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ORIGINAL RESEARCH

Belief reliability analysis of competing for failure systems with bi-uncertain variables

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Abstract
In this paper, an uncertain competing failure degradation model is proposed, in which the natural degradation process is described by an uncertain process, the time interval of shocks arrival and the size of the shocks have independent and nonidentical uncertainty distributions, respectively. The parameters in the distributions are uncertain variables. The belief reliability function and the mean time to failure of the system under three different shock models are studied according to uncertainty theory, and Micro-Electro-Mechanical System as an example is used to explain the developed models.

Keywords Competing failure processes · Uncertainty theory · Bi-uncertain variable · Uncertainty distribution · Belief reliability

1 Introduction

Reliability analysis has become very important in engineering practice. In the past decades, reliability analysis has been mainly applied to the chemical industry, machinery, aerospace, electronics industry, communication system, power system, transportation, etc., and has made considerable achievements (Rackwitz 2001; Faulin et al. 2010; Finkelstein and Cha 2013). In the traditional reliability analysis, a complex system experiences both internal degradation and random external shock (Klutke and Yang 2002). All of the internal deterioration due to wear, corrosion, weathering and so on,there are usually three ways to describe the degradation process: degradation amount distribution (Huang and Askin 2003), degradation path (Lu and Meeker 1993) and stochastic process. Gamma process (Pan and Balakrishnan 2011) and Wiener process (Guan et al. 2016) are the most commonly used stochastic methods. External shock is caused by irresistible external forces such as high pressure, overload, collision, etc. Both internal degradation and random external shock may lead to system failure. They are competing with each other, and the failure caused by that is called competitive failure. Internal degradation and random external shock may be independent or dependent. Many scholars have studied on the independent competition failure model (Huang and Askin 2004; Wang and Zhang 2005; Li and Pham 2005; Keedy and Feng 2012) and dependent competition failure model (Peng et al. 2010; Rafiee et al. 2014, 2017; Song et al. 2014; Jiang et al. 2015; Hao et al. 2017; Wang et al. 2020a, b).

Traditional reliability analysis is studied according to probability theory, which is based on large sample data with frequencies close to the probability. However, in engineering practice, we can only get limited data, or even no data such as nuclear test data, newly developed products due to restrictions of cost, technology and other factors. We have to invite experts in the relative fields to access to expert experience data. Nevertheless, Liu (2012) showed that humans will always estimate a wider range than the specific value. If we still use human belief degree to estimate the probability distribution, the result will be subjective. In this situation, probability theory is no longer suitable to compute the human belief degree.

To deal with this kind of subjective data, Liu (2007) first proposed the uncertainty theory and refined it (2010a) which was based on the normality, duality, subadditivity and product axiom. The uncertainty theory is considered as a suitable mathematical tool to model epistemic uncertainty (Liu 2012). To describe the uncertain phenomena over time, Liu

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(2008) put forward Liu process in 2008. After several years of development, uncertain theory has been widely used in many fields such as reliability analysis, option pricing, interest rate, decision making, statistics analysis, and so on.

Many scholars apply uncertainty theory to reliability analysis. Liu (2010b) first proposed the uncertainty measure to describe the system reliability, and defined the uncertainty reliability index in the boolean system. Zeng et al. (2013) named the metric of products reliability as belief reliability under uncertainty theory and presented numerical evaluation methods to evaluate the belief reliability of systems. Zeng et al. (2017) developed belief reliability considering epistemic and aleatory uncertainty and calculated the belief reliability of coherent systems by a minimal cut set-based method (Zeng et al. 2018). Gao et al. (2018) studied an uncertain weighted k-out-of-n system whose weights are estimated by uncertain variables instead of exact numbers. Cao et al. (2019) studied discrete time series–parallel systems with uncertain parameters. Sheng and Ke (2020) discussed a multiple state uncertain weighted k-out-of-n system.

All of the above studies are based on the assumption that the lifetime of components is an uncertain variable. However, in practical engineering, a complex system will be impacted by irresistible external shocks, the causes of each shock are different, and the type of shock and the damage caused by a shock to the system is also different. To describe the different damage degree of the system caused by different types of shock, it is necessary to introduce a bi-uncertain variable. Bi-uncertain variables are proposed by Liu et al. (2020) in the reliability analysis of general systems. It is assumed that the lifetime of each component has independent and nonidentical uncertainty distributions, and the parameters in the uncertainty distribution are also uncertain variables. Using bi-uncertain variables to describe different types of shock have more practical significance in a complex system with limited information.

In this paper, under the environment of uncertainty theory, we consider a system has experienced both internal degradation and external shock. The internal degradation is described by Liu process. The time interval of external shock arrival and the damage caused by a shock to the system is described by different bi-uncertain variables, respectively. Three types of shock model are considered: (1) extreme shock model: when the magnitude of an shock exceeds a specific threshold value, the system fails; (2) cumulative shock model: the damage caused by cumulative shock exceeds a critical value, the system fails; (3) δ shock model: a system fails when the inter-arrival time between two successive shocks is less than a threshold value δ. The uncertain internal degradation and uncertain external shock are independent of each other. The scientific contribution of this paper to existing engineering practice and theoretical research is summarized as: (1) the continuous degradation process is described by a Liu process; (2) the shock model is an uncertain renewal reward process; (3) the interarrival times of shocks arrival and the shock sizes are assumed to be bi-uncertain variables, respectively. The time interval of shocks and the size of the shocks have independent and nonidentical uncertainty distributions with uncertain parameters. Stents implanted in the human body can be used as the application background of the model proposed. The stent experiences cyclic stress and a variety of overloads. The cyclic stresses include contractions and dilations due to heartbeat, and the overloads are mainly caused by patient's excessive activities.

The rest of this paper is arranged as follows. In Sect. 2, competing failure reliability models are developed. In Sect. 3, the belief reliability function and mean time to failure are investigated with extreme shock model, cumulative shock model and δ shock model according to the uncertainty theory, respectively. To illustrate the applications of the established models, some numerical examples are presented in Sect. 4. Finally, a brief conclusion is made in Sect. 5.

