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ORIGINAL ARTICLE

Environmental assessment of leachate transport in saturated homogeneous media using finite element modeling

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Abstract A landfill with considerable length and width, despite its depth, was assumed, and a new two-level time-stepping iterative algorithm has been employed to develop a 1-D finite element model to predict leachate transport through a saturated homogeneous soil layer beneath the landfill. In order to verify the model, results obtained from numerical computations were compared with the analytical solution of transport equation. The comparison demonstrates a good agreement between analytical and numerical calculations, in which R^2 is higher than 98 % and maximum squared error is <10 % for all scenarios. For validation purpose, an annual measured concentration of chloride ion in Saravan landfill (Iran) has been used to compare with the computed data. The comparison shows that R^2 is more than 97 % and the mean squared error is <30 %, which demonstrates the model's ability to follow the leachate concentration for specific time and space in real-life issues. Moreover, to evaluate the role of each process in contaminant transport, a sensitivity analysis has been conducted through which the dominance of advection process has been revealed. To be more accurate, Peclet number criterion was employed to evaluate the role of each process in contaminant concentration. Analyses confirmed that even though advection process is dominant in most parts of soil layer below the landfill, the effects of diffusion term on transport equation are not negligible.

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Introduction

Monitoring and controlling leachate transport to minimize its adverse impacts on soil and groundwater is still considered as a fundamental problem in solid waste management studies. Hazardous waste and municipal solid waste's leachate seep into soil by rainfalls, flow through water table and gradually accumulate in soil and fresh water resources (Aldecy et al. 2008). After some decades, as a result of human activities, multiple untapped freshwater resources are heavily contaminated, as if they cannot be treated for use as drinking water (Qiu 2011). Since the current problem has made a violent threat against human life and ecosystems, numerous research works have been conducted to evaluate the long- and short-term effects of leachate migration (Christensen et al. 1998; Lee et al. 1986; Ogundiran and Afolabi 2008). In order to achieve these goals, an overall understanding about the basic mechanisms as well as chemical and hydraulic properties of materials is necessary (Jhamnani and Singh 2009). The physical phenomena such as dispersion, advection and diffusion and the chemical phenomena including adsorption, desorption (reaction) and ionic exchange are all fundamental mechanisms of pollutant movement through the soil (Chakraborty and Ghosh 2010). Extensive research works have been undertaken to describe various contaminant fate processes (Foose et al. 2001, 2002; Opdyke and Loehr 2002; Rowe 1988; Shackelford and Daniel 1991a, b).

One of the major aspects of monitoring is to assess leachate transport rate. Several research works have also been pursued leakage rate and contaminant migration through clay liners (Barroso et al. 2006; Çelik et al. 2009;

Giroud and Touze-Foltz 2005; Touze-Foltz and Giroud 2003). For instance, Nakhaei et al. (2015) have recently performed a research on groundwater quality using Visual HELP and HYDRUS programs to determine hydrologic properties and contaminant transport in their study area and to evaluate the rate of leachate percolation.

Providing an effective barrier in order to make a separation between the pollution and underground water can mitigate the potential risk of pollutant migration into reservoirs. Many research works focused on developing an appropriate soil layer against leachate movement (Benson and Daniel 1994; Benson et al. 1999; Benson and Trast 1995). Also, variety of composite liners such as geomembrane and compacted clay layer or geomembrane and geosynthetic clay have been used to create a barrier for leachate migration (Hoor and Rowe 2013; Liu et al. 2013; Park et al. 2011; Varank et al. 2011).

In order to simulate the leakage rate and liner's performance, governing equations of contaminant transport should be solved in porous media. Common approaches for solving these equations include analytical approach, semi-analytical approach and numerical approaches (finite difference, finite element, limited layer) (Opdyke and Loehr 2002). Rowe and Brachman (2004) presented a semi-analytical 1-D model to evaluate contaminant transport. In some research works, a one-dimensional solution has been employed to evaluate the liner capability against organic contaminant and volatile organic compound transport through a composite liner (Xie et al. 2013a, b, 2014). Chen et al. (2009) used finite domain method contaminant diffusion through multilayered media. Also, Cleall and Li (2011) proposed a model for diffusion of VOCs through liners. Furthermore, finite element methods have been employed to assess the performance of composite liners with intact and leaking geomembranes (El-Zein et al. 2012; El-Zein and Rowe 2008; El-Zein 2008).

Although leakage rate, which indicates leachate's advective strength, is a significant factor to assess the risk of clay liner's performance, it should not be considered as the only actor of the contaminant fate and transport, since other processes such as molecular diffusion, mechanical dispersion, sorption and decay are of great importance in measuring the risk factor of liner's functionality (El-Zein et al. 2012). For instance, the Ontario regulation emphasizes that "The design must consider both advective and diffusive contaminant transport and must include examination of the effect of the failure in any engineered facilities when their service lives are reached" (Ontario Ministry of Environment 2010).

