

Non-Darcian flow toward a larger-diameter partially penetrating well in a confined aquifer

Zhang Wen · Kai Liu · Hongbin Zhan

Received: 20 October 2013 / Accepted: 15 May 2014 / Published online: 3 June 2014
© Springer-Verlag Berlin Heidelberg 2014

Abstract In this study, non-Darcian flow to a larger-diameter partially penetrating well in a confined aquifer was investigated. The flow in the horizontal direction was assumed to be non-Darcian and described by the Izbash equation, and the flow in the vertical direction was assumed to be Darcian. A linearization procedure was used to approximate the nonlinear governing equation. The Laplace transform associated with the finite cosine Fourier transform was used to solve such non-Darcian flow model. Both the drawdowns inside the well and in the aquifer were analyzed under different conditions. The results indicated that the drawdowns inside the well were generally the same at early times under different conditions, and the features of the drawdowns inside the well at late times were similar to those of the drawdowns in the aquifer. The drawdown in the aquifer for the non-Darcian flow case was larger at early times and smaller at late times than their counterparts of Darcian flow case. The drawdowns for a partially penetrating well were the same as those of a fully penetrating well at early times, and were larger than those for a fully penetrating well at late times. A longer well screen resulted in a smaller drawdown in the aquifer at late times. A larger power index n in the Izbash equation resulted in a larger drawdown in the aquifer at early times and led to a smaller drawdown in the aquifer at late times. A larger well radius led to a smaller drawdown at early times, but it had

little impact on the drawdown at late times. The wellbore storage effect disappears earlier when n is larger.

Keywords Non-Darcian flow · Wellbore storage · Partially penetrating well · Laplace transform · Finite cosine Fourier transform

Introduction

Pumping test is one of the most important investigation ways to estimate aquifer parameters in the hydrogeology (Barker and Herbert 1982; Wang et al. 2013; Li et al. 2014). Because of the large storage capacity, large-diameter pumping wells have been commonly used all over the world especially in developing countries such as China and India (Wang et al. 2005; Sushil 2006). The hydraulics of groundwater flow to a larger-diameter well might be somewhat different from that of flow to an infinitesimal well because the pumping rate mainly comes from the storage of the well at early times for a large-diameter pumping well. In the past few decades, groundwater flow to an infinitesimal well has been extensively investigated (Theis 1935; Hantush and Jacob 1955; Hantush 1959; Neuman and Witherspoon 1969). The first mathematical model for unsteady groundwater flow to an infinitesimal pumping well was the well-known Theis model (Theis 1935). Later, numerous models had been established for different aquifer systems (Hantush and Jacob 1955; Hantush 1959; Neuman and Witherspoon 1969; Salcedo et al. 2013; Yeh and Chang 2013). For instance, Hantush and Jacob (1955) investigated the groundwater flow to an infinitesimal well in a leaky aquifer. Boulton (1954) developed a mathematical model for groundwater flow to a pumping well in an unconfined aquifer. Meanwhile,

Z. Wen (✉) · K. Liu · H. Zhan
School of Environmental Studies, China University of
Geosciences, Wuhan 430074, Hubei, People's Republic of China
e-mail: wenz@cug.edu.cn

H. Zhan
Department of Geology and Geophysics, Texas A&M
University, College Station, TX 77843-3115, USA
e-mail: zhan@geos.tamu.edu

research on flow to a finite-diameter pumping well had also been reported (Papadopoulos and Cooper 1967; Lai and Su 1974; Fenske 1977; Sen 1982; Patel and Mishra 1983; Cimen 2001).

All the above-mentioned studies were based on the assumption that the flow was Darcian. However, flow could be non-Darcian under certain circumstances when the pumping rate is relatively large (Sen 1990; Wen et al. 2008a, 2009), meaning that the relationship between the specific discharge and hydraulic gradient was no longer linear. Non-Darcian flow can be classified into two general types, i.e., pre-linear flow and post-linear flow. When the specific discharge (or the hydraulic gradient) is less than a critical value and the linear relationship between the specific discharge and the hydraulic gradient becomes invalid, the flow can be regarded as pre-linear. On the other hand, if the specific discharge (or the hydraulic gradient) is larger than a critical value and such a linear relationship is also invalid, the flow can be regarded as post-linear. The pre-linear flow often happens in petroleum engineering or groundwater flow in silt–clay aquitards because of the low flow velocities, while the post-linear flow often happens in fractured media (Kolditz 2001) or near the pumping well because of the high velocities of flow (Sen 1987, 1989).

During the past few decades, non-Darcian flow to a pumping well has caught the attention of many investigators. However, progress on non-Darcian flow is still very limited because of the nonlinearity nature of the problem. Sen (1987, 1989, 1990) might be one of the first hydrologists to investigate non-Darcian flow to a pumping well. The methodology used by him was one type of the similarity methods, called the Boltzmann transform. The basic idea of the Boltzmann transform was to convert the partial differential equation of unsteady groundwater flow to an ordinary differential equation by combining the spatial and the temporal coordinates into a single Boltzmann variable (Özisik 1989). For example, Sen (1987) used the Boltzmann transform to solve non-Darcian flow to a pumping well in a confined aquifer based on the assumption that the non-Darcian flow can be described by the Izbash equation. Sen (1990) also used the Boltzmann transform to solve non-Darcian flow to a large-diameter well on the basis of the Forchheimer equation. Although Sen used Boltzmann transform to solve non-Darcian flow to a pumping well under different conditions and a series of papers had been published, the Boltzmann transform had been proven to be problematic for some problems (Mathias et al. 2008; Wen et al. 2008a). In a rigorous sense, the Boltzmann transform was only applicable for an infinitesimal pumping well. It may bring in errors when the pumping well radius was not infinitesimal. Especially, when the wellbore storage was considerable, the errors associated with such a transform might be unacceptable. Recently, Wen et al. (2008a, b) developed a linearization procedure which can deal with the nonlinear

