

# Evaluation of groundwater monitoring network of Kodaganar River basin from Southern India using entropy

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**Abstract** A discrete entropy-based approach is used to assess the groundwater monitoring network that exists in Kodaganar River basin of Southern India. Since any monitoring system is essentially an information collection system, its technical design and evaluation require a quantifiable measure of information and this measure can be derived using entropy. The use of information-based measures of groundwater table shows that the existing monitoring network contains a sufficient number of wells but is not well designed for the measurement of groundwater level. Entropy-based results show that 15 wells are vital to measure regional groundwater level, not 28 wells which are being monitored effectively in this basin.

**Keywords** Groundwater monitoring network · Discrete entropy · Marginal entropy · Joint entropy · Transinformation · Information transfer index · Water resources management · Hard rock area

## Introduction

Entropy is a measure of information or uncertainty associated with a random variable or its probability distribution, and can be used for measuring the information content of realizations of any random variable. It is first developed by Shannon (1948) and has been applied in many different fields such as ecology (Ricotta et al. 2003; Carranza et al. 2007; Nicolae et al. 2009), biology (Rojdestvenski and Cottam 2000; Karamanos 2009), mining industry (Sy 2001; Siradeghyan et al. 2008), economics (Herrmann 2009; Zhou et al. 2010), financial time-series analysis (Darbellay and Wuertz 2000; Sato 2008), etc. It has also been applied extensively in hydrology and water resources for measuring information contents of random variables and models, evaluating information transfer between hydrological processes, evaluating data acquisition systems, and designing water quality monitoring (WQM) networks (Uslu and Tanriover 1979; Krastanovic and Singh 1992; Yang and Burn 1994; Harmancioglu et al. 1999; Mogheir and Singh 2002; Mogheir et al. 2004; Masoumi and Kerachian 2008; Karamouz et al. 2009, Singh 2010; Mondal and Singh 2010).

Uslu and Tanriover (1979) analyzed the entropy concept for the delineation of optimum sampling intervals in data collection systems both in space and time. Harmancioglu (1981) investigated the transfer of information between observations of two stream gauging stations. Optimal design of water quality networks has also been studied by many researchers during the past decades. Tirsch and Male (1984) proposed a measure of monitoring precision as a function of sampling location and time frequencies. This measure is defined using the coefficient of determination of a multivariate linear regression model. Harmancioglu and Alpaslan (1992) proposed a statistical procedure based on

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the entropy theory to address the assessment of both network efficiency and cost-effectiveness. Woldt and Bogardi (1992) and Karamouz et al. (2009) proposed different models for the optimal design of WQM systems combining geostatistical and Multiple Criteria Decision Making (MCDM) methods.

Ozkul et al. (2000) extended the work of Harmancioglu and Alpaslan (1992) to better define the zones with high monitoring data uncertainties along a river. This model can only be used for reducing redundant stations or decreasing the sampling frequency in an existing or a primary monitoring system. Mogheir and Singh (2002) and Mogheir et al. (2004) developed a methodology for design of an optimal groundwater monitoring network using entropy (or information) theory. We know that a monitoring system is essentially an information collection system and its technical design requires a quantifiable measure of information which can be achieved through application of the information theory. They applied this theory to describe the spatial variability of synthetic data that can represent spatially correlated groundwater quality data. The application involves calculating information measures such as transinformation, the information transfer index (ITI) and the correlation coefficient. These measures are calculated using discrete and analytical approaches. Later, Mogheir et al. (2005) evaluated the monitoring cycle in the Gaza Strip using the entropy theory. This article proposes a flowchart, which is used to evaluate the relation between the objectives, the tasks, the data and the monitoring activities.

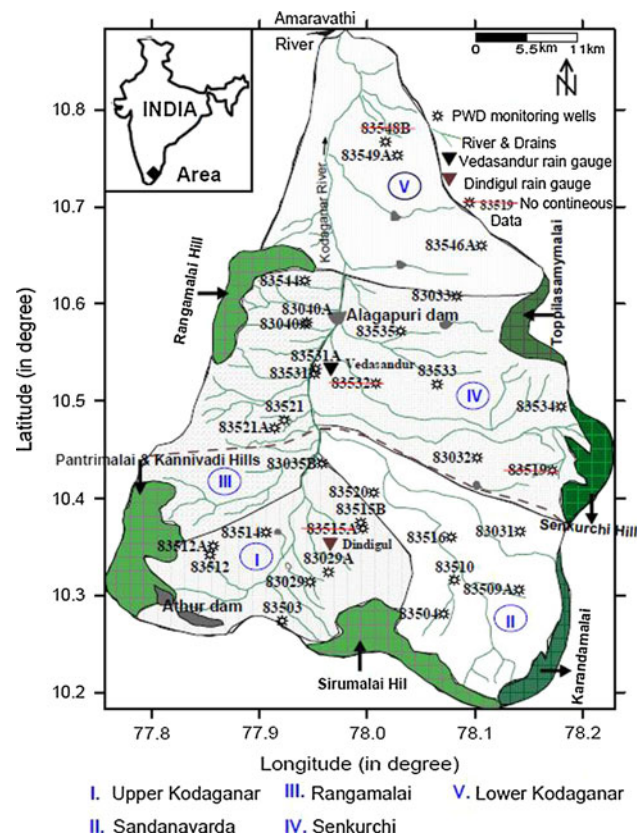
Recently, Khalil and Ouarda (2009) critically reviewed the assessment and redesign of surface WQM networks along with other statistical approaches. The various monitoring objectives, related procedures and their appropriatenesses, used for the assessment and redesign of long-term surface WQM networks, are discussed. But advantages and disadvantages of each statistical approach are found from a network design perspective. Although many studies have sought to improve the performance of monitoring networks, most have focused mainly on only one aspect of the network design. Few researchers have examined the optimization of different aspects simultaneously.

