}{\sqrt{H \cdot ctg\alpha - y}} \right) + \Phi \left(q \cdot \frac{b - m - y}{\sqrt{H \cdot ctg\alpha - y}} \right) \right],$$

where H- seam depth; α-angle of seam dip; parameters: $p = \frac{p_0}{\sqrt{\sin \alpha}}; p_0 = q_0; q_0 = (1 - \lambda \sin \alpha); \lambda = \frac{\cos \theta}{\sin(\alpha + \theta)}; q = \frac{q_0}{\sqrt{\sin \alpha}}; b = \frac{l \cos \alpha}{1 - \lambda \sin \alpha}; m = \frac{H \lambda \cos \alpha}{1 - \lambda \sin \alpha}; \theta$-angle of full subsidence.
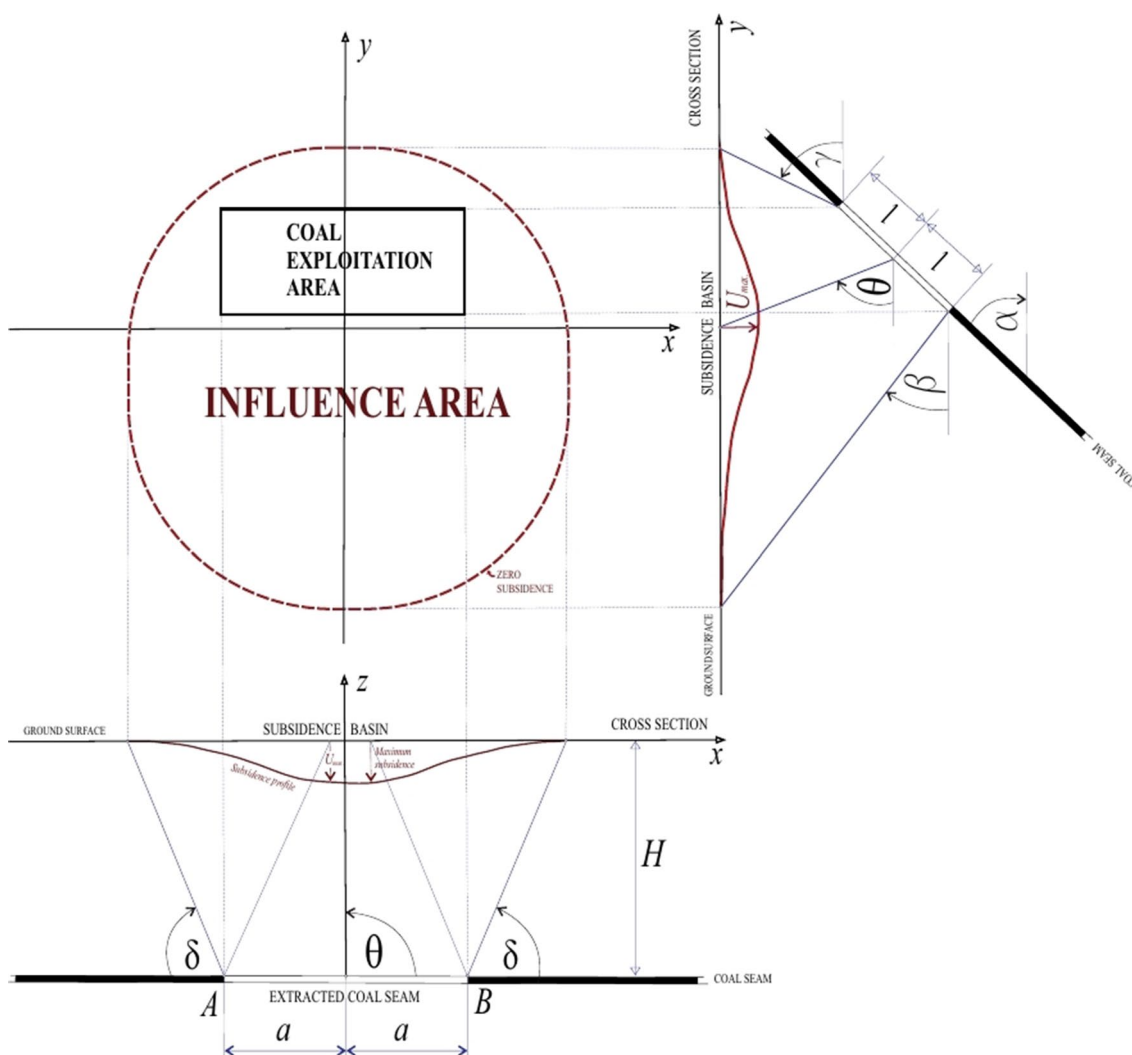
The subsidence curve will be the same in any profile by the dip, so it is a plane subsidence:

$$U(y) = U_0 Y(y) \tag{14}$$

where $Y(y)$ is calculated from the Eq. (13).

Figure 6 presents subsidence during the excavation of the inclined coal seam.

In the case that $\alpha = 0$, the formula (13) is reduced to the basic formula (11) for calculating the subsidence of the horizontal seam.

Fig. 6 Subsidence during excavation of the inclined coal seam with parameters: H-seam depth; 2a, 2l-dimension of excavation area; α-angle of seam dip; U_{max}-maximum subsidence; δ, β, γ-angles of the draw; θ-angle of full subsidence

Geometric interpretation of parameters for the subsidence predictive calculation

The subsidence calculation formulas based on the stochastic method, formulate two types of parameters:

Parameters a, l, H, d, b that are known and determined;

Parameters U_0, p, m, q, δ that are not determined in the physical sense. These parameters describe the behavior of an undermined rock massif during consolidation and need to be determined previously to find their relationship with some unit that can be measured.

The terms, definitions, and symbols that follow are adopted and presented in Fig. 7.

Dimensions of the excavation panel a, l[m] are determined from the project and depend on the exploitation area.

Seam depth (H) represents the mean seam depth for each excavation panel. It is calculated from:

$$H = H_T - H_S,$$

where H_T is the elevation of the terrain over the excavated seam [m] and H_s is the depth of the excavation panel [m].

Angle of seam dip α [°] varies. For the predictive calculation of the movement parameters, the mean values of the seam dip for each excavated area are taken.

Maximum subsidence U_0 [mm] defines the maximum possible subsidence that can occur at the full excavation surface. Only one point M at the ground surface has a maximum

possible subsidence. It is calculated from the expression (Pataric and Stojanovic 1994):

$$U_0 = d \cdot q_u \cdot i \cdot \cos \alpha, \tag{15}$$

where: q is the coefficient of subsidence of the hanging wall and is taken as a real value based on the analysis of former excavation. The most commonly adopted value is $q = 0.7$–0.9 which corresponds to the value that would be taken based on an analogy from a basin with the similar physico-mechanical properties of the excavated area. The parameter ***q*** is calculated from the expression (Pataric and Stojanovic 1994):

$$q = \frac{f + 2}{\sqrt{300 \sin \alpha}} \, \exp \left[\left(0.027 - 0.0076 \, \sqrt{f} \right) \alpha \right],$$

where: f is a strength coefficient of the rock massif by Protodjakonov; i is a coal deposit recovery determined with losses $g(\%)$, which corresponds to the designed excavation method, whereby $i = 1 - g$.

Parameter p [m^{-1}] depends on the relationship between the side a and the hypothetical element h. It is calculated from the expression (Pataric and Stojanovic 1994):

$$p = \frac{f + 2}{\sqrt{300 \, tg\alpha}}. \tag{16}$$

Parameter m to determine this parameter, it is necessary to know the position of the point with the maximum subsidence U_0 which is in the point M, with the ordinate $Y_M = -m$.
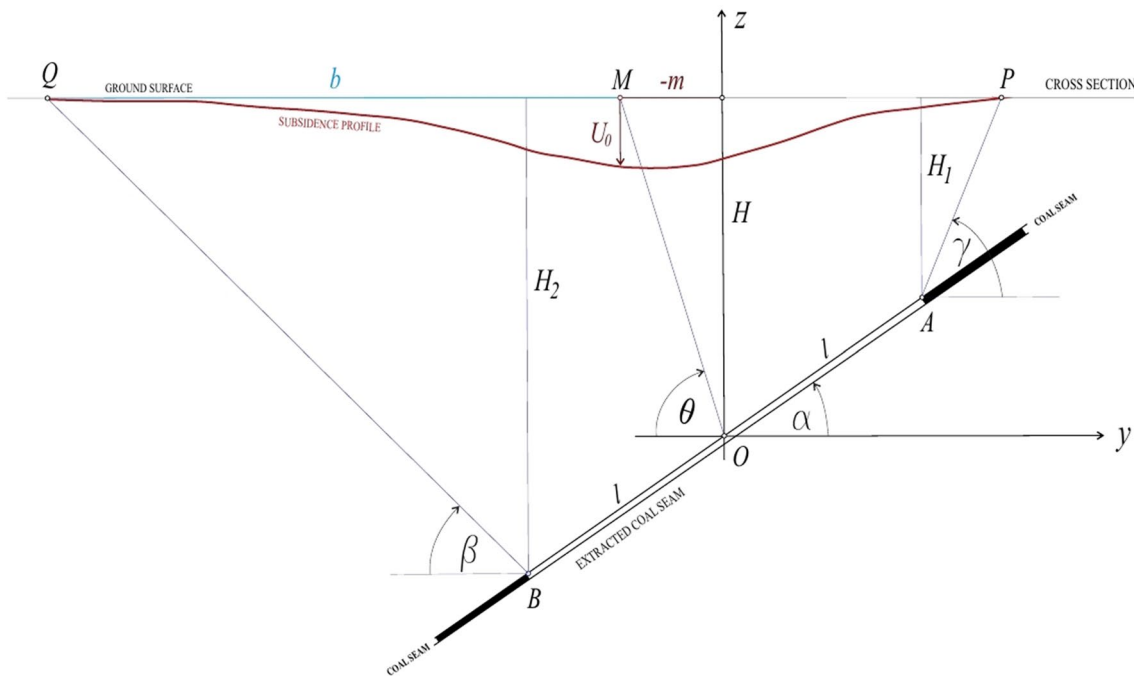