In this paper, the advection–diffusion–reaction equation (ADRE) has been solved by two-level step time-dependent finite element approach. This model was developed by C++ compiler and tested against analytical solution to verify their accuracy and performances. Also, collected data from Saravan landfill located in North of Iran have been compared with the computed data to validate the model. In addition, an investigation about the flow advection and molecular diffusion contributions into the process of leachate transport has been conducted by sensitivity analysis. Moreover, the comprehensive behavior of contaminant transport has been justified by Peclet criteria.

Method and materials

Experimental site

The main center of landfill processing in Gilan Province (the regional state on North of Iran and South of Caspian Sea.) is located at 37°04′18″N longitude and 49°37′53″E latitude with 15 km distance from capital city (Rasht). Based on municipal reports, dumping wastes was started from 1983 with 800 g per capita per day. Also, service area covered by Saravan center is about 12 ha (Fig. 1), and analyses have shown that soil structure of the landfill consists of dense silty sand, gravel and cobble and very dense crushed rock (Nakhaei et al. 2015). Moreover, the area is one of the wettest climate zones of Iran with annual precipitation of 1500 mm in which the groundwater level is high enough to cause intrusion of leachate into local groundwater. Table 1 shows summary of hydrologic and geological structure of the landfill.

Mathematical model

There are numerous researches which could describe transport equation precisely. However, equations that were used to estimate the leachate transport in the study are listed below, and we leave unnecessary details for sake of brevity. Several models also were proposed to study contaminant fate and leachate transport based on Fick's law and mass balance equations. The Fick's law and its experimental results in saturated soil are described as Eq. 1 (Crooks and Quigley 1984):

$$f = \eta c v - \eta D\left(\frac{\partial c}{\partial z}\right) \quad (1)$$

Assuming steady-state flow and homogeneous saturated media, and incorporating mass balance and adsorption effects, the 1-D equation advection–diffusion–reaction model is presented by the following equation (Rowe and Booker 1985):

$$(\eta + \rho K)\left(\frac{\partial c}{\partial t}\right) = \eta D\left(\frac{\partial^2 c}{\partial z^2}\right) - \eta v\left(\frac{\partial c}{\partial z}\right) \quad (2)$$

Equation 2 can also be expressed by the following form (Chakraborty and Ghosh 2010):

Fig. 1 Location of Saravan Landfill (GoogleEarth, V.6.2.2.6613, Google Inc.)

Table 1 Hydrogeological data of Saravan landfill

Parameter	Value
Precipitation (m^3/year)	64,007
Runoff (m^3)	33.987
Evapotranspiration (m^3)	101,080
Soil water (m^3)	5,850,480
Permeability (cm/s)	10^{-3}–10^{-4}
hydraulic conductivity (cm/day)	83.4

$$R\left(\frac{\partial c}{\partial t}\right) = D\left(\frac{\partial^2 c}{\partial z^2}\right) - v\left(\frac{\partial c}{\partial z}\right) \tag{3}$$

where R is retardation factor and can be presented as follows:

$$R = 1 + \left(\frac{\rho K_d}{\eta}\right) \tag{4}$$

where K_d is distribution factor (L^3/M), D is dispersion coefficient (L^2/T), c is concentration of contaminant (M/L^3), t is time in seconds, v is discharge velocity in a particular direction (LT^{-1}), z is distance along Z axis (L), η is effective porosity of the soil and f is flux of contaminant ($ML^{-2}T^{-1}$).

Methods and techniques

In order to solve the governing equation, the time-dependent term of Eq. 4 was separated from spatial term in the first place, and afterward by using weighted residual method and Galerkin ($W_1 = N_1$), the following equation is obtained:

$$\int_{\bar{\Omega}} \left(Dc_m(t)\frac{dN_m}{dz}\frac{dN_l}{dz} + vc_m(t)\frac{dN_m}{dz}N_l + RN_m\frac{dc_m(t)}{dt}N_l\right)d\bar{\Omega} = \left[\frac{\partial \hat{c}}{\partial z}w_l{}_{z=L}^{z=0}\right] \tag{5}$$

