Journal of Ambient Intelligence and Humanized Computing (2022) 13:4089–4102
https://doi.org/10.1007/s12652-021-03509-x

ORIGINAL RESEARCH

Generalized ordered weighted harmonic averaging operator with trapezoidal neutrosophic numbers for solving MADM problems

S. Paulraj[1] · G. Tamilarasi[1]

Received: 4 February 2021 / Accepted: 13 September 2021 / Published online: 11 October 2021

Abstract
Harmonic mean is suitable for algebraic calculation and other mathematical treatments and also suitable for directly aggregated negative indicators. In many different situations, harmonic mean improves the flexibility. In this paper, we develop some new aggregation operators under neutrosophic environment and apply with multi attribute decision making (MADM) problems. First, we provide a Single valued trapezoidal neutrosophic Generalized ordered weighted harmonic averaging(SVTNGOWHA) operator which is the extension of single valued trapezoidal neutrosophic ordered weighted harmonic averaging (SVTNOWHA) operator. To fix the operators on the mount, we have tested these methods in few illustrative examples, and the results have been presented.

Keywords Single valued trapezoidal neutrosophic numbers · Harmonic averaging operator · Multi-attribute decision making

1 Introduction

Aggregation operators are mathematical functions used to combine the information. The mathematical and behavioral are properties of aggregation operators. In 1970's Multi criteria decision making (MCDM) problems active in research area. It is concerned with structure and planning of the problems and to analyzing how to solve these decisions, invoked by multiple criteria. It is the process of selecting the best alternative from the predefined alternatives. The arithmetic mean, geometric mean and harmonic mean are the most well-known aggregation operators. By the comparison of arithmetic and geometric mean operators, the advantage of harmonic mean operator is directly aggregating negative indicators. Zadeh (1965) introduced the concept of fuzzy set, which deals with vagueness and uncertainty of real world situations. Wang and Fan (2003) introduced the concept of fuzzy ordered weighted averaging (FOWA) operator. Xu and Da (2002) developed fuzzy weighted harmonic mean operator, fuzzy OWH operator and fuzzy hybrid harmonic operators, these aggregation operators reduced interval or real numbers. By considering the non-membership degree to the concept of fuzzy set, Atanssov (1986) proposed the concept of an intuitionistic fuzzy set which is characterized by membership degree and non-membership degree. Wang and Zhong (2009) proposed the concept of weighted arithmetic and geometric average operators with intuitionistic environment. Wan and Yi (2016) developed trapezoidal intuitionistic fuzzy numbers with power geometric operators. Das and Guha (2015) proposed new aggregation operators Trapezoidal intuitionistic fuzzy weighted power harmonic mean (TrIFWPHM) and discussed with some special case of TrIFWPHM operator. Wan and Zhu (2016) proposed triangular intuitionistic fuzzy Bonferroni harmonic aggregation operators. Das and Guha (2017) developed four kinds of aggregation operators which are TrIFWHM, TrIFOWHM, TrIFIOWHM, TrOFhHM based on harmonic mean operators under trapezoidal intuitionistic fuzzy numbers. Many researchers have used neutrosophic sets in decision making.

Smarandache (1998) introduced the concept of neutrosophic set theory which is an extension of fuzzy set and intuitionistic fuzzy set. Wang et al. (2005) developed interval neutrosophic sets, single valued neutrosophic sets and multi-valued neutrosophic sets. Irfan and Yusuf

✉ S. Paulraj
profspaulraj@gmail.com

G. Tamilarasi
tamiltara5@gmail.com

[1] Department of Mathematics, College of Engineering Guindy, Anna University, Chennai, Tamil Nadu 600025, India

(2014) introduced the concept of single valued trapezoidal neutrosophic weighted aggregation (SVTNWAO) operator and applied it to the multi criteria decision making problem. Ye (2015a) defined a trapezoidal neutrosophic set and its operational rules such as score and accuracy functions. He proposed trapezoidal neutrosophic number weighted arithmetic averaging (TNNWAA) and trapezoidal neutrosophic number weighted geometric averaging (TNNWGA) operators to deal with multiple attribute decision making problems. Ye (2016a) developed a multi attribute decision making method based on trapezoidal neutrosophic weighted arithmetic averaging (TNWAA) operator and Trapezoidal neutrosophic weighted geometric averaging (TNWGA) operator and investigate their properties. Ye (2015b) presented a simplified neutrosophic harmonic averaging projection measure. Ye (2016) developed expected values of neutrosophic linguistic numbers (NLN), and also he established weighted arithmetic and geometric aggregations operators with NLN. Zhikang and Ye (2017) proposed hybrid weighted arithmetic and geometric aggregation operators, hybrid ordered weighted arithmetic and geometric operator under single valued neutrosophic number information and utilized these operators to solve multiple attribute decision making problem. Deli (2018) introduced geometric and arithmetic aggregation operators including single valued trapezoidal neutrosophic (SVTN) ordered weighted geometric operator, SVTN-hybrid geometric operator, SVTN-ordered weighted arithmetic operator, SVTN-hybrid arithmetic operator and also developed an operator for multi attribute group decision making problem. Deli (2019) proposed novel defuzzification method of SV-trapezoidal neutrosophic numbers and multi-attribute decision making. Deli and Subas (2017) proposed the ratio ranking method which is the extension of the concepts of value and ambiguities ranking function with single valued trapezoidal neutrosophic numbers. Irfan and Yusuf (2017) introduced some weighted geometric operators with SVTrN numbers. Surapati and Rama (2018) extended the TrNWAA operator and Hamming distance which deals with VIKOR strategy to MAGDM problems in trapezoidal neutrosophic environment. Surapati and Rama (2019) developed TODIM strategy under neutrosophic environment.

Harish and Nancy (2018) established novel hybrid aggregation operators based on geometric and arithmetic operators under single valued and interval neutrosophic numbers and applied multi-criteria decision making problem. Pranab et al. (2018) developed distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. Pranab et al. (2018a) developed excepted value of trapezoidal neutrosophic numbers and applied multi attribute group decision making problems. Pranab et al. (2018b) established a Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) Strategy of MADM problems in neutrosophic environment. Jana et al. (2018) introduced the interval trapezoidal neutrosophic number weighted arithmetic averaging operator (ITNNWAA) and the interval trapezoidal neutrosophic number weighted geometric averaging operator (ITNNWGA) and developed a multi attribute decision making problem. Harish and Nancy (2019) defined power aggregation operators for the linguistic single valued neutrosophic set (LSVNS) and proposed a group decision making problems. Bharatraj and Anand (2019) introduced a power harmonic weighted aggregation operator with a single valued trapezoidal neutrosophic number and Interval valued neutrosophic set and developed multi criteria decision making problem. Chiranjibe et al. (2020) utilized Hamacher aggregation operators in single valued trapezoidal neutrosophic arithmetic and geometric operator and developed a multi attribute decision making problems. Surapati and Rama (2020) developed multi-objective optimisation by ratio analysis (MOORA) strategy to solve multi-attribute group decision making (MAGDM) in trapezoidal neutrosophic numbers. Tuhin and Nirmal Kumar (2020) presented centroid approach for solving linear programming problems with trapezoidal neutrosophic number environment. Broumi et al. (2020) proposed a new distance measure of trapezoidal fuzzy neutrosophic numbers and applied the measure for software selection process. Shigui et al. (2020) utilize simplified neutrosophic indeterminate elements weighted arithmetic averaging (SNIEWAA) operator and simplified neutrosophic indeterminate elements weighted geometric averaging (SNIEWGA) operator. Wang et al. (2020) have been developed possibility degree and power weighted aggregation operators of single valued trapezoidal neutrosophic numbers. He utilized power average and geometric operators to single valued trapezoidal neutrosophic numbers to deal with multi criteria decision making problems. Deli and Ozturk (2020) proposed an MCDM method based on the score functions of single valued neutrosophic numbers and reduced single valued trapezoidal neutrosophic numbers to fuzzy numbers. Garai et al. (2020) developed ranking methods for possibility mean with neutrosophic numbers and applied to multi-attribute decision making with single valued neutrosophic numbers.

Literature review reflects that no research has been carried out on weighted harmonic averaging operator with trapezoidal neutrosophic numbers for multi attribute decision making problems. To bridge the gap, we propose harmonic aggregating operators in single valued trapezoidal neutrosophic numbers, such as single valued trapezoidal neutrosophic weighted harmonic averaging (SVTNWHA) operator, single valued trapezoidal neutrosophic ordered weighted harmonic averaging (SVTNOWHA) operator, single valued trapezoidal neutrosophic generalized ordered weighted harmonic averaging (SVTNGOWHA) operator. We can also investigate some of their properties, applying a multi attributive decision making method. The main aim of this proposed operator is to choose the best alternative of the decision making under the preference value of the alternative.

