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ORIGINAL RESEARCH

Complex interval-valued intuitionistic fuzzy TODIM approach and its application to group decision making

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Abstract

Present work proposes novel fuzzy information based TODIM approaches that can deal with the evaluations under complex interval-valued intuitionistic fuzzy (CIVIF) environment. The proposed approaches have been referred to as complex interval-valued intuitionistic fuzzy-TODIM (CIVIF-TODIM) approaches. The proposed method encompasses the characteristic features of a complex intuitionistic fuzzy set, interval-valued fuzzy set, and TODIM methodology. At first, the definitions associated with CIVIF have been discussed and then the methodological steps involved in classical TODIM have been delineated. The classical TODIM approach is then extended to deal with group decision-making problems under the CIVIF environment. Robustness, effectiveness, applicability, and the improvements made to the extant fuzzy TODIM methods by the proposed methodology have been adjudged through the consideration of illustrative examples solved by the past researcher. Sensitivity analysis with respect to the attenuation factor as well as the criteria weights has been provided to justify the robustness of the proposed methods. A comparative analysis with the existing fuzzy-TODIM approaches has been delineated and a comprehensive analysis of the ranking results obtained for different distance measures at each value of attenuation factor is provided towards the end of the work. The carried out inclusive analysis on the approaches that have been proposed in the present work reveals that the proposed CIVIF fuzzy TODIM approaches are superior to the existing fuzzy TODIM methods. Therefore, the present study provides its contribution to the domain of decision-making framework through the approaches that provide for dealing with complex, uncertain, and linguistic information in an efficient manner.

Keywords Soft computing · Fuzzy logic · Complex interval-valued intuitionistic fuzzy numbers · Prospect theory · Multi-criteria decision making

1 Introduction

The process of decision making involves the performance appraisal of several alternatives under the influence of different criteria that are often conflicting in nature. The process is known as a multi-criteria decision making (MCDM) process wherein each alternative is rated in terms of precise data and subjective information provided by the decision makers (DMs). Conventionally it is believed that the information provided depicts crisp nature. However, in today's scenario of enhanced complexity, there are real-world MCDM problems wherein the information is plagued with impreciseness, vagueness, and uncertainty (Garg 2018a; b); ;. Therefore to deal with such real-world problems, fuzzy sets were introduced by Zadeh (1965) and are characterized by membership degrees. Researchers have proposed several methods that are based on the introduced fuzzy sets (Yager 1977; Chen 2000; Wang 2016). However, single membership function is not capable to capture the valuable imprecise information, and therefore (Atanassov 1986, 1989) proposed the intuitionistic fuzzy (IF) sets that characterize the information based on both the membership degree as well as non-membership degrees. Since the introduction of IFS, several new methodologies have been researched and proposed by the researchers and play their prominent role in enhancing the domain of decision-making process. Some of the proposed researches in this regard are the interval-valued intuitionistic fuzzy (IVIF) sets (Atanassov and Gargov 1989; Wu et al. 2013; Cao et al. 2018), intuitionistic triangular fuzzy numbers (Shu 2006; Jianqiang and Zhong 2009), etc. and have been

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employed to address a wide variety of MCDM problems (Zhang and Liu 2011; Tian 2018; Nan 2016).

The scientific community involved in decision sciences have worked in different domains to enhance the effectiveness of IVIF sets. One such domain is that of aggregating the information or the evaluation provided by different DMs. Aggregation operators under the IVIF environment have been proposed by researchers from time and then. Xu and Yager (2006) proposed that the conceptual framework for intuitionistic fuzzy operators i.e., the weighted and the hybrid geometric operators. Wei (2010) proposed the conceptual framework for the novel induced geometric aggregation operators for the intuitionistic environment. Induced correlated aggregation operators for IF environment was established by Wei and Zhao (2012), prioritized intuitionistic fuzzy aggregation operator (Yu and Xu 2013), intuitionistic fuzzy Dombi Hamy mean operator (Li et al. 2018), interval-valued intuitionistic fuzzy Hamy mean operators (Wu et al. 2019a), Interval-Valued Intuitionistic Fuzzy Dombi Heronian Mean Operators (Wu et al. 2020), Einstein t-norm and Hamacher t-norm operations based operators (Garg 2016a, b; Garg 2017, 2018c), Einstein hybrid weighted geometric aggregation operator (Wang and Liu 2013a, b), etc. The second domain that has enriched the decision making in IVIF environment is the laborious work on ranking methods. A generalized improved score function was proposed by Garg (2016b) to rank the different IVIF numbers. Possibility degree measure method (Garg and Kumar 2019), similarity measures based on transformation techniques (Chen and Chang 2015), similarity measures based on the connection number (Garg and Kumar 2018) and so on. Owing to the effectiveness of IVIF, several MCDM approaches have been proposed by the researchers (Singh and Garg 2017; Kaur and Garg 2019,2018; Garg and Singh 2018; Chen and Tsai 2016; Wang and Liu 2013a, b; Arora and Garg 2018; Garg and Arora 2018a, b; Xu 2007; Garg and Arora 2018a, b).

However, the various methods for the IVIF environment are only able to handle the vagueness and uncertainty in the data that exists for making evaluations. Therefore the concept of complex fuzzy set (CFS) was developed by Ramot et al. (2002), wherein the membership function has been extended from the subset of a real number to unit disc. Later on the concept of complex intuitionistic fuzzy set was put forth by Alkouri and Salleh (2012) (CIFS) that also added the degree of non-membership in addition to the membership degree. The distance between measure between two CIFS through the introduction of the conceptual framework of complex intuitionistic fuzzy relations, projections, and compositions (Alkouri and Salleh 2013). Distance and entropy measures were proposed by Kumar and Bajaj (2014) for CIFS environment. Power aggregation operators were developed by Rani and Garg (2018) to solve MCDM problems under the CIFS environment.

There are situations wherein, the evaluations cannot be provided in terms of single membership and non-membership degrees by the experts involved addressing the problems of the decision making domains. Therefore with the motivation to allow the DMs more freedom in describing their valuations in terms of interval numbers, the conceptual framework of complex interval-valued intuitionistic fuzzy (CIVIFS) set was put forth by Garg and Rani (2019). In the CIVIFS approach, complex values represent the membership and non-membership degrees and are represented in the polar coordinate system. The extent of belonging to which an object belongs to the CIVIFS is represented in the amplitude term and the phase term provides information associated with the periodicity. CIVIFS, therefore, has the potentiality to represent the complete evaluation in one set through the addition of the second dimension and hence eliminates the chance of loss in the information that takes place in the extant IVIF conceptual framework. As for instance, there is a company 'X' that needs to install biometric-based attendance system in its offices that are spread across the country. The company 'X' purchases these devices from a manufacture. The manufacturer provides information on two aspects: models of the biometric-based attendance devices and the production dates of the devices. Hence this two-step judgement process can be handled by the company 'X' in a more effective manner by representing the amplitude term to represent the decision regarding the model of the devices and the decision regarding the production dates can be represented using a phase term. CIVIFS are more generalized extensions of the already existing fuzzy set theories.

TODIM (an acronym in Portuguese for Interactive and Multicriteria Decision Making) method was proposed by Gomes and Lima Gomes and Lima (1992) and prospect theory (Kahneman and Tversky 1979; Tversky and Kahneman 1992) forms the basis of its development. Because of the potential ability of the TODIM approach to deal with the decision-making environment with due consideration to the risk appetite of the decision maker, it has been employed widely (Fan et al. 2013) to address the decision-making problems plagued with the environment of uncertainty and risk. The versatile TODIM approach has been employed to address a wide range of decision-making problems.

Owing to the versatility of the TODIM method it has been extended to address real-world decision-making problems that are characteristic of complexity as well as uncertainty. Hence, TODIM method has been expanded to consider different types of linguistic information as such intuitionistic fuzzy (Li et al. 2018; Mohagheghi 2017), interval-valued information (Jiang et al. 2017; Hu et al. 2018), probabilistic interval-valued hesitant fuzzy information (Zhang et al. 2019), probabilistic linguistic information (Nie and Wang, 2020), the unbalanced information under the hesitant fuzzy environment (Yu et al. 2017), Uncertain linguistic

Z-numbers (Tian et al. 2020), fuzzy Pythagorean evaluations (Geng et al. 2017), 2-tuple linguistic neutrosophic information (Wang et al. 2018), etc.

The various versions of the fuzzy TODIM methods that have been proposed since time and then have been adjudged for their practical application by the scientific community into the domain of decision making. A wide array of decision-making problems have been addressed including evaluation of rental evaluation (Moshkovich et al. 2011), oil spill response (Passos et al. 2014), site selection for thermal power plants (Soni et al. 2016), material selection (Zindani et al. 2017, 2018), hotel selection (Wang et al. 2020b), home-based elderly-care services (Lu et al. 2020), rural reconstruction (Lu and Wei 2019), personnel selection (Zindani et al. 2020), etc.

The existing fuzzy TODIM methods can be made more efficient through an integrated framework of decision making that combines the advantages of CIVIFS and the TODIM method. Given the aforementioned motivation, the present work proposes the CIVIF-TODIM approach for the decision-making problems and therefore forms the major contribution of the present work towards enriching the domain of decision science. The proposed framework not only aid in making decisions under multi-step judgment scenarios but also considers the psychological behavior of the decision maker towards risk while making decisions. Therefore following objectives have been considered in the present work: (1) a novel complex interval-valued intuitionistic fuzzy TODIM approach (CIVIF-TODIM) is developed to aid decision making under the complex intuitionistic environment with due consideration to the risk appetite of the experts involved in the decision making process. Two different CIVIF-TODIM approaches: CIVIFOWA fuzzy TODIM and CIVIFOWG fuzzy TODIM have been developed, (2) applicability of the proposed methods have been demonstrated through an illustrative example, (3) sensitivity analysis with respect to the attenuation factor as well as the criteria weights have been provided to justify the robustness of the proposed methods, (4) comparative analysis with the existing fuzzy-TODIM approaches have been delineated and (5) a comprehensive analysis of the ranking results obtained for different distance measures at each value of attenuation factor is provided towards the end of the work.

The rest of the work is organized into the following sections: Sect. 2 outlines the basic concepts associated with the TODIM method and CIVIFS. The aggregation operators associated with CIVIFS have been also discussed in this section. Section 3 describes the integrated framework for group decision-making problems in the form of the CIVIFS-TODIM approach. The proposed integrated CIVIFS-TODIM approach has been demonstrated for its applicability and effectiveness through illustrative examples. These illustrative examples have been discussed in Sect. 4. Section 5 represents the concluding remarks of the present work.

2 Background

The concepts associated and the definitions required in the build-up to the proposed methodologies have been presented in this section of the work. Therefore, the basic definitions as well the associated concepts that are related to the TODIM method, the CIVIFS, and the weighted and the ordered weighted averaging and geometric aggregation operators have been discussed.

2.1 TODIM method

In the TODIM method, the psychological behavior of the decision maker is given due consideration to establish the rankings of the alternatives. As such the TODIM method helps the expert involved in addressing the MCDM problem to make a more satisfactory decision while minimizing the potentiality of associated risks. The prospect theory assumes that the process of decision-making can be divided into two phases: editing and evaluation. The information is collected during the editing phase and then value function in tandem to the weighting function of subjective probability is used during the evaluation stage to determine the information. The prospect value function is an "S" shaped curve that reflects the decision maker's attitude towards the loss and benefit while dealing with a problem.

The following are the important parameters in the value function $V = \sum v(x)\pi(\rho)$: the concave and convex degrees in the losses and gains and the changes in the weighting functions. Through a large number of experimentations carried out by Kahneman and Tversky (1979) the following values for the important parameters have been defined: $\alpha = \beta = 0.88$, $\xi = 0.61$ and $\tau = 0.69$.

The procedural steps associated with the TODIM approach (Zindani et al. 2017) have been provided in the following discussion:

Step 1: In this step of the decision-making process, decision matrix $\boldsymbol{X} = (\boldsymbol{x}_{ij})$ is formed. The performance of material alternative A_i with respect to the criteria C_j is reflected in x_{ij}. The obtained decision matrix is then normalized and represented as $\boldsymbol{P} = (\boldsymbol{p}_{ij})_{m \times n}$.

