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ORIGINAL ARTICLE

Prediction of minimum factor of safety against slope failure in clayey soils using artificial neural network

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Abstract This paper presents prediction of minimum factor of safety (FS) against slope failure in clayey soils using artificial neural network (ANN). Two multilayer perceptron ANN models were used to predict the minimum factor of safety using different data sets of geometric and shear strength parameters and based on the four well-known methods of Fellenius (Ordinary), Bishop, Janbu, and Spencer, respectively. The input parameters used to train and test the two ANN models include the reciprocal of slope tangent β, angle of internal friction of soil φ (^{o}), height of the slope H (m), cohesion of the soil c (kN/m^2), unit weight of the soil γ (kN/m^3) and the stability number m ($c/\gamma H$). The output parameter for both ANN is the FS of the slope. The number of hidden layers and the number of neurons in each hidden layer were determined by trial and error to achieve the best results. It is observed that both ANN predictions are very close to the FS calculated by each of the corresponding four methods, separately. However, the ANN model with the scaled down number of input parameters showed better performance and the best one has a normalized mean square error of 0.0073, mean absolute percent error (MAPE) of 1.52 % and correlation coefficient (r) of 0.9966. It is concluded that such ANN models are reliable, simple and valid computational tools for predicting the FS and for assessing the stability of slopes of clayey soil. Six known case studies that are based on different methods were used to further test and validate the accuracy of the ANN model. It was observed that the ANN model predictions of FS of the case studies were very accurate with MAPE of 3.72 % for all methods combined. Based on the developed ANN model, a parametric study was then carried out to investigate the influence of the slope angle (β), stability number (m) and angle of internal friction (φ) on the factor of safety and slope stability of clayey soil.

Keywords Artificial neural network · Factor of safety · Clayey soils · Shear strength · Fellenius model · Bishop model · Janbu model · Spencer model

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Abbreviations

FS	Factor of safety
b	Width of the slice
H	Height of the slope
c	Cohesion of the soil
c'	Effective cohesion of the soil
m	Stability number
h	Average height of the slice
ha	Height to the center of the slice
Sm	Mobilized shear strength
W	Weight of slice
Ww	Surface water force
hL	Height of force ZL
Q	External surcharge
N'	Effective normal force
Kh	Horizontal seismic coefficient
μ	Angle of inclination of external load
U	Pore water pressure
ZL	Left inter-slice force
ZR	Right inter-slice force
δL	Left inter-slice force inclination angle

δR Right inter-slice force inclination angle
hR Height of force ZR
α Inclination of slice base
β Inclination of slice top

Introduction

The stability of slopes of clayey soil is an important characteristic for earth dams, excavation, landfills and earth moving. The slope stability of clayey soil is mainly affected by the soil shear strength parameters—cohesion and angle of internal friction, unit weight of the material, slope geometry and pore water pressure. Also, rapid drawdown and fluctuation of water levels in reservoirs is likely to be a reason for instability and rapid sliding (Pinyol et al. 2012; Alonso et al. 2010). The performance of a slope and its susceptibility to failure are usually measured by its factor of safety (FS). Accurate prediction of factor of safety of slopes, their stability and performance is not an easy task. This is mainly due to the difficulty in accurately determining the mechanical properties of the parameters that influence the stability of the slopes, their degree of influence and the complexity of their relationships. Therefore, assessment of slope stability is influenced by many sources of uncertainties (Jurado et al. 2012; Tartakovsky 2013). Slope stability and their associated factor of safety have been investigated by many researchers experimentally, analytically and numerically. The factor of safety has been calculated using several methods including limit equilibrium methods of slices and its variations, finite difference method and finite element method, among others. Duncan (1996) presented state-of-the-art review of limit state equilibrium methods and finite element analysis for the stability of slopes. He presented a detailed summary of characteristics of different limit equilibrium methods of slices and several methods for 3D slope stability analyses on which limit equilibrium was extended and/or variational principles were employed. He also provided a detailed review of deformation analyses of slopes and embankments with primary focus on the finite element method. For the finite element methods, four different types of stress–strain relationships, mainly linear elastic, multi-linear elastic, hyperbolic, and elasto-plastic were presented together with their advantages and limitations for use in practical slope stability problems. The conventional methods discussed above to predict the factor of safety (FS) against slope failure are lengthy, time consuming, iterative and require computer software.

Other methods are also developed either to find the factor of safety or to determine the critical failure slope. Cheng et al. (2007) obtained the factor of safety of slope by limit equilibrium and strength reduction approach. Kahatadeniya et al. (2009) used the ant colony optimization to determine the critical slip surface. His results were found to be compatible and in agreement with the results obtained from other well-known methods. Sengupta et al. (2009) determined the location of the critical slip surface in slope stability analysis using genetic algorithm approach. He found this approach to be superior to other optimization routines.

In the last few decades there was a growing interest in using intelligent computational systems such as artificial neural network (ANN) in geotechnical engineering (Goh 1995; Juang and Elton 1997; Juang et al. 1999; Cheng et al. 2007; Shahin et al. 2008) in general and in predicting the behavior of slopes and their susceptibility to failure under static and dynamic loading in particular. There are several successful applications of ANN that investigated slope stability and evaluated slope failure characteristics (Ni et al. 1996; Sakellariou and Ferentinou 2005; Wang et al. 2005; Ferentinou and Sakellariou 2007; Ural and Tolon 2008; Cho 2009; Lin et al. 2009). Ni et al. (1996) combined fuzzy sets theory with artificial neural networks to evaluate the stability of slopes and to predict the slope failure potential. The results of the ANN were in good agreement with the analytical results. Sakellariou and Ferentinou (2005) used back-propagation learning algorithm to estimate the factor of safety of slopes and their stability status based on several geotechnical and geometrical input parameters. The performance of the network was measured and the results were compared to those obtained by means of standard analytical methods. Wang et al. (2005) used a Back-Propagation Neural Networks model with four layers and a training data set of landslide samples to predict the stability and safety factor of slopes. Ferentinou and Sakellariou (2007) combined ANN tools with generic interaction matrix theory to estimate slope stability controlling factors. They developed an integrated method for estimating the factor of safety, slope stability and for predicting the slope performance under static and seismic loading. They concluded that computational intelligence tools are promising and should be further exploited in tackling such complex problems. Ural and Tolon (2008) used ANN to predict factor of safety of saturated slopes under earthquake. They studied the importance of the seismic coefficients for a slope stability safety and assessed the importance of the slope and dynamic input parameters in stability of slopes in the event of earthquake. Cho (2009) integrated finite difference method into a probabilistic analysis of slope stability and employed artificial neural network-based response surface to calculate the probability of failure through the first-order and second-order reliability methods and a Monte Carlo simulation technique.

He carried out a probabilistic stability assessments for a hypothetical two-layer slope for validation of the developed method. Based on results from two examples, he indicated that the ANN-based response surface can be successfully applied to slope stability probabilistic problems. Lin et al. (2009) used neural network-based model for assessing failure potential of highway slopes. They explored the degrees of influence of several factors (variables) on slope stability and used the developed ANN models to investigate the slope failure characteristics before and after earthquake. Kaunda et al. (2010) use back-propagation artificial neural network architecture to predict slip or failure surface of active landslides, among other parameters. They concluded that the neural network models predict slip surfaces better than the limit equilibrium slip surface search using the most conservative criteria. Chang et al. (2011) used Artificial Evolution Neural Network (AENN) to learn from past slope failure records and their results showed that the developed AENN can accurately predict the occurrence of slope failure with a success rate of 99 %. They further used the AENN model to accurately predict the slump rate of slopes in the study area based on the precipitation data that consisted of daily rainfall and effective rainfall. Das et al. (2011) developed several neural network models to classify the slope as stable or unstable and for prediction of the factor of safety. They compared their results with other models based on support vector machine and genetic programming and they observed that ANN model is very accurate. As presented, different variations of ANN methods have been used to predict the factor of safety and they were successful with different level of accuracy. These ANN methods were based on training data resulted from some experimental data or generated by some specific method of analysis. In this study, all the four major classical methods were used in generating the training data for the ANN and therefore its results are more inclusive than the previous ANN attempts.