This paper is organized as follows. Section 2 depicts some review of basic concepts. Section 3 presents harmonic operations on single valued trapezoidal neutrosophic numbers. Section 4 discusses method for multi attribute decision making problem. Section 5 conclusion of the paper is given.

2 Preliminaries

In this section, we review some basic concepts about the single valued trapezoidal neutrosophic numbers.

Definition 2.1 (Smarandache 1998) Let X be a non-empty set. Then a neutrosophic set $\tilde{a}$ of X is defined as

$$\tilde{a} = \left\{x, T_{\tilde{a}}(x), I_{\tilde{a}}(x), F_{\tilde{a}}(x) | x \in X\right\}, T_{\tilde{a}}(x), I_{\tilde{a}}(x), F_{\tilde{a}}(x) \in [0,1]$$

where $T_{\tilde{a}}(x), I_{\tilde{a}}(x), F_{\tilde{a}}(x)$ are truth membership function, indeterminacy membership function and falsity membership function and $0 \le T_{\tilde{a}}(x) + I_{\tilde{a}}(x) + F_{\tilde{a}}(x) \le 3$

Definition 2.2 (Smarandache 1998) A neutrosophic set $\tilde{a}$ is defined on the universal set of real numbers R is said to be neutrosophic number if it has the following properties.

1. $\tilde{a}$ is normal if there exists $x_0 \in R$, such that $T_{\tilde{a}}(x_0) = 1, I_{\tilde{a}}(x_0) = F_{\tilde{a}}(x_0) = 0$
2. A is convex set for the truth function $T_{\tilde{a}}(x)$ $T_{\tilde{a}}(\mu x_1 + (1-\mu)x_2) \ge min(T_{\tilde{a}}(x_1), T_{\tilde{a}}(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1]$
3. $\tilde{a}$ is concave set for the indeterminacy and falsity functions $I_{\tilde{a}}(x)$ and $F_{\tilde{a}}(x)$ $I_{\tilde{a}}(\mu x_1 + (1-\mu)x_2) \ge max(I_{\tilde{a}}(x_1), I_{\tilde{a}}(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1]$ $F_{\tilde{a}}(\mu x_1 + (1-\mu)x_2) \ge max(F_{\tilde{a}}(x_1), F_{\tilde{a}}(x_2)), \forall x_1, x_2 \in R, \mu \in [0,1]$

Definition 2.3 (Mohamed Abdel-Basset et al. 2019) Let $\tilde{a} =< (a_1, a_2, a_3, a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} >$ be a single valued trapezoidal neutrosophic set on the real number set R, whose truth membership, indeterminacy membership and falsity membership functions are given by

$$T_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} T_{\tilde{a}}, & \text{for } a_1 \le x \le a_2 \\ T_{\tilde{a}}, & \text{for } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} T_{\tilde{a}}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+I_{\tilde{a}}(x-a_1)}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ I_{\tilde{a}}, & \text{for } a_2 \le x \le a_3 \\ \frac{x-a_3+I_{\tilde{a}}(a_4-x)}{a_4-a_3}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{a_2-x+F_{\tilde{a}}(x-a_1)}{a_2-a_1}, & \text{for } a_1 \le x \le a_2 \\ F_{\tilde{a}}, & \text{for } a_2 \le x \le a_3 \\ \frac{x-a_3+F_{\tilde{a}}(a_4-x)}{a_4-a_3}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{otherwise.} \end{cases}$$

respectively, Where $a_1, a_2, a_3, a_4 \in R$. If $a_1 \le 0$ and at least $a_4 > 0$, then the single valued trapezoidal neutrosophic number $\tilde{a}$ is positive and it is denoted by $\tilde{a} > 0$. If $a_4 \le 0$ and at least $a_1 < 0$, then the single valued trapezoidal neutrosophic number $\tilde{a}$is negative and it is denoted by $\tilde{a} < 0$. Without loss of generality, we have considered $a_2 = a_3$. Then trapezoidal neutrosophic numbers transform to a triangular neutrosophic numbers. Where $T_{\tilde{a}}, I_{\tilde{a}}$ and $F_{\tilde{a}}$ represent the maximum degree of acceptance, an indeterminacy and minimum degree of rejection respectively, such that they satisfy the condition $0 \le T_{\tilde{a}}(x) + I_{\tilde{a}}(x) + F_{\tilde{a}}(x) \le 3, x \in \tilde{a}$.

Definition 2.4 (Mohamed Abdel-Basset et al. 2019) Let $\tilde{a} =< (a_1, a_2, a_3, a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} >$ and $\tilde{b} =< (b_1, b_2, b_3, b_4); T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}} >$ be two single valued trapezoidal neutrosophic numbers. Then

(i) $\tilde{a} + \tilde{b} =< (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}} >$

(ii) $\tilde{a} - \tilde{b} =< (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}} >$

$$\text{(iii)}\tilde{a}\tilde{b} = \begin{cases} < (a_1b_1, a_2b_2, a_3b_3, a_4b_4); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}} >, (a_4 > 0, b_4 > 0) \\ < (a_1b_4, a_2b_3, a_3b_2, a_4b_1); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}} >, (a_4 < 0, b_4 > 0) \\ < (a_4b_4, a_3b_3, a_2b_2, a_1b_1); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}} >, (a_4 < 0, b_4 < 0) \end{cases}$$

(iv)

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} <(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}}>, (a_4 > 0, b_4 > 0) \\ <(\frac{a_4}{b_4}, \frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}}>, (a_4 < 0, b_4 > 0) \\ <(\frac{a_4}{b_1}, \frac{a_3}{b_2}, \frac{a_2}{b_3}, \frac{a_1}{b_4}); T_{\tilde{a}} \wedge T_{\tilde{b}}, I_{\tilde{a}} \vee I_{\tilde{b}}, F_{\tilde{a}} \vee F_{\tilde{b}}>, (a_4 < 0, b_4 < 0) \end{cases}$$

(v) $$\lambda\tilde{a} = \begin{cases} <(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}>, (\lambda > 0) \\ <(\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}>, (\lambda < 0) \end{cases}$$

(vi) $\tilde{a}^{-1} = <(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}> (\tilde{a} \neq 0)$

Definition 2.5 (Ye 2016a) Let $\tilde{a} = <(a_1, a_2, a_3, a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}>$ be a single valued trapezoidal neutrosophic number. Then the score function of $\tilde{a}$ is defined as follows:

$$S(\tilde{a}) = \frac{1}{12}(a_1 + a_2 + a_3 + a_4)(2 + T_{\tilde{a}} - I_{\tilde{a}} - F_{\tilde{a}}), \tag{1}$$

Where $a_1, a_2, a_3, a_4 \in R$ and $0 \leq T_{\tilde{a}} + I_{\tilde{a}} + F_{\tilde{a}} \leq 3$.

For the comparison between two single valued trapezoidal neutrosophic numbers is defined as follows:

Let $\tilde{a} = <(a_1, a_2, a_3, a_4); T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}>$ and $\tilde{b} = <(b_1, b_2, b_3, b_4); T_{\tilde{b}}, I_{\tilde{b}}, F_{\tilde{b}}>$ be two single valued trapezoidal neutrosophic numbers.

(i) $S(\tilde{a}) < S(\tilde{b})$ iff $\tilde{a} < \tilde{b}$
(ii) $S(\tilde{a}) > S(\tilde{b})$ iff $\tilde{a} > \tilde{b}$
(iii) $S(\tilde{a}) = S(\tilde{b})$ iff $\tilde{a} = \tilde{b}$

3 Harmonic averaging operators of SVTN numbers

Based on the basis of harmonic operation on single valued trapezoidal neutrosophic numbers, we propose single-valued trapezoidal neutrosophic weighted harmonic averaging (SVTNWHA) operator, single-valued trapezoidal neutrosophic ordered weighted harmonic averaging (SVT-NOWHA) operator and single valued trapezoidal neutrosophic generalized ordered weighted harmonic averaging (SVTNGOWHA) operator. In this case,the reordering step is developed with order-inducing variables that reflect a more complex reordering process.

Definition 3.1 Let $\tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j}>, (j = 1, 2, \dots, n)$ be a collection of single valued trapezoidal neutrosophic numbers. Then, SVTNWHA operator is a function SVTNWHA : $R^n \to R$ is defined as

$$SVTNWHA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{\tilde{a}_j}\right)} \tag{2}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighted vector of $\tilde{a}_j$ and $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$. Especially, when $\omega_i = 0$ and $\omega_j = 1, (i \neq j, i = 1, 2, \dots, n)$, we have SVTNWHA$(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \tilde{a}_j$; when $\omega = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})^T$, we have SVTNWHA$(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{1}{\tilde{a}_j}\right)}$

Definition 3.2 Let $\tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j}>$ be a collection of single valued trapezoidal neutrosophic numbers. Then, SVTNOWHA operator is a function SVTNOWHA : $R^n \to R$ is defined as

$$SVTNOWHA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{\tilde{b}_j}\right)} \tag{3}$$

Where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighted vector and $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$. Where $\tilde{b}_j$ is the largest j^{th} element in the collection of $\tilde{a}_j, j = (1, 2, \dots, n)$.