2 System description

2.1 Notation

H	The threshold level for soft failure
D	The threshold level for hardware failure
e	The minimum time lag between two consecutive shocks
μ	Degradation speed
σ	Diffusion coefficients
T	The lifetime of the system
τ_z	The first arrival time of $X(t)$ to z
T_H	The first arrival time of $X(t)$ to H
ξ_k	The uncertain time interval of the $k-1$th uncertain shock and the kth uncertain shock
η_k	The damage size caused by the kth uncertain shock
$X(t)$	Amount of continuous degradation at time t
$N(t)$	Number of uncertain shocks that have arrived by time t
$\Phi_t(x)$	The uncertainty distribution function of $X(t)$
$\zeta(z)$	The uncertainty distribution of τ_z
$F(x)$	The uncertainty distribution of T_H
$\varphi_k(\lambda_{k1},\dots,\lambda_{kn_k};x)$	The uncertainty distribution of the uncertain time interval ξ_k

$\phi_k(\mu_{k1},\dots,\mu_{kn_k};x)$	The uncertainty distribution of the uncertain shock η_k
$\Lambda_{ij}(\theta_{i1},\dots,\theta_{im_j};x)$	The uncertainty distribution of the uncertain variable λ_{ij}
$\Upsilon_{ij}(\omega_{i1},\dots,\omega_{im_j};x)$	The uncertainty distribution of the uncertain variable μ_{ij}
θ_{ij}	Parameters in the uncertainty distribution Λ_{ij} of uncertain variables λ_{ij}
ω_{ij}	Parameters in the uncertainty distribution Υ_{ij} of uncertain variables μ_{ij}
$\psi^{-1}(\alpha)$	Inverse uncertainty distribution of the uncertain variable $\sum_{i=1}^{k+1}\xi_i$
$\Psi^{-1}(\alpha)$	Inverse uncertainty distribution of the uncertain variable $\sum_{i=1}^{k}\eta_i$
$R(t)$	Belief reliability function at time t
NHF_t	The hardware failure does not occur by time t
NSF_t	The software failure does not occur by time t
$MTTF$	Mean time to failure

2.2 Preliminaries

In this section, we introduce some results in uncertainty theory, which are applied throughout the paper.

Definition 1 (Liu 2010a) Let ξ be an uncertain variable with regular uncertainty distribution $\Phi(x)$.Then the inverse function $\Phi^{-1}(\alpha)$ is called the inverse uncertainty distribution of ξ.

Theorem 1 (Liu 2010a) *Let $\xi_1,\xi_2,\dots,\xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1,\Phi_2,\dots,\Phi_n$, respectively. If $f(\xi_1,\xi_2,\dots,\xi_n)$ is a strictly increasing with respect to $\xi_1,\xi_2,\dots,\xi_m$ and strictly decreasing with respect to $\xi_{m+1},\xi_{m+2},\dots,\xi_n$, then*

$$\xi = f(\xi_1,\xi_2,\dots,\xi_n) \tag{1}$$

has an inverse uncertainty distribution

$$\psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha),\dots\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\dots,\Phi_n^{-1}(1-\alpha)). \tag{2}$$

Definition 2 (Liu 2007) Let ξ be an uncertain variable, then the expected value of ξ is defined by

$$E(\xi) = \int_0^{\infty} M\{\xi \ge x\}dx - \int_{-\infty}^{0} M\{\xi \le x\}dx, \tag{3}$$

provided that at least one of the integrals is finite.

Theorem 2 (Liu 2007) *Let ξ be an uncertain variable with regular uncertainty distribution Φ. Then,*

$$E(\xi) = \int_0^1 \Phi^{-1}(\alpha)d\alpha. \tag{4}$$

Theorem 3 (Liu and Ha 2010) *Assume $\xi_1,\xi_2,\dots,\xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1,\Phi_2,\dots,\Phi_n$, respectively. If $f(\xi_1,\xi_2,\dots,\xi_n)$ is a strictly increasing with respect to $\xi_1,\xi_2,\dots,\xi_m$ and strictly decreasing with respect to $\xi_{m+1},\xi_{m+2},\dots,\xi_n$, then the uncertain variable $\xi = f(\xi_1,\xi_2,\dots,\xi_n)$ has an expected value*

$$E(\xi) = \int_0^1 f(\Phi_1^{-1}(\alpha),\dots,\Phi_m^{-1}(\alpha),\Phi_{m+1}^{-1}(1-\alpha),\dots,\Phi_n^{-1}(1-\alpha))d\alpha. \tag{5}$$

Definition 3 (Liu 2009) An uncertain process C_t is said to be a Liu process if

(i) $C_0 = 0$ and almost all sample paths are Lipschitz continuous;
(ii) C_t has stationary and independent increments;
(iii) every increment $C_{s+t} - C_s$ is a normal uncertain variable with expected value 0 and variance t^2.

For an uncertain variable ξ with uncertainty distribution $\varphi(\lambda_1,\lambda_2,\dots,\lambda_n)$, if the parameters $\lambda_1,\lambda_2,\dots,\lambda_n$ are independent uncertain variables, we call it a bi-uncertain variable.

From an uncertainty space perspective, we can give a definition of bi-uncertain variable as follows.

Definition 4 Let (Γ_k,L_k,M_k) be uncertainty spaces for $k=0,1,2,\dots,n$, (Γ,L,M) is the product uncertainty space of (Γ_k,L_k,M_k) ,if $\xi(\omega|\varphi(\lambda_1,\lambda_2,\dots,\lambda_n))$ is a measurable function from uncertainty space (Γ,L,M) to the set of R^{n+1} (R is the set of real numbers), and ξ has uncertainty distribution $\varphi(\lambda_1,\lambda_2,\dots,\lambda_n)$, then ξ is called a bi-uncertain variable.

Let ξ be an bi-uncertain variable with uncertainty distribution function $\varphi(\lambda_1,\lambda_2,\dots,\lambda_n;x)$, whose parameters $\lambda_i, i=1,2,\dots,n$ are independent uncertain variables with uncertainty distributions $\Lambda_i, i=1,2,\dots,n$.

Theorem 4 *Let ξ be a bi-uncertain variable defined on the uncertainty space (Γ,L,M). If the uncertainty distribution function $\varphi(\lambda_1,\lambda_2,\dots,\lambda_n;x)$ of ξ is strictly increasing with respect to $\lambda_1,\lambda_2,\dots,\lambda_p$ and strictly decreasing with respect to $\lambda_{p+1},\lambda_{p+2},\dots,\lambda_n$, then*

$$M\{\xi \le x\} = \int_0^1 \varphi(\Lambda_1^{-1}(\alpha),\dots,\Lambda_p^{-1}(\alpha),\Lambda_{p+1}^{-1}(1-\alpha),\dots,\Lambda_n^{-1}(1-\alpha);x)d\alpha. \tag{6}$$

Proof Assume ξ is the lifetime of the component, according to Theorem 5 in reference Liu et al. (2020), we have

$$R(t) = \int_0^1 (1 - \varphi(\Lambda_1^{-1}(\alpha), \ldots, \Lambda_p^{-1}(\alpha), \Lambda_{p+1}^{-1}(1-\alpha), \ldots, \Lambda_n^{-1}(1-\alpha);t))d\alpha, \tag{7}$$

so

$$\begin{aligned} M\{\xi \le x\} &= 1 - M\{\xi > x\} \\ &= 1 - R(x) \\ &= \int_0^1 \varphi(\Lambda_1^{-1}(\alpha), \ldots, \Lambda_p^{-1}(\alpha), \Lambda_{p+1}^{-1}(1-\alpha), \ldots, \Lambda_n^{-1}(1-\alpha);x)d\alpha. \end{aligned} \tag{8}$$

The proof is complete.

