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ORIGINAL ARTICLE

Assessment of discontinuous rock slope stability with block theory and numerical modeling: a case study for the South Pars Gas Complex, Assalouyeh, Iran

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Abstract In this study, a geotechnical model has been used to analyze the stability of a discontinuous rock slope. The main idea behind block theory is that it disregards many different combinations of discontinuities and directly identifies and considers critical rock blocks known as "key blocks". The rock slope used as a case study herein is situated in the sixth phase of the gas flare site of the South Pars Gas Complex, Assalouyeh, Iran. In order to analyze the stability of discontinuous rock slopes, geotechnical modeling which was divided into geometrical sub-modeling and mechanical sub-modeling has been utilized. This model has been established upon the KGM (key-group method) algorithm which was based on the limit equilibrium method and block theory and prepared and coded by the Mathematica software. According to the results of the stability analysis, the analyzed slope was determined to be in the category of "needs attention," and the security level, calculated through the FORM (first-order reliability method) analysis, was estimated to be 1.16. In order to verify the model, the results obtained from the model were compared with those of the UDEC software, which is a numerical method based on distinct components. As a conclusion, it was determined that the results of the model agreed well with those of the numerical method.

Keywords Discontinuous rock slope stability · Block theory · Numerical modeling · South Pars Gas Complex · Assalouyeh

Introduction

Discontinuities are among the common features of rock slope engineering. Once discontinuities approach the ground surface, they often intersect each other and split the rock masses into rock blocks of various sizes and shapes (Warburton 1981). The instability phenomenon of discontinuous rock slopes may be due to sliding, rotation, or toppling. Rotational slides generally occur in closely spaced discontinuous (heavily fractured) rock. Sliding motion usually follows the pre-existing discontinuity plane(s) leading to either planar or wedge failure. Toppling conditions are attained when the discontinuous bodies are so high and thin that the gravity vectors of the discontinuities fall outside their bases. Both static and dynamic methods can be utilized for stability analyses whenever the blocky system is isolated and defined (Lin and Fairhurst 1988).

Static analysis investigates the kinematic mechanism of sliding or toppling of a block with a face exposed on the rock slope (Kuen et al. 2003; Yarahmadi-Bafghi 2003; Khanizadeh Bahabadi et al. 2014; Azarafza and Asghari-Kaljahi 2016; Shahami et al. 2016). The acting (driving) and resisting forces are calculated, and then the equilibrium equations are solved to show whether the block is stable or not (Azarafza 2013). By using numerical techniques, advances in the characterization of complex rock slope

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failure and deformation have shown a significant potential for enhancing the understanding of the mechanisms and processes involved and of the associated risk. Rock slope stability analyses are usually performed and directed in order to assess safe equilibrium conditions for natural trenches or slopes (Azarafza et al. 2013). In the last 30 years, the key-block method has been developed to analyze the instability of discontinuous rock masses. This method has been successful because of its simplicity and resolution speed as compared to more complex discontinuous analyses conducted with the finite difference method (FDM), distinct element method (DEM), and finite element method (FEM). The key-block analysis has mainly been implemented in two forms: the vectorial technique which was developed by Warburton (1981) and the graphical technique which was developed by Goodman and Shi (1985). Primarily, a key block is a block around an excavation; if it is not reinforced, it can become unstable and lead to progressive instability of other blocks. A key block is defined by four main conditions (Yarahmadi-Bafghi 2003; Yarahmadi-Bafghi and Verdel 2003):

- Active (in contact with excavation),
- Finite (limited by rock discontinuities and the excavation),
- Geometrically mobile,
- Key for the other block movements.

For a key block, limit equilibrium analysis can be performed to assess its mechanical condition (whether mechanically movable or not). The instability of such a block can cause larger rock mass movement. This movement (key-block movement condition) can be analyzed by means of an iterative process. It is assumed by the key-block method that the blocks are rigid and their surfaces are perfect planes. There are some methods, devoted to the stability analysis of discontinuous media, which may resolve these problems (Yarahmadi-Bafghi and Verdel 2003). However, the application of these methods is much more complex and their calculation is very time-consuming where examples include the distinct element method (Goodman and Shi 1985; Warburton 1981), the method that involves discontinuous deformation analyses (Yeung 1991; Brady and Brown 1993), and the relaxation technique (Brady and Brown 1993). Researchers have worked on the stability of wedges, prisms, and arbitrary rock blocks since the 1950s (John 1968; Warburton 1981; Hoek and Brown 1980; Hoek 1987; Goodman 1995; Mauldon et al. 1997; Hoek et al. 2002; Yoon et al. 2002; Yarahmadi-Bafghi and Verdel 2003, 2005; Azarafza et al. 2013, 2014a, b, c, 2015). By using the block theory, the stability of a rock slope with a number of discontinuity plane sets can be analyzed (Goodman and Shi 1985). Duncan considered the momentous aspects of 24

publications about limit equilibrium approaches (Duncan 1996). Through these approaches, the failure mass is divided into columns with vertical interfaces; and the conditions for static equilibrium are utilized to find the safety factor. Chen and Chameau (1983), Hungr (1987), Hungr et al. (1989), and Lam and Fredlund (1993) have expanded Bishop's simplified method, Spencer and Morgenstern's method, and Price's method from two to three dimensions, respectively.

The key-block analysis method, originally developed by Goodman and Shi (1985), is the most well known among the limit equilibrium approaches. This method can be performed in two different ways: graphical implementation on the basis of stereographic projections and analytical implementation on the basis of vector methods. Intensive studies of key-block analysis have been conducted by Giani (1992), Mauldon and Ureta (1996), Mauldon et al. (1997), Tonon (1998), Sagaseta et al. (2001) and Yarahmadi-Bafghi (2003).

Block theory

By the block theory, the stability of a rock slope with several discontinuity sets can be examined (Goodman and Shi 1985). However, both the location and orientation of each individual discontinuity plane are needed when conducting analyses which use the block theory. The main idea behind the block theory is that it disregards many different combinations of discontinuities and directly identifies and considers the critical rock blocks which are known as "key blocks." The blocks can be divided into finite and infinite blocks (Fig. 1). Infinite blocks (type V), as illustrated in
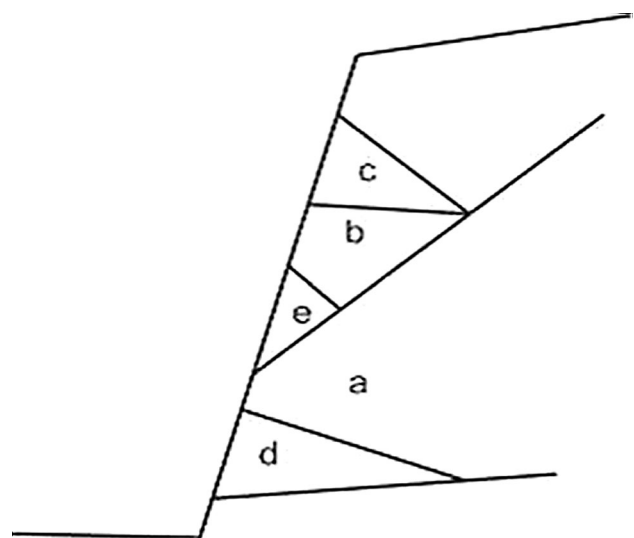

Fig. 1 Rock slope blocks in a trench cut: **a** infinite; **b** tapered; **c** stable; **d** potential; **e** key block, respectively (Kulatilake et al. 2011)

Fig. 1a, are not dangerous as long as they are not capable of internal cracking. Finite blocks can be classified into removable and non-removable blocks. An example of a type IV non-removable tapered block is presented in Fig. 1b. This block is finite, but because of its tapered shape it cannot come out to free space (Jeongi-gi and Kulatilake 2001). Finite and removable blocks are divided into three categories, including type I, type II, and type III. The identification of these blocks plays an important part in rock slope design. As illustrated in Fig. 1c, a type III block is stable without any friction under gravity alone. A type II block, as shown in Fig. 1d, is stable inasmuch as the sliding force on the block is less than its frictional resistance. Type II blocks, which are stable only under gravitational loading, are also known as potential key blocks. Finally, as demonstrated in Fig. 1e, a type I block can slide into free space under gravitational loading without any external forces unless a proper support system is provided. Thus, as shown in Fig. 1, the identification of key blocks is one of the most important steps in the stability analysis of a rock slope (Kulatilake et al. 2011).