Fig. 7 Parameters for mine subsidence prediction

The parameter m is in direct connection with the angle of full subsidence θ (Fig. 7), which is related to the angle of draw δ, ie. with auxiliary angles β and γ. Parameter m is calculated according to the empirical formula (Pataric and Stojanovic 1994):

$$m = H \, ctg\theta. \tag{17}$$

Parameter b is not independent and is calculated from known data: the seam dip α, dimension of the excavation field l, seam depth H and parameter m (Fig. 7). It can be expressed through the angle θ and calculations according to an empirical formula (Pataric and Stojanovic 1994):

$$b - l \cos \alpha = \frac{m}{H} \sin \alpha,$$

by entering the value $m = H \, ctg\theta$ it will be: $b - l \cos \alpha = l \sin \alpha \, ctg\theta$, so the parameter b is calculated from the expression:

$$b - l \frac{\sin (\alpha + \theta)}{\sin \theta}. \tag{18}$$

When the coordinates of the points P and Q (Fig. 7), given as:

$$y_P = l \cdot \cos \alpha + H_1 \cdot ctg\gamma,$$

$$y_Q = -l \cdot \cos \alpha - H_2 \cdot ctg\beta,$$

are included in Eq. (13), the subsidence in these points is obtained (Pataric and Stojanovic 1991):

Draw angles, safety angles and fracture angles are related to the points out of the excavated space, and full movement angles to the points above the excavated space. Since the angular parameters are geometric characteristics of the same process and depend on the physico-mechanical properties and mining-geological factors in the same rock massif, certain dependencies among them can be established (Djordjevic et al. 2011).

Angles of draw (δ_1, β_1, γ_1) are angular characteristics of the moving process that determine the ground surface area affected by displacements caused by excavation, assuming that deformations can be neglected beyond that displacement boundary. These are angles between the horizontal and the line joining the excavation boundary in the pit with some boundary point at the land surface (Fig. 8).

The boundary point is defined by the measured displacement value, which has millimeter values of horizontal movements and vertical subsidence movements. Angles of the draw are the base for calculating the displacement parameters and their accuracy influence the subsidence prediction precision. For the limit subsidence at the boundary point (marked as 1 in Fig. 8), a value of 10 mm is adopted (Pataric and Stojanovic 1992).

Angle of full subsidence (θ). For plane subsidence, Eq. (13) is valid, so $Y'(m) = 0$, and therefore $y = -m$ is the ordinate of the point M with the maximum subsidence. The position M is also determined by θ (Fig. 7). The value of θ is considered as independent on the size of the excavation field without a significant error.

Using coupled angles provides determining the angle θ

$$U_P = \frac{U_0}{2} \cdot \left[\Phi \left(q \cdot \frac{b + m + l \cdot \cos \alpha + H_1 \cdot ctg\gamma}{\sqrt{H \cdot ctg\alpha - l \cdot \cos \alpha - H_1 \cdot ctg\gamma}} \right) + \Phi \left(q \cdot \frac{b - m - l \cdot \cos \alpha + H_1 \cdot ctg\gamma}{\sqrt{H \cdot ctg\alpha - l \cdot \cos \alpha - H_1 \cdot ctg\gamma}} \right) \right], \tag{19}$$

$$U_Q = \frac{U_0}{2} \cdot \left[\Phi \left(q \cdot \frac{b + m - l \cdot \cos \alpha - H_2 \cdot ctg\beta}{\sqrt{H \cdot ctg\alpha + l \cdot \cos \alpha + H_2 \cdot ctg\beta}} \right) + \Phi \left(q \cdot \frac{b - m + l \cdot \cos \alpha + H_2 \cdot ctg\beta}{\sqrt{H \cdot ctg\alpha + l \cdot \cos \alpha + H_2 \cdot ctg\beta}} \right) \right]. \tag{20}$$

Angular parameters of the movement process are geometric characteristics of conditional or physical character which determine the state of the undermined massif. They serve as basic data for designing grids in the form of profile lines to monitor subsidence, determine protective pillars under natural or technical objects, as well as to calculate subsidence process parameters. Angular parameters are determined on vertical cross sections by the seam strike and by the seam dip.

Figure 8 presents angular parameters of undermined ground movement process.

even for more inclined seams, when $\alpha < \gamma$. Besides, when the values of an angle θ and of one of the coupled angles (γ) are known, the other coupled angle (β) can be calculated (Djordjevic et al. 2011).

By this method, a geometric relationship between the draw and safety angles is achieved, which enables the safety angle to be calculated using a given draw angle for any adopted deformation value. In practice, this provides calculating the appropriate safety angle for a certain type of object, ie for the deformation that is the limit in terms of the object's vulnerability, as well as a more rational placing of protective pillars. Practically, for any type of object, any category of protection, any deformation parameter, and any
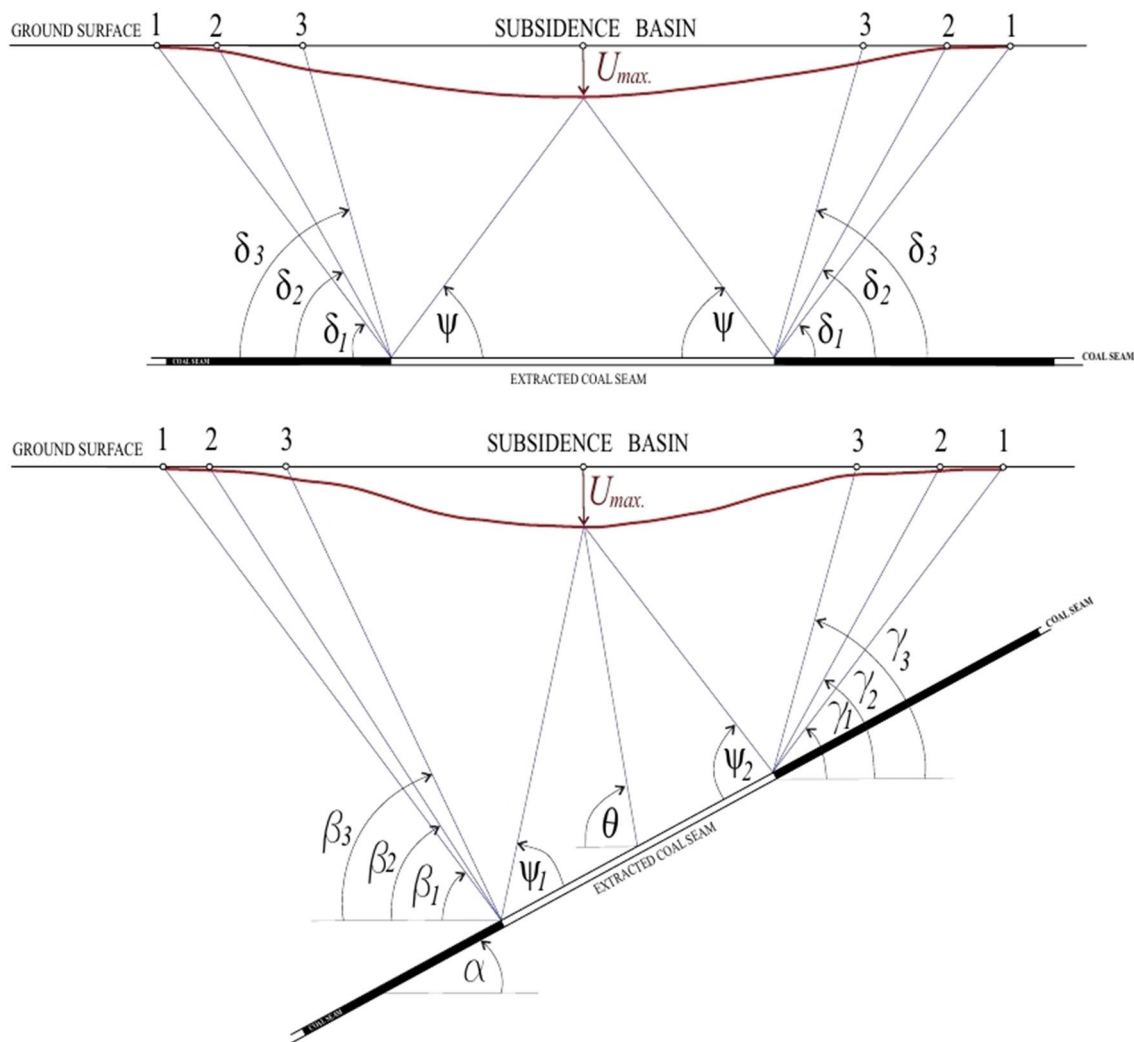
Fig. 8 Angular parameters of undermined ground movement process: δ_1, β_1, γ_1—angles of the draw; δ_2, β_2, γ_2—safety angles; δ_3, β_3, γ_3—fracture angles; ψ, ψ_1, ψ_2—full movement angles; θ—angle of full subsidence; α—angle of seam dip

specific geometrical conditions of the deposit, appropriate values of safety angles can be determined, which has not been known in mining practice.

In the predictive calculations for mines with the unexamined moving process, as it is the case in Serbia, the problem of adopting angles of draw arises. Therefore, Pataric and Stojanovic provided a unique table with angles of the draw. Interdependencies among angles δ_1, β_1, γ_1, θ which can be adopted based on the strength coefficient of the rock massif by Protodjakonov f and seam dip α for the predictive calculations (Djordjevic et al. 2011).