Step 2: Subsequently with the aid of the normalized decision matrix, the importance weights (w_j) associated with each of the criteria are obtained. The reference weight (w_r) is then determined and is usually the one that has the highest weight value. Using the reference weights, the relative weights (w_{jr}) associated with each criterion is obtained using Eq. (1):

$$\omega_{jr} = \frac{w_j}{w_r} \tag{1}$$

Step 3: In the next step, the dominance degree (ϕ_j) of each alternative over the other under particular criteria is evaluated. This is accomplished using Eq. (2):

$$\varphi_j(M_i, M_k) = \begin{cases} \sqrt{\frac{w_{jr}(p_{ij} - p_{kj})}{(\sum_{j=1}^{n} w_{jr})}}, & if\ p_{ij} - p_{kj} > 0 \\ 0, & if\ p_{ij} - p_{kj} = 0 \\ \frac{-1}{\theta}\sqrt{\frac{(\sum_{j=1}^{n} w_{jr}.(p_{ij} - p_{kj})}{w_{jr}}} & if\ p_{ij} - p_{kj} < 0 \end{cases} \tag{2}$$

where the attenuation factor is represented by θ. The attenuation factor exhibits the gain and loss of one alternative $A_i$over the other alternative A_j. The term $p_{ij} - p_{kj} \geq 0$ represents the gain and $p_{ij} - p_{kj} < 0$ represents the loss. The prospect function curve shows different shapes depending on the value of the attenuation factor.

$$A = \{(x, [\mu_A^-(x), \mu_A^+(x)], [\nu_A^-(x), \nu_A^+(x)]) : x \in U\}, \tag{5}$$

where the upper and lower bounds of membership and non-membership degrees are represented by $\mu_A^-(x), \mu_A^+(x)$ and $\nu_A^-(x), \nu_A^+(x)$ respectively and can be described using a set of Eq. (6):

$$\left.\begin{aligned} \mu_A^-(x) = z_1^- = r_A^-(x)e^{iw_{rA}^-(x)} \\ \mu_A^+(x) = z_1^+ = r_A^+(x)e^{iw_{rA}^+(x)} \\ \nu_A^+(x) = z_2^+ = k_A^+(x)e^{iw_{kA}^+(x)} \\ \nu_A^-(x) = z_2^- = k_A^-(x)e^{iw_{kA}^-(x)} \end{aligned}\right\} \tag{6}$$

such that $|z_1^-| \leq |z_1^+|$ and $|z_2^-| \leq |z_2^+|$ holds and the amplitude terms $r_A^-, r_A^+, k_A^-, k_A^+ \in [0, 1]$ and satisfies the following conditions: $r_A^- \leq r_A^+, k_A^- \leq k_A^+$ and $r_A^+(x) + k_A^+(x) \leq 1 \forall x \in U$. The phase terms, on the other hand, have been denoted by $w_{rA}^-, w_{rA}^+, w_{kA}^-, w_{kA}^+ \in [0, 2\pi]$ and are real-valued and satisfy the following conditions: $w_{rA}^- \leq w_{rA}^+, k_{kA}^+ \leq w_{kA}^-$ and $w_{rA}^+(x) + w_{kA}^+(x) \leq 2\pi\ \forall x \in U$. Therefore, CIVIFS can be redefined mathematically as:

$$A = \left\{ \left(x, [r_A^-(x), r_A^+(x)] e^{i[w_{r_A}^-(x), w_{r_A}^+(x)]}, [k_A^-(x), k_A^+(x)] e^{i[w_{k_A}^-(x), w_{k_A}^+(x)]} \right) : x \in U \right\}, \tag{7}$$

Step 4: In this step of the TODIM approach, Eq. (3) is employed to obtain the overall dominance degree (δ) for each of the material alternative over the other.

$$\delta(A_i, A_j) = \sum_{j=1}^{n} \varphi_j(A_i, A_j) \tag{3}$$

Step 5: Next Eq. (4) is employed to calculate the prospect value, ξ

$$\xi(A_i) = \frac{(\sum_{k=1}^{m} \delta(A_i, Aj)) - \min_i(\sum_{k=1}^{m} \delta(A_i, A_j))}{\max_i(\sum_{k=1}^{m} \delta(A_i, Aj)) - \min_i(\sum_{k=1}^{m} \delta(A_i, Aj))} \tag{4}$$

Step 6: The material alternatives are then ranked based on the overall prospect value. The material alternative with the highest value of prospect value is ranked as the best candidate alternative.

2.2 The Complex interval-valued intuitionistic fuzzy sets (CIVIFS)

The Definitions (2.2.1–2.2.7) associated with the CIVIFs (Garg and Rani 2019) have now been discussed:

Definition 2.2.1 CIVIFS on a universe of discourse U can be defined as:

Definition 2.2.2 Equation (8) provides the definition for the complex interval-valued intuitionistic fuzzy number (CIVIFN):

$$\beta = \left([r^-, r^+] e^{i[w_r^-, w_r^+]}, [k^-, k^+] e^{i[w_k^-, w_k^+]} \right) \tag{8}$$

where $r^-, r^+ \in [0, 1]$, $k^-, k^+ \in [0, 1]$, $r^+ + k^+ \leq 1$; and $w_r^-, w_r^+ \in [0, 2\pi]$, $w_k^-, w_k^+ \in [0, 2\pi]$, $w_r^+ + w_k^+ \leq 2\pi$.

Definition 2.2.3 The score function for the aforementioned defined CIVIFN is obtained using Eq. (9):

$$S(\beta) = \frac{1}{2}\left[(r^- + r^+) - (k^- + k^+) + \frac{1}{2\pi}\left[(w_r^- + w_r^+) - (w_k^- + w_k^+)\right]\right] \tag{9}$$

Definition 2.2.4 Accuracy function for CIVIFN is calculated using Eq. (10):

$$H(\beta) = \frac{1}{2}\left[(r^- + r^+) + (k^- + k^+) + \frac{1}{2\pi}\left[(w_r^- + w_r^+) + (w_k^- + w_k^+)\right]\right] \tag{10}$$

Definition 2.2.5 For two CIVIFNs belonging to the two CIVIFSs A and B the following will hold:

- If $S(\beta) < S(\gamma)$, then,$\beta < \gamma$
- If $S(\beta) = S(\gamma)$, then

If $H(\beta) < H(\gamma)$, then, $\beta < \gamma$.
If $H(\beta) = H(\gamma)$, then, $\beta = \gamma$.

Definition 2.2.6 For any two CIVIFNs $\beta_j = \left(\left[r_j^-, r_j^+\right] e^{i[w_{rj}^-, w_{rj}^+]}, [k_j^-, k_j^+] e^{i[w_{kj}^-, w_{kj}^+]}\right) (j = 1, 2)$ the following can be defined:

- $\beta_1 \subseteq \beta_2$ *if* $r_1^- \le r_2^-, r_1^+ \le r_2^+, k_1^- \ge k_2^-, k_1^+ \ge k_2^+$ and $w_{r1}^- \le w_{r2}^-, w_{r1}^+ \le w_{r2}^+, w_{k1}^- \ge w_{k2}^-, w_{k1}^+ \ge w_{k2}^+$
- $\beta_1 = \beta_2$ if and only if $\beta_1 \subseteq \beta_2$ and $\beta_1 \supseteq \beta_2$

Definition 2.2.7 Few main operations between two CIVIFNs $\beta_j = \left(\left[r_j^-, r_j^+\right] e^{i[w_{rj}^-, w_{rj}^+]}, [k_j^-, k_j^+] e^{i[w_{kj}^-, w_{kj}^+]}\right) (j = 1, 2)$ can be defined as follows:

- $$\beta_1 \oplus \beta_2 = \begin{pmatrix} \left[1 - \prod_{j=1}^{2}\left(1 - r_j^-\right), 1 - \prod_{j=1}^{2}\left(1 - r_j^+\right)\right] e^{i\left[2\pi\left(1-\prod_{j=1}^{2}\left(1-\frac{w_{rj}^-}{2\pi}\right)\right), 2\pi\left(1-\prod_{j=1}^{2}\left(1-\frac{w_{rj}^+}{2\pi}\right)\right)\right]}, \\ \left[\prod_{j=1}^{2} k_j^-, \prod_{j=1}^{2} k_j^+\right] e^{i\left[2\pi\left(\prod_{j=1}^{2}\frac{w_{kj}^-}{2\pi}\right), 2\pi\left(\prod_{j=1}^{2}\frac{w_{kj}^+}{2\pi}\right)\right]} \end{pmatrix} \tag{11}$$

- $$\beta_1 \otimes \beta_2 = \begin{pmatrix} \left[\prod_{j=1}^{2} r_j^-, \prod_{j=1}^{2} r_j^+\right] e^{i\left[2\pi\left(\prod_{j=1}^{2}\left(1-\frac{w_{rj}^-}{2\pi}\right)\right), 2\pi\left(\prod_{j=1}^{2}\left(1-\frac{w_{rj}^+}{2\pi}\right)\right)\right]}, \\ \left[1 - \prod_{j=1}^{2}\left(1 - k_j^-\right), 1 - \prod_{j=1}^{2}\left(1 - k_j^+\right)\right] e^{i\left[2\pi\left(1-\prod_{j=1}^{2}\left(1-\frac{w_{kj}^-}{2\pi}\right)\right), 2\pi\left(\prod_{j=1}^{2}\left(1-\frac{w_{kj}^+}{2\pi}\right)\right)\right]} \end{pmatrix} \tag{12}$$

- $$\lambda\beta_1 = \begin{pmatrix} \left[1 - \left(1 - r_1^-\right)^\lambda, 1 - \left(1 - r_1^+\right)^\lambda\right] e^{i\left[2\pi\left(1-\left(1-\frac{w_{r1}^-}{2\pi}\right)\right), 2\pi\left(1-\left(1-\frac{w_{r1}^+}{2\pi}\right)\right)\right]}, \\ \left[\left(k_1^-\right)^\lambda, \left(k_1^+\right)^\lambda\right] e^{i\left[2\pi\left(\frac{w_{k_1}^-}{2\pi}\right)^\lambda, 2\pi\left(\frac{w_{k_1}^+}{2\pi}\right)^\lambda\right]} \end{pmatrix} \tag{13}$$

- $$\beta_1^\lambda = \begin{pmatrix} \left[\left(r_1^-\right)^\lambda \left(r_1^+\right)^\lambda\right] e^{i\left[2\pi\left(\left(\frac{w_{rj}^-}{2\pi}\right)\right), 2\pi\left(\left(\frac{w_{rj}^+}{2\pi}\right)\right)\right]}, \\ \left[1 - \left(1 - k_1^-\right)^\lambda, 1 - \left(\left(1 - k_1^+\right)^\lambda\right)\right] e^{i\left[2\pi\left(1-\left(1-\frac{w_{kj}^-}{2\pi}\right)\right), 2\pi\left(\left(1-\frac{w_{kj}^+}{2\pi}\right)\right)\right]} \end{pmatrix} \tag{14}$$

Definition 2.2.8 The distance between two CIVIFNs can be obtained through the concept of following distance measures between the two complex interval-valued complex fuzzy numbers (CIVIFNs) $\beta_j = \left(\left[r_j^-, r_j^+\right] e^{i[w_{rj}^-, w_{rj}^+]}, [k_j^-, k_j^+] e^{i[w_{kj}^-, w_{kj}^+]}\right)$ $(j = 1, 2)$ (Dai et al. 2019). The Hamming distance, Euclidean distance, normalized Hamming distance, and normalized Euclidean distances have been depicted in Eq. (15–18) respectively.