Network assessment and redesign requires combining the assessment of the monitoring objectives, the variables to measure, the sampling frequency and the sampling locations into one framework. The three main aspects in monitoring design should be assessed simultaneously, and may be linked by a criterion of either cost or information. Finally, the monitoring network design needs to be periodically re-assessed and modified accordingly due to changing environmental situations and/or shifts in management priorities. In order to facilitate the periodic assessment of the monitoring network performance, the design or the assessment and redesign should be well

documented. Keeping this view in mind, a groundwater monitoring network, used for measurement of regional groundwater level in Indian context, is considered for assessing network evaluation and design for the first time. So far the entropy concept does not seem to have been applied to evaluate the network for measurement of regional groundwater table in India, and especially to groundwater monitoring wells in Kodaganar River basin from Southern India. Thus, the objective of this paper is to use discrete information measures, and describe the spatial variability of measured water level in the existing monitoring network using entropy.

## Study area

Kodaganar River basin is a drought prone hard rock area. It lies between  $77^{\circ}45'32''$  and  $78^{\circ}13'46''$ E longitude and  $10^{\circ}11'10''$ – $10^{\circ}52'54''$  N latitude (Fig. 1) with an area of about 2,250 km<sup>2</sup> (Singh et al. 2003). This area is characterized by undulating topography with main hills located in the southern (Sirumalai), southeastern (Karandamalai), eastern (Senkurchi and Toppilasamymalai) and western (Rangamalai) parts slopping towards north and northeast. The elevation ranges from 360 m above mean sea level



**Fig. 1** Location map of Kodaganar River basin from Southern India

(amsl) in the southern part to 120 m (amsl) in the northern part in plain area. Most of the tributaries of Kodaganar River originate from these hills, which enclose the basin from three sides. Therefore, the entire rainfall-generated runoff drains towards its confluence with Amaravathi River in the north (Mondal et al. 2005; Mondal and Singh 2011a). There are two surface water reservoirs, i.e., one at Attur in the southern corner upstream and another at Alagapuri downstream. No perennial streams exist in this area, except for short distance streams encompassing second and third order drainages (Mondal et al. 2002). Runoff from rainfall within the area ends in small streams flowing towards the main Kodaganar River. There are two rain gauge stations located at Dindigul in the upper basin and Vedasandur in the lower basin (Fig. 1). For a period of 2000–2007 annual average rainfall is about 875.8 mm at Dindigul and 607.6 mm at Vedasandur rain gauge stations. Temperature increases slowly to a maximum in summer months up to May, after which it drops slowly. The mean of maximum temperature ranges from 36.5 to 41.8°C, whereas the mean of minimum temperature varies from 17.4 to 24°C.

### Geological and hydrogeological characteristics of the study area

Granite and gneisses occupy most of the basin except in hilly areas where charnockite hills form the drainage boundary (Balasubramanian 1980; Chakrapani and Manickyan 1988). The larger part is occupied by metamorphic crystalline rocks, which are highly folded, fractured and jointed (Krishnan 1982). Quartzite and pyroxenite occur in patches. Some dykes are present north-east of the Vedasandur area, and they run in the NW–SE direction. There is a major fault running in the NNE–SSW direction for several kilometers situated northeast of Dindigul town (Mondal and Singh 2004). Lineaments are found to a limited extent in the entire area but they are oriented mainly in the NNE–SSW, NEE–SWW, and NW–SE directions. Shear zones are also found near Vedasandur. The denudational terrain surrounded by structural hills, as described above, occur in the form of pediments. Shallow pediments and buried pediments are major geomorphic units (Public Works Department 2000). The thickness and intensity of this landform vary, depending upon the slope and structural disturbances. The area covered by pediment (mostly in northern part) exhibits rock outcrops with or without soil cover. These areas are basically runoff zones and groundwater potential in these areas is considered as poor (Singh et al. 2003). In shallow pediment area groundwater potential is considered as moderate. The areas of low relief constituting buried pediments are most favorable for groundwater potential. Potential aquifers are formed along

the sides of river or tributaries and flood plains of recent origin. A limited extent of valley fills is also found in this basin.

Groundwater occurs in weathered portions and at depth in jointed and fractures (Singh et al. 2003; Mondal and Singh 2011b). It is being exploited through dug wells tapping the weathered zone, and bore and dug-cum-bore wells tapping fracture aquifers. The straight courses of *nalas* and streams indicate those structural features, such as lineaments; faults and joints have controlled sources. The weathered zone facilitates the movement and storage of groundwater through a network of joints, faults and lineaments. The area is interesting in that only a few dug wells function in the central and northern parts, while the presence of sheared zones and lineaments controls the groundwater system. Aquifer parameters, namely, transmissivity ( $T$ ) and storage coefficient ( $S$ ), were estimated at 28 existing dug wells through pumping tests. The pumping test data (both pumping and recovery phases) had been interpreted, considering field conditions, for evaluating aquifer parameters (Thangarajan and Singh 1998). The calculated  $T$  values vary from 4 and 1,166 m<sup>2</sup>/day and  $S$  values from 0.00001 to 0.099.

