Journal of Ambient Intelligence and Humanized Computing (2021) 12:8285–8303
https://doi.org/10.1007/s12652-020-02563-1

ORIGINAL RESEARCH

Multi-criteria decision support systems based on linguistic intuitionistic cubic fuzzy aggregation operators

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Received: 12 May 2020 / Accepted: 17 September 2020 / Published online: 17 October 2020

Abstract

This article is an advanced approach to linguistic intuitionistic fuzzy variable through application of cubic set theory. For instance, we establish the idea of the linguistic intuitionistic cubic fuzzy variable (LICFV) theory and define several operations for LICFV; also establish a series of weighted aggregation operators under linguistic intuitionistic cubic fuzzy information, so called linguistic intuitionistic cubic fuzzy weighted averaging (LICFWA) operator, linguistic intuitionistic cubic fuzzy order weighted averaging (LICFOWA) operator, linguistic intuitionistic cubic fuzzy weighted geometric (LICFWG) operator, linguistic intuitionistic cubic fuzzy order weighted geometric (LICFOWG) operator, linguistic intuitionistic cubic fuzzy hybrid averaging (LICFHA) operator, and linguistic intuitionistic cubic fuzzy hybrid geometric (LICFHG) operator; and further study their fundamental properties and showed the relationship among these aggregation operators. In order to demonstrate the feasibility and practicality of the mentioned new technique, we develop multi-criteria decision-making algorithm under linguistic intuitionistic cubic fuzzy environment. Further, the proposed method applied to mobile phone selection, consider numerical application of mobile phone. Comparing the proposed techniques with other pre-existing aggregation operators, we concluded that the proposed technique is better, reliable, and effective.

Keywords Linguistic intuitionistic cubic fuzzy variable · Linguistic intuitionistic cubic fuzzy weighted averaging and geometric operators · Multi-criteria decision-making

1 Introduction

Multi-criteria decision-making (MCDM) has played an significant role in everyday activities, such as economic, engineering, education, medical, and so on. In MCDM, one of the problems involves gathering multiple sources of information, i.e. finite alternatives giving the final result by aggregating process according to the attribute values of different alternatives (Chen and Tan 1994; Hong and Choi 2000; Li 2014; Merigo and Casanovas 2010; Xia and Xu 2013; Zhang and Liu 2010). One critical question in the decision process is how to communicate the value of the attribute. Due to the difficulty of the decision-making problems, it is often difficult to represent the attributes by crisp numbers. Because decision-makers may take decisions at a certain stage, due to the complexities of these decision-making issues and management environments, they can have concerns regarding their interpretations. In 1965 Zadeh (1965) presented the idea of the Fuzzy Set (FS) to deal with such an uncertain situation, Zadeh assigned membership grades to elements of a set in the interval [0, 1] by offering the idea of Fuzzy Sets. Zadeh research in this direction is noteworthy, as many of the set theoretical properties of crisp cases for fuzzy sets were established. After many implementations of the fuzzy set theory,

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Atanassov found that this theory includes many shortcomings. In other words, there might be a degree of uncertainty, which is too important to concentrate on, while at the same time creating perfectly beneficial models and problems. This form of hesitation is appropriately portrayed by intuitionistic fuzzy values, rather than accurate numbers. Zadeh fuzzy sets Zadeh (1965) are generalized in the form of the intuitionistic fuzzy sets (IFSs) (Atanasav 1986). An element of an IFS is represented by an ordered pair consisting of positive membership function and negative membership function, where the sum of the two functions described is less than or equal to one, thus sketching the fuzzy information characteristic in a more thorough and comprehensive way compared to the fuzzy set, which is distinguished only by membership function. Several researchers have made important contributions to the extension of IFS generalization and its application to different fields, resulting in IFSs great success in theoretical and technological aspects.

A major part of MCGDM with IFSs is the aggregation of intuitionistic fuzzy information (Garg and Rani 2019; Kou et al. 2016; Li 2010a, b, 2011; Li et al. 2010; Nayagam et al. 2011; Shuqi et al. 2009; Wei 2010; Ye 2018; Zhao et al. 2010; Zhou et al. 2013; Zhou and Chen 2014). In undetermined or firm cases, IFNs are too easy to divulge a decision-maker's confidential details about items. The aggregation of IFNs is an important step to obtain the outcome of a decision problem. For this purpose, a number of operators have been introduced recently to aggregate IFNs which are known as intuitionistic fuzzy hybrid aggregation (IFHA) operator, intuitionistic fuzzy hybrid geometric (IFHG) operator, intuitionistic fuzzy ordered weighted averaging (IFOWA) operator, intuitionistic fuzzy ordered weighted geometric (IFOWG) operator, intuitionistic fuzzy weighted averaging (IFWA) operator, and intuitionistic fuzzy weighted geometric (IFWG) operator (Beliakov et al. 2011; Kim and Ahn 1999; Liu and Wang 2017; Liang et al. 2017; Lindahl and Ramon 2010; Rani and Garg 2018; Shakeel 2018; Xu and Yager 2006; Xu 2007; Xu and Xia 2011; Yager et al. 2011; Yang and Yuan 2014). Chen et al. (2016) defined the similarity measure between intuitionistic fuzzy numbers (IFNs), using the centroid points of transformed right-angled triangular fuzzy numbers. Hassaballah and Ghareeb (2017) implemented a method for utilizing similarity measures on intuitive fuzzy sets in the field of image processing, especially for comparison of images. Garg (2016, 2017) implemented some new collaborative aggregation operators for the various IFNs. Kaur and Garg (2018a) have introduced certain aggregation operators for cubic IF set. Under the IFS setting, Ye (2017) provided some hybrid average and geometric aggregation operators to fix MCDM problems. All the preference relations reviewed above require experts to offer quantitative judgments. However, some authors Herrera et al. (1996), Herrera and Herrera-Viedma (1996), Herrera et al. (1997a, b) have noted, decision-making problems could be too complex for the experts to offer quantitative judgments.

Herrera et al. (2015) and Herrera and Viedma (2000) suggested an algorithm to address the problems of linguistic decision making problem. Next, Xu (2004b) defined some linguistic aggregation operators, such as geometric, weighted geometric and geometric hybrid operators for group decision-making with linguistic preference relationships. Xu (2006a) developed a linguistic hybrid average group decision-making operator for language multi-attribute groups. Xu (2004a) introduced uncertain linguistic weighted averaging operator and hybrid aggregation operator and tested on uncertain linguistic group decisions making. Xu (2006b) also defined induced uncertain linguistic ordered weighted average operators for decision-making problems. Wei (2009) introduced an unknown geometric mean linguistic hybrid operator and tested for the group decision-making of multi-attributes with uncertain linguistic informations. In addition, some authors Park et al. (2011), Wei et al. (2013), Zhang (2015) have suggested some uncertain linguistic aggregation operators for decision-making under uncertain linguistic informations, as Bonferroni mean, power, harmonic mean operators. Yager (2015) applied IFN operations to linguistic intuitionist style fuzzy sets and investigated the ordinal LIF aggregate operators. Zhang et al. (2017) defined LIFVs which uses the linguistic variable term to denote the experts preferred and non-preferred qualitative judgments, respectively.

1.1 Literature review

However, the intuitionist fuzzy collection does not clarify the problems of ambiguity. Then, Jun (2011) implemented a cubic fuzzy system (CFS) to address this challenge. This theory enabled us to tackle uncertainty problems. Cubic set theory also describes the satisfied, unsatisfied, and uncertain information not explained by fuzzy sets theory and intuitionistic fuzzy set theory (Fahmi et al. 2017, 2018a, b c, d, 2019a, b; Amin et al. 2018; Kaur and Garg 2019; Mehmood et al. 2016; Riaz and Tehrim 2019; Shakeel 2018; Zhan et al. 2017). Cubic set has details more attractive than FS and IFS (Kaur and Garg 2018a, b, 2019). It is one of the generalized types of fuzzy set and IFS, just like IFS, each element of a cubic fuzzy set is defined as a pair structure characterized by positive membership function and negative membership function. Negative membership is similar to the normal fuzzy set, while positive membership function is grip in the form of an interval.

Also, the linguistic intuitionistic collection does not clarify the problems of ambiguity. To address this obstacle

LCVs (linguistic cubic variables) introduced by Jun et al. (2018). This theory helped us to tackle uncertainty problems. The theory of linguistic cubic variable also describes the satisfactory, unsatisfied and ambiguous knowledge that was not clarified by linguistic intuitionistic theory. LCV has more desirable information than LFS and LIFS.

1.2 Motivation and objective

Due to the motivation and inspiration of the above discussion in this study, we have given a new approach to LICFS through application of linguistic cubic set theory. For instance, the concept of linguistic intuitionistic cubic fuzzy set (LICFS) is introduced . Each element of which consists function of linguistic membership and function of linguistic non-membership. Linguistic membership function is cubic fuzzy set and linguistic non-membership function is also cubic fuzzy set. LICFS is the hybrid set which can contain much more information to express a LCFS and an LIFS simultaneously for handling the uncertainties in the data.

In this article, firstly we give the conceptual information of linguistic intuitionistic cubic fuzzy variables (LICFVs), and initiate some fundamental laws of LICFVs. We also establish the concepts of accuracy function and score function of LICFVs, on the basis of these functions a simple procedure for ranking of LICFVs is introduced. Since an aggregation operator is an important mathematical tool in decision making problems, we introduce the aggregation proficiency for linguistic intuitionistic cubic fuzzy information and establish several aggregation operators, such as the linguistic intuitionistic cubic fuzzy weighted averaging (LICFWA) operator, linguistic intuitionistic cubic fuzzy order weighted averaging (LICFOWA) operator, linguistic intuitionistic cubic fuzzy hybrid averaging (LICFHA) operator, linguistic intuitionistic cubic fuzzy weighted geometric (LICFWG) operator, linguistic intuitionistic cubic fuzzy order weighted geometric (LICFOWG) operator, linguistic intuitionistic cubic fuzzy hybrid geometric (LICFHG) operator and present a number of properties of the mentioned operators.

To complete the said task, the remaining study is organized accordingly. In Sect. 2, firstly we review some fundamental concepts of fuzzy set, intuitionistic fuzzy set, cubic variable and linguistic term set. In Sect. 3, the concept of linguistic intuitionistic cubic fuzzy variable is presented and some valuable fundamental properties are studied. In Sect. 4, a number of LICF aggregation operators are introduced such as LICFWA operator, LICFOWA operator, LICFHA operator, LICFWG operator, LICFOWG operator and LICFHG operator and discussed their few properties. In Sect. 5, the mentioned operators are used to resolve a decision-making problem under LICF environment. Also, a numerical application related to the selection of suitable supplier for the purchase of components by a company is given to illustrate the feasibility and practicality of the mentioned new techniques. In Sect. 6, the comparison of suggested LICF averaging operators to the pre-existing averaging operators are discussed, and finally in the last section, the conclusions are presented.

2 Preliminaries

In this section, we introduce some elementary definitions of fuzzy set, intuitionistic fuzzy set, and their precious properties. In order to develop a new concept, first we review the basic definitions and properties for understanding the new concept.

Definition 1 Zadeh (1965) Let $\mathbb{R} \neq \phi$ are the general set. A fuzzy set $\Re$ is described as,

$$\Re = \{(r, \mu_{\Re}(r)) | r \in \mathbb{R}\}, \tag{2.1}$$

where $\mu_{\Re} : \mathbb{R} \to [0,1]$ is the membership grade of a fuzzy set $\Re$.

Atanassov give the idea of positive membership and negative membership function with the restriction that addition of both function is bounded by one. In next definition, the intuitionistic fuzzy set (IFS) is defined.

Definition 2 Atanasav (1986) Let $\mathbb{R} \neq \phi$ are the general set. An intuitionistic fuzzy set $\Re$ is described as,

$$\Re = \{(r, \mu_{\Re}(r), \nu_{\Re}(r) | r \in \mathbb{R}\}, \tag{2.2}$$

where the functions $\mu_{\Re}(r) : \mathbb{R} \to [0,1]$ and $\nu_{\Re}(r) : \mathbb{R} \to [0,1]$ represent the grade of positive and negative membership of each number, with $0 \leq \mu_{\Re}(r) + \nu_{\Re}(r) \leq 1$ for all $r \in \mathbb{R}$.

Furthermore, we have $\pi_{\Re}(r) = 1 - \mu_{\Re}(r) - \nu_{\Re}(r)$, is the hesitancy of IFNs of r to $\Re$ Szmidt and Kacprzyk (2000).

In 2011, Jun develop a new concept to cover the uncertainty, in next definition, the Jun concept is defined. In the Jun definition, the membership information are interval valued fuzzy information and the non-membership are fuzzy information.

Definition 3 Jun et al. (2011) A cubic set $\Re$ on a universal set $\mathbb{R} \neq \phi$ is given as following,

$$\Re = \{(x, [\mu^{-}_{\Re}(r), \mu^{+}_{\Re}(r)], \nu_{\Re}(r)) | r \in \mathbb{R}\}, \tag{2.3}$$

in which $\mu^{-}_{\Re}, \mu^{+}_{\Re}$ is an IVF numbers and $\nu_{\Re}$ is a fuzzy number in $\mathbb{R}$.

Definition 4 Phong and Cuong (2015); Herrera and Herrera-Viedma (2000) Let $\acute{S} = (\acute{s}_1, ..., \acute{s}_\ell)$ be the finite and absolutely order distinct term set. Then, $\acute{S}$ is the linguistic term set, and ℓ show the odd value, e.g., 3, ..., when $\ell = 5$, then $\acute{S}$ can be written as $\acute{S} = (\acute{s}_1, \acute{s}_2, \acute{s}_3, \acute{s}_4, \acute{s}_5) = ($ poor, slightly poor, fair, slightly good, good).

Also, satisfy the below characteristics;

(1). Ordered : $\acute{s}_\imath \prec \acute{s}_l, \Leftrightarrow \imath \prec l$;
(2). Negation : neg $(\acute{s}_\imath) = \acute{s}_{\ell-1-\imath}$;
(3). Maximum: $(\acute{s}_\imath, \acute{s}_l) = \acute{s}_\imath$, iff $\imath \geq l$;
(4). Minimum: $(\acute{s}_\imath, \acute{s}_l) = \acute{s}_l$,iff $\imath \leq l$.