2.3 Modeling of hardware failure due to uncertain shocks

The system is suffered from external shocks, and the damage size of shock was described as a random variable in the traditional reliability model. In practical engineering application, some components in the system are newly developed products with limited historical data. In this situation, it is no longer suitable to use random variables to describe external shocks. For systems with small samples of data, uncertain variables are more appropriate to describe external shocks. Due to different reasons, the types of shocks suffered by the system are different, each shock has a different distribution, and under limited sample data, in many cases, the parameters in the distribution can only estimate a range rather than a specific value. In this case, it can be assumed that the parameters in the distribution are uncertain variables.

Assume the time interval of shock arrival is a non-negative bi-uncertain variable ξ_i. Let ξ_k be the uncertain time interval of the $k-1$th shock and the kth shock, and $\xi_1, \xi_2, \ldots$ are independent variables and nonidentical distributions with different uncertain parameters. Let uncertainty distributions of uncertain variables $\xi_1, \xi_2, \ldots$ be $\varphi_1(\lambda_{11}, \lambda_{12}, \ldots, \lambda_{1n_1};x), \varphi_2(\lambda_{21}, \lambda_{22}, \ldots, \lambda_{2n_2};x), \ldots$, respectively, where parameters $\lambda_{ij}, i = 1, 2, \ldots, j = 1, 2, \ldots$ are uncertain variables which have uncertainty distributions $\Lambda_{ij}(\theta_{i1}, \theta_{i2}, \ldots, \theta_{im_i};x), i = 1, 2, \ldots, j = 1, 2, \ldots$, and parameters $\theta_{ij}, i = 1, 2, \ldots, j = 1, 2, \ldots$ are constants. Assume $\varphi_i(\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{in_i};x)$ is a strictly increasing with respect to $\lambda_{i1}, \ldots, \lambda_{ip}$ and strictly decreasing with respect to $\lambda_{i(p+1)}, \ldots, \lambda_{in_i}$.

The damage size of shock is a non-negative bi-uncertain variable η_i. Let η_k be the size of kth uncertain shock, and $\eta_1, \eta_2, \ldots$ are independent variables and nonidentical uncertainty distributions with different uncertain parameters. Let uncertainty distributions of uncertain variables $\eta_1, \eta_2, \ldots$ be $\phi_1(\mu_{11}, \mu_{12}, \ldots, \mu_{1n_1};x), \phi_2(\mu_{21}, \mu_{22}, \ldots, \mu_{2n_2};x), \ldots$, respectively, where parameters $\mu_{ij}, i = 1, 2, \ldots, j = 1, 2, \ldots$ are uncertain variables which have uncertainty distributions $\Upsilon_{ij}(\omega_{i1}, \omega_{i2}, \ldots, \omega_{im_i};x), i = 1, 2, \ldots, j = 1, 2, \ldots$, and parameters $\omega_{ij}, i = 1, 2, \ldots, j = 1, 2, \ldots$ are constants. Assume $\phi_i(\mu_{i1}, \mu_{i2}, \ldots, \mu_{in_i};x)$ is a strictly increasing with respect to $\mu_{i1}, \ldots, \mu_{ip}$ and strictly decreasing with respect to $\mu_{i(p+1)}, \ldots, \mu_{in_i}$.

According to Theorem 4, we have

$$M\{\xi_i \le x\} = \int_0^1 \varphi_i(\Lambda_{i1}^{-1}(\alpha), \ldots, \Lambda_{ip}^{-1}(\alpha), \Lambda_{i(p+1)}^{-1}(1-\alpha), \ldots, \Lambda_{in_i}^{-1}(1-\alpha);x)d\alpha. \tag{9}$$

$$M\{\eta_i < x\} = \int_0^1 \phi_i(\Upsilon_{i1}^{-1}(\alpha), \ldots, \Upsilon_{ip}^{-1}(\alpha), \Upsilon_{i(p+1)}^{-1}(1-\alpha), \ldots, \Upsilon_{in_i}^{-1}(1-\alpha);x)d\alpha. \tag{10}$$

Assume the time interval of uncertain shock arrival ξ_i and the damage size of uncertain shock η_i are independent, the number of uncertain shock at time t is $N(t)$. Hardware fails if the damage size of the uncertain shock exceeds the threshold value D.

2.4 Modeling of software failure due to uncertain degradation

In the traditional reliability model, the natural degradation process is often described by stochastic process, Wiener process is a kind of point process widely used to describe the degradation process (Ye and Xie 2015; Liu et al. 2016). Although almost all sample paths of Wiener process are continuous, they are not Lipschitz continuous (Liu 2013a). Nevertheless almost all sample paths of Liu process are Lipschitz continuous functions (Liu 2013a), so it is more appropriate to use Liu process to describe degradation process when there is only a small amount of historical fault data in engineering practice.

To describe the sample path of the degradation process, the uncertain differential equation (Liu 2009) is introduced

$$dX(t) = f(t, X(t))dt + g(t, X(t))dC(t). \tag{11}$$

Without loss of generality, assume

$$f(t, X(t)) = e, g(t, X(t)) = \sigma, e \in R, \sigma > 0, \tag{12}$$

e, σ are both constants, then the solution of the above differential equation under the initial value condition $X(0) = 0$ is

$$X(t) = et + \sigma C(t). \tag{13}$$

Thus, $X(t)$ obeys the uncertain normal distribution $N(et, \sigma t)$, and the uncertainty distribution function of $X(t)$ is

$$\Phi_t(x) = (1 + \exp(\frac{\pi(et - x)}{\sqrt{3}\sigma t}))^{-1}, x \in R. \tag{14}$$

Lemma 1 (Liu 2013b) *Let $X(t)$ be an uncertain process, whose uncertainty distribution function is $\Phi_t(x)$, and $X(0) = x_0, \quad x_0 > 0$, we define*

$$\tau_z = \inf\{t \geq 0 | X(t) = z\}, \tag{15}$$

τ_z is the first arrival time of $X(t)$ to z, the uncertainty distribution of τ_z is:

$$\zeta(z) = \begin{cases} 1 - \inf\limits_{0 \leq t \leq z} \Phi_t(z), & z > x_0, \\ \sup\limits_{0 \leq t \leq z} \Phi_t(z), & z < x_0. \end{cases} \tag{16}$$

Theorem 5 *Software failure occurs when the total degradation exceeds the threshold level H. The continuous degradation path is assumed*

$$X(t) = et + \sigma C(t), \tag{17}$$

which has an initial value $X(0) = 0$. Then, the uncertainty distribution function of the first arrival time T_H is

$$F(x) = (1 + \exp(\frac{\pi(H - ex)}{\sqrt{3}\sigma x}))^{-1}, x > 0. \tag{18}$$

Proof When $x > 0$, we have

$$\begin{aligned} F(x) &= M\{T_H \leq x\} = 1 - \inf_{0 \leq t \leq x} \Phi_t(H) \\ &= 1 - \inf_{0 \leq t \leq x} (1 + \exp(\frac{\pi(et - H)}{\sqrt{3}\sigma t}))^{-1} \\ &= 1 - (1 + \exp(\frac{\pi(ex - H)}{\sqrt{3}\sigma x}))^{-1} \\ &= (1 + \exp(\frac{\pi(H - ex)}{\sqrt{3}\sigma x}))^{-1}. \end{aligned} \tag{19}$$