Therefore, the aim of this study is to develop an ANN model to predict the factor of safety (FS) of slopes using results of different classical methods and use the developed model to carry out sensitivity analysis to investigate the influence of several parameters on the FS of slopes and their susceptibility to failure. A total of 160 data sets with different geometric and shear strength parameters were used to generate minimum factor of safety based on four well-known methods of Fellenius, Bishop, Janbu, and Spencer, respectively. The input data and the results obtained from the analysis were used to train and test two Multilayer Perceptron (MLP) ANN models to predict the minimum FS for each method. Three parameters were used as input for the first ANN—the slope angle β (°), angle of internal friction φ (°) and the stability number m ($c/\gamma H$), while five parameters were used as input for the second ANN which are the slope angle β (°), height of the slope H (m), angle of internal friction φ (°), cohesion of the soil c (kN/m^2) and the unit weight of the soil γ (kN/m^3). The output parameter for both ANN is the FS of the slope. A parametric study was then carried out to investigate the influence of these parameters on the factor of safety of slope and the stability of slopes of clayey soil.

Slope stability of clayey soil: background

According to Mohr's failure theory, the shear strength of the soil is a function of cohesion, angle of internal friction, and the applied normal stress. Based on this theory, the shear strength of the clayey soil is given by Eq. (1) (Cernica 1982):

$$\tau_f = c' + \sigma' \tan\phi' \tag{1}$$

where, τ_f is the shear strength, c' is the effective cohesion of the soil and σ' is the normal stress and φ' is the effective angle of internal friction of the soil.

Figure 1 shows a typical slip or failure surface and forces acting on a typical slice for a slope stability problem based on the method of slices. The main purpose of slope stability analysis is to find the minimum FS and the corresponding location of the slip surface. Many methods in the literature have been used to analyze and find the minimum FS. All the methods deal with the shear strength parameters developed along the potential slip surface in their analysis.

In general, the FS with respect to strength of slope is given by Eq. (2) (Das 2008) as the ratio between the resisting and driving force.

$$\mathrm{FS} = \frac{\text{Resisting force}}{\text{Driving force}} = \frac{\text{Shear strength}}{\text{Shear stress}}$$

$$\mathrm{FS} = \frac{\tau_f}{\tau_d} = \frac{c' + \sigma' \tan\phi'}{c'_d + \sigma' \tan\phi'_d} \tag{2}$$

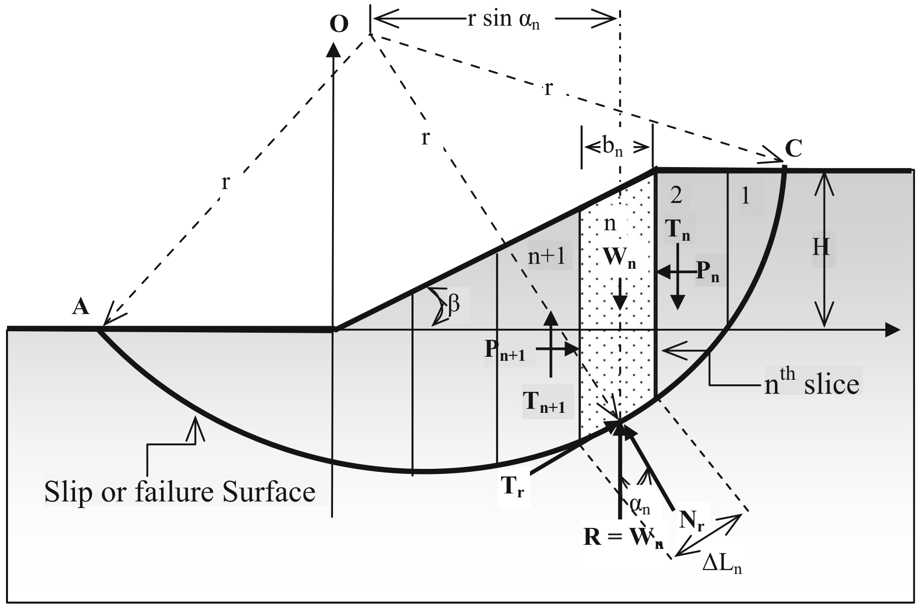
where, τ_f is the Shear strength of the soil, τ_d is the average shear stress developed along the potential failure surface, c' is the effective cohesion of the soil, σ' is the effective normal stress on the potential failure surface, ϕ' is the Effective angle of internal friction, c_d' is the Effective cohesion that develop along the potential failure surface, ϕ_d' is the Angle of friction that develop along the potential failure surface.

The FS for an infinite slope can be written as given by Eq. (3) (Das 2008) and it is a function of five parameters (c', ϕ', γ, H, β).

$$\mathrm{FS} = \frac{c'}{\gamma H \cos^2\beta \tan\beta} + \frac{\tan\phi'}{\tan\beta} \tag{3}$$

where, H is the height of the slope and β is the angle of the slope.

Fig. 1 Stability of slope of clayey soil (Das 2008)



Or alternatively as given by Eq. (4) (Das 2008) which is a function of thee parameters (m, ϕ', β).

$$\mathrm{FS} = \frac{m}{\cos^2\beta\tan\beta} + \frac{\tan\phi'}{\tan\beta} \tag{4}$$

where, m is the stability number and is given by $m = \frac{c'}{\gamma H}$.

There are several methods developed for analysis of slopes, excavations and embankments. The most widely used method for computing the FS of finite slopes is the limit equilibrium method of slices and its variations which can be classified into two categories (Nash 1987): (1) *linear methods*; and (2) *nonlinear methods*. Linear methods include wedge analysis, circular arc methods (ordinary method) (Fellenius 1936). Nonlinear methods include Bishop's simplified method (1955), Janbu's simplified method (1954), Spencer's method (1967), Morgenstern-Price's method, Janbu's generalized analysis, among others. Duncan and Wright (1980) assessed the accuracy of limit equilibrium methods and Fredlund et al. (1981, 1984) showed the relationship among limit equilibrium methods of slices. In addition, several 3D nonlinear methods were developed thereafter by Seed et al. (1990), Leshchinsky and Huang (1992) and Michalowski (1995). Among these methods Fellenius or ordinary method, Janbu simplified method, Bishop simplified method and Spencer method will be used in this study.

Ordinary method (Fellenius method)

The FS for the Ordinary Method of slices is given by Eq. (5) (Malkawi and Hassan 2003):

$$\mathrm{FS} = \frac{\sum_{i=1}^{n}\left[c'.b\sec\alpha + [W\cos\alpha + Q\cos(u-\alpha) + W_w\cos(\beta-\alpha) + k_hW\sin\alpha - Ub]\tan\phi'\right]}{\sum_{i=1}^{n}(W + W_w\cos\beta + Q\cos\mu)\sin\alpha - \sum_{i=1}^{n}(W_w\sin\beta + Q\sin\mu)\left(\cos\alpha - \frac{h}{R}\right) + \sum_{i=1}^{n}k_hW\left(\cos\alpha - \frac{h_a}{R}\right)} \tag{5}$$

where, FS is the factor of safety, B is the width of the slice, c' is the effective cohesion of the soil, h is the Average height of the slice, ha is the height to the center of the slice, W is the Weight of slice, Ww is the Surface water force, Q is the External surcharge, Kh is the Horizontal seismic coefficient, μ is the Angle of inclination of external load, U is the Pore water pressure, α is the Inclination of slice base, β is the Inclination of slice top.