The basis of the key-block method (KBM) is to study those key blocks which, from the perspective of rock mass stability, are proven to be critical. Under the block theory (Goodman 1995), the key-block stability depends merely on the direction of the applied loading (usually gravitational), frictional discontinuity strength, and discontinuity orientations (Shi 1988). The analysis of these blocks is on the basis of a computed factor of safety (FOS) that serves to exhibit either stability, FOS ≥ 1.0, or instability, FOS < 1.0 (Goodman and Shi 1985). In analyzing the stability of a fractured rock mass, not only single key blocks but also groups of blocks have to be considered. These blocks are considered in their entirety and could eventually make a "key group" more hazardous than single individual blocks. To develop such a key group, an initial key block has to be included. The second block candidate for the combination must be either another key block or a block with movement-hindering faces exposed to one or more of the single key blocks (Yarahmadi-Bafghi 1994).

The key-group method (KGM) is a method developed by Yarahmadi-Bafghi and Verdel in 2003 that considers not only individual key blocks but also groups of collapsible blocks into an iterative and progressive analysis of the stability of discontinuous rock slopes. The basic principle of KGM is based on key-block identification and creation of key groups of the unstable rock blocks. A key group must first be identified as a key block (i.e., active, finite, and geometrically movable block). Then groups of key blocks (the second block around the key block) generate key-group blocks. The key group contains a minimum of two blocks and at least three discontinuity sets. The key grouping process is shown in Fig. 2.

In simple terms, the grouping method or KGM by using the Goodman and Shi block theory starts by detecting key blocks which if not reinforced can become unstable and lead to the progressive instability of other blocks. The limit equilibrium analysis of such a block can thus be conducted to allow assessing its mechanical status (whether mechanically movable or not). As mentioned before, in a stability analysis of a discontinuous rock mass slope, not only the key block but also the group of neighboring blocks should be considered. When both of them are considered, the hazard potential will be more than considering a single key block alone. Thus, identification of key groups is the first step in a stability analysis with KGM (Yarahmadi-Bafghi 2003).

Noroozi et al. (2011) established a procedure for implementing key-group analysis based on the main steps of key-block analysis, followed by a specific key-group analysis as follows:

- Identification of the key blocks using the vector technique;
- Removal of the unstable single key blocks when the FOS computed from a limit equilibrium analysis is less than a given limit, and returning to the previous step until there are no more unstable key blocks;
- Identification of the neighbors that are common to one or several remaining single and stable key blocks (those that are not eliminated in the previous steps);
- Building all possible groups using the remaining single key blocks and each of the previously identified neighbors;
- Performing a stability analysis of the groups (limit equilibrium method) and removal of the unstable groups or keeping only the stable groups having minimum FOS;
- Iteration using the new geometry (from the first step) until no more blocks capable of being combined are left (Yarahmadi-Bafghi and Verdel 2003).

Figure 3 is an example of a KGM that graphically displays the successive stages of a key-group analysis performed in a discontinuous rock slope. Key blocks and block candidates for regrouping at the beginning are illustrated in Fig. 3a. When applying the classical key-block method, the analysis would stop at this point because all the key blocks are stable. With the key-group method (KGM), a key group, as shown in Fig. 3a, is searched and selected among candidate key groups (i.e., combinations involving 1 + 5, 2 + 3, 2 + 6, 3 + 7, and 4 + 8). The figure shows that regrouping of key blocks 2 and 3 yields the only unstable key group that is possible for this particular example. Therefore, in the following stage, this group is deleted (Fig. 3b). Afterward, groups 1 + 5, 6 + 7, and 4 + 8 in Fig. 3b become candidates for further
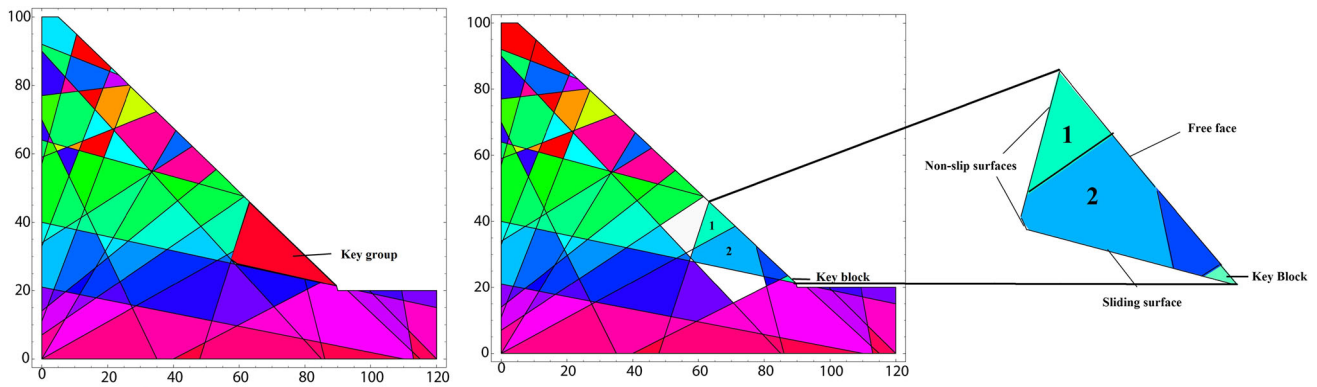
Fig. 2 Key grouping process (Yarahmadi-Bafghi and Verdel 2003)
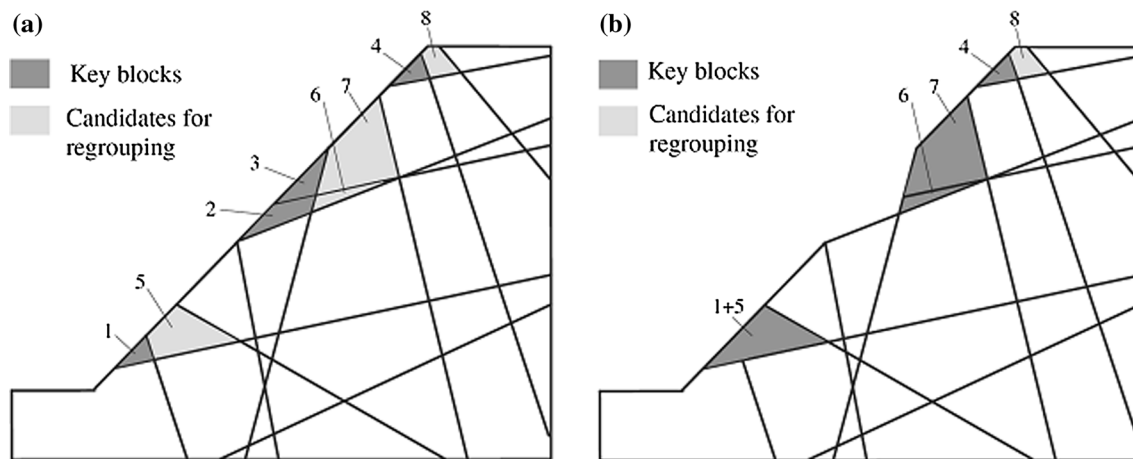