The extended version of the discrete term set $\acute{S}$ is known as a continues linguistic term set and defined as $\acute{S}^* = \{\acute{s}_\psi | \acute{s}_0 \leq \acute{s}_\psi \leq \acute{s}_g, \psi \in [0, \ell]\}$, and if $\acute{s}_\psi \in \acute{S}^*$ then $\acute{s}_\psi$ is known as original term, otherwise, virtual term.

In next definition, the linguistic information added to cubic set theory and defined linguistic cubic variable.

Definition 5 Ye (2018) A linguistic cubic variable $\Re$ in $\mathbb{R} \neq \phi$ is given as following,

$$\Re = \{(r, [\acute{s}_{\mu_\Re^-}(r), \acute{s}_{\mu_\Re^+}(r)], \acute{s}_{v_\Re}(r)) | r \in \mathbb{R}\}, \tag{2.4}$$

in which $\acute{s}_{\mu_\Re^-}, \acute{s}_{\mu_\Re^+}$ is an LIVF numbers and $\acute{s}_{v_\Re}$ is a linguistic fuzzy number in $\mathbb{R}$.

Definition 6 Zhang (2014) Let $\mathbb{R} \neq \phi$ and $\acute{S}^* = \{\acute{s}_\psi | \acute{s}_0 \leq \acute{s}_\psi \leq \acute{s}_g, \psi \in [0, \ell]$ be a continues linguistic set. Then, the linguistic intuitionistic fuzzy set (LIFS) is described as,

$$\Re = \{\langle r, \acute{s}_\mu(r), \acute{s}_v(r) \rangle | r \in \mathbb{R}\}, \tag{2.5}$$

where $\langle \acute{s}_\mu(r), \acute{s}_v(r) \rangle \in \acute{S}^*$ denotes the linguistic positive grade and negative grade of the number $r \in \mathbb{R}$. We represent pair $\langle \acute{s}_\mu(r), \acute{s}_v(r) \rangle$ as $\Re = \langle \mathfrak{s}_\mu, \mathfrak{s}_v \rangle$ and known as linguistic intuitionistic fuzzy variable (LIFV). $\mu + v \leq \ell$ is always true, and $\pi(r) = \acute{s}_{\ell-\mu-v}$ represent the refusal grade of r to $\mathbb{R}$.

3 Linguistic intuitionistic cubic fuzzy variable and its basic relations and operations

In this section, we define the linguistic intuitionistic cubic fuzzy variable and also describe its fundamental relations and different operations.

Definition 7 Let $\acute{S}^* = \{\acute{s}_\psi | \acute{s}_0 \leq \acute{s}_\psi \leq \acute{s}_g, \psi \in [0, \ell]$ be a continues linguistic set. Then, a LICFV is defined as,

$$\Re = \{\langle [\acute{s}_{\mu^-}, \acute{s}_{\mu^+}], \acute{s}_\phi \rangle, \langle [\acute{s}_{v^-}, \acute{s}_{v^+}], \acute{s}_\varkappa \rangle\}, \tag{3.1}$$

where $\langle [\acute{s}_{\mu^-}, \acute{s}_{\mu^+}], \acute{s}_\phi \rangle, \langle [\acute{s}_{v^-}, \acute{s}_{v^+}], \acute{s}_\varkappa \rangle$ denote the exact grade of positive and negative membership grade respectively, $[\acute{s}_{\mu^-}, \acute{s}_{\mu^+}, \acute{s}_{v^-}, \acute{s}_{v^+}] \in \acute{S}^*$ are the uncertain linguistic numbers and $[\acute{s}_\phi, \acute{s}_\varkappa] \in \acute{S}^*$ are the linguistic variables for $\acute{s}_{\mu^-} \leq \acute{s}_{\mu^+}, \acute{s}_{v^-} \leq \acute{s}_{v^+}$. If $\mu^- + v^- \leq \phi + \varkappa \leq \mu^+ + v^+$, then $\Re = \{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \langle [\mathfrak{s}_{v^-}, \mathfrak{s}_{v^+}], \mathfrak{s}_\varkappa \rangle\}$ is an internal LICFVs, and if $\phi + \varkappa < \mu^- + v^-$ or $\phi + \varkappa > \mu^+ + v^+$, then $\Re = \{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \langle [\mathfrak{s}_{v^-}, \mathfrak{s}_{v^+}], \mathfrak{s}_\varkappa \rangle\}$ is an external LICFVs.

On the basis of linguistic intuitionistic cubic fuzzy values, we established a score function $Sc(\Re)$ which estimates the compliance degree that an alternative satisfies the need of a decision-maker.

Definition 8 Let $\Re = \{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \langle [\mathfrak{s}_{v^-}, \mathfrak{s}_{v^+}], \mathfrak{s}_\varkappa \rangle\}$ be a LICFV. Then, the score value are;

$$Sc(\Re) = \frac{\mu^- + \mu^+ + \phi + v^- + v^+ + \varkappa}{6\ell}, \quad Sc(\Re) \in [-,]. \tag{3.2}$$

Definition 9 Let $\Re = \left\{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \left\langle [\acute{s}_{v_1^-}, \acute{s}_{v_1^+}], \acute{s}_{\varkappa_1} \right\rangle\right\}$ and $\Re = \left\{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \left\langle [\acute{s}_{v_2^-}, \acute{s}_{v_2^+}], \acute{s}_{\varkappa_2} \right\rangle\right\}$ be the two linguistic intuitionistic cubic fuzzy variables, their expected values comparison are defined as;

· If $Sc(\Re) > \mathfrak{Sc}(\Re)$, then $\Re > \Re$
· If $Sc(\Re) < \mathfrak{Sc}(\Re)$, then $\Re < \Re$
· If $Sc(\Re) = \mathfrak{Sc}(\Re)$, then $\Re = \Re$.

Definition 10 Let $\Re = \left\{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \left\langle [\acute{s}_{v_1^-}, \acute{s}_{v_1^+}], \acute{s}_{\varkappa_1} \right\rangle\right\}$ and $\Re = \left\{\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_\phi \rangle, \left\langle [\acute{s}_{v_2^-}, \acute{s}_{v_2^+}], \acute{s}_{\varkappa_2} \right\rangle\right\}$ be the two linguistic intuitionistic cubic fuzzy variables and $\lambda > 0$. Then, the operational laws are defined as;

(1)

$$\Re \oplus \Re = \left\{ \begin{array}{c} \left(\left[\acute{s}_{\mu_1^- + \mu_2^- - \frac{\mu_1^- \mu_2^-}{\ell}}, \acute{s}_{\mu_1^+ + \mu_2^+ - \frac{\mu_1^+ \mu_2^+}{\ell}} \right], \acute{s}_{\phi_1 + \phi_2 - \frac{\phi_1 \phi_2}{\ell}} \right), \\ \left(\left[\acute{s}_{\frac{v_1^- v_2^-}{\ell}}, \acute{s}_{\frac{v_1^+ v_2^+}{\ell}} \right], \acute{s}_{\frac{\varkappa_1 \varkappa_2}{\ell}} \right) \end{array} \right\};$$

(2)

$$\Re \otimes \Re = \left\{ \begin{array}{c} \left(\left[\acute{s}_{\frac{\mu_1^- \mu_2^-}{\ell}}, \acute{s}_{\frac{\mu_1^+ \mu_2^+}{\ell}} \right], \acute{s}_{\frac{\phi_1 \varphi_2}{\ell}} \right), \\ \left(\left[\acute{s}_{v_1^- + v_2^- - \frac{v_1^- v_2^-}{\ell}}, \acute{s}_{v_1^+ + v_2^+ - \frac{v_1^+ v_2^+}{\ell}} \right], \acute{s}_{\varkappa_1 + \varkappa_2 - \frac{\varkappa_1 \varkappa_2}{\ell}} \right) \end{array} \right\};$$

(3)

$$\lambda\mathfrak{R}=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\lambda}},\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\lambda}}\right],\acute{s}_{\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\lambda}}\right),\\ \left(\left[\acute{s}_{\ell\left(\frac{v_1^-}{\ell}\right)^{\lambda}},\acute{s}_{\ell\left(\frac{v_1^+}{\ell}\right)^{\lambda}}\right],\acute{s}_{\ell\left(\frac{\varkappa_1}{\ell}\right)^{\lambda}}\right)\end{array}\right\};$$

(4)

$$\mathfrak{R}^{\lambda}=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell\left(\frac{\mu_1^-}{\ell}\right)^{\lambda}},\acute{s}_{\ell\left(\frac{\mu_1^+}{\ell}\right)^{\lambda}}\right],\acute{s}_{\ell\left(\frac{\phi_1}{\ell}\right)^{\lambda}}\right),\\ \left(\left[\acute{s}_{\ell-\ell\left(1-\frac{v_1^-}{\ell}\right)^{\lambda}},\acute{s}_{\ell-\ell\left(1-\frac{v_1^+}{\ell}\right)^{\lambda}}\right],\acute{s}_{\ell-\ell\left(1-\frac{\varkappa_1}{\ell}\right)^{\lambda}}\right)\end{array}\right\}.$$

4 Aggregation operators on linguistic intuitionistic cubic fuzzy variables

We introduced a number of linguistic intuitionistic cubic fuzzy aggregation operators and discussed some of their characteristics in this section.

4.1 Linguistic intuitionistic cubic fuzzy averaging operators

This subsection contains the definitions of LICFWA operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 11 Let $\mathfrak{R}_{\jmath}=\left\{\left\langle[\mathfrak{s}_{\mu_{\jmath}^-},\mathfrak{s}_{\mu_{\jmath}^+}],\mathfrak{s}_{\phi_{\jmath}}\right\rangle,\left\langle[\acute{s}_{v_{\jmath}^-},\acute{s}_{v_{\jmath}^+}],\acute{s}_{\varkappa_{\jmath}}\right\rangle\right\}(\jmath=1,...,n)$ are the set of LICFVs, and LICFWA is a mapping $LICFWA:\Omega^n\rightarrow\Omega$ if;

$$LICFWA_{\Lambda}(\mathfrak{R},...,\mathfrak{R}_{\mathfrak{n}})=\sum_{\jmath=1}^{n}\Lambda_{\jmath}\mathfrak{R}_{\jmath},\tag{4.1}$$

then, LICFWA operator is known as linguistic intuitionistic fuzzy weighted average operator with the dimension n and $\Lambda=(\Lambda_1,...,\Lambda_n)^T$ be the weights of $\mathfrak{R}_{\jmath}(\jmath=,...,\mathfrak{n})$ with $\Lambda_{\jmath}\in[0,1]$ and $\Sigma_{\jmath=1}^{n}\Lambda_{\jmath}=1$. Specially, if $\Lambda=(1/n,...,1/n)^T$, then the LICFWA operator reduced to LICFA operator with the dimension n, such as;

$$LICA_{\Lambda}(\mathfrak{R},...,\mathfrak{R}_{\mathfrak{n}})=1/n(\mathfrak{R}\oplus,...,\oplus\mathfrak{R}_{\mathfrak{n}}).\tag{4.2}$$

Theorem 1 *Let* $\mathfrak{R}_{\jmath}=\left\{\left\langle[\mathfrak{s}_{\mu_{\jmath}^-},\mathfrak{s}_{\mu_{\jmath}^+}],\mathfrak{s}_{\phi_{\jmath}}\right\rangle,\left\langle[\mathfrak{s}_{v_{\jmath}^-},\mathfrak{s}_{v_{\jmath}^+}],\mathfrak{s}_{\varkappa_{\jmath}}\right\rangle\right\}$ $(\jmath=1,...,n)$ *are the set of LICFVs. Then, there aggregated value by utilizing the LICFWA operator is also a LICFVs, and*

$$LICFWA_{\Lambda}(\mathfrak{R},...,\mathfrak{R}_{\mathfrak{n}})=\sum_{\jmath=}^{\mathfrak{n}}\Lambda_{\jmath}\mathfrak{R}_{\jmath}$$

$$=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\mu_{\jmath}^-}{\ell}\right)^{\Lambda_{\jmath}}},\acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\mu_{\jmath}^+}{\ell}\right)^{\Lambda_{\jmath}}}\right],\acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\phi_{\jmath}}{\ell}\right)^{\Lambda_{\jmath}}}\right),\\ \left(\left[\acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{v_{\jmath}^-}{\ell}\right)^{\Lambda_{\jmath}}},\acute{s}_{\ell\prod_{\jmath=1}^{n}\left(1-\frac{v_{\jmath}^+}{\ell}\right)^{\Lambda_{\jmath}}}\right],\acute{s}_{\ell\prod_{\jmath=1}^{n}\left(1-\frac{\varkappa_{\jmath}}{\ell}\right)^{\Lambda_{\jmath}}}\right)\end{array}\right\},\tag{4.3}$$

where $\Lambda=(\Lambda_1,...,\Lambda_n)^T$ *is the weights of* $\mathfrak{R}_{\jmath}(\jmath=,...,\mathfrak{n})$ *with* $\Lambda_{\jmath}\in[0,1]$ *and* $\Sigma_{\jmath=1}^{n}\Lambda_{\jmath}=1$.