$$d_H(\beta_1, \beta_2) = \begin{pmatrix} \frac{1}{2}\left(\frac{1}{2}\left|r_1^- - r_2^-\right| + \frac{1}{2}\left|r_1^+ - r_2^+\right| + \frac{1}{2\pi}\left|w_{r1}^- - w_{r2}^+\right|\right) + \\ \frac{1}{2}\left(\frac{1}{2}\left|k_1^- - k_2^-\right| + \frac{1}{2}\left|k_1^+ - k_2^+\right| + \frac{1}{2\pi}\left|w_{k1}^- - w_{k2}^+\right|\right) \end{pmatrix} \tag{15}$$

$$d_E(A,B)=\begin{pmatrix}\frac{1}{2}\left(\frac{1}{2}\left|r_1^- - r_2^-\right|^2+\frac{1}{2}\left|r_1^+ - r_2^+\right|^2+\frac{1}{4\pi^2}\left|w_{r1}^- - w_{r2}^+\right|^2\right)+\\ \frac{1}{2}\left(\frac{1}{2}\left|k_1^- - k_2^-\right|^2+\frac{1}{2}\left|k_1^+ - k_2^+\right|^2+\frac{1}{2\pi}\left|w_{k1}^- - w_{k2}^+\right|^2\right)\end{pmatrix}^{\frac{1}{2}} \tag{16}$$

$$d_{nH}(A,B)=\frac{1}{n}\begin{pmatrix}\frac{1}{2}\left(\frac{1}{2}\left|r_1^- - r_2^-\right|+\frac{1}{2}\left|r_1^+ - r_2^+\right|+\frac{1}{2\pi}\left|w_{r1}^- - w_{r2}^+\right|\right)+\\ \frac{1}{2}\left(\frac{1}{2}\left|k_1^- - k_2^-\right|+\frac{1}{2}\left|k_1^+ - k_2^+\right|+\frac{1}{2\pi}\left|w_{k1}^- - w_{k2}^+\right|\right)\end{pmatrix} \tag{17}$$

$$d_{nE}(A,B)=\sqrt{\frac{1}{n}\begin{pmatrix}\frac{1}{2}\left(\frac{1}{2}\left|r_1^- - r_2^-\right|^2+\frac{1}{2}\left|r_1^+ - r_2^+\right|^2+\frac{1}{4\pi^2}\left|w_{r1}^- - w_{r2}^+\right|^2\right)+\\ \frac{1}{2}\left(\frac{1}{2}\left|k_1^- - k_2^-\right|^2+\frac{1}{2}\left|k_1^+ - k_2^+\right|^2+\frac{1}{2\pi}\left|w_{k1}^- - w_{k2}^+\right|^2\right)\end{pmatrix}^{\frac{1}{2}}} \tag{18}$$

Definition 2.2.9 the normalized weighted Hamming distance and normalized Euclidean distance between the two complex interval-valued complex fuzzy numbers (CIVIFNs) will aid in the calculation of normalized weighted Hamming distance and normalized Euclidean distance between the two CIVIFNs (Dai et al. 2019). This is given by Eq. (19) and Eq. (20) respectively:

$$d_{nH}\left(\beta_1,\beta_2\right)=\begin{pmatrix}\frac{\xi_1}{2}\left(\frac{1}{2}\left|r_1^- - r_2^-\right|+\frac{1}{2}\left|r_1^+ - r_2^+\right|+\frac{1}{2\pi}\left|w_{r1}^- - w_{r2}^+\right|\right)+\\ \frac{\xi_2}{2}\left(\frac{1}{2}\left|k_1^- - k_2^-\right|+\frac{1}{2}\left|k_1^+ - k_2^+\right|+\frac{1}{2\pi}\left|w_{k1}^- - w_{k2}^+\right|\right)\end{pmatrix} \tag{19}$$

$$d_{nE}\left(\beta_1,\beta_2\right)=\sqrt{\frac{1}{n}\begin{pmatrix}\frac{\xi_1}{2}\left(\frac{1}{2}\left|r_1^- - r_2^-\right|^2+\frac{1}{2}\left|r_1^+ - r_2^+\right|^2+\frac{1}{4\pi^2}\left|w_{r1}^- - w_{r2}^+\right|^2\right)+\\ \frac{\xi_2}{2}\left(\frac{1}{2}\left|k_1^- - k_2^-\right|^2+\frac{1}{2}\left|k_1^+ - k_2^+\right|^2+\frac{1}{2\pi}\left|w_{k1}^- - w_{k2}^+\right|^2\right)\end{pmatrix}^{\frac{1}{2}}} \tag{20}$$

where $\xi=\left(\xi_1,\xi_2,\ldots,\xi_n\right)^T$ is the weight vector associated with the CIVIFN β_j. The following condition holds for the weight vector: $\xi_j>0$ and $\sum_{j=1}^{n}\xi_j=1$.

2.3 Aggregation operators

In the following section, the developed weighted averaging operators, as well as the geometric aggregation operators for aggregation of CIVIFNs (Garg and Rani 2019), have been discussed.

2.3.1 Weighted averaging operator

Let the collection of CIVIFNs be described as $\beta_j,(j=1,2,\ldots,n)$ then CIVIF weighted averaging (CIVIFWA) operator can be defined using Eq. (21):

$$CIVIFWA(\beta_1,\beta_2,\ldots,\beta_n)=\xi_1\beta_1\oplus\xi_2\beta_2\oplus\ldots\oplus\xi_n\beta_n \tag{21}$$

Therefore, if $\beta_j=\left(\left[r_j^-,r_j^+\right]e^{i[w_{rj}^-,w_{rj}^+]},[k_j^-,k_j^+]e^{i[w_{kj}^-,w_{kj}^+]}\right)$ $(j=1(1)n)$, then the aggregated value obtained by employing a CIVIFWA operator can be defined using Eq. (11), Eq. (13) and Eq. (22) as follows:

$$CIVIFWA\left(\beta_1,\beta_2,\ldots,\beta_n\right)=\begin{pmatrix}\left[1-\prod_{j=1}^{n}\left(1-r_j^-\right)^{\xi_j},1-\prod_{j=1}^{n}\left(1-r_j^+\right)^{\xi_j}\right]e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{w_{rj}^-}{2\pi}\right)^{\xi_j}\right),2\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{w_{rj}^+}{2\pi}\right)^{\xi_j}\right)\right]},\\ \left[\prod_{j=1}^{n}\left(k_j^-\right)^{\xi_j},\prod_{j=1}^{n}\left(k_j^+\right)^{\xi_j}\right]e^{i\left[2\pi\left(\prod_{j=1}^{n}\frac{w_{kj}^-}{2\pi}\right)^{\xi_j},2\pi\left(\prod_{j=1}^{n}\frac{w_{kj}^+}{2\pi}\right)^{\xi_j}\right]}\end{pmatrix} \tag{22}$$

The complex interval-valued intuitionistic fuzzy ordered weighted averaging (CIVIFOWA) operator can now be defined using Eq. (23):

$$CIVIFOWA(\beta_1,\beta_2,\ldots,\beta_n)=\xi_1\beta_{\sigma(1)}\oplus\xi_2\beta_{\sigma(2)}\oplus\ldots\oplus\xi_n\beta_{\sigma(n)} \tag{23}$$

i.e.

$$CIVIFOWA(\beta_1, \beta_2, \dots, \beta_n) = \left(\begin{array}{l} \left[1 - \prod_{j=1}^{n}\left(1 - r^-_{\sigma(j)}\right)^{\xi_j}, 1 - \prod_{j=1}^{n}\left(1 - r^+_{\sigma(j)}\right)^{\xi_j}\right] e^{i\left[2\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{w^-_{r_{\sigma(j)}}}{2\pi}\right)^{\xi_j}\right), 2\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{w^+_{r_{\sigma(j)}}}{2\pi}\right)^{\xi_j}\right)\right]}, \\ \left[\prod_{j=1}^{n}\left(k^-_{\sigma(j)}\right)^{\xi_j}, \prod_{j=1}^{n}\left(k^+_{\sigma(j)}\right)^{\xi_j}\right] e^{i\left[2\pi\left(\prod_{j=1}^{n}\left(\frac{w^-_{k_{\sigma(j)}}}{2\pi}\right)^{\xi_j}\right), 2\pi\left(\prod_{j=1}^{n}\left(\frac{w^+_{k_{\sigma(j)}}}{2\pi}\right)^{\xi_j}\right)\right]} \end{array} \right) \tag{24}$$

where the permutation of (1,2,…,n) is represented by σ and the permutation is such that, $\beta_{\sigma(j-1)}$ is superior to $\beta_{\sigma(j)}$ for $j=2,3,\dots,n$

2.3.2 Weighted geometric operator

The following discussion depicts the concept that is related to the weighted geometric aggregation operator that is employed for the aggregation of CIVIFNs.

$$CIVIFWG(\beta_1, \beta_2, \dots, \beta_n) = \bigotimes_{j=1}^{n} \beta_j^{\xi_j} \tag{25}$$

$$= \left(\begin{array}{l} \left[\prod_{j=1}^{n}\left(r^-_j\right)^{\xi_j}, \prod_{j=1}^{n}\left(r^+_j\right)^{\xi_j}\right] e^{i\left[2\pi\prod_{j=1}^{n}\left(\frac{w^-_{rj}}{2\pi}\right)^{\xi_j}, 2\pi\prod_{j=1}^{n}\left(\frac{w^+_{rj}}{2\pi}\right)^{\xi_j}\right]}, \\ \left[1 - \prod_{j=1}^{n}\left(1 - k^-_j\right)^{\xi_j}, 1 - \prod_{j=1}^{n}\left(1 - k^+_j\right)^{\xi_j}\right] e^{i\left[2\pi\left(1-\prod_{j=1}^{n} 1-\frac{w^-_{kj}}{2\pi}\right)^{\xi_j}, 2\pi\left(1-\prod_{j=1}^{n} 1-\frac{w^+_{kj}}{2\pi}\right)^{\xi_j}\right]} \end{array} \right) \tag{26}$$

where $\xi = \left(\xi_1, \xi_2, \dots, \xi_n\right)^T$ is the weight vector associated with the CIVIFN β_j. The following condition holds for the weight vector: $\xi_j > 0$ and $\sum_{j=1}^{n} \xi_j = 1$.

Complex interval-valued intuitionistic fuzzy ordered weighted geometric (CIVIFOWG) operator is defined using Eq. (23):

$$CIVIFOWG(\beta_1, \beta_2, \dots, \beta_n) = \bigotimes_{j=1}^{n} (\beta_{\sigma(j)})^{\xi_j} \tag{27}$$

$$= \left(\begin{array}{l} \left[\prod_{j=1}^{n}\left(r^-_{\sigma(j)}\right)^{\xi_j}, \prod_{j=1}^{n}\left(r^+_{\sigma(j)}\right)^{\xi_j}\right] e^{i\left[2\pi\prod_{j=1}^{n}\left(\frac{w^-_{r_{\sigma(j)}}}{2\pi}\right)^{\xi_j}, 2\pi\prod_{j=1}^{n}\left(\frac{w^+_{r_{\sigma(j)}}}{2\pi}\right)^{\xi_j}\right]}, \\ \left[1 - \prod_{j=1}^{n}\left(1 - k^-_{\sigma(j)}\right)^{\xi_j}, 1 - \prod_{j=1}^{n}\left(1 - k^+_{\sigma(j)}\right)^{\xi_j}\right] e^{i\left[2\pi\left(1-\prod_{j=1}^{n} 1-\frac{w^-_{k_{\sigma(j)}}}{2\pi}\right)^{\xi_j}, 2\pi\left(1-\prod_{j=1}^{n} 1-\frac{w^+_{k_{\sigma(j)}}}{2\pi}\right)^{\xi_j}\right]} \end{array} \right)$$

3 Fuzzy TODIM approach based on Complex Interval-valued Intuitionistic fuzzy sets

The present section now proposes novel fuzzy TODIM approaches based on the normal wiggly hesitant fuzzy linguistic power hammy mean operators. Two different approaches in the form of CIVIFOWA-TODIM and CIVIFOWG-TODIM have been proposed and discussed. Figure 1 depicts the procedural steps of the proposed decision-making frameworks. Let $\{A_1, A_2, \dots, A_n\}$ be a collection of alternatives (m), $\{C_1, C_2, \dots, C_n\}$ denote the collection of n number of criteria. Let $w = (w_1, w_2, \dots, w_n)$ be the weight vector associated with the criteria associated with the problem under consideration. The value of weights must satisfy $\sum_{i=1}^{n} w_i = 1$. Let $D = \{D_1, D_2, \dots, D_q\}$ be the set of q decision makers, and the weight vector associated with the experts be denoted by $w = (w_1, w_2, \dots, w_q)$. This weight vector must satisfy the following $\sum_{k=1}^{n} w_k = 1$.