By reorganizing Eq. 5 with respect to $c_m(t)$, we have:

$$\boldsymbol{C}\frac{dc(t)}{dt} + \boldsymbol{K}c(t) = \boldsymbol{f}_l \tag{6}$$

in which **K** and **C** are, respectively, as follows:

$$\boldsymbol{k_{lm}} = \int_{\bar{\Omega}} \left(D\frac{dN_m}{dx}\frac{dN_l}{dz} + v\frac{dN_m}{dz}N_l\right)d\bar{\Omega} \tag{7}$$

$$\boldsymbol{c_{lm}} = \int_{\bar{\Omega}} RN_mN_l d\bar{\Omega} \tag{8}$$

After substituting the shape functions for each i and j element, **K** and **C** matrices are calculated as follows:

$$K_{lm}^e = \int_0^h \begin{bmatrix} \left(D\left(\frac{dN_i}{dx}\right)^2 + v\frac{dN_i}{dz}N_i\right) & \left(D\frac{dN_i}{dx}\frac{dN_j}{dz} + v\frac{dN_j}{dz}N_i\right) \\ \left(D\frac{dN_i}{dx}\frac{dN_j}{dz} + v\frac{dN_i}{dz}N_j\right) & \left(D\left(\frac{dN_j}{dx}\right)^2 + v\frac{dN_j}{dz}N_j\right) \end{bmatrix} dz \tag{9}$$

$$c_{lm}^e = R\int_0^h \begin{bmatrix} N_i^2 & N_iN_j \\ N_iN_j & N_j^2 \end{bmatrix} dz \tag{10}$$

Thus, the general separated equation can be calculated as follows:

$$C_{lm}\begin{bmatrix}\frac{dc_1}{dt}\\ \frac{dc_2}{dt}\\ \vdots\\ \frac{dc_{m-1}}{dt}\\ \frac{dc_m}{dt}\end{bmatrix}+K_{lm}\begin{bmatrix}c_1\\ c_2\\ \vdots\\ c_{m-1}\\ c_m\end{bmatrix}=\begin{bmatrix}-\frac{\partial c_1}{\partial z}_{z=0}\\ 0\\ \vdots\\ 0\\ \frac{\partial c_m}{\partial z}_{z=L}\end{bmatrix} \tag{11}$$

In the above system of differential equations, the unknowns are as follows:

$$\left\{c_1, c_2, \ldots, c_{m-1}, c_m, \frac{\partial c_1}{\partial z}_{z=0}, \frac{\partial c_m}{\partial z}_{z=1}\right\} \tag{12}$$

Utilizing allocated points (Zienkiewicz et al. 2006), $c(t)$ was divided into time elements (Δt_n).Considering proper shape function, Eq. 13 is obtained as:

$$\left\{\frac{C}{\Delta t_n}+\gamma_n K\right\}c^{n+1}+\left\{-\frac{C}{\Delta t_n}+(1-\gamma_n)K\right\}c^n = (1-\gamma_n)f^n+\gamma_n f^{n+1} \tag{13}$$

In Eq. 13, $0 \leq \gamma_n \leq 1$ and the unknown c^{n+1} can be calculated by substituting c^n and incrementing through the time. Equation 13 is solved for $\gamma_n = 0$ (explicit method) and $\gamma_n = 1$ (implicit method).

Verification

In order to verify the method, results are compared with the analytical solution, which is presented as follows (Ogata 1970; Ogata and Banks 1961):

$$c(z,t) = (c_0/2)\left\{\operatorname{erf}\left[\frac{Rz-vt}{2\sqrt{DRt}}\right]+\exp(vz/D) + \exp(vz/D)\operatorname{erf}\left[\frac{Rz-vt}{2\sqrt{DRt}}\right]\right\} \tag{14}$$

where c_0 is initial concentration and assumed to be constant by considering perpendicular motion and $c(z, t)$ is contaminant concentration in depth (z) and time (t).

In this study, landfill with large width and length, rather than depth, has been assumed. Also, it is supposed that the distance between beneath of landfill and groundwater is 2.5 m and the leachate can seep through this layer. The liner is assumed saturated and homogeneous with low permeability. The other assumptions are as follows (Chakraborty and Ghosh 2010): (1) The 1-D contaminant transport equation in porous media governs in at hand problem. And (2) the flow is steady state and the Darcy's law is valid.

The boundary conditions are as follows:

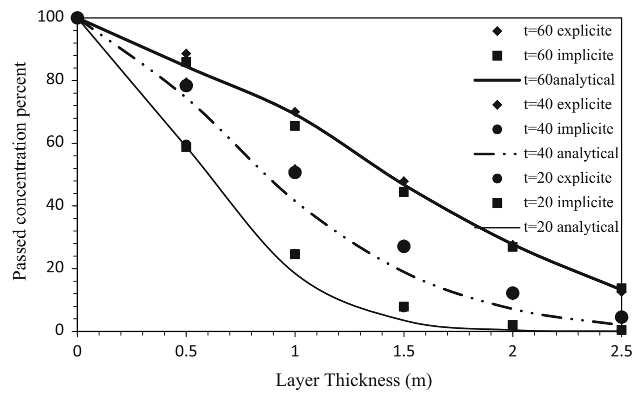

Fig. 2 Comparison between analytical and finite element models for passed concentration percentage

$C(z,0) = C_0 \quad \text{at } z = 0;$

$C(z,0) = 0 \quad \text{at } z > 0;$

$C(0,t) = C_0 \quad \text{at } t > 0;$

$C(z,t) = 0 \quad \text{at } t > 0 \quad \text{and } z > 2.5;$

where $C(z, t)$ is the passed concentration percentage in z meter below the landfill in time t.