Theorem 6 *The uncertain measure of software failure does not occur by time t is*

$$M(NSF_t) = (1 + \exp(\frac{\pi(et - H)}{\sqrt{3}\sigma t}))^{-1}. \tag{20}$$

Proof The uncertain measure of software failure does not occur by time t is

$$M(NSF_t) = M\{T_H > t\} = 1 - F(t) = (1 + \exp(\frac{\pi(et - H)}{\sqrt{3}\sigma t}))^{-1}. \tag{21}$$

3 Belief reliability analysis

The system suffers uncertain internal degradation and uncertain external shock, uncertain internal degradation and the uncertain external shock are independent, the belief reliability of the system is defined as the uncertain measure that the total uncertain degradation does not exceed a threshold value H and the uncertain shocks do not cause the system fails by time t,

$$R(t) = M\{NHF_t \cap NSF_t\}. \tag{22}$$

3.1 Case 1: extreme shock model

In the extreme shock model, hardware failure occurs when the damage size of shock exceeds a hard failure threshold D at first.

Theorem 7 *If the uncertain internal degradation process $X(t)$ for the system follows $X(t) = et + \sigma C(t)$, for $t \geq 0$. The uncertain external shock pattern is an extreme shock model, then the belief reliability of the system is*

$$\begin{aligned} R(t) = &[\max_{k \geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha); t)d\alpha) \\ &\wedge \min_{1 \leq i \leq k}(\int_0^1 \phi_i(Y_{i1}^{-1}(\alpha), \ldots, Y_{ip}^{-1}(\alpha), Y_{i(p+1)}^{-1}(1-\alpha), \ldots, Y_{in_i}^{-1}(1-\alpha); D)d\alpha)] \\ &\wedge (1 + \exp(\tfrac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}, \end{aligned} \tag{23}$$

and the mean time to failure of the system is

$$MTTF = \int_0^{+\infty} \{[\max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \wedge \min_{1\leq i\leq k}(\int_0^1 \phi_i(\Upsilon_{i1}^{-1}(\alpha), \ldots, \Upsilon_{ip}^{-1}(\alpha), \Upsilon_{i(p+1)}^{-1}(1-\alpha), \ldots, \Upsilon_{in_i}^{-1}(1-\alpha);D)d\alpha)] \wedge (1+\exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}\}dt. \tag{24}$$

Proof

$$R(t) = M\{NHF_t \cap NSF_t\} = M\{(\bigcap_{i=1}^{N(t)} \eta_i < D) \cap (T_H > t)\} = M\{\bigcap_{i=1}^{N(t)} \eta_i < D\} \wedge M\{T_H > t\}. \tag{25}$$

Since the uncertain events $\left\{\bigcap_{k=1}^{N(t)} \eta_k < D\right\}$ and the uncertain events $\left\{\bigcup_{k=0}^{\infty} (N(t)=k), (\bigcap_{i=1}^{k} \eta_i < D)\right\}$ are equivalent, then, we have

$$M\left\{\bigcap_{i=1}^{N(t)} \eta_i < D\right\} = M\left\{\bigcup_{k=0}^{\infty} (N(t)=k), (\bigcap_{i=1}^{k} \eta_i < D)\right\} = \max_{k\geq 0} M\left\{N(t)=k, \bigcap_{i=1}^{k} \eta_i < D\right\} = \max_{k\geq 0} M\{N(t)\leq k\} \wedge M\left\{\bigcap_{i=1}^{k} \eta_i < D\right\}, \tag{26}$$

where

$$M\{N(t)\leq k\} = 1 - M\left\{\sum_{i=1}^{k+1} \xi_i \leq t\right\}. \tag{27}$$

According to Theorem 1, $\sum_{i=1}^{k+1} \xi_i$ has inverse uncertainty distribution

$$\psi^{-1}(\alpha) = \sum_{i=1}^{k+1} \varphi_i^{-1}(\alpha). \tag{28}$$

According to Theorem 4, we have

$$M\left\{\sum_{i=1}^{k+1} \xi_i \leq t\right\} = \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha. \tag{29}$$

$$M\left\{\bigcap_{k=1}^{N(t)} (\eta_k < D)\right\} = \max_{k\geq 0}(1 - M\left\{\sum_{i=1}^{k+1} \xi_i \leq t\right\}) \wedge \min_{1\leq i\leq k} M\{\eta_i < D\} = \max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \wedge \min_{1\leq i\leq k}(\int_0^1 \phi_i(\Upsilon_{i1}^{-1}(\alpha), \ldots, \Upsilon_{ip}^{-1}(\alpha), \Upsilon_{i(p+1)}^{-1}(1-\alpha), \ldots, \Upsilon_{in_i}^{-1}(1-\alpha);D)d\alpha). \tag{30}$$

So

$$\begin{aligned} R(t) &= M\left\{\bigcap_{i=1}^{N(t)} \eta_i < D\right\} \wedge M\{T_H > t\} \\ &= [\max_{k\geq 0}(1-\int_0^1 \psi(\Lambda_{11}^{-1}(\alpha),\ldots,\Lambda_{1m_1}^{-1}(1-\alpha),\Lambda_{21}^{-1}(\alpha),\ldots,\Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \min_{1\leq i\leq k}(\int_0^1 \phi_i(Y_{i1}^{-1}(\alpha),\ldots,Y_{ip}^{-1}(\alpha),Y_{i(p+1)}^{-1}(1-\alpha),\ldots,Y_{in_i}^{-1}(1-\alpha);D)d\alpha)] \\ &\wedge \left(1+\exp\left(\frac{\pi(et-H)}{\sqrt{3}\sigma t}\right)\right)^{-1}. \end{aligned} \tag{31}$$

Let T be the lifetime of the system, according to Definition 2, the mean time to failure of the system is

$$\begin{aligned} MTTF &= \int_0^{+\infty} M\{T > t\}dt = \int_0^{+\infty} R(t)dt \\ &= \int_0^{+\infty} \{[\max_{k\geq 0}(1-\int_0^1 \psi(\Lambda_{11}^{-1}(\alpha),\ldots,\Lambda_{1m_1}^{-1}(1-\alpha),\Lambda_{21}^{-1}(\alpha),\ldots,\Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \min_{1\leq i\leq k}(\int_0^1 \phi_i(Y_{i1}^{-1}(\alpha),\ldots,Y_{ip}^{-1}(\alpha),Y_{i(p+1)}^{-1}(1-\alpha),\ldots,Y_{in_i}^{-1}(1-\alpha);D)d\alpha)] \\ &\wedge (1+\exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}\}dt. \end{aligned} \tag{32}$$

3.2 Case 2: cumulative shock model

In the cumulative shock model, hardware failure occurs when the damage size of the cumulative shock exceeds a hard failure threshold D.