Fig. 3 Consecutive regrouping steps in the key-group method analysis (Noroozi et al. 2011)

regrouping. Finally, during the neighboring block analysis, blocks 1 and 5 are regrouped to obtain an unstable key group (Emami-Meybodi et al. 2008; Goodarzi and Yarahmadi-Bafghi 2013; 2014; Noroozi et al. 2011). The KGM algorithm flow chart is presented in Fig. 4.

Case study

The block theory/key-block method used in this study is based on the Mathematica high-level programming language (Wolfram 1999), which considerably simplifies scientific computing time. Geotechnical modeling was divided into two parts, where the first part was intended for geometrical modeling and the second part was intended for two-dimensional mechanical modeling on the basis of the limit equilibrium analysis and the 2D key-group method principles (Yarahmadi-Bafghi and Verdel 2003). A computer program based on the presented algorithm, Mathematica, was used to execute the proposed method on a real case, namely to a discontinuous rock slope situated in the Phase 6 gas flare site of the South Pars Gas Complex, Assalouyeh, Iran. The studied slope is located 3 m north of

the phase 6 gas flare site, with coordinates 27° 53′ 36″N and 52° 35′ 14″E (Fig. 5). A view of this slope is shown in Fig. 6. Figure 7 presents a Rose diagram of the discontinuous rock mass of the studied slope.

The geotechnical model based on the key-group method was used for calculating the FOS of the slope. Three steps have been followed in utilizing the geotechnical model, namely geometric modeling, behavior definition model, and assignment of the geomechanical properties. Finally, mechanical modeling and stability analysis have been performed.

Geometrical modeling

A concept or model of the geometry of the discontinuity is needed in order to develop pilot plans and interpret their results. Such a model would be ideally specified by a limited number of parameters. In addition, it would be simple enough to be distinguished from typical field observations (Baecher 1984).

The simplest simulation of slopes with geometrical modeling is the Monte Carlo method. In this method,

Fig. 4 The Key Group Method (KGM) algorithm flowchart (Yarahmadi-Bafghi and Verdel 2003)
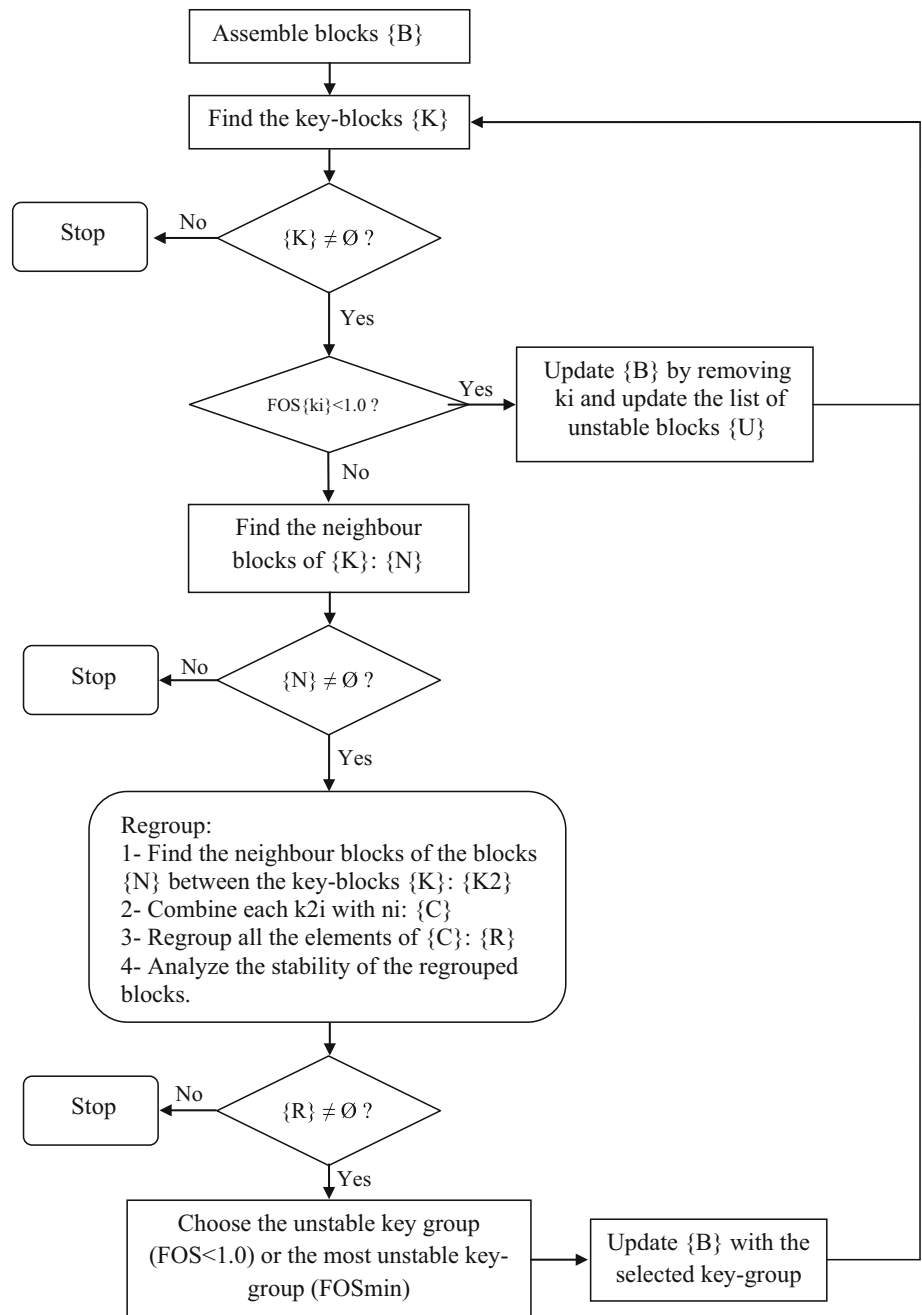
simulation is easily performed by means of random numbers obtained from the distribution function and the effective parameters in the model (Amini and Yarahmadi-Bafghi 2007). The simulation of the geometry of the rock mass discontinuities by the Monte Carlo method may be accomplished in two ways (Azarafza et al. 2013):

- Sequential and infinite discontinuities systems, and
- Random-disk discontinuities systems.