The comparison between analytical method and numerical analyses is shown in Fig. 2. It shows that the passed concentration of contaminants through 20, 40 and 60 years is predicted. Also, following values were used so that we will be able to compare the models with the analytical solutions: $R = 1.45$, $D = 0.014$ (m^2/a) and $v = 0.025$ (m/a).

Results show that the presented model demonstrates good consistency with the analytical solution. The calculated regression coefficient (RSQ) for all scenarios is more than 98 %. Also, the mean squared error (MSE) is <10 %.

Validation

In order to validate the model, annual measured mean chloride concentration in Saravan landfill for a 10-year interval has been used. The data are provided from $z = 0$, $z = 10$, $z = 20$ and $z = 30$ m below the landfill, and the groundwater level is lower than 10 m. Hence, soil layer is assumed to be saturated. In addition, the conservative characteristic of chloride ion, which is not under the effect of adsorption and biological degradation, provides an equilibrium transport condition that ADRE could be used to simulate leachate transport characteristics.

The chloride concentration was assumed to be constant at initial condition. Also, the concentration at exactly beneath the landfill ($z = 0$) and $z = 30$ were used as boundary conditions. Tables 1 and 2 represent values used to simulate the chloride concentration through the soil.

Table 2 Summary of other input values used for computations

Parameter	Description	Value
D (m^2/s)	Dispersion coefficient	3.71×10^{-9} (Rotaru et al. 2014)
n (–)	Porosity	0.67 (Nakhaei et al. 2015)
K (m/s)	Hydraulic conductivity	2.73×10^{-8} (Huysmans and Dassargues 2005)
grad h (–)	Hydraulic head gradient	0.02 (Huysmans and Dassargues 2005)
K_d (cm^3/g m)	Distribution coefficient	10 (Chakraborty and Ghosh 2010)
ρ (kg/m)	Density of chloride	2165

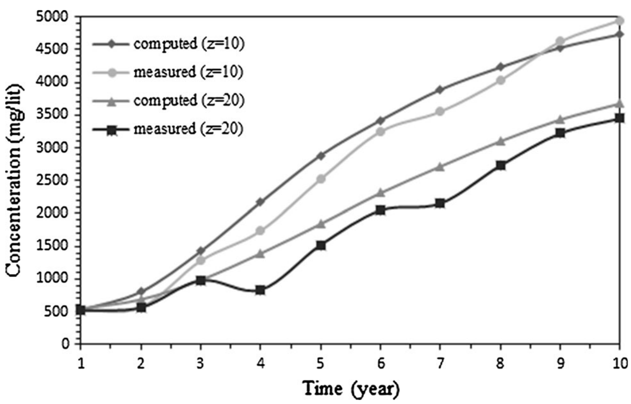

Fig. 3 Comparison between passed mean annual chloride concentrations of computed data with measured data for $z = 10$ and 20 m for 10 years

Figure 3 shows the comparison between measured and computed data in $z = 10$ and $z = 20$ ms from below the landfill. As it can be seen in the diagrams, the chloride concentration raised during 10 years even though the growth is not consistent. However, the model shows a consistent growth of chloride concentration in this period.

Table 3 shows the extent of agreement between the model and the real data. As it can be seen in this table, the R^2 values are more than 97 % which demonstrates the model is capable of simulating the contaminant transport in real conditions. The maximum square errors are <30 %. The reason for this difference is that we assumed that soil is homogeneous and the hydraulic conductivity and porosity are the same everywhere. However, the trends for both computation and measurements show that the chloride concentration has grown in these years for both 10 and 20 m depths.

Discussion about fate and transport mechanisms

Although the effects of mechanical advection due to flow of leachate have been considered as the major mechanism responsible for contaminant fate and transport, the pollutant migration is a complicated process and depends on numerous factors. For instance, in flows with lower velocities, the molecular diffusion might gain dominance over advection, while by increasing the velocity the balance changes in favor of mechanical advection. Also, when the permeability is low, determining the performance of these characters could be difficult (Fetter and Fetter 1999). Given this uncertainties, in order to identify effects of advection and diffusion on contaminant transport in this study, first a sensitivity analysis has been performed on each term of governed equation. Then by using Peclet number criterion, more investigation about effective domain of advection and diffusion processes will be pursued.