Proof We used mathematical induction to prove this Theorem;

(1). If $n=2$, then using the developed operational laws, we obtain

$$\Lambda_1\mathfrak{R}=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}},\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}}\right],\acute{s}_{\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}}\right),\\ \left(\left[\acute{s}_{\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}},\acute{s}_{\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}}\right],\acute{s}_{\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}}\right)\end{array}\right\}$$

$$\Lambda_2\mathfrak{R}=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}},\acute{s}_{\ell-\ell\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}}\right],\acute{s}_{\ell-\ell\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}}\right),\\ \left(\left[\acute{s}_{\ell\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}},\acute{s}_{\ell\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}}\right],\acute{s}_{\ell\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}}\right)\end{array}\right\}$$

Based on the operational law (1), we get

$$LICFWA_{\Lambda}(\mathfrak{R},\mathfrak{R})=\Lambda\mathfrak{R}\oplus\Lambda\mathfrak{R}$$

$$=\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}-\frac{\left(\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}\right)\left(\ell-\ell\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}\right)}{\ell}},\\ \acute{s}_{\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}-\frac{\left(\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}\right)\left(\ell-\ell\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}\right)}{\ell}}\end{array}\right],\\ \acute{s}_{\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}-\frac{\left(\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}\right)\left(\ell-\ell\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}\right)}{\ell}}\end{array}\right),\\ \left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}-\frac{\left(\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}\right)\left(\ell\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}\right)}{\ell}},\\ \acute{s}_{\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}-\frac{\left(\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}\right)\left(\ell\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}\right)}{\ell}}\end{array}\right],\\ \acute{s}_{\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}-\frac{\left(\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}\right)\left(\ell\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}\right)}{\ell}}\end{array}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}-\left(\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}-\ell\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}+\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}\right)}, \\ \acute{s}_{\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}-\left(\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}-\ell\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}+\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}\right)}\end{array}\right], \\ \acute{s}_{\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}-\left(\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}-\ell\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}+\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}\right)}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}-\left(\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}-\ell\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}+\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}\right)}, \\ \acute{s}_{\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}-\left(\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}-\ell\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}+\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}\right)}\end{array}\right], \\ \acute{s}_{\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}-\left(\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}-\ell\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}+\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}\right)}\end{array}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\left(1-\frac{\mu_1^-}{\ell}\right)^{\Lambda_1}\left(1-\frac{\mu_2^-}{\ell}\right)^{\Lambda_2}}, \acute{s}_{\ell-\ell\left(1-\frac{\mu_1^+}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\mu_2^+}{\ell}\right)^{\Lambda_2}}\right], \acute{s}_{\ell-\ell\left(1-\frac{\phi_1}{\ell}\right)^{\Lambda_1}+\ell-\ell\left(1-\frac{\phi_2}{\ell}\right)^{\Lambda_2}}\right), \\ \left(\left[\acute{s}_{\ell\left(\frac{v_1^-}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{v_2^-}{\ell}\right)^{\Lambda_2}}, \acute{s}_{\ell\left(\frac{v_1^+}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{v_2^+}{\ell}\right)^{\Lambda_2}}\right], \acute{s}_{\ell\left(\frac{\varkappa_1}{\ell}\right)^{\Lambda_1}+\ell\left(\frac{\varkappa_2}{\ell}\right)^{\Lambda_2}}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{2}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{2}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{2}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}}\right), \\ \left(\left[\acute{s}_{\ell\prod_{\jmath=1}^{2}\left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell\prod_{\jmath=1}^{2}\left(\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell\prod_{\jmath=1}^{2}\left(\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}}\right)\end{array}\right\} \qquad (A)$$

(2). If $n=\kappa$, by applying Eq. (4.3), we get

$$LICFWA_{\Lambda}(\mathfrak{R},...,\mathfrak{R}_\kappa)=\sum_{\jmath=}^{\kappa}\Lambda_\jmath\mathfrak{R}_\jmath$$

$$=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}}\right), \\ \left(\left[\acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}}\right)\end{array}\right\} \qquad (B)$$

(3). If $n=\kappa+1$, then using Eq. (A) and (B), we obtain

$$=\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}+\ell-\ell\left(1-\frac{\mu_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}-\frac{\left(\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}\right)\left(\ell-\ell\left(1-\frac{\mu_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}{\ell}}, \\ \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}+\ell-\ell\left(1-\frac{\mu_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}-\frac{\left(\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}\right)\left(\ell-\ell\left(1-\frac{\mu_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}{\ell}}\end{array}\right], \\ \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}+\ell-\ell\left(1-\frac{\phi_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}-\frac{\left(\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}\right)\left(\ell-\ell\left(1-\frac{\phi_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}{\ell}}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}+\ell\left(\frac{v_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}-\frac{\left(\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}\right)\left(\ell\left(\frac{v_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}{\ell}}, \\ \acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}+\ell\left(\frac{v_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}-\frac{\left(\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}\right)\left(\ell\left(\frac{v_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}{\ell}}\end{array}\right], \\ \acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}+\ell\left(\frac{\varkappa_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}-\frac{\left(\ell\prod_{\jmath=1}^{\kappa}\left(\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}\right)\left(\ell\left(\frac{\varkappa_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}{\ell}}\end{array}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\begin{array}{c}\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}+\ell-\ell\left(1-\frac{\mu_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}} \\ -\left(\begin{array}{c}\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}-\ell\left(1-\frac{\mu_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}} \\ +\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\mu_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}\end{array}\right)\end{array}}, \\ \acute{s}_{\begin{array}{c}\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}+\ell-\ell\left(1-\frac{\mu_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}} \\ -\left(\begin{array}{c}\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}-\ell\left(1-\frac{\mu_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}} \\ +\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\mu_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}\end{array}\right)\end{array}}\end{array}\right], \\ \acute{s}_{\begin{array}{c}\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}+\ell-\ell\left(1-\frac{\phi_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}} \\ -\left(\begin{array}{c}\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}-\ell\left(1-\frac{\phi_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}} \\ +\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\phi_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}\end{array}\right)\end{array}}\end{array}\right), \\ \left(\begin{array}{c}\left[\begin{array}{c}\acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}+\ell\left(\frac{v_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}-\left(\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}-\ell\left(\frac{v_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}+\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}\left(\frac{v_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}, \\ \acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}+\ell\left(\frac{v_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}-\left(\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}-\ell\left(\frac{v_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}+\ell\prod_{\jmath=1}^{\kappa}\left(\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}\left(\frac{v_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}\end{array}\right], \\ \acute{s}_{\ell\prod_{\jmath=1}^{\kappa}\left(\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}+\ell\left(\frac{\varkappa_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}-\left(\ell\prod_{\jmath=1}^{\kappa}\left(\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}-\ell\left(\frac{\varkappa_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}+\ell\prod_{\jmath=1}^{\kappa}\left(\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}\left(\frac{\varkappa_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}\right)}\end{array}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(\begin{array}{c}\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\mu_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\mu_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}}\right], \\ \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\phi_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}}\end{array}\right), \\ \left(\begin{array}{c}\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{v_{\kappa+1}^-}{\ell}\right)^{\Lambda_{\kappa+1}}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{v_{\kappa+1}^+}{\ell}\right)^{\Lambda_{\kappa+1}}}\right], \\ \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa}\left(1-\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}\left(1-\frac{\varkappa_{\kappa+1}}{\ell}\right)^{\Lambda_{\kappa+1}}}\end{array}\right)\end{array}\right\}$$

$$=\left\{\begin{array}{c}\left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa+1}\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa+1}\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa+1}\left(1-\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}}\right), \\ \left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa+1}\left(1-\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa+1}\left(1-\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{\kappa+1}\left(1-\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}}\right)\end{array}\right\}.$$

Which shows that Eq. (4.3) holds for all values of n.

□

Proposition 1 *Let*

$$\mathfrak{R}_\jmath=\left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-},\mathfrak{s}_{\mu_\jmath^+}],\mathfrak{s}_{\phi_\jmath}\right\rangle,\left\langle[\mathfrak{s}_{v_\jmath^-},\mathfrak{s}_{v_\jmath^+}],\mathfrak{s}_{\varkappa_\jmath}\right\rangle\right\}(\jmath=,...,\mathfrak{n})$$

be the set of LICFVs, and $\Lambda=(\Lambda_1,...,\Lambda_n)^T$ *be the weight vector of* $\mathfrak{R}_\jmath,(\jmath=,...,\mathfrak{n})$ *with* $\Lambda_\jmath\in[0,1]$ *and* $\Sigma_{\jmath=1}^{n}\Lambda_\jmath=1$. *Then, we have below properties.* ***Idempotency:*** *If all*

$$\mathfrak{R}_\jmath=\left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-},\mathfrak{s}_{\mu_\jmath^+}],\mathfrak{s}_{\phi_\jmath}\right\rangle,\left\langle[\mathfrak{s}_{v_\jmath^-},\mathfrak{s}_{v_\jmath^+}],\mathfrak{s}_{\varkappa_\jmath}\right\rangle\right\}(\jmath=,...,\mathfrak{n})$$

are equal, i.e., $\mathfrak{R}_\jmath=\mathfrak{R}$ *for all* $\jmath=1,...,n$, *then*

$$LICFWA_{\Lambda}(\mathfrak{R},...,\mathfrak{R}_\mathfrak{n})=\mathfrak{R}. \qquad (4.4)$$

Proof

$$LICFWA_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) = \sum_{\jmath=}^{\mathfrak{n}} \Lambda_\jmath \Re_\jmath$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell-\ell \prod_{\jmath=1}^{n} \left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell-\ell \prod_{\jmath=1}^{n} \left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}} \right], \acute{s}_{\ell-\ell \prod_{\jmath=1}^{n} \left(1-\frac{\varphi_\jmath}{\ell}\right)^{\Lambda_\jmath}} \right), \\ \left(\left[\acute{s}_{\ell \prod_{\jmath=1}^{n} \left(\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell \Pi_{\jmath=1}^{n} \left(1-\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}} \right], \acute{s}_{\ell \prod_{\jmath=1}^{n} \left(1-\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}} \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell-\ell\left(1-\frac{\mu_\jmath^-}{\ell}\right)^{\sum_{\jmath=1}^{n}\Lambda_\jmath}}, \acute{s}_{\ell-\ell\left(1-\frac{\mu_\jmath^+}{\ell}\right)^{\sum_{\jmath=1}^{n}\Lambda_\jmath}} \right], \acute{s}_{\ell-\ell\left(1-\frac{\varphi_\jmath}{\ell}\right)^{\sum_{\jmath=1}^{n}\Lambda_\jmath}} \right), \\ \left(\left[\acute{s}_{\ell\left(\frac{v_\jmath^-}{\ell}\right)^{\sum_{\jmath=1}^{n}\Lambda_\jmath}}, \acute{s}_{\ell\left(1-\frac{v_\jmath^+}{\ell}\right)^{\sum_{\jmath=1}^{n}\Lambda_\jmath}} \right], \acute{s}_{\ell\left(1-\frac{\varkappa_\jmath}{\ell}\right)^{\sum_{\jmath=1}^{n}\Lambda_\jmath}} \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell-\ell\left(1-\frac{\mu^-}{\ell}\right)}, \acute{s}_{\ell-\ell\left(1-\frac{\mu^+}{\ell}\right)} \right], \acute{s}_{\ell-\ell\left(1-\frac{\varphi}{\ell}\right)} \right), \\ \left(\left[\acute{s}_{\ell\left(\frac{v^-}{\ell}\right)}, \acute{s}_{\ell\left(1-\frac{v^+}{\ell}\right)} \right], \acute{s}_{\ell\left(1-\frac{\varkappa}{\ell}\right)} \right) \end{array} \right\}$$

$$= \left\{ \left\langle [\acute{s}_{\mu^-}, \acute{s}_{\mu^+}], \acute{s}_{\phi} \right\rangle, \left\langle [\acute{s}_{v^-}, \acute{s}_{v^+}], \acute{s}_{\varkappa} \right\rangle \right\} = \Re$$

proved. □

Boundary: Let $\Re^- = \{[\min_\jmath \mathfrak{s}_{\mu_\jmath^-}, \min_\jmath \mathfrak{s}_{\mu_\jmath^+}], \min_\jmath \mathfrak{s}_{\phi_\jmath}),$ $([\max_\jmath \acute{s}_{v_\jmath^-}, \max_\jmath \acute{s}_{v_\jmath^-}], \max_\jmath \acute{s}_{\varkappa_\jmath})\}$ and $\Re^+ = \{[\max_\jmath \mathfrak{s}_{\mu_\jmath^-},$ $\max_\jmath \acute{s}_{\mu_\jmath^+}], \max_\jmath \acute{s}_{\phi_\jmath}), ([\min_\jmath \acute{s}_{v_\jmath^-}, \min_\jmath \acute{s}_{v_\jmath^-}], \min_\jmath \acute{s}_{\varkappa_\jmath})\}$ are the set of LICFVs for every Λ. Then,

$$\Re^- \leq \mathfrak{LICFWA}_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) \leq \Re^+. \tag{4.5}$$

Proof Since, the min of LICFVs are $\Re^-$ and the max are $\Re^+$, there is $\Re^- \leq \Re_\jmath \leq \Re^+$. Thus, there exist $\sum_{\jmath=1}^{n} \Lambda_\jmath \Re^- \leq \sum_{\jmath=}^{\mathfrak{n}} \Lambda_\jmath \Re_\jmath \leq \sum_{\jmath=}^{\mathfrak{n}} \Lambda_\jmath \Re^+$. Using the above property (1), we have $\Re^- \leq \sum_{\jmath=}^{\mathfrak{n}} \Lambda_\jmath \Re^- \leq \Re^-$, i.e.,

$$\Re^- \leq \mathfrak{LICFWA}_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) \leq \Re^+.$$

□

(3). Monotonicity: Let $\Re^* = \left\{ ([\mu_\jmath^{*^-}, \mu_\jmath^{*^+}], \phi_\jmath^*), \right.$ $([v_\jmath^{*^-}, v_\jmath^{*^+}],\ \varkappa_\jmath^*)\}(\jmath = 1, ..., n)$ be the set of linguistic intuitionistic cubic fuzzy variables $[\mu_\jmath^-, \mu_\jmath^+] \leq [\mu_\jmath^{*^-}, \mu_\jmath^{*^+}]$, $\phi_\jmath \leq \phi_\jmath^*$, $[v_\jmath^{*^-}, v_\jmath^{*^+}] \leq [v_\jmath^-, v_\jmath^+]$ and $\varkappa_\jmath^* \leq \varkappa_\jmath$, for all $\jmath$. Then, there exist

$$LICFWA_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) \leq \mathfrak{LICFWA}_{\Lambda}(\Re^*, ..., \Re_{\mathfrak{n}}^*). \tag{4.6}$$