The stochastic Paratic-Stojanovic method provides reliable results for displacement and deformations prediction at any point of undermined terrain for the seam dip $10^o \leq \alpha \leq 60^o$, even in cases when the footwall is affected by movement process.

A case study

The problem of subsidence and protecting objects above the mining works has been present for decades at the "Strmosten" deposit of underground coal mine "Rembas"-Resavica, Serbia. This study presents a predictive subsidence calculation based on the stochastic Pataric-Stojanovic method and the GIS model using input data from this mine (Vusovic 2018).

Having in mind that the displacement of an undermined terrain is very complex, it is difficult for any prediction method to completely include all characteristics of the subsidence process. Since the occurrence of cracks- discontinuities in the rock mass is a precondition for starting the displacement, the equations of continuous media mechanics cannot be applied. The limit equilibrium theory answers whether the displacement will start, when cracks occur and

how they are arranged, but it does not indicate displacement of points in a massif. Therefore, the clastic sedimentary rocks fragmented by cracks must be considered and also the displacement of parts of the rock massif as a stochastic, that is a random process. In this sense, the stochastic method for the calculation of subsidence and deformation of the ground surface can provide answers to these questions, regardless the horizontal coal seam is excavated or the slope of the seam is up to a limit of 60°. For all methods, the angles of draw and angles of full subsidence are regarded as independent magnitudes. However, these angles are part of a unique process, so they are mutually dependent, and this dependence can be determined regardless of the change in seam dip, which is provided by the stochastic method.

Characteristics of the research site

The "Rembas" coal mine is engaged in the exploitation of high-quality brown coal with a tradition of more than 150 years. It is part of the Public company "Resavica", the most important underground coal mining center of the Balkan. At the "Strmosten"deposit, a coal seam with a complex structure has been developed. In the northeastern region of the deposit, this coal seam is divided into three parts, while in the central, eastern and western regions of the deposit it is divided into two parts.

The "Strmosten" coal deposit as well as its wider environment is formed of rocks with the different lithological composition. Based on previous geological studies of the Strmosten deposit and its surroundings, according to the distribution of rocks that participate in the structure of the deposit, three types of rocks can be distinguished: Permian sandstones, paleorelief rocks (limestones and dacites) and coal-bearing Miocene series (Fig. 9).

The Pliocene and Quaternary sediments lie transgressively across the Permian sandstones that represent the coal seam roof. The floor seam consists of clayey sandstones, clays, conglomerates and rarely of marl.

The carboniferous Miocene series is inserted between the cretaceous limestones and andesites, which form the paleorelief, and the Permian red sandstones. The sediments based on the structural and textural characteristics of the "Strmosten" deposit, porosity and hydrogeological properties, can be classified into two groups: rocks in which the aquifer formation is possible and conditionally anhydrous rocks (Vusovic 2018).

Based on the hydrogeological research and mining performed so far in the "Strmosten" pit, water flow was 0.5–1.7 l/s in the part with the water-permeable rocks deposits. According to the data from the geological service of the mine, the average water inflow at the pit exit in 2019 was 3.8 l/s. For 2020, the estimated water inflow in EP2, in the northern part of the field, is in the range 1.1–2.25 l/s, which certainly would not affect the surface subsidence, since the coal seam in that part of the deposit is at a depth of 390 m.

The coal seam thickness (d) is taken from the geological interpretation of the deposit and isolines of the main seam; it ranges from 2 to 8 m. The seam dip (α) is determined as
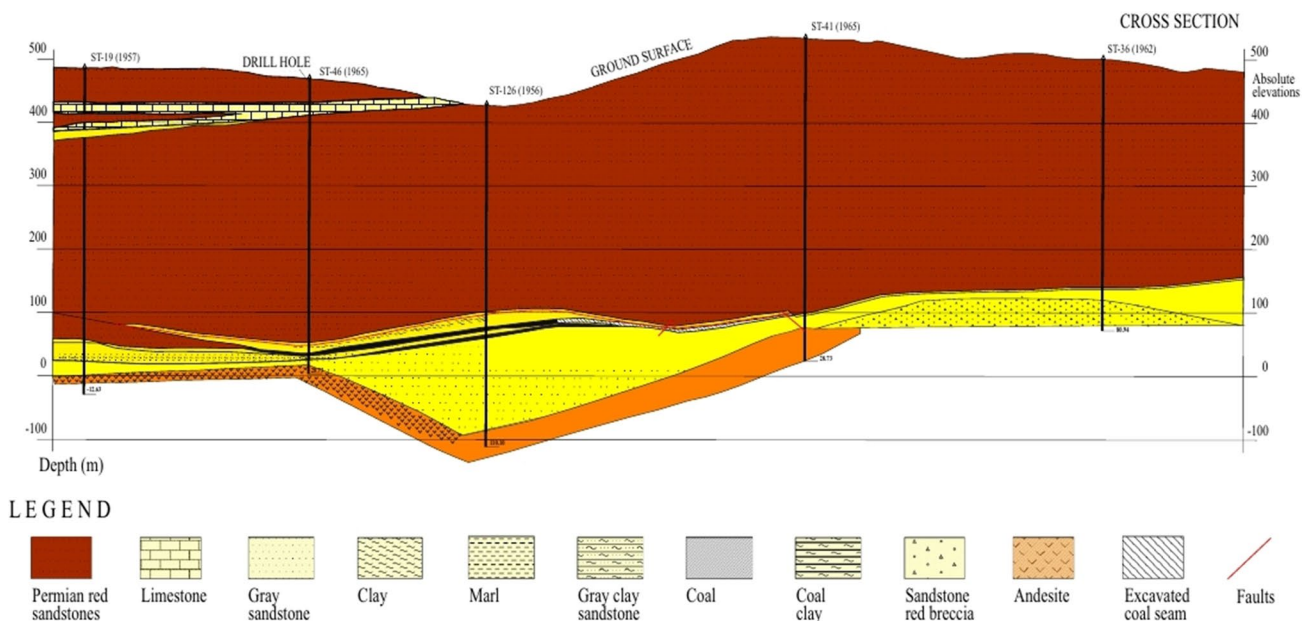

Fig. 9 Structural geological profile of the "Strmosten" deposit
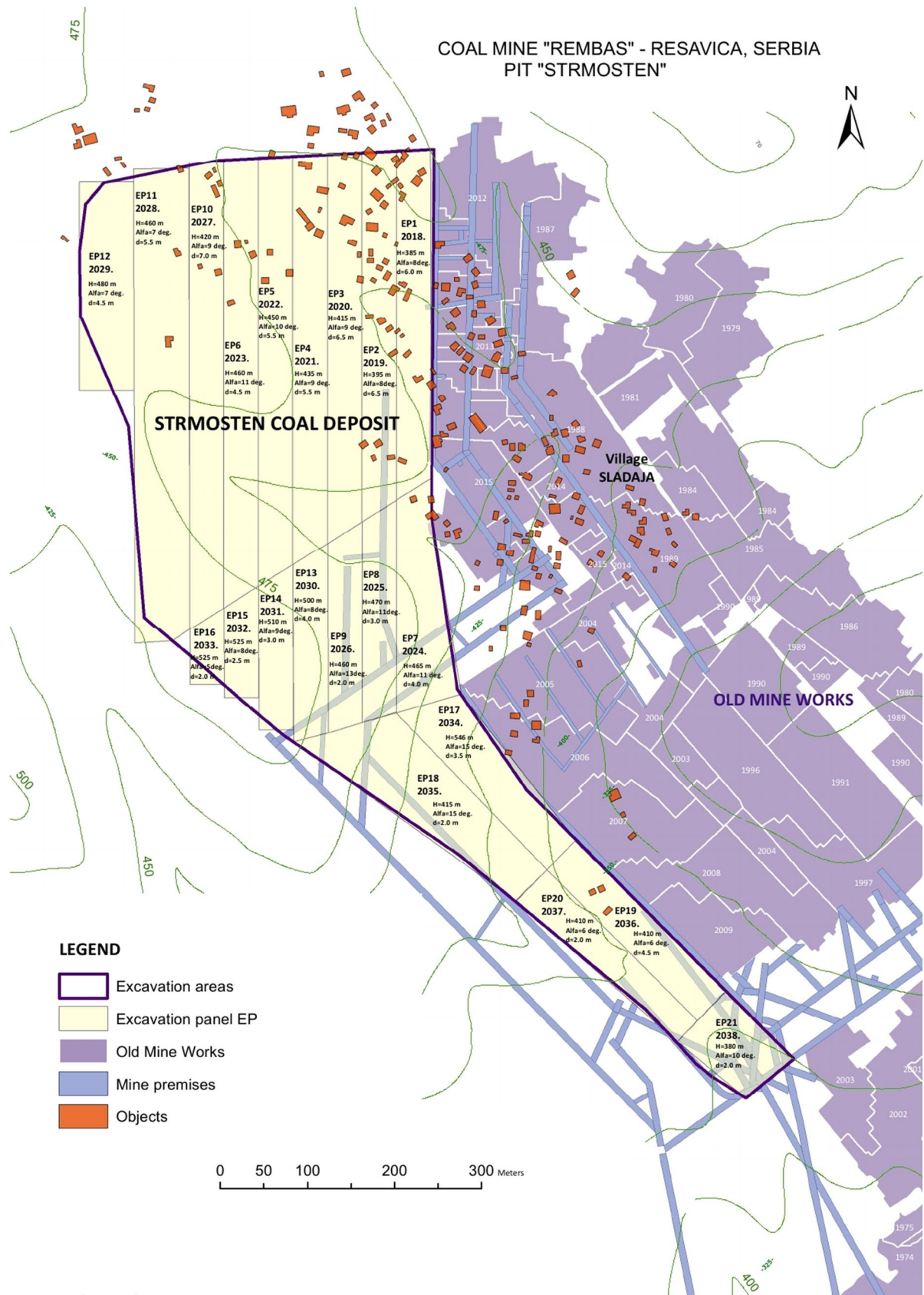
Fig. 10 Excavation panels (EP) mined successively in the excavation field of the "Strmosten" pit

the mean value of certain parts of the excavation field, with values in the range of 5–15°.