Table 3 Extent of Agreement between the model and the measured data

Depth (m)	Maximum square error (%)	R^2 (%)
$Z = 10$	23	98
$Z = 20$	27	97

Sensitivity analysis

In order to analyze the advection effect in Eq. 4, the diffusion coefficient is kept constant and equal to 0.014 (m^2/a), and the advective velocity changed during the test for half and twice of the current value of the advection coefficient. Also, for analyzing the diffusion term, the reverse process was considered and the advection coefficient was taken as constant value equals to 0.025 (m/a).

Figure 4 shows that by increasing the advection coefficient in initial layers, differences between concentrations values are more than that of lower layers (Fig. 4a). Also, as time passes (Figs. 4b, 3c), the aforementioned process intensifies. Conversely, by using the advection coefficient equals to half of the initial value, the reverse of the above process with less intensity is observable (Fig. 4a–c). By both decreasing and increasing the diffusion coefficient, all diagrams have an overlap on initial layers. At the lower layers of the liner, however, the difference between these two diagrams intensified, and they have become completely separated at the end of the liner (Fig. 5a–c).

The following results can be concluded from the above observations: (1) The mechanical advection term plays a more significant role on the passed concentration of

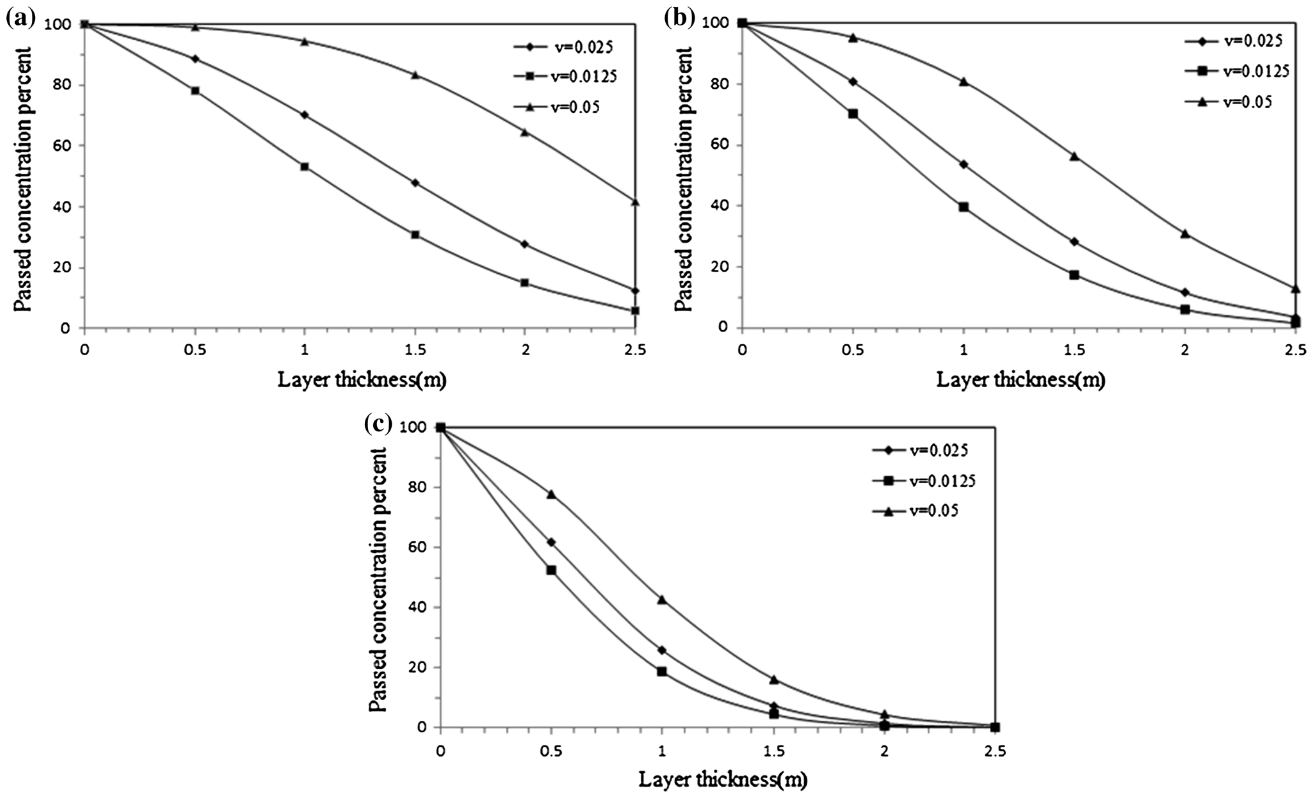

Fig. 4 Sensitivity analysis for advection process through the liner: **a** after 60 years, **b** after 40 years and **c** after 20 years

leachate transport in general. (2) By increasing the thickness, the molecular diffusion role becomes more important. However, on lower layer the effects fade out. And (3) as time goes by, the effects of both advection and diffusion will become stronger.