Definition 3.3 Let $\tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j}>$ be a collection of single valued trapezoidal neutrosophic numbers. Then, SVTNGOWHA operator is a function SVTNGOWHA : $R^n \to R$ is defined as

$$SVTNGOWHA(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{\tilde{b}_j^{\lambda}}\right)^{\frac{1}{\lambda}}} \tag{4}$$

where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weighted vector with $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$.

Where $\tilde{b}_j$ is the largest j^{th} element in the collection of $\tilde{a}_j$. $\tilde{b}_j = <(b_{j1}, b_{j2}, b_{j3}, b_{j4}); T_{\tilde{b}j}, I_{\tilde{b}j}, F_{\tilde{b}j}>$ is reordering of the individual collection of $\tilde{a}_j$. where $\lambda \in R$ is a parameter.

By using arithmetic operations on single valued trapezoidal neutrosophic numbers, we get the following theorem.

Theorem 3.4 *Let $\tilde{a}_j = <(a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j}>, (j = 1, 2, \dots, n)$ be a collection of single valued trapezoidal Neutrosophic number and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be a weighted vector of $\tilde{a}_j$, $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$ and the parameter $\lambda \in R$, then the aggregation value by utilizing the operator is defined as*

$$SVTNGOWHA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{\tilde{b}_j^{\lambda}}\right)^{\frac{1}{\lambda}}}$$

$$= \left\langle \left(\frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j1})^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j2})^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j3})^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j4})^{\lambda}}\right)^{\frac{1}{\lambda}}} \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

Proof This theorem can be proved by mathematical inductions.

Consider $\lambda > 0$, When n=2, then SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2)$is calculated as follows:

$$\frac{\omega_1}{\tilde{b}_1^{\lambda}} = \frac{\omega_1}{<(b_{11}^{\lambda}, b_{12}^{\lambda}, b_{13}^{\lambda}, b_{14}^{\lambda}); T_{\tilde{b}1}, I_{\tilde{b}1}, F_{\tilde{b}1}>}$$

and

$$\frac{\omega_2}{\tilde{b}_2^{\lambda}} = \frac{\omega_2}{<(b_{21}^{\lambda}, b_{22}^{\lambda}, b_{23}^{\lambda}, b_{24}^{\lambda}); T_{\tilde{b}2}, I_{\tilde{b}2}, F_{\tilde{b}2}>}$$

$$\frac{\omega_1}{\tilde{b}_1^{\lambda}} + \frac{\omega_2}{\tilde{b}_2^{\lambda}} = \frac{\omega_1}{<(b_{11}^{\lambda}, b_{12}^{\lambda}, b_{13}^{\lambda}, b_{14}^{\lambda}); T_{\tilde{b}1}, I_{\tilde{b}1}, F_{\tilde{b}1}>} + \frac{\omega_2}{<(b_{21}^{\lambda}, b_{22}^{\lambda}, b_{23}^{\lambda}, b_{24}^{\lambda}); T_{\tilde{b}2}, I_{\tilde{b}2}, F_{\tilde{b}2}>}$$

$$\frac{1}{\frac{\omega_1}{\tilde{b}_1^{\lambda}} + \frac{\omega_2}{\tilde{b}_2^{\lambda}}} = \frac{1}{\frac{\omega_1}{<(b_{11}^{\lambda}, b_{12}^{\lambda}, b_{13}^{\lambda}, b_{14}^{\lambda}); T_{\tilde{b}1}, I_{\tilde{b}1}, F_{\tilde{b}1}>} + \frac{\omega_2}{<(b_{21}^{\lambda}, b_{22}^{\lambda}, b_{23}^{\lambda}, b_{24}^{\lambda}); T_{\tilde{b}2}, I_{\tilde{b}2}, F_{\tilde{b}2}>}}$$

$$= \frac{1}{\omega_1 <(\frac{1}{b_{14}^{\lambda}}, \frac{1}{b_{13}^{\lambda}}, \frac{1}{b_{12}^{\lambda}}, \frac{1}{b_{11}^{\lambda}}); T_{\tilde{b}1}, I_{\tilde{b}1}, F_{\tilde{b}1}> + \omega_2 <(\frac{1}{b_{24}^{\lambda}}, \frac{1}{b_{23}^{\lambda}}, \frac{1}{b_{22}^{\lambda}}, \frac{1}{b_{21}^{\lambda}}); T_{\tilde{b}2}, I_{\tilde{b}2}, F_{\tilde{b}2}>}$$

$$= \frac{1}{<(\frac{\omega_1}{b_{14}^{\lambda}} + \frac{\omega_2}{b_{24}^{\lambda}}), (\frac{\omega_1}{b_{13}^{\lambda}} + \frac{\omega_2}{b_{23}^{\lambda}}), (\frac{\omega_1}{b_{12}^{\lambda}} + \frac{\omega_2}{b_{22}^{\lambda}}), (\frac{\omega_1}{b_{11}^{\lambda}} + \frac{\omega_2}{b_{21}^{\lambda}}); \min(T_{\tilde{b}1}, T_{\tilde{b}2}), \max(I_{\tilde{b}1}, I_{\tilde{b}2}), \max(F_{\tilde{b}1}, F_{\tilde{b}2})>}$$

$$=< (\frac{1}{(\frac{\omega_1}{b_{11}^{\lambda}} + \frac{\omega_2}{b_{21}^{\lambda}})}, \frac{1}{(\frac{\omega_1}{b_{12}^{\lambda}} + \frac{\omega_2}{b_{22}^{\lambda}})}, \frac{1}{(\frac{\omega_1}{b_{13}^{\lambda}} + \frac{\omega_2}{b_{23}^{\lambda}})}, \frac{1}{(\frac{\omega_1}{b_{14}^{\lambda}} + \frac{\omega_2}{b_{24}^{\lambda}})}); \min(T_{\tilde{b}1}, T_{\tilde{b}2}), \max(I_{\tilde{b}1}, I_{\tilde{b}2}), \max(F_{\tilde{b}1}, F_{\tilde{b}2}) >$$

Therefore SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2) = \frac{1}{(\frac{\omega_1}{\tilde{b}_1^{\lambda}} + \frac{\omega_2}{\tilde{b}_2^{\lambda}})^{\frac{1}{\lambda}}}$

$$= \left\langle \left(\frac{1}{\left(\frac{\omega_1}{b_{11}^{\lambda}} + \frac{\omega_2}{b_{21}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\frac{\omega_1}{b_{12}^{\lambda}} + \frac{\omega_2}{b_{22}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\frac{\omega_1}{b_{13}^{\lambda}} + \frac{\omega_2}{b_{23}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\frac{\omega_1}{b_{14}^{\lambda}} + \frac{\omega_2}{b_{24}^{\lambda}}\right)^{\frac{1}{\lambda}}}); \min(T_{\tilde{b}1}, T_{\tilde{b}2}), \max(I_{\tilde{b}1}, I_{\tilde{b}2}), \max(F_{\tilde{b}1}, F_{\tilde{b}2} \right) \right\rangle$$

Then the result is true for n = 2 and it is assumed that the result holds for n = k.

SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_k) =$

$$= \left\langle \left(\frac{1}{\left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j1}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j2}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j3}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j4}^{\lambda}}\right)^{\frac{1}{\lambda}}} \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

For n = k + 1, using the above result and arithmetic operations laws, we have

SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_k, \tilde{a}_{k+1}) = \frac{1}{(\sum_{j=1}^{k} \frac{\omega_j}{\tilde{b}_j^{\lambda}} + \frac{\omega_{k+1}}{\tilde{b}_{k+1}^{\lambda}})^{\frac{1}{\lambda}}; \alpha, \beta, \gamma}$

$$= \frac{1}{\left\langle \left(\left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j4}^{\lambda}} + \frac{\omega_{k+1}}{b_{k+14}^{\lambda}} \right), \left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j3}^{\lambda}} + \frac{\omega_{k+1}}{b_{k+13}^{\lambda}} \right), \left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j2}^{\lambda}} + \frac{\omega_{k+1}}{b_{k+12}^{\lambda}} \right), \left(\sum_{j=1}^{k} \frac{\omega_j}{b_{j1}^{\lambda}} + \frac{\omega_{k+1}}{b_{k+11}^{\lambda}} \right) \right); \alpha, \beta, \gamma \right\rangle}$$

Where $\alpha = \min(T_{\tilde{b}j}, T_{\tilde{b}(k+1)}), \beta = \max(I_{\tilde{b}j}, I_{\tilde{b}(k+1)}), \gamma = \max(F_{\tilde{b}j}, F_{\tilde{b}(k+1)})$

$$= \left\langle \left(\frac{1}{\sum_{j=1}^{k+1} \left(\frac{\omega_j}{b_{j1}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\sum_{j=1}^{k+1} \left(\frac{\omega_j}{b_{j2}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\sum_{j=1}^{k+1} \left(\frac{\omega_j}{b_{j3}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\sum_{j=1}^{k+1} \left(\frac{\omega_j}{b_{j4}^{\lambda}}\right)^{\frac{1}{\lambda}}} \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

Then the result is true for all n. Similarly the result is considered for $\lambda < 0$, theorem can be proved easily. Thus, mathematical induction method, the proof of theorem is completed.