For calculating the subsidence parameters, the mean values of the seam dip for each excavation panel (EP) are taken. Seam depths (H) were determined based on the geological interpretation of the deposit, using isohipsas of the terrain and isolines of the layer for each EP individually. The values range from 380 to 525 m (Fig. 10).

So far, during the exploitation of the "Strmosten" deposit, several excavation methods have been used. Coal deposit recovery (i) is determined with losses of $g = 35\%$, which corresponds to the projected excavation method and its value is confirmed to $i = 65\%$. The rate of caving (q) is adopted based on the analysis of the previous studying of the displacement process of the undermined rock massif in the deposit. The value $q = 0.70$ is accepted.

Software solution for mine subsidence prediction based on the stochastic method

Since the subsidence prognosis using equations of the stochastic Pataric-Stojanovic method would be very complicated and time-consuming in case of manual data processing, the original computer program package with the title MITSOUKO has been created based on this method and formulas given in "Stochastic method by Pataric and Stojanovic". MITSOUKO is designed in the Python programming language and provides the mine subsidence and deformations calculation, based on the simulating excavation process according to the geometry of the excavation panels of polygonal shape and the adopted dynamics, at any point of the land surface and representing the results owing to their integration and further processing in GIS.

If the excavation area when mining coal seams is of irregular shape, as a rule, it is divided into excavation panels of polygonal shape, whereby in the predictive calculation the coordinate origin is placed at the intersection point of the diagonals of the rectangle- the coal seam that is excavated.

The basic approach in the surface subsidence calculation is based on the adopted exploitation dynamics in the excavation field and on the original stochastic method's formulas integrated with a graphical presentation of the subsidence contour lines in GIS. The excavation field, with a total area of 29 ha, consists of 21 excavation panels (EP), which are exploited successively from 2018 to 2038 (Fig. 10).

In the initial phase of mine design and estimating the impact of mining works on the ground surface, the results of subsidence measurements are not known, so the only possibility is to perform a predictive calculation using theoretical and empirical methods.

Empirical formulas in the Pataríć-Stojanović stochastic method, contain parameters dependent on the excavated space (a, l, H, d, b) (given in "Geometric interpretation of parameters for the subsidence predictive calculation") and included in the mining project, as well as parameters (U_0, p, m, q), which are not determined in the physical meaning, but define the behavior of the undermined terrain during consolidation and have to be brought into connection with the geometric characteristics of the displacement process in the rock massif. Angles of draw ($\delta_1, \beta_1, \gamma_1$), which limit the zone affected by movements on the ground surface, and angle of full subsidence (θ), as the basic angular parameters of the displacement process, are among them. These angles are obtained by connecting the excavation boundary with the boundary points on the terrain surface. The boundary points are determined by measuring the value of subsidence values on the ground surface.

When deriving empirical formulas for the predictive subsidence calculation by the Patarić-Stojanović method, all points of the massif were observed, although the application of the formulas is limited only to its surface- undermined terrain, since long-term systematic subsidence monitoring in mines over the world were performed mainly at the surface of the undermined terrain.

It consists of two modules: *PARAMETERS* and *SUBSIDENCE* which represent individual independent functions and are mutually connected by a hierarchical structure. Each module starts with form according to a textual description explaining its name and function, the tags of the input data that are loaded (*Read parameters*) or computed (*Calculate parameters*) into a particular module, and the data values returned by a module through the control loop. The modules have control functions, describing what the program does according to the algorithm shown in Fig. 11.

Firstly, by entering the MITSOUKO program, in the *PARAMETERS* module, the function $\Phi(t) = \frac{2}{\sqrt{2\pi}} \cdot \int_0^t e^{-\frac{1}{2} \cdot s^2} ds$ (Eq. 3) is initialized, according to the tabular data (Djordjevic et al. 2011). Then, through the menu, the data obtained based on geometric characteristics for each excavation panel are entered: *ID*, dimensions (a, l), seam depth (H), seam thickness (d) and seam dip angle (α). In the next step, in a specified subroutine of the *PARAMETERS* module, the following is calculated for each panel: maximum subsidence (U_0), parameters (m, b, p), rate of caving (q), angles of draw (δ, β, γ), and angle of full subsidence (θ).

The local coordinate system is situated symmetrically with respect to the first excavation panel (EP1), with x-axis in the direction of the seam strike, y- axis in the direction of the seam dip and the coordinate origin in the diagonal intersection of this panel. The positions of all excavation panels (x_i, y_i, F_i) are determined with respect to the defined local coordinate system. During the calculation for each excavation panel (EP), it is necessary to rotate its coordinate axes for the value of the angle F_i (expressed in degrees) to the direction of axes of the local coordinate system. The

Fig. 11 Stochastic prediction algorithm for subsidence calculation using the MITSOUKO software
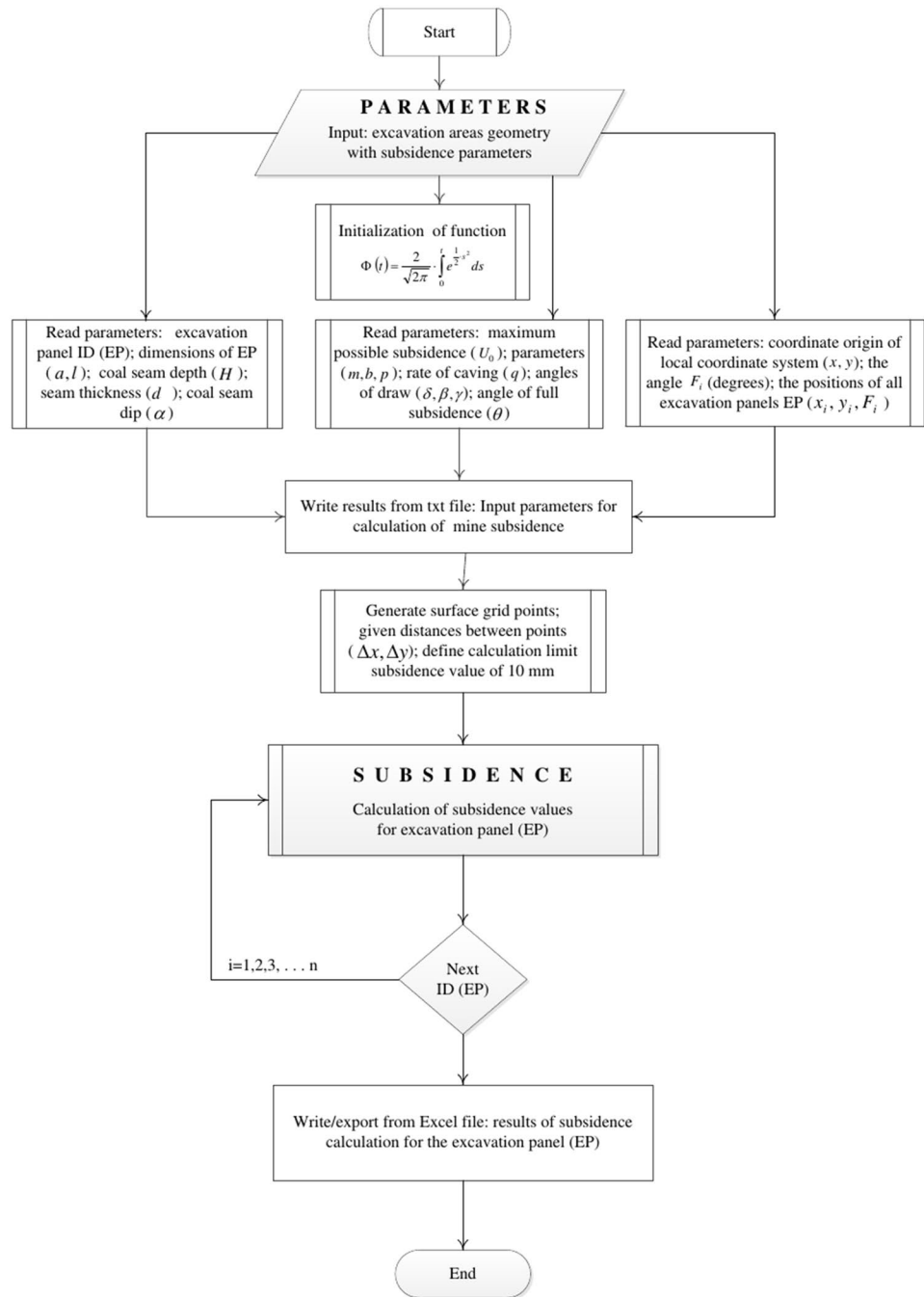
calculated parameters, together with the geometric characteristics of each EP and the information concerning their positions in the local coordinate system (x_i, y_i, F_i), represent the input data for the subsidence calculation in the *SUBSIDENCE* computational module (Table 1).

Subsidences are calculated in the *SUBSIDENCE* module. A certain subroutine allows entering the coordinates of points in a grid of a given density, through the assigned distances between points $(\Delta x, \Delta y)$, in the x and y axes

directions of the local coordinate system. In this way, it is possible to define calculation limits for all panels up to a limit subsidence value of 10 mm. Further, the subsidence values after mining each panel are calculated cumulatively, according to the projected mining dynamics of the excavation field (Fig. 10). Also, the program allows storing and printing the input parameters.