Ignoring the effect of surface water force (W_w), external surcharge (Q), horizontal seismic force (k_h) and pore water pressure (U), the FS of slope can be written as shown in Eq. (6) (Das 2008):

$$\mathrm{FS} = \frac{\sum_{n=1}^{n=p}(c'b_n/\cos\alpha_n + W_n\cos\alpha_n\tan\phi')}{\sum_{n=1}^{n=p}W_n\sin\alpha_n} \tag{6}$$

Janbu simplified method

Janbu (1954) developed a method presented in Eq. (7) which satisfies the vertical force equilibrium for each slice and the overall horizontal equilibrium force for the entire mass slices and the method is applicable to failure surfaces of any shape.

$$\mathrm{FS} = \frac{\sum_{i=1}^{n}(c'b\sec\alpha + N'\tan\phi')\cos\alpha}{\sum_{i=1}^{n}(ub\sin\alpha + Wk_h - W_W\sin\beta - Q\sin\mu) + \sum_{i=1}^{n}N'\sin\alpha} \tag{7}$$

where, N' is the effective normal force. Ignoring the effect of surface water force (W_w), external surcharge (Q), horizontal seismic force (k_h) and pore water pressure (U), the factor of safety of slope can be written as shown in Eq. (8) (Das 2008):

$$\mathrm{FS} = \frac{\sum_{i=1}^{n}(c'b\sec\alpha + N'\tan\phi')\cos\alpha}{\sum_{i=1}^{n}(N'\sin\alpha)} \tag{8}$$

Bishop simplified method

Bishop (1955) developed a modified method of slices for calculating the FS. It satisfies the equation of equilibrium with respect to moment only (Das 2008). The FS for the Bishop Method is given by Eq. (9):

$$\mathrm{FS} = \frac{\sum_{i=1}^{n}(c'b\sec\alpha + N'\tan\phi')}{\sum_{i=1}^{n}(W + W_w\cos\beta + Q\cos\mu)\sin\alpha - \sum_{i=1}^{n}(W_w\sin\beta + Q\sin\mu)\left(\cos\alpha - \frac{h}{R}\right) + \sum_{i=1}^{n}k_hW\left(\cos\alpha - \frac{h_a}{R}\right)} \tag{9}$$

Ignoring the effect of surface water force (W_w), external surcharge (Q), horizontal seismic force (k_h) and pore water pressure (U), the FS of slope can be written as shown in Eq. (10) (Das 2008):

$$\mathrm{FS} = \frac{\sum_{n=1}^{n=p}(c'b_n + W_n\tan\phi')\frac{1}{m_{\alpha(n)}}}{\sum_{n=1}^{n=p}W_n\sin\alpha_n} \tag{10}$$

where $m_{\alpha(n)}$ is given by:

$$m_{\alpha(n)} = \cos\alpha_n + \frac{\tan\phi'\sin\alpha_n}{FS} \tag{11}$$

Spencer method

Spencer (1967) developed a method of slices that satisfies both the equation of equilibrium with respect to both moments and forces. Spencer's method (1967) assumes that the angle of inclination of the inter-slice forces is constant for all slices. It is a special case of the Morgenstern-Price method. According to Spencer's assumptions for all slides as shown in Fig. 2.

$$\begin{aligned} &\delta_R = \delta_L = \delta \text{ (for all slices)} \\ &Z_R = Z_L + \frac{\mathrm{FS}\times W\sin\alpha - c'.b\sec\alpha - W\cos\alpha\tan\phi'}{\sin(\delta-\alpha)\tan\phi' - FS\cos(\delta-\alpha)} + \frac{ub\sec\alpha\tan\phi' + Wk_h(\mathrm{FS} - \tan\phi'\tan\alpha)\cos\alpha}{\sin(\delta-\alpha)\tan\phi' - FS\cos(\delta-\alpha)} \\ &+ \frac{Q[\mathrm{FS}\sin(\alpha-u) - \cos(\alpha-u)\tan\phi']}{\sin(\delta-\alpha)\tan\phi' - FS\cos(\delta-\alpha)} + \frac{W_W[\mathrm{FS}\sin(\alpha-\beta) - \cos(\alpha-\beta)\tan\phi']}{\sin(\delta-\alpha)\tan\phi' - \mathrm{FS}\cos(\delta-\alpha)} \end{aligned} \tag{12}$$

where, FS is the factor of safety, Q is the external surcharge, δL is the left inter-slice force inclination angle, δR is the Right inter-slice force inclination angle, α is the inclination of slice base.

Ignoring the effect of surface water force (W_w), external surcharge (Q), horizontal seismic force (k_h) and pore water pressure (U), the factor of safety of slope can be written as shown in Eq. (13).

$$Z_R = Z_L + \frac{FS\times W\sin\alpha - c'.b\sec\alpha - W\cos\alpha\tan\phi'}{\sin(\delta-\alpha)\tan\phi' - FS\cos(\delta-\alpha)} \tag{13}$$

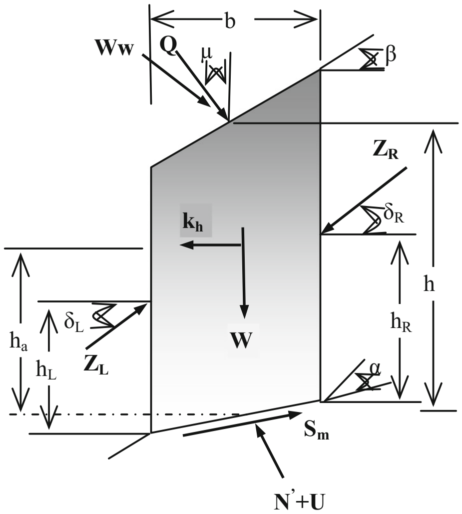


Fig. 2 Forces acting on one slice (Malkawi A and Hassan 2003)

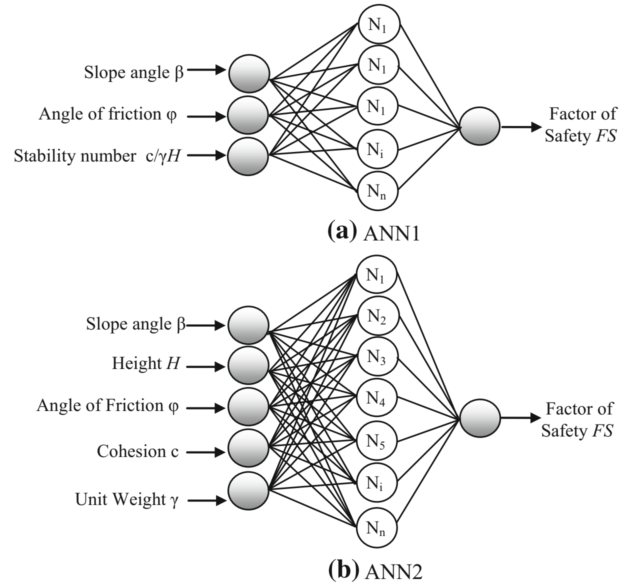


Fig. 3 Architecture of the two MLP ANN models

where, *ZL* is the left inter-slice force, *ZR* is the right inter-slice force.