In the sequential and infinite discontinuities method, the discontinuities are assumed to be infinite and the absence of a continuum and the creation order causes the discontinuity to continue to the model boundaries. In the case of assuming continuity, the dominant discontinuity sets are assumed to cut off the model boundary and the secondary discontinuity sets are assumed to be limited to the initial discontinuity. However, neglecting the expansion of the discontinuities is one of the disadvantages of this system and produces an unrealistic number of blocks in the block set of modeling (Yarahmadi-Bafghi and Verdel 2003, 2004). The random-disk model idealizes discontinuities as bounded planar features of random size and

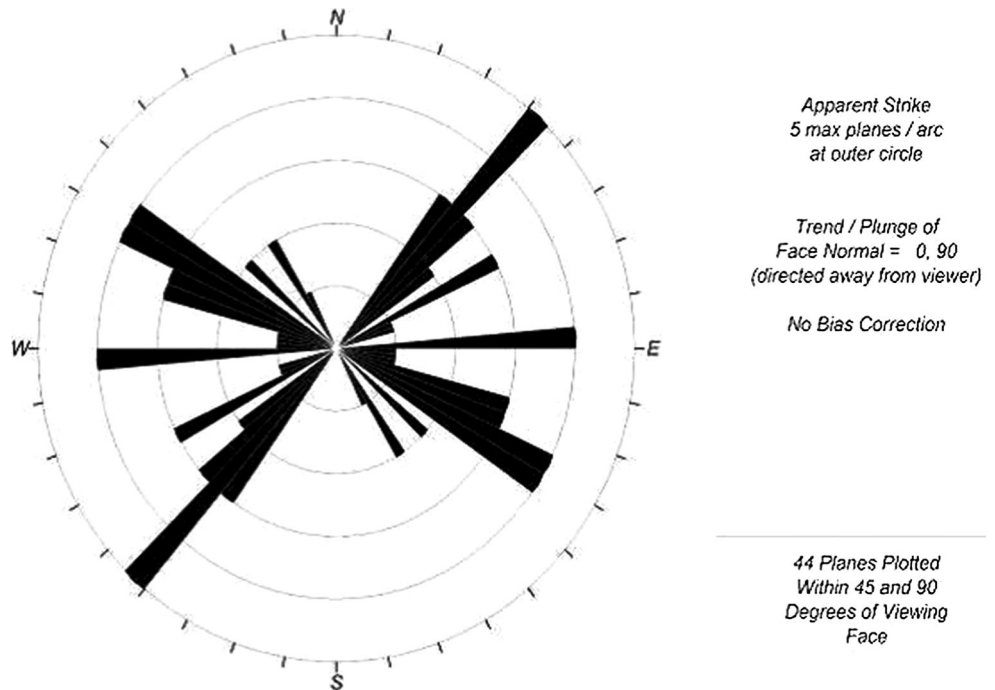**Fig. 5** Location of Assalouyeh and the study area

Fig. 6 A view of the discontinuous rock slope

orientation, which are randomly positioned in a 3D space. The shape of these features may be fixed (e.g., circles) or allowed to vary within restricted families (e.g., ellipses). The model is defined and specified by an intensity measure (e.g., number of discontinuity centers per rock volume),

size distribution parameters, and orientation distribution parameters. The random-disk simulation method begins with the localization of the rock mass discontinuities based on the Poisson distribution, and then each disk is simulated with the assumption that the discontinuity plates are disk-shaped with an assigned orientation (dip amount and dip direction) and extension (i.e., the diameter of the discontinuity). In this simulation, the disk centers were selected by a 3D Poisson process where the orientations and dimensions of the discontinuities were determined by the Fisher distribution function and the log-normal distribution function, respectively. In this simulation system, all discontinuities were assumed to be independent of each other (Yarahmadi-Bafghi and Verdel 2004).

According to the geometrical characteristics of the rock mass discontinuity systems identified in the rock slope of the gas flare site, a sequential and infinite discontinuity system was selected for modeling. Geometrical slope models of FORM (first-order reliability method) and KGM algorithm that run in the Mathematica software (Wolfram 1999) were prepared (Yarahmadi-Bafghi and Verdel 2003). The dimensions of the model were chosen

Fig. 7 Rose diagram of the studied slope in the case study

Apparent Strike
5 max planes / arc
at outer circle

Trend / Plunge of
Face Normal = 0, 90
(directed away from viewer)

No Bias Correction

44 Planes Plotted
Within 45 and 90
Degrees of Viewing
Face

in their real scale metric sizes using the surface survey information.

The KGM algorithm has been described previously. The FORM is a first-order reliability method where the name "first-order reliability method" comes from the fact that the performance function g(x) is approximated by the first-order Taylor expansion (linearization). The performance function of the FORM system can be written as:

$$g(x) = g(a_1, a_2, \ldots, a_n) \begin{cases} > 0 & \rightarrow & \text{safe state} \\ = 0 & \rightarrow & \text{limit state} \\ < 0 & \rightarrow & \text{failure state.} \end{cases} \quad (1)$$

If $g(x)$ is zero, i.e., $P\{g(x) = 0\}$, this is known as a limit state surface and each x indicates the basic load or resistance variable. Usually, a number of limit states can be identified for a system with each representing a state of ultimate system failure, system unserviceability, or operational malfunction x such that $P\{g(x) = 0\}$ is the level set of $g(-)$ at level 0. When $g(x)$ is greater than zero, i.e., $P\{g(x) > 0\}$, the random variables of $x = \{a_1, a_2, \ldots, a_n\}$ are in the safe region. The probability that the random variables of x are in the failure region is defined by $g(x) < 0$ or $P\{g(x) < 0\}$.

The limit state and linearized limit state of the performance function are shown in Fig. 8. It should be noted that different equivalent formulations of performance function will not change the failure (limit state) surface because the equivalency is based on $g(-) = 0$ or $P\{g(x) = 0\}$. However, the linearized limit state depends on what formulation or performance function is used in the mean value of the
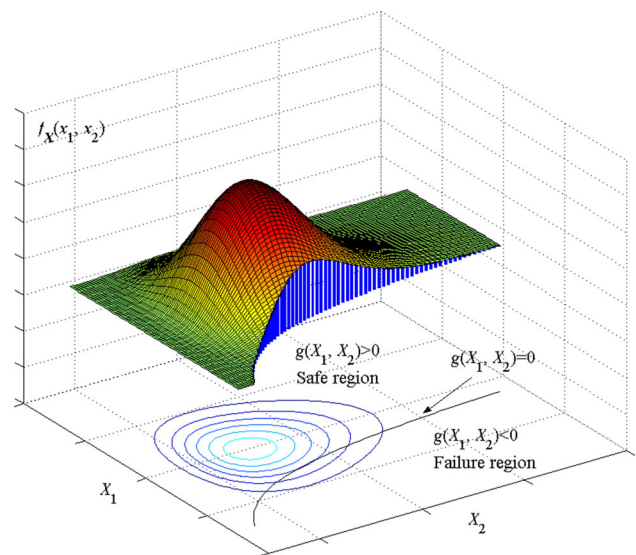

Fig. 8 Limit state and linearized limit state of performance function in FORM (first-order reliability method) (Bhattacharya 2012)

Taylor series expansion. This is because the mean values are not on the failure surface and the two performance functions are different and are away from the failure surface.

Definition of the behavioral model and assignment of geomechanical properties

After designing the geometrical model of the slope, an attempt was made to define the behavioral model and to assign the geomechanical properties of the model.

Terms controlling the behavior and strength of rock masses

The relationships ruling on strength and behavior are generally well known as empirical relationships, and the strength parameters of the rock mass are determined by rock mass classification systems (Hack 1998). The Slope Mass Rating (SMR) method proposed by Romana et al. (2003) and Rock Mass Rating (RMR) classification system of Bieniawski (Bieniawski 1974, 1989) have been utilized to determine the rock mass strength parameters.