Peclet criteria

In this study, in order to distinct the dominancy of the mechanical advection and molecular diffusion, Peclet criteria have been employed. Peclet is a dimensionless number and has been defined as the ratio of advection rate of a physical quantity to diffusion rate of that quantity. Unfortunately, a number of Peclet number definitions can be found in the literature. The assumptions for simplification of the solution transport equation are the main reason for this broad variety. Strictly speaking, these differences are as a result of the variation between the effective porosity n_e, which is the accessible porosity for fluid flow or advection and diffusion available porosity n, which in turn is the fraction of the total water filled porosity available for diffusive transport (Fetter and Fetter 1999). In fact, when the clay is compacted, these two values (i.e., n and n_e) are not necessarily equal to each other, and many research works have proved that diffusion accessible porosity is smaller than the total porosity due to size-exclusion effect, which is like charge interaction that leads to some ions that are not allowed to permeate through the pores (Horseman et al. 1996).

The general three-dimensional transport equation (advection–diffusion–reaction) is as follows (Marsily 1986):

$$n\frac{\partial c}{\partial t} = \mathrm{div}(nD_e \cdot \mathrm{grad}c + n_e \cdot D \cdot \mathrm{grad}c - n_e v_e c) \qquad (15)$$

where n is the diffusion accessible porosity, n_e is the effective porosity, V_e is the effective advection velocity (m/s) and D_e is the effective diffusion coefficient (m^2/s) which in turn is defined as:

$$D_e = \omega D_d \qquad (16)$$

in which ω is related to tortuosity, D_d is the molecular diffusion coefficient (m^2/s) and D is the dispersion coefficient (m^2/s). Table 4 demonstrates four variations of Peclet number.

The first Peclet number occurs when the transport equation is solved by the analytical solution, which is presented in this paper (Fetter and Fetter 2001). For the second one, if the value of Pe_2 is much smaller than 1, the diffusion process is dominant (Remenda et al. 1996). If the problem is advection dominated, the sharpness of concentration can be measured by the grid Peclet number Pe_3 (Zheng and Bennett 2002). To evaluate the role of