3.1 Analyzing the weighted vector

If the weighted vector $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then we have single valued trapezoidal neutrosophic generalized order weighted harmonic averaging (SVTNGOWHA) operator which is reduced to generalized harmonic mean operator with neutrosophic environment.

If the weighted vector $\omega = (1, 0, 0, \ldots, 0)^T$, then we get maximum value SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \max_j \{\tilde{a}_j\}$ and if the weighted vector $\omega = (0, 0, 0, \ldots, 1)^T$, then we get minimum value SVTNGOWHA$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \min_j \{\tilde{a}_j\}$

3.2 Analyzing the parameter λ

(i) If the parameter $\lambda = 1$, then the operator SVTNGOWHA reduces to ordered weighted harmonic averaging (OWHA) operator with trapezoidal neutrosophic number.

$$SVTNOWHA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{\tilde{b}_j}\right)} \quad (5)$$

SVTNOWHA

$$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left\langle \left(\frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j1})}\right)}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j2})}\right)}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j3})}\right)}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j4})}\right)} \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

(ii) If the parameter $\lambda = 2$, then the operator SVTNGOWHA reduces to generalized ordered weighted quadratic harmonic averaging (GOWQHA) operator with trapezoidal neutrosophic number.

$$SVTNGOWQHA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{b_j^2}\right)^{\frac{1}{2}}} \quad (6)$$

SVTNGOWQHA

$$= \left\langle \left(\frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j1})^2}\right)^{\frac{1}{2}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j2})^2}\right)^{\frac{1}{2}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j3})^2}\right)^{\frac{1}{2}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j4})^2}\right)^{\frac{1}{2}}} \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

(iii) If the parameter $\lambda = -1$, then the operator SVTNGOWHA reduces to ordered weighted averaging (OWA) operator with trapezoidal neutrosophic number.

$$SVTNOWA(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left(\sum_{j=1}^{n} \frac{\omega_j}{\tilde{b}_j} \right) \quad (7)$$

SVTNOWA

$$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \left\langle \left(\left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j1})} \right), \left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j2})} \right), \left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j3})} \right), \left(\sum_{j=1}^{n} \frac{\omega_j}{(\tilde{b}_{j4})} \right) \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

(iv) If the parameter $\lambda \to 0$, then the operator SVTNGOWHA reduces to ordered weighted geometric average (OWGA) operator with trapezoidal neutrosophic number and which operator is based on the L' Hospital's rule.

(v) If the parameter $\lambda \to -\infty$, then SVTNGOWHA$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \max_j \tilde{a}_j$

(vi) If the parameter $\lambda \to +\infty$, then SVTNGOWHA$(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \min_j \tilde{a}_j$

Theorem 3.5 (Monotonicity) *Let* $\tilde{a}_j =< (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j} >$ *and* $\tilde{a}'_j =< (a'_{j1}, a'_{j2}, a'_{j3}, a'_{j4}); T'_{\tilde{a}j}, I'_{\tilde{a}j}, F'_{\tilde{a}j} >, (j = 1, 2, \ldots, n)$ *be two collections of SVTN numbers. If* $\tilde{b}_j \leq \tilde{b}'_j$ *for* $j = 1, 2, \ldots, n$*. Then SVTNGOWHA* $(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) \leq$ *SVTNGOWHA* $(\tilde{a}'_1, \tilde{a}'_2, \ldots, \tilde{a}'_n)$

Proof **Case I:** For $\lambda > 0$

Since $b_{j1} \leq b'_{j1} \Rightarrow b_{j1}^{\lambda} \leq (b'_{j1})^{\lambda} \Rightarrow \frac{\omega_j}{b_{j1}^{\lambda}} \geq \frac{\omega_j}{(b'_{j1})^{\lambda}}, (\omega_j > 0) \forall j$ $\Rightarrow (\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}} \geq (\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda}})^{\frac{1}{\lambda}} \Rightarrow \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda}})^{\frac{1}{\lambda}}}$

Similarly,
$\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j2}^{\lambda}})^{\frac{1}{\lambda}}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j2})^{\lambda}})^{\frac{1}{\lambda}}}$ and $\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j3}^{\lambda}})^{\frac{1}{\lambda}}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j3})^{\lambda}})^{\frac{1}{\lambda}}}$
$\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j4}^{\lambda}})^{\frac{1}{\lambda}}} \leq \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j4})^{\lambda}})^{\frac{1}{\lambda}}}$

Since $T_{\tilde{b}j} \leq T'_{\tilde{b}j} \Rightarrow \min_j (T_{\tilde{b}j}) \leq \min_j (T'_{\tilde{b}j}), \forall j$,
$I_{\tilde{b}j} \geq I'_{\tilde{b}j} \Rightarrow \max_j (I_{\tilde{b}j}) \geq \max_j (I'_{\tilde{b}j}), \forall j$,
$F_{\tilde{b}j} \geq F'_{\tilde{b}j} \Rightarrow \max_j (F_{\tilde{b}j}) \geq \max_j (F'_{\tilde{b}j}), \forall j$,
Hence,

$$\left\langle \left(\frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{b_{j2}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{b_{j3}^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{b_{j4}^{\lambda}}\right)^{\frac{1}{\lambda}}} \right); \min_j (T_{\tilde{b}j}), \max_j (I_{\tilde{b}j}), \max_j (F_{\tilde{b}j}) \right\rangle \le$$

$$\left\langle \left(\frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j2})^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j3})^{\lambda}}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j4})^{\lambda}}\right)^{\frac{1}{\lambda}}} \right); \min_j (T'_{\tilde{b}j}), \max_j (I'_{\tilde{b}j}), \max_j (F'_{\tilde{b}j}) \right\rangle$$

$\Rightarrow$SVTNGOWHA$(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \le$ SVTNGOWHA$(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n)$

Case II: For $\lambda < 0$

Since $b_{j1} \le b'_{j1} \Rightarrow b_{j1}^{\lambda} \ge (b'_{j1})^{\lambda}, \forall j$

$\Rightarrow \frac{\omega_j}{b_{j1}^{\lambda}} \le \frac{\omega_j}{(b'_{j1})^{\lambda}}, (\omega_j > 0) \Rightarrow (\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}} \ge (\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda}})^{\frac{1}{\lambda}}$

$\Rightarrow \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{(b'_{j1})^{\lambda}})^{\frac{1}{\lambda}}}$

In the same way as case I it can be proved.

Theorem 3.6 (Idempotency) *Let $\tilde{a}_j =< (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j} >, (j = 1, 2, \dots, n)$ be a collection of single valued trapezoidal neutrosophic number. If all $\tilde{a}_j$ are equal, $\tilde{a}_j = \tilde{a}, (j = 1, 2, \dots n)$, then SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) =$ SVTNGOWHA $(\tilde{a}, \tilde{a}, \dots, \tilde{a}) = \tilde{a}$.*

Theorem 3.7 (Commutativity) *If $(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n)$ is any permutation of $(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$, then SVTNGOWHA $(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) =$ SVTNGOWHA $(\tilde{a}'_1, \tilde{a}'_2, \dots, \tilde{a}'_n)$.*

Theorem 3.8 (Boundedness) *Let $\tilde{a}_j =< (a_{j1}, a_{j2}, a_{j3}, a_{j4}); T_{\tilde{a}j}, I_{\tilde{a}j}, F_{\tilde{a}j} >, (j = 1, 2, \dots, n)$ be a collection of SVTN numbers and Let $\tilde{a}_j^+ = \langle(\min_j b_{j1}, \min_j b_{j2}, \min_j b_{j3}, \min_j b_{j4}); \min_j (T_{\tilde{b}j}), \max_j (I_{\tilde{b}j}), \max_j (F_{\tilde{b}j})\rangle$*

$\tilde{a}_j^- = \langle(\max_j b_{j1}, \max_j b_{j2}, \max_j b_{j3}, \max_j b_{j4}); \min_j (T_{\tilde{b}j}), \max_j (I_{\tilde{b}j}), \max_j (F_{\tilde{b}j})\rangle$

Then $\tilde{a}^- \le$ SVTNGOWHA$(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) \le \tilde{a}^+$.