Theorem 8 *If the uncertain internal degradation process $X(t)$ for the system follows $X(t) = et + \sigma C(t)$ for $t \geq 0$. The uncertain external shock pattern is a cumulative shock model, then belief reliability of the system is*

$$\begin{aligned} R(t) =& [\max_{k\geq 0}(1-\int_0^1 \psi(\Lambda_{11}^{-1}(\alpha),\ldots,\Lambda_{1m_1}^{-1}(1-\alpha),\Lambda_{21}^{-1}(\alpha),\ldots,\Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \int_0^1 \Psi(Y_{11}^{-1}(\alpha),\ldots,Y_{1m_1}^{-1}(1-\alpha),Y_{21}^{-1}(\alpha),\ldots,Y_{km_k}^{-1}(1-\alpha);D)d\alpha] \\ &\wedge (1+\exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}, \end{aligned} \tag{33}$$

and the mean time to failure of the system is

$$\begin{aligned} MTTF = \int_0^{+\infty} & \{[\max_{k\geq 0}(1-\int_0^1 \psi(\Lambda_{11}^{-1}(\alpha),\ldots,\Lambda_{1m_1}^{-1}(1-\alpha),\Lambda_{21}^{-1}(\alpha),\ldots,\Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \int_0^1 \Psi(Y_{11}^{-1}(\alpha),\ldots,Y_{1m_1}^{-1}(1-\alpha),Y_{21}^{-1}(\alpha),\ldots,Y_{km_k}^{-1}(1-\alpha);D)d\alpha] \\ &\wedge (1+\exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}\}dt. \end{aligned} \tag{34}$$

Proof

$$R(t) = M\{NHF_t \cap NSF_t\}=M\{(\sum_{i=1}^{N(t)} \eta_i < D) \cap (T_H > t)\} = M\{\sum_{i=1}^{N(t)} \eta_i < D\} \wedge M\{T_H > t\}, \quad (35)$$

where

$$\begin{aligned} M\{\sum_{i=1}^{N(t)} \eta_i < D\} &= M\{\bigcup_{k=0}^{\infty} (N(t) = k), (\sum_{i=1}^{k} \eta_i < D)\} \\ &= \max_{k\geq 0} M\{N(t) = k, \sum_{i=1}^{k} \eta_i < D\} \\ &= \max_{k\geq 0} M\{N(t) \leq k\} \wedge M\{\sum_{i=1}^{k} \eta_i < D\} \\ &= \max_{k\geq 0}(1-M\{\sum_{i=1}^{k+1} \xi_i \leq t\}) \wedge M\{\sum_{i=1}^{k} \eta_i < D\}. \end{aligned} \quad (36)$$

According to Theorem 1, $\sum_{i=1}^{k} \eta_i$ has inverse uncertainty distribution

$$\Psi^{-1}(\alpha) = \sum_{i=1}^{k} \phi_i^{-1}(\alpha). \quad (37)$$

According to Theorem 4, we have

$$\begin{aligned} &M\{\sum_{i=1}^{k} \eta_i < D\} \\ &= \int_0^1 \Psi(\Upsilon_{11}^{-1}(\alpha), \ldots, \Upsilon_{1m_1}^{-1}(1-\alpha), \Upsilon_{21}^{-1}(\alpha), \ldots, \Upsilon_{km_k}^{-1}(1-\alpha); D) d\alpha. \end{aligned} \quad (38)$$

$$\begin{aligned} M\{\sum_{i=1}^{N(t)} \eta_i < D\}&=M\{\bigcup_{k=0}^{\infty} (N(t) = k), \sum_{i=1}^{k} \eta_i < D\} \\ &= \max_{k\geq 0} M\{N(t) = k, \sum_{i=1}^{k} \eta_i < D\} \\ &= \max_{k\geq 0} M\{N(t) \leq k\} \wedge M\{\sum_{i=1}^{k} \eta_i < D\} \\ &= \max_{k\geq 0}(1 - M\{\sum_{i=1}^{k+1} \xi_i \leq t\}) \wedge \int_0^1 \Psi(\Upsilon_{11}^{-1}(\alpha), \ldots, \Upsilon_{1m_1}^{-1}(1-\alpha), \Upsilon_{21}^{-1}(\alpha), \ldots, \Upsilon_{km_k}^{-1}(1-\alpha); D) d\alpha \\ &= \max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha); t) d\alpha) \\ &\wedge \int_0^1 \Psi(\Upsilon_{11}^{-1}(\alpha), \ldots, \Upsilon_{1m_1}^{-1}(1-\alpha), \Upsilon_{21}^{-1}(\alpha), \ldots, \Upsilon_{km_k}^{-1}(1-\alpha); D) d\alpha. \end{aligned} \quad (39)$$

So

$$\begin{aligned} R(t) &= M\{\sum_{i=1}^{N(t)} \eta_i < D\} \wedge M\{T_H > t\} \\ &= [\max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha); t) d\alpha) \\ &\wedge \int_0^1 \Psi(\Upsilon_{11}^{-1}(\alpha), \ldots, \Upsilon_{1m_1}^{-1}(1-\alpha), \Upsilon_{21}^{-1}(\alpha), \ldots, \Upsilon_{km_k}^{-1}(1-\alpha); D) d\alpha] \\ &\wedge (1 + \exp(\frac{\pi(et - H)}{\sqrt{3}\sigma t}))^{-1}. \end{aligned} \quad (40)$$

According to Definition 2, the mean time to failure of the system is

$$\begin{aligned} MTTF &= \int_0^{+\infty} M\{T > t\}dt = \int_0^{+\infty} R(t)dt \\ &= \int_0^{+\infty} \{[\max_{k\geq 0}(1 - \int_0^1 \psi(\varLambda_{11}^{-1}(\alpha), \dots, \varLambda_{1m_1}^{-1}(1-\alpha), \varLambda_{21}^{-1}(\alpha), \dots, \varLambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \int_0^1 \varPsi(\varUpsilon_{11}^{-1}(\alpha), \dots, \varUpsilon_{1m_1}^{-1}(1-\alpha), \varUpsilon_{21}^{-1}(\alpha), \dots, \varUpsilon_{km_k}^{-1}(1-\alpha);D)d\alpha] \\ &\wedge (1 + \exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}\}dt. \end{aligned} \tag{41}$$

3.3 Case 3: δ-shock model

In the δ shock model, hardware failure occurs when the interval time of two adjective consecutive shocks is less than a constant δ.