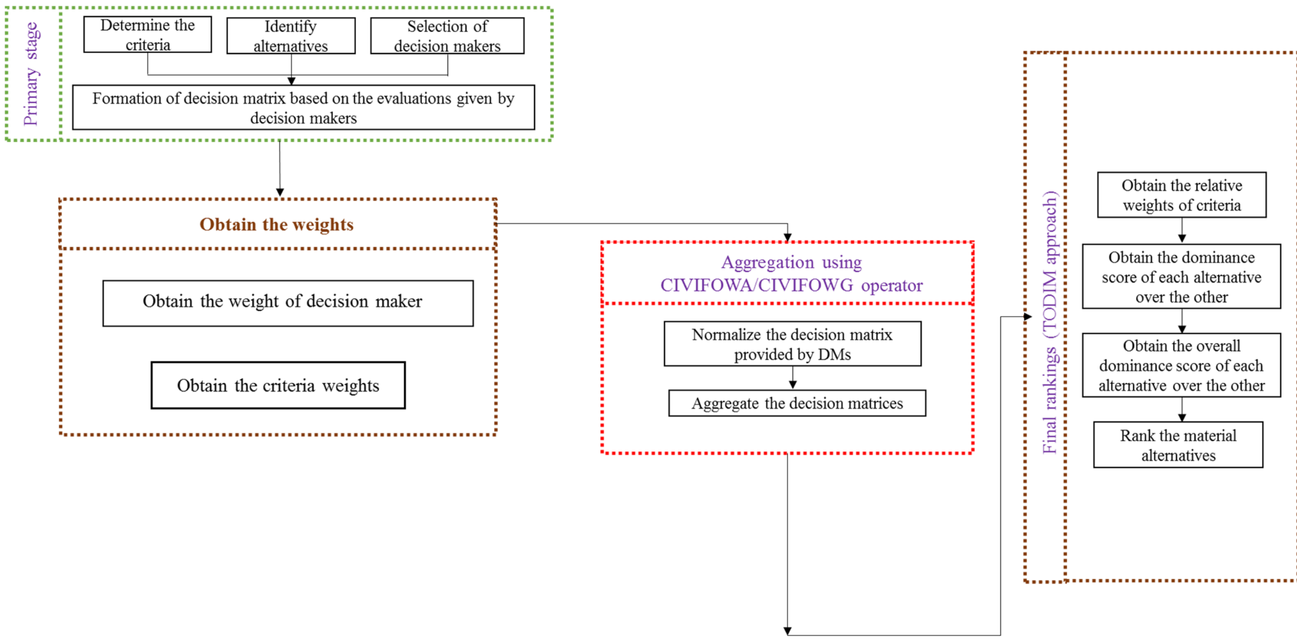


Fig. 1 Steps involved in the proposed CIVIFOWA-TODIM and CIVIFOWG-TODIM approach

3.1 The fuzzy TODIM method based on the complex interval-valued intuitionistic fuzzy ordered weighted averaging operator (CIVIFOWA-TODIM)

The procedural steps for the CIVIFOWA-TODIM approach have been delineated in the ensuing discussion.

Step 1 In this step of the proposed CIVIFOWA-TODIM approach, the d decision makers are allowed to make performance evaluation of each alternative with respect to each of the considered criteria. The information collected is in the form of complex intuitionistic fuzzy matrices. Let the decision matrix be represented by $R_{pq}^{T} = \left(\beta_{pq}^{(t)}\right)_{m\times n} = \left(\left[\left(s_{pq}^{(t)}\right)^-, \left(s_{pq}^{(t)}\right)^+\right] e^{i\left[\left(u_{pq}^{(t)}\right)^-, \left(u_{pq}^{(t)}\right)^+\right]}, \left[\left(t_{pq}^{(t)}\right)^-, \left(t_{pq}^{(t)}\right)^+\right] e^{i\left[\left(l_{pq}^{(t)}\right)^-, \left(l_{pq}^{(t)}\right)^+\right]}\right)_{m\times n}$ $(k = 1, 2, \ldots, q)$, where $t = 1,2,\ldots,d$, $p = 1,2,\ldots,m$ and $q = 1,2,\ldots,n$. The decision matrix can be represented as follows:

$$R_{pq}^{T} = \begin{pmatrix} \beta_{11}^{(t)} & \beta_{12}^{(t)} & \cdots & \beta_{1n}^{(t)} \\ \beta_{21}^{(t)} & \beta_{22}^{(t)} & & \beta_{2n}^{(t)} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{m1}^{(t)} & \beta_{m2}^{(t)} & \cdots & \beta_{mn}^{(t)} \end{pmatrix} \tag{28}$$

Step 2 the weights of decision makers are determined or provided. It is worth noting that weight for decision makers can be calculated through any available method for weight determination. One such instance is the weight model discussed by Ye (2013).

Step 3 In this step of the proposed decision-making framework, the decision matrices obtained from the DMs are aggregated through the aid of the CIVIFOWA operator depicted in Eq. (24).

Steps 4 The weights of criteria are determined or provided as per the subjective evaluations of the decision maker. It is worth noting that any of the algorithms available for weight determination of the criteria can be employed following the situation of the problem that is being addressed as such that proposed by Das et al. (2016).

Step 5 The relative weights (w_{jr}) associated with the criteria under consideration are obtained next. This is accomplished using Eq. (29):

$$\omega_{jr} = \frac{w_j}{w_r} \tag{29}$$

The reference weight is reflected in w_r. The largest value of criteria weights is considered to be the reference weight.

Step 6 Next Eq. (30) is employed to obtain the values of the dominance degree for the alternative A_i over the rest of the other alternatives A_j for a particular criteria C_n:

$$\delta(A_i, A_j) = \sum_{j=1}^{n} \varphi_j(A_i, A_j) \tag{30}$$

where,

$$\varphi_j(A_i,A_k)=\begin{cases}\sqrt{\dfrac{w_{jr}(p_{ij}-p_{kj})}{(\sum_{j=1}^{n} w_{jr})}}, & if\ p_{ij}-p_{kj}>0\\ 0, & if\ p_{ij}-p_{kj}=0\\ \dfrac{-1}{\theta}\sqrt{\dfrac{(\sum_{j=1}^{n} w_{jr-}.(p_{ij}-p_{kj})}{w_{jr}}} & if\ p_{ij}-p_{kj}<0\end{cases}$$

Step 7 The overall dominance degree, $\xi_i(A_i)$ is obtained using Eq. (31)

$$\xi(A_i)=\frac{(\sum_{k=1}^{m}\delta(A_i,A_k))-\min_i(\sum_{k=1}^{m}\delta(A_i,A_k))}{\max_i(\sum_{k=1}^{m}\delta(A_i,A_k))-\min_i(\sum_{k=1}^{m}\delta(A_i,A_k))} \tag{31}$$

Step 8 The overall dominance degree is employed to rank the considered alternatives within the decision-making problem. The material with the highest-ranking corresponds to the highest value of the overall dominance degree.

3.2 The fuzzy TODIM method based on the complex interval-valued intuitionistic fuzzy ordered weighted geometric operator (CIVIFOWG-TODIM)

Within this approach, the steps (1–2) and (4–9) are similar to those depicted in Sect. 3.1. Only the aggregation process is different and has been highlighted as follows:

Step 3 In this step of the proposed decision-making framework, the decision matrices obtained from the DMs are aggregated through the aid of the CIVIFOWG operator depicted in Eq. (27).

4 Case study: Performance evaluation of the proposed CIVIFOWA-TODIM and CIVIFOWG-TODIM methods

In the present section of the work, and evaluation into the performance of the proposed methods has been carried out. A comparative analysis of the proposed approaches with other fuzzy TODIM approaches has also been carried out. The comparative analysis aided in revealing the advantages and the limitations of the proposed fuzzy TODIM methodologies. The section also presents the robustness of the proposed approaches through the sensitivity analysis.

4.1 Illustrative examples

4.1.1 Ranking of materials obtained through the application of the proposed CIVIFOWA-fuzzy TODIM method

The example has been taken from Garg and Rani (2019) wherein the entrepreneur is required to purchase a new machine for the company. There are four models for the machine under the influence of four criteria: *reliability* (C_1), *safety* (C_2), *flexibility* (C_3), and *productivity* (C_4). Three DMs were selected to evaluate the alternatives with respect to the considered criteria. The evaluations were provided in the form of CIVIFNs. The following weight vector was associated with the three DMs: w = $(0.243, 0.514, 0.243)^T$. The criteria weights associated with the case study were taken as $\xi = (0.3,0.2,0.1,0.4)^T$.

Following *Step 1* of the proposed methodology, the decision matrices are obtained from the DMs in the form of CIVIFNs. Theses matrices have been depicted in the work carried out by Garg and Rani (2019) and have not been shown here for the sake of conciseness. The obtained CIVIFNs obtained can be explained as follows: suppose the decision maker agrees that for a particular alternative under criteria from 30 to 40% and disagrees on it from 20 to 50% and let the phase term reflect sub-criteria wherein the decision maker agrees from 10 to 50% and disagrees from 30 to 40%, then the information in terms of CIVIFN for the alternative under criteria and sub-criteria can be represented as: $\left([0.3,0.4]e^{i[2\pi(0.2),2\pi(0.5)]},[0.10,0.50]e^{i[2\pi(0.3),2\pi(0.4)]}\right)$.

Following *Step 2* of the proposed decision-making framework, the weights are obtained for each of three DMs. Here, in the present illustrative example, the weight vector associated with the DMs is w = $(0.243, 0.514, 0.243)^T$.

Next, through *Step 3,* the aggregated decision matrix is obtained using a CIVIFOWA operator and is tabulated in Table 1.

In *Step 4*, the criteria weights are either provided or determined through any of the available weight determination methods. In the present case study, the criteria weights were already specified and have been presented in the aforementioned discussion.

Relative criteria weights are determined next using Eq. (29) delineated in *Step 5*. $w_{nr} = (0.75, 0.5, 0.25, 1)^T$ is the calculated relative weight vector.

Step 6 dominance score of each alternative over the other alternative is obtained. This has been depicted in Table 2. The value of attenuation factor has been kept at 1 while arriving at the dominance score. The distance measures between the two CIVIFNs have been obtained using the

Table 1 Aggregated decision matrix obtained using CIVIFOWA-fuzzy TODIM approach

	C_1	C_2	C_3	C4
A_1	$\left([0.4141, 0.6432]e^{i[2\pi(0.2769),2\pi(0.5049)]}, [0.1306, 0.2915]e^{i[2\pi(0.1690),2\pi(0.3938)]}\right)$	$\left([0.3035, 0.5884]e^{i[2\pi(0.2324),2\pi(0.4509)]}, [0.1546, 0.3217]e^{i[2\pi(0.2413),2\pi(0.3901)]}\right)$	$\left([0.2255, 0.3257]e^{i[2\pi(0.2324),2\pi(0.4563)]}, [0.2207, 0.4416]e^{i[2\pi(0.1546),2\pi(0.3396)]}\right)$	$\left([0.3337, 0.4799]e^{i[2\pi(0.2559),2\pi(0.4328)]}, [0.1000, 0.3000]e^{i[2\pi(0.2856),2\pi(0.4416)]}\right)$
A_2	$\left([0.3035, 0.5384]e^{i[2\pi(0.1533),2\pi(0.3890)]}, [0.1306, 0.2915]e^{i[2\pi(0.1690),2\pi(0.3730)]}\right)$	$\left([0.3320, 0.5586]e^{i[2\pi(0.2531),2\pi(0.5264)]}, [0.1000, 0.2207]e^{i[2\pi(0.1759),2\pi(0.3152)]}\right)$	$\left([0.2656, 0.5264]e^{i[2\pi(0.4041),2\pi(0.6270)]}, [0.2000, 0.3901]e^{i[2\pi(0.1183),2\pi(0.2719)]}\right)$	$\left([0.2769, 0.5340]e^{i[2\pi(0.1772),2\pi(0.4773)]}, [0.1183, 0.2207]e^{i[2\pi(0.1690),2\pi(0.3730)]}\right)$
A_3	$\left([0.2324, 0.6155]e^{i[2\pi(0.3000),2\pi(0.5542)]}, [0.1183, 0.2367]e^{i[2\pi(0.2297),2\pi(0.3730)]}\right)$	$\left([0.3337, 0.4799]e^{i[2\pi(0.3320),2\pi(0.5542)]}, [0.1183, 0.2367]e^{i[2\pi(0.1865),2\pi(0.3217)]}\right)$	$\left([0.2899, 0.4943]e^{i[2\pi(0.1533),2\pi(0.5049)]}, [0.1690, 0.3217]e^{i[2\pi(0.1690),2\pi(0.3078)]}\right)$	$\left([0.1533, 0.4733]e^{i[2\pi(0.2035),2\pi(0.5049)]}, [0.2000, 0.3730]e^{i[2\pi(0.1306),2\pi(0.2915)]}\right)$
A_4	$\left([0.2031, 0.4260]e^{i[2\pi(0.2531),2\pi(0.4041)]}, [0.1183, 0.3535]e^{i[2\pi(0.1306),2\pi(0.2957)]}\right)$	$\left([0.2531, 0.4737]e^{i[2\pi(0.2503),2\pi(0.4773)]}, [0.1000, 0.2612]e^{i[2\pi(0.1401),2\pi(0.4183)]}\right)$	$\left([0.2198, 0.5264]e^{i[2\pi(0.2031),2\pi(0.4773)]}, [0.2000, 0.3000]e^{i[2\pi(0.1183),2\pi(0.2612)]}\right)$	$\left([0.3337, 0.4563]e^{i[2\pi(0.2559),2\pi(0.4328)]}, [0.1000, 0.3568]e^{i[2\pi(0.1690),2\pi(0.4000)]}\right)$

Table 2 Dominance degree and overall dominance score of alternative A_i over alternative A_j obtained using CIVIFOWA-fuzzy TODIM approach

φ(A$_1$, A$_2$) – 3.43	**φ (A$_1$, A$_3$)** – 2.15	**φ (A$_1$, A$_4$)** – 2.81	**δ(A$_1$)** – 8.39	**ξ(A$_1$)** 0.00
φ (A$_2$, A$_1$) 1.11	**φ (A$_2$, A$_3$)** 0.97	**φ (A$_2$, A$_4$)** 0.93	**δ(A$_2$)** 3.02	**ξ(A$_2$)** 1.00
φ (A$_3$, A$_1$) – 2.01	**φ (A$_3$, A$_2$)** – 2.73	**φ (A$_3$, A$_4$)** – 1.18	**δ(A$_3$)** – 5.92	**ξ(A$_3$)** 0.22
φ (A$_4$, A$_1$) – 1.12	**φ (A$_4$, A$_2$)** – 4.19	**φ (A$_4$, A$_3$)** – 2.24	**δ(A$_3$)** – 7.55	**ξ(A$_4$)** 0.07

Hamming distance. This aided in determining the dominance score of each alternative over the other.