The individual results for each panel can be exported (write) in tabular form in an Excel file, which is also the

Table 1 Input parameters for subsidence calculation

[EP] ID	a [m]	l [m]	H [m]	d [m]	α (°)	U_0 [mm]	M [m]	b [m]	p	q	θ (°)	x_i [m]	y_i [m]	F_i (°)
EP1	20.0	195.0	385	6.0	8	3151	26.92	197.00	0.73923	0.52316	86	0.00	0.00	0
EP2	20.0	206.0	395	6.5	8	3414	27.62	208.00	0.73923	0.52316	86	−40.00	−13.50	0
EP3	20.0	218.0	415	6.5	9	3405	29.02	220.70	0.69634	0.49346	86	−79.70	−24.80	0
EP4	20.0	229.0	435	5.5	9	2881	30.42	228.69	0.69634	0.49346	86	−119.00	−42.00	0
EP5	20.0	241.0	450	5.5	10	2873	39.37	241.00	0.65997	0.46836	86	−159.00	−54.00	0
EP6	20.0	252.0	460	4.5	11	2343	40.24	251.58	0.62857	0.44680	86	−199.00	−67.00	0
EP7	20.0	263.0	465	4.0	11	3667	29.37	125.83	0.69634	0.49346	86	4.50	−284.00	0
EP8	32.0	272.0	470	3.0	11	2895	32.17	118.13	0.79088	0.55907	86	−41.00	−339.00	0
EP9	31.5	120.0	460	2.0	13	2369	33.56	112.12	0.79088	0.55907	86	−80.00	−359.00	0
EP10	28.0	126.0	420	7.0	9	2083	40.68	262.56	0.62857	0.31888	86	−238.50	−81.50	0
EP11	23.5	118.0	460	5.5	7	1562	41.12	271.54	0.62857	0.44680	86	−291.00	−92.00	0
EP12	23.0	112.0	480	4.5	7	1034	48.35	119.76	0.57676	0.41150	86	−353.00	38.00	0
EP13	24.0	105.0	500	4.0	8	2101	34.96	105.00	0.73923	0.52316	86	−120.00	−383.00	0
EP14	23.0	87.0	510	3.0	9	1572	35.66	86.88	0.69634	0.49346	86	−161.00	−384.00	0
EP15	20.0	57.0	525	2.5	8	1313	36.71	57.00	0.73923	0.52316	86	−200.00	−382.00	0
EP16	20.0	36.0	525	2.0	5	1057	27.51	36.03	0.93692	0.66110	86	−242.00	−384.00	0
EP17	115.0	30.0	415	3.5	15	1793	50.96	29.93	0.53537	0.38363	86	110.00	−546.00	−46
EP18	140.0	40.0	415	2.0	15	1025	50.96	39.91	0.53537	0.38363	86	35.00	−557.00	−46
EP19	123.0	23.0	410	4.5	6	2374	21.49	23.00	0.85481	0.60367	86	254.00	−702.00	−46
EP20	123.0	21.0	410	2.0	6	1055	21.49	21.00	0.85481	0.60367	86	227.00	−732.00	−46
EP21	38.0	53.0	380.0	2.0	10	1045	33.25	53.00	0.65997	0.46836	86	370.00	−841.00	−136

preparation for graphical presentation and spatial analysis in GIS.
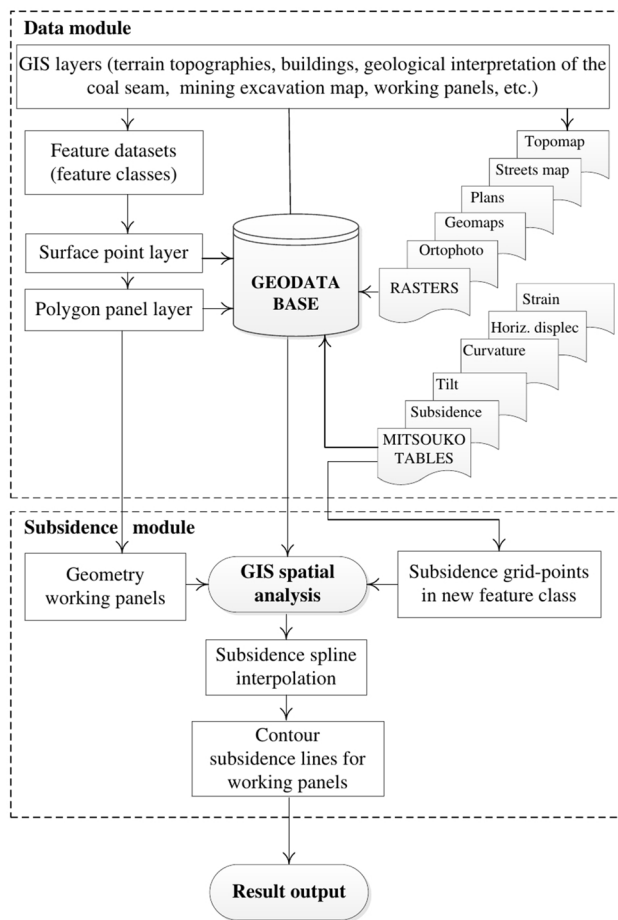
Spatial analysis in GIS

Spatial analysis in GIS is based on the integration of the subsidence prognosis results obtained using MITSOUKO computer program package and data processing in ArcGIS. The main steps in this integration are transfer of mining subsidence prediction tabular data from MITSOUKO to GIS, building a geodatabase, spatial data analysis, and combination of maps layers to predict subsidence and deformations (Wang and Kuan 2004; Banerjee et al. 2014).

GIS is used for creating complex geodatabase, converting numerical data, imported from the *SUBSIDENCE* module, in feature classes and graphical data as well as for performing the spatial analyses of subsidence and deformations (Cai et al. 2016; Cao et al. 2019). Since the "Strmosten" pit consists of multiple EPs with complex geometry, using the stochastic method for spatial analysis of subsidence requires a long time without GIS because for each EP subsidence must be presented cumulatively, assuming all previous and the current EP (Blachowski 2016).

Implementing the stochastic method for spatial analyses of subsidence in GIS is performed in two steps:

The first step, ***Data module,*** involves creating a geodatabase of the "REMBAS" coal mine in ArcCatalog, within the ArcGIS application (ESRI Software Inc. http://www.esri.com/software/arcgis/) with feature classes, tables, and rasters. The feature class is a set of homogeneous spatial attributes in the form of digitized vector data, in the same National Coordinate System (MGI Balkans7). To integrate feature classes thematically and spatially into the mine model, within the given excavation panels (EP), feature datasets have been created, in which all types of feature classes are entered. Feature datasets with feature classes related to spatial geometry: terrain topographies, buildings, mining facilities, old mining works and new exploitation field in "Strmosten" pit, excavation panels with mining dynamics by years, active and old mining premises, and geological interpretation of the coal seam are created in geodatabase of "Rembas"- Resavica mine. Outside the feature datasets, tables with subsidence, tilt, curvature, horizontal displacement and strain, calculations from the *SUBSIDENCE* module in the MITSOUKO program, rasters for the subject area in the form of orthophoto, geographic maps, situational plans of mine and photographs are imported in the geodatabase.

Figure 12 presents the algorithm connecting stochastic prediction analysis with the GIS spatial analysis function. Feature Datasets have been created, in which all types of

Fig. 12 Algorithm connecting stohastic prediction analysis with the GIS spatial analysis function

feature classes are entered. Also, using ArcMap, an integrated part of the ArcGIS software package, to create layers for displaying feature classes from ArcCatalog (ESRI) is included. All the tables of the subsidence calculations have been transferred from the geodatabase coal mine to ArcMap.

The second step, ***Subsidence module*** (Fig. 12), involves using ArcMap, which is an integrated part of the ArcGIS software package, to create layers for displaying feature classes taken from ArcCatalog (ESRI). All the tables of the subsidence calculations have been transferred from the geodatabase "Rembas" coal mine to ArcMap.

Using the Display XY Data command, the selected table of e.g. EP10, which contains the x,y coordinates of the points and the calculated subsidence values, is added as a new layer in the Table of Contents. Thereby, a new feature class, panel ID10_events, was formed, which for EP10 contains 21,500 points with values of x and y coordinates and associated subsidences, georeferenced to the adopted coordinate system MGI_Balkans7 (Vusovic 2018).

Following the same procedure, new feature classes were created for all panels. The created feature classes contain x and y coordinates of all points in the grid 10×10 m with associated subsidence values. All the calculation results can be stored into GIS point-grid. Each calculation point such as vertical displacement, slope, curvature, horizontal displacement and horizontal strain can be transformed into a GIS raster by a surface interpolation as well. The Spline interpolation method from the Spatial Analyst Tools palette is then used to create new layers with contour subsidence lines for each panel, by cumulative subsidence transformation from the previous to the new state. Figure 13 presents the interpolation of subsidence contour lines after mining EP10.

This provides successive following of the subsidence process on the map at all stages of the excavation in the "Strmosten" pit. Digitization and formation of feature classes for all excavation panels in the global MGI Balkans7 coordinate system are performed in map.

Subsidence analysis during mining the excavation panels in the "Strmosten" pit

The stochastic method was used to calculate the mine subsidence values in the MITSOUKO program package. The subsidence contour lines are graphically presented in the ArcGIS software package by interpolation and cumulative transition from the previous to the new state, formed on the data of the seam, and according to the adopted excavation dynamics (Esaki et al. 2008; Cai et al. 2016). Based on the predicted subsidence values, the impact of mining works for all EPs was analyzed. With the progress of mining, the subsidence value for each EP is obtained cumulatively, that is, by superimposing the subsidence effects of all previously excavated areas and the actual one.