term in the governing equation for non-Darcian flow successfully. The linearization procedure worked generally well when the pumping time was relatively large meaning that the flow approaches a quasi-steady state, while it would underestimate the drawdown at early times (Wen et al. 2009). As mentioned before, the solutions obtained by the Boltzmann transform and the linearization procedure were approximate analytical solutions. It is very difficult, if not impossible, to obtain closed-form analytical solutions for non-Darcian flow problems because of the nonlinear nature of the problem. Meanwhile, numerical approaches had also been used to solve non-Darcian flow problems (Ewing et al. 1999; Mathias et al. 2008; Wen et al. 2009).

After a careful check on the literature, one can find that most of the studies on non-Darcian flow to a pumping well focus on the fully penetrating wells. Research on non-Darcian flow to a partially penetrating well was limited. In fact, partially penetrating wells were commonly used in the real-field applications, especially in the site where the thickness of the aquifer was large (Shu et al. 2013). Wen et al. (2013) firstly investigated non-Darcian flow toward a partially penetrating well in a confined aquifer, and some approximate analytical solutions were obtained. In addition, the wellbore storage should also be considered when the radius of the well is relatively large. Therefore, it was necessary and interesting to investigate the non-Darcian flow to a larger-diameter partially penetrating well. This study was an extension of a previous study (Wen et al. 2013), which did not consider the wellbore storage. The main purpose of this study was to analyze the impact of the wellbore storage under non-Darcian flow and partially penetrating condition.

In this study, non-Darcian flow to a partially penetrating well was investigated taking into account the wellbore storage. Both the Izbash equation and the Forchheimer equation describing the non-Darcian flow were used widely. The Izbash equation was used in this study, and the Forchheimer equation will be investigated separately elsewhere. The Izbash equation had been proven to describe non-Darcian flow very well in coarse porous media (Bordier and Zimmer 2000; Moutsopoulos et al. 2009). The linearization procedure associated with the Laplace transform and the finite cosine Fourier transform were used to solve the mathematical model. The drawdowns inside the well and in the aquifer had been thoroughly analyzed for different possible field cases.

Problem statement and semi-analytical solutions

Problem statement

The schematic diagram of the problem investigated here is shown in Fig. 1. In order to simplify the flow system and

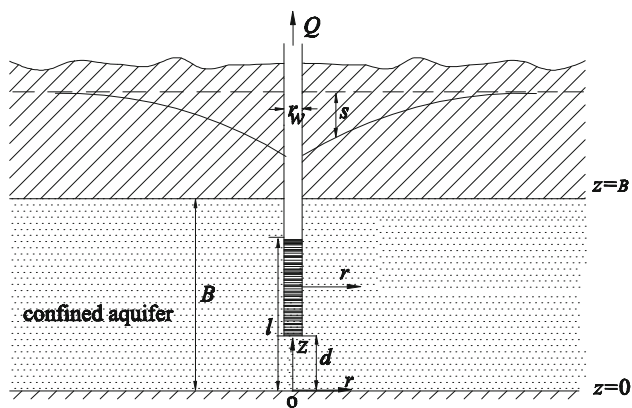


Fig. 1 Schematic diagram of the flow system

make the problem mathematically tractable, the following assumptions have been used in this study: (1) the aquifer is homogenous, horizontally isotropic, and laterally infinite with a constant thickness; (2) the well partially penetrates the aquifer, and the radius of the well is relatively large and equal to r_w , meaning that the wellbore storage cannot be ignored; (3) the pumping rate is constant and equal to Q (positive for pumping); (4) the whole system is hydrostatic before the pumping starts; (5) the horizontal flow is assumed to be non-Darcian, while the flow in the vertical direction is assumed to be Darcian (the dominating flow component is generally along the horizontal direction, supplemented by a secondary vertical flow component close to the well. The vertical flow component drops quickly when moving away from the well). Based on these assumptions, the flow mathematical model can be established:

$$\frac{\partial q_r}{\partial r} + \frac{q_r}{r} + \frac{\partial q_z}{\partial z} = S_s \frac{\partial s(r, z, t)}{\partial t}, \tag{1}$$

$$s(r, z, 0) = 0, \tag{2}$$

$$s(\infty, z, t) = 0, \tag{3}$$

$$\frac{\partial s(r, 0, t)}{\partial z} = 0, \tag{4}$$

$$\frac{\partial s(r, B, t)}{\partial z} = 0, \tag{5}$$

$$\lim_{r \rightarrow r_w} 2\pi r(l-d)q_r - \pi r_w^2 \frac{ds_w(t)}{dt} = -Q[U(z-d) - U(z-l)], \quad 0 \leq z \leq B, \tag{6}$$

where q_r is the radial specific discharge [m/h]; q_z is the vertical specific discharge [m/hr]; r is the radial coordinate [m]; z is the vertical coordinate [m]; $s(r, z, t)$ is the drawdown [m]; S_s is the specific storage coefficient [m^{-1}]; Q is the pumping rate [m^3/h]; B is the thickness of the aquifer [m]; r_w is the radius of the pumping well [m]; $s_w(t)$ is the drawdown inside the well [m]; l is the distance from the top

of the well screen to the bottom of the aquifer [m]; d is the distance from the bottom of the well screen to the bottom of the aquifer [m]; $U(\cdot)$ is the Heaviside step function, $U(z-d)$ is zero when z is less than d , otherwise $U(z-d)$ equals one. The origin of coordinates is located at the intercept of the vertical axis through the center of the well and the bottom of the aquifer.