Equation (35), depicted in *Step 7*, is employed to arrive at the overall dominance degree $\xi(A_i)$ of the alternatives. The calculated overall dominance degree has been depicted in Table 2.

Attenuation factor (θ) reflects the risk appetite of the decision maker or in other words the psychological behavior of the decision maker towards risk. In the present case, to adjudge the effect of the risk appetite of the decision maker on the final ranking results of the considered alternatives, following values of attenuation factor have been considered: θ = 0.1, 1, 2, 3,4,5,6,7,8,9, 10, 50, 100. The low value of attenuation factor i.e., θ = 0.1 reflects a higher risk perspective of the decision maker. An attenuation factor value of 1 is reflective of the neutral behavior of the expert towards the risk. A neutral risk perspective signifies that the magnitude associated with the gains and losses of the alternative under consideration are equal. When the value of θ = 2 then it signifies a variety of neutral to less risk perspective of the decision maker while making evaluations. A higher risk attitude is reflected again when the value of θ is 10 or more than 10. In such cases, the experts involved in the decision-making process don't concentrate their evaluations on the negative impacts associated with the losses.

The calculated values of the overall dominance score have been tabulated in Table 3. The tabulated values comprise the data for the overall dominance score at distinct values of the attenuation factor. The ranking results for different values of attenuation factor have been presented in Table 4. For a more clear visualization of the ranking results, the variation on the ranking results with distinct values of the attenuation factor has been provided in Fig. 2. As observed from the results, that the ranking results obtained through the proposed framework ($A_2 > A_3 > A_4 > A_1$) are different from those derived in the past study (Garg and Rani 2019) i.e., $A_2 > A_3 > A_1 > A_4$. As such, the risk perspective of the expert clearly has a dominating effect on the performances and hence the rankings of the alternatives under consideration. Therefore, the importance of the risk appetite of the

Table 3 ξ (A_i) of the four material alternatives with different θ values

	0.1	1	2	3	4	5	6	7	8	9	10	50	100
A_1	0.00	0.00	0.00	0.00	0.00	0.03	0.05	0.06	0.07	0.08	0.09	0.19	0.22
A_2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A_3	0.29	0.22	0.16	0.11	0.08	0.07	0.06	0.05	0.04	0.04	0.03	0.00	0.00
A_4	0.12	0.07	0.04	0.01	0.001	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.04

decision maker cannot be ignored during the decision making process. However, the study conducted by Garg and Rani (2019) ignored this quintessential aspect of the decision making process. Figure 2 and Table 4 reflects the changing ranking results with the distinct values of the attenuation factor. However, the majority of the rank results are $A_2 > A_3 > A_4 > A_1$ and therefore, following the majority rule (Wang et al. 2017), the final order of alternative rankings that can be adopted is: $A_2 > A_3 > A_4 > A_1$.

4.1.2 Ranking of materials obtained through the application of the proposed CIVIFOWG-fuzzy TODIM method

The case study under consideration has also been solved through the employability of the CIVIFOWG-fuzzy TODIM approach. Table 5 presents the obtained aggregated decision matrix using the CIVIFOWG operator. This has been obtained through *Steps* (1–2) and *Step 3* delineated in Sects. 3.1.1 and 3.1.2, respectively. The dominance score values associated with the different considered alternatives and therefore the overall dominance score has then been obtained through *Steps* (4–8). Table 6 represents the determined values of the dominance score as well as the overall dominance score associated with the different alternatives under consideration.

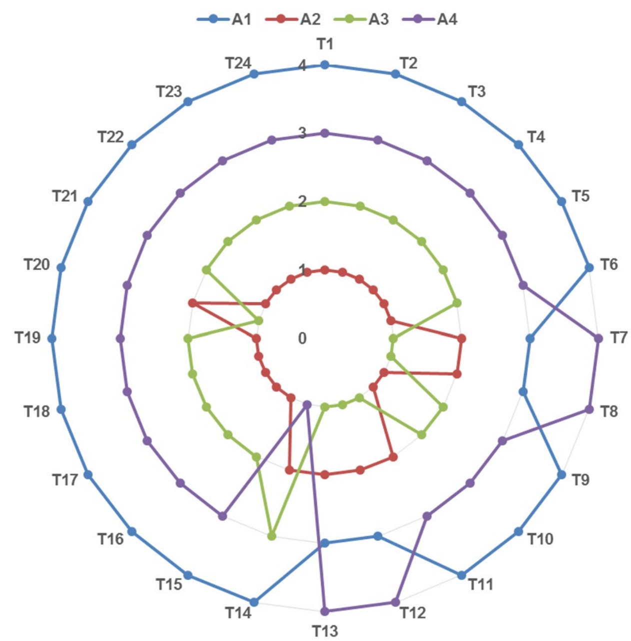


Fig. 2 Rankings obtained with CIVIFOWA fuzzy TODIM approach for different test cases

Table 4 Rank results of the material alternatives based on ξ (A_i) at each θ value

	θ	Ranking results
Our proposal	0.1	$A_2 > A_3 > A_4 > A_1$
	1	$A_2 > A_3 > A_4 > A_1$
	2	$A_2 > A_3 > A_4 > A_1$
	3	$A_2 > A_3 > A_4 > A_1$
	4	$A_2 > A_3 > A_4 > A_1$
	5	$A_2 > A_3 > A_1 > A_4$
	6	$A_2 > A_3 > A_1 > A_4$
	7	$A_2 > A_1 > A_3 > A_4$
	8	$A_2 > A_1 > A_3 > A_4$
	9	$A_2 > A_1 > A_3 > A_4$
	10	$A_2 > A_1 > A_3 > A_4$
	50	$A_2 > A_1 > A_4 > A_3$
	100	$A_2 > A_1 > A_4 > A_3$

The calculated values of the overall dominance score for different values of the attenuation factor have been reported in Table 7. The performance score of the material alternatives with different values of the attenuation factor has been reported in Table 8 and Fig. 3. The ranking results delineated in Table 8 shows that $A_2 > A_3 > A_4 > A_1$ is the ranking order of alternatives and remains unaffected with the change in the values of attenuation factor. However, the ranking results obtained are different from those obtained in the study conducted by Garg and Rani (2019), but the impact of DMs psychological behavior is reflected clearly in the proposed decision-making framework. The importance of DMs psychological behavior has been ignored in the study conducted by Garg and Rani (2019).

On analysis of the proposed CIVIF-fuzzy TODIM approaches, more consistency in the obtained results has been depicted by the CIVIFOWG-fuzzy TODIM method

Table 5 Aggregated decision matrix obtained using CIVIFOWG-fuzzy TODIM approach

	C_1	C_2	C_3	C4
A_1	$\left([0.3791, 0.6156]e^{i[2\pi(0.2719,2\pi(0.4951))]}, [0.1533, 0.3035]e^{i[2\pi(0.1768,2\pi(0.4041))]}\right)$	$\left([0.2915, 0.5645]e^{i[2\pi(0.2000),2\pi(0.4458))]}, [0.1772, 0.3257]e^{i[2\pi(0.2899),2\pi(0.4112))]}\right)$	$\left([0.2207, 0.3217]e^{i[2\pi(0.2000),2\pi(0.4414))]}, [0.2255, 0.4574]e^{i[2\pi(0.1772),2\pi(0.3550))]}\right)$	$\left([0.3078, 0.4660]e^{i[2\pi(0.2297),2\pi(0.4183))]}, [0.1000, 0.3000]e^{i[2\pi(0.3379,2\pi(0.4574))]}\right)$
A_2	$\left([0.2915, 0.5140]e^{i[2\pi(0.1306,2\pi(0.3550))]}, [0.1533, 0.3035]e^{i[2\pi(0.1768,2\pi(0.4169))]}\right)$	$\left([0.3152, 0.5437]e^{i[2\pi(0.2463),2\pi(0.5227))]}, [0.1000, 0.2255]e^{i[2\pi(0.2091),2\pi(0.3320))]}\right)$	$\left([0.2111, 0.5227]e^{i[2\pi(0.3938),2\pi(0.6229))]}, [0.2000, 0.4112]e^{i[2\pi(0.1254),2\pi(0.2769))]}\right)$	$\left([0.2719, 0.5201]e^{i[2\pi(0.1546),2\pi(0.5000))]}, [0.1254, 0.2255]e^{i[2\pi(0.1768),2\pi(0.3771))]}\right)$
A_3	$\left([0.2000, 0.5944]e^{i[2\pi(0.3000,2\pi(0.5491))]}, [0.1254, 0.2540]e^{i[2\pi(0.2559,2\pi(0.3771))]}\right)$	$\left([0.3078, 0.4660]e^{i[2\pi(0.3152),2\pi(0.5491))]}, [0.1254, 0.2540]e^{i[2\pi(0.2031,2\pi(0.3257))]}\right)$	$\left([0.2413, 0.4594]e^{i[2\pi(0.1306),2\pi(0.4951))]}, [0.1768, 0.3257]e^{i[2\pi(0.1768),2\pi(0.3337))]}\right)$	$\left([0.1306, 0.4736]e^{i[2\pi(0.1706),2\pi(0.4951))]}, [0.2000, 0.3771]e^{i[2\pi(0.1533),2\pi(0.3035))]}\right)$
A_4	$\left([0.1865, 0.4223]e^{i[2\pi(0.2463,2\pi(0.3938))]}, [0.1183, 0.3535]e^{i[2\pi(0.1533,2\pi(0.3345))]}\right)$	$\left([0.2463, 0.4273]e^{i[2\pi(0.2436,2\pi(0.4736))]}, [0.1000, 0.2778]e^{i[2\pi(0.1501,2\pi(0.4328))]}\right)$	$\left([0.1479, 0.5227]e^{i[2\pi(0.1865),2\pi(0.4736))]}, [0.2000, 0.3000]e^{i[2\pi(0.1254),2\pi(0.2778))]}\right)$	$\left([0.3078, 0.4414]e^{i[2\pi(0.2297),2\pi(0.4183))]}, [0.1000, 0.3844]e^{i[2\pi(0.1768),2\pi(0.4000))]}\right)$

Table 6 Dominance degree and overall dominance score of alternative A_i over alternative A_j obtained using CIVIFOWG-fuzzy TODIM approach

$\varphi(A_1, A_2)$ − 3.64	$\varphi(A_1, A_3)$ − 3.38	$\varphi(A_1, A_4)$ − 3.23	$\delta(A_1)$ − 10.25	$\xi(A_1)$ 0.00
$\varphi(A_2, A_1)$ − 0.25	$\varphi(A_2, A_3)$ − 0.44	$\varphi(A_2, A_4)$ 0.97	$\delta(A_2)$ 0.28	$\xi(A_2)$ 1.00
$\varphi(A_3, A_1)$ − 0.34	$\varphi(A_3, A_2)$ − 2.74	$\varphi(A_3, A_4)$ − 1.25	$\delta(A_3)$ − 4.32	$\xi(A_3)$ 0.56
$\varphi(A_4, A_1)$ − 0.71	$\varphi(A_4, A_2)$ − 4.37	$\varphi(A_4, A_3)$ − 2.21	$\delta(A_3)$ − 7.30	$\xi(A_4)$ 0.28

in comparison to the CIVIFOWA-fuzzy TODIM method. This is clearly reflected in Figs. 2 and 3, respectively. Furthermore, the overall dominance scores obtained using CIVIFOWG-fuzzy TODIM approach are higher than that obtained using CIVIFOWA-fuzzy TODIM approach. This is significant of the fact that the CIVIFOWG-fuzzy TODIM approach preserves the information to a greater extent in comparison to the CIVIFOWA-fuzzy TODIM approach.