Artificial neural network model and methodology

Artificial neural networks are computational systems that can be trained to model physical phenomenon. There are several ANN architectures that have been used in civil engineering applications in general (Flood and Kartam 1994; Abdalla et al. 2013) and in geotechnical engineering in particular Shahin et al. (2001, 2008). In this study, after several trials of different ANN architectures using Neuro-solutions (Neurodimensions 2011) and upon comparisons of their performance criteria, two Multilayer Perceptron (MLP) ANN models with an input layer, one hidden layer and an output layer were used. The hidden layers of the two ANN have four nodes. The first ANN model (ANN1) has three input parameters—slope angle, angle of internal friction and stability number (Fig. 3a) and the second ANN model (ANN2) has five input parameters—slope angle, slope height, angle of internal friction, cohesion and soil unit weight (Fig. 3b). The output parameter for each ANN is the factor of safety. A tangent hyperbolic (*tanh*) transfer function and four processing elements are used for the hidden layer. A linear bias transfer function was used for the output layer. Each of the ANN was trained using 10 runs with 5,000 epoch for each run Neurodimensions (2011). A total of 160 × 4 data sets, for the four methods Ordinary (Fellenius), Bishop, Janbu and Spencer, with different input and output parameters were used to train and test the ANN models. The training and testing data were randomly chosen as 80 % (128 data set) and 20 % (32 data set) of the total data, respectively. The trained ANN models were presented with the test data of the four methods and values of FS were predicted. These values were then compared with the FS values using Fellenius, Bishop, Janbu, and Spencer methods based on certain performance criteria.

Tables 1 and 2 show the range of input parameters for the training and testing data, respectively for both ANN models. For the given range of slope run to unit rise ratio, the angle of slope ranges from 18.43° to 33.69°. The range of slope height is from 7 m to 10 m, the angle of internal friction from 15° to 30°, the cohesion from 10 to 18 kN/m^2, the soil unit weight from 12 to 16 kN/m^3 and the stability number from 0.074 to 0.275.

Results and discussion

As previously indicated, the two ANN models (ANN1 and ANN2) were trained with 10 cycles with 5,000 epochs in each training cycle. The performances of the learning rate of each ANN model for the first 100 epochs are shown in Fig. 4. There is sharp drop in the training NMSE of the best runs during the first 10 epochs, especially for ANN1.

The performance of the four method based on the performance criteria shown in Tables 3 and 4 indicates that Bishop simplified method has better performance than the other methods. It has the lowest Mean Square Error (MSE), Normalized Mean Square Error (NMSE), Mean Absolute Error (MAE), Mean Absolute Percent Error (MAPE) and better correlation coefficient than Ordinary, Janbu, and Spencer when ANN1 model with three input parameters (β, φ, m) is used. It should also be noted from Table 3 that

Table 1 Range of training and testing of input parameters for ANN1 and ANN2

Input parameter	Training data		Testing data	
	Maximum	Minimum	Maximum	Minimum
Slope run for unit rise ratio (β)	3	1.5	3	1.5
Height (H) (m)	10	7	10	7
Angle of internal friction (φ) (°)	30	15	30	15
Cohesion (c) (kN/m^2)	18	10	15	10
Unit weight (γ) (kN/m^3)	16	12	16	12
Stability number ($m = c/\gamma H$)	0.2747	0.0741	0.2404	0.0833

Table 2 Range of training and testing of output parameters for ANN1 and ANN2

Output parameter	Training data		Testing data	
	Maximum	Minimum	Maximum	Minimum
Ordinary factor of safety	4.730	1.308	4.071	1.492
Janbu factor of safety	4.699	1.290	4.098	1.471
Bishop factor of safety	4.699	1.365	4.207	1.553
Spencer factor of safety	4.777	1.182	4.112	1.514

the Janbu simplified method outperformed the other three methods in these performance criteria (MSE, NSME, MAE and MAPE) when ANN2 model with five input parameters (β, φ, c, γ, H) is used.

Figure 5 shows a comparison between the calculated and predicted ANN1 and ANN2 FS for the data generated by the four methods (Ordinary, Janbu, Bishop and Spencer) for the test data. It is observed from Fig. 5 that ANN1 model predictions for all methods are very close to the calculated ones as compared to the ANN2 predictions. The scaling and the reduction of the number of parameters in ANN1 resulted in better performance which could be attributed to the number of sufficient data in the solution space of ANN1.

Figure 6 shows a comparison between the accuracy of prediction of ANN1 and ANN2 and the calculated FS based on the four methods for the test data that is used to test the performance of the ANN models. It is clear that the Bishop method showed the best performance among other methods.

Figure 7 shows the error in prediction of factor of safety of all four methods. It is observed that the relative errors of the majority of the ANN predictions are less than 5 %. Specifically, for the Ordinary Method, 96.9 % of the tested data have relative error less than 5 % based on ANN1 prediction while only 78.1 % of the tested data have relative error of 5 % or less based on ANN2 prediction. For the Janbu Method, all the tested data have relative error less than 5 % based on ANN1 prediction while only 81.3 % of the tested data have relative error of 5 % or less based on ANN2 prediction. For the Bishop Method, all the tested data have relative error less than 5 % based on ANN1 prediction while about 81.3.8 % of the tested data have relative error of 5 % or less based on ANN2 prediction. For the Spencer Method, 93.8 % of the tested data have relative error less than 5 % based on ANN1 prediction while about 78.1 % of the tested data have relative error of 5 % or less based on ANN2 prediction. It is clear that the ANN1 and ANN2 prediction based on the Janbu and the Bishop methods are more accurate than the other two and ANN1 prediction is more accurate ANN2.

Case studies

Six case studies were collected from the literature and used to validate the ANN model predictions. The case studies used different analysis methods including the Finite Element Method (FE), the friction circular method (FCM) and the ordinary method of slice (OMS). Table 5 shows the results of validating the ANN model using the six case studies. It is observed from these case studies that the ANN model predictions were very accurate and with MAPE of 3.72 % for all methods combined.

Parametric study

It is evident from the results presented in the previous section that, within the data range of applicability, the ANN models are capable of predicting the FS of clayey soil accurately for the test data that it has not been trained. Therefore, such trained ANN can be use to carry out a parametric study in which the parameters influencing the FS of clayey soil can be varied and the effect of these parameters can be investigated. The Bishop ANN1 model was selected to carry out the parametric study due to its performance and accuracy as compared to others as shown in Table 3. The parameters that were varied to carry out the parametric study are the stability number (m), the angle of internal friction of the clayey soil (φ) and the slope angle (β).