### Materials and methods

#### Groundwater monitoring network data

Monitoring of groundwater level is an important component of groundwater survey. The water level fluctuations reflect the change in groundwater storage. Ground Water Resource Estimation Committee (GWREC) (1996) recommended that the size of a watershed unit could be about 100–300 km<sup>2</sup> area and there should be at least three spatially well-distributed observation wells in the unit, or one observation well per 100 km<sup>2</sup>, whichever is more. For this purpose, 32 control wells (Fig. 1) spread over the entire Kodaganar River Basin, Tamil Nadu (Southern India) have been used for observing water level during the first week of every month (Public Works Department 2000). The water level fluctuation is needed for investigating the time wise depth to water level, recharge and discharge periods, hydraulic gradient, and rate of water level increase or decrease (Todd 1980; GWREC 1996). Groundwater samples have also been collected from the same wells once in 6 months to determine the suitability of groundwater for domestic, agricultural and industrial purposes. To evaluate this monitoring network using entropy, 28 wells were selected for a period of 7 years between January 2000 and December 2007, because they have continuous data. The missing data from these wells was calculated using a moving average method (Medhi 2005).

## Entropy theory

Clausius coined the word ‘Entropy’ from the Greek meaning transformation. Thus, entropy originated in physics and occupies an exceptional position among physical quantities. Its nature is, rather, a statistical or probabilistic one, and it can be interpreted as a measure of the amount of chaos within a quantum mechanical mixed state (Wehri 1978). In physical sciences, entropy relates macroscopic and microscopic aspects of nature and determines the behaviour of macroscopic systems in equilibrium (or close to equilibrium). Entropy is not an observable that means there does not exist an operator with the property (Wehri 1978). It is, rather, a function of a state. Entropy is viewed in three different but related contexts and is hence typified by three forms such as (1) thermodynamic entropy given by Clausius in 1850 (Gull 1991), (2) statistical–mechanical entropy given by Boltzmann in 1866 (Lebowitz 1993), and (3) information–theoretical entropy (Shannon 1948).

This entropy is a measure of information or uncertainty associated with a random variable or its probability distribution. It can be used for measuring the information content of realizations of the random variable (measurement of groundwater level at a gauging station). The measures of information are marginal entropy, conditional entropy, joint entropy and transinformation. Marginal entropy  $H(X)$  can be defined as the average information content of a random variable  $X$  with the probability distribution  $p(x)$  and is used as a measure of uncertainty. It is also referred to as a measure of uncertainty or information content. This is logical because uncertainty represents the potential information in the sense that when a random variable takes on a value, we gain information and lose uncertainty. Therefore, for a discrete random variable  $X$  with probability distribution  $p(x)$ , entropy is its potential information. For two random variables  $X$  and  $Y$  conditional entropy  $H(X|Y)$  is a measure of the information content of  $X$  which is not contained in  $Y$ . The joint entropy  $H(X, Y)$  is the total information content in both  $X$  and  $Y$ . The mutual information,  $T(X, Y)$ , also called transinformation, is the information common between  $X$  and  $Y$ . It can be defined as the information content of  $X$  which is contained in  $Y$  and can be interpreted as the reduction in uncertainty in  $X$  due to the knowledge of random variable  $Y$ . Entropy measures can be expressed using both discrete and analytical approaches (Lubbe 1996; Singh 1998). Discrete forms of these entropies can be expressed as

$$H(X) = - \sum_{i=1}^n p(x_i) \log_2 p(x_i) \quad (1)$$

$$H(X, Y) = - \sum_{i=1}^n \sum_{j=1}^m p(x_i, y_j) \log_2 p(x_i, y_j) \quad (2)$$

$$H(X|Y) = - \sum_{i=1}^n \sum_{j=1}^m p(x_i, y_j) \log_2 p(x_i|y_j) \quad (3)$$

$$T(X, Y) = T(Y, X) = \sum_{i=1}^n \sum_{j=1}^m p(x_i, y_j) \log_2 \left[ \frac{p(x_i, y_j)}{p(x_i)p(y_j)} \right] \quad (4)$$

where  $X$  and  $Y$  are two discrete variables taking on values  $x_i$ ,  $i = 1, 2, \dots, n$ ;  $y_j$ ,  $j = 1, 2, \dots, m$ ;  $n$  is the number of values that  $X$  takes on;  $m$  is the number of values that  $Y$  takes on;  $p_i$  is the discrete probability of occurrence;  $p(x_i, y_j)$  is the joint probability and  $p(x_i|y_j)$  is the conditional probability.  $X$  and  $Y$  are defined on the same probability space, each of which has a discrete probability of occurrence  $p_i$ . Note that  $H(X, Y) = H(Y, X)$ , and  $T(X, Y) = 0$ , if  $X$  and  $Y$  are independent (Jessop 1995). Transinformation is an indicator of the capability of information transmission. Although transinformation indicates the dependence of two variables, it is not a good index of dependence because its upper bound varies from site to site (it varies from 0 to marginal entropy,  $H(x)$ ). Therefore, an ITI is defined by normalizing transinformation, which then indicates the standardized information transferred from one site to another

$$\text{ITI} = \frac{T(X, Y)}{H(X, Y)} \quad (5)$$

The unit of information measures depends on the logarithmic base used. If a base 2 is used, then the unit is ‘bit’; for a logarithmic base 10 the unit is decibel, and it is nat (natural units) if the logarithmic base is  $e$  (Caselton and Husain 1980; Mogheir et al. 2003). However, some researchers (Harmancioglu and Yevjevich 1987) have used base  $e$  to express information measures in Napiers. Provided that the logarithmic base is used consistently the choice of units is not critical. For convenience, base 2 and unit ‘bit’ were used here in computations.