4.2 Sensitivity analysis

To adjudge the robustness of the proposed material selection framework, an investigation into the influence of attenuation factor and the criteria weights on the ranking orders of the material alternatives has been carried out in this section. Global values of the material alternatives for different values of attenuation factor have already been depicted in Tables 3 and 7 for the CIVIFOWA-fuzzy TODIM method and the CIVIFOWG-fuzzy TODIM method respectively. The ranking results that have been derived for different values of attenuation factors are delineated in Tables 4 and 8, respectively for the aforementioned two approaches. From the results, it can be observed that although the values of global values of alternatives differ with the attenuation factor, the rankings of material alternative remains stable.

Table 9 depicts different tests carried out by exchanging criteria weights and hence the ranking results are obtained using CIVIFOWA-fuzzy TODIM and the CIVIFOWG-fuzzy TODIM methodologies. Figure 2 depicts that candidate alternative A_2 is revealed to be the best candidate alternative in 19 tests and is therefore sufficient enough to conclude that A_2 is the best material alternative. Furthermore, alternative A_1 has been revealed to be the worst candidate alternative in 20 tests and therefore, can be selected as the worst candidate alternative. Similar tests were carried out for the CIVIFOWG-fuzzy TODIM approach and the ranking results have been shown in Fig. 3. It is observed that alternative A_2 is revealed to be the best candidate alternative in all the tests and is therefore sufficient enough to conclude that A_2 is the best candidate alternative. Furthermore, alternative

Table 7 ξ (A_i) of the four material alternatives with different θ values

	0.1	1	2	3	4	5	6	7	8	9	10	50	100
A_1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A_2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A_3	0.61	0.56	0.52	0.49	0.47	0.45	0.43	0.42	0.41	0.39	0.38	0.28	0.26
A_4	0.29	0.28	0.27	0.26	0.26	0.25	0.25	0.24	0.24	0.24	0.24	0.21	0.20

Table 8 Rank results of the material alternatives based on ξ (A_i) at each θ value

	θ	Ranking results
Our proposal	0.1	$A_2 > A_3 > A_4 > A_1$
	1	$A_2 > A_3 > A_4 > A_1$
	2	$A_2 > A_3 > A_4 > A_1$
	3	$A_2 > A_3 > A_4 > A_1$
	4	$A_2 > A_3 > A_4 > A_1$
	5	$A_2 > A_3 > A_4 > A_1$
	6	$A_2 > A_3 > A_4 > A_1$
	7	$A_2 > A_3 > A_4 > A_1$
	8	$A_2 > A_3 > A_4 > A_1$
	9	$A_2 > A_3 > A_4 > A_1$
	10	$A_2 > A_3 > A_4 > A_1$
	50	$A_2 > A_3 > A_4 > A_1$
	100	$A_2 > A_3 > A_4 > A_1$

A_1 has been revealed to be the worst candidate alternative in twenty-three tests and therefore can be selected as the worst candidate alternative. The robustness of CIVIFOWA-fuzzy TODIM and the CIVIFOWG-fuzzy TODIM is therefore revealed through the conducted tests, therefore, it can be employed for solving decision-making problems are under risk and uncertainty. As observed, the CIVIFOWG-fuzzy TODIM approach is relatively more robust in comparison to the CIVIFOWA-fuzzy TODIM approach.

4.3 Comparative study

The above case study has been addressed using different versions of existing fuzzy-TODIM approaches: interval-valued fuzzy-TODIM, intuitionistic fuzzy-TODIM, interval-valued intuitionistic fuzzy-TODIM. The comparison to other fuzzy-TODIM approaches has not been considered since the

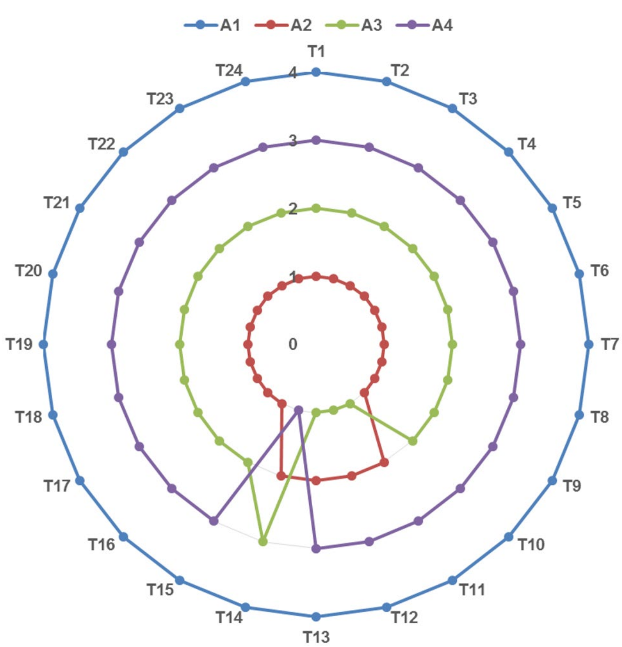


Fig. 3 Rankings obtained with CIVIFOWG fuzzy TODIM approach for different test cases

Table 9 Different test scenarios for sensitivity analysis

Tests	C_1	C_2	C_3	C_4
Test 1	0.3	0.4	0.1	0.2
Test 2	0.3	0.4	0.2	0.1
Test 3	0.3	0.1	0.2	0.4
Test 4	0.3	0.1	0.4	0.2
Test 5	0.3	0.2	0.4	0.1
Test 6	0.3	0.2	0.1	0.4
Test 7	0.2	0.4	0.1	0.3
Test 8	0.2	0.4	0.3	0.1
Test 9	0.2	0.1	0.3	0.4
Test 10	0.2	0.1	0.4	0.3
Test 11	0.2	0.3	0.4	0.1
Test 12	0.2	0.3	0.1	0.4
Test 13	0.1	0.4	0.2	0.3
Test 14	0.1	0.4	0.3	0.2
Test 15	0.1	0.1	0.2	0.3
Test 16	0.1	0.1	0.3	0.2
Test 17	0.1	0.3	0.4	0.2
Test 18	0.1	0.3	0.2	0.4
Test 19	0.4	0.3	0.2	0.1
Test 20	0.4	0.3	0.1	0.2
Test 21	0.4	0.1	0.3	0.2
Test 22	0.4	0.1	0.2	0.3
Test 23	0.4	0.2	0.1	0.3
Test 24	0.4	0.2	0.3	0.1

context is of intuitionistic fuzzy sets. The results obtained for overall values as well as the ranking orders have been tabulated in Table 10. The values have been obtained keeping $\theta = 1$. From the results, it is clear that the best-selected alternative is similar in all the cases but a change in the preferences of the other alternatives has been observed. This may be attributed to the different decision-making environments in which the decisions are made.

It's worth noted that the procedural steps involved in the proposed methodology are different from other fuzzy-TODIM approaches under intuitionistic and interval-valued intuitionistic environments. The extant fuzzy-TODIM methods under IF/IVIF environment only consider the amplitude term and hence results in loss into information. However, the proposed method is much closer to the real decision-making process as it considers the two-dimensional evaluations into a single fuzzy set simultaneously. Furthermore, the method also takes into account the hesitancy between the non-membership and membership degree.

Table 11 provides a comparative analysis for the proposed approach to that with the other similar models reported in various literature. It is observed that higher values of overall dominance scores are reflected by the CIVIFOWG-fuzzy TODIM approach in comparison to the other methods. This is significant of the fact that the proposed CIVIFOWG-fuzzy TODIM approach has the potential ability to maintain the fuzzy information closer to the real-life situation and hence the rankings are more reliable.

Table 10 A comparative analysis with the extant fuzzy TODIM methods

Method	Overall dominance score value				Ranking results
	A_1	A_2	A_3	A_4	
Interval-valued fuzzy-TODIM	0.00	1.00	0.12	0.04	$A_2 > A_3 > A_4 > A_1$
Intuitionistic fuzzy-TODIM	0.00	1.00	0.15	0.04	$A_2 > A_3 > A_4 > A_1$
Interval-valued intuitionistic fuzzy-TODIM	0.00	1.00	0.19	0.06	$A_2 > A_3 > A_4 > A_1$
CIVIFOWA-fuzzy TODIM	0.00	1.00	0.22	0.07	$A_2 > A_3 > A_4 > A_1$
CIVIFOWG-fuzzy TODIM	0.00	1.00	0.56	0.28	$A_2 > A_3 > A_4 > A_1$

4.4 Analysis concerning the different distance measures

The rankings arrived are based on Hamming distances. An analysis of the derived values of the dominance score and therefore the rankings of the alternatives under consideration obtained with the employability of other distance measures, depicted through Eqs. (16–20), at each value of the attenuation factor have been undertaken in this section of the work. Table 12 and Fig. 4 depict the results for the CIVIFOWA-fuzzy TODIM approach. As observed, there is consistency in the ranking results for a given value of attenuation factor except in cases where the value of the attenuation factor is very high. Also, the rankings show variations with the distance measures at a higher value of attenuation factor. The best and the worst candidate alternatives are consistent with the distance measures as well as the attenuation factor. Higher values of dominance scores are depicted by the Euclidean distances which signifies a greater information retention capacity by these distance measures.

The results obtained using the CIVIFOWG-fuzzy TODIM approach for different distance measures have been reported in Table 13 and Fig. 5. The rankings are almost stable with the attenuation factor as well as the distance measures. Furthermore, the dominance scores are higher for the Euclidean distances. Higher values of dominance scores are retained in

Table 11 Comparison analysis between the proposed CIVIF-TODIM approach and the existing fuzzy TODIM approaches

Method	Periodicity	Falsity	Hesitancy	Multi-dimensional data	Psychological behaviour of DM
Fuzzy-TODIM	✗	✗	✗	✗	✓
Interval-valued fuzzy-TODIM	✗	✗	✓	✗	✓
Intuitionistic fuzzy-TODIM	✗	✓	✓	✗	✓
Interval-valued intuitionistic fuzzy-TODIM	✗	✓	✓	✗	✓
Complex interval-valued intuitionistic	✓	✓	✓	✓	✗
Proposed model: Complex interval-valued intuitionistic fuzzy-TODIM	✓	✓	✓	✓	✓

Table 12 Dominance scores obtained using CIVIFOWA fuzzy TODIM approach for the candidate alternatives based on ξ (Ai) for different distance measures at each θ value