Proof Due to $\Re_\jmath \ \leq \Re_\jmath^*$ for $\jmath = 1, ..., n$, there exists $\sum_{\jmath=1}^{n} \Lambda_\jmath \Re_\jmath \leq \sum_{\jmath=}^{\mathfrak{n}} \Lambda_\jmath \Re_\jmath^*$, i.e.,

$$LICFWA_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) \leq \mathfrak{LICFWA}_{\Lambda}(\Re^*, ..., \Re^*).$$

proved. □

4.2 Linguistic intuitionistic cubic fuzzy order weighted averaging operators

We introduce LICFOWA operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 12 Let $\Re_\jmath = \left\{ \left\langle [\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath} \right\rangle, \right.$ $\left\langle [\acute{s}_{v_\jmath^-}, \acute{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath} \right\rangle \}(\jmath = 1, ..., n)$ are the set of LICFVs. Then, a LICF order weighted averaging operator is a mapping $LICFWA : \Omega^n \to \Omega$, such as

$$LICFOWA_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) = \sum_{\jmath=1}^{n} \Lambda_\jmath \Re_{\sigma_{(\jmath)}}, \tag{4.7}$$

then LICFOWA operator is called a linguistic intuitionistic cubic fuzzy order weighted average operator of dimension n, and $(\sigma_{(1)}, ..., \sigma_{(n)})$ is a permutation of (1, ..., n) such that $\Re_{\sigma_{(\jmath-)}} \geq \ \Re_{\sigma_{(\jmath)}} \forall \jmath$. Also, $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the weight vector of $\Re_\jmath, (\jmath = , ..., \mathfrak{n})$ with $\Lambda_\jmath \in [0, 1]$ and $\Sigma_{\jmath=1}^{n} \Lambda_\jmath = 1$. Furthermore, specially if $\Lambda = (1/n, ..., 1/n)^T$, then the LICOWA operator reduced to a LICFA operator with dimension n, and described as:

$$LICFA_{\Lambda}(\Re, ..., \Re_{\mathfrak{n}}) = 1/n(\Re \oplus, ..., \oplus \Re_{\mathfrak{n}}). \tag{4.8}$$

Theorem 2 *Let* $\Re_\jmath = \left\{ \left\langle [\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath} \right\rangle, \left\langle [\mathfrak{s}_{v_\jmath^-}, \mathfrak{s}_{v_\jmath^+}], \mathfrak{s}_{\varkappa_\jmath} \right\rangle \right\}$ $(\jmath = 1, ..., n)$ *are the set of LICFVs. Then, there aggregated value by utilizing the LICFOWA operator is also an LICFVs, and*

$$LICFOWA_{\Lambda}(\mathfrak{R},...,\mathfrak{R}_{\mathfrak{n}}) = \sum_{\jmath=}^{\mathfrak{n}} \Lambda_{\jmath}\mathfrak{R}_{\sigma(\jmath)}$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\mu^{-}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\mu^{+}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\varphi_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right), \\ \left(\left[\acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{v^{-}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}}, \acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{v^{+}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right], \acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\varkappa_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right) \end{array} \right\}, \tag{4.9}$$

where $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ is the weight vector of $\mathfrak{R}_{\jmath}(\jmath = , ..., \mathfrak{n})$ with $\Lambda_{\jmath} \in [0, 1]$ and $\Sigma^{n}_{\jmath=1}\Lambda_{\jmath} = 1$.

Let $\mathfrak{R}_{\jmath} = \left\{\left\langle[\mathfrak{s}_{\mu^{-}_{\jmath}}, \mathfrak{s}_{\mu^{+}_{\jmath}}], \mathfrak{s}_{\phi_{\jmath}}\right\rangle, \left\langle[\mathfrak{s}_{v^{-}_{\jmath}}, \mathfrak{s}_{v^{+}_{\jmath}}], \mathfrak{s}_{\varkappa_{\jmath}}\right\rangle\right\}$ $(\jmath = 1, ..., n)$ be the set of LICFVs, and $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the weight vector of $\mathfrak{R}_{\jmath}, (\jmath = , ..., \mathfrak{n})$ with $\Lambda_{\jmath} \in [0, 1]$ and $\Sigma^{n}_{\jmath=1}\Lambda_{\jmath} = 1$. Then, we have below properties.

Idempotency: If all $\mathfrak{R}_{\jmath} = \left\{\left\langle[\mathfrak{s}_{\mu^{-}_{\jmath}}, \mathfrak{s}_{\mu^{+}_{\jmath}}], \mathfrak{s}_{\phi_{\jmath}}\right\rangle, \left\langle[\acute{s}_{v^{-}_{\jmath}}, \acute{s}_{v^{+}_{\jmath}}], \acute{s}_{\varkappa_{\jmath}}\right\rangle\right\}(\jmath = 1, ..., n)$ are equal, i.e., $\mathfrak{R}_{\jmath} = \mathfrak{R}$ for all $\jmath = 1, ..., n$, then

$$LICFOWA_{\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_{\mathfrak{n}}) = \mathfrak{R}. \tag{4.10}$$

Boundary: Let $\mathfrak{R}^{-} = \{[\min_{\jmath} \mathfrak{s}_{\mu^{-}_{\jmath}}, \min_{\jmath} \mathfrak{s}_{\mu^{+}_{\jmath}}], \min_{\jmath} \mathfrak{s}_{\phi_{\jmath}}), ([\max_{\jmath} \acute{s}_{v^{-}_{\jmath}}, \max_{\jmath} \acute{s}_{v^{-}_{\jmath}}], \max_{\jmath} \acute{s}_{\varkappa_{\jmath}})\}$ and $\mathfrak{R}^{+} = \{[\max_{\jmath} \mathfrak{s}_{\mu^{-}_{\jmath}}, \max_{\jmath} \acute{s}_{\mu^{+}_{\jmath}}], \max_{\jmath} \acute{s}_{\phi_{\jmath}}), ([\min_{\jmath} \acute{s}_{v^{-}_{\jmath}}, \min_{\jmath} \acute{s}_{v^{-}_{\jmath}}], \min_{\jmath} \acute{s}_{\varkappa_{\jmath}})\}$ are the set of LICFVs for every Λ. Then,

$$\mathfrak{R}^{-} \leq \mathfrak{LICFOWA}_{\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_{\mathfrak{n}}) \leq \mathfrak{R}^{+}. \tag{4.11}$$

Monotonicity: Let $\mathfrak{R}^{*} = \left\{([\mu^{*^{-}}_{\jmath}, \mu^{*^{+}}_{\jmath}], \phi^{*}_{\jmath}), ([v^{*^{-}}_{\jmath}, v^{*^{+}}_{\jmath}], \varkappa^{*}_{\jmath})\right\}$ $(\jmath = 1, ..., n)$ be the set of linguistic intuitionistic cubic fuzzy variables $[\mu^{-}_{\jmath}, \mu^{+}_{\jmath}] \leq [\mu^{*^{-}}_{\jmath}, \mu^{*^{+}}_{\jmath}], \phi_{\jmath} \leq \phi^{*}_{\jmath}$, $[v^{*^{-}}_{\jmath}, v^{*^{+}}_{\jmath}] \leq [v^{-}_{\jmath}, v^{+}_{\jmath}]$ and $\varkappa^{*}_{\jmath} \leq \varkappa_{\jmath}$, for all $\jmath$. Then, there exist

$$LICFOWA_{\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_{\mathfrak{n}}) \leq \mathfrak{LICFOWA}_{\Lambda}(\mathfrak{R}^{*}, ..., \mathfrak{R}^{*}_{\mathfrak{n}}). \tag{4.12}$$

4.3 Linguistic intuitionistic cubic fuzzy hybrid averaging operator

Here, we introduce LICFHA operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 13 Let $\mathfrak{R}_{\jmath} = \left\{\left\langle[\mathfrak{s}_{\mu^{-}_{\jmath}}, \mathfrak{s}_{\mu^{+}_{\jmath}}], \mathfrak{s}_{\phi_{\jmath}}\right\rangle, \left\langle[\mathfrak{s}_{v^{-}_{\jmath}}, \mathfrak{s}_{v^{+}_{\jmath}}], \mathfrak{s}_{\varkappa_{\jmath}}\right\rangle\right\}$, $(\jmath = 1, ..., n)$ be the set of LICFVs. An hybrid averaging operator of dimension n is a mapping $LICFHA : \Omega^{n} \to \Omega$, on linguistic intuitinistic cubic fuzzy variables, such that

$$LICFHA_{w,\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_{\mathfrak{n}}) = \sum_{\jmath=1}^{n} \Lambda_{\jmath}\widetilde{\mathfrak{R}}_{\sigma(\jmath)}, \tag{4.13}$$

where $\widetilde{\mathfrak{R}}_{\sigma_{(\jmath)}}$ is the $\jmath$th largest of the weighted linguistic intuitionistic cubic fuzzy variables $\mathfrak{R}_{\jmath}$. i.e., $\widetilde{\mathfrak{R}}_{\jmath} = nw_{\jmath}\mathfrak{R}_{\jmath} = \left\langle([\acute{s}_{\widetilde{\mu^{-}_{\jmath}}}, \acute{s}_{\widetilde{\mu^{+}_{\jmath}}}], \acute{s}_{\widetilde{\phi_{\jmath}}}), ([\acute{s}_{\widetilde{v^{-}_{\jmath}}}, \acute{s}_{\widetilde{v^{+}_{\jmath}}}], \acute{s}_{\widetilde{\varkappa_{\jmath}}})\right\rangle, (\jmath = 1, ..., n)$. Where, $\acute{s}_{\widetilde{\mu^{-}_{\jmath}}} = \acute{s}_{\ell-\ell\left(1-\frac{\mu^{-}_{\jmath}}{\ell}\right)^{nw_{\jmath}}}$, $\acute{s}_{\widetilde{\mu^{+}_{\jmath}}} = \acute{s}_{\ell-\ell\left(1-\frac{\mu^{+}_{\jmath}}{\ell}\right)^{nw_{\jmath}}}$, $\acute{s}_{\widetilde{\phi_{\jmath}}} = \acute{s}_{\ell-\ell\left(1-\frac{\phi_{\jmath}}{\ell}\right)^{nw_{\jmath}}}$, $\acute{s}_{\widetilde{v^{-}_{\jmath}}} = \acute{s}_{\ell\left(\frac{v^{-}_{1}}{\ell}\right)^{nw_{\jmath}}}$, $\acute{s}_{\widetilde{v^{+}_{\jmath}}} = \acute{s}_{\ell\left(\frac{v^{+}_{1}}{\ell}\right)^{nw_{\jmath}}}$, and $\acute{s}_{\widetilde{\varkappa_{\jmath}}} = \acute{s}_{\ell\left(\frac{\varkappa_{1}}{\ell}\right)^{nw_{\jmath}}}$. Also, $w = (w_1, ..., w_n)^T$ be the associated weighting vector of $\widetilde{\mathfrak{R}}_{\jmath}(\jmath = 1, ..., n)$ with $w_{\jmath} \in [0, 1]$ and $\Sigma^{n}_{\jmath=1}w_{\jmath} = 1$, and n is the balancing coefficient. Especially, if $w = (1/n, ..., 1/n)^T$, then the LICFHA operator reduced to LICFA operator with dimension n.

Theorem 3 *Let $\mathfrak{R}_{\jmath} = \left\{\left\langle[\mathfrak{s}_{\mu^{-}_{\jmath}}, \mathfrak{s}_{\mu^{+}_{\jmath}}], \mathfrak{s}_{\phi_{\jmath}}\right\rangle, \left\langle[\mathfrak{s}_{v^{-}_{\jmath}}, \mathfrak{s}_{v^{+}_{\jmath}}], \mathfrak{s}_{\varkappa_{\jmath}}\right\rangle\right\}$ $(\jmath = 1, ..., n)$ are the set of LICFVs. Then, there aggregated value by utilizing the LICFHA operator is also an LICFV and of the form,*

$$LICFHA_{\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_{\mathfrak{n}}) = \sum_{\jmath=}^{\mathfrak{n}} \Lambda_{\jmath}\widetilde{\mathfrak{R}}_{\sigma(\jmath)}$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\widetilde{\mu^{-}}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\widetilde{\mu^{+}}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\widetilde{\phi}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right), \\ \left(\left[\acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\widetilde{v^{-}}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}}, \acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\widetilde{v^{+}}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right], \acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\widetilde{\varkappa}_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_{\jmath}}} \right) \end{array} \right\}, \tag{4.14}$$

where $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ is the weights of $\mathfrak{R}_{\jmath}, (\jmath = , ..., \mathfrak{n})$ with $\Lambda_{\jmath} \in [0, 1]$ and $\Sigma^{n}_{\jmath=1}\Lambda_{\jmath} = 1$.