The geometric distance ( $d$ ) between two wells was calculated as

$$d = \sqrt{(A_1 - A_2)^2 + (B_1 - B_2)^2} \quad (6)$$

where  $(A_1, B_1)$  are the coordinates of the first well (1) where marginal entropy is maximum,  $(A_2, B_2)$  are the coordinates of second any corresponding well (2) in the network, and  $d$  is the distance between wells 1 and 2. The marginal entropy, joint entropy, transinformation, ITI, and distance between pairs of wells were calculated with the aid of Eqs. 1–6.

**Results and discussion**

Regional groundwater level measurements

Details of 32 PWD well inventories are given in Table 1. All open wells are rectangular shaped, except for 10 circular structures, with depth ranging from 10.60 to 28.50 m below the ground level (bgl). The depth to water bearing zone varied from 1.00 to 9.50 m (bgl) and the thickness ranged from 1.54 to 17.45 m under phreatic conditions. Groundwater was extracted mainly through the bucket and

pulley method for domestic and gardening purposes. Monthly water levels were monitored during the first week of every month (Public Works Department 2008) since January 2000 to December 2007 from these PWD wells, which are uniformly spread over the basin (Fig. 1).

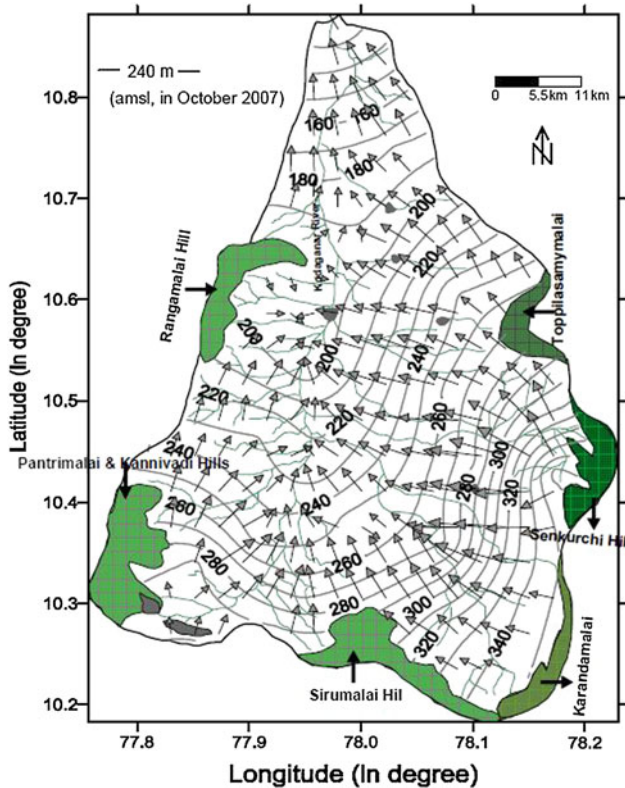
The water level fluctuation is needed for investigating the time wise depth to water level, recharge and discharge periods, hydraulic gradient, and rate of water level increase or decrease. The water level contours were prepared using the kriging method for October 2007 with the help of Surfer version 8.00 (2002) Surface Mapping System,