$\xi(A_i)$	Alternatives	Dominance score					
		d_E	d_H	d_{nE}	d_{nE}	d_{wnE}	d_{wnH}
θ = 0.1	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.38	0.29	0.44	0.38	0.34	0.44
	A_4	0.08	0.12	0.13	0.08	0.13	0.13
θ = 1	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.28	0.22	0.33	0.28	0.28	0.33
	A_4	0.03	0.07	0.07	0.03	0.11	0.07
θ = 2	A_1	0.03	0.00	0.00	0.03	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.21	0.16	0.23	0.21	0.22	0.23
	A_4	0.00	0.04	0.02	0.00	0.10	0.02
θ = 3	A_1	0.07	0.00	0.03	0.07	0.00	0.03
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.16	0.11	0.17	0.16	0.18	0.17
	A_4	0.00	0.01	0.00	0.00	0.08	0.00
θ = 4	A_1	0.11	0.00	0.08	0.11	0.00	0.08
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.12	0.08	0.12	0.12	0.14	0.12
	A_4	0.00	0.001	0.00	0.00	0.07	0.00
θ = 5	A_1	0.14	0.03	0.11	0.14	0.00	0.11
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.09	0.07	0.09	0.09	0.10	0.09
	A_4	0.00	0.00	0.00	0.00	0.06	0.00
θ = 6	A_1	0.17	0.05	0.14	0.17	0.00	0.14
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.06	0.06	0.06	0.06	0.07	0.06
	A_4	0.00	0.00	0.00	0.00	0.05	0.00
θ = 7	A_1	0.19	0.06	0.17	0.19	0.00	0.17
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.04	0.05	0.03	0.04	0.05	0.03
	A_4	0.00	0.00	0.00	0.00	0.04	0.00
θ = 8	A_1	0.21	0.07	0.19	0.21	0.00	0.19
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.02	0.04	0.01	0.02	0.03	0.01
	A_4	0.00	0.00	0.00	0.00	0.04	0.00
θ = 9	A_1	0.23	0.08	0.21	0.23	0.00	0.21
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.00	0.04	0.00	0.00	0.01	0.00
	A_4	0.00	0.00	0.01	0.00	0.03	0.01
θ = 10	A_1	0.2566	0.09	0.24	0.2566	0.01	0.24
	A_2	1.0000	1.00	1.00	1.0000	1.00	1.00
	A_3	0.0000	0.03	0.00	0.0000	0.00	0.00
	A_4	0.0136	0.00	0.02	0.0136	0.04	0.02
θ = 50	A_1	0.52	0.19	0.52	0.52	0.18	0.52
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.00	0.00	0.00	0.00	0.00	0.00
	A_4	0.17	0.03	0.18	0.17	0.15	0.18
θ = 100	A_1	0.57	0.22	0.56	0.57	0.21	0.56
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.00	0.00	0.00	0.00	0.00	0.00
	A_4	0.19	0.04	0.20	0.19	0.17	0.20

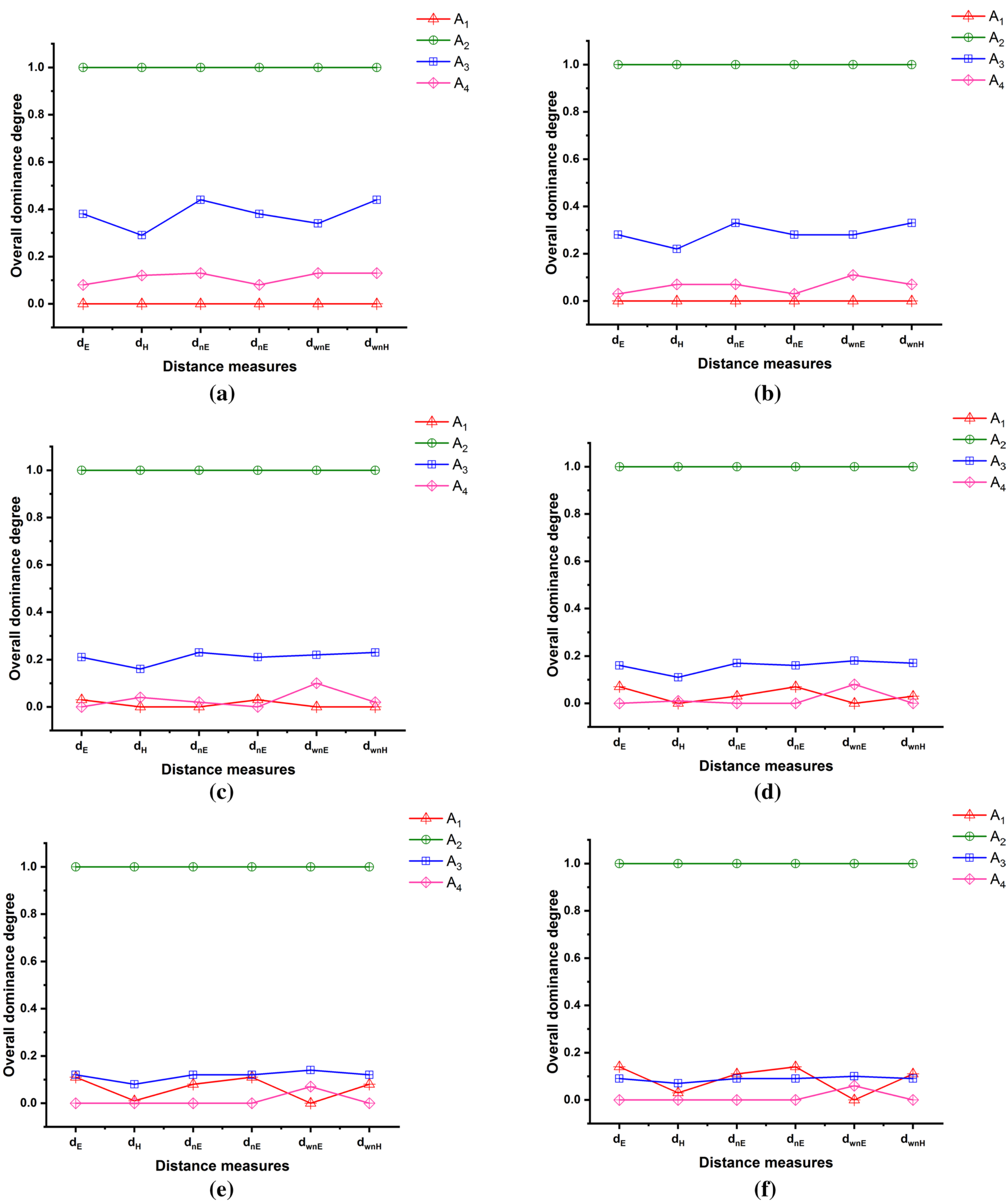


Fig. 4 ξ (A_i) of the four material alternatives at each distance measure with different θ values obtained using CIVIFOWA fuzzy TODIM approach for **a** θ = 0.1, **b** θ = 1, **c** θ = 2, **d** θ = 3, **e** θ = 4, **f** θ = 5, **g** θ = 6, **h** θ = 7, **i** θ = 8, **j** θ = 9, **k** θ = 10, **l** θ = 50, **m** θ = 100

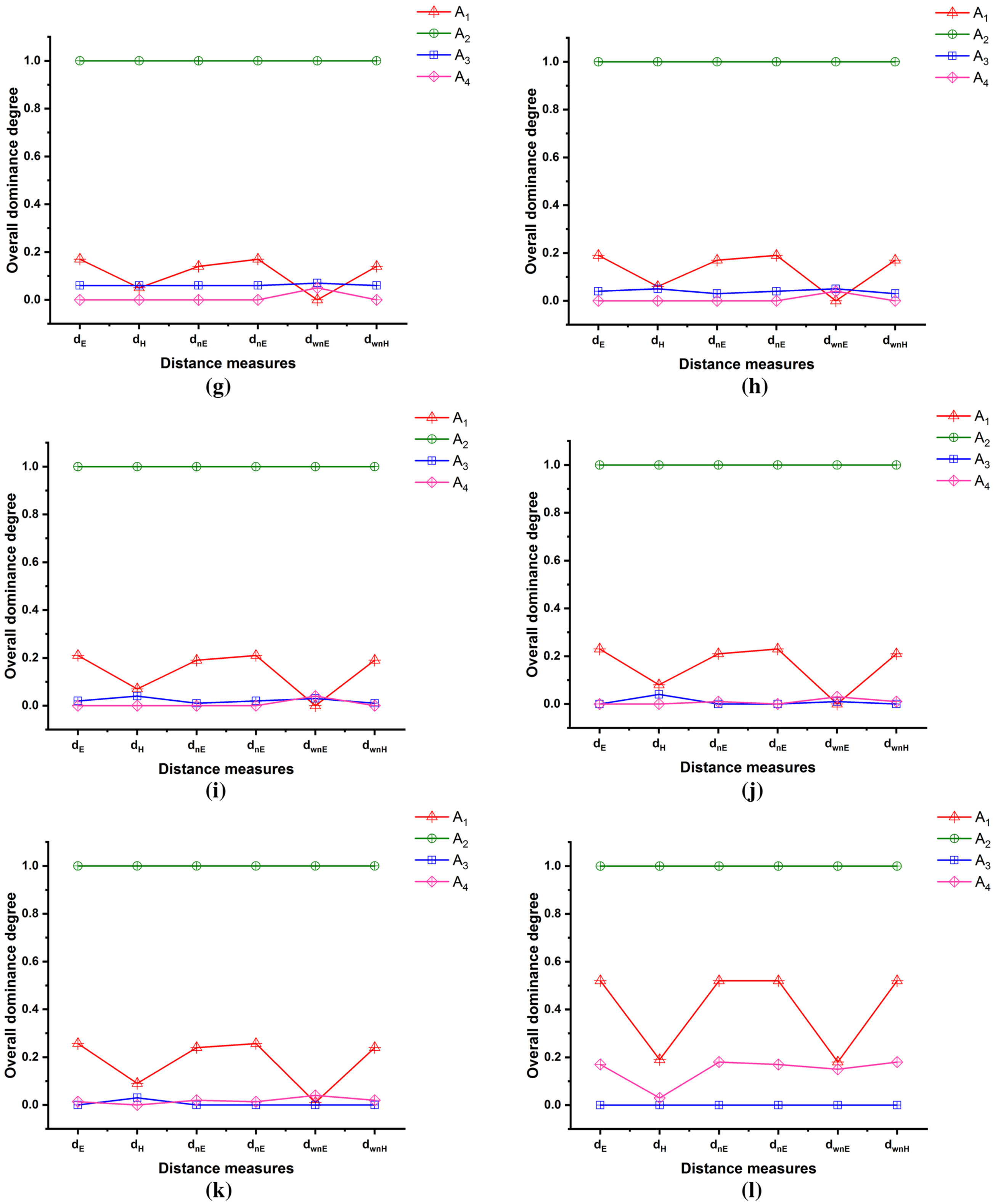
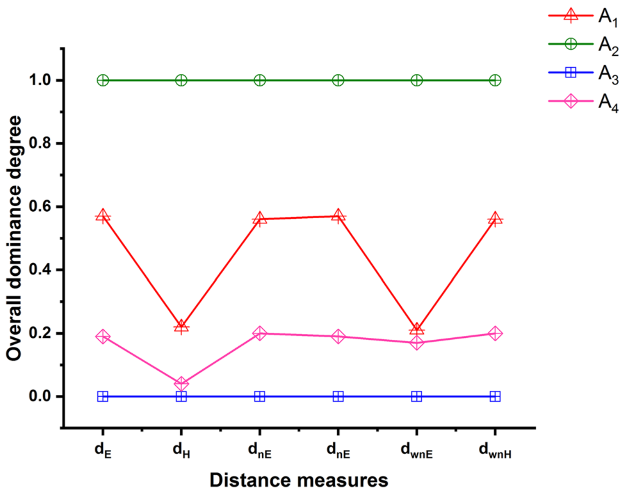


Fig. 4 (continued)



(m)

Fig. 4 (continued)

all the cases of the results obtained using CIVIFOWG-fuzzy TODIM methodology in comparison to the other proposed CIVIFOWA-fuzzy TODIM method.