The flow in the radial direction is assumed to be non-Darcian and can be described by the Izbash equation, that is,

$$q_r |q_r|^{n-1} = K_r \frac{\partial s(r, z, t)}{\partial r}, \tag{7}$$

where n is a power index, K_r is an empirical coefficient [m^n/h^n], and it can be regarded as an apparent radial hydraulic conductivity when n is 1. As q_r is negative, which is the case for a pumping scenario, the above equation can be rewritten as:

$$-|q_r|^n = -(-q_r)^n = K_r \frac{\partial s(r, z, t)}{\partial r} \tag{8}$$

The flow in the vertical direction is assumed to be Darcian described as:

$$q_z = K_z \frac{\partial s(r, z, t)}{\partial z}, \tag{9}$$

where K_z is the vertical hydraulic conductivity [m/h].

Approximate analytical solutions in Laplace–Fourier domain

Substituting Eqs. (8) and (9) to Eq. (1), one can obtain

$$\frac{K_r}{n(-q_r)^{n-1}} \frac{\partial^2 s(r, z, t)}{\partial r^2} + \frac{K_r}{r(-q_r)^{n-1}} \frac{\partial s(r, z, t)}{\partial r} + K_z \frac{\partial^2 s(r, z, t)}{\partial z^2} = S_s \frac{\partial s(r, z, t)}{\partial t} \tag{10}$$

The above equation has a nonlinear term $(-q_r)^{n-1}$, which is very difficult to handle analytically. Fortunately, Wen et al. (2008a) proposed a linearization procedure which can be used to solve such nonlinear equation successfully. In this study, the linearization procedure was also used to deal with the nonlinear term in the above equation. According to Wen et al. (2008a), one has the following approximation:

$$(-q_r)^{n-1} \approx \left(\frac{Q}{2\pi r B} \right)^{n-1}. \tag{11}$$

Then, Eq. (10) can be transformed as:

$$\frac{\partial^2 s(r, z, t)}{\partial r^2} + \frac{n}{r} \frac{\partial s(r, z, t)}{\partial r} + \frac{K_z n}{K_r} \left(\frac{Q}{2\pi r B} \right)^{n-1} \frac{\partial^2 s(r, z, t)}{\partial z^2} \approx \frac{n}{K_r} \left(\frac{Q}{2\pi r B} \right)^{n-1} S_s \frac{\partial s(r, z, t)}{\partial t}. \tag{12}$$

Eq. (12) indicates that the flow rate at any cylindrical cross section is equal to the pumping rate Q . It should be pointed out that this is exactly true only at the center of the pumping well (the inner boundary). Obviously, for the area far away from the pumping well, the flow rate at the cylindrical cross section is less than the pumping rate Q . It has been proven that such a linearization procedure will underestimate the drawdown at early times and it works generally well at late times (Wen et al. 2009).

Eq. (12) can be solved by the Laplace transform associated with the finite cosine Fourier transform along the z -direction. When considering the initial and boundary conditions, the final solution for Eq. (12) in Laplace domain can be expressed as:

$$\bar{s}(r, z, p) = \frac{Q(l-d)r^{\frac{1-n}{2}}K_{\frac{1-n}{3-n}}\left(\frac{2}{3-n}r^{\frac{3-n}{2}}\sqrt{\delta'}\right)}{Bp\left\{\frac{[2\pi r_w(l-d)]^n}{Q^{n-1}}K_r r_w^{1-n}\sqrt{\delta'}K_{\frac{2}{3-n}}\left(\frac{2}{3-n}r_w^{\frac{3-n}{2}}\sqrt{\delta'}\right) + \pi r_w^{\frac{5-n}{2}}pK_{\frac{1-n}{3-n}}\left(\frac{2}{3-n}r_w^{\frac{3-n}{2}}\sqrt{\delta'}\right)\right\}} + \frac{2Q}{p\pi} \sum_{N=1}^{\infty} \frac{[\sin(\frac{N\pi l}{B}) - \sin(\frac{N\pi d}{B})]r^{\frac{1-n}{2}}K_{\frac{1-n}{3-n}}\left(\frac{2}{3-n}r^{\frac{3-n}{2}}\sqrt{\delta}\right)\cos(\frac{N\pi z}{B})}{N\left\{\frac{[2\pi r_w(l-d)]^n}{Q^{n-1}}K_r r_w^{1-n}\sqrt{\delta}K_{\frac{2}{3-n}}\left(\frac{2}{3-n}r_w^{\frac{3-n}{2}}\sqrt{\delta}\right) + \pi r_w^{\frac{5-n}{2}}pK_{\frac{1-n}{3-n}}\left(\frac{2}{3-n}r_w^{\frac{3-n}{2}}\sqrt{\delta}\right)\right\}} \tag{13}$$

$$\bar{s}(r, p) = \frac{Qr^{\frac{1-n}{2}}K_{\frac{1-n}{3-n}}\left(\frac{2}{3-n}r^{\frac{3-n}{2}}\sqrt{\delta'}\right)}{\frac{(2\pi r_w B)^n}{Q^{n-1}}K_r p r_w^{1-n}\sqrt{\delta'}K_{\frac{2}{3-n}}\left(\frac{2}{3-n}r_w^{\frac{3-n}{2}}\sqrt{\delta'}\right) + \pi p^2 r_w^{\frac{5-n}{2}}K_{\frac{1-n}{3-n}}\left(\frac{2}{3-n}r_w^{\frac{3-n}{2}}\sqrt{\delta'}\right)} \tag{15}$$

in which, $\delta' = \frac{S_p p n}{K_r} \left(\frac{Q}{2\pi B}\right)^{n-1}$, $\delta = \frac{B^2 S_p p n + K_r n N^2 \pi^2}{K_r B^2} \left(\frac{Q}{2\pi B}\right)^{n-1}$. N is the finite cosine Fourier transform variable. p is Laplace transform variable, $K_{\frac{1-n}{3-n}}$ and $K_{\frac{2}{3-n}}$ are the second kind of Bessel functions with order $\frac{1-n}{3-n}$ and $\frac{2}{3-n}$, respectively. The details for the derivation of the solution can be found in the “Appendix.”