Figure 14 shows excavations panels (from EP1 to EP20) at the coal mine "Rembas" Resavica- Serbia with the adopted excavation dynamics, subsidence contour lines obtained by simulation of mine subsidence process by stochastic prediction method and maximum subsidence values (Vusovic 2018).

As seen in Fig. 14 the predicted maximum subsidence values on the ground surface continuously increase from $U_{\max.} = -1893$ mm reached after the excavation of EP3 in 2020 to $U_{\max.} = -2927$ mm after excavation of EP16 in 2033. Value of $U_{\max.} = -2927$ mm remains unchanged with further excavation, ending with EP20 in 2037.

Finally, Fig. 15 presents mining operation plan and predicted subsidence contour lines with the maximum subsidence value ($U_{\max.} = -2927$ mm) after mining 21 excavation panels in 2038, that is the entire excavation field at the "Rembas" Resavica- Serbia coal mine (Vusovic 2018).
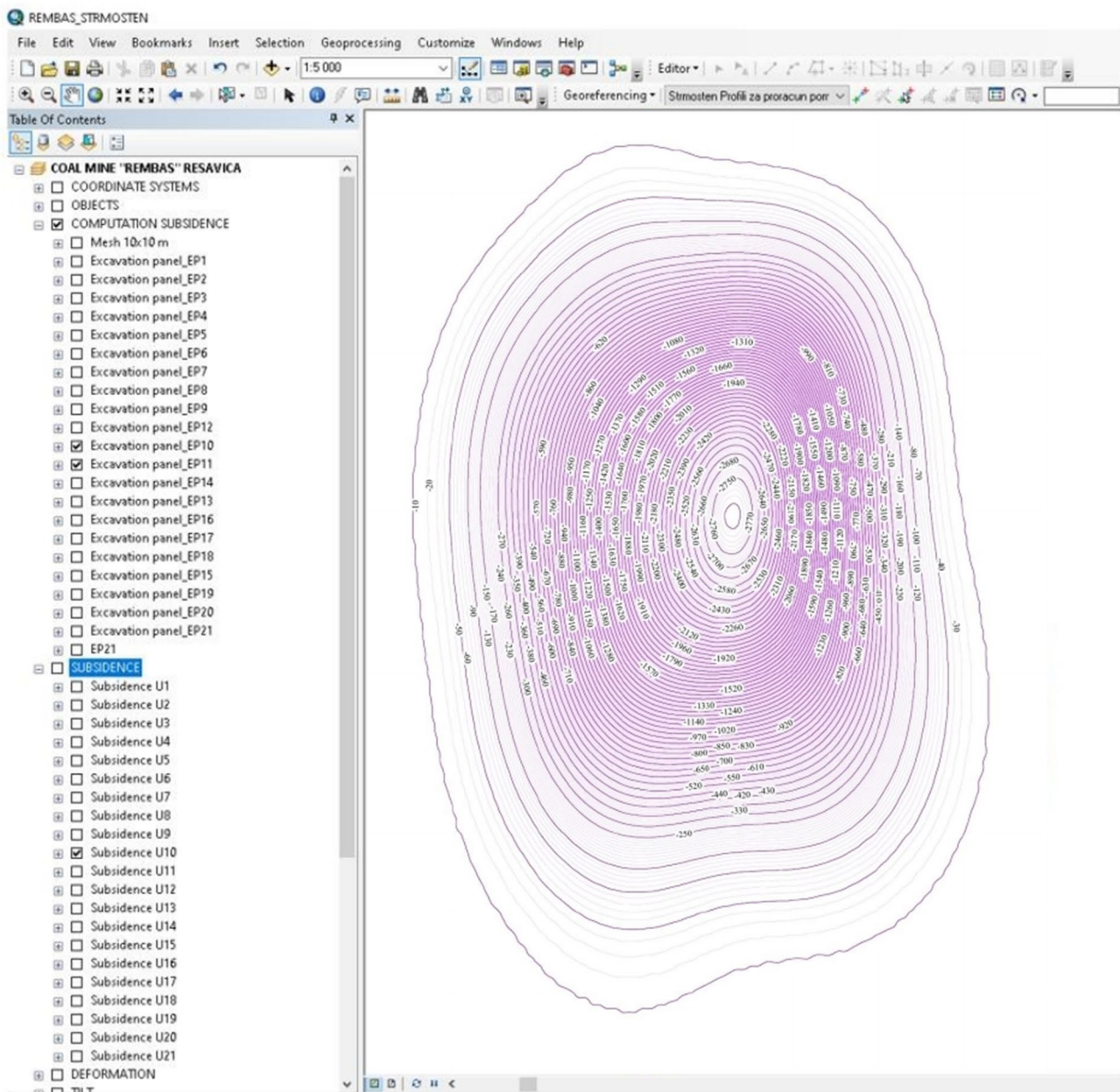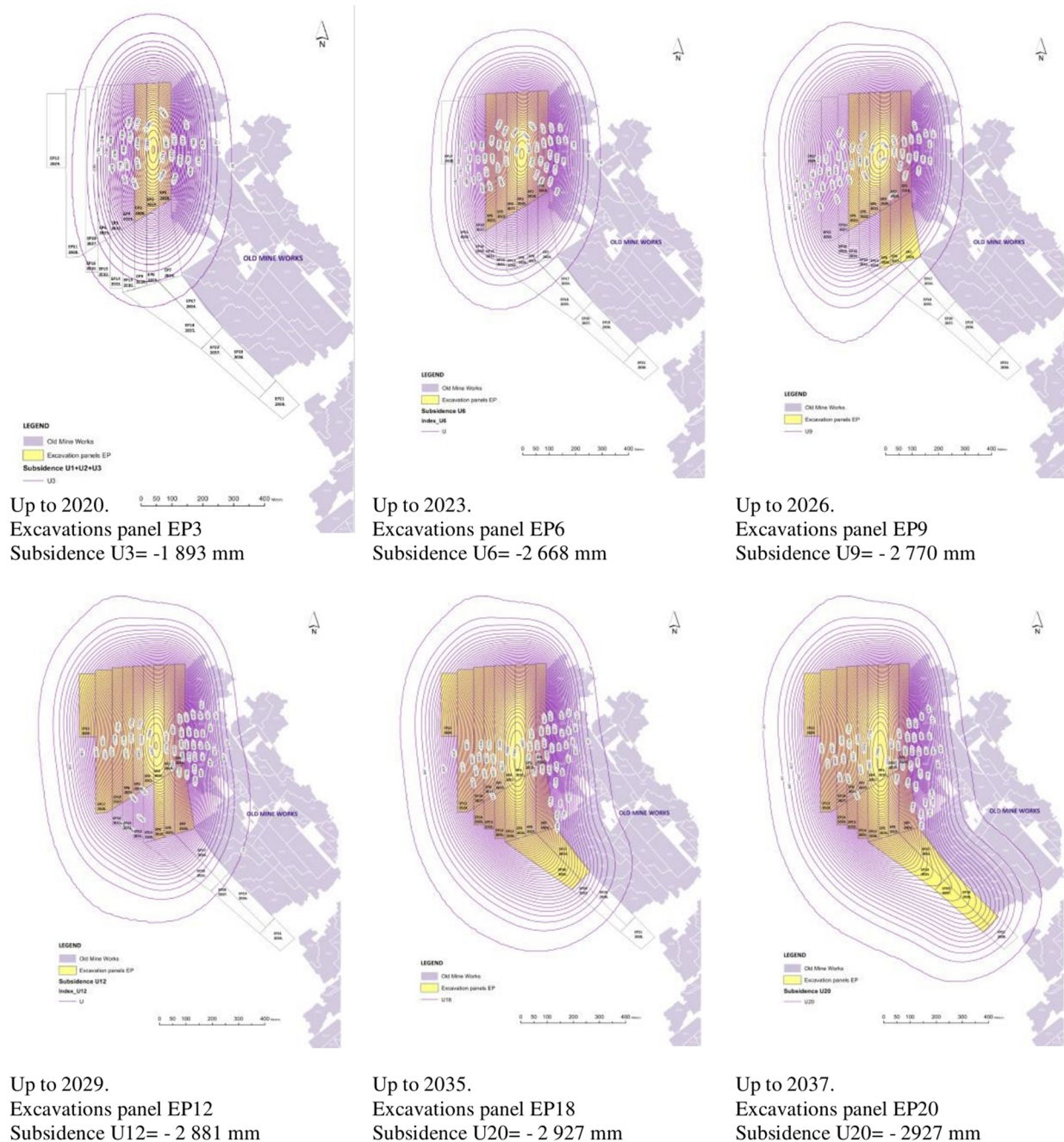
Fig. 13 Interpolation of subsidence contour lines after mining excavation panels EP10 (Vusovic 2018)

According to the predictive calculations, the maximum relative subsidence was determined as the highest subsidence value ($U_{max.} = -2925$ mm) after exploitation of excavation panels EP1-EP15. With the progression of mining operations, this value grows towards the value of maximum absolute subsidence. Maximum absolute subsidence, $U_{max.} = -2927$ mm, is the highest subsidence value, achieved after mining EP16, which does not increase with further mining works (EP16-EP21), but flat-bottomed subsidence trough, collapsed for that value, appears. The shape of the subsidence curve on the profile by the seam dip is infundibuliform. On the profile by the seam strike, the shape of the subsidence curve is infundibuliform, but with a flat bottom in the narrow central part of the curve, formed because several points have the same value of the maximum absolute subsidence, $U_{max.} = -2927$ mm(Figs. 14, 15).