Let $\mathfrak{R}_{\jmath} = \left\{\left\langle[\mathfrak{s}_{\mu^{-}_{\jmath}}, \mathfrak{s}_{\mu^{+}_{\jmath}}], \mathfrak{s}_{\phi_{\jmath}}\right\rangle, \left\langle[\mathfrak{s}_{v^{-}_{\jmath}}, \mathfrak{s}_{v^{+}_{\jmath}}], \mathfrak{s}_{\varkappa_{\jmath}}\right\rangle\right\}$ $(\jmath = 1, ..., n)$ be the set of LICFVs, and $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the weight vector of $\mathfrak{R}_{\jmath}, (\jmath = , ..., \mathfrak{n})$ with $\Lambda_{\jmath} \in [0, 1]$ and $\Sigma^{n}_{\jmath=1}\Lambda_{\jmath} = 1$. Then, we have below properties. **Idempotency:** If all $\mathfrak{R}_{\jmath} = \left\{\left\langle[\mathfrak{s}_{\mu^{-}_{\jmath}}, \mathfrak{s}_{\mu^{+}_{\jmath}}], \mathfrak{s}_{\phi_{\jmath}}\right\rangle, \left\langle[\mathfrak{s}_{v^{-}_{\jmath}}, \acute{s}_{v^{+}_{\jmath}}], \acute{s}_{\varkappa_{\jmath}}\right\rangle\right\}(\jmath = 1, ..., n)$ are equal, i.e., $\mathfrak{R}_{\jmath} = \mathfrak{R}$ for all $\jmath = 1, ..., n$, then

$$LICFHA_{\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_{\mathfrak{n}}) = \mathfrak{R}. \tag{4.15}$$

Boundary: Let $\mathfrak{R}^{-} = \{[\min_{\jmath} \mathfrak{s}_{\mu^{-}_{\jmath}}, \min_{\jmath} \mathfrak{s}_{\mu^{+}_{\jmath}}], \min_{\jmath} \mathfrak{s}_{\phi_{\jmath}}), ([\max_{\jmath} \acute{s}_{v^{-}_{\jmath}}, \max_{\jmath} \acute{s}_{v^{-}_{\jmath}}], \max_{\jmath} \acute{s}_{\varkappa_{\jmath}})\}$ and $\mathfrak{R}^{+} = \{[\max_{\jmath} \mathfrak{s}_{\mu^{-}_{\jmath}},$

$\max_\jmath \acute{s}_{\mu_\jmath^+}], \max_\jmath \acute{s}_{\phi_\jmath}), ([\min_\jmath \acute{s}_{v_\jmath^-}, \min_\jmath \acute{s}_{v_\jmath^-}], \min_\jmath \acute{s}_{\varkappa_\jmath})\}$ are the set of LICFVs for every Λ. Then,

$$\mathfrak{R}^- \leq \mathfrak{LICFHA}_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq \mathfrak{R}^+. \tag{4.16}$$

Monotonicity: Let $\mathfrak{R}^* = \left\{([\mu_\jmath^{*^-}, \mu_\jmath^{*^+}], \phi_\jmath^*), ([v_\jmath^{*^-}, v_\jmath^{*^+}], \varkappa_\jmath^*)\right\}(\jmath = 1, ..., n)$ be the set of linguistic intuitionistic cubic fuzzy variables $[\mu_\jmath^-, \mu_\jmath^+] \leq [\mu_\jmath^{*^-}, \mu_\jmath^{*^+}], \phi_\jmath \leq \phi_\jmath^*$, $[v_\jmath^{*^-}, v_\jmath^{*^+}] \leq [v_\jmath^-, v_\jmath^+]$ and $\varkappa_\jmath^* \leq \varkappa_\jmath$, for all $\jmath$. Then, there exist

$$LICFHA_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq LICFHA_\Lambda(\mathfrak{R}^*, ..., \mathfrak{R}^*_\mathfrak{n}). \tag{4.17}$$

4.4 Linguistic intuitionistic cubic fuzzy weighted geometric operator

Here, we introduce LICFWG operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 14 Let $\mathfrak{R}_\jmath = \left\{\left\langle[s_{\mu_\jmath^-}, s_{\mu_\jmath^+}], s_{\phi_\jmath}\right\rangle, \left\langle[\acute{s}_{v_\jmath^-}, \acute{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath}\right\rangle\right\}$ $(\jmath = 1, ..., n)$ are the set of LICFVs and LICFWG is a mapping $LICFWG : \Omega^n \rightarrow \Omega$, if

$$LICFWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \prod_{\jmath=1}^{n} (\mathfrak{R}_\jmath)^{\Lambda_\jmath}, \tag{4.18}$$

then, LICFWG operator is called the linguistic intuitionistic cubic fuzzy weighted geometric operator of dimension n, and $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the weights of $\mathfrak{R}_\jmath, (\jmath = , ..., \mathfrak{n})$ with $\Lambda_\jmath \in [0, 1]$ and $\Sigma_{\jmath=1}^n \Lambda_\jmath = 1$. Specially, if $\Lambda = (1/n, ..., 1/n)^T$, then the LICFWG operator reduced to an LICFG operator with the dimension n , have defined as;

$$LICFG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = 1/n(\mathfrak{R} \otimes, ..., \otimes \mathfrak{R}_\mathfrak{n}). \tag{4.19}$$

Theorem 4 *Let* $\mathfrak{R}_\jmath = \left\{\left\langle[s_{\mu_\jmath^-}, s_{\mu_\jmath^+}], s_{\phi_\jmath}\right\rangle, \left\langle[s_{v_\jmath^-}, s_{v_\jmath^+}], s_{\varkappa_\jmath}\right\rangle\right\}$ $(\jmath = 1, ..., n)$ *are the set of LICFVs. Then, there aggregated value by utilizing the LICFWG operator is also an LICFVs, and*

$$LICFWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \prod_{\jmath=}^{\mathfrak{n}} (\mathfrak{R}_\jmath)^{\Lambda_\jmath}$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell \prod_{\jmath=1}^{n} \left(\frac{\mu_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell \prod_{\jmath=1}^{n} \left(\frac{\mu_\jmath^+}{\ell}\right)^{\Lambda_\jmath}} \right], \acute{s}_{\ell \prod_{\jmath=1}^{n} \left(\frac{\phi_\jmath}{\ell}\right)^{\Lambda_\jmath}} \right), \\ \left(\left[\acute{s}_{\ell - \ell \prod_{\jmath=1}^{n} \left(1-\frac{v_\jmath^-}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell - \ell \prod_{\jmath=1}^{n} \left(1-\frac{v_\jmath^+}{\ell}\right)^{\Lambda_\jmath}} \right], \acute{s}_{\ell - \ell \prod_{\jmath=1}^{n} \left(1-\frac{\varkappa_\jmath}{\ell}\right)^{\Lambda_\jmath}} \right) \end{array} \right\}, \tag{4.20}$$

where $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ *is the weights of* $\mathfrak{R}_\jmath(\jmath = , ..., \mathfrak{n})$ *with* $\Lambda_\jmath \in [0, 1]$ *and* $\Sigma_{\jmath=1}^n \Lambda_\jmath = 1$.

Proof The proof is same as Theorem 1. □

Proposition 2 *Let* $\mathfrak{R}_\jmath = \left\{\left\langle[s_{\mu_\jmath^-}, s_{\mu_\jmath^+}], s_{\phi_\jmath}\right.\right.$ *ngle*, $\left\langle[\acute{s}_{v_\jmath^-}, \acute{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath}\right\rangle\}, (\jmath = 1, ..., n)$ *are the set of LICFVs, and* $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ *be the weight vector of* $\mathfrak{R}_\jmath, (\jmath = , ..., \mathfrak{n})$ *with* $\Lambda_\jmath \in [0, 1]$ *and* $\Sigma_{\jmath=1}^n \Lambda_\jmath = 1$. *Then, we have the below properties.*

Idempotency: If all $\mathfrak{R}_\jmath = \left\{\left\langle[s_{\mu_\jmath^-}, s_{\mu_\jmath^+}], s_{\phi_\jmath}\right\rangle, \left\langle[\acute{s}_{v_\jmath^-}, \acute{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath}\right\rangle\right\}$, $(\jmath = 1, ..., n)$ are equal, i.e., $\mathfrak{R}_\jmath = \mathfrak{R}$ for all $\jmath = 1, ..., n$, then

$$LICFWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \mathfrak{R}. \tag{4.21}$$

Boundary: Let $\mathfrak{R}^- = \{[\min_\jmath s_{\mu_\jmath^-}, \min_\jmath s_{\mu_\jmath^+}], \min_\jmath s_{\phi_\jmath}), ([\max_\jmath \acute{s}_{v_\jmath^-}, \max_\jmath \acute{s}_{v_\jmath^-}], \max_\jmath \acute{s}_{\varkappa_\jmath})\}$ and $\mathfrak{R}^+ = \{[\max_\jmath s_{\mu_\jmath^-}, \max_\jmath \acute{s}_{\mu_\jmath^+}], \max_\jmath \acute{s}_{\phi_\jmath}), ([\min_\jmath \acute{s}_{v_\jmath^-}, \min_\jmath \acute{s}_{v_\jmath^-}], \min_\jmath \acute{s}_{\varkappa_\jmath})\}$ are the set of linguistic intuitionistic cubic fuzzy variables. Then,

$$\mathfrak{R}^- \leq \mathfrak{LICFWG}_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq \mathfrak{R}^+. \tag{4.22}$$

Monotonicity: Let $\mathfrak{R}^* = \left\{([\mu_\jmath^{*^-}, \mu_\jmath^{*^+}], \phi_\jmath^*), ([v_\jmath^{*^-}, v_\jmath^{*^+}], \varkappa_\jmath^*)\right\}$ $(\jmath = 1, ..., n)$ are the set of LICFVs $[\mu_\jmath^-, \mu_\jmath^+] \leq [\mu_\jmath^{*^-}, \mu_\jmath^{*^+}]$, $\phi_\jmath \leq \phi_\jmath^*$, $[v_\jmath^{*^-}, v_\jmath^{*^+}] \leq [v_\jmath^-, v_\jmath^+]$ and $\varkappa_\jmath^* \leq \varkappa_\jmath$, for all $\jmath$. Then, there exist

$$LICFWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq \mathfrak{LICFWG}_\Lambda(\mathfrak{R}^*, ..., \mathfrak{R}^*_\mathfrak{n}). \tag{4.23}$$

4.5 Linguistic intuitionistic cubic fuzzy order weighted geometric operator

Here, we introduce LICFOWG operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 15 Let $\mathfrak{R}_\jmath = \left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath}\right\rangle, \left\langle[\mathfrak{s}_{v_\jmath^-}, \mathfrak{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath}\right\rangle\right\}(\jmath = 1, ..., n)$ be the set of LICFVs. A linguistic intuitinistic cubic fuzzy order weighted geometric operator is a mapping $LICFOWG : \Omega^n \to \Omega$, such as;

$$LICFOWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \prod_{\jmath=1}^{n}\left(\mathfrak{R}_{\sigma_{(\jmath)}}\right)^{\Lambda_\jmath}. \tag{4.24}$$

Then, LICFOWG operator is called linguistic intuitionistic fuzzy order weighted geometric operator of dimension n, and $(\sigma_{(1)}, ..., \sigma_{(n)})$ denote the permutation of (1, ..., n) such as $\mathfrak{R}_{\sigma_{(\jmath-)}} \geq \mathfrak{R}_{\sigma_{(\jmath)}}$ for all $\jmath$. Also, $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the weights of $\mathfrak{R}_\jmath(\jmath = , ..., \mathfrak{n})$ with $\Lambda_\jmath \in [0, 1]$ and $\Sigma_{\jmath=1}^{n}\Lambda_\jmath = 1$. Furthermore, specially if $\Lambda = (1/n, ..., 1/n)^T$, then the LICFOWG operator reduced to LICFG operator with dimension n, have defined as;

$$LICFOG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = 1/n(\mathfrak{R}\otimes, ..., \otimes\mathfrak{R}_\mathfrak{n}). \tag{4.25}$$

Theorem 5 *Let* $\mathfrak{R}_\jmath = \left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath}\right\rangle, \left\langle[\mathfrak{s}_{v_\jmath^-}, \mathfrak{s}_{v_\jmath^+}], \mathfrak{s}_{\varkappa_\jmath}\right\rangle\right\}$ $(\jmath = 1, ..., n)$ *are the set of LICFVs. Then, there aggregated value by utilizing the LICFOWG operator is also an LICFVs, and*

$$LICFOWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \prod_{\jmath=}^{\mathfrak{n}}\left(\mathfrak{R}_{\sigma(\jmath)}\right)^{\Lambda_\jmath}$$

$$= \left\{\begin{array}{c} \left(\left[\acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\mu^-_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\mu^+_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell\prod_{\jmath=1}^{n}\left(\frac{\phi_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_\jmath}}\right), \\ \left(\left[\acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{v^-_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_\jmath}}, \acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{v^+_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_\jmath}}\right], \acute{s}_{\ell-\ell\prod_{\jmath=1}^{n}\left(1-\frac{\varkappa_{\sigma(\jmath)}}{\ell}\right)^{\Lambda_\jmath}}\right) \end{array}\right\}, \tag{4.26}$$

where $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ *is the weight vector of* $\mathfrak{R}_\jmath(\jmath = , ..., \mathfrak{n})$ *with* $\Lambda_\jmath \in [0, 1]$ *and* $\Sigma_{\jmath=1}^{n}\Lambda_\jmath = 1$.

Proposition 3 *Let* $\mathfrak{R}_\jmath = \left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath}\right\rangle, \left\langle[\mathfrak{s}_{v_\jmath^-}, \mathfrak{s}_{v_\jmath^+}], \mathfrak{s}_{\varkappa_\jmath}\right\rangle\right\}$, $(\jmath = 1, ..., n)$ *are the set of LICFVs, and* $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ *be the weight vector of* $\mathfrak{R}_\jmath, (\jmath = , ..., \mathfrak{n})$ *with* $\Lambda_\jmath \in [0, 1]$ *and* $\Sigma_{\jmath=1}^{n}\Lambda_\jmath = 1$. *Then, we have the below properties.*

Idempotency: If all $\mathfrak{R}_\jmath = \left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath}\right\rangle, \left\langle[\acute{s}_{v_\jmath^-}, \acute{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath}\right\rangle\right\}, (\jmath = 1, ..., n)$ are equal, i.e., $\mathfrak{R}_\jmath = \mathfrak{R}$ for all $\jmath = 1, ..., n$, then

$$LICFOWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \mathfrak{R}. \tag{4.27}$$

Boundary: Let $\mathfrak{R}^- = \{[\min_\jmath \mathfrak{s}_{\mu_\jmath^-}, \min_\jmath \mathfrak{s}_{\mu_\jmath^+}], \min_\jmath \mathfrak{s}_{\phi_\jmath}), ([\max_\jmath \acute{s}_{v_\jmath^-}, \max_\jmath \acute{s}_{v_\jmath^-}], \max_\jmath \acute{s}_{\varkappa_\jmath})\}$ and $\mathfrak{R}^+ = \{[\max_\jmath \mathfrak{s}_{\mu_\jmath^-}, \max_\jmath \acute{s}_{\mu_\jmath^+}], \max_\jmath \acute{s}_{\phi_\jmath}), ([\min_\jmath \acute{s}_{v_\jmath^-}, \min_\jmath \acute{s}_{v_\jmath^-}], \min_\jmath \acute{s}_{\varkappa_\jmath})\}$ are the set of linguistic intuitionistic cubic fuzzy variables. Then,

$$\mathfrak{R}^- \leq \mathfrak{LICFOWG}_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq \mathfrak{R}^+. \tag{4.28}$$

Monotonicity: Let $\mathfrak{R}^* = \left\{([\mu_\jmath^{*^-}, \mu_\jmath^{*^+}], \phi_\jmath^*), ([v_\jmath^{*^-}, v_\jmath^{*^+}], \varkappa_\jmath^*)\right\}(\jmath = 1, ..., n)$ are the set of LICFVs $[\mu_\jmath^-, \mu_\jmath^+] \leq [\mu_\jmath^{*^-}, \mu_\jmath^{*^+}], \phi_\jmath \leq \phi_\jmath^*$, $[v_\jmath^{*^-}, v_\jmath^{*^+}] \leq [v_\jmath^-, v_\jmath^+]$ and $\varkappa_\jmath^* \leq \varkappa_\jmath$, for all $\jmath$. Then, there exist

$$LICFOWG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq LICFOWG_\Lambda(\mathfrak{R}^*, ..., \mathfrak{R}^*_\mathfrak{n}). \tag{4.29}$$

4.6 Linguistic intuitionistic cubic fuzzy hybrid geometric operator

Here, we have introduced LICFHG operator and studied its fundamental properties, i.e., idempotency property, boundedness property, and monotonicity property.