**Table 1** Well inventory of PWD wells in Kodaganar River basin from Southern India

Well Ids.	Village names	Latitude	Longitude	Dimension (m)	Total depth (m, bgl)	Lining depth (m)	Water bearing (m, bgl)	Elevation of ground level (m, amsl)	MP (m)	Depth to water level (m) (Oct 2007)
83029A	A.Vellodu	10°19'26"	77°57'50"	3.93	28.50	3.0	3.00–7.00	282.80	0.45	20.33
83029	A.Vellodu	10°18'50"	77°56'50"	3.02	22.00	3.0	2.60–8.10	279.76	1.00	16.05
83515B	Dindigul	10°22'30"	77°59'40"	3.50 × 3.25	18.00	4.0	5.85–17.50	258.97	1.25	2.45
83503	Ambathurai	10°16'25"	77°55'14"	3.85 × 3.00	16.65	4.0	4.20–9.00	300.39	0.66	8.76
83514	Sinthalakundu	10°21'55"	77°54'20"	2.41 × 2.47	14.00	5.0	5.00–8.00	258.60	0.90	12.40
83520	Seelapadi	10°24'22"	78°00'23"	2.82 × 2.93	18.40	4.0	3.80–18.40	259.92	0.78	11.57
83512A	Palayakannivadi	10°21'05"	77°51'24"	5.65 × 6.80	19.55	5.0	9.50–19.55	286.86	0.60	5.15
83512	Palayakannivadi	10°20'30"	77°51'15"	3.15	16.70	4.0	4.40–7.30	300.55	0.76	10.19
83035B	Chettinaickenpatti	10°26'11"	77°57'28"	2.70	16.60	3.0	3.70–13.45	232.94	0.75	11.80
83504	Shanarpatti	10°16'53"	78°04'15"	4.76 × 5.40	16.80	4.0	4.33–12.39	330.79	0.96	12.34
83510	Ragalapuram	10°18'57"	78°04'53"	2.27 × 2.27	13.20	5.0	5.40–10.56	301.53	1.24	2.61
83516	Madur	10°21'35"	78°04'40"	3.05 × 2.99	15.10	3.0	3.40–6.50	284.68	0.85	14.90
83521A	Alagupatti	10°28'18"	77°54'50"	2.70 × 2.70	12.25	6.0	6.00–10.15	237.95	0.85	9.95
83521	Alagupatti	10°28'48"	77°55'23"	4.80 × 4.59	11.25	3.0	3.45–7.00	231.54	0.44	8.06
83509A	Silvarpatti	10°18'23"	78°08'26"	4.60	15.95	2.0	8.50–15.50	353.21	1.15	14.75
83032	Vadamadurai	10°26'30"	78°06'08"	4.00 × 5.35	22.00	4.0	4.15–6.00	284.99	1.00	6.40
83031	Ammapatti	10°21'58"	78°08'30"	2.00	12.50	6.0	6.00–10.80	341.16	1.00	12.05
83531	Vedasantur	10°31'40"	77°57'03"	6.65 × 7.65	23.40	3.0	3.20–9.00	213.40	0.70	18.20
83531A	Vedasantur	10°31'57"	77°57'08"	2.40 × 2.20	17.50	2.0	1.65–14.04	211.72	0.85	16.82
83533	Thennampatti	10°31'00"	78°03'55"	3.38 × 3.48	17.50	2.0	1.95–6.00	263.84	0.67	8.33
83040	Kaithienkottai	10°34'45"	77°56'32"	7.40 × 7.40	20.65	4.0	4.47–10.00	217.38	0.80	17.93
83535	Usilampatti	10°34'20"	78°01'50"	2.68 × 2.68	12.00	5.0	4.76–6.60	229.71	0.78	10.62
83040A	Kaithienkottai	10°34'53"	77°56'40"	2.14 × 2.48	18.40	5.0	5.15–8.00	220.45	0.72	10.98
83534	Kollapatti	10°29'38"	78°10'48"	2.45 × 2.45	10.60	4.0	4.75–10.60	317.74	0.84	9.16
83544	Kalvarpatti	10°37'27"	77°56'30"	2.90	27.60	2.0	2.23–10.00	220.06	0.74	22.91
83033	R.Kombai	10°36'30"	78°04'58"	2.80 × 2.80	12.60	4.0	3.76–5.30	257.47	1.30	10.20
83546A	R.Kombai	10°39'35"	78°06'22"	2.67	20.50	6.0	4.74–8.12	242.76	1.10	17.15
83549A	R.Vellodu	10°45'12"	78°01'42"	2.05	19.15	2.0	1.70–19.15	191.55	0.85	12.75
83515A	Dindigul	10°22'10"	77°59'45"	7.36 × 6.08	14.65	3.0	2.54–11.10	258.44	1.05	9.35
83519	Puthur	10°25'45"	78°10'19"	2.17	11.50	2.0	1.00–8.63	372.37	0.89	10.40
83532	Marambadi	10°31'05"	78°00'30"	2.30 × 2.30	19.90	3.0	3.40–8.40	235.46	0.77	17.80
83548B	Ayyampatti	10°46'00"	78°01'00"	2.38 × 1.87	22.00	2.0	2.14–5.80	156.19	0.72	19.98

Well type: dug well; Type of aquifer: phreatic; Geology: granite and gneisses; Stratigraphy: Archaean; Water lifting device: bucket and pulley method

bgl below ground level, MP measuring point, amsl above mean sea level



**Fig. 2** Water level contours map in October 2007

Golden Software, Inc., as shown in Fig. 2. This exhibits a general trend of the flow direction but not the micro level characteristics of the aquifer. When water level hydrographs with rainfall were plotted, there was approximately one month time lag in the response of water table to rainfall. Well hydrographs were prepared corresponding to rainfall recorded by the nearby rain gauge station and they showed that water table fluctuations due to rainfall have been used to determine areas where there has been a good response. These hydrographs closely follow the rainfall trend. In general, the water level in most of the cases returns to its original position after good rainfall. This phenomenon may be due to rapid recharge that results from heavy rainfall and also irrigation return flows. All these PWD wells were representative monitoring wells for groundwater level measurements, which were not used for water extraction. If there was any water extraction that was negated by sufficient recharge at the individual well area.

#### Entropy-based measures

For computation of information measures, the value of  $p(x)$  was calculated based on frequency analysis of the available water level data for each well. Then, joint probabilities were computed using a contingency table for individual well pairs in the basin. A total of 96 events were used

for constructing contingency tables. A two-dimensional contingency table is illustrated for monthly water level data for PWD wells 83029A & 83544 in Table 2. It was considered that the depth to water table of well 83029A had a range of values (0–5, 5–10, 10–15, 15–20 and 20–25 m) consisting of five categories (class intervals), whereas the water level of well 83544 was also assumed to have five categories (class intervals) with a range of 5.00 m (bgl). The cell density or the joint frequency for  $(i, j)$  was denoted by  $f_{ij}$ ,  $i = 1, 2, \dots, 5$ ;  $j = 1, 2, \dots, 5$ , where the first subscript refers to the column (water level of well 83029A) and the second subscript to the row (water table of well 83544). Marginal frequencies were denoted by  $f_i$  and  $f_j$  for the column and the row values of these two set water levels, respectively. Then, marginal entropies of individual wells and total entropy,  $H(X, Y)$  were calculated with the aid of Eqs. 1 and 2. Transinformation,  $T(X, Y)$ , given by Eq. 4, was also calculated for each pair of PWD wells. Then, ITI was estimated with the aid of Eq. 5. The same procedure was followed to calculate information for other pairs of wells.