Theorem 9 *If the uncertain internal degradation process $X(t)$ for the system follows $X(t) = et + \sigma C(t)$ for $t \geq 0$. The uncertain external shock pattern is a δ shock model, then the belief reliability of the system is*

$$\begin{aligned} R(t) = &\{\max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \dots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \dots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \min_{1\leq i\leq k}[1 - \int_0^1 \varphi_i(\Lambda_{i1}^{-1}(\alpha), \dots, \Lambda_{ip}^{-1}(\alpha), \Lambda_{i(p+1)}^{-1}(1-\alpha), \dots, \Lambda_{in_i}^{-1}(1-\alpha);\delta)d\alpha]\} \\ &\wedge(1 + \exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}, \end{aligned} \tag{42}$$

and the mean time to failure of the system is

$$\begin{aligned} MTTF = &\int_0^{+\infty} \{[\max_{k\geq 0}(1 - \int_0^1 \psi(\varLambda_{11}^{-1}(\alpha), \dots, \varLambda_{1m_1}^{-1}(1-\alpha), \varLambda_{21}^{-1}(\alpha), \dots, \varLambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha);t)d\alpha) \\ &\wedge \min_{1\leq i\leq k}[1 - \int_0^1 \varphi_i(\varLambda_{i1}^{-1}(\alpha), \dots, \varLambda_{ip}^{-1}(\alpha), \varLambda_{i(p+1)}^{-1}(1-\alpha), \dots, \varLambda_{in_i}^{-1}(1-\alpha);\delta)d\alpha]] \\ &\wedge (1 + \exp(\frac{\pi(et-H)}{\sqrt{3}\sigma t}))^{-1}\}dt. \end{aligned} \tag{43}$$

Proof

$$R(t) = M\{NHF_t \cap NSF_t\}=M\{(\bigcap_{i=1}^{N(t)} \xi_i > \delta) \cap (T_H > t)\} = M\{\bigcap_{i=1}^{N(t)} \xi_i > \delta\} \wedge M\{T_H > t\}, \tag{44}$$

where

$$
\begin{aligned}
&M\{\bigcap_{k=1}^{N(t)}(\xi_k > \delta)\} = M\{\bigcup_{k=0}^{\infty}(N(t)=k), \bigcap_{i=1}^{k}\xi_i > \delta\} \\
&= \max_{k\geq 0} M\{N(t)=k, \bigcap_{i=1}^{k}\xi_i > \delta\} \\
&= \max_{k\geq 0} M\{N(t)\leq k\} \wedge M\{\bigcap_{i=1}^{k}\xi_i > \delta\} \\
&= \max_{k\geq 0}(1 - M\{\sum_{i=1}^{k+1}\xi_i \leq t\}) \wedge \min_{1\leq i\leq k} M\{\xi_i > \delta\}.
\end{aligned} \tag{45}
$$

$$
\begin{aligned}
&M\{\xi_i > \delta\} = 1 - M\{\xi_i \leq \delta\} \\
&= 1 - \int_0^1 \varphi_i(\Lambda_{i1}^{-1}(\alpha), \ldots, \Lambda_{ip}^{-1}(\alpha), \Lambda_{i(p+1)}^{-1}(1-\alpha), \ldots, \Lambda_{in_i}^{-1}(1-\alpha); \delta) d\alpha
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
&M\{\bigcap_{i=1}^{N(t)}\xi_i > \delta\} \\
&= \max_{k\geq 0}(1 - M\{\sum_{i=1}^{k+1}\xi_i \leq t\}) \wedge M\{\bigcap_{i=0}^{k}\xi_i > \delta\} \\
&= \max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha); t) d\alpha) \\
&\wedge \min_{1\leq i\leq k}[1 - \int_0^1 \varphi_i(\Lambda_{i1}^{-1}(\alpha), \ldots, \Lambda_{ip}^{-1}(\alpha), \Lambda_{i(p+1)}^{-1}(1-\alpha), \ldots, \Lambda_{in_i}^{-1}(1-\alpha); \delta) d\alpha].
\end{aligned} \tag{47}
$$

So

$$
\begin{aligned}
R(t) &= M\{\bigcap_{i=1}^{N(t)}\xi_i > \delta\} \wedge M\{T_H > t\} \\
&= \{\max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \ldots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \ldots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha); t) d\alpha) \\
&\wedge \min_{1\leq i\leq k}[1 - \int_0^1 \varphi_i(\Lambda_{i1}^{-1}(\alpha), \ldots, \Lambda_{ip}^{-1}(\alpha), \Lambda_{i(p+1)}^{-1}(1-\alpha), \ldots, \Lambda_{in_i}^{-1}(1-\alpha); \delta) d\alpha]\} \\
&\wedge (1 + \exp(\frac{\pi(et - H)}{\sqrt{3}\sigma t}))^{-1}.
\end{aligned} \tag{48}
$$

According to Definition 2, the mean time to failure of the system is

$$\begin{aligned} MTTF &= \int_0^{+\infty} M\{T > t\}dt = \int_0^{+\infty} R(t)dt \\ &= \int_0^{+\infty} \{[\max_{k\geq 0}(1 - \int_0^1 \psi(\Lambda_{11}^{-1}(\alpha), \dots, \Lambda_{1m_1}^{-1}(1-\alpha), \Lambda_{21}^{-1}(\alpha), \dots, \Lambda_{(k+1)m_{(k+1)}}^{-1}(1-\alpha); t)d\alpha) \\ &\wedge \min_{1\leq i\leq k}[1 - \int_0^1 \varphi_i(\Lambda_{i1}^{-1}(\alpha), \dots, \Lambda_{ip}^{-1}(\alpha), \Lambda_{i(p+1)}^{-1}(1-\alpha), \dots, \Lambda_{in_i}^{-1}(1-\alpha); \delta)d\alpha]] \\ &\wedge (1 + \exp(\frac{\pi(et - H)}{\sqrt{3}\sigma t}))^{-1}\}dt. \end{aligned} \tag{49}$$

4 Numerical examples

In this section, using Micro-Electro-Mechanical System (MEMS) as an example to explain the proposed model. A micro-engine consists of several orthogonal linear comb drive actuators which are mechanically joined to a rotating gear. The wear on the rubbing surface between the gear and the pin joint usually causes a broken pin, which is the dominant reason for micro-engines failure. Additionally, in shock tests on micro-engines, the springs fracture is observed when the magnitude of shocks is larger than a certain threshold (Tanner et al. 2000). Therefore, the micro-engine is subject to two competing failure processes: soft failure due to wear degradation and hard failure due to springs fracture caused by external shocks. The failure mode of the micro-engine is modeled by employing uncertainty theory in this paper.

Assume the uncertain degradation process is $X(t) = et + \sigma C(t)$, the time interval of shock arrival is a non-negative bi-uncertain variable ξ_i, the size of shock is a non-negative bi-uncertain variable η_i, assume

$$\begin{aligned} &\xi_i \sim N(\lambda_i, 0.05^i), \lambda_i \sim L(0,1), i = 1,2,\dots, \\ &\eta_i \sim N(\mu_i, 0.5^i), \mu_i \sim L(1,2), i = 1,2,\dots. \end{aligned} \tag{50}$$

and the above distributions are independent,the uncertain degradation process and the uncertain shock are independent, other parameters are as follows in Table 1.