Terms controlling the behavior of the rock mass at the flare site

In the studied rock slope, the structural conditions were determined to be the main causes of sliding, since the discontinuities played a main role in the instability of the rock slope. The Mohr–Coulomb criterion has been used for obtaining the strength properties, and the Barton–Bandis criterion has been used to satisfy the part of the significant gaps, which is required in constructing the flare 6 site. Using the Mohr–Coulomb failure criterion, the reliability of a simple plane slide or the FOS of a dry block without considering seismic forces may be computed from Eq. 2 (Hoek 1987):

$$FOS = \frac{cA + R_n \tan \varphi}{R_t} \qquad (2)$$

where, c is cohesion; A is the sliding surface length per unit width; ϕ is friction angle; and R_n and R_t are normal and tangential components of the outcome force exerted on the sliding surface (R), respectively.

The factor of safety (FOS) for wedge failure or collapse can be calculated through the decomposition of the R force to the vertical or tangential component on the sliding surface (Fig. 9) by using Eq. (3) (Hoek 1987):

$$FOS_{Wedge} = \frac{c_i A_i + N_i \tan \varphi + c_j A_j + N_j \tan \varphi}{T_{ij}} \qquad (3)$$

where $N_i = -\vec{R} \cdot \vec{n_i}$, $N_j = -\vec{R} \cdot \vec{n_j}$ and $T_{ij} = R \cdot \vec{i_{ij}}$, N_i and N_j are the respective normal components of the R outcome vector; T_{ij} is the tangential component of R; c is cohesion, and ϕ is internal friction angle.

Mechanical modeling and stability analysis

The proposed key-group method (Yarahmadi-Bafghi 2003; Yarahmadi-Bafghi and Verdel 2003) that is based on the equilibrium analysis was used as the analysis method in the mechanical model. After determining the geometrical properties of the discontinuity statistically, a 2D geometrical modeling was performed through the Mathematica computer program (Wolfram 1999). A mechanical model
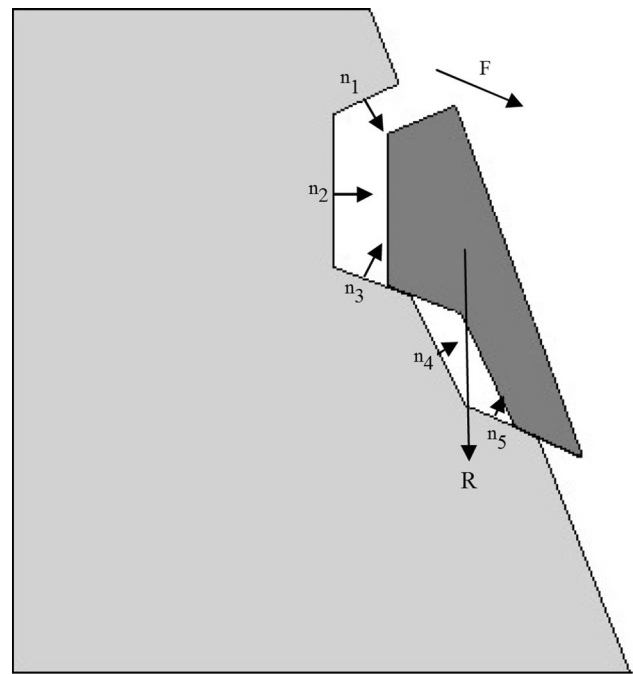

Fig. 9 Scheme of wedge failure mechanism

of the sections was added to the geometrical model, and the analysis was presented statically under limit equilibrium condition based on the block theory. Moreover, probabilistic modeling of the rock slope was performed to evaluate and reduce the ambiguities that existed in the mechanical model. Variables considered in the modeling included:

- First- and second-order statistical moments (average and standard deviation);
- Strength parameters of the discontinuities (c and ϕ);
- Unit weight of the rock material (γ_d); and
- Pairwise covariance values of these components (Cov).

This information was assessed based on the statistical data of the above components and was used in the analysis. Regarding the geotechnical modeling of the slope, KGM and FORM algorithms were used to analyze the reliability and to determine the FOS values. Tables 1, 2, 3, 4 and 5 present the information used in the modeling.

Slope modeling and stability analysis

The studied rock slope was a discontinuous marlstone slope that is located in the Aghajari formation. According to the stability analysis performed in the geotechnical modeling by the KGM and FORM, the slope was classified as "needs attention." The geometrical modeling of the slope is illustrated in Fig. 10.

According to the engineering geological field survey and geometric modeling performed for the slope, the

Table 1 Criteria for factor of safety (FOS) utilized in the stability analysis of rock slope (Azarafza et al. 2013)

Slope class	Slope importance	Slope type	Criteria 1	Criteria 2	Criteria 3
			Minimum allowable FOS	Maximum allowable sliding probability (FOS < 1) (%)	Maximum allowable sliding probability (FOS < 1.5) (%)
A	Very important	Stable	2.00	0.30	5
B	Important	Semi-stable	1.60	1.0	10
C	Unimportant	Provisional	1.30	10	20

Table 2 Guide of slope behavior analysis versus slope stability (Azarafza et al. 2013)

No.	Criteria[a]	Stability	Description
1	All three criteria are satisfied	Stable	Slope is stable
2	The first and one of the two criteria are inappropriate	Needs attention	Slope needs indispensable attention
3	All are inappropriate	Unstable	Slope is unstable

[a] Criteria defined in Table 1

Table 3 Rock mass mechanical properties

Parameter	c (MPa)	ϕ (°)	γ (kN/m^3)	E (GPa)	G (MPa)	K (MPa)	T (MPa)	υ	K_n (GPa)	K_s (GPa)
Value	1.1	29	18.70	70	28	45	0.097	0.24	3.9	25

c cohesion, ϕ angle of internal friction, γ unit weight, E Young's modulus, G shear modulus, K bulk modulus, T tensile strength, υ Poisson's ratio, K_n & K_s Normal and shear joint stiffness coefficients

Table 4 Engineering classification of the rock mass

SMR (Romana et al. 2003)	RMR (Bieniawski 1989)	Classification
55.6 (IIIa)	43 (IV)	Description[a]

[a] 43 (IV) fair rock in RMR classification system, 55.6 (IIIa) fair rock in SMR classification system

most possible rock slope failure mechanism was determined to be wedge type of failure. In wedge failure, there needs to be at least two intersecting discontinuity planes with their line of intersection (ψ_i) angle greater than the internal friction angle (ϕ) (Azarafza 2013). The parameters used in the slope stability analysis are presented in Table 6.

The unit weight and shear strength parameters (c, ϕ) of the rock specimens reported in Table 6 were determined in accordance with ISRM (1981). The mechanical modeling of the slope is presented in Fig. 11. Table 7 presents the stability analysis results based on the key-group theory. As shown in Fig. 11, the red colored polygons reflect the sliding (unstable) masses of the slope with a FOS less than 1. The other colorful parts reflect the other benefits of the Mathematica software. In modeling with the block theory method, the region between the discontinuities which are named as polygons (single intact blocks) may be detected and colored.