#### Marginal entropy of the existing monitoring network

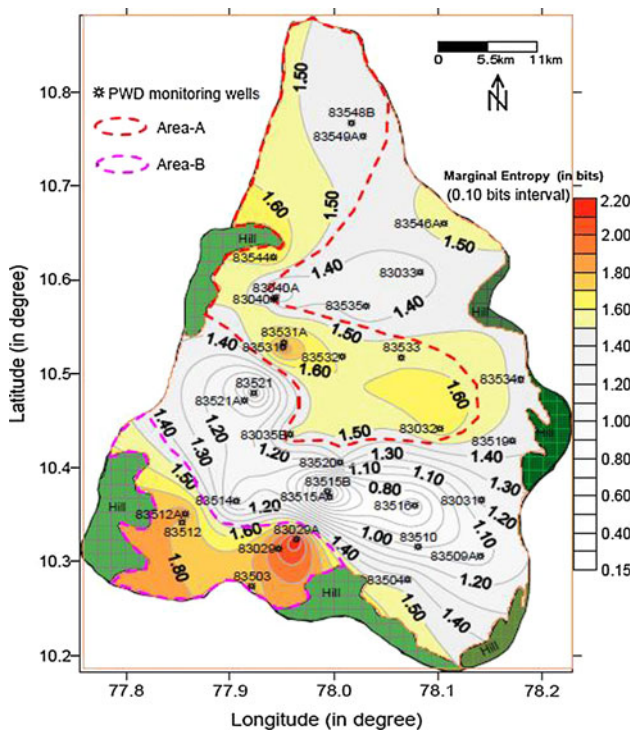
The calculated marginal entropy varied from 0.15 to 2.20 bits with a mean value of 1.40 bits. This marginal entropy for 43% monitoring wells in the existing network was less than the average marginal entropy (1.40 bits) and its contour map (Fig. 3) presents the landscape of entropy. At area-A (in and around wells 83531, 83544 and 83032), the information content, in general, increased as small scale urbanization is being developed in the presence of Alagapuri dam (Public Works Department 2000). However, such is also the case at area-B (in and around wells 83512, 83503 and 83029), information increased in the presence of Athur dam as well as agricultural and human activities (Singh et al. 2003). In general, urbanization, man-made ecosystem and agricultural activities increase entropy, i.e., increase uncertainty.

#### Transinformation, joint entropy and ITI

The distance matrix, transinformation matrix, ITI matrix and joint entropy matrix were computed. The distance matrix consists of the well number in rows and columns; each cell of the matrix contains the distance between the wells in rows and columns. It shows that there is no existing well within 7.00 km in and around PWD wells 83032, 83533, 83535, 83534 and 83549A. But there are a few wells having minimum distances of 0.31, 0.57, 1.06, 1.34, 2.20 and 3.70 km between wells 83040 & 83040A, 83531 & 83531A, 83512 & 83512A, 83521 & 83521A, 83029 & 83029A, and 83520 & 83515B, respectively. The transinformation matrix also consists of well numbers in

**Table 2** Absolute frequency contingency table for depth to water tables (m, bgl) at PWD wells 83029A and 83544 during January 2000 to December 2007

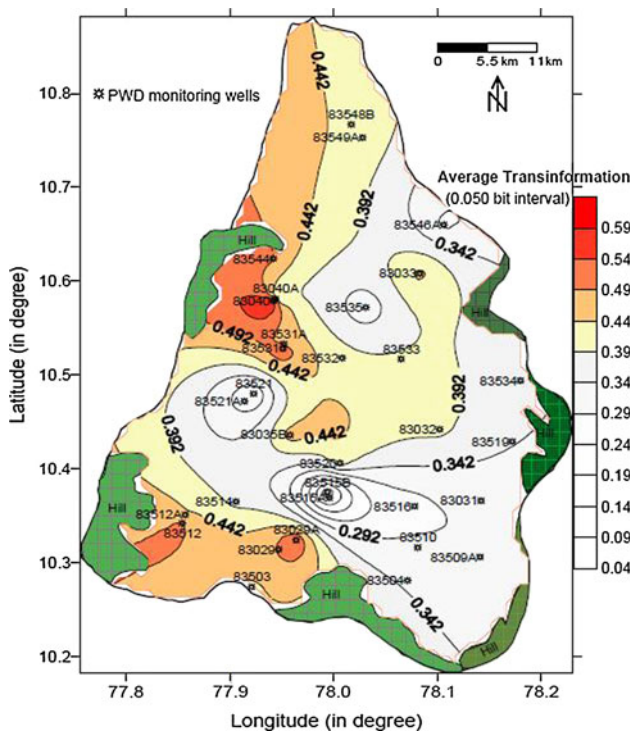
Depth to water level (m, bgl) at well 83544		Depth to water level (m, bgl) at well 83029A					Total	$p_j$	$\ln p_j$	$p_j \times \ln p_j$
		$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$				
		0–5	5–10	10–15	15–20	20–25				
$j = 1$	0–5	4	1	0	1	0	6	0.063	-4.000	-0.250
$j = 2$	5–10	0	8	5	3	0	16	0.167	-2.585	-0.431
$j = 3$	10–15	0	1	6	7	0	14	0.146	-2.778	-0.405
$j = 4$	15–20	0	0	1	1	1	3	0.031	-5.000	-0.156
$j = 5$	20–25	2	8	10	10	27	57	0.594	-0.752	-0.447
Total		6	18	22	22	28	96			
$p_i$		0.063	0.188	0.229	0.229	0.292	–			
$\ln p_i$		-4.000	-2.415	-2.126	-2.126	-1.778				
$p_i \times \ln p_i$		-0.250	-0.453	-0.487	-0.487	-0.518				
$P_{i,j}$		0.042	0.010	0.000	0.010	0.000				
		0.000	0.083	0.052	0.031	0.000				
		0.000	0.010	0.063	0.073	0.000				
		0.000	0.000	0.010	0.010	0.010				
		0.021	0.083	0.104	0.104	0.281				
$P_{i,j} \times \ln P_{i,j}$		-0.191	-0.069	0.000	-0.069	0.000				
		0.000	-0.299	-0.222	0.000	0.000				
		0.000	-0.069	-0.250	-0.275	0.000				
		0.000	0.000	-0.069	-0.069	-0.069				
		-0.116	-0.299	-0.340	-0.340	-0.515				