4.1 Belief reliability analysis

4.1.1 Case 1: extreme shock model

Firstly, we introduce the numerical function graphs of the system under the extreme shock model as shown in Fig. 1. From Fig. 1, we can see the belief reliability function changes when the soft failure threshold increases from $H = 1.5$ to $H = 2.5$. In general, the belief reliability function increases with the increase of H. The reason for this phenomenon is that the greater the soft failure threshold, the less the uncertain measure of software failure, so the belief

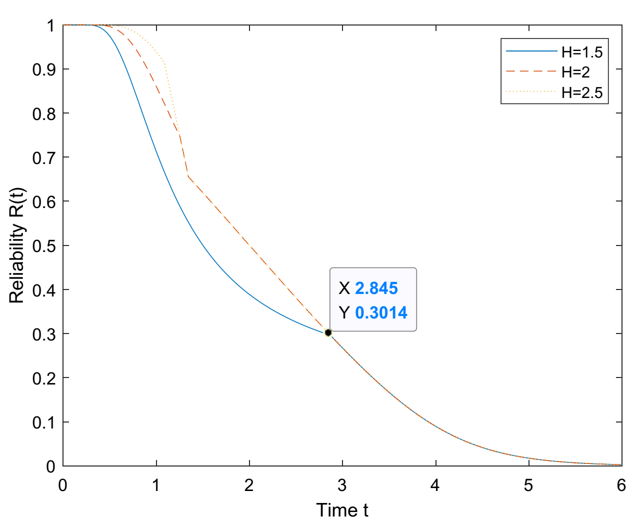


Fig. 1 Sensitivity analysis of $R(t)$ on H for Case 1

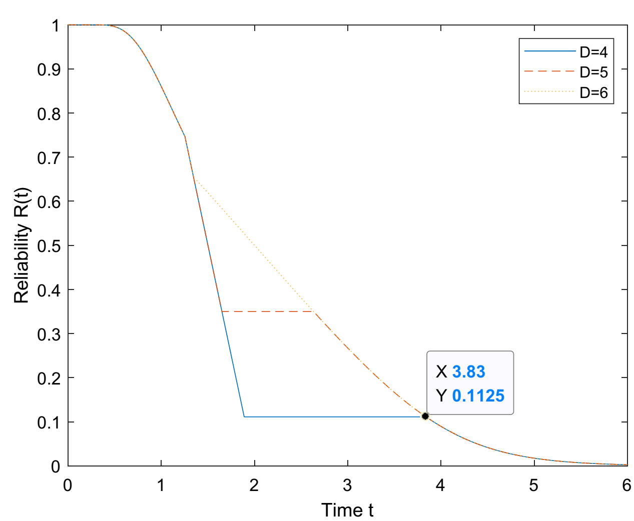


Fig. 2 Sensitivity analysis of $R(t)$ on D for Case 2

reliability increases. The belief reliability decreases rapidly before the change point $t = 2.845$, but decreases slowly after the change point, and before this change point the value of H is different from the belief reliability function, but after this change point the belief reliability function coincidence. This is because the system is affected by both the uncertain internal degradation and the uncertain shock, the change of software failure threshold influences the uncertain internal degradation, but has no influence on the uncertain external shock. When $t > 2.845$, compared with the uncertain internal degradation, the impact of uncertain external shock is greater, and the impact of uncertain external shock plays a leading role. Therefore, the belief reliability function will not change with the change of software failure threshold.

4.1.2 Case 2: cumulative shock model

Then, under the cumulative shock model, the function graphs of belief reliability are simulated in Fig. 2. Figure 2 shows the belief reliability function changes when the hardware failure threshold increases from $D = 4$ to $D = 6$. In general, the belief reliability function increases with the increase of D. The reason for this phenomenon is that the greater the hardware failure threshold, the less the uncertain measure of hardware failure, so the belief reliability increases. The three belief reliability function curves decrease rapidly before the first change point, and slowly after the first change point, some of them even remain unchanged for a certain period, but after $t = 3.83$, the three curves coincide, and the belief reliability gradually tends to 0. When $t > 3.83$, compared with the uncertain external shock, the impact of uncertain internal degradation is greater, and the impact of uncertain internal degradation plays a leading role. Therefore, the belief reliability function will not change after with the change of hardware failure threshold.

4.1.3 Case 3: δ-shock model

Finally, the belief reliability function graphs of δ shock model are simulated in Fig. 3. Figure 3 shows the belief reliability function changes when the δ increases from $\delta = 1$ to $\delta = 2$ through step 3. In general, the belief reliability function decreases with the increase of δ. The reason for this phenomenon is that the greater the δ, the bigger the uncertain measure of hardware failure, so the belief reliability decreases. The three belief reliability function curves decrease rapidly before the first change point, and after the first change point some of them even remain unchanged for a certain period, but after $t = 3.221$, the three curves coincide, and the belief reliability gradually tends to 0. Because there are two parts of system failure, software failure and hardware failure. The change of δ will only cause hardware failure. When $t > 3.221$, the failure of the system is mainly caused by software failure. Therefore, no matter how δ changes, the belief reliability of the system will not change.

Table 1 Model parameter values

Parameters	Values	Sources
e	1	Assumption
σ	1	Assumption
H	2	Assumption
D	5	Assumption
δ	1.5	Assumption