Table 5 Results of the statistical analysis of the discontinuity sets of the studied slope

Discontinuity set no.	Parameter	Mean values
Discontinuity set 1	Dip	63°
	Dip direction	56°
	Spacing	0.78 m
Discontinuity set 2	Dip	47°
	Dip direction	184°
	Spacing	0.60 m
Discontinuity set 3	Dip	71°
	Dip direction	319°
	Spacing	1.0 m
Discontinuity set 4	Dip	84°
	Dip direction	139°
	Spacing	2.5 m
Rock mass	Cohesion	1.1 MPa
	Internal friction angle	29°
	Unit weight	18.70 kN/m^3

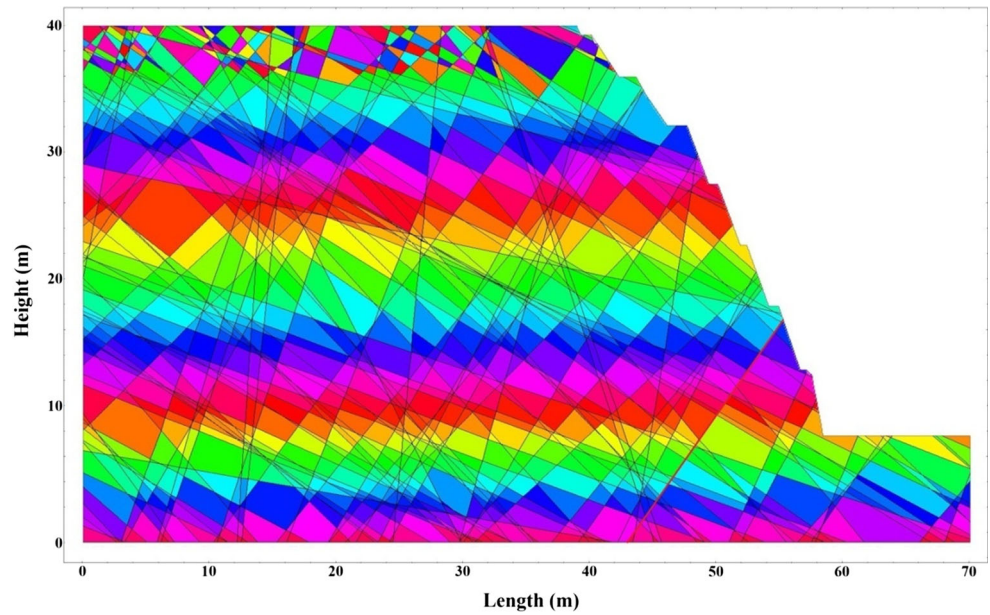**Fig. 10** Geometrical modeling of the slope

Table 6 Parameters used in the static slope stability analysis and the calculation method

Parameter	Calculation method	Average	Standard deviation	Distribution
RMR	Rock mass rating (Bieniawski 1989)	43	–	–
Equivalent elastic modulus (MPa)	$E_s = 10^{\left(\frac{RMR-10}{40}\right)}$ (Bieniawski 1974)	70	–	–
Unit weight (kN/m³)	ISRM (1981)	18.70	–	Normal
Cohesion (MPa)	ISRM (1981)	1.1	0.001	Normal
Friction angle (°)	ISRM (1981)	29	3.4	Normal
Covariance C_j and ϕ_j	Cov[ϕ_j, C_0]	0.276	–	Monte Carlo

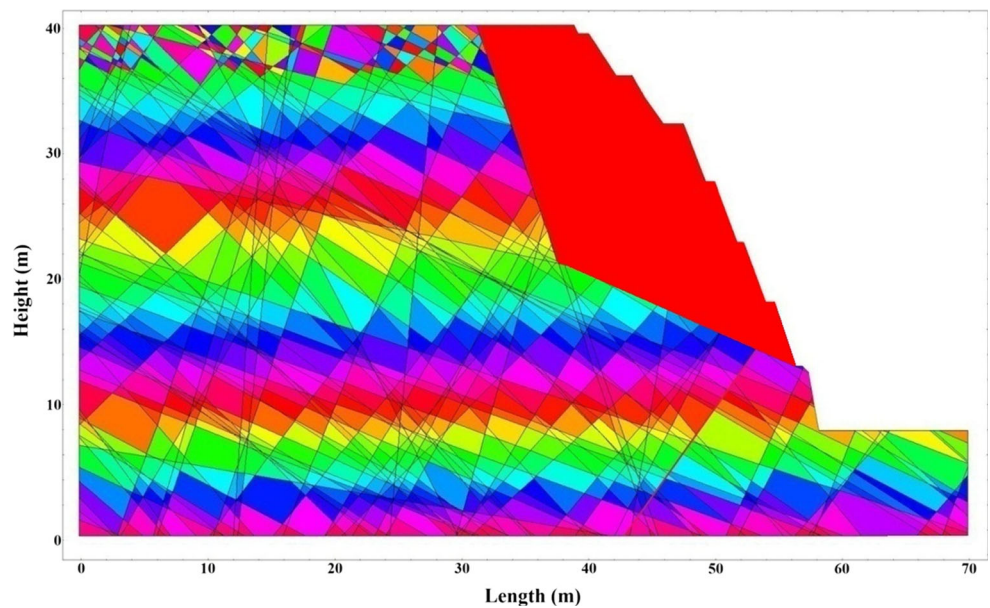**Fig. 11** Mechanical modeling of the slope

Table 7 Safety analysis of the slope by the key-group method (KGM)

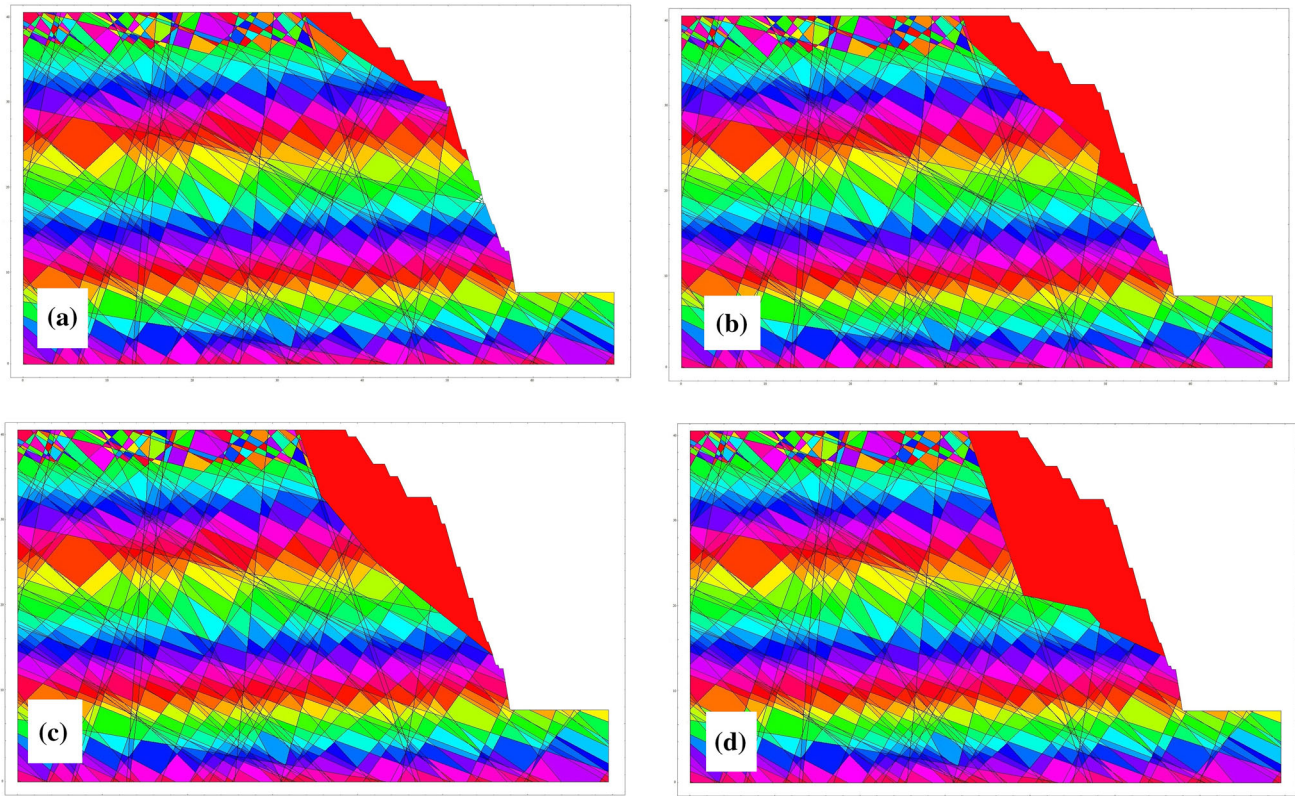| Stability	Weight of unstable group (Tonnes)	P (FOS > 1.5)	P (FOS > 1.0)	Factor of safety (FOS)	Analysis method
Needs attention	6.68	0.00984	0.971	1.16	FORM