The stochastic subsidence prediction and the spatial data analysis in GIS enable the calculation and presentation of surface subsidence at any point in the grid with a large number of numerical data which requires a long time of processing and interpretation of the obtained results by standard data processing method. The answer to the set task lies in

Up to 2020.
Excavations panel EP3
Subsidence U3= -1 893 mm

Up to 2023.
Excavations panel EP6
Subsidence U6= -2 668 mm

Up to 2026.
Excavations panel EP9
Subsidence U9= - 2 770 mm

Up to 2029.
Excavations panel EP12
Subsidence U12= - 2 881 mm

Up to 2035.
Excavations panel EP18
Subsidence U20= - 2 927 mm

Up to 2037.
Excavations panel EP20
Subsidence U20= - 2927 mm

Fig. 14 Simulation of mine subsidence process at the coal mine "Rembas" Resavica- Serbia (Vusovic 2018)

the MITSOUKO program package, intended for the subsidence calculation by stochastic method, which allows the user to process a large amount of data, thus excluding the time factor, and the obtained numerical data can be quickly and easily graphically processed and displayed in GIS.

Comparative analysis of predicted and measured subsidence values

The values of the predicted parameters for the ground surface subsidence should be verified throughout the entire period of coal seam excavation. This requires systematic geodetic surveying. The main goal of these surveying is to realize the surface subsidence process in space and time
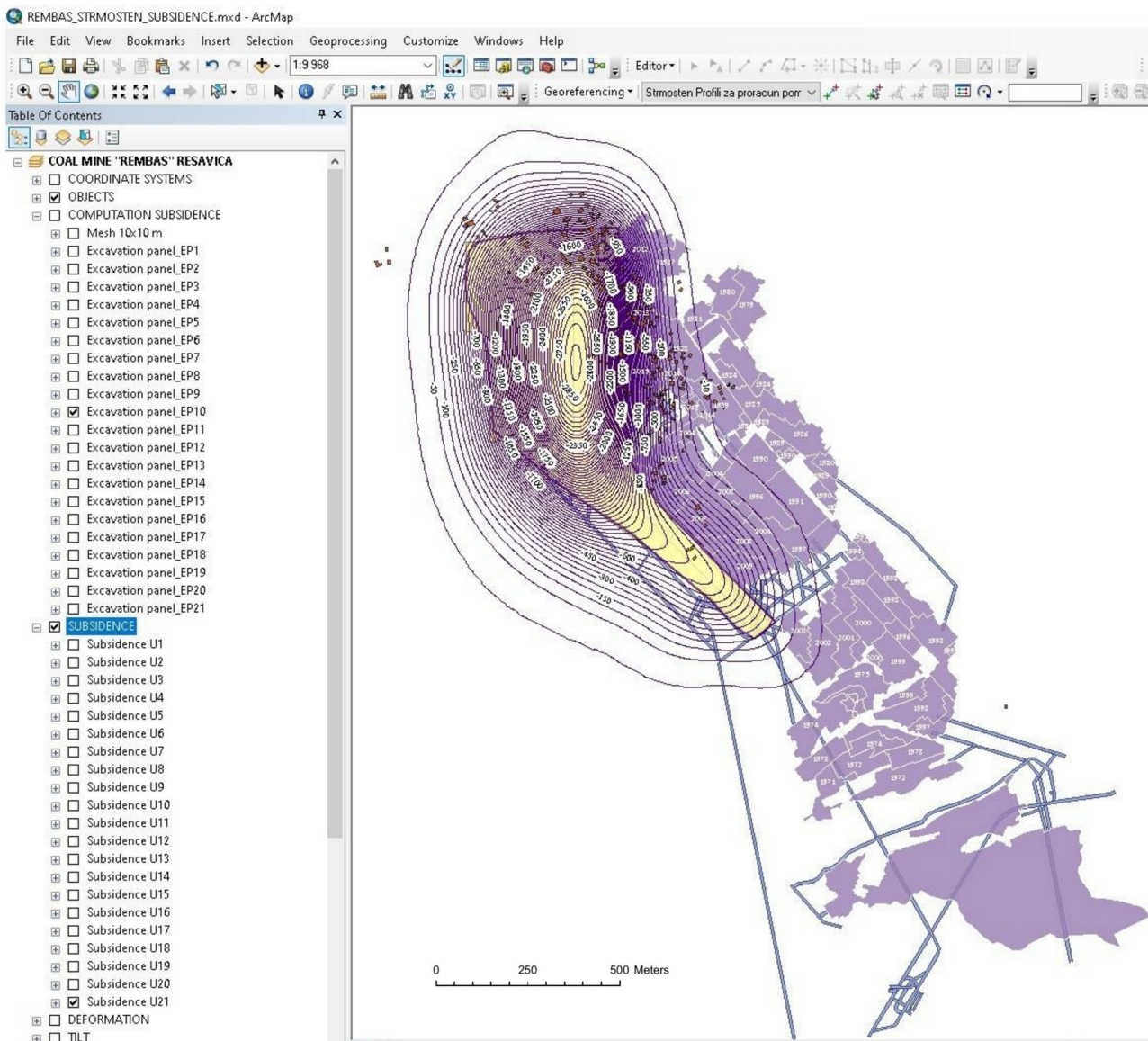
Fig. 15 Subsidence contour lines after mining the entire excavation field at the coal mine "Rembas" Resavica- Serbia (Vusovic 2018)

during the excavation of the coal seam (Cui et al. 2001; Malinowska and Hejmanowski 2009).

The basic grid for surveying consists of profile lines I–I and II–II arranged in directions of seam dip and seam strike. This grid must satisfy the requirement of long-term subsidence observations, which will last more than twenty years so that the stability of the benchmark should not be compromised. Working benchmarks are stabilized in the zone to be affected by the subsidence process and by periodic defining the benchmark position, the intensity, and state of the subsidence process for a certain period of time.

Figure 16 shows the profile lines I–I and II–II for coal seam strike and coal seam dip with predicted subsidence curves at cross sections.

Geodetic surveying on profile lines gradually follows the general development of the subsidence process through the data that define the beginning of the displacement, the beginning of the intensive displacement phase, the intensive displacement phase, the completion of this phase and the process regression phase. The extent, type of measurements, field conditions and the required accuracy of measurements demand the application of modern geodetic surveying methods and processing of measurement results (Miao et al. 2008; Hay-Man Ng et al. 2010; Ma et al. 2012).