Proof **Case I:** For $\lambda > 0$

Since $\min\{b_{j1}\} \le b_{j1} \le \max\{b_{j1}\}$

$\Rightarrow \frac{\omega_j}{\min\{b_{j1}\}^{\lambda}} \ge \frac{\omega_j}{b_{j1}^{\lambda}} \ge \frac{\omega_j}{\max\{b_{j1}\}^{\lambda}}, (\omega_j > 0)$

$\Rightarrow (\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}} \ge (\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}} \ge (\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}}$

$\Rightarrow \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}}}$

Similarly, $\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j2}\}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j2}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j2}\}^{\lambda}})^{\frac{1}{\lambda}}}$

$\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j3}\}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j3}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j3}\}^{\lambda}})^{\frac{1}{\lambda}}}$

$\frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j4}\}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j4}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j4}\}^{\lambda}})^{\frac{1}{\lambda}}}$

Also $\min\{T_{\tilde{b}j}\} \le T_{\tilde{b}j} \le \max\{T_{\tilde{b}j}\}, \forall j$ and $\min\{I_{\tilde{b}j}\} \le I_{\tilde{b}j} \le \max\{I_{\tilde{b}j}\}, \forall j$
$\min\{I_{\tilde{b}j}\} \le I_{\tilde{b}j} \le \max\{I_{\tilde{b}j}\}, \forall j$ By using the properties of monotonicity and Idempotency, we get SVTNGOWHA $(\tilde{a}_j^-) \le$ SVTNGOWHA $(\tilde{a}_j) \le$ SVTNGOWHA$(\tilde{a}_j^+) \Rightarrow \tilde{a}_j^- \le \tilde{a}_j \le \tilde{a}_j^+, \forall j$.

Case II: For $\lambda < 0$

Table 1 Decision matrix provided by expert d_1

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	< (2, 4, 6, 8); 0.5, 0.4, 0.8 >	< (2, 4, 6, 7); 0.7, 0.2, 0.5 >	< (17, 18, 19, 20); 0.6, 0.3, 0.4 >	< (3, 4, 6, 7); 0.7, 0.1, 0.4 >
$\tilde{a}_2$	< (3, 5, 6, 7); 0.6, 0.3, 0.4 >	< (15, 17, 19, 20); 0.7, 0.2, 0.4 >	< (3, 4, 5, 6); 0.7, 0.2, 0.6 >	< (4, 5, 6, 7); 0.6, 0.4, 0.3 >
$\tilde{a}_3$	< (1, 2, 3, 4); 0.7, 0.2, 0.5 >	< (2, 3, 4, 5); 0.5, 0.4, 0.3 >	< (2, 4, 5, 6); 0.6, 0.4, 0.2 >	< (15, 16, 18, 20); 0.8, 0.1, 0.2 >

Table 2 Decision matrix provided by expert d_2

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(15,16,17,20); 0.9,0.1,0.4>$	$<(2,4,5,7); 0.5,0.3,0.6>$	$<(2,5,6,8); 0.7,0.2,0.5>$	$<(3,5,6,7); 0.8,0.1,0.3>$
$\tilde{a}_2$	$<(4,5,6,7); 0.6,0.3,0.4>$	$<(16,17,19,20); 0.8,0.2,0.1>$	$<(3,4,5,6); 0.7,0.2,0.5>$	$<(4,5,6,9); 0.6,0.3,0.5>$
$\tilde{a}_3$	$<(1,3,5,6); 0.6,0.4,0.3>$	$<(2,3,4,6); 0.6,0.3,0.4>$	$<(2,3,4,5); 0.6,0.4,0.2>$	$<(17,18,19,20); 0.6,0.3,0.7>$

Table 3 Decision matrix provided by expert d_3

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(4,5,6,8); 0.5,0.4,0.3>$	$<(1,2,3,4); 0.7,0.2,0.5>$	$<(17,18,19,20); 0.6,0.25,0.3>$	$<(3,4,5,6); 0.7,0.1,0.4>$
$\tilde{a}_2$	$<(3,5,6,7); 0.6,0.2,0.4>$	$<(2,3,4,6); 0.6,0.3,0.8>$	$<(3,4,5,6); 0.7,0.2,0.6>$	$<(16,17,19,20); 0.8,0.2,0.1>$
$\tilde{a}_3$	$<(16,17,18,20); 0.8,0.1,0.3>$	$<(4,5,6,7); 0.5,0.4,0.3>$	$<(2,4,5,6); 0.6,0.4,0.1>$	$<(3,4,6,7); 0.7,0.2,0.5>$

Table 4 Normalized decision matrix provided by expert d_1

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(0.05,0.108, 0.2, 0.33); 0.5,0.4,0.8>$	$<(0.05,0.108, 0.2, 0.29); 0.5,0.4,0.8>$	$<(0.405,0.49, 0.63, 0.83); 0.5,0.4,0.8>$	$<(0.07,0.108, 0.2, 0.29); 0.5,0.4,0.8>$
$\tilde{a}_2$	$<(0.08,0.139, 0.194, 0.28); 0.6,0.4,0.6>$	$<(0.38,0.47, 0.613, 0.8); 0.6,0.4,0.6>$	$<(0.08,0.11, 0.16, 0.24); 0.6,0.4,0.6>$	$<(0.1,0.139, 0.194, 0.28); 0.6,0.4,0.6>$
$\tilde{a}_3$	$<(0.02,0.07, 0.12, 0.2); 0.5,0.4,0.5>$	$<(0.04,0.1, 0.16, 0.25); 0.5,0.4,0.5>$	$<(0.04,0.13, 0.2, 0.3); 0.5,0.4,0.5>$	$<(0.33,0.53, 0.72, 0.8); 0.5,0.4,0.5>$

Table 5 Normalized decision matrix provided by expert d_2

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(0.36,0.47, 0.57, 0.91); 0.5,0.3,0.6>$	$<(0.048,0.12, 0.17, 0.32); 0.5,0.3,0.6>$	$<(0.048,0.147, 0.2, 0.364); 0.5,0.3,0.6>$	$<(0.07,0.15, 0.2, 0.32); 0.5,0.3,0.6>$
$\tilde{a}_2$	$<(0.1,0.139, 0.194, 0.26); 0.6,0.3,0.5>$	$<(0.4,0.47, 0.61, 0.7); 0.6,0.3,0.5>$	$<(0.07,0.111, 0.16, 0.22); 0.6,0.3,0.5>$	$<(0.1,0.139, 0.194, 0.33); 0.6,0.3,0.5>$
$\tilde{a}_3$	$<(0.027,0.09, 0.185, 0.273); 0.6,0.4,0.7>$	$<(0.05,0.09, 0.148, 0.273); 0.6,0.4,0.7>$	$<(0.05,0.09, 0.148, 0.227); 0.6,0.4,0.7>$	$<(0.459,0.563, 0.704, 0.91); 0.6,0.4,0.7>$

Since $\min\{b_{j1}\} \le b_{j1} \le \max\{b_{j1}\} \Rightarrow \frac{\omega_j}{\min\{b_{j1}\}^{\lambda}} \le \frac{\omega_j}{b_{j1}^{\lambda}} \le \frac{\omega_j}{\max\{b_{j1}\}^{\lambda}}, (\omega_j > 0) \Rightarrow (\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}} \ge (\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}} \ge (\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}}$

$$\Rightarrow \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\min\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{b_{j1}^{\lambda}})^{\frac{1}{\lambda}}} \le \frac{1}{(\sum_{j=1}^{n} \frac{\omega_j}{\max\{b_{j1}\}^{\lambda}})^{\frac{1}{\lambda}}}$$

In the same way as case I it can be proved.