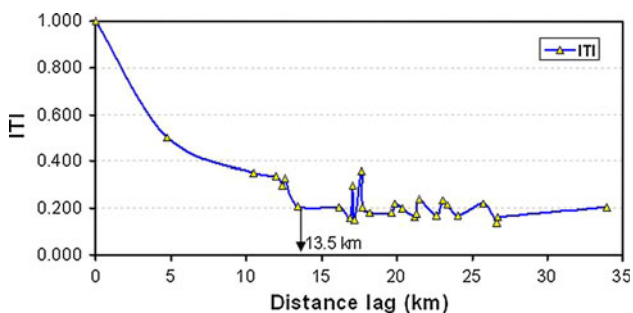
**Fig. 3** Marginal entropy map for measurement of regional groundwater level

rows and columns and each cell of the matrix contains the value of transinformation between wells in rows and columns. A contour map of the average groundwater level transinformation was prepared and is shown in Fig. 4. It indicates that the value of transinformation varies from 0.04 to 0.59 bits with a mean value of 0.39 bits. There is no independent well at which the groundwater level is being measured by PWD. The calculated transinformation is comparatively less where the marginal entropy is low.

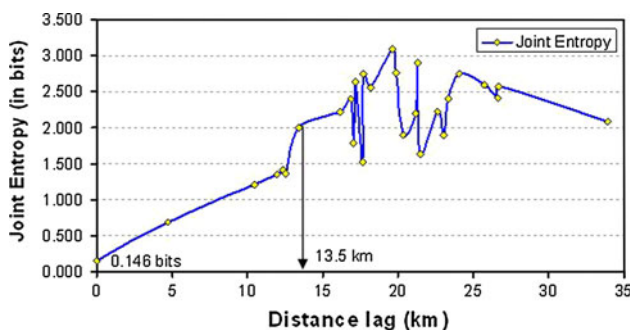
In the same way, the joint entropy matrix and ITI matrix were computed. Then, ITI and joint entropy were plotted against distance in Figs. 5, 6, respectively. Figure 5 shows that there is a sharp decrease of ITI, when the distance lag is large. With further increase in the distance lag, ITI became essentially constant. Therefore, what is important for the spatial distribution design is selecting the distance lag at which the transinformation has a minimum steady value. Figure 6 shows that the joint entropy has opposite variation (increasing) compared to the ITI curve, because the high joint entropy gives low transinformation and the low joint entropy gives high transinformation. Note that when the distance is zero, the redundant information will be maximized and the transinformation will equal the marginal entropy. While at the zero distance as indicated in Fig. 6, the joint entropy will be equal to the minimum value (0.146 bits)



**Fig. 4** Average transformation (in bits) of groundwater level in Kodaganar River basin



**Fig. 5** Information Transfer Index (ITI) with distance in the study area



**Fig. 6** Joint entropy with distance in Kodaganar River basin

since the redundant information is maximized in the existing network.

*Assessment of the network*

By analyzing the monitoring water level data in Kodaganar River basin, the entropy map and the transinformation (or joint entropy) measure can be used for assessing the existing monitoring network. Analysis of monitoring wells indicates that the well, which has high marginal entropy, gives the highest redundant information. Accordingly, a marginal entropy contour map was used to classify the monitoring area from which new information was extricated or the area which has redundant information. The marginal entropy contour map (Fig. 3) along with transinformation map (Fig. 4) indicates that the area or the zone, which has the low redundant entropy (information) value should be the second priority for collecting more information. These zones are in the area of wells 83521, 83521A, 83515B, 83516, 83510, and 83535. The highest marginal entropy (redundant information) area can be allocated a first priority (i.e., the area around the wells 83029A, 83029, 83512, 83531, 83544 and 83032). Thus, from the marginal entropy map the first priority area for monitoring can be distinguished. The existing wells in that area should have the first priority for monitoring.

The ITI measure can be used for reducing the number of wells in a dense monitoring network (in both first and second priority areas). For example, in Fig. 5, the maximum distance lag relating to the minimum steady ITI value is about 13.50 km. An association among the measured groundwater levels at the same distance is related to the maximum steady joint entropy (Fig. 6). Therefore, the distance between wells should be on average at least 13.50 km, which reduces the number of existing monitoring wells. There is only one well 83549A in the basin having a distance more than 13.50 km, but 23 monitoring wells having distances less than 7.00 km. In the first priority area, if new observation wells are needed (expansion of the existing monitoring network), then the transinformation measure can also be used to determine the maximum distance between wells by using the related minimum transinformation as well considering the hydrogeologic characteristics.