5 Conclusions

In the present work, a novel fuzzy TODIM approach has been proposed to deal with the fuzzy information with the complex interval-valued intuitionistic characteristics. The proposed method combines the advantages of both the TODIM method and also the inherent ability of complex interval-valued intuitionistic fuzzy sets to handle complex linguistic information. The proposed fuzzy TODIM approaches have the potential ability to depict the complex linguistic information, which is particularly two-dimensional information, into a single set. Due to this approach, the information or the evaluations provided by the decision

Table 13 Dominance scores obtained using CIVIFOWG fuzzy TODIM approach for the candidate alternatives based on ξ (Ai) for different distance measures at each θ value

$\xi(A_i)$	Alternatives	Dominance score					
		d_E	d_H	d_{nE}	d_{nH}	d_{wnE}	d_{wnH}
θ = 0.1	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.62	0.61	0.65	0.73	0.63	0.73
	A_4	0.27	0.29	0.29	0.29	0.16	0.29
θ = 1	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.57	0.56	0.59	0.68	0.58	0.48
	A_4	0.25	0.28	0.28	0.28	0.14	0.24
θ = 2	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.52	0.52	0.54	0.64	0.53	0.43
	A_4	0.24	0.27	0.26	0.27	0.11	0.22
θ = 3	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.49	0.49	0.50	0.60	0.49	0.39
	A_4	0.23	0.26	0.25	0.26	0.09	0.21
θ = 4	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.45	0.47	0.47	0.58	0.46	0.36
	A_4	0.22	0.26	0.24	0.26	0.08	0.20
θ = 5	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.43	0.45	0.44	0.55	0.43	0.33
	A_4	0.22	0.25	0.23	0.25	0.07	0.19
θ = 6	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.41	0.43	0.42	0.53	0.41	0.31
	A_4	0.21	0.25	0.23	0.25	0.05	0.19
θ = 7	A_1	0.00	0.00	0.00	0.00	0.00	0.00

Table 13 (continued)

$\xi(A_i)$	Alternatives	Dominance score					
		d_E	d_H	d_{nE}	d_{nH}	d_{wnE}	d_{wnH}
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.37	0.42	0.40	0.52	0.39	0.29
	A_4	0.20	0.24	0.22	0.24	0.04	0.18
$\theta = 8$	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.37	0.41	0.38	0.51	0.37	0.27
	A_4	0.20	0.24	0.21	0.24	0.04	0.18
$\theta = 9$	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.35	0.39	0.36	0.49	0.36	0.26
	A_4	0.20	0.24	0.21	0.24	0.03	0.17
$\theta = 10$	A_1	0.00	0.00	0.00	0.00	0.00	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.34	0.38	0.35	0.48	0.34	0.25
	A_4	0.19	0.24	0.21	0.24	0.02	0.17
$\theta = 50$	A_1	0.00	0.00	0.00	0.00	0.05	0.00
	A_2	1.00	1.00	1.00	1.00	1.00	1.00
	A_3	0.19	0.28	0.19	0.36	0.23	0.12
	A_4	0.15	0.21	0.16	0.21	0.00	0.13
$\theta = 100$	A_1	0.00	0.00	0.000	0.00	0.07	0.00
	A_2	1.00	1.00	1.000	1.00	1.00	1.00
	A_3	0.16	0.26	0.153	0.34	0.21	0.09
	A_4	0.14	0.20	0.147	0.20	0.00	0.13

makers are maintained as close as possible to the real-world scenario. The decisions taken accounts for the risk appetite of the experts involved in the process of decision making. The risk appetite of the experts is an inevitable component and should not be ignored during the decision making process. On the basis of two aggregation operators, CIVIFOWA fuzzy TODIM and CIVIFOWG fuzzy TODIM approaches have been proposed with a motive to improve the existing lacuna in the decision making process and hence contribute towards the decision making process. The developed decision-making frameworks have been adjudged for their feasibility through an illustrative example. The ranking results demonstrate the effect of psychological behaviour of the decision maker in the decision making process. Sensitivity analysis depicts the robustness of the proposed methodologies. Furthermore, a comprehensive analysis of the effect of distance measures on the ranking results also justify the efficiency and robustness of the proposed methodologies. However, CIVIFOWG fuzzy TODIM approach was revealed to be more stable in comparison to the CIVIFOWA fuzzy TODIM approach owing to the inherent characteristic advantages of aggregating the data by the geometric operators. A comparative analysis with the extant fuzzy TODIM approaches clearly depicts the superiority of the proposed approaches and hence justify the value addition to the decision making sciences. As a future scope of the present study, the applicability of the developed methodologies to different application domains must be extended. Different MCDM tools as such VIKOR (Wu et al. 2019b) as well as other information representations (Shen et al. 2019; Wang et al. 2020a) may be integrated with the proposed frameworks.

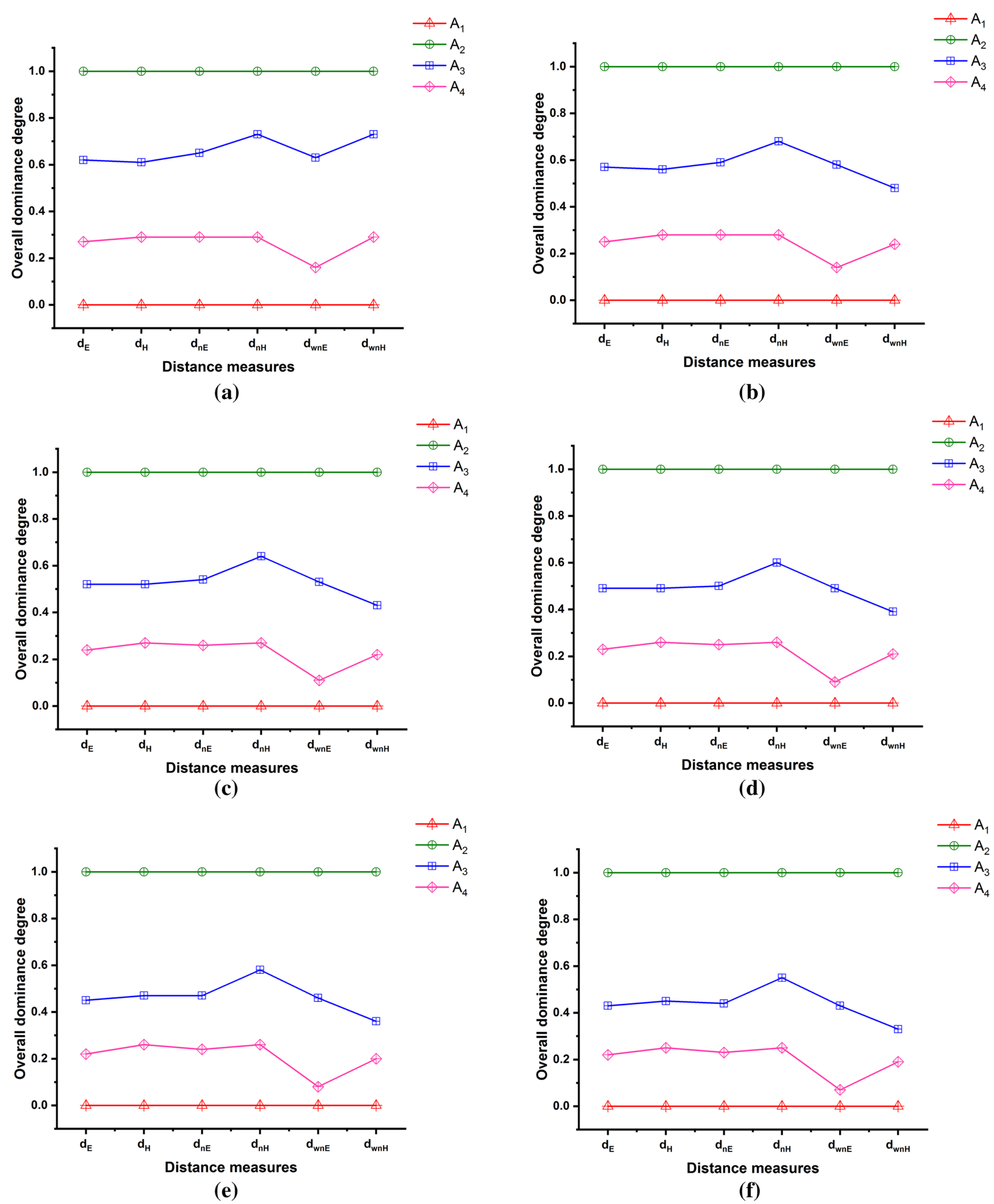


Fig. 5 ξ (A_i) of the four material alternatives at each distance measure with different θ values obtained using CIVIFOWG fuzzy TODIM approach for **a** θ = 0.1, **b** θ = 1, **c** θ = 2, **d** θ = 3, **e** θ = 4, **f** θ = 5, **g** θ = 6, **h** θ = 7, **i** θ = 8, **j** θ = 9, **k** θ = 10, **l** θ = 50, **m** θ = 100

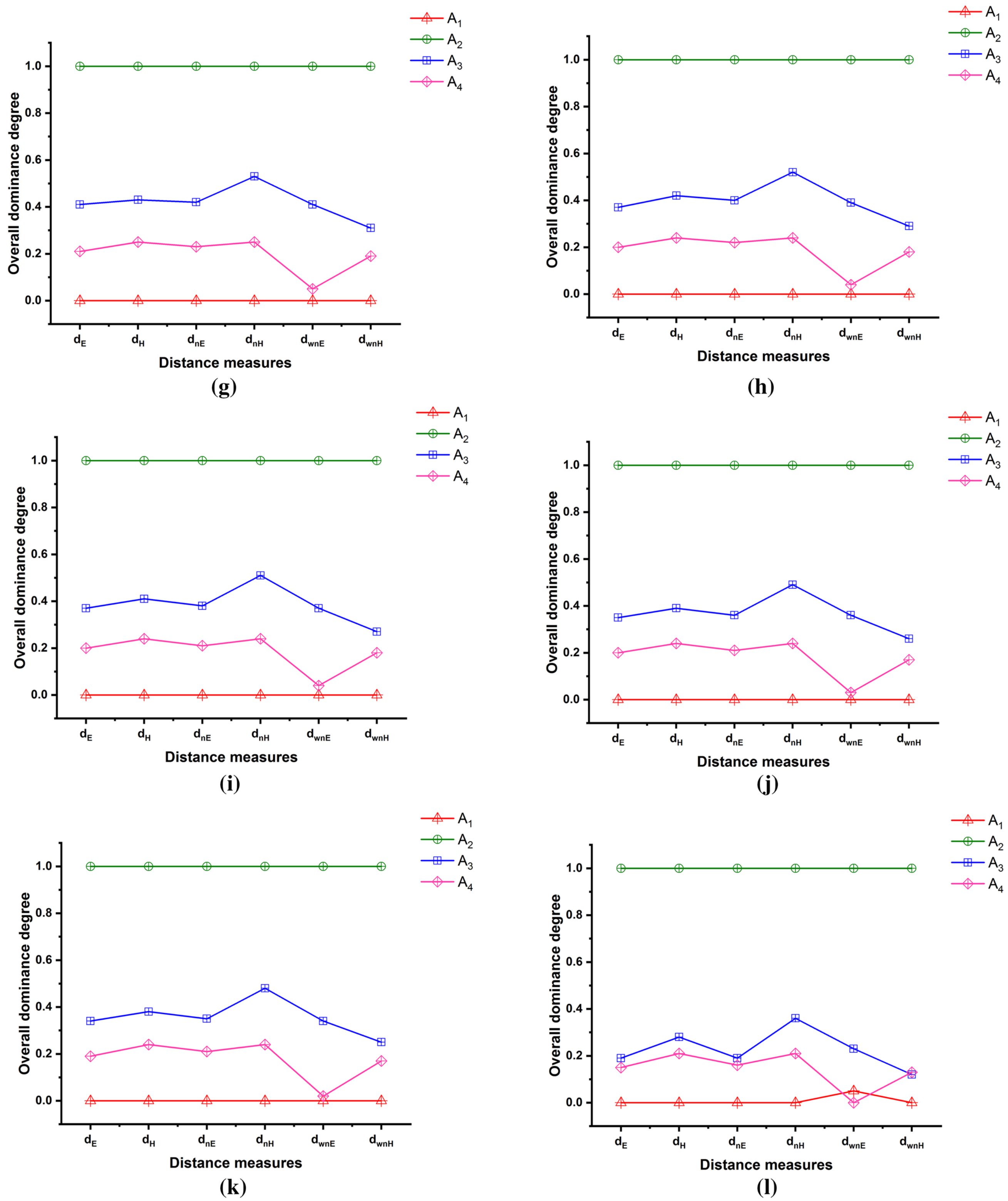


Fig. 5 (continued)

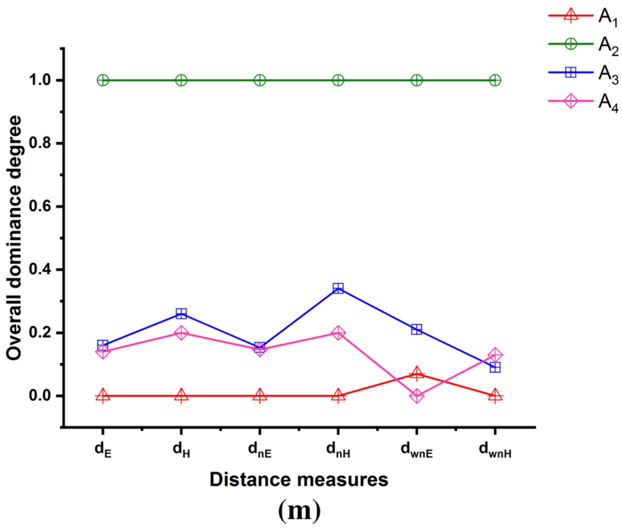


Fig. 5 (continued)

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Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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