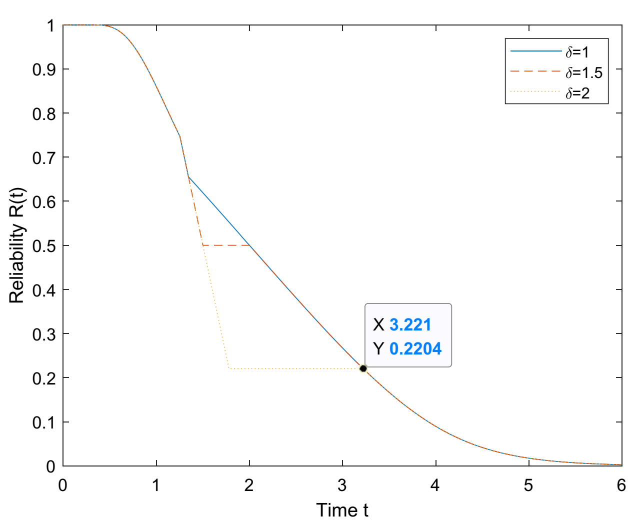


Fig. 3 Sensitivity analysis of $R(t)$ on δ for Case 3

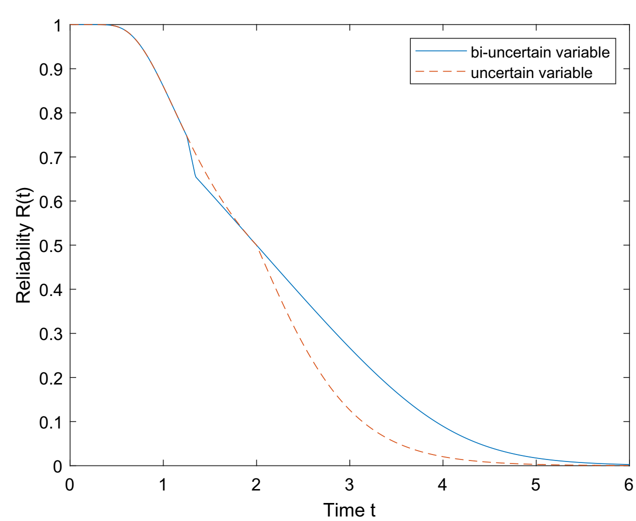
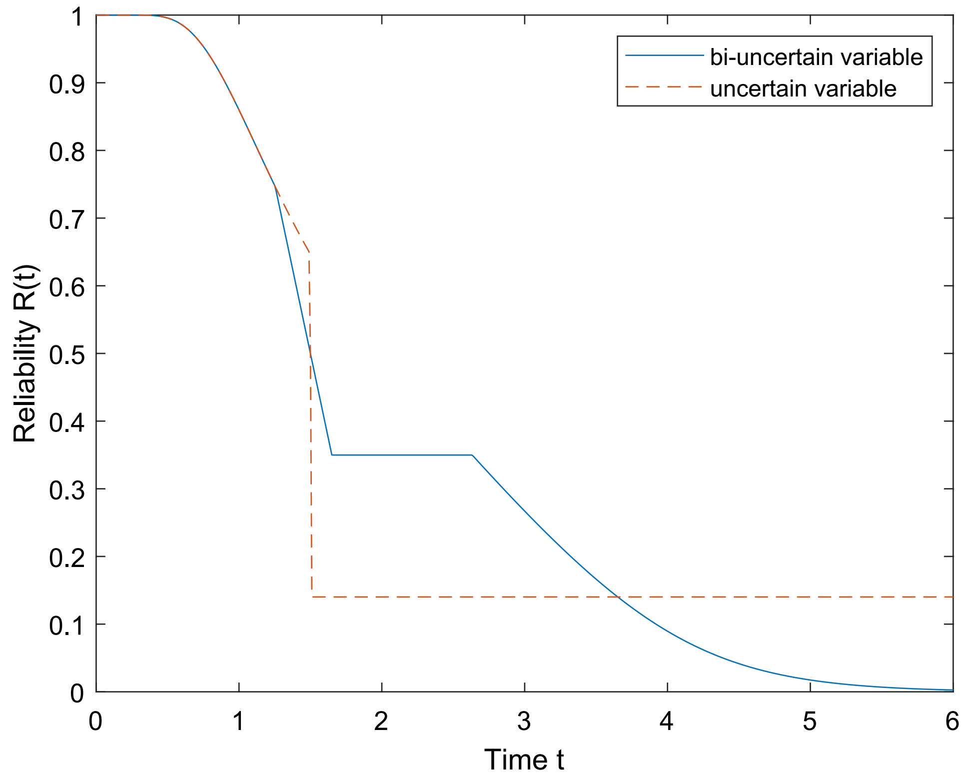


Fig. 4 The belief reliability curves of the bi-uncertain variable and uncertain variable for Case 1

Fig. 5 The belief reliability curves of the bi-uncertain variable and uncertain variable for Case 2



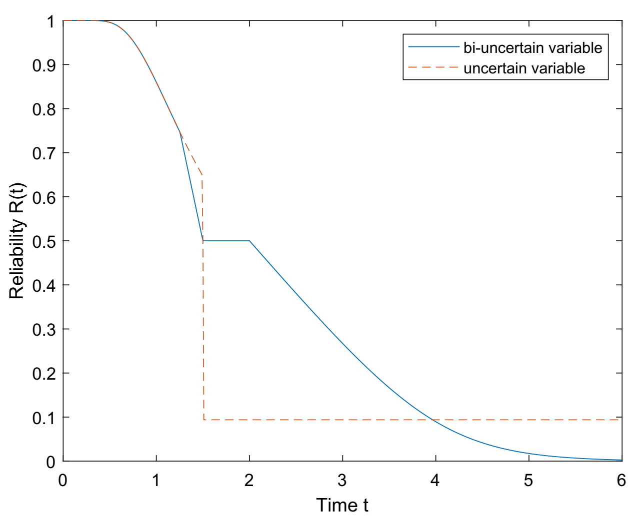


Fig. 6 The belief reliability curves of the bi-uncertain variable and uncertain variable for Case 3

4.2 Comparison the belief reliability between the bi-uncertain variable and uncertain variable

To compare the reliability of the bi-uncertain variable system with that of the uncertain variable system, assume the uncertain degradation process is $X(t) = et + \sigma C(t)$, the time

Table 2 MTTFs of the system

system	Extreme shock	Cumulative shock	δ shock
MTTFs of the system with bi-uncertain variables	2.2241	2.0740	2.1850
MTTFs of the system with uncertain variables	1.9888	1.9789	1.7708

interval of shock arrival is a non-negative uncertain variable ξ_i^*, the size of shock is a non-negative uncertain variable η_i^*, assume

$$\xi_i^* \sim N(0.5, 0.05^i), i = 1, 2, \ldots$$

$$\eta_i^* \sim N(1.5, 0.5^i), i = 1, 2, \ldots \tag{51}$$

other parameters are shown in Table 1.

In the three cases, the belief reliability function graphs of bi-uncertain variable system and uncertain variable system are respectively shown in Figs. 4, 5, 6. From the Figs. 4, 5, 6, it is easy to see that the belief reliability function of the system is indeed affected by the parameters with bi-uncertain and uncertain variables. It's worth mentioning that the general shape is similar under uncertain variables and constants. The common feature of the three graphs is that no matter which shock model, the belief reliability functions with uncertain variables and constant parameters are sometimes the same, sometimes it is larger than the uncertain variable under the constant parameters, sometimes it is smaller than the uncertain variable.

4.3 Comparison the MTTFs of the system between the bi-uncertain variable and uncertain variable

To illustrate the relationship between the uncertain variables and constants parameters, we make a comparison of MTTFs in Table 2. According to Theorems 7–9 and the parameters are uncertain variables and constants, we obtain MTTFs with bi-uncertain variables and uncertain variables, and MTTFs of the system with bi-uncertain variables are higher than that of the uncertain variables. This is consistent with the results in reference Liu et al. (2020).

5 Conclusions

In this paper, we introduce a reliability model in which the degradation process is Liu process and the external shock is a bi-uncertain variable. The degradation process and external shock are independent competitive failure models. The belief reliability and the mean time to failure of the system under the three models are discussed according to the uncertainty theory. A numerical example is given to show the belief reliability of the three models. The belief reliability and MTTFs are compared between the bi-uncertain variables and the uncertain variables. In the time interval $[0, t]$, the belief reliability of bi-uncertain variables is sometimes greater, sometimes smaller, sometimes equal than that of uncertain variables. However, the MTTFs of bi-uncertain variables is greater than that of uncertain variables.

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