Simplified solutions

If the power index n in the Izbash equation happens to be 1, the Izbash equation then becomes Darcy’s law. For such a special case, the solution of Eq. (12) becomes

$$\bar{s}(r, z, p) = \frac{Q(l-d)K_0\left(r\sqrt{\frac{S_p p}{K_r}}\right)}{\pi p B r_w \left[2(l-d)K_r \sqrt{\frac{S_p p}{K_r}} K_1\left(r_w \sqrt{\frac{S_p p}{K_r}}\right) + p r_w K_0\left(r_w \sqrt{\frac{S_p p}{K_r}}\right)\right]} + \frac{2Q}{p\pi^2 r_w} \sum_{N=1}^{\infty} \frac{[\sin(\frac{N\pi l}{B}) - \sin(\frac{N\pi d}{B})]K_0(r\sqrt{\delta''})\cos(\frac{N\pi z}{B})}{N[2(l-d)K_r \sqrt{\delta''} K_1(r_w \sqrt{\delta''}) + p r_w K_0(r_w \sqrt{\delta''})]} \tag{14}$$

where $\delta'' = \frac{B^2 S_p p + K_r N^2 \pi^2}{K_r B^2}$. Equation (14) is the same as the solution obtained by Mishra et al. (2012) who investigated Darcian flow to a partially penetrating well with storage in an anisotropic confined aquifer.

If $l = B$ and $d = 0$, then the partially penetrating well becomes a fully penetrating well, and the solution of Eq. (12) becomes

Equation (15) is the same as the solution obtained by Wen et al. (2009) who investigated non-Darcian flow to a fully penetrating well with considering wellbore storage.

If $l = B$, $d = 0$, and $n = 1$, the problem investigated here is the same as that of Papadopoulos and Cooper (1967). Then, Eq. (13) can be simplified as:

$$\bar{s}(r, z, p) = \frac{QK_0\left(r\sqrt{\frac{S_p p}{K_r}}\right)}{\pi p r_w \left[2BK_r \sqrt{\frac{S_p p}{K_r}} K_1\left(r_w \sqrt{\frac{S_p p}{K_r}}\right) + p r_w K_0\left(r_w \sqrt{\frac{S_p p}{K_r}}\right)\right]} \tag{16}$$

which is the solution obtained by Papadopoulos and Cooper (1967).

Results and discussions

Drawdowns inside the well

For a large-diameter well, it is very interesting to analyze the drawdown inside the well because the pumping rate is mainly from the wellbore storage at early times.

Firstly, this solution was compared with some benchmark analytical solutions, i.e., the solution of Papadopoulos and Cooper (1967) for Darcian flow to a fully penetrating well with wellbore storage, the solution of Wen et al. (2008a, b) for non-Darcian flow to a fully penetrating well with wellbore storage, and the solution of Mishra et al. (2012) for Darcian flow to a partially penetrating well with wellbore storage. The default values of parameters are given in Table 1. As shown in Fig. 2, it can be seen that all the curves for different cases converge to the same asymptotic value at early times, meaning that all the pumping rate comes from the water stored inside the well. At late times, the drawdown for Darcian flow case is larger than that for non-Darcian flow case, and the drawdown for the partially penetrating case is larger than that for the fully penetrating case, if the other conditions are the same. The impacts of different parameters on the drawdown inside the well have also been analyzed and one found that the early-time behavior was the same as that reflected in Fig. 2, while the late-time behavior is the same as those of the drawdown in the aquifer, which will be explained in more detail below.

Drawdowns in the aquifer

Figure 3 shows the comparison between the drawdown calculated at a radial distance of 5 m from the well, using

Table 1 Default value of parameters used in this study

Parameter name	Symbol	Default value
Pumping rate	Q	50 m ³ /h
Non-Darcian factor	n	1.5
Radial hydraulic conductivity	K_r	0.1 m/h
Vertical hydraulic conductivity	K_z	0.01 m/h
Specific storage coefficient	S_s	0.0001 m ⁻¹
Thickness of the aquifer	B	20 m
Radius of the pumping well	r_w	1 m
Radial coordinate	r	5 m
Vertical coordinate	z	10 m
Distance from the bottom of the well screen to the bottom of the aquifer	d	5 m
Distance from the top of the well screen to the bottom of the aquifer	l	15 m

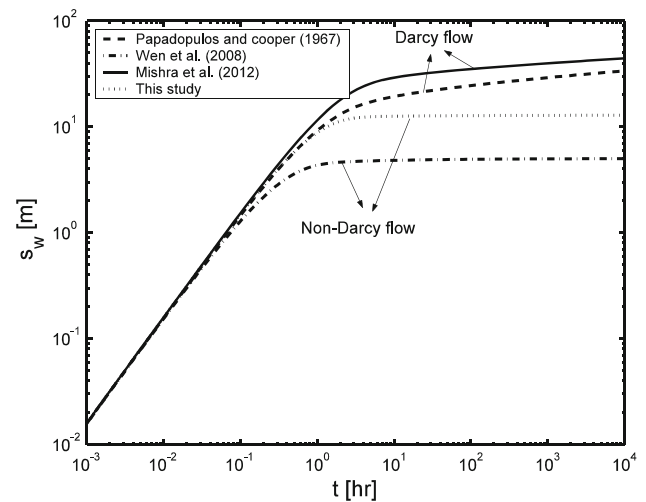


Fig. 2 Comparison between the solution for the drawdown inside the well of this study and those of the previous studies

the model developed in this study and model that have been analyzed in Fig. 2. The default values of parameters are given in Table 1. The late-time behavior shown in this figure is the same as that in Fig. 2. However, at early times, the drawdown for the non-Darcian flow case is larger than that of the Darcian flow case. It is also found that the drawdown for a partially penetrating well is the same as that of a fully penetrating well at early times.