Definition 16 Let $\mathfrak{R}_\jmath = \left\{\left\langle[\mathfrak{s}_{\mu_\jmath^-}, \mathfrak{s}_{\mu_\jmath^+}], \mathfrak{s}_{\phi_\jmath}\right\rangle, \left\langle[\acute{s}_{v_\jmath^-}, \acute{s}_{v_\jmath^+}], \acute{s}_{\varkappa_\jmath}\right\rangle\right\}(\jmath = 1, ..., n)$ be the set of LICFVs. A linguistic intuitinistic cubic fuzzy hybrid geometric operator of dimension n is a mapping $LICFHG : \Omega^n \to \Omega$, such as

$$LICFHG_{w,\Lambda}(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \prod_{\jmath=1}^{n}\left(\widetilde{\mathfrak{R}}_{\sigma_{(\jmath)}}\right)^{\Lambda_\jmath}, \tag{4.30}$$

where $\widetilde{\mathfrak{R}}_{\sigma_{(\jmath)}}$ is the $\jmath$th largest of the weighted LICFVs $\mathfrak{R}_\jmath$. i.e., $\widetilde{\mathfrak{R}}_\jmath = nw_\jmath\mathfrak{R}_\jmath = \left\langle([\mathfrak{s}_{\widetilde{\mu_\jmath^-}}, \mathfrak{s}_{\widetilde{\mu_\jmath^+}}], \mathfrak{s}_{\widetilde{\phi_\jmath}}), \quad ([\acute{s}_{\widetilde{v_\jmath^-}}, \acute{s}_{\widetilde{v_\jmath^+}}], \acute{s}_{\widetilde{\varkappa_\jmath}})\right\rangle$ $(\jmath = 1, ..., n)$. Where, $\acute{s}_{\widetilde{\mu_\jmath^+}} = \acute{s}_{\ell\left(\frac{\mu_1^+}{\ell}\right)^{nw_\jmath}}, \acute{s}_{\widetilde{\mu_\jmath^+}} = \acute{s}_{\ell\left(\frac{\mu_1^+}{\ell}\right)^{nw_\jmath}}$, $\acute{s}_{\widetilde{\phi_\jmath}} = \acute{s}_{\ell\left(\frac{\phi_1}{\ell}\right)^{nw_\jmath}}, \acute{s}_{\widetilde{v_\jmath^-}} = \acute{s}_{\ell-\ell\left(1-\frac{v_\jmath^-}{\ell}\right)^{nw_\jmath}}, \acute{s}_{\widetilde{v_\jmath^+}} = \acute{s}_{\ell-\ell\left(1-\frac{v_\jmath^+}{\ell}\right)^{nw_\jmath}}$ and $\acute{s}_{\widetilde{\varkappa_\jmath}} = \acute{s}_{\ell-\ell\left(1-\frac{\varkappa_\jmath}{\ell}\right)^{nw_\jmath}}$. Also, $w = (w_1, ..., w_n)^T$ be the associated weighting vector of $\widetilde{\mathfrak{R}}_\jmath, (\jmath = 1, ..., n)$ with $w_\jmath \in [0, 1]$ and $\Sigma_{\jmath=1}^{n}w_\jmath = 1$, and n is the balancing coefficient.

Specially, if $w = (1/n, , ..., 1/n)^T$, then the LICFHG operator reduced to LICFG operator with dimension n.

Theorem 6 *Let* $\mathfrak{R}_J = \left\{\left\langle [\mathfrak{s}_{\mu_J^-}, \mathfrak{s}_{\mu_J^+}], \mathfrak{s}_{\phi_J} \right\rangle, \left\langle [\mathfrak{s}_{v_J^-}, \mathfrak{s}_{v_J^+}], \mathfrak{s}_{\varkappa_J} \right\rangle\right\}$, *$(J = 1, ..., n)$ be the set of LICFVs. Then, there aggregated value by using the LICFHG operator is also an LICFVs and of the form,*

$$LICFHG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \prod_{J=}^{\mathfrak{n}} \left(\widetilde{\mathfrak{R}}_{\sigma_{(J)}}\right)^{\Lambda_J}$$

$$= \left\{ \begin{array}{c} \left(\left[\acute{s}_{\ell \prod_{J=1}^{n} \left(\frac{\widetilde{\mu}_{\sigma(J)}^-}{\ell}\right)^{\Lambda_J}}, \acute{s}_{\ell \prod_{J=1}^{n} \left(\frac{\widetilde{\mu}_{\sigma(J)}^+}{\ell}\right)^{\Lambda_J}} \right], \acute{s}_{\ell \prod_{J=1}^{n} \left(\frac{\widetilde{\phi}_{\sigma(J)}}{\ell}\right)^{\Lambda_J}} \right), \\ \left(\left[\acute{s}_{\ell - \ell \prod_{J=1}^{n} \left(1 - \frac{\widetilde{v}_{\sigma(J)}^-}{\ell}\right)^{\Lambda_J}}, \acute{s}_{\ell - \ell \prod_{J=1}^{n} \left(1 - \frac{\widetilde{v}_{\sigma(J)}^+}{\ell}\right)^{\Lambda_J}} \right], \acute{s}_{\ell - \ell \prod_{J=1}^{n} \left(1 - \frac{\widetilde{\phi}_{\sigma(J)}}{\ell}\right)^{\Lambda_J}} \right) \end{array} \right\}, \tag{4.31}$$

where $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ is the weight vector of $\mathfrak{R}_J (J = , ..., \mathfrak{n})$ with $\Lambda_J \in [0, 1]$ and $\Sigma_{J=1}^n \Lambda_J = 1$.

Proposition 4 *Let* $\mathfrak{R}_J = \left\{\left\langle [\mathfrak{s}_{\mu_J^-}, \mathfrak{s}_{\mu_J^+}], \mathfrak{s}_{\phi_J} \right\rangle, \left\langle [\mathfrak{s}_{v_J^-}, \mathfrak{s}_{v_J^+}], \mathfrak{s}_{\varkappa_J} \right\rangle\right\}$, *$(J = 1, ..., n)$ are the set of LICFVs, and $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ be the weight vector of $\mathfrak{R}_J, (J = , ..., \mathfrak{n})$ with $\Lambda_J \in [0, 1]$ and $\Sigma_{J=1}^n \Lambda_J = 1$. Then, we have the below properties.*

Idempotency: If all $\mathfrak{R}_J = \left\{\left\langle [\mathfrak{s}_{\mu_J^-}, \mathfrak{s}_{\mu_J^+}], \mathfrak{s}_{\phi_J} \right\rangle, \left\langle [\acute{s}_{v_J^-}, \acute{s}_{v_J^+}], \acute{s}_{\varkappa_J} \right\rangle\right\}, (J = 1, ..., n)$ are equal, i.e., $\mathfrak{R}_J = \mathfrak{R}$ for all $J = 1, ..., n$, then

$$LICFHG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) = \mathfrak{R}. \tag{4.32}$$

Boundary: Let $\mathfrak{R}^- = \{[\min_J \mathfrak{s}_{\mu_J^-}, \min_J \mathfrak{s}_{\mu_J^+}], \min_J \mathfrak{s}_{\phi_J}), ([\max_J \acute{s}_{v_J^-}, \max_J \acute{s}_{v_J^-}], \max_J \acute{s}_{\varkappa_J})\}$ and $\mathfrak{R}^+ = \{[\max_J \mathfrak{s}_{\mu_J^-}, \max_J \mathfrak{s}_{\mu_J^+}], \max_J \acute{s}_{\phi_J}), ([\min_J \acute{s}_{v_J^-}, \min_J \acute{s}_{v_J^-}], \min_J \acute{s}_{\varkappa_J})\}$ are the set of linguistic intuitionistic cubic fuzzy variables. Then,

$$\mathfrak{R}^- \leq \mathfrak{LICFHG}_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq \mathfrak{R}^+. \tag{4.33}$$

Monotonicity: Let $\mathfrak{R}^* = \left\{([\mu_J^{*-}, \mu_J^{*+}], \phi_J^*), ([v_J^{*-}, v_J^{*+}], \varkappa_J^*)\right\}$ $(J = 1, ..., n)$ are the set of LICFVs $[\mu_J^-, \mu_J^+] \leq [\mu_J^{*-}, \mu_J^{*+}], \phi_J \leq \phi_J^*$, $[v_J^{*-}, v_J^{*+}] \leq [v_J^-, v_J^+]$ and $\varkappa_J^* \leq \varkappa_J$, for all J. Then, there exist

$$LICFHG_\Lambda(\mathfrak{R}, ..., \mathfrak{R}_\mathfrak{n}) \leq LICFHG_\Lambda(\mathfrak{R}^*, ..., \mathfrak{R}_\mathfrak{n}^*). \tag{4.34}$$

5 Approach of linguistic intuitionistic cubic fuzzy variables for multi-criteria decision making problem

In this portion, we use linguistic intuitionistic cubic fuzzy averaging and geometric aggregation operators for multi-criteria decision making problem.

Let, there are n alternatives $\mathbb{Z} = \{\mathbb{Z}_1, ..., \mathbb{Z}_n\}$ and m criteria $\mathbb{N} = \{\mathbb{N}_1, ..., \mathbb{N}_m\}$ to be evaluated with associated weighs are $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$, such that $\Lambda_J \in [0, 1]$ and $\Sigma_{J=1}^n \Lambda_J = 1$. Let the rating of alternatives $\mathbb{Z}_i$ on criteria $\mathbb{N}_J$, given by the experts be linguistic intuitionistic cubic fuzzy variables in $\mathbb{R} : \mathbb{Z}_J = \left\{\left\langle [\acute{s}_{\mu_J^-}, \acute{s}_{\mu_J^+}], \acute{s}_{\phi_J} \right\rangle, \left\langle [\acute{s}_{v_J^-}, \acute{s}_{v_J^+}], \acute{s}_{\varkappa_J} \right\rangle\right\}$ $(i = 1, ..., n; J = 1, ..., m)$. Let $\left\langle [\acute{s}_{\mu_J^-}, \acute{s}_{\mu_J^+}], \acute{s}_{\phi_J} \right\rangle$ represents the grade of alternative $\mathbb{Z}_i$ satisfying the criteria $\mathbb{N}_J$ and $\left\langle [\acute{s}_{v_J^-}, \acute{s}_{v_J^+}], \acute{s}_{\varkappa_J} \right\rangle$ represents the grade of alternative $\mathbb{Z}_i$ not satisfying the criteria $\mathbb{N}_J$, with the condition that $[\mu_{iJ}^-, \mu_{iJ}^+] \subset [0, \ell]$, $[v_{iJ}^-, v_{iJ}^+] \subset [0, \ell]$, $\phi_{iJ} : \mathbb{R} \to [0, \ell]$ and $\varkappa_{iJ} : \mathbb{R} \to [0, \ell]$. Subject to $\sup[\mu_{iJ}^-, \mu_{iJ}^+] + \sup[v_{iJ}^-, v_{iJ}^+] \leq \ell$ and $\phi_{iJ} + \varkappa_{iJ} \leq \ell, (i = 1, ..., n; J = 1, ..., m)$. Thus, a MCDM problem can be concisely expressed in linguistic intuitionistic cubic fuzzy decision matrix $D = (\mathbb{Z}_{iJ})_{n \times m} = \left\{\left\langle [\acute{s}_{\mu_{iJ}^-}, \acute{s}_{\mu_{iJ}^+}], \acute{s}_{\phi_{iJ}} \right\rangle, \left\langle [\acute{s}_{v_{iJ}^-}, \acute{s}_{v_{iJ}^+}], \quad \acute{s}_{\varkappa_{iJ}} \right\rangle\right\}_{n \times m} (i = 1, ..., n; J = 1, ..., m)$. To aggregate the given data, we apply the following steps.

Step 1. Make linguistic intuitionistic cubic fuzzy variables decision matrix. $D = (\mathbb{Z}_{iJ})_{n \times m} = \left\{\left\langle [\acute{s}_{\mu_{iJ}^-}, \acute{s}_{\mu_{iJ}^+}], \acute{s}_{\phi_{iJ}} \right\rangle, \left\langle [\acute{s}_{v_{iJ}^-}, \acute{s}_{v_{iJ}^+}], \acute{s}_{\varkappa_{iJ}} \right\rangle\right\}_{n \times m} (i = 1, ..., n; J = 1, ..., m)$. Usually the criteria can be classified into two types, benefit criteria and cost criteria. If all the criteria have same type, then normalization are not needed. And if the decision matrix contains both types, in such a case we can transform the cost type criteria into benefit type criteria by the following formula;

$$D_{iJ} = \langle r_{iJ}, t_{iJ} \rangle = \left\{ \begin{array}{l} d_{iJ}, \text{ if the criteria is of benefit type} \\ d_{iJ}^c, \text{ if the criteria is of cost type} \end{array} \right\},$$

d_{iJ}^c is the complement of d_{iJ}. Hence, we get the normalized linguistic intuitionistic cubic fuzzy variables decision matrix. The normalized linguistic intuitionistic cubic fuzzy variables decision matrix is denoted by D^N.

Step 2. Using the proposed aggregation operators to find the LICFVs for the alternatives $\mathbb{Z}_i (i = 1, ..., n)$. i.e., the developed operators to stem the collective overall preference values $\mathbb{Z}_i (i = 1, ..., n)$ of the alternatives $\mathbb{Z}_i$, where $\Lambda = (\Lambda_1, ..., \Lambda_n)^T$ is the weight vector of the criteria.

Step 3. By the using of Eq. (3.2) , to find the scores $Sc(\mathbb{Z}_i)$ of all the values $\mathbb{Z}_i$.

Step 4. Select the best alternative, on the score value base.