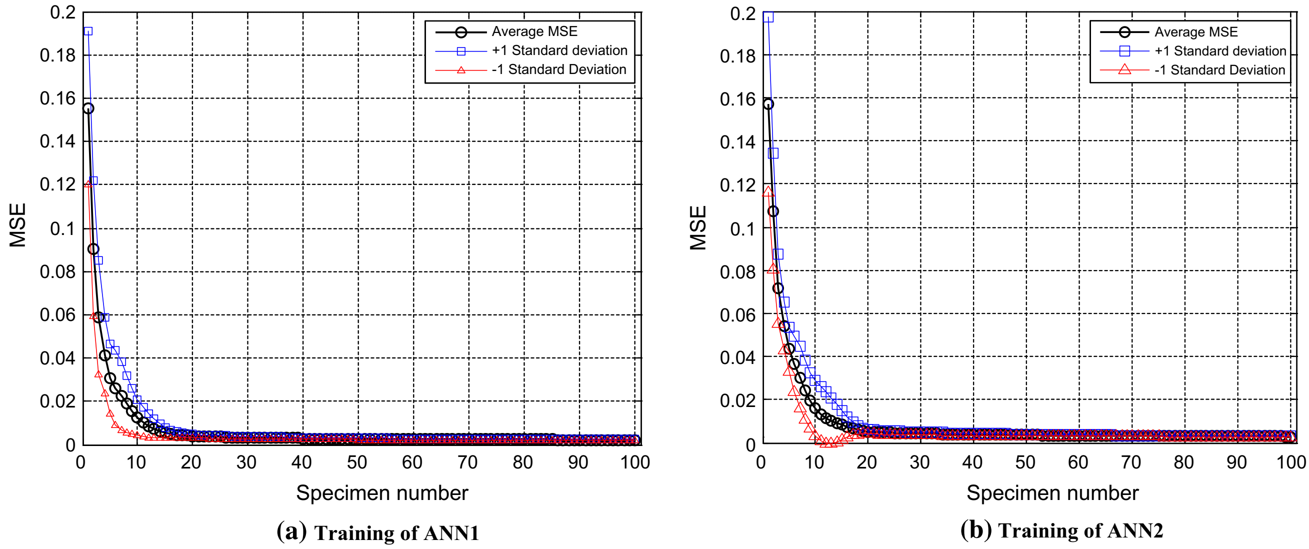


Fig. 4 Learning rate of ANN1 and ANN2—average MSE and ± 1 SD for training set

Table 3 Performance of ANN1 (three input parameters) on the test data

Performance criterion	MSE	NMSE	MAE	MAPE	Minimum absolute error	Maximum absolute error	Correlation coefficient (r)
Ordinary	0.00371	0.00908	0.04396	1.681	0.00061	0.16183	0.99598
Janbu	0.00353	0.00832	0.04595	1.841	0.00235	0.161478	0.99597
Bishop	0.00325	0.00727	0.04056	1.522	0.00032	0.16113	0.99664
Spencer	0.00367	0.00888	0.04413	1.750	0.00098	0.15587	0.99596

Table 4 Performance of ANN2 (five input parameters) on the test data

Performance criterion	MSE	NMSE	MAE	MAPE	Minimum absolute error	Maximum absolute error	Correlation coefficient (r)
Ordinary	0.01538	0.03764	0.07981	3.133	0.00599	0.51483	0.98268
Janbu	0.01402	0.03304	0.07516	2.925	0.00013	0.48333	0.98489
Bishop	0.01512	0.03385	0.08005	3.009	0.001789	0.49042	0.98367
Spencer	0.01670	0.04036	0.08415	3.337	0.00082	0.53245	0.98138

Influence of friction angle (φ) and stability number (m) on FS

Figure 8 shows the variation of the FS of stability of clayey soil with the stability number (m) for different angles of internal friction (φ) of 15°, 20°, 25° and 30° at different values of the slope angle (β). Figure 8a shows the FS for large slope ($\beta = 33.69^{\circ}$), Fig. 8b, c shows the FS for intermediate slopes with ($\beta = 26.573^{\circ}$ and $\beta = 21.80^{\circ}$), while Fig. 8d shows the FS for small slope ($\beta = 18.43^{\circ}$). It is observed in Fig. 8a, b, c and d that the FS increases almost linearly with the increase in the stability number (m). Also the factor of safety increases almost linearly with the increase in the angle of internal friction while it decreases with the increase in the slope angle. For example, for large slope ($\beta = 33.69^{\circ}$) the FS ranges from 1.16 for a small angle of internal friction ($\varphi = 15^{\circ}$) and a small stability number ($m = 0.075$) to 3.50 for a large angle of internal friction ($\varphi = 30^{\circ}$) and a large stability number ($m = 0.275$). However, for small slope angle ($\beta = 18.43^{\circ}$) the FS ranges from 1.87 for a small angle of internal friction ($\varphi = 15^{\circ}$) and small stability number ($m = 0.075$) to 4.84 for a large angle of internal friction ($\varphi = 30^{\circ}$) and a large stability number ($m = 0.275$). For intermediate values of slope angles ($\beta = 26.573^{\circ}$ and $\beta = 21.80^{\circ}$) and at the same values of internal angle of frictions and stability numbers, the FS assumes values in between those of the boundary values.