Fig. 12 Progressive stability analysis by the proposed algorithm (Azarafza et al. 2013)

As it has been pointed out previously, the failure mechanism of the studied slope was wedge type. This type of sliding movement (transfer of the unstable wedge) occurred in 3D space, and for analyzing this type of sliding in 2D space, the possible sliding critical surface was considered on intersection of discontinuities. One of the benefits of the KGM and the used algorithm is the possibility of analyzing progressive mass sliding. Hence, cyclic analysis and classification of the reliability of the slope was possible which is presented in Fig. 12.

The stability analysis of the studied slope indicated that a wedge type of sliding was possible along the line of intersection of the discontinuity sets. In this slope, as a result of discontinuity sets system operation, the act of crushing of the rock was blocked. Three intersecting discontinuity sets and non-systematic discontinuities by the phenomenon of crack propagation might have led to slope instability. Based on the field studies, it was observed that the rock slope was highly weathered, which led to a low shear strength of the rock mass. The analysis led to a determination that the slope required immediate attention as it was classified in the "needs attention" category.

Model controlling

For numerical modeling of a discontinuous rock slope, discontinuum methods or discontinuum modeling is utilized. In the discontinuum methods, the rock slope is treated as a discontinuous rock mass by considering it as an assemblage of rigid or deformable blocks. The analysis includes sliding along and opening/closure of rock discontinuities controlled principally by the normal and shear joint stiffness values. Discontinuum modeling constitutes of an applied numerical

Fig. 13 Representation of a fractured rock mass: **a** the fractured rock mass, **b** model by DEM (Jing 2003)
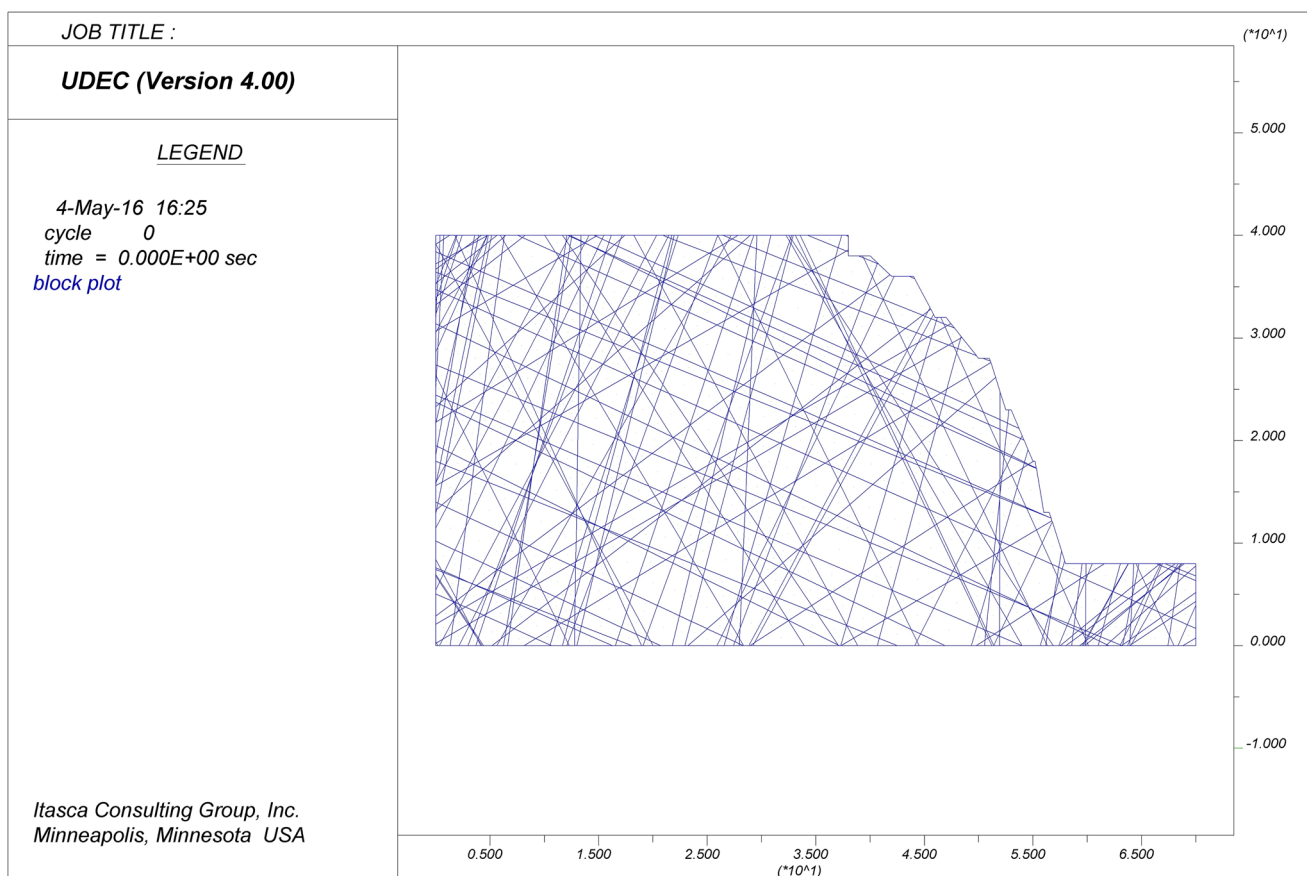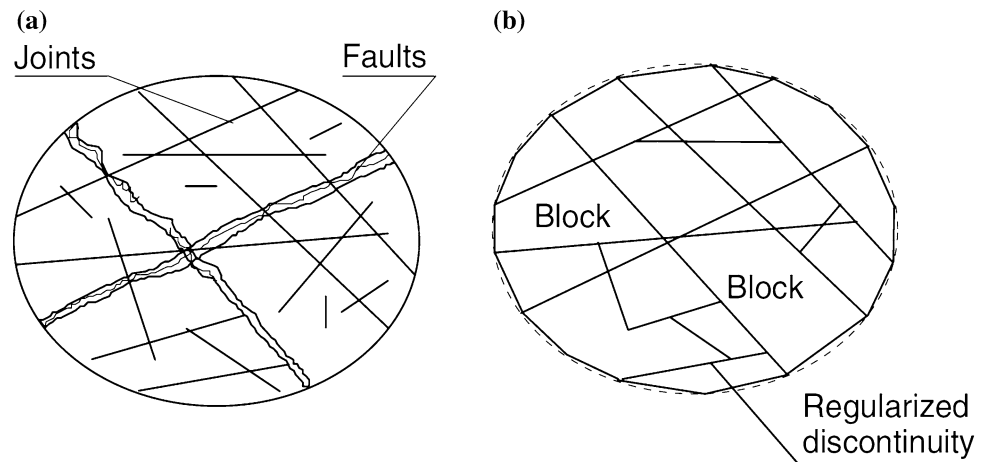