Figure 4 shows the impact of n value on the drawdown in the aquifer with $n = 1.0, 1.2, 1.5,$ or 1.8 . Other default values of parameters are given in Table 1. From this figure, it is found that a larger value of n leads to a larger drawdown at early times, while a larger value of n will lead to a smaller drawdown at late times. A larger n means greater deviation from Darcian flow. For early times, a larger n implies greater resistance to flow; therefore, a larger drawdown was found. For the late times, the flow approaches a quasi-steady state. The elastic release of the aquifer is nearly completed, and the pumping rate is mainly from the recharge from the area where it is relatively far away from the pumping well. It can also be found that the flow approach the quasi-steady state earlier when n is larger in this figure. The impact of n value on the drawdown inside the well has also been analyzed, as shown in Fig. 5. It is evident to see that the wellbore storage effect at early times, as shown in this figure, different n values led to the same drawdowns at early times. The late-time behavior is the same as that of Fig. 4, which has been explained before.

In order to analyze the impact of non-Darcian flow on the wellbore storage effect, one can draw the curves for different r_w values with different n values at the same figure. As shown in Fig. 6, one can also see the wellbore storage effect at early times, and a larger n led to a smaller

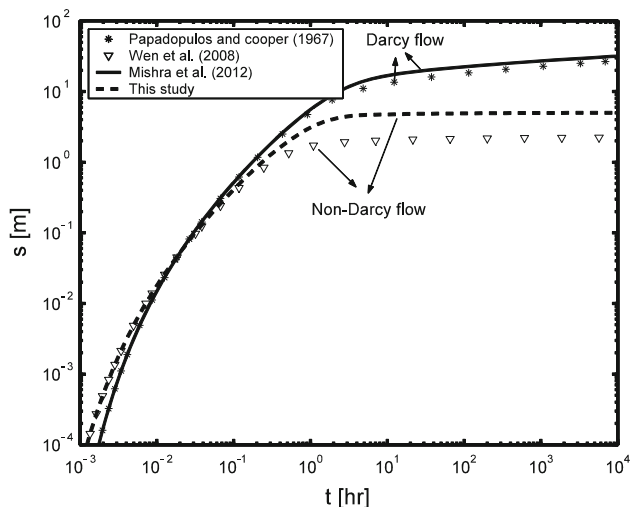


Fig. 3 Comparison between the solution for the drawdown in the aquifer of this study and those of the previous studies

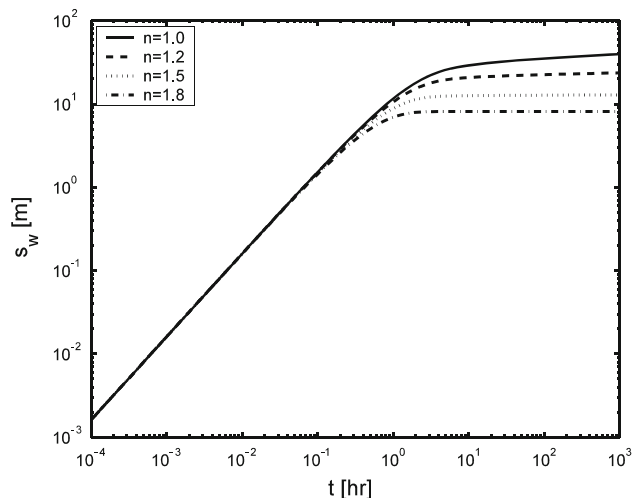


Fig. 5 Impact of n value on the drawdown in the well with $n = 1.0, 1.2, 1.5,$ or 1.8

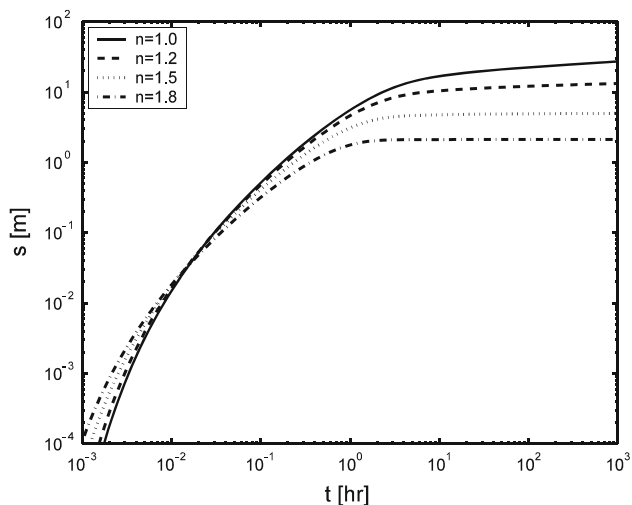


Fig. 4 Impact of n value on the drawdown in the aquifer with $n = 1.0, 1.2, 1.5,$ or 1.8

drawdown at late times. Interesting enough, if comparing the curves for different n values, one can see that the wellbore storage effect disappears earlier when n is larger (the disappearing time is 32.5 h for $n = 1$, 5.0 h for $n = 1.5$, and 2.1 h for $n = 1.8$).