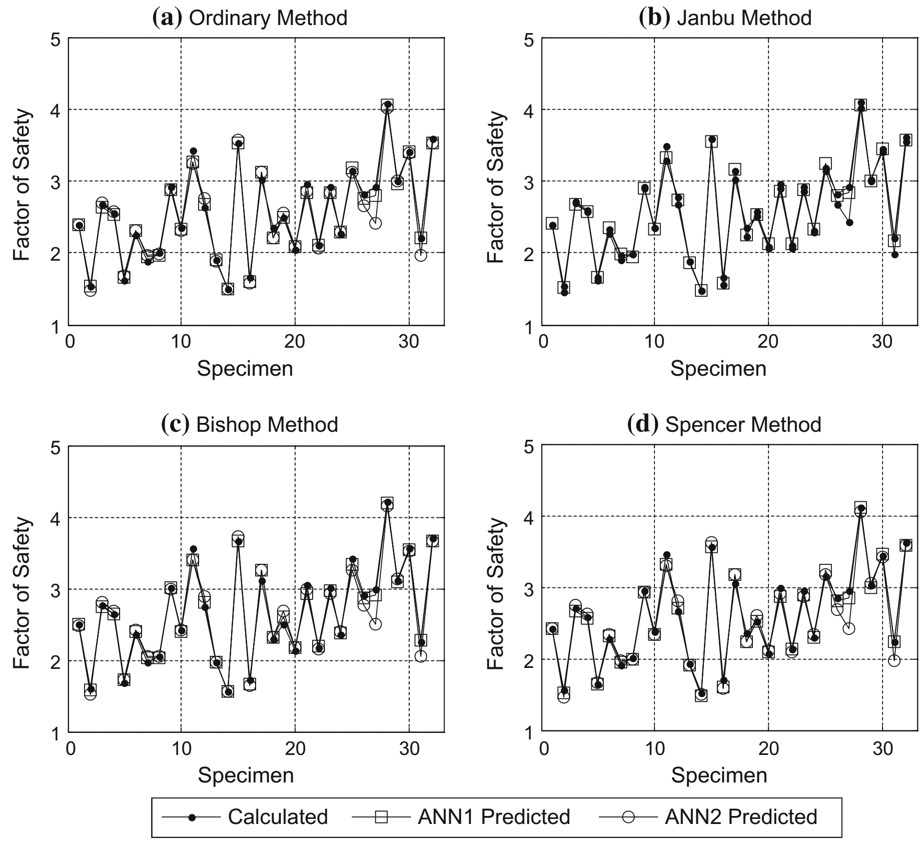


Fig. 5 Prediction of factor of safety (FS) for test data for different methods

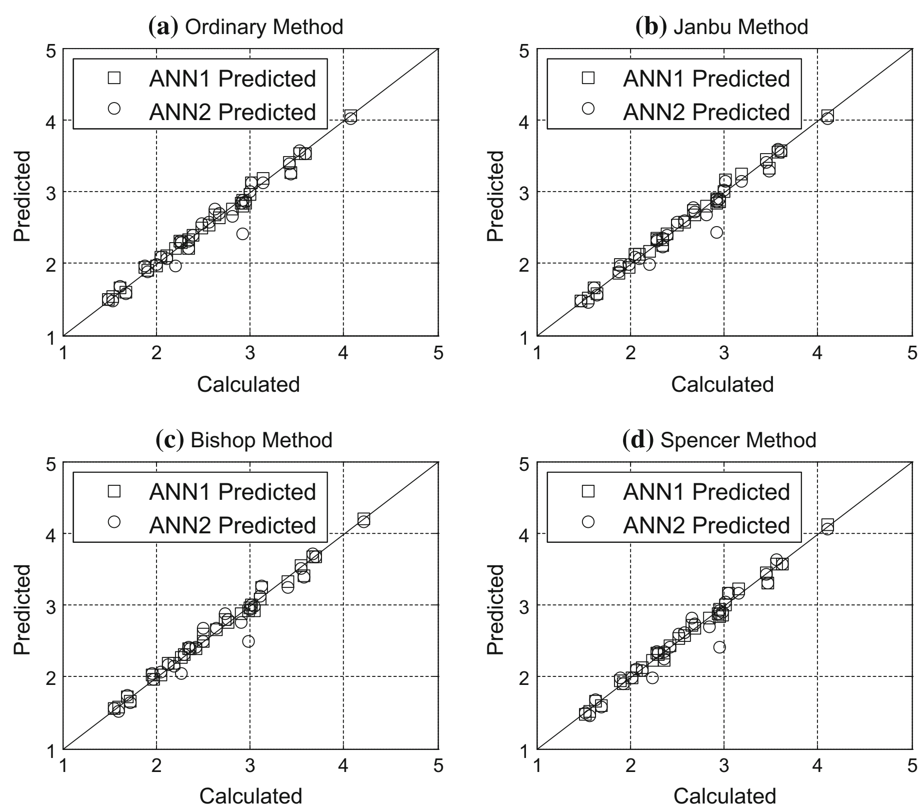


Fig. 6 Accuracy of prediction of factor of safety (FS) for test data for different methods

Fig. 7 Error of prediction of factor of safety (FS) for different methods

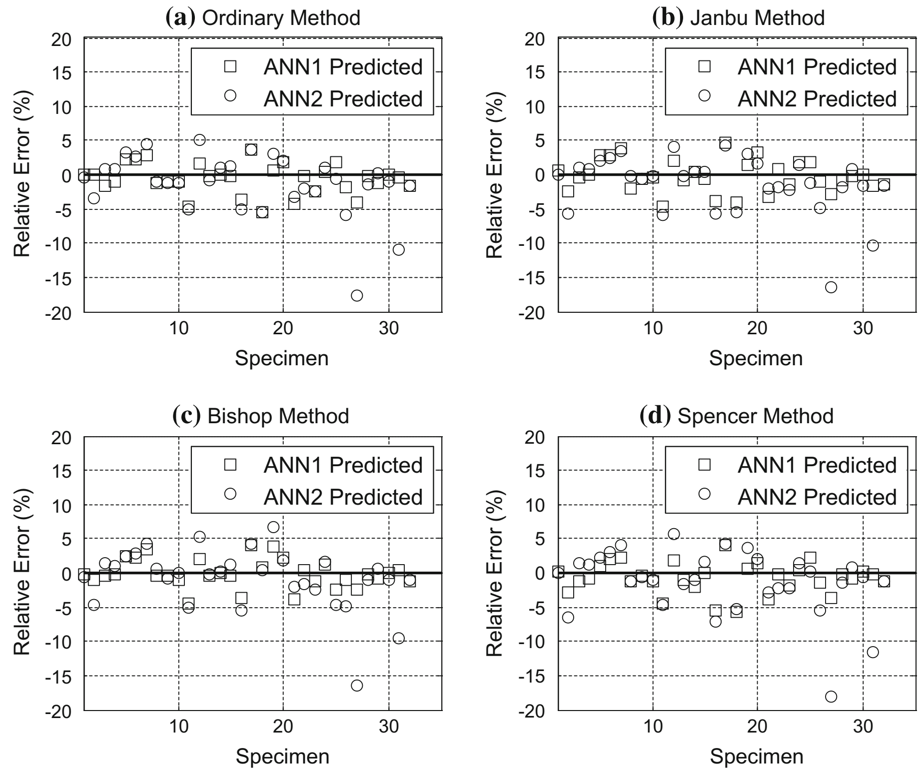


Table 5 Performance of ANN1 (three input parameters) on the case studies data

	$1/\beta$	$m = c/\gamma.H$	φ	Case study prediction FS	ANN Prediction of FS (% error w.r.t. case study)			
					Ordinary	Janbu	Bishop	Spencer
Case 1 (FE) (Chock 2008)	1.00	0.05	30	1.12	1.119 (−0.09 %)	1.036 (−7.5 %)	1.112 (−0.71 %)	1.130 (0.89 %)
Case 2 (FE) (Martins et al. 2011)	2.00	0.037	32	1.75	1.829 (4.52 %)	1.808 (3.31 %)	1.893 (8.17 %)	1.832 (4.69 %)
Case 3 (FCM) (Das 2007)	1.732	0.104	20	1.73	1.645 (−4.91 %)	1.631 (−5.72 %)	1.703 (−1.56 %)	1.632 (−5.66 %)
Case 4 (FCM) (Das 2010)	1.00	0.127	20	1.40	1.354 (−3.29 %)	1.305 (−6.79 %)	1.380 (−1.43 %)	1.354 (−3.29 %)
Case 5 (OMS) (Das 2007)	1.732	0.089	20	1.55	1.528 (−1.42 %)	1.507 (−2.77 %)	1.579 (1.87 %)	1.503 (−3.03 %)
Case 6 (OMS) (Cernica 1982)	1.753	0.143	18	2.00	1.891 (−5.45 %)	1.894 (−5.30 %)	1.965 (−1.75 %)	1.897 (−5.15 %)

FE finite element method; *FCM* friction circular method; *OMS* ordinary method of slice

Influence of friction angle (φ) and slope (β) on FS

Figure 9 shows the variation of the factor of safety of stability of clayey soil with the slope angle (β) for different angle of internal friction (φ) of 15°, 20°, 25° and 30° at different values of the stability number (m). Figure 9a shows the FS for a small stability number ($m = 0.075$), Fig. 9b and 9c shows the FS for intermediate stability numbers ($m = 0.175$ and $m = 0.225$) while Fig. 9d shows the factor of safety for a large stability number ($m = 0.275$). It is observed from Fig. 9a, b, c, d that the FS decreases nonlinearly with the increase in the slope angle (β). Also the FS increases almost linearly with the increase in the angle of internal friction and also with the increase in the stability number. For example, for a small stability number ($m = 0.075$) the FS ranges from 1.16 for a small angle of internal friction ($\varphi = 15°$) and a large slope angle ($\beta = 33.68°$) to 2.97 for a large angle of internal friction ($\varphi = 30°$) and a small slope angle ($\beta = 18.43°$). However, for a large stability number ($m = 0.275$), the FS ranges