Fig. 14 Geometrical modeling of the slope by UDEC

approach to rock slope stability analysis where the most common method is the DEM (Itasca 2008).

The most significant explicit DEM codes developed for simulation of discontinuous rock masses are the UDEC (2D) and 3DEC (3D) codes for analysis of block system problems (Itasca 2000, 2008). The DEM formulation is used in rock engineering due to three issues (Jing and Stephansson 2007):

- Identification of rock blocks, material, fracture systems, and system topology
- Formulation and solution of equations
- Detection and updating of varying contacts.

The UDEC is two-dimensional numerical software to model the static or dynamic response to loading of a media
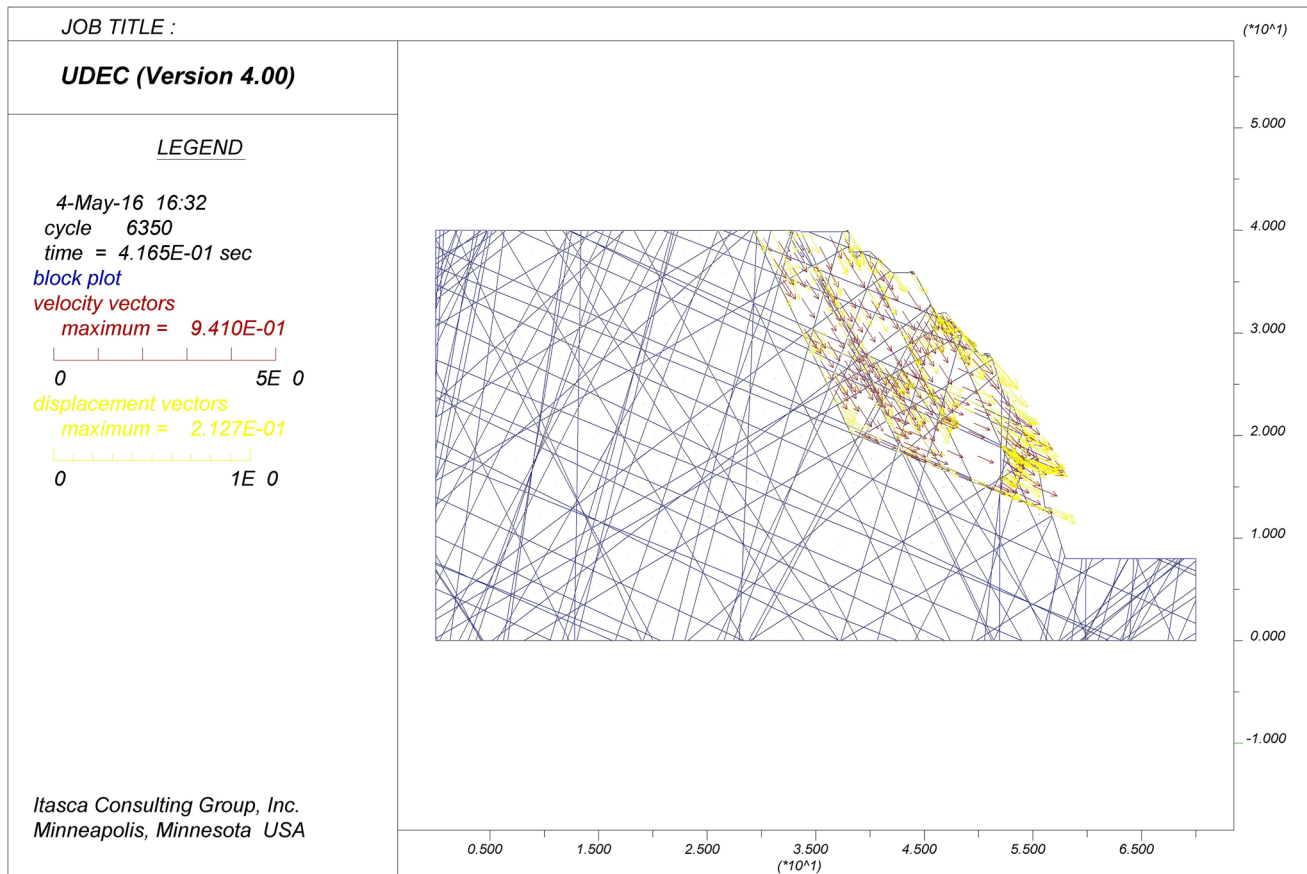
Fig. 15 Mechanical modeling of the slope by UDEC

containing multiple, intersecting joint structures. This software considers a blocky environment that is discretized by discontinuities that behave as boundary conditions between blocks. The block displacement and rotation occurs along discontinuities as shown in Fig. 13.

To control the accuracy of the proposed approach, an attempt was made to analyze this slope using the UDEC software (Itasca 2008) based on the engineering geological parameters listed in Tables 3, 4 and 5. The designed geometrical model of the slope is shown in Fig. 14. Figure 15 indicates the mechanical model of the slope after analyzing the reliability and assigning the geomechanical properties and behavioral model of the slope.

According to these results, it may be concluded that there is an excellent agreement between the numerical analysis results and the model related with the block theory results. In addition, the superiority of the used algorithm for analyzing critical sliding zones and progressive failure analysis is revealed beyond the ability of the numerical analysis method. It is also necessary to mention that the used algorithm which is according to the key-group method (KGM) has a higher processing speed than the numerical method.

Conclusions

The research findings, the analyses and interpretation of the modeling results, and the field observations led to the following conclusions:

1. The key-group method (KGM) algorithm, because of its relatively high accuracy and its good agreement with the results of the numerical analysis, may be used as an alternative to or to complement other statistical methods.
2. Comparing the used algorithm based on the KGM with the DEM via the UDEC software, a highly analogous geometry of the failing system is determined by both of these models. However, the main advantage of using the algorithm is that it is faster than DEM in data processing, in sliding surface determination, and in stability analysis.
3. Since in the KGM all movable groups are studied, there is no possibility of loss of falling key groups.
4. As the analysis was performed to control the slope stability, the FOS of the slope was determined to be within a range of 1.0–1.5. Thus, the slope required

immediate attention as it was classified as "needs attention".

5. The numerical analysis results and the model utilized are consistent with the block theory results. In addition, the superiority of the used algorithm for analyzing critical sliding zones and progressive failure analysis beyond the ability of the numerical analysis method was revealed. Besides, it was found that the algorithm was faster than the numerical method.

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