5.1 Example

In 2009 the first cell phone was invented by Q Mobile Company. At present, cell phone feature and usage have undergone enormous changes that have become one of the most significant daily necessities. Pakistan's cell phone revenues surpass 200 million in the first half of 2016. According to the sales, there are four major brands of mobile phones in Pakistan, including $\mathbb{N}_1$: HUAWEI, $\mathbb{N}_2$: OPPO, $\mathbb{N}_3$: APPLE, and $\mathbb{N}_4$: VIVO, which account for about 63% of the total sales. However, when we evaluate these four brands of mobile phones, several factors should be considered, such as: $(\mathbb{Z}_1)$ appearance, $(\mathbb{Z}_2)$ price, $(\mathbb{Z}_3)$ performance, and $(\mathbb{Z}_4)$ quality. We believe that an expert team is called upon to determine these four mobile phone brands. In order to fully articulate the experts' awareness, linguistic variables may be added within the predefined linguistic term collection $\acute{S} = (\acute{s}_1$: extremely bad; $\acute{s}_1$: very bad; $\acute{s}_3$: bad; $\acute{s}_4$: relatively bad; $\acute{s}_5$: fair; $\acute{s}_6$: relatively good; $\acute{s}_7$: good; $\acute{s}_8$: very good; $\acute{s}_9$: extremely good) . In addition, the experts are allowed to express preferred and unprepared opinions for each pair of cell phone brands. When they are unwilling or unable to offer some judgments, missing values are permitted. With respect to these four brands of mobile phones for each criterion weight vector are $\Lambda = (0.3, 0.2, 0.1, 0.4)^T$, and the decision matrix in the form of LICFVs are listed in Tables 1.

In the following, to choose the best alternative (Mobile), we use the proposed operators.

5.1.1 By LICFWA operator

Step 1. The experts give their decisions in Table 1. As all the criteria have the same type (benefit), so the normalization are not needed.

Step 2. Utilizing LICFWA operator in Eq. (4.3), having $\Lambda = (0.2, 0.3, 0.1, 0.4)^T$ weight vector, we obtain the collective LICFVs for the alternatives $\mathbb{Z}_i (i = 1, ..., 4)$.

$$\begin{aligned}
\mathbb{Z}_1 &= (\langle[2.42703, 3.91018], 4.73519\rangle, \langle[3.55234, 5.08982], \\ &\quad 3.16979\rangle) \\
\mathbb{Z}_2 &= (\langle[4.42909, 5.87857], 2.43710\rangle, \langle[1.80885, 3.12143], \\ &\quad 3.25862\rangle) \\
\mathbb{Z}_3 &= (\langle[2.54593, 3.92875], 4.30683\rangle, \langle[2.42081, 3.88218], \\ &\quad 2.39246\rangle) \\
\mathbb{Z}_4 &= (\langle[2.58192, 3.81674], 3.53818\rangle, \langle[2.70192, 4.11914], \\ &\quad 2.41507\rangle)
\end{aligned}$$

Step 3. Using Eq. (3.2) , find the scores $Sc(\mathbb{Z}_i)$ of $\mathbb{Z}_i (i = 1, ..., 4)$ as follows;

$$Sc(\mathbb{Z}_1) = 0.4238, Sc(\mathbb{Z}_2) = 0.3876, Sc(\mathbb{Z}_3) = 0.3607, \\ Sc(\mathbb{Z}_4) = 0.3551$$

Step 4. According to the score values, we have, $\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$. Thus, $\mathbb{Z}_1$ is the best choice.

5.1.2 By LICFOWA operator

Step 1. The aggregated information are taken from the Table 1.

Step 2. Utilizing LICFOWA operator in Eq. (4.9), having $\Lambda = (0.2, 0.3, 0.1, 0.4)^T$ weight vector, we obtain the collective LICFVs for the alternatives $\mathbb{Z}_i (i = 1, ..., 4)$.

Table 1 The linguistic intuitionistic cubic fuzzy variables decision matrix

		$\mathbb{N}_1$	$\mathbb{N}_2$	$\mathbb{N}_3$	$\mathbb{N}_4$
D=	$\mathbb{Z}_1$	$\begin{pmatrix}\langle[\acute{s}_4, \acute{s}_6], \acute{s}_5\rangle, \\ \langle[\acute{s}_2, \acute{s}_3], \acute{s}_4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_2, \acute{s}_3], \acute{s}_1\rangle, \\ \langle[\acute{s}_4, \acute{s}_6], \acute{s}_5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_3, \acute{s}_4], \acute{s}_3\rangle, \\ \langle[\acute{s}_4, \acute{s}_5], \acute{s}_4\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_1, \acute{s}_2], \acute{s}_6\rangle, \\ \langle[\acute{s}_5, \acute{s}_7], \acute{s}_2\rangle\end{pmatrix}$
	$\mathbb{Z}_2$	$\begin{pmatrix}\langle[\acute{s}_1, \acute{s}_2], \acute{s}_2\rangle, \\ \langle[\acute{s}_5, \acute{s}_7], \acute{s}_3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_5, \acute{s}_7], \acute{s}_3\rangle, \\ \langle[\acute{s}_1, \acute{s}_2], \acute{s}_2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_3, \acute{s}_5], \acute{s}_4\rangle, \\ \langle[\acute{s}_3, \acute{s}_4], \acute{s}_2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_6, \acute{s}_7], \acute{s}_2\rangle, \\ \langle[\acute{s}_1, \acute{s}_2], \acute{s}_5\rangle\end{pmatrix}$
	$\mathbb{Z}_3$	$\begin{pmatrix}\langle[\acute{s}_3, \acute{s}_4], \acute{s}_4\rangle, \\ \langle[\acute{s}_3, \acute{s}_4], \acute{s}_2\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_1, \acute{s}_3], \acute{s}_7\rangle, \\ \langle[\acute{s}_2, \acute{s}_5], \acute{s}_1\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_2, \acute{s}_5], \acute{s}_1\rangle, \\ \langle[\acute{s}_4, \acute{s}_6], \acute{s}_3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_3, \acute{s}_4], \acute{s}_3\rangle, \\ \langle[\acute{s}_2, \acute{s}_3], \acute{s}_4\rangle\end{pmatrix}$
	$\mathbb{Z}_4$	$\begin{pmatrix}\langle[\acute{s}_2, \acute{s}_3], \acute{s}_3\rangle, \\ \langle[\acute{s}_4, \acute{s}_5], \acute{s}_5\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_3, \acute{s}_4], \acute{s}_4\rangle, \\ \langle[\acute{s}_2, \acute{s}_3], \acute{s}_3\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_5, \acute{s}_7], \acute{s}_2\rangle, \\ \langle[\acute{s}_1, \acute{s}_2], \acute{s}_6\rangle\end{pmatrix}$	$\begin{pmatrix}\langle[\acute{s}_2, \acute{s}_3], \acute{s}_4\rangle, \\ \langle[\acute{s}_3, \acute{s}_5], \acute{s}_1\rangle\end{pmatrix}$

$$\begin{aligned}\mathbb{Z}_1 &= (\langle[2.78173, 4.22802], 3.43844\rangle, \langle[3.32232, 4.77201], 4.08057\rangle)\\ \mathbb{Z}_2 &= (\langle[4.73519, 6.39599], 2.84691\rangle, \langle[1.46326, 2.60401], 2.74171\rangle)\\ \mathbb{Z}_3 &= (\langle[2.53261, 4.23772], 3.34953\rangle, \langle[2.67028, 4.11721], 2.78078\rangle)\\ \mathbb{Z}_4 &= (\langle[3.17237, 4.76263], 3.26353\rangle, \langle[2.19464, 3.60913], 2.63604\rangle)\end{aligned}$$

Step 3. Using Eq. (3.2), find the scores $Sc(\mathbb{Z}_i)$ of $\mathbb{Z}_i(i = 1, ..., 4)$ as follows;

$$Sc(\mathbb{Z}_1) = 0.4189, Sc(\mathbb{Z}_2) = 0.3849, Sc(\mathbb{Z}_3) = 0.3646, Sc(\mathbb{Z}_4) = 0.3636.$$

Step 4. According to the score values, we have, $\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$. Thus, $\mathbb{Z}_1$ is the best choice.

5.1.3 By LICFHA operator

Step 1. The aggregated information are taken from the Table 1.

Step 2. Utilizing LICFHA operator in Eq. (4.14), having $\Lambda = (0.2, 0.3, 0.1, 0.4)^T$ weights, and $w = (0.2, 0.3, 0.3, 0.2)^T$ associated weights, we obtain the collective LICFVs for the alternatives $\mathbb{Z}_i(i = 1, ..., 4)$.

$$\begin{aligned}\mathbb{Z}_1 &= (\langle[2.25006, 3.60545], 4.17312\rangle, \langle[3.88128, 5.39401], 3.60703\rangle)\\ \mathbb{Z}_2 &= (\langle[4.17327, 5.68839], 2.38934\rangle, \langle[2.00131, 3.31058], 3.33290\rangle)\\ \mathbb{Z}_3 &= (\langle[2.23515, 3.66914], 4.28193\rangle, \langle[2.70211, 4.31142], 2.51024\rangle)\\ \mathbb{Z}_4 &= (\langle[2.56392, 3.80008], 3.29922\rangle, \langle[2.79055, 4.15304], 2.83117\rangle)\end{aligned}$$

Step 3. Using Eq. (3.2), to find the scores $Sc(\mathbb{Z}_i)$ of $\mathbb{Z}_i(i = 1, ..., 4)$ as follows;

$$Sc(\mathbb{Z}_1) = 0.4242, Sc(\mathbb{Z}_2) = 0.3869, Sc(\mathbb{Z}_3) = 0.3649, Sc(\mathbb{Z}_4) = 0.3599.$$

Step 4. According to the score values, we have, $\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$. Thus, $\mathbb{Z}_1$ is the best choice.

5.1.4 By LICFWG operator

Step 1. The aggregated information are taken from the Table 1.

Step 2. Utilizing LICFWG operator in Eq. (4.20), having $\Lambda = (0.2, 0.3, 0.1, 0.4)^T$ weight vector, we obtain the collective LICFVs for the alternatives $\mathbb{Z}_i(i = 1, ..., 4)$.

$$\begin{aligned}\mathbb{Z}_1 &= (\langle[1.94328, 3.23212], 3.70394\rangle, \langle[3.94124, 5.76788], 3.52935\rangle)\\ \mathbb{Z}_2 &= (\langle[3.15331, 4.64799], 2.32462\rangle, \langle[2.68625, 4.35201], 3.65683\rangle)\\ \mathbb{Z}_3 &= (\langle[2.31253, 3.86156], 3.47125\rangle, \langle[2.53749, 4.11263], 2.81221\rangle)\\ \mathbb{Z}_4 &= (\langle[2.37707, 3.45865], 3.42354\rangle, \langle[2.97052, 4.41244], 3.43844\rangle)\end{aligned}$$

Step 3. Using Eq. (3.2), to find the scores $Sc(\mathbb{Z}_i)$ of $\mathbb{Z}_i(i = 1, ..., 4)$ as follows;

$$Sc(\mathbb{Z}_1) = 0.4096, Sc(\mathbb{Z}_2) = 0.3856, Sc(\mathbb{Z}_3) = 0.3538, Sc(\mathbb{Z}_4) = 0.3718$$

Step 4. According to the score values, we have, $\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$. Thus, $\mathbb{Z}_1$is the best choice

5.1.5 By LICFOWG operator

Step 1. The aggregated information are taken from the Table 1.

Step 2. Utilizing LICFOWG operator in Eq. (4.26), having $\Lambda = (0.2, 0.3, 0.1, 0.4)^T$ weight vector, we obtain the collective LICFVs for the alternatives $\mathbb{Z}_i(i = 1, ..., 4)$.

$$\begin{aligned}\mathbb{Z}_1 &= (\langle[2.49146, 3.75672], 2.41507\rangle, \langle[3.59099, 5.24328], 4.27046\rangle)\\ \mathbb{Z}_2 &= (\langle[4.05936, 5.77385], 2.70192\rangle, \langle[1.95308, 3.22615], 3.17237\rangle)\\ \mathbb{Z}_3 &= (\langle[2.38003, 4.15565], 2.48754\rangle, \langle[2.86421, 4.48771], 3.07945\rangle)\\ \mathbb{Z}_4 &= (\langle[2.74171, 3.98115], 3.06735\rangle, \langle[2.59541, 4.07302], 3.95804\rangle)\end{aligned}$$

Step 3. Using Eq. (3.2), to find the scores $Sc(\mathbb{Z}_i)$ of $\mathbb{Z}_i(i = 1, ..., 4)$ as follows;

$$Sc(\mathbb{Z}_1) = 0.4031, Sc(\mathbb{Z}_2) = 0.3867, Sc(\mathbb{Z}_3) = 0.3602, Sc(\mathbb{Z}_4) = 0.3780.$$

Step 4. According to the score values, we have, $\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$. Thus, $\mathbb{Z}_1$ is the best choice.

5.1.6 By LICFHG operator

Step 1. The aggregated information are taken from the Table 1.

Step 2. Utilizing LICFHG operator in Eq. (4.29), having $\Lambda = (0.2, 0.3, 0.1, 0.4)^T$ weights and $w = (0.2, 0.3, 0.3, 0.2)^T$ associated weights, we obtain the collective LICFVs for the alternatives $\mathbb{Z}_i (i = 1, \ldots, 4)$.

$$\begin{aligned}
\mathbb{Z}_1 &= (\langle [3.60704, 3.51706], 3.55044\rangle, \langle [3.70951, 5.48294], 3.46736\rangle)\\
\mathbb{Z}_2 &= (\langle [3.33292, 5.07836], 3.48547\rangle, \langle [2.39347, 3.92153], 3.24481\rangle)\\
\mathbb{Z}_3 &= (\langle [2.50285, 4.09115], 3.76947\rangle, \langle [2.67237, 4.04718], 2.49915\rangle)\\
\mathbb{Z}_4 &= (\langle [2.83044, 5.11484], 5.33485\rangle, \langle [2.62751, 3.96908], 3.32638\rangle)
\end{aligned}$$

Step 3. Using Eq. (3.2), to find the scores $Sc(\mathbb{Z}_i)$ of $\mathbb{Z}_i (i = 1, \ldots, 4)$ as follows;

$$Sc(\mathbb{Z}_1) = 0.4321, Sc(\mathbb{Z}_2) = 0.3973, Sc(\mathbb{Z}_3) = 0.3626, Sc(\mathbb{Z}_4) = 0.4296.$$

Step 4. According to the score values, we have, $\mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_2 > \mathbb{Z}_3$. Thus, $\mathbb{Z}_1$ is the best choice (Table 2) and graphically representation is given in Fig. 1.