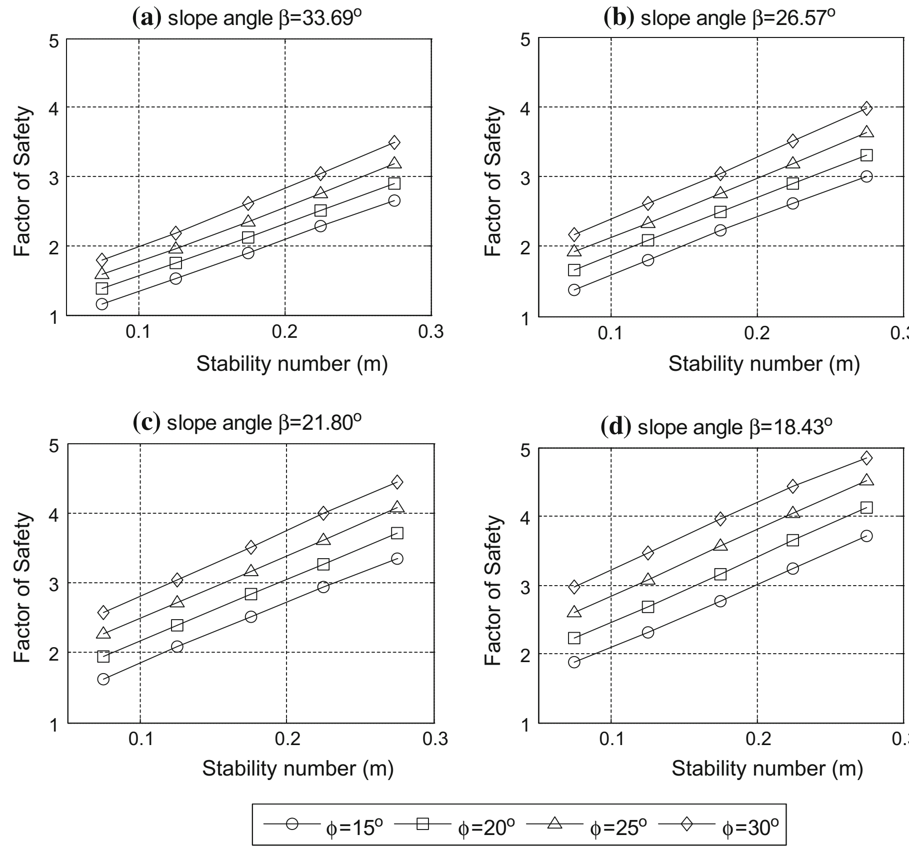


Fig. 8 Variation of factor of safety (FS) with stability number (m) for different angles of friction (φ) at different slope angles (β)

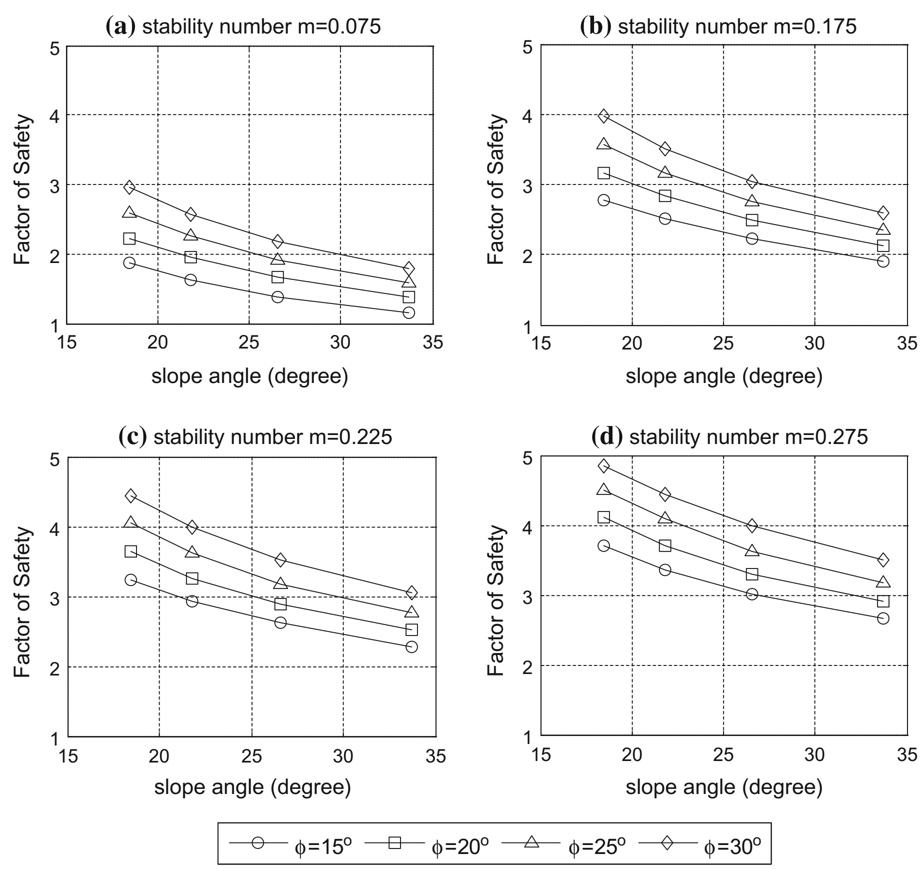


Fig. 9 Variation of factor of safety with slopes angle (β) for different angle of friction (φ) at different stability number (m)

from 2.66 for a small angle of internal friction ($\varphi = 15^{\circ}$) and a large slope angle ($\beta = 33.68^{\circ}$) to 4.84 for a large angle of internal friction ($\varphi = 30^{\circ}$) and a small slope angle ($\beta = 18.43^{\circ}$). For intermediate values of stability numbers ($m = 0.175$ and $m = 0.225$) and at the same values of internal angle of frictions and slope angles, the FS assumes values in between those of the boundary values.

Influence of stability number (m) and slope (β) on FS

Figure 10 shows the variation of the factor of safety of stability of clayey soil with the stability number (m) for different slope angles (β) of 18.4°, 21.80°, 26.573° and 33.69° for clayey soils with different angles of internal friction (φ). Figure 10a shows the FS for clayey soil with a small angle of internal friction ($\varphi = 15^{\circ}$), Fig. 10b, c shows the FS for clayey soils with intermediate angle of internal frictions ($\varphi = 20^{\circ}$ and $\varphi = 25^{\circ}$) while Fig. 10d shows the FS for clayey soil with large angle of internal friction ($\varphi = 30^{\circ}$). It is observed from Figs. 10a, b, c and d that the FS increases almost linearly with the increase in the stability number. Also the factor of safety decreases with the increase in the slope angle while it increases almost linearly with the increase in the angle of internal friction. For example, for small angle of internal friction ($\varphi = 15^{\circ}$) the FS ranges from 1.16 for a large slope angle ($\beta = 33.69^{\circ}$) and a small stability number ($m = 0.075$) to 3.71 for a small slope angle ($\beta = 18.43^{\circ}$) and a large stability number ($m = 0.275$). However, for a large angle of internal friction ($\varphi = 30^{\circ}$) the FS ranges from 1.78 for a large slope angle ($\beta = 33.69^{\circ}$) and small stability number ($m = 0.075$) to 4.84 for a small slope angle ($\beta = 18.43^{\circ}$) and a large stability number ($m = 0.275$). For intermediate values of internal angles of friction ($\varphi = 20^{\circ}$ and $\varphi = 25^{\circ}$) and at the same values of slope angles and stability numbers, the FS assumes values in between those of the boundary values.