Figure 7 is the comparison between the fully penetrating case and the partially penetrating case with different r_w values. As shown in this figure, the drawdowns for the fully penetrating case are larger than those of the partially penetrating case. This is because a longer well screen can transmit water more easily, which is somewhat like the aquifer has a larger hydraulic conductivity. Thus, a smaller drawdown will be found at late times.

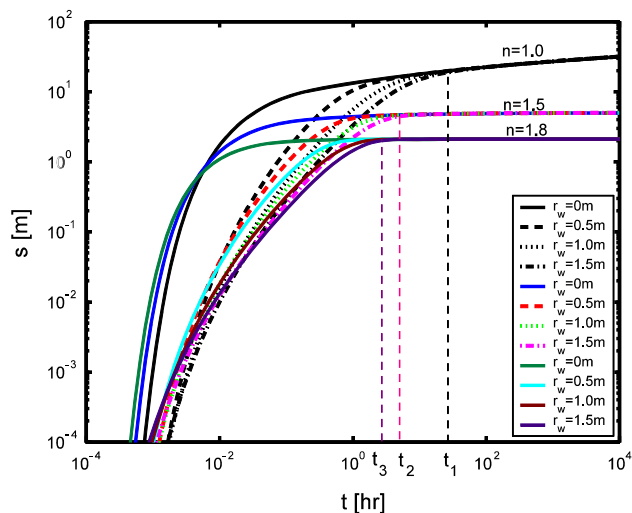


Fig. 6 Impact of different r_w values with different n values on the drawdown in the aquifer

The impact of S_s , K_r , and the length of the well screen $l - d$ on the drawdowns in the aquifer was investigated, and the features were similar to those without considering the wellbore storage (Wen et al. 2013), which will not be repeated here.

Conclusions

In this study, non-Darcian flow to a large-diameter partially penetrating well in a confined aquifer was investigated. The wellbore storage was considered. The horizontal non-Darcian flow was assumed to follow the Izbash equation, while the vertical flow was still assumed to be Darcian. The Laplace transform associated with the finite cosine Fourier

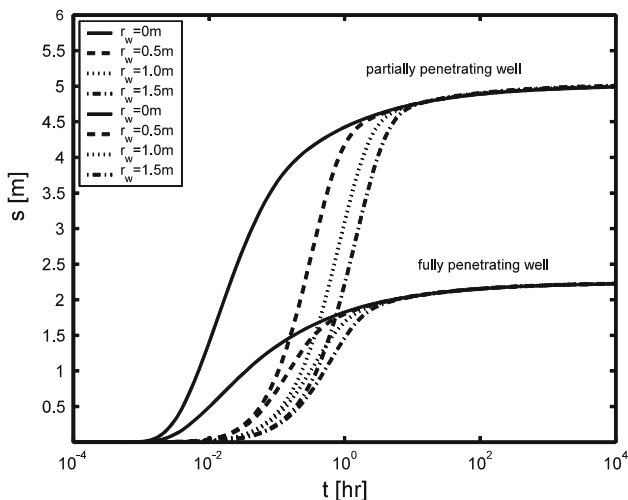


Fig. 7 Impact of different r_w values under partially penetrating pumping well and fully penetrating pumping well on the drawdown in the aquifer

transform was used to solve such non-Darcian flow model. Both the drawdowns inside the well and in the aquifer were analyzed under different conditions. The main findings can be concluded as follows:

1. The drawdowns inside the well are generally the same at early times under different aquifer conditions, indicating the wellbore storage effect. The features of the drawdowns inside the well at late times are the same as those of the drawdowns in the aquifer.
2. The drawdown in the aquifer for the non-Darcian flow case is larger than that of the Darcian flow case at early times, while the drawdown in the aquifer for the non-Darcian flow case is smaller than that of the Darcian flow at late times.
3. The drawdowns for a partially penetrating well are the same as those of a fully penetrating well at early times, while the drawdowns for the partially penetrating case are larger than those for the fully penetrating case at late times, and a longer well screen results in a smaller drawdown in the aquifer at late times, if the other conditions are the same.
4. From the curves for different r_w values with different n values, one can see that a larger n led to a smaller drawdown at late times, and the wellbore storage effect disappears earlier when n is larger.

Acknowledgments This research was partially supported by the National Natural Science Foundation of China (Grant Numbers: 41002082, 41372253), the National Basic Research Program of China (Grant Number: 2010CB428802), and the Special Fund for Basic Scientific Research of Central Colleges, China University of Geosciences (Wuhan) (Grant Numbers: CUG140503, CUG120113).

Appendix: Derivation of the analytical solution in the Laplace domain

Applying Laplace transform to Eq. (12) with considering the initial condition Eq. (2), one can obtain

$$\frac{\partial^2 \bar{s}(r, z, p)}{\partial r^2} + \frac{n}{r} \frac{\partial \bar{s}(r, z, p)}{\partial r} + \frac{K_z n}{K_r} \left(\frac{Q}{2\pi r B} \right)^{n-1} \frac{\partial^2 \bar{s}(r, z, p)}{\partial z^2} \approx \frac{n}{K_r} \left(\frac{Q}{2\pi r B} \right)^{n-1} S_s p \bar{s}(r, z, p), \tag{17}$$

in which p is the Laplace variable, over bar means the variable in Laplace domain. After using the finite cosine Fourier transform to deal with the second-order derivative of z , it can be expressed as:

$$F_c \left(\frac{\partial^2 \bar{s}(r, z, p)}{\partial z^2}, z \rightarrow N \right) = \int_0^B \frac{\partial^2 \bar{s}(r, z, p)}{\partial z^2} \cos \frac{N\pi z}{B} dz = -\frac{N^2 \pi^2}{B^2} \bar{s}(r, N, p) - \frac{\partial \bar{s}(r, 0, p)}{\partial z} - (-1)^N \frac{\partial \bar{s}(r, B, p)}{\partial z} \tag{18}$$