Surface movement and deformations in wider areas have been simpler monitored by introducing airborne, satellite-based remote sensing techniques, GPS and UAV systems (drones). Precise GPS provides placing control points
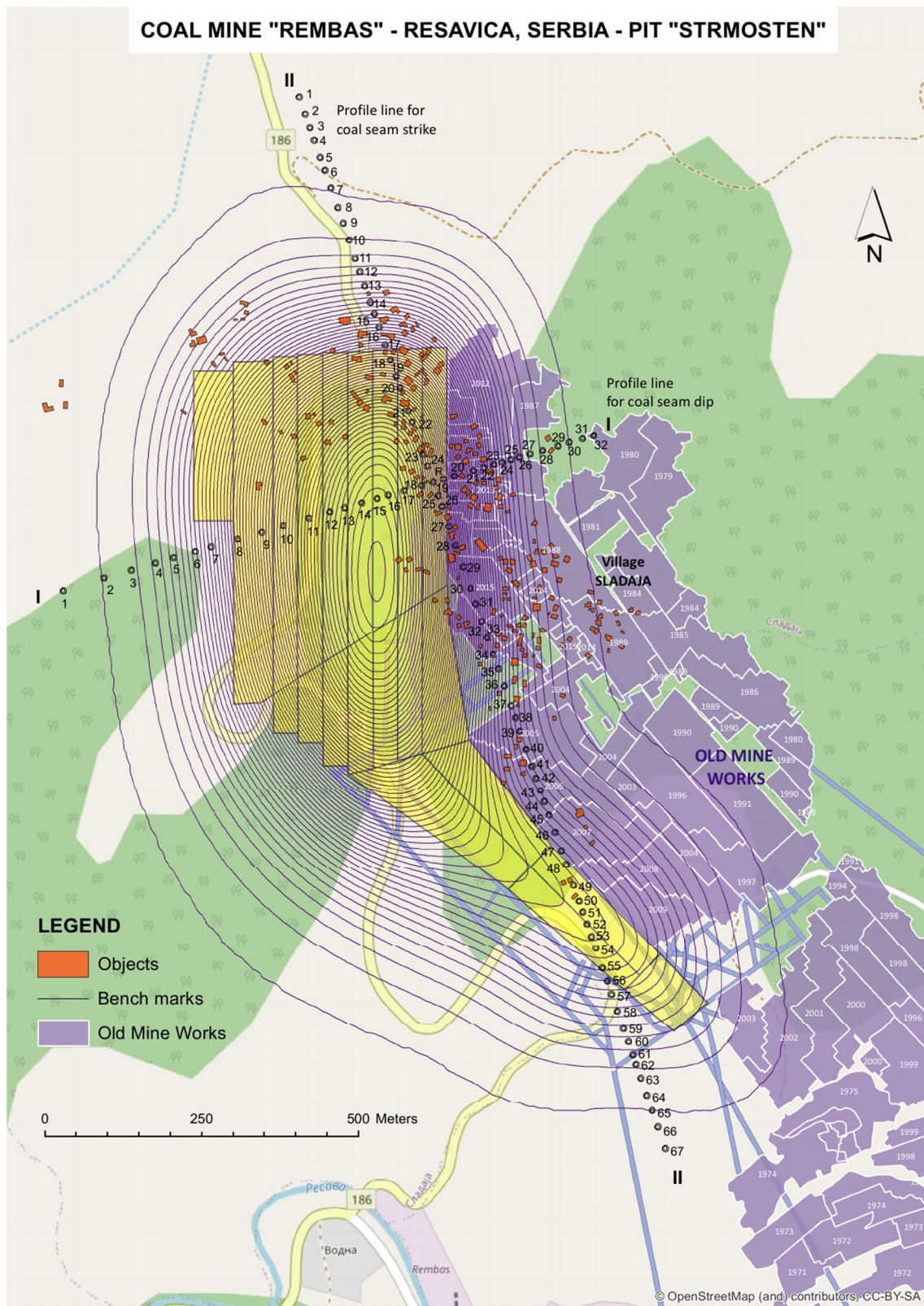
Fig. 16 Profile lines I–I and II–II for dip and strike of the seam with the associated predicted subsidence curves at the coal mine "Rembas" Resavica, Serbia (Vusovic 2018)
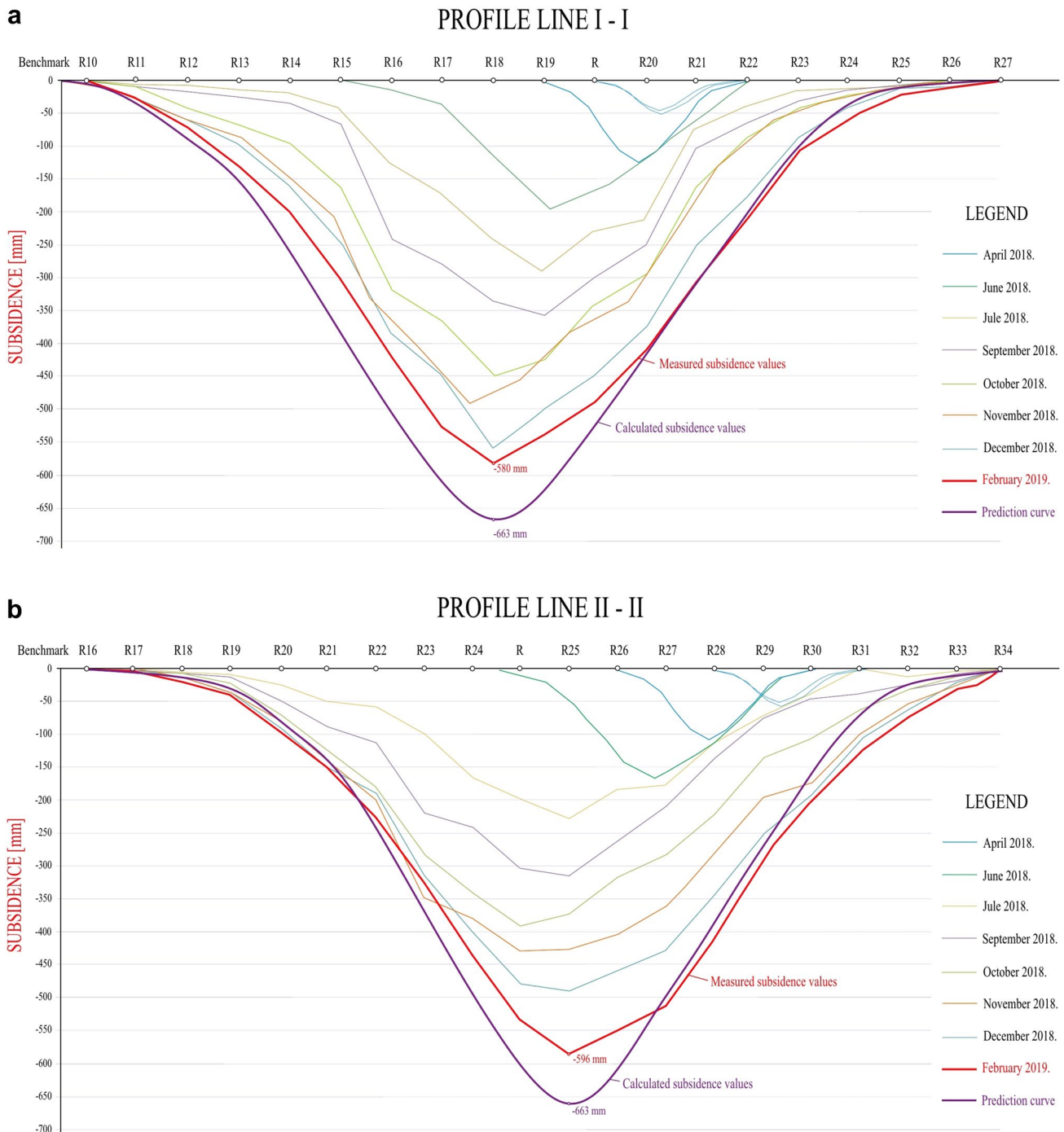
Fig. 17 **a** Time curves of the benchmark subsidence on profile line I–I for the coal seam dip. **b** Time curves of the benchmark subsidence on profile line II–II for the coal seam strike

significantly out of mining impact as it is known that discrete horizontal movements take place at least 100 m from the formerly adopted subsidence limit of 10 mm (Anderson et al. 2007). UAV systems offer pre-mining baseline topography and differential level information through repeated measurements with an accuracy of ±1 cm/pixel (Mills 2011). The

rapid development of remote sensing techniques such as LiDAR is leading to very high accuracy and cost-effectiveness and, therefore, more viable option.

The tabular and graphical processing of the measurement results give the values of characteristic displacement parameters: subsidence, horizontal displacement, slope,

Fig. 18 Sladaja village: **a** Damaged object above the old exploitation works of the "Strmosten" pit; **b** Surface subsidence above the actual exploitation works in the "Strmosten" pit

deformation and radius of curvature. The subsidence trend during time is tracked on charts. There are two types of charts along the profile lines- the first are the subsidence curves while the second are the parameters of the displacement process in the form of diagrams of horizontal displacements, diagrams of slope change and diagrams of change of curvature which serve for estimating the vulnerability degree of objects (Sheorey et al. 2000; Cui et al. 2001; Vusovic and Svrkota 2004; Esaki et al. 2008; Diyab 2011; Chang et al. 2015; Vusovic 2018).

On the ground surface above the "Strmosten" pit, a system of 32 benchmarks on profile line I–I and 67 benchmarks on profile line II-II is installed (Fig. 16). The surface subsidence during the mining in EP1 is followed by the periodic leveling of the benchmark along these profile lines. The surveying program covered the benchmark leveling from zero measurements in February 2018, to February 2019 with a total of eight series of measurements. The performed geodetic surveying, as well as the tabular and graphical processing of the results of leveling along the profile lines, enable comparing the measured and predicted subsidence values after the termination of exploitation works in EP1 in the north-eastern part of the excavation field.

By processing the measurement results, the subsidence values for all benchmarks on the profile lines were determined. The R18 benchmarks on profile line I–I and R25 benchmarks on profile line II–II with measured maximum subsidence values of -580 mm and -596 mm, respectively, exhibited the highest subsidence tendency. The predicted maximum subsidence value using the stochastic method is -663 mm.

Comparative presentation of the calculated subsidence curve and curves based on measurements on profile lines I–I and II–II are given in Fig. 17a and b.

The obtained measurement results and time curves of the benchmark subsidence by series of measurements on profile lines I-I and II-II after the end of exploitation works in EP1 of the "Strmosten" pit, and their comparison with the values on the predicted subsidence curve, confirm the possibility of using the stochastic method for prognostic subsidence calculation in practice, which is a necessity for modern mining.

Particularly important are the data obtained from the geodetic surveying, which relates to the approaching of the subsidence process to objects and the need to observe deformations on them. Measurements provide to monitor the behavior of objects affected by the subsidence influence and to assess the damage degree according to the criteria of permissible deformations for a certain type of object.

The effects of old and actual mining works of the "Strmosten" pit on objects and ground surface in Sladaja village is presented in Fig. 18.

Conclusion

In this study, a GIS- based dynamic model, based on a theoretical Pataric-Stojanovic stochastic method for the subsidence prediction, supported by an original MIT-SOUKO program package and integrated into GIS, has been proposed to successfully simulate ground movement propagation during mining out 21 excavation panels.

Benefits of this approach are:

- Reliability of the method for calculation of subsidence for inclined seams;
- Equations for inclined seams, in cases when $\alpha = 0$ give solutions for horizontal seams;
- Correct results for predicting displacement at any point of undermined terrain for the seam dip angle $10° \leq \alpha \leq 60°$, even when the footwall is disturbed by the movement process;
- Using coupled angles (β, γ) provides determining the angle of full subsidence (θ) even for more inclined seams, when $\alpha < \gamma$. Also, if the values of an angle θ and of one of the coupled angles (γ) are known, the other coupled angle (β) can be calculated;
- Successful determination of angular parameters in mines with unknown ground movement process but in regular mining and geological characteristics as well as control of already determined angular parameters;
- Comparative analysis of calculated and measured subsidence values along the profile lines allows correction of input parameters for the prognosis by the stochastic method and corrective calculations by the same procedure using the MITSOUKO program package and spatial analyses in GIS, especially in cases of non-compliance with the projected dynamics of coal seam excavation.

Results provided in this study have the accuracy needed for prediction of ground movement process and protection of surface objects from underground coal mining.

This work presents an important research study, actual and significant for the mining profession and practice.

The calculation of subsidence by stochastic Pataric-Stojanovic method and originally developed MITSOUKO program package integrated with the GIS are powerful tools for predicting subsidence and monitoring the impact of underground mining works on the land surface.

Author contributions NV: Conceptualization, methodology, software, validation, formal analysis, investigation, data curation, visualization, supervision. MV: Conceptualization, methodology, software, resources, writing—original draft, writing—review and editing. DK: Resources, project administration.

Data availability All data and materials as well as software application support published claims.

Code availability Not applicable.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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