6 Analyses and comparisons

6.1 Comparison 1

In the upcoming contents, proposed MADM approach will be analyzed their comparisons with existing approaches also be investigated.

We contrasted our proposed advanced aggregation operators with pre-existing fuzzy aggregation operators and suggested the conclusion of our work. Given the fact that the LIF set theory has an enormous impact in various fields, there are still some real world problems that LIFS and IVLIFS could not solve. Term in LICFVs consists of the linguistic positive grade and linguistic negative grade. If we take the numerical problem described in Sec. V, as LICFVs is the most advanced structure, it is therefore not possible for the other developed aggregation operators to solve the data contained in this problem, showing the restricted approach of the current approaches. But if we take some problem with the interval-valued fuzzy information, we can easily solve by the LICFVs, converting the data from the interval-valued to LICFVs, by taking the values outside the interval in LICFVs is zero.

Table 2 Ranking of the alternatives using different operators

Operators	Score value				Ranking
	$Sc(\mathbb{Z}_1)$	$Sc(\mathbb{Z}_2)$	$Sc(\mathbb{Z}_3)$	$Sc(\mathbb{Z}_4)$	
LICFWA	0.4238	0.3876	0.3607	0.3551	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
LICFOWA	0.4189	0.3849	0.3646	0.3636	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
LICFHA	0.4242	0.3869	0.3649	0.3599	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
LICFWG	0.4096	0.3856	0.3538	0.3718	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$
LICFOWG	0.4031	0.3867	0.3602	0.3780	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$
LICFHG	0.4321	0.3973	0.3626	0.4296	$\mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_2 > \mathbb{Z}_3$

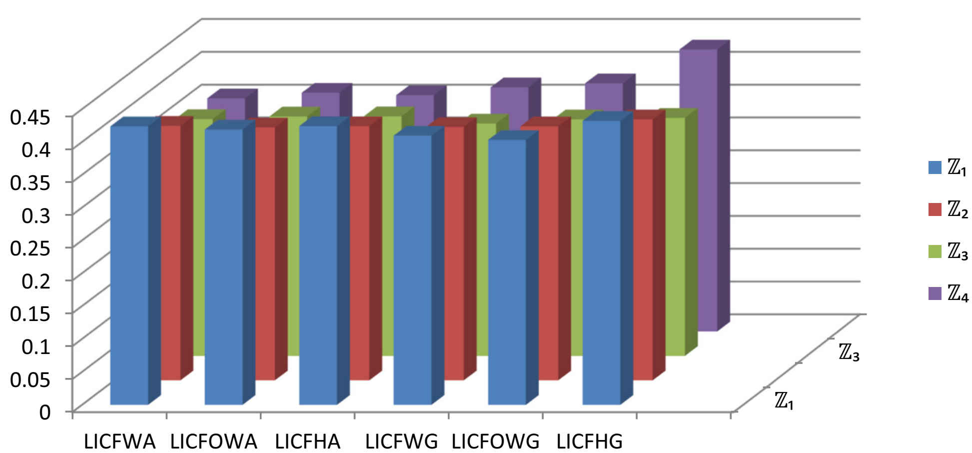


Fig. 1 Ranking of alternative by different aggregation operator

Now, we compare our developed approach to the approaches of (Garg and Kumar 2019; Chen et al. 2015; Liu et al. 2017; Fahmi et al. 2018a; Kaur and Garg 2018b; Khan et al. 2019 and Qiyas et al. 2019) . To compare our proposed method with other (Chen et al. 2015; Fahmi et al. 2018a; Garg and Kumar 2019; Kaur and Garg 2018b; Khan et al. 2019; Liu and Liu 2017; Qiyas et al. 2019) methods, in which each linguistic term or fuzzy term has one positive and negative grades. So, if we consider only the positive and negative grade we neglect the cubic term, then the LICFVs decrease to the LIVIF or IVIF variables. We take $\Lambda = (0.3, 0.2, 0.1, 0.4)^T$ are the criteria weight vector to facilitate the comparison. Using the given preferences and information, the existing methods (Chen et al. 2015; Fahmi et al. 2018a; Garg and Kumar 2019; Kaur and Garg 2018b; Khan et al. 2019; Liu and Liu 2017; Qiyas et al. 2019) are applied to the data being considered, and then the final scores of the alternatives $\mathbb{Z}_\imath(\imath = 1, \ldots, 4)$ is shown in Table 3. The Table 3 show that $\mathbb{Z}_1$ is the best alternative in any approach. Compared with these existing approaches with general linguistic intuitionistic sets (LIVIFSs or LIFSs), the proposed decision-making method under linguistic intuitionistic cubic fuzzy set environment contains much more evaluation information on the alternatives by considering both the IVIFSs and IFSs simultaneously, while the existing approaches contain either LIFS or LIVIFS information. Therefore, the approaches under the LIVIFSs or LIFSs may lose some useful information, either LIVIFNs or LIFNs, of alternatives which may affect the decision results. Furthermore, it is noted from the study that the computational procedure of the proposed approach is different from the existing approaches under the different information, but the proposed result in this paper is more rational to reality in the decision process due to the consideration of the consistent priority degree between the pairs of the arguments. There are some variations in the remaining alternatives, however due to different evaluations. Thus, the below comparative analysis table, we say that our proposed LICF aggregation operators are more effective and reliable than previous aggregation operators. The graphical representation of the Table 3 are shown in Fig. 2.

6.2 Comparison 2

In addition, if we consider the number problem discussed in Sec. 5, then instead of utilizing the score function of LICFVs, we used the score function of linguistic cubic

Table 3 Ranking of the comparative study

Authors	Score values				Ranking
	$Sc(\mathbb{Z}_1)$	$Sc(\mathbb{Z}_2)$	$Sc(\mathbb{Z}_3)$	$Sc(\mathbb{Z}_4)$	
Garg and Kumar (2019)	6.6341	5.7925	4.3551	4.0473	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
Chen et al. (2015)	2.8921	1.9932	0.8746	1.3712	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$
Liu et al. (2017)	4.5379	4.0642	3.6734	3.1933	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
Fahmi et al. (2018a)	0.8414	0.6321	0.6972	0.5411	$\mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_2 > \mathbb{Z}_4$
Kaur and Garg (2018b)	−1.328	−0.685	−0.893	−1.126	$\mathbb{Z}_1 > \mathbb{Z}_4 > \mathbb{Z}_2 > \mathbb{Z}_3$
Khan et al. (2019)	0.8731	0.7163	0.7936	0.5673	$\mathbb{Z}_1 > \mathbb{Z}_3 > \mathbb{Z}_2 > \mathbb{Z}_4$
Qiyas et al. (2019)	0.4915	0.4372	0.3629	0.2841	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$

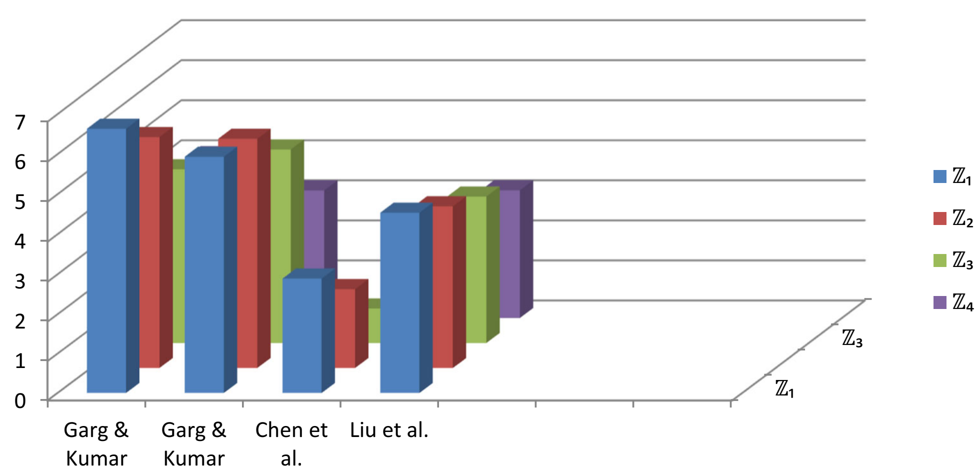
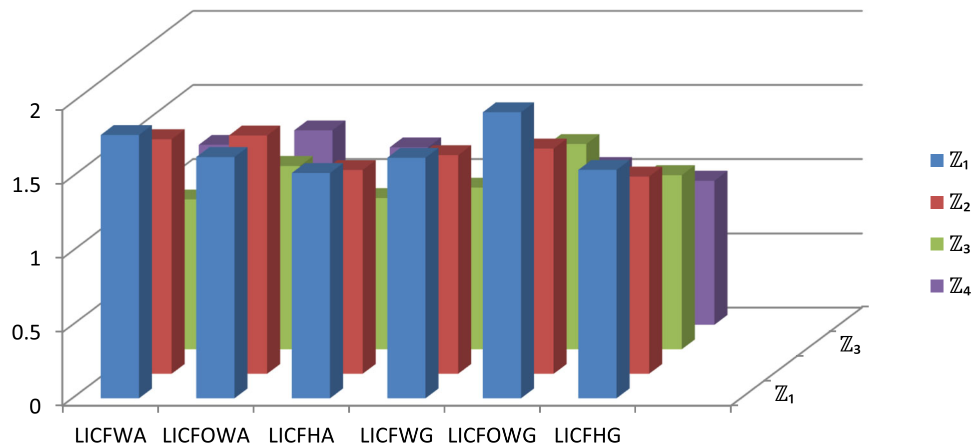


Fig. 2 Comparative of proposed operator

Table 4 Comparison (Ranking of the alternatives using different operators)

Operators	Score value				Ranking
	$Sc(\mathbb{Z}_1)$	$Sc(\mathbb{Z}_2)$	$Sc(\mathbb{Z}_3)$	$Sc(\mathbb{Z}_4)$	
LICFWA	1.782	1.586	1.012	1.213	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$
LICFOWA	1.633	1.611	1.238	1.311	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_4 > \mathbb{Z}_3$
LICFHA	1.525	1.378	1.021	1.196	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
LICFWG	1.628	1.478	1.092	1.065	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
LICFOWG	1.935	1.523	1.386	1.063	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$
LICFHG	1.546	1.334	1.176	0.971	$\mathbb{Z}_1 > \mathbb{Z}_2 > \mathbb{Z}_3 > \mathbb{Z}_4$

Fig. 3 Cmp



fuzzy variables by considering LICFVs membership then non-membership functions as individual linguistic cubic fuzzy variables i.e., here we take a LCFV $\left\langle [\acute{s}_{\mu^-}, \acute{s}_{\mu^+}], \acute{s}_{\phi} \right\rangle, \left\langle [\acute{s}_{\nu^-}, \acute{s}_{\nu^+}], \acute{s}_{\chi} \right\rangle$ as the collection of two linguistic cubic numbers $\mathfrak{R} = \left\langle [\mathfrak{s}_{\mu^-}, \mathfrak{s}_{\mu^+}], \mathfrak{s}_{\phi} \right\rangle$ and $\mathfrak{R} = \left\langle [\mathfrak{s}_{\nu^-}, \mathfrak{s}_{\nu^+}], \mathfrak{s}_{\chi} \right\rangle$ calculate the score value of each individually by the score function of LCFVs Ye (2018), and then by taking the average of both $\frac{1}{2}(Sc(\mathfrak{R}) + \mathfrak{Sc}(\mathfrak{R})) = \frac{1}{2}\left(\frac{\mu^- + \mu^+ - \phi}{3} + \frac{\nu^- + \nu^+ - \chi}{3}\right)$, we get the ranking results of the alternatives, which are given in Table 4, and find the same result as given in Table 2 by using LICF score function, i.e., again $\mathbb{Z}_1$ is the best choice among all alternatives as shown in the following Fig. 3.

7 Conclusion

We have established an advanced approach to LIFS through application of linguistic cubic fuzzy variable theory and introduced the concept of an linguistic intuitionistic cubic fuzzy variable. Also, we have defined accuracy degree and score function for the comparison of two linguistic intuitionistic cubic fuzzy variables. We defined some connectivity of two linguistic intuitionistic cubic fuzzy variables, i.e., the operational laws of linguistic intuitionistic cubic fuzzy variables introduced. Some LICF operational laws have been developed. We also established a number of linguistic intuitionistic cubic fuzzy aggregation operators, i.e., we proposed LICFWA operator, LICFOWA, LICFHA, LICFWG, LICFOWG and LICFHG operator under LICF environment; discussed some properties of these operators like idempotency, boundary, and monotonicity, and showed relationships among these developed operators. The operator is characterized by considering information about the relationship among the linguistic intuitionistic cubic fuzzy numbers LICFNs being aggregated. To demonstrate the performance of these new techniques, we develop a MCDM based on the proposed operators under the LICF information. Resolving the problem of evaluation and ranking the potential suppliers has become a key strategic element for the company. As the intelligent and automated information systems were developed in the information era, more effective decision-making methods have become necessary. For instance, a numerical application related to the selection of suitable supplier of the proposed operators under the LICF information has been presented, which shows that the suggested operators delivers an alternative way to solve decision-making process in a more actual way. Finally, we have provided comparison of the proposed operators to the existence operators to show the validity, practicality, and effectiveness of the proposed methods. Our proposed method is different from all the previous techniques for group decision-making due to the fact that the proposed method uses linguistic intuitionistic cubic fuzzy information, which will not cause any loss of information in the process. So it is efficient and feasible for real-world decision-making applications.

In the future, more aggregation operators will be formed under the LICF details, such as Dombi aggregation operators, Himachar aggregation operators, Dombi Bonferroni mean operators, Heronian mean operators and others.

Acknowledgements The authors would like to thank the Deanship of Scientific Research at Umm Al-Qura University for supporting this work by Grant Code: 19-SCI-1-01-0041.

Compliance with ethical standards

Conflict of interest The authors declare no conflict of interest.

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