$(N = 0, 1, 2, 3, \dots)$

The over ^sign means the variable in Fourier domain. The vertical boundary equations Eqs. (4) and (5) can be rewritten after the Laplace transform as:

$$\frac{\partial \bar{s}(r, 0, p)}{\partial z} = 0, \tag{19}$$

$$\frac{\partial \bar{s}(r, B, p)}{\partial z} = 0, \tag{20}$$

Substituting Eqs. (19) and (20) to (18), one has

$$F_c \left(\frac{\partial^2 \bar{s}(r, z, p)}{\partial z^2}, z \rightarrow N \right) = -\frac{N^2 \pi^2}{B^2} \bar{s}(r, N, p), \tag{21}$$

$N = 0, 1, 2, \dots$

With the finite cosine Fourier transform, Eq. (17) can be changed to the following equation after some simplifications:

$$\frac{d^2 \bar{s}(r, N, p)}{dr^2} + \frac{n}{r} \frac{d \bar{s}(r, N, p)}{dr} = \frac{n K_z N^2 \pi^2 + B^2 n p S_s}{K_r B^2} \left(\frac{Q}{2\pi r B} \right)^{n-1} \bar{s}(r, N, p). \tag{22}$$

Eq. (22) is a Bessel equation, whose general solution can be expressed as:

$$\bar{s}(r, N, p) = r^{\frac{1-n}{2}} \left[C_1 I_{\frac{1-n}{2}} \left(\frac{2}{3-n} r^{\frac{3-n}{2}} \sqrt{\delta} \right) + C_2 K_{\frac{1-n}{2}} \left(\frac{2}{3-n} r^{\frac{3-n}{2}} \sqrt{\delta} \right) \right], \tag{23}$$

in which $\delta = \frac{n K_z N^2 \pi^2 + B^2 n p S_s}{K_r B^2} \left(\frac{Q}{2\pi B} \right)^{n-1}$, $I_\nu(x)$ and $K_\nu(x)$ are the first and second kinds of modified Bessel functions with

order v , respectively. C_1 and C_2 are constants, which depend on the boundary conditions. The radial boundary conditions Eqs. (3) and (6) can be changed to

$$\bar{s}(\infty, N, p) = 0, \quad (24)$$

and after using the Laplace transform and the finite cosine Fourier transform, one has

$$\begin{aligned} & \frac{2\pi r_w(l-d)K_r d\bar{s}(r_w, N, p)}{\left[\frac{Q}{2\pi r_w(l-d)}\right]^{n-1} dr_w} - \pi r_w^2 p \bar{s}(r_w, N, p) \\ &= -\frac{Q \sin\left(\frac{N\pi l}{B}\right) - \sin\left(\frac{N\pi d}{B}\right)}{p N\pi/B}. \end{aligned} \quad (25)$$

It should be pointed out that the linearization procedure has also been used in Eq. (25). With the consideration of Eq. (24) and the properties of the Bessel functions, one has $C_1 = 0$. Then, the solution of Eq. (23) can be rewritten as:

$$\bar{s}(r, N, p) = C_2 r^{\frac{1-n}{2}} K_{\frac{1-n}{2}} \left(\frac{2}{3-n} r^{\frac{3-n}{2}} \sqrt{\delta} \right). \quad (26)$$

With Eqs. (25) and (26), one can obtain C_2 as:

$$C_2 = \frac{QB \left[\sin\left(\frac{N\pi l}{B}\right) - \sin\left(\frac{N\pi d}{B}\right) \right]}{pN\pi \left\{ \frac{[2\pi r_w(l-d)]^n}{Q^{n-1}} K_r r_w^{1-n} \sqrt{\delta} K_{\frac{3-n}{2}} \left(\frac{2}{3-n} r_w^{\frac{3-n}{2}} \sqrt{\delta} \right) + \pi r_w^{\frac{3-n}{2}} p K_{\frac{1-n}{2}} \left(\frac{2}{3-n} r_w^{\frac{3-n}{2}} \sqrt{\delta} \right) \right\}} \quad (27)$$

It should be pointed out that the following properties of the second kind of modified Bessel functions have been used to obtain C_2 : $x dK_\nu(x)/dx + \nu K_\nu(x) = -xK_{\nu-1}(x)$, $K_\nu(x) = K_{-\nu}(x)$, $K_\nu(x) \approx \frac{\Gamma(\nu)}{2} \left(\frac{x}{2}\right)^{-\nu}$, $\nu > 0, x \rightarrow 0$. Therefore, the solution in Laplace–Fourier domain can be expressed as:

$$\bar{s}(r, N, p) = \frac{QB \left[\sin\left(\frac{N\pi l}{B}\right) - \sin\left(\frac{N\pi d}{B}\right) \right] r^{\frac{1-n}{2}} K_{\frac{1-n}{2}} \left(\frac{2}{3-n} r^{\frac{3-n}{2}} \sqrt{\delta} \right)}{pN\pi \left\{ \frac{[2\pi r_w(l-d)]^n}{Q^{n-1}} K_r r_w^{1-n} \sqrt{\delta} K_{\frac{3-n}{2}} \left(\frac{2}{3-n} r_w^{\frac{3-n}{2}} \sqrt{\delta} \right) + \pi r_w^{\frac{3-n}{2}} p K_{\frac{1-n}{2}} \left(\frac{2}{3-n} r_w^{\frac{3-n}{2}} \sqrt{\delta} \right) \right\}} \quad (28)$$