Table 6 Normalized decision matrix provided by expert d_3

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(0.11, 0.15, 0.21, 0.32); 0.5, 0.4, 0.5>$	$<(0.03, 0.06, 0.1, 0.16); 0.5, 0.4, 0.5>$	$<(0.45, 0.55, 0.66, 0.8); 0.5, 0.4, 0.5>$	$<(0.08, 0.12, 0.17, 0.24); 0.5, 0.4, 0.5>$
$\tilde{a}_2$	$<(0.08, 0.15, 0.21, 0.29); 0.6, 0.3, 0.8>$	$<(0.05, 0.09, 0.14, 0.25); 0.6, 0.3, 0.8>$	$<(0.08, 0.12, 0.17, 0.25); 0.6, 0.3, 0.8>$	$<(0.4, 0.59, 0.66, 0.83); 0.6, 0.3, 0.8>$
$\tilde{a}_3$	$<(0.4, 0.49, 0.6, 0.8); 0.5, 0.4, 0.5>$	$<(0.1, 0.14, 0.2, 0.28); 0.5, 0.4, 0.5>$	$<(0.05, 0.11, 0.167, 0.24); 0.5, 0.4, 0.5>$	$<(0.08, 0.11, 0.2, 0.28); 0.5, 0.4, 0.5>$

Table 7 Individual overall attributes values with SVTNNGOWHA operator

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(0.086, 0.143, 0.216, 0.342); 0.5, 0.4, 0.8>$	$<(0.041, 0.089, 0.144, 0.240); 0.5, 0.4, 0.8>$	$<(0.136, 0.3, 0.4, 0.607); 0.5, 0.4, 0.8>$	$<(0.071, 0.118, 0.19, 0.276); 0.5, 0.4, 0.8>$
$\tilde{a}_2$	$<(0.081, 0.140, 0.195, 0.275); 0.6, 0.4, 0.8>$	$<(0.138, 0.22, 0.323, 0.475); 0.6, 0.4, 0.8>$	$<(0.08, 0.111, 0.161, 0.235); 0.6, 0.4, 0.8>$	$<(0.105, 0.147, 0.203, 0.328); 0.6, 0.4, 0.8>$
$\tilde{a}_3$	$<(0.026, 0.088, 0.168, 0.259); 0.5, 0.4, 0.7>$	$<(0.048, 0.099, 0.159, 0.267); 0.5, 0.4, 0.7>$	$<(0.047, 0.105, 0.163, 0.240); 0.5, 0.4, 0.7>$	$<(0.182, 0.263, 0.421, 0.538); 0.5, 0.4, 0.7>$

4 Decision making method with trapezoidal neutrosophic number

In this section, we present an approach to multi attribute decision making based on the SVTNGOWHA operator with the help of score function for trapezoidal neutrosophic numbers.

Let $\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_m)$ be a set of m attributes and $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n)$ be the set of n attributes related to alternatives weighted vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ being its weighting vector, which is used to represent the importance weights of different attributes,

where $0 \leq \omega_j \leq 1 (j = 1, 2, \ldots, n)$ and $\sum_{j=1}^{n} \omega_j = 1$. Let $D = (d_1, d_2, \ldots, d_t)$ be the set of decision makers (Expert) with weighting vector $\eta = (\eta_1, \eta_2, \ldots, \eta_t)^T$,

where $0 \leq \eta_k \leq 1 (k = 1, 2, \ldots, t)$ and $\sum_{k=1}^{t} \eta_k = 1$. In this problem, decision makers evaluate each alternative with trapezoidal neutrosophic numbers according to each criterion. Thus, we obtain trapezoidal neutrosophic decision matrix as follows

$\tilde{a}^k = (\tilde{a}^k_{ij})_{m \times n} = (< (a^k_{ij1}, a^k_{ij2}, a^k_{ij3}, a^k_{ij4}); T^k_{ij}, I^k_{ij}, F^k_{ij} >)_{(m \times n)}$ provided by an expert decision maker D. where T_{ij}, I_{ij}, F_{ij} defined on truth, an indeterminacy and falsity membership function and $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$ with $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3, a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4} \in R$. Multi-attribute decision making problem contains benefit and cost attribute.

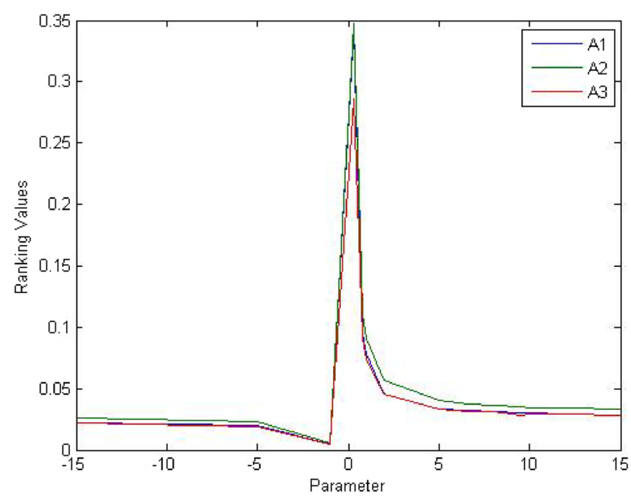

Fig. 1 Ranking values of alternatives with respect to parameter λ in SVTNGOWHA operator

In this procedure, we consider linear scale transformation (sum) which divides the performance ratings of each attribute by the sum of performance ratings for that attribute. Herein, the following algorithm is proposed to obtain the solution of the multi-attribute decision-making problem with the trapezoidal neutrosophic numbers information by using SVTNGOWHA operator with score function.

Table 8 Decision-making results of different aggregation operators

Method	Operator	Ranking order	Best alternative
Ye (2016a)	TNWAA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
	TNWGA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
Bharatraj and Anand (2019)	PHWAOSVTrNN	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
Chiranjibe et al. (2020)	SVTNHWA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
	SVTNHWGA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
Wang et al. (2020)	SVTNPA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
	SVTNPG	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
Proposed Method	SVTNWHA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
	SVTNOWHA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$
	SVTNGOWHA	$\tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$	$\tilde{a}_2$

Table 9 Linguistic values of trapezoidal neutrosophic numbers for the linguistic term set

Linguistic term	Linguistic value
Extremely low priority (ELP)	$<(0,0,0.1,0.2);0.6,0.2,0.4>$
Low priority (LP)	$<(0.1,0.11,0.2,0.3);0.5,0.1,0.3>$
Simple priority (SP)	$<(0.2,0.3,0.4,0.5);0.8,0.2,0.2>$
Medium priority (HP)	$<(0.4,0.5,0.6,0.7);0.9,0.2,0.1>$
High priority (HP)	$<(0.6,0.7,0.8,0.9);0.9,0.1,0.1>$

Algorithm:

Step 1: Compute the normalized decision making matrix

For benefit attributes, the normalized value r_{ij}^k is obtained by

$r_{ij}^k = \frac{\tilde{a}_{ij}^k}{\sum_{j=1}^n \tilde{a}_{ij}^k}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$ and $k = 1,2,\ldots,t$

For cost attributes, the normalized value r_{ij}^k is obtained by

$r_{ij}^k = \frac{\frac{1}{\tilde{a}_{ij}^k}}{\sum_{j=1}^n (\frac{1}{\tilde{a}_{ij}^k})}, i = 1,2,\ldots,m, j = 1,2,\ldots,n$ and $k = 1,2,\ldots,t$

Step 2: Utilize SVTNGOWHA aggregation operator which computes the individual overall ratings of all the alternatives.

$$\text{SVTNGOWHA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n) = \left\langle \left(\frac{1}{\left(\sum_{j=1}^n \frac{\omega_j}{(\tilde{b}_{j1})^\lambda}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^n \frac{\omega_j}{(\tilde{b}_{j2})^\lambda}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^n \frac{\omega_j}{(\tilde{b}_{j3})^\lambda}\right)^{\frac{1}{\lambda}}}, \frac{1}{\left(\sum_{j=1}^n \frac{\omega_j}{(\tilde{b}_{j4})^\lambda}\right)^{\frac{1}{\lambda}}} \right); \min_j T_{\tilde{b}j}, \max_j I_{\tilde{b}j}, \max_j F_{\tilde{b}j} \right\rangle$$

Step 3: Utilizing SVTNGOWHA aggregation operator we obtain the comprehensive attribute value of the alternatives value. Then the collection of single valued trapezoidal neutrosophic number decision matrix $S = (S_{ij})_{m\times n}$ is as follows:

$S_i(\omega) = \frac{1}{\sum_{j=1}^n (\frac{\omega_j}{S_{ij}})}, i = 1,2,\ldots,m$

Step 4: Ranking of the alternatives.

Rank the comprehensive attribute value $S_i(\omega)$ evaluated on the scoring function based on single valued trapezoidal neutrosophic number.

Step 5: End.