Soils with small internal angle of friction and low cohesion

The trained ANN has been used to predict factor of safety of soils with low values of internal angle of friction (5°, 7.5 °, 10 °, 12.5 °) and low cohesion values close to zero, i.e., with stability number ranging from 0.03 to 0.07 and different slope angles (β) of 12 °, 14°, 16° and 18°. Figure 11 shows the variation of the factor of safety with the stability number (m) and slopes angle (β) for different internal angle of friction (φ) for such extreme soils that represent unfavorable conditions. It is observed that for very small stability number ($m = 0.03$) (cohesion) and very small internal angle of friction ($\varphi = 5^{\circ}$) (Fig. 11a), the FS

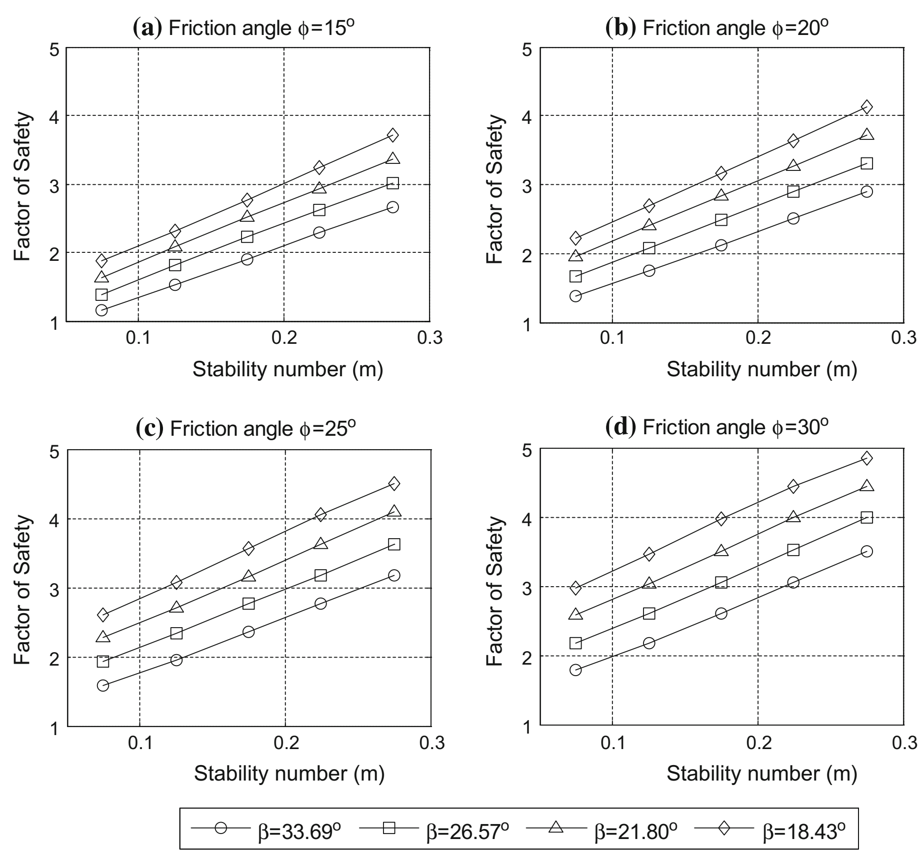


Fig. 10 Variation of factor of safety with the stability number (m) for different slopes angle (β) at different angle of friction (φ)

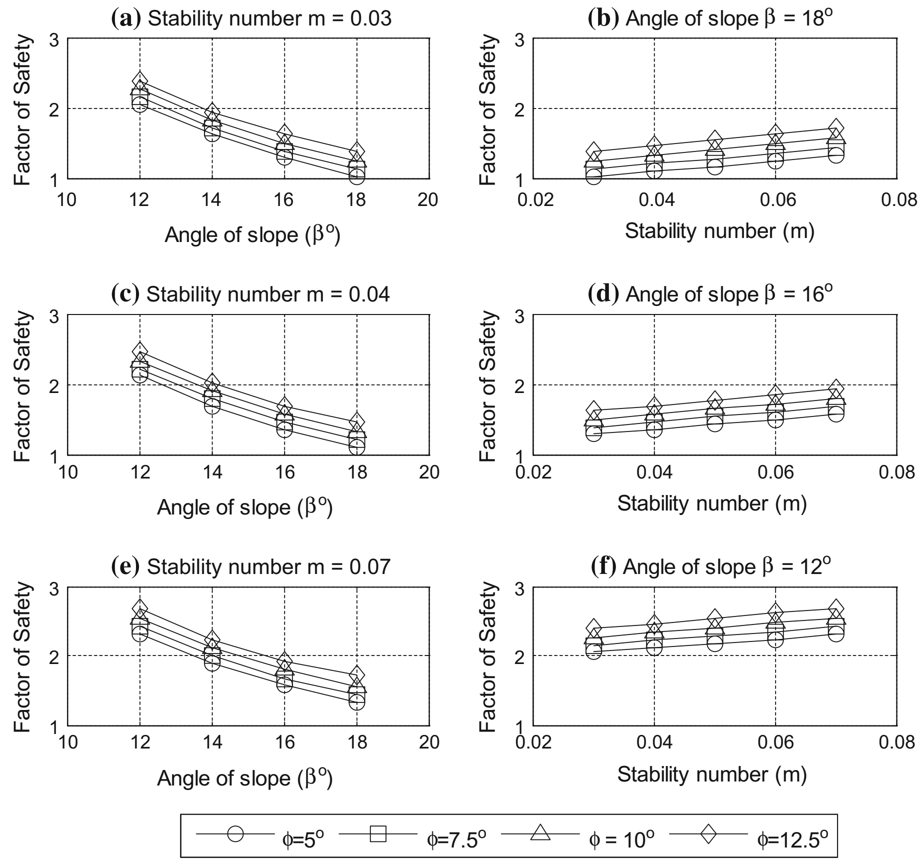


Fig. 11 Variation of factor of safety with the stability number (m) and slopes angle (β) for different angle of friction (φ) for extreme soils

becomes very small and approaches one as the slope angles increases. When the slope angle exceeds 18^{o}, the FS drops below one. In this case, the driving forces increase and exceed the shear resistance of the soil and accordingly, failure will take place.

Summary and conclusion

This paper employed ANN to predict the factor of safety of clayey soil based on four widely used methods known as the Fellenius (ordinary), Bishop, Janbu, and Spencer methods, respectively. Two models of ANN were used with different input parameters. It is observed that predictions of factor of safety by both ANN models are very close to the factor of safety calculated by each of the corresponding four widely methods. It is concluded that such ANN models are reliable, simple and valid computational tools for predicting the factor of safety and also for assessing the stability of slopes of clayey soil. Accordingly, a parametric study was carried out based on the developed ANN model, to investigate the influence of the slope angle (β), the stability number (m) and the angle of internal friction (φ) on the factor of safety of slopes of clayey soil. The following observations and conclusion are made from the results of this study;

1. ANN1 model that uses three input parameters, namely the stability number, slope angle and angle of internal friction performed better than ANN2 model that uses five parameters as input. The scaling and the reduction of the number of parameters in ANN1 resulted in better performance. This could be attributed to the number of sufficient data in the solution space of ANN1.
2. The NMSE for ANN1 are very small and they ranged from 0.0073 to 0.0091 for the four methods, and ranged from 0.0330 to 0404 for ANN2. Likewise, the MAPE is also very small and they ranged from 1.52 to 1.84 % for ANN1 and ranged from 2.93 to 3.34 % for ANN2. The correlation coefficients for all four methods ranged from 0.996 to 0.997 for ANN1 and ranged from 0.981 to 0.985 for ANN2.
3. Although the performance of ANN1 based on the four different methods are very close to each other, however, the best performance was that obtained by the model based on Bishop calculated factor of safety with NMSE = 0.0073, MAPE = 1.52 % and a correlation coefficient $r = 0.997$.

4. Also, although the performance of ANN2 based on the four different methods are very close to each other, however, the best performance was that obtained by the model based on Janbu calculated factor of safety with NMSE $= 0.0330$, MAPE $= 2.93$ % and a correlation coefficient $r = 0.985$.
5. From the parametric study, it is concluded that the factor of safety is almost linearly related to the stability number for different values of slope angle (β) and angle of friction (φ).
6. From the parametric study, it is concluded that the factor of safety of slope decreased significantly if the clayey soil has small angle of internal friction and low cohesion. These small values of shear strength parameters cannot produce enough shear resistance to encounter the driving forces of the slope and thus failure will take place.

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