With inverse Fourier transform, one can obtain the solution in Laplace domain finally:

$$\bar{s}(r, z, p) = \frac{1}{B} \bar{s}(r, 0, p) + \frac{2}{B} \sum_{N=1}^{\infty} \bar{s}(r, N, p) \cos\left(\frac{N\pi z}{B}\right) \quad (29)$$

References

- Barker JA, Herbert R (1982) Pumping tests in patchy aquifer. *Ground Water* 20(2):150–155
- Bordier C, Zimmer D (2000) Drainage equations and non-Darcian modeling in coarse porous media or geosynthetic materials. *J Hydrol* 228(3–4):174–187
- Boulton NS (1954) The drawdown of the water table under nonsteady conditions near a pumped well in an unconfined formation. *Proc Inst Civil Eng* 3(2):564–579
- Cimen M (2001) A simple solution for drawdown calculation in a large-diameter well. *Ground Water* 39(1):144–147

- Ewing RE, Lazarov RD, Lyons SL, Papavassiliou DV, Pasciak J, Qin G (1999) Numerical well model for non-Darcy flow through isotropic porous media. *Comp Geosci* 3(3–4):185–204
- Fenske PR (1977) Radial flow with discharging-well and observation-well storage. *J Hydrol* 32(1–2):87–96
- Hantush MS (1959) Nonsteady flow to flow wells in leaky aquifers. *J Geophys Res* 64(8):1043–1052
- Hantush MS, Jacob CE (1955) Non-steady radial flow in an infinite leaky aquifer. *Am Geophys Union Trans* 36(1):95–100
- Kolditz O (2001) Non-linear flow in fractured rock. *Int J Numer Method Heat Fluid Flow* 11(5–6):547–575
- Lai RY, Su CW (1974) Nonsteady flow to a large well in a leaky aquifer. *J Hydrol* 22(3–4):333–345
- Li PY, Qian H, Wu JH (2014) Determining the optimal pumping duration of transient pumping test for estimating hydraulic properties of leaky aquifers using global curve-fitting method: a simulation approach. *Environ Earth Sci* 71(1):293–299
- Mathias S, Butler A, Zhan HB (2008) Approximate solutions for Forchheimer flow to a well. *J Hydraul Eng* 134(9):1318–1325
- Mishra PK, Vesselinov VV, Neuman SP (2012) Radial flow to a partially penetrating well with storage in an anisotropic confined aquifer. *J Hydrol* 448–449(2):255–259
- Moutsopoulos KN, Papaspyros NE, Tsihrintzis VA (2009) Experimental investigation of inertial flow processes in porous media. *J Hydrol* 374(3–4):242–254
- Neuman SP, Witherspoon PA (1969) Applicability of current theories of flow in leaky aquifers. *Water Resour Res* 5(4):817–829
- Özisk MN (1989) Boundary value problems of heat conduction. Dover, New York
- Papadopoulos IS, Cooper HH (1967) Drawdown in a well of large diameter. *Water Resour Res* 3(1):241–244
- Patel SC, Mishra GC (1983) Analysis of flow to a large-diameter well discrete kernel approach. *Ground Water* 21(5):573–576
- Salcedo ER, Vicenta EM, Garrido HS (2013) Groundwater optimization model for sustainable management of the Valley of Puebla Aquifer, Mexico. *Environ Earth Sci* 70(1):337–351
- Sen Z (1982) Type curves for large-diameter wells near barriers. *Ground Water* 20(3):274–277
- Sen Z (1987) Non-Darcian flow in fractured rocks with a linear flow pattern. *J Hydrol* 92(1–2):43–57
- Sen Z (1989) Nonlinear flow toward wells. *J Hydrol* 115(2):193–209
- Sen Z (1990) Nonlinear radial flow in confined aquifers toward large-diameter wells. *Water Resour Res* 26(5):1103–1109
- Shu LC, Deng MJ, Wang XH (2013) Evaluation of unconfined aquifer parameters from flow to partially penetrating wells in Tailan River basin, China. *Environ Earth Sci* 69(3):799–809
- Sushil KS (2006) Simplified kernel method for flow to larger diameter wells. *J Irrig Drain Eng ASCE* 132(1):77–79
- Theis CV (1935) The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage. *Am Geophys Union Trans* 16:519–524
- Wang XJ, Xie ZH, Zhou X, Shao JL (2005) Simulation of artificial injection with large-diameter wells in the western outskirts of Beijing. *Hydrogeol Eng Geol* 32(1):71–74
- Wang JX, Feng B, Yu HP, Guo TP, Yang GY, Tang JW (2013) Numerical study of dewatering in a large deep foundation on pit. *Environ Earth Sci* 69(3):863–872
- Wen Z, Huang GH, Zhan HB (2008a) An analytical solution for non-Darcian flow in a confined aquifer using the power law function. *Adv Water Res* 31(1):44–55

- Wen Z, Huang GH, Zhan HB, Li J (2008b) Two-region non-Darcian flow toward a well in a confined aquifer. *Adv Water Res* 31(5):818–827
- Wen Z, Huang GH, Zhan HB (2009) A numerical solution for non-Darcian flow to a well in a confined aquifer using the power law function. *J Hydrol* 364(1–2):99–106
- Wen Z, Liu K, Chen XL (2013) Approximate analytical solution for non-Darcian flow toward a partially penetrating well in a confined aquifer. *J Hydrol* 498:124–131
- Yeh HD, Chang YC (2013) Recent advances in modeling of well hydraulics. *Adv Water Resour* 51:27–51