Table 10 Evaluation of criteria by three experts using linguistic variables

Expert 1	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	ELP	1/SP	MP	1/LP
$\tilde{a}_2$	SP	ELP	MP	1/LP
$\tilde{a}_3$	1/MP	1/MP	ELP	1/MP
$\tilde{a}_4$	LP	LP	MP	ELP
Expert 2	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	ELP	1/LP	SP	1/LP
$\tilde{a}_2$	LP	ELP	SP	ELP
$\tilde{a}_3$	1/SP	1/SP	ELP	1/LP
$\tilde{a}_4$	LP	ELP	LP	ELP
Expert 3	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	ELP	1/LP	LP	1/SP
$\tilde{a}_2$	LP	ELP	MP	LP
$\tilde{a}_3$	1/LP	1/MP	ELP	1/SP
$\tilde{a}_4$	SP	1/LP	SP	ELP

Table 11 Individual overall attributes values with SVTNNGOWHA operator

Alternatives	$\tilde{c}_1$	$\tilde{c}_2$	$\tilde{c}_3$	$\tilde{c}_4$
$\tilde{a}_1$	$<(0,0,0.1,0.2);0.6,0.2,0.4>$	$<(2.81,3.95,6.19,7.89);0.5,0.2,0.3>$	$<(0.16,0.209,0.32,0.43);0.8,0.2,0.3>$	$<(2.81,3.95,6.194,7.893);0.5,0.2,0.3>$
$\tilde{a}_2$	$<(0.104,0.115,0.207,0.308);0.5,0.2,0.3>$	$<(0,0,0.1,0.2);0.6,0.2,0.4>$	$<(0.316,0.424,0.529,0.633);0.8,0.2,0.1>$	$<(0,0,0.125,0.237);0.5,0.2,0.4>$
$\tilde{a}_3$	$<(1.852,2.274,2.918,4.054);0.5,0.2,0.3>$	$<(1.458,1.708,2.054,2.587);0.8,0.2,0.2>$	$<(0,0,0.1,0.2);0.6,0.2,0.4>$	$<(1.85,2.274,2.918,4.054);0.5,0.2,0.3>$
$\tilde{a}_4$	$<(0.12,0.1395,0.24,0.346);0.5,0.2,0.3>$	$<(0,0,0.167,0.281);0.5,0.2,0.4>$	$<(0.162,0.209,0.321,0.4315);0.5,0.2,0.3>$	$<(0,0,0.1,0.2);0.6,0.2,0.4>$

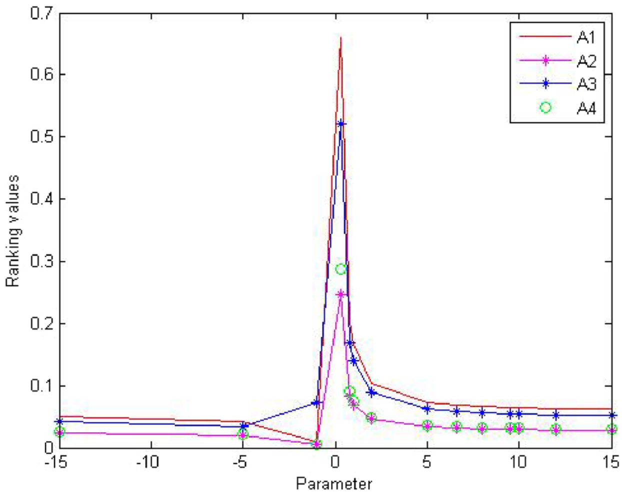

Fig. 2 Ranking values of Alternatives w.r.to parameter λ in SVTNGOWHA operator

Table 12 Decision-making results of different aggregation operators

Method	Operator	Ranking order	Best alternative
Ye (2016a)	TNWAA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
	TNWGA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
Bharatraj and Anand (2019)	PHWAOSVTrNN	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
	SVTNHWGA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
Wang et al. (2020)	SVTNPA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
	SVTNPG	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
Proposed Method	SVTNWHA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
	SVTNOWHA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$
	SVTNGOWHA	$\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$	$\tilde{a}_1$

4.1 Illustrative examples

In this section, we are going to develop an enterprise selection problem in order to illustrate the new approach. The following problem is adapted from Das and Guha (2017) and applied with SVTNGOWHA operator.

Example 1

Enterprise Selection Problem: A company wants to form a co-operative alliance with some potential enterprises to fulfill the market demand. After pre-evaluation, three enterprises $\tilde{a}_i, (i = 1, 2, 3)$ are selected for further evaluation. The expert unit selects the best enterprise on the basis of the following four attributes; $\tilde{c}_1$-Producing ability, $\tilde{c}_2$-the technology capability, $\tilde{c}_3$-Capital currency,$\tilde{c}_4$-Research ability. Let $\omega = (0.15, 0.35, 0.3, 0.2)^T$ be the weight vector of these four attributes. We obtain the decision matrices are listed in Tables 1, 2 and 3.

Step 1: Since the given attributes are benefit criteria. So, we need to normalize the decision matrix. Then the computations of normalization matrices are given by Tables 4, 5 and 6.

Step 2: Utilize SVTNGOWHA aggregation operator. Assume the parameter $\lambda = 1$ and the associated weighted vector $W = (0.067, 0.666, 0.267)^T$ which can be obtained by the fuzzy linguistic quantifier "most" with the pair of$(\alpha, \beta) = (0.3, 0.8)$. Then final aggregated values are given Table 7.

Step 3: Utilize the SVTNNGOWHA aggregation operator.

$S_1(\omega) =< (0.0653, 0.130, 0.203, 0.320);0.5, 0.4, 0.8 >$

$S_2(\omega) =< (0.099, 0.149, 0.213, 0.316);0.6, 0.4, 0.8 >$

$S_3(\omega) =< (0.049, 0.113, 0.185, 0.285);0.5, 0.4, 0.7 >$

Step 4: Finally, the aggregation results are obtained by method of score function of single valued trapezoidal neutrosophic number.

$S_1(\omega) = 0.078$, $S_2(\omega) = 0.0907$, $S_3(\omega)$ =0.0737

Then, we can rank the alternatives $\tilde{a}_j \in \tilde{a}$according to $S_i(\omega), i = 1, 2, 3, \tilde{a}_2 > \tilde{a}_1 > \tilde{a}_3$, we say that the enterprise $\tilde{a}_2$

will be first choice,$\tilde{a}_1$ and $\tilde{a}_3$ are second and third choice. Hence, the best enterprise is $\tilde{a}_2$.

Furthermore, we analyze the different parameter λ that deals with the aggregation results provided by best decision maker. We can consider different values of $\lambda : -15, -5, -1, \ldots, 0.8, 1, \ldots, 6.6, 10, \ldots 12, 15$ which are provided by the best decision maker. The variation of the aggregation results with parameter λ is shown in Fig. 1. We observed that the aggregation results, if λ decreases, $(\lambda < 0)$ the values will increase and if λ increases, $(\lambda > 0)$ the values will decrease. If we compared with different type of parameter λ the aggregation results of decision maker chosen the best alternative is $\tilde{a}_2$. Compared with other operators, we find that the main advantage of using the SVTNGOWHA operator we can consider a range. In this paper, the different values of parameter λ are considered sufficiently while Bharatraj and Anand (2019), Chiranjibe et al. (2020), Wang et al. Wang et al. (2020) did not consider the decision makers preference.

Table 8 will show that existing works and the proposed method which develop decision making approach using single valued trapezoidal neutrosophic numbers.

Example 2 To identify the effective allocation of the COVID-19 vaccine for priority groups, decision-makers must involve experts from multiple fields to get benefit from their experiences in setting priorities and principle guidelines. After pre-evaluation four alternatives $\tilde{a}_i, (i = 1, 2, 3, 4)$ are selected for further evaluations. The expert select the best priority group of the basis of the following four attributes:

$\tilde{c}_1$-Old,Adult and kids peoples with health problems,$\tilde{c}_2$ -People with high risk health problems, $\tilde{c}_3$-Breastfeeding problem, $\tilde{c}_4$-Healthcare personnel and Essential workers.

$\tilde{a}_1$ Age index (AC)

$\tilde{a}_2$ Health state index (HS)

$\tilde{a}_3$ Women state index (WS)

$\tilde{a}_4$ Job kind index (JK)

Let $\omega = (0.15, 0.35, 0.3, 0.2)^T$ be the weight vector of these four attributes. We obtain the decision matrices are listed in Tables 9 and 10.

Step 1: Since the given attributes are normalized benefit criteria. So, we need not normalize the decision matrix.

Step 2: Utilize SVTNGOWHA aggregation operator. Assume the parameter $\lambda = 1$ and the associated weighted vector $W = (0.067, 0.666, 0.267)^T$ which can be obtained by the fuzzy linguistic quantifier "most" with the pair of$(\alpha, \beta) = (0.3, 0.8)$. Then final aggregated values are given Table 11.

Step 3: Utilize the SVTNNGOWHA aggregation operator.

$S_1(\omega) =< (0, 0, 0.3965, 0.6601);0.5, 0.2, 0.4 >$

$S_2(\omega) =< (0, 0, 0.1564, 0.2812);0.5, 0.2, 0.4 >$

$S_3(\omega) =< (0, 0, 0.3039, 0.5808);0.5, 0.2, 0.4 >$

$S_4(\omega) =< (0, 0, 0.1767, 0.2963);0.5, 0.2, 0.4 >$

Step 4: Finally, the aggregation results are obtained by method of score function of single valued trapezoidal neutrosophic number.

$S_1(\omega) = 0.1673$, $S_2(\omega) = 0.0693$, $S_3(\omega)$ =0.1401, $S_4(\omega)$ =0.0749

Then, we can rank the alternatives $\tilde{a}_j \in \tilde{a}$according to $S_i(\omega), i = 1, 2, 3,$ $\tilde{a}_1 > \tilde{a}_3 > \tilde{a}_4 > \tilde{a}_2$, Hence, the best enterprise is $\tilde{a}_1$.