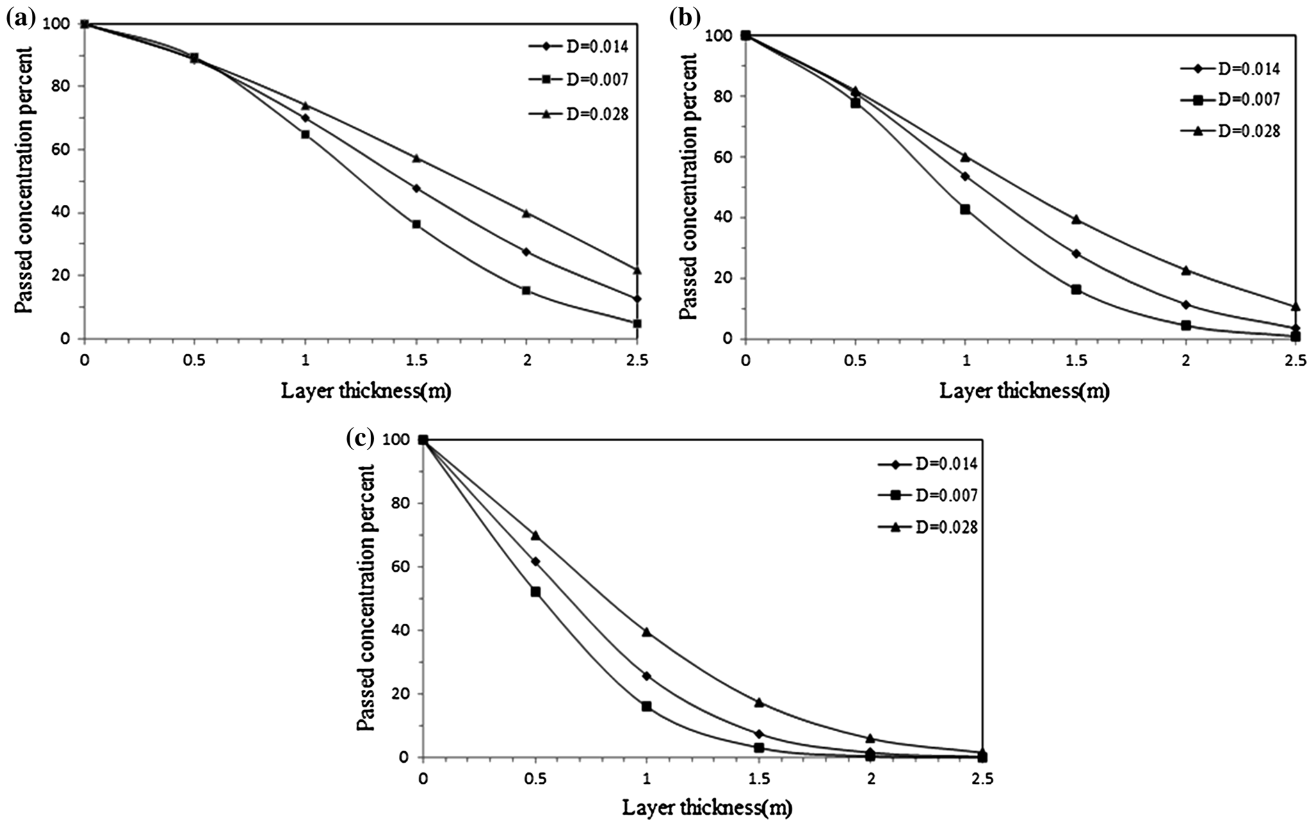

Fig. 5 Sensitivity analysis for molecular diffusion through the liner: **a** after 60 years, **b** after 40 years and **c** after 20 years

Table 4 Variations of Peclet number definitions

Equations	Variable definitions
$Pe_1 = \frac{V_e L}{D_L} = \frac{V_D L}{n_e D_L}$	V_e: effective groundwater velocity, L: reference length (m), D_L: longitudinal hydrodynamic (m^2/s), V_D: Darcy velocity (m/s)
$Pe_2 = \frac{V_e^2 T}{D_h}$	V_e: average linear groundwater velocity, T: total duration of process, $D_h = D + D_e$: coefficient of hydraulic dispersion
$Pe_3 = \frac{V_e \Delta m}{D_h}$	Δm: grid spacing (m)
$Pe_4 = \frac{V_D x}{n D_e}$	x: distance, n: diffusion accessible porosity

advection and diffusion through transport path in a low permeability clay, the fourth Peclet number can be utilized. In this case, for $Pe < 1$ diffusion dominates at the distance x from the source (Wemaere et al. 2002).

Table 5 shows the necessary input values for calculations. Pe_1 is >1 which indicates that the advection term is dominant in the problem. However, in the calculation of this number the effective porosity (n_e) is assumed to be equal to diffusive porosity (n). With this simplifying assumption, the value of diffusive coefficient dramatically decreases because as it is given in Table 5, the effective porosity is about 200 times greater than diffusion porosity. This result cannot be realistic owing to our problem assumptions, i.e., the clay liner permeability is low and due to size-exclusion effect, which is mentioned above. Thus, there should not be that much differences between these two values. The calculated Peclet numbers are given in Table 6.

If the calculated value of Pe_2 is much smaller than 1, we can argue that the advective transport is negligible. On the contrary, our values are more than 2 and we cannot figure out which term is dominant in our problem. On the other hand, the third Peclet number (Pe_3) is <1. This case is usually applicable while the problem is dominated by advection and there is oscillation due to spatial discretization of applying numerical method. Nevertheless, in this problem no oscillation is observed during the process of solving the transport equation, and the reason is that the advection is not completely dominant in the problem.

Table 5 Parameters and values used for Peclet number calculations

Parameter	Description	Value
L (m)	Distance to the source	0.25–2.75
D_e (m^2/year)	Effective diffusion coefficient	2×10^{-4}–3.5×10^{-3} (Huysmans and Dassargues 2005)
T (year)	Time	10
n_e (–)	Effective porosity	0.001–0.1 (Huysmans and Dassargues 2005)
n (–)	Diffusion accessible porosity	0.2–0.4 (Huysmans and Dassargues 2005)
α_L (m)	Longitudinal dispersivity	0.01–10 (Huysmans and Dassargues 2005)
ω (–)	Factor related to tortuosity	0.1 (Huysmans and Dassargues 2005)
Δm (m)	Grid spacing	0.5

Table 6 Calculated Peclet number range for leachate transport problem in this study

Peclet number	Min	Max	Mean
Pe_1	6.187500	6.194571	6.191036
Pe_2	2.142857	2.640845	2.391851
Pe_3	0.714286	0.880282	0.797284
Pe_4	0.196875	6.194571	3.195723

Pe_4 scenario shows the minimum Peclet number corresponds to initial levels of thickness ($x = 0.25$ m) and the maximum number is computed on the end of the supposed thickness ($x = 2.75$). This criterion, unlike other three, has the capability of determining the dominancy of contaminant process through the thickness of liner layer. Regarding this criteria, if we solve the equation of $Pe_4 = 1$, we can discover the transition border where the dominancy of processes changes. By replacing the variables, the transition border will be obtained in range between 0.443937 and 1.269841. Also, mean value of Pe_4 indicates that the advection process is dominant in most part of soil layer below the landfill; however, diffusion process has significant effect on leachate concentration in some locations in transition zone. Thus, we are not allowed to eliminate one of these terms to simplify the contaminant transport, and both advection and diffusion effects should be taken into account.