For Kodaganar basin area of 2,250 km<sup>2</sup>, on average the required number of monitoring wells was estimated as 12. At present PWD is using effectively 28 monitoring wells in this basin for the measurement of regional groundwater level. To evaluate this network the basin was divided into five sub-watersheds based on drainage patterns, lineaments and structural controls, etc. They are (I) Upper Kodaganar, (II) Sandanavarda, (III) Rangamalai, (IV) Senkurchi, and (V) Lower Kodaganar (Fig. 1). There are eight exiting monitoring wells in Upper Kodaganar (I) of area 373 km<sup>2</sup> but



on average it required at least two monitoring wells. The marginal entropy for water level measurements varied from 0.146 to 2.195 bits with an average of 1.530 bits in this sub-watershed. The average distance of individual wells was 8.89 km, and average joint entropy and ITI were 1.960 bits and 0.430, respectively (Figs. 7, 8). It indicates that wells 83515B, 83514 and 83035 could be neglected for further monitoring. Sandanavarda (II) of area 450 km<sup>2</sup> required only three wells. The average marginal entropy was 1.120 bits, but the joint entropy varied from 0.622 to 2.087 bits (Fig. 7). Thus, out of the six existing wells, this network could be reduced to three wells by neglecting wells 83516, 83510 and 83509A. It was necessary about two wells in Rangamalai (III) watershed due to its area of 379 km<sup>2</sup>. The marginal entropy varied from 0.650 to 1.986 bits among five existing wells; whereas joint entropy was varied from 0.650 to 2.445 bits (Fig. 7). Wells 83040 and 83544 were given first priority for the measurement of groundwater level in this watershed. There were six monitoring wells in Senkurchi (IV) of area 554 km<sup>2</sup> and on average at least four wells were required. The marginal entropy varied from 1.312 to 1.9375 bits with average 1.633 bits and the average distance of individual wells was 11.75 km. The joint entropy varied from 1.312 to 2.811 bits (Fig. 7). Then the four existing wells (i.e.,

83032, 83533, 83531 and 83531A) are being continued to monitor by neglecting the wells 83535 and 83534. The area of Lower Kodaganar (V) is 494 km<sup>2</sup> and it was sought at least three wells. But the estimated marginal entropy among the exiting three wells varied from 1.354 to 1.560 bits with an average of 1.440 bits whereas the joint entropy from 1.354 to 2.200 bits (Fig. 7). So the well 83546A was given only first priority in this sub-watershed for measurement of regional groundwater level.

**Conclusions**

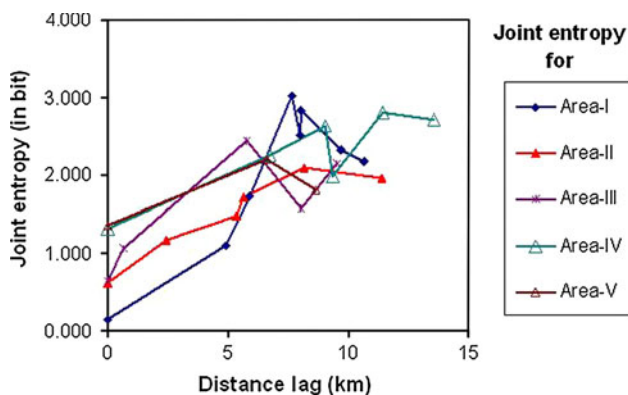
This study presents a preliminary framework for assessing the existing groundwater monitoring network in Kodaganar River basin from Southern India. Entropy is applied to determine the first priority area to be monitored. The well, having the highest marginal entropy, gives the highest transinformation with other wells, this well should be considered as a first priority to measure regional groundwater level. The well area that has the lowest marginal entropy gives the lowest transinformation with other wells to be considered as a second priority area.

The average ITI of this network shows that the maximum distance lag relating to the minimum steady transinformation value is about 13.50 km. Therefore, the distance between wells should be on average at least 13.50 km, which reduces/increases the number of existing PWD monitoring wells. For the basin area of 2,250 km<sup>2</sup>, on average the required monitoring wells is estimated as 12. The joint entropy and ITI are also used to evaluate each sub-watershed considering hydrogeologic characteristics. It shows that 15 wells are essential for groundwater level measurement instead of 28 PWD wells. Thus, the monitoring wells of Kodaganar River basin can be evaluated using entropy for spatial design of wells. Further the design of this network needs to be periodically re-assessed and modified accordingly due to changing environmental conditions and/or shifts in management priorities, as it will be incorporated in an ongoing research.

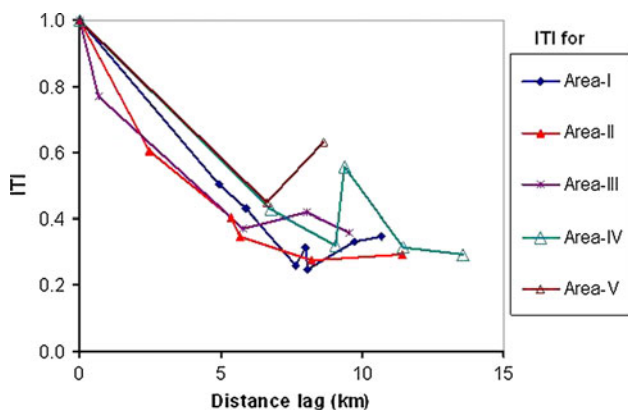
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**Fig. 7** Responses of joint entropies for sub-watersheds in Kodaganar River basin



**Fig. 8** Changes of ITI in different sub-watersheds

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