Furthermore, we analyze the different parameter λ that deals with the aggregation results provided by best decision maker.The variation of the aggregation results with parameter λ is shown in Fig. 2.

Table 12 will show that existing works and the proposed method which develop decision making approach using single valued trapezoidal neutrosophic numbers.

5 Conclusion

This paper introduced the single valued trapezoidal neutrosophic numbers with generalized ordered weighted harmonic averaging (SVTNGOWHA) operator, which provides general formulation that includes a wide range of aggregation operators and it combines with the generalized mean and the weighted harmonic averaging operator under single valued trapezoidal neutrosophic numbers. It can be applied in the selection of financial products, engineering, soft computing a decision theory under neutrosophic environment. The main advantages of the decision making approach based on the SVTNGOWHA operator is that the decision maker can obtain a complete view of decision making problem. The application of enterprise selection problem shows the feasibility and effectiveness for multi attribute decision making problems. In the future research, we can establish approaches of aggregation operators with single valued neutrosophic number and apply them in the fields of medical diagnosis, forecasting and project investment.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

Atanssov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96

Bera T, Mahapatra NK (2020) An approach to solve the linear programming problem using single valued trapezoidal neutrosophic

number. Int J Neutrosophic Sci. https://doi.org/10.5281/zenodo.3740647
Bharatraj J, Anand MCJ (2019) Power harmonic weighted aggregation operator on single valued trapezoidal neutrosophic numbers and interval-valued neutrosophic sets. Fuzzy Multi-criteria Decision Making using neutrosophic sets. Stud Fuzziness Soft Comput. https://doi.org/10.1007/978-3-030-00045-5-3
Biswas P, Pramanik S, Giri BC (2018) Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. Neutrosophic Sets Syst 19:20
Biswas P, Pramanik S, Giri BC (2018a) Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. New Trends Neutrosophic Theory Appl II:20
Biswas P, Pramanik S, Giri BC (2018b) TOPSIS strategy for multi-attribute decision making with TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. Neutrosophic Sets Syst 19:25
Das S, Guha D (2015) Power Harmonic aggregation operator with trapezoidal intuitionistic fuzzy numbers for solving MAGDM problems. Iran J Fuzzy Syst 12(6):41–74
Das S, Guha D (2017) Family of harmonic aggregation operators under intuitionistic fuzzy environment. Sci Iran E 24(6):3308–3323. https://doi.org/10.24200/sci.2017.4400
Deli I (2018) Operators on single valued trapezoidal neutrosophic numbers and SVTN-group decision making. Neutrosophic Sets Syst 22:20
Deli I (2019) A novel defuzzication method of SV-trapezoidal neutrosophic numbers and multi-attribute decision making: a comparative analysis. Soft Comput 23:12529–12545. https://doi.org/10.1007/s00500-019-03803-z
Deli I, Ozturk EK (2020) A defuzzification method on single-valued trapezoidal neutrosophic numbers and multiple attribute decision making. Cumhuriyet Sci J 41(1):22–37. https://doi.org/10.17776/csj.574518
Deli I, Subas Y (2014) Single valued neutrosophic numbers and their applications to multicriteria decision making problem. Neutrosophic Sets Syst 20:20
Deli I, Subas Y (2017) A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. J Mach Learn Cyber Int. https://doi.org/10.1007/s13042-0160-0505-3
Deli I, Yusuf S (2017) Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision making problems. J Intell Fuzzy Syst 32(1):291–301. https://doi.org/10.3233/JIFS-151677
Garai T, Garg H, Roy TK (2020) A ranking method based on possibility mean for multi-attribute decision making with single valued neutrosophic numbers. J Ambient Intell Human Comput 11:5245–5258. https://doi.org/10.1007/s12652-020-01853-y
Harish G Nancy (2018) Some hybrid weighted aggregation operators under neutrosophic set environment and their applications to multi criteria decision-making. Appl Intell. https://doi.org/10.1007/s10489-018-1244-9
Jana C, Pal M, Karaaslan F, Wang J (2018) Trapezoidal neutrosophic aggregation operators and its application in multiple attribute decision making process. Sci Iran. https://doi.org/10.24200/sci.2018.51136.2024
Jana C, Muhiuddin G, Pal M (2020) Multiple-attribute decision making problems based on SVTNH methods. J Ambient Intell Human Comput. https://doi.org/10.1007/s12652-019-01568-9
Jing W, Jian-qiang W, Yin-xiang M (2020) Possibility degree and power aggregation operators of single-valued trapezoidal neutrosophic numbers and applications to multi-criteria group-decision making. Cogn Comput. https://doi.org/10.1007/s12559-020-09736-2
Jinpei L, Huayou C, Ligang Z (2013) Group decision making approach based on the generalized hybrid harmonic averaging operators. J Appl Sci 13(8):1185–1191. https://doi.org/10.3923/jas.2013.1185.1191**(ISSN 1812–5654)**
Jinpei L, Huayou C, Ligang Z, Zhifu T, Yingdong H (2014) On the properties of the generalized OWHA operators and their application to group decision making. J Intell Fuzzy Syst 27:2077–2089. https://doi.org/10.3233/IFS-141173
Jun Y (2015) Simplified neutrosophic harmonic averaging projection based method for multiple for multiple attribute decision-making problems. J Mach Learn Cyber Int. https://doi.org/10.1007/s13042-015-0456-0
Mohamed Abdel-Basset M, Gunasekaran MM, Smarandache F (2019) A novel method for solving the fully neutrosophic linear programming problems. Neural Comput Appl 31:1595–1605. https://doi.org/10.1007/s00521-018-3404-6
Nancy HG (2019) Linguistic single-valued neutrosophic power aggregation operators and their applications to group decision-making problems. IEEE CAA J Autom Sin 20:20
Pramanik S, Mallick R (2018) VIKOR based MAGDM strategy with trapezoidal neutrosophic numbers. Neutrosophic Sets Syst 22:20
Pramanik S, Mallick R (2019) TODIM strategy for multi-attributes group decision making in trapezoidal in neutrosophic number environment. Complex Intell Syst 5:379–389. https://doi.org/10.1007/s40747-019-0110-7
Pramanik S, Mallick R (2020) MULTIMOORA strategy for solving multi-attribute group decision making (MAGDM) in trapezoidal neutrosophic number environment. CAAI Trans Intell Technol 5(3):150–156. https://doi.org/10.1049/trit.2019.0101
Said B, Lathamaheswari M, Tan R, Nagarajan D, Mohamed T, Smarandache F, Bakali A (2020) A new distance measure for trapezoidal fuzzy neutrosophic numbers based on the centroids. Neutrosophic Sets Syst 35:20
Shigui D, Ye J, Yong R, Zhang F (2020) Simplified neutrosophic indeterminate decision making method with decision makers' indeterminate ranges. J Civ Eng Manag 26(6):590–598. https://doi.org/10.3846/jcem.2020.12919**(ISSN 1392-3730/eISSN 1822-3605)**
Smarandache F (1998) Unifying field in logics. Neutrosophy: neutrosophic probability set and logic. American Research Press, Rehoboth
Smarandache F (2005) Neutrosophic set, a generalization of the intuitionistic fuzzy sets. Int J Pure Appl Math 24:287–297
Wan SP, Yi Z (2016) Power average of trapezoidal intuitionistic fuzzy numbers using strict t-norms and t-conorms. IEEE Trans Fuzzy Syst 24(5):1035–1047. https://doi.org/10.1109/TFUZZ.2015.2501408
Wan SP, Zhu YJ (2016) Triangular intuitionistic fuzzy triple Bonferroni harmonic mean operators and application to multi-attribute group decision making. Iran J Fuzzy Syst 13(5):117–145
Wang X, Fan Z (2003) Fuzzy ordered weighted averaging (FOWA) operator and its application. Fuzzy Syst Math 17(4):67–72
Wang J, Zhong Z (2009) Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems. J Syst Eng Electron 20(2):321–326
Wang H, Smarandache F, Zhang YQ, Sunderraman R (2005) Interval neutrosophic sets and logic: theory and applications in computing. Hexis, Phoenix
Wang H, Smarandache F, Zhang YQ, Sunderraman R (2010) Single valued neutrosophic sets. Multisp Multistruct 4:410–413
Xu Z, Da Q (2002) The ordered weighted geometric averaging operators. Int J Intell Syst 17(7):709–716
Ye J (2015) Trapezoidal neutrosophic set and its application to multiple attribute decision making. Neural Comput Appl 26:1157–1166. https://doi.org/10.1007/s00521-0140-1787-6

Ye J (2016) Aggregation operators of neutrosophic linguistic numbers for multiple attribute group decision making. Springer Plus 5:1691

Ye J (2016a) Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method. Informatica 20:20

Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353

Zhikang L, Ye J (2017) Single valued neutrosophic hybrid arithmetic and geometric aggregation operators and their decision making method. Information 8:84. https://doi.org/10.3390/info8030084