Conclusions

The present study is a state-of-art technique to predict leakage transport seeping from beneath of landfills toward water table, which is determining factor to optimize landfill's designing, controlling and monitoring systems. In order to solve the fate and contaminant transport equation, a finite element model has been developed by applying weak form of integration to weighted residual method and separating time-dependent component from spatial ones and solving the ADRE equation by an iterative algorithm. Since the algorithm takes advantage of a powerful finite element scheme, it could be suitable to be employed in landfills with complex geometry by defining proper shape functions to evaluate contaminant concentration in both temporal and spatial dimensions. Another preference of this algorithm is its relatively short run time of calculations stems from using simplified form of algebraic equation (i.e., explicit and implicit scheme) in the numerical computations.

To verify the model, a landfill with simple geometry and boundary conditions is assumed and the analytical solution of ADRE equation has been compared with the presented method. Results show that the proposed model has good consistency with the analytical one ($RSQ > 98$ % and $MSE < 10$ %). To highlight the presented model's performance, we have performed an analogy between the proposed model and CONTAMINATE model (Chakraborty and Ghosh 2010) which has been employed FDM method to solve the contaminant transport equation for a landfill with considerable width and length relative to its height. Figures 6 and 7 demonstrate that the model has proper results compared with the CONTAMINATE model.

Also, to validate the model, mean annual passed chloride concentration obtained from Saravan disposal center, located in North of Iran, compared with the computed data for a 10-year duration in 10 and 20 m below the landfill. Agreement between the results shows that the model has good consistency with the real data and it is capable of applying on real-life conditions.

In order to determine the role of advection and diffusion processes during the time through the liner layer, an investigation has been conducted on each term. At first, a sensitive analysis on both variables has been performed. The results show that the advection term is dominant generally, but the diffusion term affects the pollutant transport in some areas. To be more precise, the Peclet number criterion has been employed and four variations of Peclet number have been calculated. The calculations show that though mechanical advection has stronger impact in leachate transport, the diffusion process plays a key part on

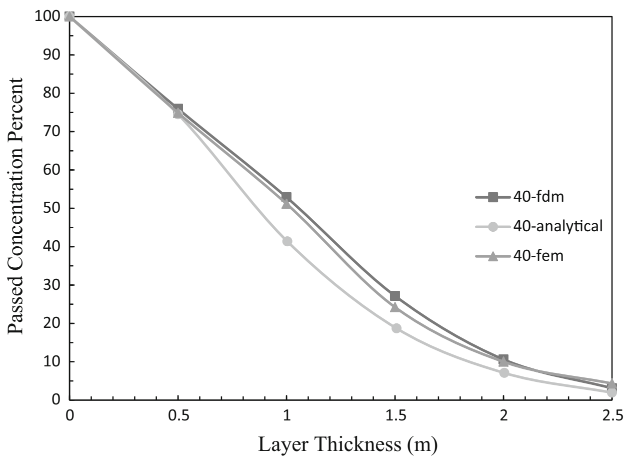

Fig. 6 Comparison between proposed model (i.e., fem) and CONTAMINATE model (i.e., fdm) for 40 years

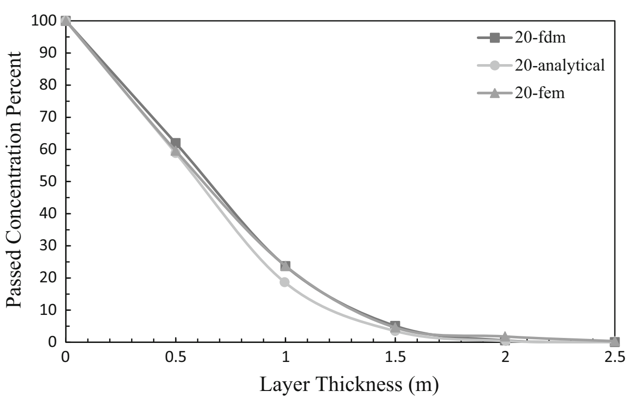

Fig. 7 Comparison between proposed model (i.e., fem) and CONTAMINATE model (i.e., fdm) for 20 years

leachate transport at initial layers; nonetheless, its effects will gradually be undermined by taking distance from the contaminant source.

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